Optimal Managerial Compensation and Financial Hedging in Commodity Procurement

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The procurement of commodities, such as basic raw materials and energy sources, is an important operations management activity that can significantly impact the value of a firm. Managing commodities may also include trading of commodities’ contracts on the financial markets. The finance and economics literature suggests that financial hedging is a valuable risk management strategy because it can reduce the agency cost. We elaborate on this idea by studying the interaction between financial trading and physical trading decisions within a firm from the principal-agent perspective. This interaction creates tension between the compensation that the firm provides to the procurement manager responsible for physical trading and the financial hedging decision of the firm: a higher bonus rate makes the procurement manager exert more effort, which reduces the variability of the commodity price and reduces the need for hedging. As a result of this tension, we find that the bonus rate is non-monotonic in several parameters that characterize the firm and the manager, such as the average commodity price and the ability level of the manager. We also find that only medium sized companies engaged in commodity procurement should use financial hedging to reduce agency costs.

Key words: principal-agent model, financial hedging, managerial compensation, commodity, procurement

1. Introduction

The procurement of commodities, such as basic raw materials and energy sources, is an important operations management activity that can significantly impact the value of a firm (Kleindorfer and Wu 2003). In response to volatile commodity prices, firms have adopted financial hedging as an integral part of their commodity management strategies. Finance and economics research
establishes that financial hedging is a risk management strategy that can add value to the firm (Froot et al. 1993, Stulz 1996, Smith and Stulz 1985, Leland 1998, Tirole 2006) and empirically documents its wide use in practice (Guay and Kothari 2003, Berkman and Bradbury 1996, Graham and Rogers 2002). This is consistent with anecdotal evidence from industry literature. For example, aggressive hedging strategies helped Southwest Airlines Co. decrease its fuel expenses by $455 million in 2004, which led to an annual profit of $313 million, when the airline industry lost about $4 billion in the same year because of high oil prices (Donnelly 2005). Moreover, the use of financial hedging seems to be increasing over the years. According to a study by Greenwich Associates, large companies from the U.S., Europe, and Asia have hedged 55% of their energy commodities exposure in the year 2008 as opposed to 45% in the year 2007 (The Economist 2009).

Jensen and Meckling (1976) have challenged the Miller and Modigliani theory that financial hedging cannot increase the value of a firm (Modigliani and Miller 1958) over three decades ago by showing that the mismatch of objectives between the firm shareholders and its managers creates a link between the capital structure and production decisions of the firm. This mismatch of objectives between the firm shareholders and managers is known as the principal-agent problem and the cost that the principal bears to induce the manager to act in the best interest of the firm is referred to as the agency cost. The importance of agency cost and the link between the agency cost and the corporate use of financial hedging has been repeatedly mentioned in the literature (Smith and Stulz 1985, DeMarzo and Duffie 1995, Tirole 2006).

In particular, financial hedging can add value to the firm by reducing the agency cost in commodity procurement problem as follows: suppose that the commodity procurement decisions of the firm are made by risk-averse procurement managers. The compensation of such a manager includes a risk premium that depends on the amount of risk that this manager is exposed to. The firm can reduce this risk premium by using financial hedging. In turn, the procurement manager can exert effort to reduce the firm’s total spend on commodities and this effort can also affect the value of the firm. The level of this effort chosen optimally by a risk averse procurement manager depends
on the firm’s profit, which includes the firm’s financial performance. Hence, procurement actions and financial activities are inherently linked.

Clearly the agency problem in coordinating physical commodity procurement and financial hedging is relevant, however, to the best of our knowledge, this issue has not been addressed in the operations management literature and it is not clear when and how financial hedging should be used to reduce the agency cost. We are interested in closing this gap by understanding the interaction between the financial hedging and physical procurement and studying the impact of this interaction on the optimal financial hedging strategy for a firm procuring commodities.

In order to address this issue we develop and analyze a stylized principal-agent model that captures the joint impact of hedging and procurement strategies on the total profit of the firm. Our model helps us in answering many questions pertaining to the interaction between financial hedging and physical procurement, such as 1) What is the impact of the hedging strategy on the procurement manager’s effort level? 2) How is the optimal design of the procurement manager’s compensation package in the presence of hedging affected by various business and market parameters? 3) How should a firm jointly design financial hedging strategies and the compensation package of the procurement manager to optimize the value of the firm? etc.

From the analysis of our model we find several managerially relevant results. First, we find that the procurement manager’s optimal effort decreases as the firm hedges more. This can be explained as follows. Since the procurement manager receives a share of the firm’s total profit, he shares part of the risk that the firm is exposed to. The risk is reduced by the firm’s financial hedging activity and also by the procurement manager’s effort, since this effort reduces the total commodity spend. Hence, when the level of financial hedging increases, the risk that the manager is exposed to decreases, the marginal benefit of the procurement effort decreases, and the manager exerts lower level of effort. We find that when the financial hedging policy is fixed (which holds for some industries where financial hedging is highly regulated, e.g. energy) the bonus rate included in an optimal compensation package of the procurement manager is non-monotonic in the expected commodity price and in the manager’s skill level, with the latter quantity modeling the effect of
the procurement manager’s effort on the firm’s profit. It is intuitive to expect that the bonus rate offered to the procurement manager increases as the average commodity price increases because the marginal benefit of the manager’s effort increases. This initial intuition holds true until a certain threshold for the average commodity price, after which it is optimal for the firm to lower the bonus rate again. This counterintuitive behavior can be explained as follows. Fixing the firm’s financial hedging decision implies that the firm “eliminates” risk for a certain quantity of its commodity risk exposure. The procurement manager can decrease the commodity risk exposure by exerting high level of effort. If the firm’s exposure to commodity risk after the procurement manager’s effort is lower than the hedged amount, the firm faces the risk on this gap between the hedged amount and the procured amount. When the average commodity price is high and the marginal benefit of the procurement manager’s effort is high, the manager is likely to exert high level of effort, which is going to reduce the commodity risk exposure. The risk that the firm faces is then increased and the firm wants to provide a disincentive for the manager to work that hard by lowering the bonus rate. A similar argument explains the relationship between the optimal bonus rate and the manager’s skill level.

We find that not only the firm should never speculate, but it should always hedge less than the total quantity of the commodity needed. This result is again driven by the fact that the firm actually faces risk on the gap between the hedged value and the procured value of the commodity. Hence, at equilibrium, the optimal hedge should be equal to the procured value after the procurement manager exerts his effort and this value is less than the total amount of the commodity needed. We also find that only medium sized companies engaged in commodity procurement should use financial hedging to reduce agency costs. Small companies have small commodity spend and hence, do not need to motivate the manager to exert high effort and consequently do not need financial hedging to reduce the compensation; medium companies want to induce the manager to exert higher level of effort and hence, provide a larger bonus rate and benefit from using financial hedging to reduce the compensation. Large firms, however, can get a larger discount with the same level of effort because of large bargaining power; hence, the discounted procured value decreases with the
size of the firm and consequently the need for hedging decreases as well. This result that connects the optimal level of hedging with the size of the firm could be empirically tested.

