Efficient Financial Crises

Ariel Zetlin-Jones
University of Minnesota, azj@cmu.edu

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ABSTRACT

I analyze the causes of financial crises and policies designed to mitigate their effects. I provide new evidence that the capital structure of financial institutions is significantly more illiquid than that of non-financial businesses. I develop a theory in which such differences in capital structure arise from the differences in information lenders have about the assets of financial and non-financial businesses. I use the theory to show that the illiquid capital structure used by financial institutions leads such institutions to be inherently fragile and that government interventions during a crisis, such as bailouts, are not desirable.

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1. Introduction

Banks and other financial firms typically rely heavily on short-term debt to finance their assets. Non-financial firms typically do not. The short-term debt heavy capital structure of banks and other financial firms naturally exposes them to runs or other panic-like phenomena as in Diamond and Dybvig (1983) for example. This paper analyzes the reasons why banks, in particular, choose to expose themselves to bank runs and the role short-term debt, as opposed to other types of capital structures, plays in achieving this exposure. In this paper, I develop and analyze a model in which financial firms rely more heavily on short-term debt than do non-financial firms. In the model, such differences in capital structure arise from differences in the kind and quality of information lenders have about the balance sheets of financial and non-financial firms. Lenders, in the model, cannot commit to a full set of contingent contracts. I show how short-term debt with many small lenders introduces ex-post coordination problems that effectively commit the lenders to implement the optimal ex-ante contract under commitment. The reliance of financial firms on short-term debt leads to occasional bank runs, or financial crises. Such equilibrium outcomes are optimal in the sense that any alternative arrangement leads to lower welfare. Consequently, government interventions to mitigate financial crises are not desirable.

The simple observation that financial firms rely heavily on short-term debt without being required to do so suggests that such capital structures play a desirable social role. This implies that in order to analyze the welfare implications of policy interventions aimed at mitigating or eliminating financial crises, one needs a model in which it is optimal for financial firms to issue short-term debt. One common view in the literature on banking suggests that short-term debt can be useful because the threat of bank runs provide discipline to bank managers (see Calomiris and Kahn (1991) or Diamond and Rajan (2001) for examples of this role). However, data on publicly traded corporations in the United States since 1970 suggests that heavy reliance on short-term debt is, by and large, only a property of the capital structure of financial firms. In other words, non-financial firms do not rely heavily on short-term debt. Therefore, such a model should have the property that the gains to a short-term debt heavy capital structure are specific to the business of financial firms.

Following the literature in corporate finance, I develop a model in which the capital structure of firms is optimally designed to solve incentive problems (see Biais et al. (2007) for an example). Specifically, I develop a model in which lenders to a firm must design compensation contracts for managers to provide incentives to exert effort in a dynamic environment. The manager is protected by limited liability. In the model, this effort affects the distribution of both future outcomes and future signals lenders receive about the manager’s previous effort level. High effort implies that good outcomes and good signals are likely, and
low effort implies that poor outcomes and poor signals are likely (see H¨ olmstrom (1979) for an example of this kind of incentive problem). One way of providing incentives to exert high effort is to commit to dismissing the manager if both outcomes and signals are poor. Such dismissal, typically, is costly not just for the manager but for the lenders as well.

In the model, differences in the capital structure of financial and non-financial firms arise due to differences in the information lenders have about the balance sheets of financial and non-financial firms. One difference between financial and non-financial firms is that financial firms can change the types of assets they hold on their balance sheets more quickly than non-financial firms. It is also more difficult for outside lenders to monitor the types of assets a financial institution holds than for non-financial firms. For example, it is more difficult to know the quality of, say a portfolio of mortgage backed securities acquired by a financial institution than it is to tell whether a non-financial firm has built a plant in a particular location.

I interpret this difference in the ability of lenders to infer the investment actions of the manager in the model as implying that the signals lenders receive are more informative for non-financial firms than for financial firms. I show that the optimal provision of incentives depends critically on the quality of the signals lenders receive. When signal quality about managerial effort is relatively low, it is efficient to dismiss the manager when outcomes and signals are poor in spite of the costs of such dismissal to the lenders. When signal quality is high, however, it is no longer efficient to provide incentives to the manager with dismissal after poor outcomes and signals. I then demonstrate that this result implies that it is efficient for lenders in a financial firm to lend via short-term debt contracts, while for non-financial firms lenders will prefer long-term debt contracts.

To see these results, consider more specifically the tradeoffs involved with dismissing the manager after poor outcomes. In any history in which the lenders continue the project, the manager receives strictly positive expected value (net of effort costs), or a “rent,” due to the combination of moral hazard and limited liability. As signals become more precise the value of the manager’s rent decreases – intuitively, the moral hazard problem becomes less severe as lenders can more easily observe the manager’s effort choice. As the manager’s rent declines, the expected profit the lenders can receive by continuing the project increases. Since the loss to lenders from dismissal after poor outcome and signal histories is the forgone profit from continuing the investment opportunity, the loss becomes larger as signal quality improves.

The benefit to lenders from such a dismissal strategy is cost savings in terms of providing the manager with incentives to exert effort. When the lenders dismiss the manager, the manager does not receive the rents involved with continuing the investment. By dismiss-
ing the manager after poor outcomes, lenders align the manager’s ex-ante incentives with their own, and, therefore, can save on how much they must compensate the manager after good outcomes and signals in order to provide appropriate incentives for effort. Because the manager’s rent is decreasing in signal quality, this incentive savings is also decreasing in signal quality. As a result, the total benefit to using such a dismissal strategy becomes smaller as signal quality improves.

Thus, when signals are very precise, the value of dismissal after poor outcomes is low and lenders prefer to always continue the investment, using the compensation scheme only to provide the manager with incentives to exert effort. When signals are very imprecise, however, the value of such a dismissal strategy is highest, and, under sufficient conditions on the returns of the project, is the optimal way for lenders to provide incentives to the manager.

I go on to show how these differences in the optimal provision of incentives can be used to explain why financial institutions rely more heavily on short-term debt than do non-financial businesses. To allow for a meaningful distinction between short and long-term debt, I analyze my environment under an assumption that lenders cannot commit to the entirety of their long-term contract. A consequence of assuming that lenders lack full commitment, however, is that implementing a contract which calls for dismissal after poor outcomes may not be feasible. The reason is that after such outcomes, both the lenders and the manager stand to gain by renegotiating their contract and allowing the manager to continue. If the manager and the lenders expect such renegotiation, then the manager rationally chooses a low level of effort (ex ante). Thus, if managers and lenders cannot commit to carrying out their contracts, outcomes on average are worse than with commitment. In this sense, lack of commitment creates a time inconsistency problem (see Kydland and Prescott (1977) for an example of this problem).

My main theoretical contribution in this paper is to show that the use of short-term debt introduces a coordination problem among lenders that can help solve the time inconsistency problem. In particular, if an agreement to renegotiate the contract requires all or a substantial fraction of lenders to agree to a renegotiation, I show that the time inconsistency problem can be resolved. The basic idea is that such an agreement to renegotiate creates incentives on the part of each lender to threaten to disagree unless that lender is paid a large fraction of the firm’s assets. Such incentives make it difficult for lenders to renegotiate the terms of the contract and help ensure that the original contract is implemented even when it is undesirable from the perspective of the collective interests of the lenders. Since the manager anticipates the likelihood of such disagreement, the manager expects the original contract to be implemented and rationally chooses to exert high effort to reduce the
likelihood of poor outcomes and signals.

Coordination problems of the kind studied here are well known in the literature on the problem of providing public goods which serve common purposes such as military defense or pollution control (see Rob (1989) or Mailath and Postlewaite (1990) for examples). This literature has emphasized that requiring all or most citizens to agree to an appropriate level of defense or pollution control is difficult and has emphasized that government action might be desirable in such circumstances. The theoretical result, that coordination problems can be used to resolve time inconsistency problems demonstrates how such coordination problems can actually serve a desirable social role.

Short-term debt plays a desirable social role by introducing a coordination problem among lenders that allow them to, in effect, commit to dismiss the manager after poor project outcomes. This is only (ex ante) efficient when future signal quality about manager effort is low enough. As a result, it implies that in some cases (when the project yields a poor outcome) along the equilibrium path, outcomes trigger actions that look like they could not be part of an ex ante efficient contract – e.g., bank runs. Each lender refuses to roll-over their debt even though it is in the collective interest of the lenders to do so.

The same short-term debt contracts, however, are not optimal when the optimal provision of incentives calls for lenders to never dismiss the manager. When signal quality is high, short-term debt contracts introduce the same coordination problem and prevent lenders from continuing profitable investment opportunities. As a result, when signal quality is high, lenders prefer to use long-term debt, which, in effect, commits them to continue the project after all outcome histories.

To the extent that the information lenders have about the balance sheets of financial businesses is less precise than those of non-financial businesses, this paper provides one reason that financial institutions are more exposed to coordination problems than non-financial institutions. As a result, not surprisingly, financial institutions are more fragile and more susceptible to crises. In the model, such fragility and susceptibility serves an important purpose by providing managers of financial institutions the incentives needed to achieve good outcomes. Equilibrium outcomes in the model are efficient in the sense that no planner confronted with the same informational structures as other agents could achieve a better outcome. In this sense, government interventions can only be harmful. In future work, I plan to introduce various spillover effects which are likely to imply that equilibrium outcomes are inefficient. Such an extended model could be useful for analyzing the best way to mitigate the probability of financial crises and to address them appropriately when they do occur.
Related Literature

This paper is related to an extensive literature on bank runs and the role of demand deposits or short-term debt (see Cole and Kehoe (2000) and Diamond and Dybvig (1983)). The theoretical results on the use of short-term debt as a commitment device are closest in nature to those found in Diamond and Rajan (2001), Diamond and Rajan (2000), Calomiris and Kahn (1991), and Bolton and Scharfstein (1990), and I view my results as an important generalization of the ones in these papers. Specifically, in Diamond and Rajan (2001), bank runs do not occur along the equilibrium path; in Diamond and Rajan (2000), inefficient dismissal of the manager is not a feature of the optimal contract; and in Calomiris and Kahn (1991), when the optimal contract is, in their terminology, “short-term debt,” dismissal of the manager is desirable from the perspective of the collective interest of the lenders. Bolton and Scharfstein (1990) consider an arbitrary ex-post coordination game between two lenders. When terminating the firm is costly from the collective interests of the lenders, the lenders have strong incentives to coordinate and roll over their debt. In my paper, I allow the lenders to coordinate to develop arbitrary incentive-feasible contracts that induce the lenders as a whole to roll their debt over and demonstrate that even when the option to do so exists, short-term debt prevents them from doing so. Generalizing these results to the case where bank runs occur in equilibrium and are a feature of the optimal contract is a necessary first step in building a framework to analyze the effects of regulatory policy on the capital structure of banks.

The idea that bank runs may be a feature of optimal lending arrangements is related to results in Allen and Gale (1998) and Allen and Gale (2004). In these papers, when intermediaries are restricted to offer demand deposits, bank default or crises allow intermediaries to share risk and effectively offer fully state-contingent contracts. In this paper, I analyze general optimal contracts and show under what conditions particular frictions of moral hazard on the part of the bank manager and incomplete information regarding lenders’ liquidity shocks give rise to crises as a feature of optimal lending arrangements.

The idea that a coordination problem can resolve a time inconsistency problem is related to the results in Laffont and Tirole (1988) and Netzer and Scheuer (2010). In their environments, a risk-neutral principal wants to provide both incentives for effort and insurance to a risk-averse principal. Under commitment, the principal provides incentives by delivering less than full insurance to the agent. In both of these papers, when the principal (or markets) lacks commitment, the optimal contract introduces an adverse selection problem ex-post, which limits the ability of the principal to provide full insurance after effort has been provided. This adverse selection problem allows the principal to commit to deliver less than full insurance and is the efficient way to provide ex-ante incentives. In my problem,
because the agent or manager is risk-neutral, a different type of ex-post informational problem is necessary for the principal (in my case, lenders) to commit to deliver the appropriate incentives.

Additionally, this paper provides new results regarding the optimality of short-term contracts in long term agency relationships. Fudenberg et al. (1990) develop conditions under which spot contracts implement optimal commitment outcomes in a long-run relationship. One key condition for their result is that the utility frontier describing payoffs of the principal and payoffs of the agent must be decreasing. In other words, after each history, continuation utilities for the principal and the agent lie on the set of efficient continuation allocations. The main result in this paper demonstrates that short-term contracts may implement long-run commitment outcomes even when long-run commitment outcomes feature histories where continuation outcomes are ex-post inefficient. In this sense, my results differ from those found in Brunnermeier and Oehmke (2010), where a lack of commitment causes short-term contracts to deliver worse outcomes than long-run commitment outcomes.

Lastly, this paper is related to an extensive literature on the optimal maturity structure of firm debt (see Diamond (1991), Flannery (1986), Myers (1977) for examples). Each of these papers is primarily concerned with variation of maturity across non-financial firms, whereas I focus on differences in maturity structure between financial and non-financial firms.

The remainder of the paper is organized as follows. Section 2 contains a benchmark moral hazard problem between a single principal and an agent where managerial effort affects the distribution over outcomes and signals. This sections demonstrates how optimal incentive provision depends on signal quality. Section 3 contains the same model with a large number of lenders, each of which receive liquidity shocks. This section contains the main result about efficiency of short-term debt. In section 4, I demonstrate empirically that financial firms in the U.S. do in fact have a much more fragile capital structure than do non-financial firms in the U.S.. Section 5 concludes.

2. A Moral Hazard Model with Signals of Managerial Actions

In this section I study a benchmark moral hazard problem. In this model, a single principal must design compensation contracts in order to provide the agent, or manager of the investment, with incentives to exert effort. This effort affects the distribution over future outcomes and signals the principal will receive about the manager’s previous effort level. I show that when signal quality is sufficiently high, the principal does not dismiss the manager after any history (in the optimal contract) but when signal quality is sufficiently low, (in the optimal contract) the principal dismisses the manager after a poor project outcome.
2A. The Model with 1 Lender, Direct Signals of Managerial Effort, and Full Commitment

There are three periods, indexed by \( t = 0, 1, 2 \). There is a single principal and a single agent I call a manager. Both the principal and the manager are risk neutral, but they gave different discount rates. The manager’s preferences over consumption streams are given by \( c_0 + c_1 + \beta c_2 \). The principal’s preferences are given by \( c_0 + c_1 + \delta c_2 \). I restrict consumption of the manager to be positive in all periods and assume \( \delta > \beta \).

The manager has access to a project which, in period 0, requires \( I \) units of resources and effort of the manager \( e_0 \in \{ \pi_l, \pi_h \} \) which causes disutility to the manager \( q(e_0) \). In period 1, the project yields both a gross return \( I + y_1, y_1 \in \{ y_l, y_h \} \) and a signal of the manager’s effort level \( s_1 \). If the project is continued from period 1 to 2, it again requires \( I \) units of resources and effort of the manager, \( e_1 \). In period 2, the project yields gross output \( y_2 \) and another signal, \( s_2 \). Effort affects the distribution over returns, \( y_t \) and signals \( s_t \) (\( t \geq 1 \)) according to

\[
y_{t+1} = \begin{cases} 
    y_h & \text{w/prob } e_t \\
    y_l & \text{w/prob } 1 - e_t
\end{cases}
\]

and

\[
s_{t+1} = \begin{cases} 
    s_h & \text{w/prob } \sigma_{e_t} \\
    s_l & \text{w/prob } 1 - \sigma_{e_t}
\end{cases}
\]

where \( \sigma_{\pi_h} \geq \frac{1}{2} \geq \sigma_{\pi_l} \). For simplicity, assume \( \sigma_h = 1 - \sigma_l \). The timeline of events in the model can be depicted as follows:

I restrict attention to direct revelation mechanisms in which the principal recommends an action to the agent, and the contract is designed so that the recommended action is incentive compatible for the manager. A contract (with initial investment) in this environment consists of the following collection of functions:

\[ \{ e_0, c_1(y_1, s_1), x(y_1, s_1), e_1(y_1, s_1), c_2(y_1, s_1, y_2, s_2) \} \]

which specify effort in period \( t, e_t \), the continuation rule, \( x \), and consumption of the manager.
\[ c_t \text{ as functions of the relevant history.} \]

The value of an investment contract to the principal is given by
\[
E_{e_0} \left[ y_1 + I - c_1(y_1, s_1) + x(y_1, s_1) \left( -I + \delta E_{e_1(y_1, s_1)} \left[ y_2 - c_2(y_1, s_1, y_2, s_2) \right] \right) \right].
\]

The value to the agent is
\[
-q(e_0) + E_{e_0} \left[ c_1(y_1, s_1) + x(y_1, s_1) \left( -\psi(e_1(y_1, s_1)) + \beta E_{e_1(y_1, s_1)} c_2(y_1, s_1, y_2, s_2) \right) \right]
\]

Let \( \psi = q(\pi_h) \) and normalize \( q(\pi_l) = 0 \). Assuming effort is valuable, the optimal investment contract solves the following problem.

\[
\max E_{\pi_h} \left[ y_1 + I - c_1(y_1, s_1) + x(y_1, s_1) \left( -I + \delta E_{\pi_h} \left[ y_2 - c_2(y_1, s_1, y_2, s_2) \right] \right) \right]
\]

subject to
\[
\beta E_{\pi_h} c_2(y_1, s_1, y_2, s_2) - \psi \geq \beta E_{\pi_l} c_2(y_1, s_1, y_2, s_2)
\]
\[
U_1(y_1, s_1) = x(y_1, s_1) \left[ \beta E_{\pi_h} c_2(y_1, s_1, y_2, s_2) - \psi \right]
\]
\[
E_{\pi_h} \left[ c_1(y_1, s_1) + U_1(y_1, s_1) \right] - \psi \geq E_{\pi_l} \left[ c_1(y_1, s_1) + U_1(y_1, s_1) \right]
\]
\[
c_t \geq 0
\]

The first constraint contains the incentive constraint for the manager in period 1 for every period 1 history. The second and third constraints contain the incentive constraint for the manager in period 0. The final constraint \( c_t \geq 0 \) is the limited liability constraint that arises by restricting the manager’s consumption to be positive. This constraint plays a key role, as is standard in the moral hazard literature, in the sense that it ensures that the full information optimum is not attainable. I now characterize key features of the optimal contract in this environment.