The rest of the paper is organized as following. We position our work within existing literature in Section 2 and present the model in Section 3. Then, we start our analysis by describing the optimal behavior of the procurement manager for a given compensation contract in Section 4.1. Next, we analyze the optimal compensation contract when the hedging quantity is exogenous in Section 4.2, and then, endogenize the hedging quantity in Section 4.3. We discuss our main results and propose directions for future research in Section 5.

2. Literature Review

Our paper draws upon literature from several research streams. The principal-agent problem has been recognized in accounting and information economics literature for a long time. See Holmstrom (1979), Harris and Raviv (1979), Mirrlees (1976) for classical reference on this problem. Since Jensen and Meckling (1976) and Smith and Stulz (1985) have recognized the principal-agent conflict as one of the determinants of corporate use of financial derivatives, there have appeared several studies that analyzed the hedging decisions of firms that contract with risk-averse managers (see, for example, DeMarzo and Duffie (1995), DaDalt et al. (2002), Nan (2008)); these studies, however, analyze optimal hedging policies when either the firm or the manager can affect the risk. The distinguishing feature of our model is that in commodity procurement setting both principal and agent have levers to affect the variability: the firm can reduce variability by performing financial hedging and the manager can decrease variability by reducing the price on the spot market.

The principal-agent problem has also attracted attention of the operations management community in the past decade (Porteus and Whang 1991, Jerath et al. 2007, Chen 2000, Plambeck and Zenios 2000). Most of the papers that adapted the principal-agent framework in the operations management field are either on the interface between operations management and marketing (Porteus and Whang 1991, Jerath et al. 2007) or in the supply chain contracting (Corbett and DeCroix 2001, Corbett et al. 2005). To the best of our knowledge, there is no work that addresses the agency costs in commodity procurement and financial hedging.
Our work is also related to the fast growing body of literature on the interface between operations and financial hedging, e.g. Van Mieghem (2007), Huchzermeier and Cohen (1996), Ding et al. (2007), Gaur and Seshadri (2004). These papers study the interaction between operational hedging and financial hedging, assuming that financial hedging is valuable for the firm because of the firm’s risk aversion. In our setting, however, the firm is risk neutral and is concerned with risk reduction for the purpose of reducing the agency cost.

3. The Model

Definitions

Before presenting our model, we introduce several financial terms that we use extensively in the model description. The commodity trading firm in our model performs financial hedging by trading futures contracts. Futures contract is a contractual agreement to buy or sell a particular commodity at a pre-determined price in the future. Futures contracts detail the quantity of the commodity; we will focus only on the contracts that do not call for physical delivery of the commodity and are settled in cash. Futures contracts are traded on a monthly basis and have a futures maturity date, which is a date at which the financial settlement is due on the futures contract. The dates are fixed by the exchange and occur before the delivery period. The price at which the commodity is traded on the futures maturity date is called the futures price. The procurement manager performs physical trading of the commodity, the market price for the physical delivery of the commodity on the date of purchase is called the spot price.

Model setup and sequence of events

We consider a single-period setting with two players: a value maximizing firm and a risk-averse procurement manager. We formulate a principal-agent model, where the manager plays the agent’s role and the firm plays the principal’s role. The objective of the firm (the principal) is to maximize the expected value of the firm. The objective of the procurement manager (the agent) is to maximize his expected utility, which depends on his compensation contract and the personal cost of effort. First, we present a brief outline of the model using a sequence of events happening within the firm during a single period.
Figure 1 displays the sequence of events. The front row represents the events that are performed by or observed by the manager, the back row represents the events that involve the firm. The events that involve both players, are pictured in between the rows. Event 0 occurs before the start of the period - the firm sets a compensation contract with the procurement manager; the contract consist of two parameters: the fixed wage \((W^1)\) and the bonus that is represented as a fraction \((\beta)\) of the total profit. The next four events occur sequentially during the period that starts at \(t = 0\). At event 1, the firm decides on the number of futures contracts to trade \((h)\). Financial trading impacts the commodity price risk to which the firm and the manager are exposed to, which impacts the effort level decision that the procurement manager has to make next. Next, at event 2, the procurement manager exerts effort \((e)\), which leads to a lower procurement price. The manager chooses his level of effort \(e\) by maximizing his expected utility for a given compensation contract. At the futures contracts maturity date (event 3), the firm observes the futures price realization \(f\). At the physical procurement date (event 4), the procurement manager purchases the commodity on the spot market at the realized spot price \(s\) with a percentage discount \(D(e)\) negotiated earlier (event 2). Finally, at the end of the period \((t=1)\), event 5 occurs and the firm compensates the procurement manager based on the total profit according to the contract signed at event 0. Next, we elaborate on the details of the model and our assumptions.

1 All the notation is summarized in Table 2 in the Appendix (Section 6)
Financial trading performed by the firm

For simplicity, we assume that the physical procurement task is delegated to the procurement manager while the financial hedging decision is performed by the firm. This assumption is consistent with empirical evidence on hedging in the industry (Zhou 2009). The firm can engage in financial hedging by trading futures contracts on the commodity. The futures price of the commodity at maturity date (event 3) is denoted by a random variable $F$ that is normally distributed with mean $\mu$ and standard deviation $\sigma$. The value of the futures contract at the maturity date (event 3) is $h(F - \mu)$, where $h$ is the quantity of futures contracts ($h > 0$ implies that the firm purchases futures contracts and $h < 0$ implies that the firm sells futures contracts). We assume that there is a per unit cost $\lambda$ for the futures contracts transaction (a similar way to model transaction cost for futures contracts can be found in Krokhmal et al. (2002)). Therefore, the financial profit of the firm is:

$$\pi_f(F,h) = h(F - \mu) - \lambda|h|.$$  

Physical trading performed by the procurement manager

The procurement manager is in charge of sourcing a commodity at the physical delivery date (event 4). Total need for the commodity at this time is $Q$ and the random spot price of the commodity at that time is $S$. Since the maturity date of the futures contract (event 3) occurs before the physical delivery date (event 4), the spot price at the physical delivery date is not equal to the futures price at the maturity date. We assume that $S$ and $F$ are independent random variables that have the same probability distribution function. We assume that the random variables are independent because of the extra information that is revealed on the market during the time lag between events 3 and 4. The manager can exert effort $e$ at a cost of $\frac{e^2}{2}$ to obtain a percentage discount ($D(e)$). 

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2 According to the aforementioned study performed by Greenwich Associates, companies are moving hedging decisions to the highest levels of the organization; e.g., the percentage of companies that reported that high level executive officers were in charge of hedging policies have increased from 75% in the year 2007 to 96% in the year 2008. In 55% of the companies surveyed, the hedging decisions were made by the CFOs in 2008, which is an increase from 40% in 2007 (Zhou 2009).

3 Schwartz (1997) and Schwartz and Smith (2000) mention that due to high uncertainty of the actual spot price, empiricists use futures prices with the maturity date closest to the spot transaction date as a proxy metric for the spot price. Hence, it is reasonable to assume that the firm uses the same distribution function for the spot prices as for the futures prices.
on the commodity that is negotiated at event 2 before the maturity date of the futures contract (event 3). This cost of effort has decreasing marginal returns that go to zero.