**2B. Characterization of the Optimal Contract**

Given that the principal is more patient than the agent, standard front-loading arguments (the principal retains as much utility as possible in the final period) allow me to reduce to problem to one of only determining the continuation rule in different histories. In front-loading the contract, the principal only delivers consumption to the manager after a high output and a high signal realization in period 2. Then, I use the period 1 incentive constraint of the manager to define consumption after such a history in period 2. In other words, I have the following set of lemmas.

**Lemma 1.** The optimal contract satisfies \( c_2(y_1, s_1, y_2, s_2) = 0 \) if \((y_2, s_2) \neq (y_h, s_h)\) and
\[ c_2(h_1, y_h, s_h) = \frac{\psi}{\beta(\pi_h \sigma_h - \pi_l \sigma_l)}. \]

Then, the problem reduces to choosing only period 1 consumption and continuation rule, \((c_1(h_1), x(h_1))\) to solve

\[
\max E_{\pi_h} \left[ y_1 + I - c_1(y_1, s_1) + x(y_1, s_1) \left( -I + \delta \left( E_{\pi_l} y_2 - \frac{\psi \pi_h \sigma_h}{\beta (\pi_h \sigma_h - \pi_l \sigma_l)} \right) \right) \right]
\]

subject to

\[
E_{\pi_h} \left[ c_1(y_1, s_1) + x(y_1, s_1) \frac{\psi \pi_l \sigma_l}{\pi_h \sigma_h - \pi_l \sigma_l} \right] - \psi \geq E_{\pi_l} \left[ c_1(y_1, s_1) + x(y_1, s_1) \frac{\psi \pi_l \sigma_l}{\pi_h \sigma_h - \pi_l \sigma_l} \right]
\]

\[ c_1 \geq 0 \]

It is useful to define \(\Psi\) as the expected rent the manager receives in any continuation contract (arising from limited liability):

\[
\Psi = \frac{\pi_l \sigma_l}{\pi_h \sigma_h - \pi_l \sigma_l} \psi
\]

and a term \(\zeta\), the expected gain to the principal of continuing the project under high effort of the manager,

\[
\zeta = -I + \delta \left( E_{\pi_h} y_2 - \frac{\pi_l \sigma_l \psi}{\beta (\pi_h \sigma_h - \pi_l \sigma_l)} \right)
\]

Then, the next result characterizes \(c_1\):

**Lemma 2.** The optimal contract satisfies \(c_1(y_h, s_l) = c_1(y_l, s_1) = 0\) and

\[
(\pi_h \sigma_h - \pi_l \sigma_l) c_1(y_h, s_h) = \psi - x(y_h, s_h)(\pi_h \sigma_h - \pi_l \sigma_l) \Psi
- x(y_h, s_l)(\pi_h (1 - \sigma_h) - \pi_l (1 - \sigma_l)) \Psi
- x(y_l, s_h)((1 - \pi_h) \sigma_h - (1 - \pi_l) \sigma_l) \Psi
- x(y_l, s_l)((1 - \pi_h) (1 - \sigma_h) - (1 - \pi_l) (1 - \sigma_l)) \Psi
\]
Using the above lemma to simplify the problem yields an optimization problem only in $x(h_1)$:

$$\max_{x(h_1)} E_{\pi_h} y_1 + I - \frac{\pi_h \sigma_h}{\pi_h \sigma_h - \pi_l \sigma_l} \psi + x(y_h, s_h) \left[ \pi_h \sigma_h (\zeta + \Psi) \right]$$

$$+ x(y_l, s_l) \left[ (1 - \pi_h)(1 - \sigma_h) \zeta + \frac{\pi_h \sigma_h}{\pi_h \sigma_h - \pi_l \sigma_l} ((1 - \pi_h)(1 - \sigma_h) - (1 - \pi_l)(1 - \sigma_l)) \Psi \right]$$

Now, I consider extreme cases, when the signal is uninformative, $\sigma_h = \sigma_l = \frac{1}{2}$ and when the signal is perfectly informative.

**Lemma 3.** If the signal of effort is perfectly informative, i.e. $\sigma_h = 1$ and $\sigma_l = 0$, then $x(y_1, s_1) = 1$.

When $\sigma_h = 1$, $\Psi = 0$, the terms multiplying $x$ must be positive. In this extreme case, effort is observable (through the signal) and therefore there is no moral hazard problem. In this sense, this result is not surprising. What is important is that in a neighborhood of perfect signal quality the same result holds, as I demonstrate below.

In the other extreme, signal quality is completely uninformative. In this case, there is no reason to condition the continuation rule on the signal, so the problem simplifies to

$$\max_{x(y_1)} E_{\pi_h} y_1 + I - \frac{\pi_h \psi}{\pi_h - \pi_l} + x(y_h) \pi_h (\zeta + \Psi) + x(y_l) [(1 - \pi_h) \zeta - \pi_h \Psi]$$

This equation illustrates the tradeoffs involved with continuation after different histories. Typically, the term $\zeta$, the expected gain to continuing the project net of the manager’s rent will be positive. Thus, continuation after either a high or low outcome will yield this continuation gain, $\zeta$. However, the incentive costs of continuing the investment after a high or low outcome are asymmetric. Continuing after a high outcome and dismissing the manager after a low outcome relaxes the period 0 incentive constraint of the manager because it introduces more spread in the manager continuation payoffs. This spread provides incentives for the manager to exert high effort in period 0. The cost of introducing this kind of spread, however, is forgone continuation value for the principle, $\zeta$, which occurs with probability $(1 - \pi_h)$ if the manager exerts high effort in period 0. These kind of tradeoffs, in the sense that the ex-ante optimal contract may require ex-post inefficient outcomes are standard in
the moral hazard literature.

I place sufficient conditions on the technologies so that dismissal after a low outcome is optimal. In other words, I assume that

\[(1 - \pi_h)\zeta < \pi_h \Psi\]

so that when signals are uninformative, the optimal contract calls for dismissal after low project outcomes. I then have the following lemma.

**Lemma 4.** Suppose

\[(1 - \pi_h)\zeta < \pi_h \Psi\]

If \(\sigma_h = \sigma_l = 1/2\), then \(x(y_h, s_1) = 1, x(y_l, s_1) = 0\).

I now demonstrate that results similar to the previous two lemmas hold in neighborhood of these signal quality cases.

**Proposition 1.** Suppose

\[\delta E_{\pi_h y_2} - I \leq \frac{\pi_h \psi}{\pi_h - \pi_l} \left[ \frac{\delta}{\beta} + \frac{\pi_l}{1 - \pi_h} \right] \]

and \(\delta(1 - \pi_h) \geq \beta\). There exists \(\bar{\sigma}_1 < \bar{\sigma}_2 \in [1/2, 1] \) s.t. when \(\sigma_h \in \left[\frac{1}{2}, \bar{\sigma}_1\right]\), the optimal contract satisfies

\[x(y_h, s_1) = 1, x(y_l, s_1) = 0.\]

Furthermore, when \(\sigma_h \in [\bar{\sigma}_2, 1]\), the optimal contract satisfies

\[x(y_1, s_h) = 1, x(y_1, s_l) = 1.\]

That a lower portion exists is straightforward from continuity.\(^1\) For the upper portion, the key idea is that as \(\sigma_h\) gets large, \(\zeta\) increases and \(\Psi\) decreases. In words, the limited liability rent the manager receives (in both periods) declines as signals become more informative because the principal need only deliver consumption to the manager when observing high output and a high signal. As the rent declines in period 2, the loss from discontinuing the project from period 1 to 2 becomes larger. Moreover, because the rent the manager receives in future periods is lower, continuing the project in period 1 has lower ex-ante costs (from the principal’s perspective) with respect to providing incentives for the manager to exert

\(^1\)More stringent sufficient conditions can be developed to show fully characterize the optimal continuation rules in general, but they do not alter the key content of the characterization.
effort in period 0. These effects reinforce each other, and a sufficient condition ensures that for \( \sigma_h \) near 1, the gains outweigh the costs.

One minor difficulty that complicates the proof arises because as \( \sigma_h \) increases, the likelihood of the history \((y_l, s_l)\) (when the agent exerts effort in period 0) converges to zero, so the expected opportunity cost of such a dismissal strategy also converges to zero. In the extreme, the costs of liquidation after a low signal of effort are exactly zero because this history never occurs (under high effort) and the incentive costs are zero because the manager receives zero rents. The additional condition, which relates \( \delta \) to \( \beta \) and \( \pi_h \) simply ensures that for \( \sigma_h \) near 1, the expected loss from not continuing the project is larger relative to the expected savings in incentives and guarantees the existence of an upper dominance region.

2C. The Value of Commitment

In this type of moral hazard model, there are two primary tools which the principal can use to provide incentives for the manager to exert effort. First, the principal can create spread in transfers following different histories, rewarding the manager with high transfers after good outcomes and punishing the manager with low transfers after poor outcomes. However, because the manager receives “rents” in subsequent periods when the project is continued, there is a limit to the amount of spread that principal can deliver using only transfers. When these spreads are not sufficient, the principal must rely on dismissing the manager (and terminating the investment) to provide incentives. I have developed sufficient conditions under which when signal quality is sufficiently low, such dismissal is necessary for providing incentives, while when signal quality is high, such dismissal is not necessary.

One problem with contracts that call for dismissal is that they require commitment on the part of the principal. As is well known in these types of moral hazard problems, lack of commitment can be a severe problem and lead to underinvestment ex-ante. To be precise, consider the case of uninformative signals and suppose that

\[
\delta E_{\pi_h, y_2} - I \geq \frac{\delta \pi_h \psi}{\beta \pi_h - \pi_l}.
\]

Then, if the principal cannot commit to the continuation rule and is free to re-write the contract in period 1 following any history of \( y_1 \), under the above condition, the principal will always continue the project (since it is profitable to do so even though the principal must pay the manager additional compensation so that the manager will exert effort in period 1). As a result, the value of the optimal contract (including the initial investment cost) when
the principal cannot commit to dismiss the manager is given by

\[ E_{\pi_h} y_1 - \frac{\pi_h}{\pi_h - \pi_l} \psi + \pi_h (\zeta + \Psi) + (1 - \pi_h) \zeta - \pi_h \Psi. \]

When the principal can commit to dismiss the manager, this value is given by

\[ E_{\pi_h} y_1 - \frac{\pi_h}{\pi_h - \pi_l} \psi + \pi_h (\zeta + \Psi). \]

Hence, the value of commitment can be defined as

\[ \pi_h \Psi - (1 - \pi_h) \zeta \]

which I have assumed to be strictly positive. The fact that the principal earns strictly higher value under commitment can be made more stark by assuming that without commitment and the ability to provide optimal incentives to the manager, the principal would not undertake any investment ex-ante. I now show that when the principal acts as a stand in for a large group of lenders, each of which must contribute to finance the investment, the lenders as a block can recapture the value of commitment even when they lack the same commitment as the principal in this example.

3. Benchmark Model of Investment with Many Lenders and Limited Commitment

In this section, I focus on the case with uninformative signals and show how short-term contracts among many small lenders can help resolve the time inconsistency problem. I set up the problem with \( N \) lenders designing optimal compensation contracts for a manager who operates the firm and is subject to moral hazard and limited liability. I develop conditions analogous to the above case under which the optimal commitment contract calls for lenders to discontinue their investment in the project after poor project outcomes. I show that when there are sufficiently many lenders and re-negotiating the contract requires all or almost all of the lenders to agree, then the lenders can effectively commit not to re-negotiate the contract.

3A. Environment of the Benchmark Model

Consider again a three period environment with \( N + 1 \) agents. Let the periods be indexed by \( t = 0, 1, 2 \). I call the \( N + 1 \)st agent a manager and the remaining agents lenders. All agents are risk neutral, but I assume that they discount consumption in the final period at different rates. The manager’s preferences over consumption streams are given by \( c_0 + c_1 + \beta c_2 \). Each lender’s utility over consumption is given by \( c_0 + c_1 + v_i c_2 \) where I restrict consumption to be
positive in all periods. I assume that $v_i$ is an i.i.d. preference shock realized in period 1 with $v_i \sim G_i(v_i)$ having support $[\underline{v}, \bar{v}]$. Let $G$ denote the joint distribution over $v$. The preference shocks can be thought of as liquidity shocks to the lenders, causing them to have a stronger preference for period 1 consumption when they realize lower values of $v_i$. Additionally, the preference shocks are privately known by each individual lender and $\beta < v_i$.

Each lender has an identical endowment stream $(k_i^0, k_i^1, k_i^2)$ with $k_i^0 = k^0(N), k_i^1 = 0, k_i^2 = 0$. For simplicity, I will further assume that $k_i^0 = I/N, k_i^1 = k_i^2 = 0$. This assumption ensures that each lender must participate in the investment project in order for the project to be undertaken. This assumption can be relaxed by appropriately modifying some of the assumptions that follow.

The manager has access to a project which requires $I$ units of resources in period 0 and effort of the manager $e_0 \in \{\pi_l, \pi_h\}$ which causes disutility $q(e_0)$ to the manager in period 0. The project yields a gross return $I + y_1 \in \{y_l, y_h\}$ in period 1 where $y_1 = y_h$ with probability $e_0$ and $y_1 = y_l$ with prob. $1 - e_0$. If the project is continued from period 1 to 2, it requires resource inputs $I$ again and additional effort of the manager $e_1$. The project then yields output $y_2 \in \{y_l^1, y_l^2\}$ with $y_h^j > y_l^j$ and is $y_2^1 = y_h^1$ with probability $e_1$. Notice, then, the problem is slightly different from the earlier example as there is some persistence in project outcomes. The time-line of the physical attributes of the environment are as follows

A contract in this environment consists of transfers (to or from the lenders and to the manager), recommended effort levels for the manager, and a continuation rule in period 1 as a function of the relevant history. I will focus on investment contracts which call for investment in period 0 and I will ensure that investment is superior to autarky from the ex-ante perspective of each of the lenders. To be specific, an investment contract consists of the following collection of functions:

$$\left\{ (k_i^0, t_i^1(y_1), t_{1n}(y_1, v), t_{1c}(y_1, v), t_2(y_1, v)), i \in \{1, ..., N\} ; x(y_1, v), c_0, e_1(y_1, v), c_1(y_1), c_2(y_1, y_2^1, v) \right\}.$$

Let $C$ denote the set of all contracts. A contract consists of transfers to lenders at each feasible history. Notice, a contract may specify transfers in period 1 both before lenders’

**Figure 2:** Timeline for the model described in Section 3A.
report their types and after. In a model with full commitment, this distinction is meaningless, however in the model with limited commitment, this distinction will play an important role which I describe carefully below. In effect, the distinction will allow me to distinguish between short and long term debt contract. In addition, I have indexed period 2 transfers independently of the period 2 cash flow outcome because lenders are risk neutral.

The additional components of the contract consist of a continuation rule in period 1 (as a function of the cash flow and lenders’ types, \( x(y_1, v) \)), recommended effort levels for the manager, \( (e_0, e_1(y_1, v)) \), and managerial consumption in period 1 (in expectation with respect to the continuation rule and the lenders’ reports) \( c_1(y_1) \), and as a function of the history in period 2 \( (c_2(y_1, y_2, v)) \). Ex-ante welfare of the lenders (net of initial investment costs) under any contract is given by

\[
E_{e_0} \sum_{i=1}^{N} \left[ t^i_1(y_1) + \int_v [x(y_1, v) (t^i_{1c}(y_1, v) + v_k t^i_{2c}(y_1, v)) + (1 - x(y_1, v)) t^i_{1n}(y_1, v)] dG(v) \right].
\]

(1)

Given the structure of the environment, a contract will have to satisfy several constraints to be considered feasible. These constraints include resource feasibility in all periods, capacity constraints of the lenders in period 1, incentive compatibility of the lenders in period 1 with respect to their privately known liquidity shock, voluntary participation of the lenders in period 0, and incentive compatibility and participation of the manager in each period (where I appeal to the revelation principle to restrict attention to incentive compatible contracts\(^2\)).