There are two factors that positively impact the discount: manager’s negotiation ability ($\alpha$) and the size of the transaction ($Q$). Moreover, when the size of the transaction is large, the procurement manager has high bargaining power and the same level of managerial ability will lead to a higher discount. Therefore, the discount on the commodity is an increasing function of $\alpha$ and $Q$, where $\alpha$ and $Q$ are proportional. The discount is bounded and we will assume that the maximum possible discount is $100\%$. We approximate the relationship between the discount and the effect of the procurement manager with the following piecewise linear function:

$$D(e) = \alpha Q \min\{e, \frac{1}{\alpha Q}\}, e \geq 0$$

![Graph](image)

**Figure 2** Discount obtained by the procurement manager on the commodity as a function of his effort.

**Parameters:** $\alpha Q = 1$.

Alternatively, we can interpret $D(e)$ as a percentage improvement in the yield that is impacted by the production manager’s effort: The production manager can work on improving the production process, such that the firm can use $Q(1 - D(e))$ units of input to produce $Q$ units of output. A similar way of modeling the effect of effort can be found in Corbett and DeCroix (2001). The total operating profit of the firm is then

$$\pi_o(S, e) := Qp - QS(1 - D(e)),$$  \hspace{1cm} (2)
where $p$ is the product selling price.

**Compensation contract and the firm’s total profit**

The total profit of the firm before paying compensation to the manager is the summation of the operating and financial profits:

$$\pi_t(S, F, e, h) = \pi_f(F, h) + \pi_o(S, e).$$

(3)

At the end of the period, the firm has to compensate the procurement manager (event 5). The firm observes the realization of $F$ that is a commodity futures price as of maturity date (event 3); hence, the firm can calculate $\pi_f(F, h)$. The firm also observes $\pi_o(S, e)$ from the financial statements at the end of the period, however, it cannot deduce the actual level of effort that the manager has exerted because the exact spot price realization at delivery $s$ is not easily observable. We assume that the procurement manager who works daily on the physical market can observe all spot prices, however, the firm cannot. Hence, the firm can observe $sD(e)$, but cannot deduce the level of effort $e$ as it has no information about the exact value of $s$.

Therefore, the firm has to write a contract based on the resulting profit and not on the effort. As is typical in practice, compensation is linked to the total profit of the firm that is a function of the effort. Compensation contract is linear: $C(W, \beta, S, F, e, h) := W + \beta \pi_t(S, F, e, h)$, where $C$ is the amount of compensation, $W$ is the fixed wage, and $\beta$ is the bonus rate assigned to the realized profit.

**Principal-Agent model**

Now, we put all the elements of the model together and formulate the firm’s problem as a Principal-Agent model. Define $\pi(W, \beta, e, h)$ as the expected net profit after compensating the manager: $\pi(W, \beta, e, h) = \mathbb{E}_{(S, F)}[\pi_t(S, F, e, h) - C(W, \beta, S, F, e, h)]$, where $\mathbb{E}_{(S, F)}[x]$ indicates the expected value.

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4 In some industries, daily spot prices can be obtained from industry newsletters and publications, such as reports published by Platts; however, this published daily spot price is only a volume-weighted average of many commodity transactions on that day as self-reported by traders to the data collector. This reported number is not an accurate representation of the actual market spot price at the time of the transaction. Schwartz (1997) and Schwartz and Smith (2000) mention that commodity spot prices are very hard to observe because the spot market is highly fragmented.

5 Since the firm is willing to reduce the agency cost using financial hedging, it includes the financial profit in the procurement manager’s contract. Note, however, that by setting $h = 0$, the firm makes the financial profit equal to zero and hence, makes the compensation contract dependent only on the operating profit. In section 4.3, we will derive conditions under which it is optimal for the firm to do that.
value of $x$ taken over two random variables ($S$ and $F$). The manager is risk-averse and has an exponential utility function with constant absolute risk aversion (CARA): $U(W, \beta, S, F, e, h) := -\exp(-r(C(W, \beta, S, F, e, h) - \xi(e)))$, where $r$ is the Arrow-Pratt measure of risk aversion. Then, the Principal-Agent model is as following:

$$
\max_{W, \beta, h} \pi(W, \beta, e^o(W, \beta, h), h)
$$

$$
e^o(W, \beta, h) \in \arg \max_{e \in \mathbb{R}_+} \mathbb{E}(S, F)[U(W, \beta, S, F, e, h)]
$$

$$
\mathbb{E}(S, F)[U(W, \beta, S, F, e^o(W, \beta, h), h)] \geq 0
$$

Constraint 4 is the Incentive Compatibility (IC) constraint and it shows that the manager chooses his effort to maximize his expected utility. Constraint 5 is the Individual Rationality (IR) constraint that makes sure that the manager’s reservation utility is met. Without loss of generality, we set the manager’s reservation compensations in such a way that his certainty equivalent must be greater than 0.

4. Analysis

For analytical convenience, we let $CE(W, \beta, e, h)$ denote the certainty equivalent for the manager for a given compensation scheme and hedging decision, which is defined as:

$$
\exp(-rCE(\cdot)) = \mathbb{E}(S, F)[\exp(-r(W + \beta \pi_t(S, F, e, h) - \frac{e^2}{2}))].
$$

The certainty equivalent is:

$$
CE(W, \beta, e, h) = W + \beta \mathbb{E}(S, F)[\pi_t(S, F, e, h)] - \frac{e^2}{2} - R(\beta, e, h),
$$

where $R(\beta, e, h) = \frac{1}{2}(h - Q(1 - \alpha eQ))^2r\sigma^2\beta^2$ represents the variance of the manager’s share of profit that is commonly referred to as risk premium. For notational convenience, we will use $CE'(\beta, e, h)$ to represent the portion of the certainty equivalent that is independent of the fixed wage and can be influenced by the procurement manager’s effort. All mathematical derivations and subsequent proofs are presented in the appendix (Section 6).
IR constraint (5) always binds at optimality because the firm can reduce the fixed part of the compensation contract \((W)\) until the certainty equivalent of the manager’s utility function is equal to the manager’s minimum reservation utility (Feltham and Xie 1994). Hence, we can find optimal \(W^*(\beta, h)\) by setting the IR constraint equal to zero and re-write the firm’s profit as

\[
\Pi(\beta, e, h) = \pi(W^*(\beta, h), \beta, e, h) = \mathbb{E}(S,F)[\pi_t(S,F,e,h)] - \frac{e^2}{2} - R(\beta, e, h), \tag{8}
\]

and the principal-agent problem as

\[
\max_{\beta, h} \Pi(\beta, e^*(\beta, h), h) \tag{9}
\]

\(e^*(\beta, h) \in \arg\max_{e \in \mathbb{R}^+} CE'(\beta, e, h).\)

From now on, we will focus on analyzing problem (9).

4.1. Optimal behavior of the procurement manager

We start our analysis by studying the behavior of the procurement manager for a given compensation contract. Understanding the optimal response of the procurement manager to the compensation scheme offered by the firm helps in building up intuition for the structure of the optimal contract. First, we study how the procurement effort \((e)\) and the number of futures contracts \((h)\) impact the risk premium \((R)\). The risk is captured by the variance term of the procurement manager’s certainty equivalent.

**Lemma 1.** 1. If \(h > (\leq)Q(1 - \alpha eQ)\), the risk premium, \(R\), increases (decreases) in the procurement manager’s effort and in the number of futures contracts \(\left(\frac{\partial R(e,h;\beta)}{\partial e} > 0\right)\text{ and }\left(\frac{\partial R(e,h;\beta)}{\partial h} > 0\right)\) if \(h > (\leq)Q(1 - \alpha eQ)\);

2. Procurement manager’s effort and financial hedging are complements with respect to the risk premium \((\frac{\partial^2 R(e,h;\beta)}{\partial e \partial h} > 0)\).

The result in Lemma 1 suggests that if the procurement manager obtains a large discount on the commodity, financial hedging may increase the variance. We illustrate this result by plotting the risk premium as a function of the number of futures contracts for a fixed level of procurement manager’s in Figure 3.
For a fixed level of procurement manager’s effort, the risk premium may increase or decrease in the number of futures contracts. Parameters: $Q = 3$, $\alpha = 1$, $r\sigma^2 = 1$, $\beta = 0.8$.

Notice that contrary to the initial intuition that the risk should decrease in the number of futures contracts for all $h < Q$, the curve has a minimum at $h = 1.2$, which is less than $Q \ (= 3)$. This result may seem not intuitive at first, but can be explained as follows. The discount on the commodity is obtained on a single transaction on the spot market between the procurement manager and the commodity supplier and it does not affect the market expectations of the commodity prices. Financial hedging, on the other hand, is performed on the financial market and hence, the value of futures contracts is based on the expectation of the market price without the discount. Hence, the risk is impacted by the gap between the hedged value and the discounted value. Buying futures contracts may expose the firm to more risk if the hedged value on the financial market exceeds the discounted value on the physical market. Recall that the procurement manager obtains a percentage discount on the commodity. Therefore we may think about the discounted value as buying a lower quantity at the full price (which is consistent with our second interpretation of the effect of operational effort: production manager improving the yield and hence, reducing the quantity of the commodity needed). To minimize the risk exposure, the firm will want to make the hedged value equal to the discounted value. The gap between the hedged value and the discounted value grows in both the procurement effort and the number of futures, hence, we can see in the last part of Lemma 1 that the marginal effect of the procurement manager’s effort on the variance.
increases in the number of futures contracts purchased and vice versa.

It is easy to see that the procurement manager’s effort will never exceed \( \frac{1}{\alpha Q} \), because the marginal benefit of effort after this point is equal to zero, but the effort is costly. In Lemma 2, we derive the conditions under which the firm can motivate the procurement manager to exert the maximum level of effort.

**Lemma 2.** There exists a threshold \( \hat{\mu}(h) \) on the average commodity price, such that

1. when \( \mu < \hat{\mu}(h) \), there is no \( \beta \in \mathbb{R} \) that makes \( e^*(\beta, h) = \frac{1}{\alpha Q} \) binding and the unique utility maximizing effort level is \( e^*(\beta, h) \in [0, \frac{1}{\alpha Q}] \);

2. otherwise, when \( \mu > \hat{\mu}(h) \), there exist \( \beta > 0 \) and \( \overline{\beta} > 0 \) (\( \beta < \overline{\beta} \)), such that \( e^*(\beta, h) = \frac{1}{\alpha Q} \) for \( \beta \in [\beta, \overline{\beta}] \).

When \( \beta < 1 \), the marginal benefit of the procurement manager’s effort increases linearly in the average commodity price (\( \frac{\partial \pi}{\partial e} = \alpha \mu Q^2 (1 - \beta) \)). Hence, when the price of the commodity is low, the marginal benefit of effort is low and there is little incentive for the manager to exert effort to get a high discount. Thus, there is no bonus rate that the firm could set to motivate the manager to obtain the maximum discount. We illustrate this behavior using a numerical example in Figure 4.

(a) \( \mu = 0.7 \). When the average commodity price is low (\( \mu < \overline{\mu} \)), there is no bonus rate that will make the manager exert the highest level of effort.

(b) \( \mu = 1 \). When the average commodity price is high (\( \mu > \overline{\mu} \)), the manager will exert the highest level of effort in the mid-range of bonus rate: \( \beta \in [\overline{\beta}, \overline{\beta}] \).

**Figure 4** Parameters: \( h = 1, Q = 1.6, \alpha = 1, r \sigma^2 = 1, \) and \( \lambda = 1 \).
Otherwise, when the price of the commodity is high, the manager would exert the highest level of effort if he receives a significant portion of the profit ($\beta \leq \beta \leq \beta$). Hence, his compensation is substantially impacted by the discount. However, if the bonus rate is too high ($\beta \geq \beta$), the manager is exposed to a large portion of the risk. Since the variance may increase in the procurement manager’s effort (Lemma 1), the manager would not choose the highest level of effort. When the manager does not choose the highest level of effort, this effort depends on the financial hedging strategy of the firm. Next, we examine how the number of futures contracts impacts the procurement manager’s effort.

**Proposition 1.** For a given compensation scheme (i.e. fixed $\beta$), when $e^*(\beta, h) < \frac{1}{\alpha Q}$, the procurement manager’s effort decreases in the number of futures contracts ($\frac{\partial e^*(\beta, h)}{\partial h} < 0$).

Proposition 1 establishes that when the procurement manager’s optimal level is not at its upper bound ($\frac{1}{\alpha Q}$), the effort decreases in the number of futures contracts traded ($h$). This can be explained as follows. Recall from Lemma 1 that the procurement manager’s effort and financial hedging are complementary in their effect on the variance, i.e. both actions impact the risk premium in the same direction. Hence, when the number of futures contracts purchased increases, the procurement manager can exert less effort and nevertheless obtain the same expected utility. And since the effort is costly for the procurement manager, it will be optimal for him to exert lower level of effort. As a result of this “substitutable” effect between the number of futures contracts and the procurement manager’s effort, the firm will have to use the bonus rate $\beta$ to mitigate this effect and benefit from both the procurement manager’s effort and financial hedging.

### 4.2. Optimal contracting when hedging quantity is exogenous

In this section, we focus on the scenario where the hedging quantity is fixed. We use this analysis mainly as a building block for understanding further results when the hedging quantity is endogenous. However, this scenario with exogenous number of future contracts may also be applicable in practice in some settings. For example, this situation occurs when hedging amount is highly regulated by the government and the firm is obliged to hedge a certain amount of commodity exposure.
For instance, the California Public Utilities Commission requires electric power providers to verify futures contracts for 90% of the customer requirements one year forward and 100% of the customer requirements one month forward (Pechman 2007).

**Lemma 3.** There exists a unique $\beta^*(h) = \arg\max_\beta \Pi(\beta, e^*(\beta, h), h)$, such that $\beta^*(h) \in (0, \min[1, \beta])]$.