A contract is **resource feasible** if it satisfies

\[
I + \sum_{i=1}^{N} t^i_0 \leq 0
\]

(2)

\[
\sum_{i=1}^{N} [t^i_1(y_1) + x(y_1, v) t^i_{1c}(y_1, v) + (1 - x(y_1, v)) t^i_{1n}(y_1, v)] + c_1(y_1) \leq y_1 + I - I x(y_1, v)
\]

\[
\sum_{i=1}^{N} t^i_2(y_1, v) \leq E_{e_1(y_1, v)} (y_2^1 - c_2(y_1, y_2^1, v)).
\]

A contract satisfies the lenders’ **capacity constraints** (or positive consumption requirement)

\(^2\)Although restricting attention to incentive compatible contracts is not without loss of generality in general in environments with limited commitment, the fact that the manager’s effort level in period 0 does not independently affect the likelihood of of period 2 outcomes ensures that the standard version of the revelation principle applies in my environment. See Sleet and Yeltekin (2006) for an extended discussion of such an application of the revelation principle.
if for \( j = c, n \)
\[
t^{i}_{1j}(y_1, v) + t^{i}_{1}(y_1) \geq 0. \tag{3}
\]

To define incentive compatibility, it is useful to define the continuation utility of an lender conditional on realizing a preference shock \( v_i \) and reporting preference shock \( \hat{v}_i \), which I denote by \( w_i(y_1, \hat{v}_i, v_i) \) and is given by
\[
w_i(y_1, \hat{v}_i, v_i) = \int_{v_i} x(y_1, \hat{v}_i, v_i) \left( t^{i}_{1c}(y_1, \hat{v}_i, v_i) + v_i t^{i}_{2}(y_1, \hat{v}_i, v_i) \right) dG_{-i}(v_{-i})
+ \int_{v_{-i}} (1 - x(y_1, \hat{v}_i, v_{-i}) t^{i}_{1n}(y_1, \hat{v}_i, v_{-i}) dG_{-i}(v_{-i})
\]

A contract is \textit{incentive compatible} with respect to the lenders if
\[
w_i(y_1, v_i, v_i) \geq \max_{\hat{v}_i \in \mathcal{W}} w_i(y_1, \hat{v}_i, v_i) \tag{4}
\]
and satisfies lender \textit{voluntary participation} if
\[
E_{e_0} \left[ t^{i}_{1}(y_1) + \int_{v_i} w_i(y_1, v_i, v_i) dG_i(v_i) \right] \geq \frac{I}{N} \tag{5}
\]

A contract is \textit{incentive compatible} with respect to the manager if
\[
\beta e_1(y_1, v) c_2(y_1, y_h, v) + \beta (1 - e_1(y_1, v)) c_2(y_1, y_i, v) - q(e_1(y_1, v)) \geq 
\max_{e'} \beta e' c_2(y_1, y_h, v) + \beta (1 - e') c_2(y_1, y_i, v) - q(e') \tag{6}
\]
\[
U_1(y_1, v) = x(y_1, v) \left[ \beta E_{e_1(y_1, v)} c_2(y_1, y_2, v) - q(e_1(y_1, v)) \right]
\]
\[
E_{e_0} \left[ c_1(y_1) + \int_{v} U_1(y_1, v) dG(v) \right] - q(e_0) \geq \max_{e'} E_{e'} \left[ c_1(y_1) + \int_{v} U_1(y_1, v) dG(v) \right] - q(e')
\]

I now describe the notion of limited commitment which I impose on the environment. I assume that in period 1, after period 1 transfers \( t^{i}_{1}(y_1) \) and \( c_1(y_1) \) have been allocated, the lenders may freely choose to alter the remaining components of the contract. The only restrictions on the new contract are that it cannot deliver negative (total) consumption to any agent in period 1 and no agent can be coerced into participating. The first constraint limits any single lender (or a small block of lenders) from fully financing the investment project in period 1. The second constraint serves to define the outside option of any individual lender, which is simply \( t^{i}_{1}(y_1) \).
Explicitly, the lenders may choose a new *continuation contract* defined as
\[ \left\{ \left( \hat{r}_{1e}(v), \hat{r}_{1n}(v), \hat{r}_2(v) \right)_{i \in \{1, \ldots, N\}}, \hat{x}(v), \hat{e}_1(v), \hat{c}_2(y_2, v) \right\}. \]

This notion of limited commitment gives rise to an *enforceability* constraint which any contract must additionally satisfy to be considered feasible. Specifically, a contract \( C \in \mathbf{C} \) is *enforceable* if there exists no sub-contract
\[ \hat{C} = \left\{ \left( \hat{r}_{1e}(v), \hat{r}_{1n}(v), \hat{r}_2(v) \right)_{i \in \{1, \ldots, N\}}, \hat{x}(v), \hat{e}_1(v), \hat{c}_2(y_2, v) \right\} \]
satisfying (6) such that
\[
\int_v \hat{x}(v) \left[ \beta E_{\hat{e}_1(v)} \hat{c}_2(y_2, v) - q(\hat{e}_1(v)) \right] dG(v) \geq U_1(y_1, v) \tag{7}
\]
\[
\sum_{i=1}^N \int_v [\hat{x}(v) (\hat{r}_{1e}(v) + v_i \hat{r}_2(v)) + (1 - \hat{x}(v)) \hat{r}_{1n}(v)] dG(v) > \sum_{i=1}^N \int_v [x(y_1, v) (t_{1e}(y_1, v) + v_i t_2(y_1, v)) + (1 - x(y_1, v)) t_{1n}(y_1, v)] dG(v) \tag{8}
\]
\[
\hat{w}^i(v_i, v_i) \geq \max_{\tilde{v}_i \in \mathbb{R}} \hat{w}_i(\tilde{v}_i, v_i) \tag{9}
\]
\[
\hat{w}_i(v_i, v_i) \geq 0 \tag{10}
\]
\[
\sum_{i=1}^N \hat{r}_2(v) \leq E_{\hat{e}_1(v)} [y_2 - \hat{c}_2(y_2, v)] \tag{11}
\]
\[
\sum_{i=1}^N \int_v [\hat{x}(v) \hat{r}_{1e}(v) + (1 - \hat{x}(v)) \hat{r}_{1n}(v)] dG(v) \leq y_1 + I - \sum_{i=1}^N t_i(y_1) - c_1(y_1) + I \int_v \hat{x}(v) dG(v) \tag{12}
\]
\[
t_1(y_1) + \hat{x}(v) \hat{r}_{1e}(v) + (1 - \hat{x}(v)) \hat{r}_{1n}(v) \geq 0 \tag{13}
\]

In other words, a contract is enforceable if there is no other contract that increases expected utility of the manager and the total expected utility of the collection of lenders in from period 1 onwards. Observe that elements of the original contract, \( C \), appear only in the objective (equation (8)), the resource constraint (equation (12)), the capacity constraint (equation (13)), and the manager’s participation constraint (equation (7)). This notion of enforceability makes clear the distinction between period 1 transfers made before agents realize their preference shock (early transfers) and period 1 transfers made after agents realize their type (late transfers). Because I assume that early transfers are made before new
continuation contracts can be designed, these transfers affect the set of feasible continuation contracts. By allocating positive early transfers in period 1, the limited liability constraints of the lenders and the resource constraints in any continuation contract become more stringent. If early transfers are all equal to zero, then the limited liability constraints are weak – any contract (including contracts that call for continuation) are feasible as long as they deliver at least 0 transfers to each agent.\footnote{Note that in this formulation, re-negotiated contracts do not necessarily have to be pareto improving. Requiring pareto improvements, that is, ensuring each lender receives at least weakly greater utility under the new contract than under the status quo, is a stronger assumption and the following results would still hold.}

3B. Optimal Contracts

The optimal contract with \emph{full} commitment maximizes ex-ante welfare in (1) and satisfies the resource constraints in (2), the incentive constraints and voluntary participation constraints in (4), (6), and (5), and satisfies the positive consumption constraint for lenders given by (3). The optimal contract with \emph{limited} commitment maximizes ex-ante welfare in (1) and satisfies the resource constraints in (2), the incentive constraints and voluntary participation constraints in (4), (6), and (5), satisfies the positive consumption constraint for lenders given by (3), and is enforceable (there exist no subcontracts satisfying (7)-(13)).

The optimal contract with full commitment provides a benchmark that is useful in characterizing optimal contracts with limited commitment. Indeed, the main result in this section provides assumptions under which the optimal commitment contract is enforceable in the environment with limited commitment. I state this result here including all additional assumptions and then discuss the outline of its proof and its significance.

**Proposition 2.** Define $C(y_1) = E_{\pi_h} y_1^2 - \frac{\pi_l \psi}{\beta \Delta}$. Suppose that $\{G^n\}$ defines a sequence of economies where $G^n = (G^n_1, \ldots, G^n_n)$ such that

(i) $\beta < v,$
(ii) $0 < \pi_h \frac{\pi_l \psi}{\pi_h - \pi_l} + (1 - \pi_h) [I - \bar{v}C(y_l)],$
(iii) $I < \bar{v}C(y_l),$
(iv) $\underline{v}C(y_l) < I < \bar{v}C(y_h),$
(v) $\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)}$ is decreasing in $v_i$
(vi) there exists $\kappa > 0$ such that $g_i^\underline{v}(v_i) > \kappa$
(vii) there exists $v^*$ such that $\bar{v}_i^n < v^*$.

Then, as $N \to \infty$, the optimal contract under commitment is enforceable.
that choosing \( x(y_l) = 0 \) is ex-post inefficient (on average) and, therefore, would typically suffer from the same time inconsistency problem discussed above in section 2C. In other words, the first three assumptions ensure that whether the optimal contract under commitment is enforceable with limited commitment is an interesting question.

Assumptions (iv) – (vii) are useful in characterizing the nature of the coordination problem that arises in period 1 following low or high outcomes. Indeed, assumption (iv) asserts that if all of the lenders have the lowest rate of time preference, then it would be ex-post inefficient for them to continue the project. Of course, as the number of agents becomes large, the probability of this outcome becomes arbitrarily small (by assumption (iii)). Nonetheless the fact that it is possible for all of the lenders to have a preference shock of \( y \) implies that each individual lender must be provided with incentives to reveal their type truthfully, and these incentives do not become arbitrarily small as \( N \to \infty \). As I will show, only by choosing \( t^*_1 \) to be sufficiently large do these incentives become relevant.