Lemma 3 suggests that the firm always pays a non-zero bonus rate to the manager and hence, it means that the marginal benefit of effort at zero bonus rate is always greater than marginal cost. Indeed, zero bonus rate implies that the procurement manager’s effort is equal to zero and the firm is purchasing the commodity at the non-discounted price. The marginal benefit of effort is then always positive and is equal to the expected discount on the commodity ($\alpha Q^2 \mu$). The marginal cost of effort at zero effort level is equal to zero. Hence, when the procurement manager’s ability level ($\alpha$), total quantity of the commodity needed ($Q$) and the average commodity price ($\mu$) are positive, the bonus rate is always positive.

**Proposition 2.** There exists a threshold $\mu(h)$ on the average commodity price such that when $\mu < \mu(h)$, the bonus rate increases in $\mu$ ($\frac{\partial \beta^*}{\partial \mu} > 0$) and when $\mu > \mu(h)$, the bonus rate decreases in $\mu$ ($\frac{\partial \beta^*}{\partial \mu} < 0$).

We find that the bonus rate included in an optimal compensation package of the procurement manager is non-monotonic in the average commodity price. This is contrary to the initial intuition that suggests that if the average commodity price is high, the firm should provide more incentives to the procurement manager to work harder to decrease the high price. We illustrate this behavior with a numerical example in Figure 5.

This counterintuitive behavior can be explained as follows. The firm’s profit after paying the manager can be written as follows:

$$\Pi(\beta, e^*(\beta, h), h) = \mathbb{E}_{(S,F)}[\pi_t(S,F,e^*(\beta, h), h)] - \frac{(e^*(\beta, h))^2}{2} - R(\beta, e^*(\beta, h), h).$$  \hspace{1cm} (10)
(a) The optimal bonus rate offered to the procurement manager is non-monotonic in the average commodity price ($\mu$).

(b) The risk premium changes non-monotonically with respect to the bonus rate ($\beta$) and the average commodity price ($\mu$). The risk premium decreases as the shading darkens.

**Figure 5** Parameters: $h = 0.1$, $Q = 1$, $\alpha = 0.5$, $r\sigma^2 = 1$, and $\lambda = 1$.

Notice that the bonus rate ($\beta$) has a direct impact on the risk premium and indirect impacts on all three terms of (10) via the procurement manager’s effort. The procurement manager’s effort increases the total profit, increases the cost of effort and may increase or decrease the risk premium depending on the number of futures contracts purchased (see Lemma 1). Recall that in our model the procurement manager can affect the total spend on the commodity by reducing the price paid, which in turn has a direct impact on the share of the firm’s profit that the manager receives. Hence, the procurement manager’s optimal effort level always increases in the commodity price. This effect may be enhanced by offering the manager a higher bonus rate, but it comes at a cost to the firm.

Let’s examine the firm’s perspective. When the commodity price is low, the procurement manager’s effort will naturally be low as the decrease in the commodity price does not have a significant impact on the procurement manager’s compensation. The firm may increase this effort by offering a high bonus rate to the procurement manager; however, the benefit of effort to the firm is low
since the commodity is not a large expense and hence, the firm keeps the bonus rate low. When the commodity price increases, the procurement effort will naturally increase as well. The benefit of procurement to the firm will also increase and hence, it is beneficial to the firm to offer a high bonus rate, motivate high procurement effort, and get a significant discount. However, when the commodity price is very high, the procurement manager’s effort is naturally very high. If the firm increases the effort even more by offering a high bonus rate, it is likely that the risk premium will increase (recall the result from Lemma 1 that the risk premium increases in the level of effort when the hedged value is higher than the discounted value). See the light area in the top right corner of Figure 5(b) that illustrates that the risk premium increases in the bonus rate ($\beta$) when the average commodity price is high. Hence, even though the discount is significant, the increase in risk premium and in the cost of effort ($\frac{(e^*(\beta,h))^2}{2} + R(\beta,e^*(\beta,h),h)$) makes it not efficient for the firm to offer a high bonus rate.

Without stating a formal proposition, we observe from numerical experiments that the ability level of the manager ($\alpha$) has a similar impact on the optimal bonus rate. We illustrate this behavior with an example in Figure 6.

Although the behavior observed in Figure 6(a) is similar to the result of Proposition 2, the reasoning behind this observation is slightly different. In Proposition 2, we argued that the procurement manager’s effort always increases in the average commodity price. In the case of the manager’s ability level, this is not always true ($\frac{\partial e^*(\beta,h)}{\partial \alpha}$ may be greater than or less than 0). In Figure 6(b), we can see that when the bonus rate is high, the procurement effort may decrease in the manager’s ability level. Hence, the optimal policy from the firm’s perspective would be not to offer a high bonus rate when the ability level is high.

4.3. Optimal contracting when hedging quantity is endogenous

In this section, we endogenize the number of future contracts traded by the firm. We start our analysis by understanding the optimal financial hedging strategy when there is no transaction cost of hedging, i.e. set $\lambda = 0$ (Part 1 of Proposition 3), which also establishes bounds on the optimal hedging strategy when the cost is non-zero (Part 2 of Proposition 3).
(a) The optimal bonus rate offered to the procurement manager is non-monotone in the manager’s ability level \((\alpha)\).

(b) The procurement manager’s effort changes non-monotonically in his ability level.

**Figure 6** Parameters: \(h = 0.1, Q = 1, \mu = 1, r\sigma^2 = 1, \text{ and } \lambda = 1.\)

**Proposition 3.**

1. When hedging is free \((\lambda = 0)\), optimal hedging quantity \(h^*_o\) is less than \(Q\) and is independent of the bonus rate \((\beta)\);
2. When cost of hedging is positive \((\lambda > 0)\), \(h^* \in [0, h^*_o]\).

Initial intuition may suggest that when the cost of hedging is sufficiently low, it would be optimal to buy futures contracts for the whole quantity needed \((Q)\). However, we show in Proposition 3 that the optimal hedging quantity is less than \(Q\) even when the cost of hedging is zero. This may be explained as following. If the cost of hedging is zero, it is optimal for the firm to hedge the amount that would minimize the risk premium that the firm has to pay to the procurement manager. As we learned in Lemma 1, the risk premium is minimized when the hedged volume is equal to the discounted volume. We also know from Lemma 3 that the bonus is always greater than 0 because the marginal benefit of the procurement manager’s effort at zero level is greater than the marginal cost and hence, the effort is always greater than zero at optimality. Since the discounted value is less than the total quantity \(Q\) when the procurement effort is greater than zero, optimal level of hedging volume is less than \(Q\) as well. We illustrate this result with a numerical example in Table
Observe in Table 1 that the variance changes non-monotonically with the number of futures contracts. When hedging value is high ($h = Q = 3$), the variance $R(e, h)$ is equal to 0.004, if we decrease the hedging value, ($h = 2$), the variance decreases to 0. Notice that at $h = 2$, the hedged value on the financial market ($h$) is equal to the discounted value on the physical market ($Q(1 - \alpha e^*(1, h)Q)$ in Column 3). Hence, it is optimal to set $h = 2$, which minimizes the variance and yields the highest profit.

Also, notice that $h^* \geq 0$, which implies that financial speculation is never optimal. This observation is consistent with our problem definition that assumes that hedging is used for reducing the managerial compensation by minimizing the risk that the manager is exposed to and is costly to the firm. Hence hedging does not add any value to the firm.