The incentive problem facing the lenders following high period 1 outcomes is different however, due primarily to the right-most inequality in assumption (iv). After high period 1 outcomes, all of the lenders know (with probability 1) that it is ex-post efficient to continue the project. As a result, even with a probability of continuation equal to 1, there exist transfers satisfying voluntary participation and incentive compatibility of the lenders. For example, a constant transfer scheme will satisfy these constraints. As a result, the lenders will always efficiently continue the project after high period 1 project outcomes; in effect, the public good problem that enforces liquidation of the project following poor period 1 outcomes is weaker (because future project returns following high period 1 outcomes are higher) and can be overcome by the lenders.

The outline of the proof of Proposition 2 can be summarized by the results:

1. Under full commitment, the optimal continuation rule satisfies \( x(y_h) = 1, x(y_l) = 0 \) (I also characterize optimal consumption of the manager),
2. There exists period 1 transfers, \( t^*_1 \) such that \( x(y_h) = 1, x(y_l) = 0 \) is enforceable as \( N \to \infty \)
3. Given the transfers \( t^*_1 \) and continuation rules \( x(y_h) = 1, x(y_l) = 0 \) (and optimal managerial consumption), the remaining optimization problem with limited commitment coincides with the optimization problem with full commitment, implying that they have the same solution.

I now discuss the proofs of each of these results in turn (in all of the following lemmas and propositions, assumptions \((i) – (vii) \) are maintained).
**Optimal Continuation and Managerial Consumption under Commitment**

The first step in demonstrating my main result is to characterize key features of the optimal contract under commitment. To do so, I first adapt techniques from the dynamic moral hazard literature to my economy and characterize optimal consumption allocations for the manager as a function of the continuation rule.

**Characterizing Consumption of the Manager.** Since the manager is more impatient than the most impatient lender, optimal contracts are front-loaded. That is, the manager’s effort constraint in the last period always binds (except possibly on a set of measure 0 realizations of lenders types). As a result, the manager’s continuation utility, conditional on the continuation probability is a fixed number independent of the history at time 1.\(^4\) Then, by assuming that the manager’s participation constraint is slack (which is satisfied, for example, if the manager’s outside option yields 0 utility), the manager’s incentive constraint must bind and implies \(c_1(y_t) = 0\). I state these results in the following lemmas. (All proofs are in the appendix).

**Lemma 5.** If \(\beta < \frac{1}{\psi}\), then it is without loss of generality to restrict attention to contracts that satisfy

\[
c_2(y_1, y_h, v) = \frac{\psi}{\beta(\pi_h - \pi_l)}
\]

and \(c_2(y_1, y_2, v) = 0\) otherwise.

This result is standard in environments with moral hazard and limited liability (see Hölmstrom (1979) for an example). Here, the result is simply extended to allow for dependence of the consumption function on the realization of \(v\). Let \(\Delta = \pi_h - \pi_l\) and define \(\Psi\) to be the expected “rent” the manager in any state in period 1 in which the project is continued. The value of this rent is given by

\[
\Psi = \beta Ec_2 - \psi = \frac{\pi_l \psi}{\Delta}.
\]

I use \(\Psi\) to characterize consumption allocated to the manager in period 1 conditional on a realization of high output.

**Lemma 6.** If \(\beta < \frac{1}{\psi}\), then it is without loss of generality to restrict attention to contracts that satisfy \(c_1(y_t) = 0\) and

\[
c_1(y_h) = \frac{\psi}{\Delta} + \Psi \int_v [x(y_t, v) - x(y_h, v)] dG(v).
\]

\(^4\)What is important is not that the manager’s utility conditional on continuation is a fixed number, but that it is strictly positive. In my model, the continuation utility is strictly positive in period 1 because the moral hazard problem is present in period 2. Alternative assumptions are interpretations could be made to ensure that the model has the same feature.
Lemma 6 illustrates the tradeoff of continuing the investment project following different period 1 project outcomes. Contracts that call for continuation after a low output realization provides the manager with utility because the manager will receive the rent in the second period. As a result, to induce high effort in period zero, the manager must be compensated with more period 1 consumption after a high output. Contracts that call for continuation after a high output realization in period 1 can deliver lower period 1 consumption to the manager because such contracts deliver future utility to the manager.

The previous two lemmas have fully specified the manager’s consumption as a function of only the continuation rule \( x \). It remains to show how the choice of the continuation rule affects lenders’ payoffs.

**The Optimal Continuation Rule.** It remains to develop additional sufficient conditions so that the optimal continuation rule has the property that when \( y_1 = y_h \), it is optimal to continue the project and when \( y_1 = y_l \), it is optimal to shut down the project. Conditions analogous to those in Section 2B ensure this is the case. To prove that this continuation rule is feasible, given the resource feasibility constraints and the lender incentive compatibility and participation constraints, I demonstrate that such a continuation rule can be implemented with transfers that depend on the outcome \( y_1 \), but not on the reported discount rate of any individual lender \( v_i \).

Assumption (iii) ensures that even after poor period 1 project outcomes, in expectation (over the lenders’ preference shocks) continuing the project is a profitable venture (even net of the cost of providing the manager with incentives to exert effort). Assumption (ii) is analogous to that assumed in Lemma (4) and ensures that the gains the incentive associated with dismissing the manager after low period 1 outcomes outweigh the resources lost by forgoing the investment for another period.

I then have the following result concerning the optimal continuation rule.

**Lemma 7.** Under the maintained assumptions, if \( U_0^s \) sufficiently small, any optimal contract under full commitment satisfies \( e_0 = e_1 = \pi_h, x(y_h, v) = 1, x(y_l, v) = 0 \).

This proof mirrors that of Lemma (4). The only additional concern is whether such a continuation rule is feasible with respect to the lenders incentive constraints. This is easily demonstrated, however, by verifying that there exist transfer schemes such that combined with the continuation rule the contracts are incentive compatible. When the contract calls for dismissal \((x(y_l, v) = 0)\) it is clear that fixed transfers will implement the optimal contract. When the contract calls for continuation \((x(y_h, v) = 1)\), again, fixed transfers are feasible,
though not necessarily optimal. The optimal transfer scheme satisfies

\[
\max \sum_i \int_v \left[ t^i_1(y_h) + t^i_{1c}(y_h, v) + v_i t^i_2(y_h, v) \right] dG(v)
\]

subject to

\[
t^i_1(y_h) + t^i_{1c}(y_h, v) \geq 0
\]

\[
\sum_i \left[ t^i_1(y_h) + t^i_{1c}(y_h, v) \right] + c_1(y_h) \leq y_h
\]

\[
\sum_i t^i_2(y_h, v) \leq E_{\pi_h} y_2 - c_2
\]

\[
w_i(v_i, \hat{v}_i) = \int_v \left[ t^i_{1c}(y_h, \hat{v}_i, v_{-i}) + v_i t^i_2(y_h, \hat{v}_i, v_{-i}) \right] dG_{-i}(v_{-i})
\]

\[
w_i(v_i, v_i) \geq w_i(v_i, \hat{v}_i)
\]

\[
w_i(v_i, v_i) \geq 0
\]

\[
\sum_i t^i_2(v) \leq E_{\pi_h} y_2 - \frac{\pi_h \psi}{\beta \Delta}
\]

**Enforcing the Optimal Commitment Continuation Rule**

In period 1, after any history, without commitment lenders are free to choose a new contract satisfying the constraints in the definition of enforceable contracts. In words, the constraints facing lenders are limited liability, which depends on \( t^i_1 \), resource constraints (continuation requires \( I \) to be re-invested, whereas liquidation can pay out the initial principal, \( I \)), the resource constraint in period 2, and incentive compatibility and voluntary participation for lenders.

It is easiest to prove enforceability by considering extreme period 1 transfers, \( t^i_1 = I/N \). In this case, following a low period 1 outcome, by limited liability, it must be that

\[
t^i_{1c} = -I/N, t^i_{1n} = 0.
\]

Consider, then, the constraints facing the lenders after a low realization of output in the first period. These constraints can be summarized as

\[
w_i(\hat{v}_i, v_i) = \int_{v_{-i}} \left[ x(\hat{v}_i, v_{-i}) \left( \frac{-I}{N} + v_i t^i_2(\hat{v}_i, v_{-i}) \right) \right] G_{-i}(dv_{-i})
\]

\[
w_i(v_i, v_i) \geq w_i(\hat{v}_i, v_i)
\]

\[
w_i(v_i, v_i) \geq 0
\]

\[
\sum_i t^i_2(v) \leq E_{\pi_h} y_2 - \frac{\pi_h \psi}{\beta \Delta}
\]

Notice, the manager’s incentive constraint is nested in the resource constraint in period 2 (in equation (18)).
Now, I appeal to results from Myerson (1981) and Myerson and Satterthwaite (1983) which allow me to characterize the global incentive constraints as local constraints and allow me to eliminate period 2 transfers from the problem (albeit under a weaker, ex-ante version of the period 2 resource constraint). These results are stated in the following lemmas.

**Lemma 8.** A contract satisfies lender incentive compatibility if and only if the function $\rho_i(v_i)$ defined by

$$
\rho_i(v_i) = \int_{v_{-i}}^{v_i} x(v_{-i}, v_i) dG_{-i}(v_{-i}) \text{test}
$$

is increasing in $v_i$ for all $i, v_i, y_1$ and

$$
u^i(v_i) \equiv w^i(v_i) = v_i \left[ \frac{u_i(v)}{v} + \frac{1}{N} \int_v^{v_i} \frac{1}{z^2} \rho_i(z) dz \right].
$$

Moreover, the contract satisfies voluntary participation if and only if $u^i(v) \geq 0$.

The proof is in the appendix. Combining this lemma with the ex-ante version of the period 2 resource constraint, I have the following result.

**Lemma 9.** Suppose that $x$ is such that $\rho^j$ is increasing in $v_i$. There exist payments $t^j_2$ such that $(t^j_2, x)$ satisfy lender incentive compatibility, voluntary participation, and the period 2 resource constraint if and only if

$$
\int_v x(v) \left[ E_{\pi, \psi} y^1 - \frac{\pi_h \psi}{\beta \Delta} - \frac{I}{N} \sum_i \left[ \frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] \right] G(dv) \geq 0.
$$

Using this lemma, following a result in Mailath and Postlewaite (1990), it is straightforward to demonstrate that as $N \to \infty$, the maximal probability of continuation for which there exist transfers so that the continuation rule and transfers satisfy the constraints of the renegotiation problem converges to zero (following low period 1 outcomes). I state this result in the following proposition, which makes use of the regularity conditions of assumptions (v)-(vii) in Proposition 2.

**Lemma 10.** Following a low period 1 outcome, the maximum probability that the project is continued, or $\hat{x}(n)$ satisfying

$$
\hat{x}(n) = \sup \left\{ E\rho(v) : \exists (t^j_2), (\rho, t^j_2) \text{ satisfies lender IC, participation, and and the RC in the n-agent economy ((15)-(18))} \right\}
$$

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converges to 0 as $n$ goes to infinity. Furthermore, the probability that it is ex-post efficient to continue the project goes to 1.

To understand the result, consider the tradeoffs a single lender faces when determining which discount rate to report, $\hat{v}_i$. The benefit of under-reporting is that the agent’s transfers become larger (if the agent were that type, for a fixed probability of continuation a higher discount rate would require higher transfers to satisfy the participation constraint of the lender). The cost to that lender is that by under-reporting, the probability of continuation declines (since $\rho_i$ is increasing).

The key idea behind the limiting result, as in Mailath and Postlewaite (1990), is that as $N \to \infty$, the cost of under-reporting shrinks to zero since the likelihood that a single lender is pivotal, and thus by under-reporting $v_i$ would cause the investment not to be undertaken, becomes arbitrarily small. On the other hand, the costs of providing incentives do not shrink to zero. As a result, in the limit the incentive costs are so large as to make the value of continuation, net of incentive costs, equivalent to that which would occur in a full information economy where all lenders had the highest discount rate; at this rate, lenders prefer not to undertake the investment.

The constraints also make plain why by using all long-term debt (i.e. $t_1 = 0$), even when lenders have the option to walk away from the contract (and force liquidation of the project) cannot implement the optimal continuation rule. With this contract, an individual lender’s payoff (from walking away) is simply 0. Thus, any contract that delivers 0 transfers in period 1 and constant transfers in period 2 will be incentive compatible, satisfy the limited liability constraints and the participation constraints. In particular, if the lenders maximize the ex-ante value of all the lenders, it will be optimal for the lenders to continue the project after low outcomes.

After high period 1 outcomes, it is straightforward to demonstrate the existence of transfers which support $x(y_h) = 1$ for arbitrary transfer rules $t_1(y_h)$. In this sense, the public goods problem that enforces dismissal of the manager following low period 1 outcomes is much weaker following high period 1 outcomes.

**Optimal Transfers with Limited Commitment**

I have demonstrated that the optimal continuation rule (and managerial consumption) under commitment is enforceable even with limited commitment as $N \to \infty$. It remains to prove that the remaining features of the optimal contract with the limited commitment coincide with those under full commitment. Conditional on the continuation rule and managerial consumption, the optimal transfer scheme with limited commitment must coincide with that under full commitment (up to the indeterminacy between $t_1$ and $t_{1c}$ and $t_{1n}$). Moreover, op-
timality of transfers under full commitment ensures that these transfers are enforceable with limited commitment (were they not enforceable, then there would exist superior transfers under full commitment). As a result, I have proved that the solutions to the commitment and limited commitment problem coincide (as $N \rightarrow \infty$).

3C. A Decentralization with Short-Term Debt Claims

TO BE COMPLETED.

3D. Discussion

I conclude this section with a summary of the above results. Note that I have proved that short-term contracts, freely negotiated in each period after project outcomes realized on behalf of the collective interests of the lenders implement long-term commitment outcomes. One feature of these outcomes is that after a realization of low period 1 project outcomes, the investment project is terminated. I call this feature of optimal contracts a bank run. There are two reasons for this. First, for almost every realization of lenders’ types, every lender could be made better off by continuing the investment project if the lenders’ types were public information. Second, lenders choose not to continue investment projects because each lender has an incentive to “run” or threaten not to roll their funds over unless given a larger share of the future project returns than the remaining lenders. Even under arbitrary general mechanisms, lenders cannot avoid the “run” outcome and enforce termination of the investment project. Of course, viewed from the perspective of period zero, these ex-post inefficient outcomes are actually ex-ante efficient, and allow for more efficient investment ex-ante.

One feature of this environment is that, under the maintained assumptions, a contract with the property that $t^i_1(y_l) = 0$ will not enforce the optimal continuation rule. In other words, if short-term contracts are not used and the period 1 resources of the firm are not remitted to the lenders before the realization of their private types, $v_i$, then the lenders can re-negotiate the contract continue with probability 1. The key difference between these contracts is that the implementability constraint in Lemma 9 is weaker as the resources $I$ are still available to the agents. In this sense, long-term lending contracts which only deliver the net return to the lenders in each period are worse than short-term claims which return the gross return to the lenders in each period. I interpret this result, in the context of Fudenberg et al. (1990), as a sample economy in which short-term contracts can implement long-term commitment outcomes even though the utility possibility frontier features ex-post inefficiencies.

Finally, in an economy with sufficiently informative signals, the optimal contract under commitment has the feature that $x(y_h) = x(y_l) = 1$. In such an economy, if the
contract returned the gross payout to the lenders in each period, the same public goods problem would arise after low period 1 project outcomes, causing lenders to “run” and not continue their investment projects. Such contracts would not implement optimal long-term outcomes. Thus, I interpret differences in signal quality as leading to predictions about optimal debt maturity. With low signal quality, the optimal contract is short-term and makes full principal payments in each period; whereas, with sufficiently informative signals, the optimal contract is longer-term, and only returns net payments to the lenders in period 1. Such a payout structure under informative signals effectively commits the lenders to continue investment projects regardless of period 1 project outcomes which is both ex-ante and ex-post efficient.

4. Facts about Debt Structure of Financial and Non-Financial Firms

In this section, I provide documentation that demonstrates how the capital structure of financial firms differs from those of non-financial firms. In particular, I show first that financial firms use much more short-term debt than do non-financial firms. Second, I show that the debt structure of financial firms is much shorter in maturity than is the maturity structure of their assets, while the opposite holds for non-financial firms.

The data are from Compustat and cover both active and inactive publicly traded firms in the United States since 1950. I define financial firms broadly as firms that are banks or undertake bank-like investment activities. Specifically, I restrict the sample of financial firms to be those in the 4 digit SIC industries 6000-6299, which cover Depository Institutions (6000), Non-depository Credit Institutions (6100), and Security And Commodity Brokers, Dealers, Exchanges, And Services (6200). Of the remaining firms, I exclude only those in public administration (9000), and label the rest non-financial firms.

I now present two sets of statistics that describe first how much short-term debt a sample of firms uses and second how mis-matched a firm’s debt and assets are. First, I define short-term debt as the sum of notes and accounts payable on the firm’s balance sheet. This definition of short-term debt explicitly excludes long-term debt due in one year as well as taxes payable and other accrued expenses. My definition of short-term assets is the sum of cash and short-term investments and receivables. I restrict attention to firms that report positive total assets, less short-term debt than total assets, positive short-term debt and positive short-term assets.