In practice, there is typically a transaction fee associated with hedging. Recall that we model the cost of hedging as a linear absolute function: $\lambda |h|$. Based on the result of Proposition 3, we restrict further discussion to $h \geq 0$, which is always true at optimality. Hence, we will replace $\lambda |h|$ with $\lambda h$ in subsequent analysis.

To illustrate the tension that the firm faces in its hedging decision, we present the following numerical example depicted in Figure 7.

Examining the profit curve on the Figure 7(a), we can see that when the number of futures contracts is low, the firm’s profit may decrease in the number of futures and the profit curve attains a local minimum at $h = 0.14$. While the marginal benefit of effort is higher than the marginal

<table>
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<tr>
<th>Number of futures</th>
<th>Procurement effort $e^*(1, h)$</th>
<th>Discounted volume $Q(1 - \alpha e^*(1, h)Q)$</th>
<th>Variance $R(e^*(1, h), h)$</th>
<th>Firm’s profit $\Pi(\beta, e^*(1, h), h)$</th>
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<td>2.910</td>
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Table 1  Observe that if the firm hedges the total quantity $Q$, variance is higher than at a lower hedging value, which causes low value for procurement effort and decreased profit. Parameters: $\lambda = 0$, $Q = 3$, $\alpha = 0.5$, $\mu = 0.0087$, $\nu \sigma^2 = 1$, $\beta = 1$, and $p = 0.3$
(a) The firm can decrease its profitability by hedging too little.
(b) The profit is maximized when the risk premium is minimized.

**Figure 7** Parameters: $Q = 0.9$, $\alpha = 0.5$, $\mu = 1$, $r\sigma^2 = 12$, $\lambda = 0.1$, and $p = 2$. 

cost, the procurement manager will be willing to increase his level of effort to bring the discounted value of the commodity closer to the hedged value; hence, the firm does not need to provide high incentives to the manager, and financial hedging brings little value to the firm, while also incurring a transaction cost. Therefore, the firm can hurt itself by hedging too little. When, the hedging is higher, though, the marginal benefit of effort for the manager decreases. Hence, the firm has to increase the bonus rate to motivate the manager to exert high level of effort. When the bonus rate is high, financial hedging is more beneficial to the firm and may outweigh its cost. In equilibrium, the firm will purchase enough futures contracts to motivate the manager to exert the level of effort that will reduce the risk premium down to zero ($h = 0.63$ on Figure 7(b)).

**PROPPOSITION 4.** There exist $Q$ and $\bar{Q}$, such that for $Q < Q$ or $Q > \bar{Q}$, $h^* = 0$ and for $Q < Q < \bar{Q}$, $h^* \in [0, h^*_o]$.

In Proposition 4, we find that only medium sized companies engaged in commodity procurement should use financial hedging to reduce agency costs. Small companies have low total commodity spend and consequently the marginal benefit of the procurement manager’s effort is low. Hence, the optimal manager’s bonus rate is low and there is no need to financially hedge to reduce the risk premium paid to this manager. Medium sized firms have larger commodity spend and want to induce a high level of effort to reduce the spend, hence, the bonus rate is high, but the procurement effort is not high enough to reduce the risk. The firm then optimally chooses to hedge to reduce
the risk that will in turn reduce the compensation to the manager. Large companies have high total commodity spend and the firm wishes to reduce this spend by obtaining a sizable discount on the spot market. At the same time, large companies have greater bargaining power and hence, can achieve the same discount with lower effort of the procurement manager. Therefore, there is no need to offer a very large bonus rate to the manager to induce a high level of effort. The risk premium decreases and the firm no longer needs to engage in financial hedging as its cost would outweigh its benefit.

Notice that when the firm sets the number of future contracts to zero, the financial profit is also equal to zero. Therefore, the total profit reduces to the operating profit and hence, the procurement manager’s compensation depends only on the operating profit. We can interpret the result in Proposition 4 as follows. Small companies \((Q < \bar{Q})\) and large companies \((Q > \bar{Q})\) should link the compensation contract to the operating profit and then, they cannot hurt themselves by hedging; however, the value of hedging in such case is zero. Medium companies \((Q < Q < \bar{Q})\) should include financial profit in the compensation contract of the procurement manager and use financial hedging to reduce the risk premium.

5. Discussion

The key contribution of this paper is the analysis of the interaction between two levers that reduce commodity risk exposure: financial hedging and effort of a procurement manager. The interaction creates tension between the compensation that the firm provides to the manager and the financial hedging decision: higher bonus rate makes the procurement manager exert more effort, which reduces the variability of the commodity price and reduces the need for hedging. As a result of this tension, we find that the bonus rate is non-monotonic in several parameters that characterize the firm and the manager, such as the average commodity price and the ability level of the manager. We also find that only medium sized companies engaged in commodity procurement should use financial hedging to reduce agency costs. Stated differently, we conclude that only medium sized firms should include financial profit in the compensation contract of the procurement manager. This result could be empirically tested.
Our model is not without limitations. We assume a LEN principal-agent model, which implies a Linear compensation contract, negative Exponential utility function for the manager and a Normal distribution for the commodity price. Linear contracts are widely used for studying principal-agent models due to mathematical tractability. However, this assumption may limit the results as linear contracts are not usually optimal. We limit the financial hedging activity of the firm to trading futures contracts, which are also linear and may not be optimal. Furthermore, we assume that hedging activity is performed by the firm, which is consistent with empirical observations on hedging practices. However, it is interesting to see what results if the firm delegates the hedging tasks to the procurement manager for the following reason. Our assumption drives the tension between the bonus rate and the financial hedging decision, because the firm makes the financial hedging decision before the procurement manager chooses his effort level and hence, the firm has no information about the actual size of the transaction on the physical market. If the financial hedging decision was delegated to the procurement manager, this tension would disappear, but instead we would get the tension between the two tasks performed by the same person that have different costs and different benefits. Hence, the problem would change significantly and may result in interesting insights. Our model does not include other aspects that can make financial hedging optimal for a value maximizing firm, such as convex tax schedules, financial distress costs, differences between the external and internal financing costs, etc. These limitations may be addressed in further research.
References


Jerath, K., S. Netessine, Z.J. Zhang. 2007. Can we all get along? incentive contracts to bridge the marketing and operations divide.


6. Appendix
6.1. Table of Notation

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<td>( \beta, \bar{\beta} )</td>
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<td>( Q, \bar{Q} )</td>
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Table 2  Table of Notation

6.2. Derive Certainty Equivalent (CE)

\[
\exp(-rCE(\cdot)) = \mathbb{E}_S[F \exp(-r(W + \beta\pi_t(S, F, e, h) - \frac{e^2}{2}))]
\]
exp(-rCE(·)) = \exp\left(\frac{1}{2} r (e^2 - 2W - 2\beta (pQ - h\lambda + \mu - Q\mu + eQ^2\alpha\mu) + r (1 + (h - Q(1 - eQ\alpha))^2) \beta^2\sigma^2)\right)

\begin{align*}
-rCE(·) &= \frac{1}{2} r (e^2 - 2W - 2\beta (pQ - h\lambda + \mu - Q\mu + eQ^2\alpha\mu) + r (1 + (h - Q(1 - eQ\alpha))^2) \beta^2\sigma^2) \\
CE(·) &= -\frac{1}{2} e^2 + W + \beta (pQ - h\lambda + \mu - Q\mu + eQ^2\alpha\mu) - \frac{1}{2} r (1 + (h - Q(1 - eQ\alpha))^2) \beta^2\sigma^2
\end{align*}

For ease of exposition, we replace \(r\sigma^2\) by \(\rho\) in the following proofs.

6.3. Proofs of Lemmas and Propositions

Proof of Lemma 1 In Lemma 1, we show the comparative statics of the risk premium with respect to the procurement effort and the number of futures traded. I.e. for the fixed number of futures contract, how does the procurement effort impact the variance and vice versa.

\[ R(\beta, e, h) = \frac{1}{2} (h - Q(1 - \alpha eQ))^2 \rho\beta^2, \quad \partial R(\beta, e, h) = \alpha Q^2 (h - Q(1 - \alpha eQ)) \rho \beta^2, \quad \text{and} \quad \partial^2 R(\beta, e, h) = (h - Q(1 - \alpha eQ)) \rho \beta^2. \]

Hence, when \(h > Q(1 - \alpha eQ)\), \(\partial R(\beta, e, h) > 0\) and \(\partial^2 R(\beta, e, h) > 0\). Since \(\partial^2 R(\beta, e, h) < 0\), \(R(\beta, e, h)\) is minimized at \(e\) that satisfies \(h - Q(1 - \alpha eQ)\) for a fixed value of \(h\); and since \(\partial^2 R(\beta, e, h) > 0\), \(R(\beta, e, h)\) is minimized at \(e\) that satisfies \(h - Q(1 - \alpha eQ)\) for a fixed value of \(e\).

Finally, since \(\partial^2 R(\beta, e, h) = \alpha Q^2 \rho \beta^2 > 0\), we conclude that the procurement effort and the number of futures contracts are complements.

Proof of Lemma 2 In Lemma 2, we derive \(e^*(\beta, h)\) and state the conditions that make the maximum discount constraint \(e^*(\beta, h) = \frac{1}{\alpha Q}\) binding. We perform the proof in 3 steps. In Step 1, we show that \(CE'\) is concave and find \(e^*(\beta, h)\) using the first order condition. In Step 2, we derive an interval for \(\mu\), in which the maximum discount constraint is never binding. In Step 3, we find the range of \(\beta\)'s that make the constraint binding.

Step 1. \(\frac{d^2 CE'(e)}{de^2} = -\rho \alpha^2 \beta^2 Q^4 - 1 < 0\), therefore, \(CE'(e)\) is concave in \(e\) and we use the first order condition to find optimal \(e^*: \frac{dCE'(e)}{de} = -\rho \alpha (h + Q(eQ\alpha - 1)) \beta^2 Q^2 + \alpha \beta \mu Q^2 - e = 0 \Rightarrow e^* = \frac{Q^2 \beta (h + Q(eQ\alpha - 1)) \beta^2 Q^2 + \alpha \beta \mu Q^2}{\rho \alpha^2 \beta^2 Q^4 + 1}.\)

Step 2 Next, we solve \(e^* = \frac{1}{\alpha Q}\) for \(\beta\) to find what \(\beta\) makes the constraint binding. \(e^* = \frac{1}{\alpha Q}\) is quadratic in \(\beta\) with the discriminant equal to \(Q^3 \alpha^2 \mu^2 - 4h \rho\). The discriminant is negative when \(-\frac{\sqrt{4\rho}}{Q^3 \alpha^2} < \mu < \frac{\sqrt{4\rho}}{Q^3 \alpha^2}\). Since \(\mu > 0\) we only need to consider the upper bound \(\Rightarrow \mu < \frac{\sqrt{4\rho}}{Q^3 \alpha^2}\).
Step 3. Finally, we solve $e^* = \frac{1}{\alpha Q}$ for $\beta$ to find the region for $\beta$ that makes the maximum discount constraint binding: $\overline{\beta} = \frac{\alpha \mu Q^3 + \sqrt{\rho \alpha^2 \mu^2 - 4hQ^3 \rho}}{2hQ^3 \rho} \omega$ and $\underline{\beta} = \frac{\alpha \mu Q^3 - \sqrt{\rho \alpha^2 \mu^2 - 4hQ^3 \rho}}{2hQ^3 \rho}$. It is easy to see that $\overline{\beta} > 0$ and $\underline{\beta} > 0$ and that $\beta \in [\underline{\beta}, \overline{\beta}]$ make the constraint binding. □

Proof of Proposition 1 We show that $e^*(\beta, h)$ decreases in $h$ by calculating the comparative static of $e^*(\beta, h)$ with respect to $h$ for a fixed $\beta$.

$$\frac{\partial e^*(\beta,h)}{\partial h} = -\frac{Q^2 \rho \alpha^2 \beta^2}{\rho \alpha^2 \beta^2 Q^4 + 1} < 0 \quad \square$$

Proof of Lemma 3 We perform the proof in X steps. In Step 1, we show that $\frac{d\pi(\beta)}{d\beta}$ has at most one positive root. In Step 2, we show that $\left.\frac{d\pi(\beta)}{d\beta}\right|_{\beta=0} > 0$ and $\left.\frac{d\pi(\beta)}{d\beta}\right|_{\beta=1} \leq 0$, hence, there exists exactly one root in $(0,1]$, in Step 3, we show that $\lim_{\beta \to -\infty} \pi(\beta) = \lim_{\beta \to \infty} \pi(\beta)$ and hence, the root in $(0,1]$ is the unique maximizer of $\pi(\beta)$, and finally in Step 4, we write $\beta^*$.

Step 1. The numerator of $\frac{d\pi(\beta)}{d\beta}$ is quadratic in $\beta$ and can be written as $A\beta^2 + B\beta + C$, where

$$J = \alpha^2 Q^4 \mu > 0$$

$$A = -\rho J^2 < 0,$$

$$B = -2\rho(h - Q) - \rho(h - Q)^2 - J\mu,$$

$$C = J\mu > 0.$$ Since $A < 0$ and $C > 0$, regardless of the sign of $B$, the polynomial has one change in sign; hence, according to the Descartes Rule of Signs, the polynomial has at most one positive root ($\beta^*$).

Step 2.

$$\left.\frac{d\pi(\beta)}{d\beta}\right|_{\beta=0} = C > 0$$

$$\left.\frac{d\pi(\beta)}{d\beta}\right|_{\beta=1} = -\rho \frac{(h + J - Q)^2}{\left(\frac{\rho \mu}{2} + 1\right)^2} \leq 0$$

Hence, the positive root $\beta^*$ is a local maximum.