Table (1) reports the ratios of short-term debt to total assets and short-term debt to short-term assets for financial and non-financial firms. The first finding is that the median use of short-term debt by financial financial firms is roughly 78% of their total assets whereas for non-financial firms it is only 9% of total assets. Figure (4) displays histograms of these
Table 1: Comparison of Financial and Non-Financial Firms.

<table>
<thead>
<tr>
<th></th>
<th>Short-Term Debt to Total Assets</th>
<th>Short-Term Debt to Short-Term Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Firms</td>
<td>78%</td>
<td>108%</td>
</tr>
<tr>
<td>Non-Financial Firms</td>
<td>9%</td>
<td>42%</td>
</tr>
</tbody>
</table>

ratios for each set of firms for the entire sample.\textsuperscript{5} Figure (3) displays empirical cumulative distributions of this ratio for each set of firms. Notice, roughly 80% of non-financial firms have less than 20% of their assets backed by short-term debt claims, whereas roughly 70% of financial firms have more than 80% of their assets backed by short-term debt claims.

Figure (5) displays histograms of the ratio of short-term debt to short-term assets of financial and non-financial firms. I have truncated the sample at a ratio of 3, as 95% of firms in both samples have a ratio below this value. Again, there is a large mass of financial firms with more short-term debt than short-term assets but relatively few non-financial firms with this property.

5. Conclusion

TO BE COMPLETED.

\textsuperscript{5}While I am including data for all years, the statistics and figures look similar if I focus on any particular year.
Appendix

A1. Figures

Figure 3: Empirical Cumulative Distributions of Short-Term Debt to Total Assets for Financial and Non-Financial Firms.
Figure 4: Short-Term Debt to Total Assets Ratios.

(a) Financial Firms

(b) Non-Financial Firms
Figure 5: Short-Term Debt to Short-Term Assets, Ratios.

(a) Financial Firms

(b) Non-Financial Firms
A2. Proofs
Proof of Lemma 8
Define

\[ \zeta_i(v_i) = \int_{v_i} x(v_i, v_{-i}) t_2^i(v_i, v_{-i}) G_{-i}(dv_{-i}) \]

\[ \rho_i(v_i) = \int_{v_i} x(v_i, v_{-i}) G_{-i}(dv_{-i}) \]

Then

\[ u_i(v_i) = -\frac{I}{N} \rho_i(v_i) + v_i \zeta_i(v_i) \]

or

\[ \frac{1}{v_i} u_i(v_i) = -\frac{I}{N v_i} \rho_i(v_i) + \zeta_i(v_i) \]

\[ \geq -\frac{I}{N v_i} \rho_i(\hat{v}_i) + \zeta_i(\hat{v}_i) \]

\[ = -\frac{I}{N v_i} \rho_i(\hat{v}_i) + \zeta_i(\hat{v}_i) + \frac{I}{N \hat{v}_i} \rho_i(\hat{v}_i) - \frac{I}{N \hat{v}_i} \rho_i(\hat{v}_i) \]

\[ = \frac{1}{\hat{v}_i} u_i(\hat{v}_i) + \rho_i(\hat{v}_i) \frac{I}{N} \left[ \frac{1}{\hat{v}_i} - \frac{1}{v_i} \right] \]

So

\[ \frac{1}{v_i} u_i(v_i) \geq \frac{1}{\hat{v}_i} u_i(\hat{v}_i) + \rho_i(\hat{v}_i) \frac{I}{N} \left[ \frac{1}{\hat{v}_i} - \frac{1}{v_i} \right] \]

\[ \frac{1}{\hat{v}_i} u_i(\hat{v}_i) \geq \frac{1}{v_i} u_i(v_i) + \rho_i(v_i) \frac{I}{N} \left[ \frac{1}{v_i} - \frac{1}{\hat{v}_i} \right] \]

So

\[ \frac{1}{\hat{v}_i} u_i(\hat{v}_i) + \rho_i(v_i) \frac{I}{N} \left[ \frac{1}{v_i} - \frac{1}{\hat{v}_i} \right] \geq \frac{1}{v_i} u_i(v_i) \geq \frac{1}{\hat{v}_i} u_i(\hat{v}_i) + \rho_i(\hat{v}_i) \frac{I}{N} \left[ \frac{1}{\hat{v}_i} - \frac{1}{v_i} \right] \]

\[ \rho_i(v_i) \frac{I}{N} \left[ \frac{1}{v_i} - \frac{1}{\hat{v}_i} \right] \geq \frac{1}{v_i} u_i(v_i) - \frac{1}{\hat{v}_i} u_i(\hat{v}_i) \geq \rho_i(\hat{v}_i) \frac{I}{N} \left[ \frac{v_i - \hat{v}_i}{v_i \hat{v}_i} \right] \]

\[ \rho_i(v_i) \frac{I}{v_i \hat{v}_i} \cdot \frac{1}{\hat{v}_i} u_i(\hat{v}_i) - \frac{1}{v_i} u_i(v_i) \geq \rho_i(\hat{v}_i) \frac{I}{v_i \hat{v}_i} \]

\[ \geq \frac{1}{v_i} u_i(v_i) - \frac{1}{\hat{v}_i} u_i(\hat{v}_i) \geq \rho_i(\hat{v}_i) \frac{I}{v_i \hat{v}_i} \]
This implies that $\rho_i(v_i)$ is increasing in $v_i$. Taking limits, we have

$$\frac{1}{v_i^2} \rho_i(v_i) \frac{I}{N} = \frac{d}{dv_i} \left[ \frac{1}{v_i} u_i(v_i) \right] = \frac{1}{v_i} u'_i(v_i) - \frac{1}{v_i^2} u_i(v_i)$$

So that

$$\frac{1}{v_i} \rho_i(v_i) \frac{I}{N} = u'_i(v_i) - \frac{1}{v_i} u_i(v_i)$$

Then, solving the differential equation we have

$$u_i(v_i) = v_i \left[ \frac{u_i(v)}{v} + \frac{I}{N} \int_{v_i}^{v} \frac{1}{z^2} \rho_i(z) dz \right]$$

This concludes the “If” portion of the proof. The “Only if” portion follows standard arguments.

**Proof of Lemma 9** First, suppose the constraints of the original problem are satisfied. Now, the resource constraint is

$$\sum_i \int_v x(v) t^i_2(v) G(dv) \leq C \int_v x(v) G(dv)$$

Focusing on the LHS, and using the definitions from the previous proof, we have

$$\int_v x(v) t^i_2(v) G(dv) = \int_{v_i} \zeta_i(v_i) G_i(dv_i) = \int_{v_i} \left[ \frac{u_i(v_i)}{v_i} + \frac{I}{N v_i} \rho_i(v_i) \right] G_i(dv_i) = \int_{v_i} \left[ \frac{u_i(v)}{v} + \frac{I}{N} \int_{v}^{v_i} \frac{1}{z} \rho_i(z) dz + \frac{I}{N v_i} \rho_i(v_i) \right] G_i(dv_i) = \frac{u_i(v)}{v} + \frac{I}{N} \int_{v_i} \int_{v}^{v_i} \frac{1}{z^2} \rho_i(z) dz G_i(dv_i) + \frac{I}{N} \int_{v_i} \rho_i(v_i) G_i(dv_i) = \frac{u_i(v)}{v} + \frac{I}{N} \left[ \int_{v_i} \rho_i(v_i) v_i^2 dv_i - \int_{v_i} G_i(v_i) \frac{\rho_i(v_i)}{v_i^2} dv_i + \int_{v_i} \frac{\rho_i(v_i)}{v_i} G_i(dv_i) \right] = \frac{u_i(v)}{v} + \frac{I}{N} \left[ \int_{v_i} \rho_i(v_i) \left[ \frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] G_i(dv_i) \right]$$

Summing over $i$ and combining with the resource constraint yields the desired result. The “Only if” can be demonstrated using a transfer scheme similar to that considered in Mailath and Postlewaite (1990).
Proof of Proposition 10 The general idea is to consider the following auxiliary problem:

\[
\max \int_v x(v)G(dv)
\]

subject to

\[
\int_v x(v) \left[ C - \frac{I}{N} \sum_i \left[ \frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] \right] G(dv) \geq 0
\]

Then, the optimal continuation rule has the property that

\[
x(v) = 1 \iff C \geq \frac{I}{N} \sum_i \left[ \frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right]
\]

In any case, forming the lagrangian, we have that \( x(v) = 1 \) if and only if the condition above modified by incorporating the inverse of the lagrange multiplier on the implementability constraint. An argument from Mailath and Postlewaite (1990) ensures that the lagrange multiplier converges to \( \infty \) as \( N \to \infty \) so that this term vanishes in the limit. Then, the term multiplying \( I \), by a law of large numbers, converges to

\[
E \left[ \frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] = \frac{1}{v_i}
\]

Thus, as \( N \to \infty \), the RHS converges to \( \frac{I}{v_i} \) and \( v \left( E_{\sigma_{vy}}^1 y^2 - \frac{\sigma_{vy}^2}{\beta \Delta} \right) < I \). Therefore, \( x(v) \to 0 \) for all \( v \).

References