Step 3.

$$\lim_{\beta \to -\infty} \pi(\beta) = -\frac{(h - Q)^2}{2\alpha^2 Q^4} + pQ - h\mu - \frac{\mu^2}{2\tau} - \lambda|h| = \lim_{\beta \to -\infty} \pi(\beta)$$

Therefore, $\pi(\beta < 0) < \pi(\beta^*)$ and $\beta^*$ is the unique maximizer of $\pi(\beta)$.
Step 4.

$$\beta^* = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

Proof of Proposition 2 We perform the proof in 3 steps. In step 1, we show that the sign of $$\frac{\partial \beta^*}{\partial \mu}$$ is equal to the sign of $$B \frac{\partial A}{\partial \mu} - A \frac{\partial B}{\partial \mu}$$, where A and B are as defined in the proof of Lemma 3. In step 2, we find $$\bar{\mu}$$ and show that $$\frac{\partial \beta^*}{\partial \mu} < (>) 0$$ when $$\mu > (<) \bar{\mu}$$. In step 3, we plug in $$\bar{\mu}$$ for $$\mu$$ and find that $$\beta^*(\bar{\mu}) = 1$$.

To make the expressions below more readable, we will use $$A', B', C'$$ to denote $$\frac{\partial A}{\partial \mu}, \frac{\partial B}{\partial \mu}, \frac{\partial C}{\partial \mu}$$.

Step 1.

$$\frac{\partial \beta^*}{\partial \mu} = \frac{A \left( \frac{2CA' + 2AC' - BB'}{\sqrt{B^2 - 4AC}} - B' \right) + A' \left( \sqrt{B^2 - 4AC} + B \right)}{2A^2}$$

$$= \frac{A(2CA' - B\sqrt{B^2 - 4AC} + 2AC') + B(A'\sqrt{B^2 - 4AC} - AB') + B^2A'}{2A^2\sqrt{B^2 - 4AC}}$$

Notice that $$A' = AC'$$, hence, we can simplify the above as:

$$\frac{\partial \beta^*}{\partial \mu} = \frac{-B' \sqrt{B^2 - 4AC} + B(A' \sqrt{B^2 - 4AC} - AB') + B^2A'}{2A^2\sqrt{B^2 - 4AC}}$$

$$= \frac{(\sqrt{B^2 - 4AC} + B)(BA' - AB')}{2A^2\sqrt{B^2 - 4AC}}$$

The denominator is positive; since $$A < 0$$, $$\sqrt{B^2 - 4AC} > |B|$$, and even when $$B < 0$$, the term $$\sqrt{B^2 - 4AC} + B > 0$$. Hence, Sign($$\frac{\partial \beta^*}{\partial \mu}$$) = Sign($$BA' - AB'$$).

Step 2. Substituting for A and B, we obtain:

$$BA' - AB' = 2Q^2 \rho^2 \alpha^4 \mu(h - Q) (h + Q^4 \alpha^2 \mu - Q).$$

Hence, since $$h < Q$$, $$\frac{\partial \beta^*}{\partial \mu} < (>) 0$$ when $$\mu > (<) \frac{Q - h}{Q^4 \alpha^2}$$.

Step 3. Since $$\frac{\partial \beta^*}{\partial \mu}_{\mu = \bar{\mu}} = 0$$, $$\beta^*$$ attains its highest value at $$\mu = \bar{\mu}$$ and $$\beta^*|_{\mu = \bar{\mu}} = 1$$.

Proof of Proposition 3 In Proposition 3, we show that the optimal number of futures contracts $$h$$ is always less than $$Q$$ and greater than 0. In Step 1, we form the Lagrangian, set $$\lambda$$ equal to zero, and solve for the optimal hedging quantity and bonus rate. In Step 2, we show that $$0 \leq h^* \leq h_*$$.
Step 1. Find optimal hedging quantity and bonus rate when $\lambda = 0$. We form the Lagrangian by adding the restriction on procurement effort to the objective function: $L(\beta, h, \theta) = \pi(e^*(\beta, h), \beta, h) - \theta(e^*(\beta, h) - \frac{1}{\alpha Q})$. We need to consider two cases: $\theta = 0$ and $\theta > 0$.

Case 1. $\theta = 0$.

$$\frac{\partial L(\beta, h, \theta)}{\partial h} = -\frac{\rho \beta^2 (h + Q^4 \alpha^2 \mu - Q)}{Q^4 \rho \alpha^2 \beta^2 + 1}$$

$$\frac{\partial^2 L(\beta, h, \theta|\lambda = 0)}{\partial h^2} = -\frac{\rho \beta^2}{Q^4 \rho \alpha^2 \beta^2 + 1} < 0$$

Notice that if we treat the question as a single-variable problem and use the first order condition to solve for $h$, $h^*_o = Q - Q^4 \alpha^2 \mu$ is independent of $\beta$. Hence, this is the optimal solution for the hedging quantity and the solution for $\beta$ is the same as in Section 4.2. We plug in $h^*_o$ into $\beta^*$ and obtain $\beta^*_o = 1$. Now, we need to make sure that $e^*(h^*_o, \beta^*_o) < \frac{1}{\alpha Q}$. Hence, this solution is feasible only if $1 > Q^3 \alpha^2 \mu$.

Case 2. $\theta > 0$. Now, solving the FOC results in the following solution: $\theta = Q^2 \alpha \mu - \frac{1}{\alpha Q^2 \mu}$, and $h^*_o = 0$. Since $\theta > 0$, this solution is feasible only if $1 < Q^3 \alpha^2 \mu$.

In either case, $h^*_o < Q$.

Step 2. Show that $0 \leq h^* \leq h^*_o$ when $\lambda > 0$. We break this step into analyzing two cases: $1 > Q^3 \alpha^2 \mu$ and $1 < Q^3 \alpha^2 \mu$.

Case 1: $1 > Q^3 \alpha^2 \mu$. First, notice that $\pi(\beta, h|\lambda > 0) = \pi(\beta, h|\lambda = 0) - \lambda |h|$. Hence, it cannot be optimal to increase $h$ above $h^*_o$, however, it may be optimal to decrease $h$, hence, $h^* \leq h^*_o$. We need to show now that it cannot be optimal to set $h$ below $0$. Let $V(h) = \max_{\beta} \pi(\beta, h, e^*(\beta, h))$. By the envelope theorem, $\frac{dV(h)}{dh} = \frac{\partial L(\beta^*(h), h, e^*(\beta^*(h), h))}{\partial h} = -\frac{\rho \beta^*(h)^2 (h - Q(1 - Q^3 \alpha^2 \mu) - \theta \alpha Q^2)}{Q^4 \rho \alpha^2 \beta^*(h)^2 + 1} + \lambda h$ when $h < 0$. It is easy to see that for all $h < h^*_o$, $\frac{dV(h)}{dh} > 0$. Hence, $h^* \geq 0$.

Case 2: $1 < Q^3 \alpha^2 \mu$. In this case, the optimal solution when $\lambda = 0$ is equal to $h^*_o = 0$ (from Step 1). Since, $\pi(\beta, h|\lambda > 0) = \pi(\beta, h|\lambda = 0) - \lambda |h|$, it is easy to see that any $h \neq 0$ will only decrease the objective function value. Hence, $h^* = h^*_o = 0$. □

Proof of Proposition 4 Given the result of Proposition 3, we will only focus on $h \geq 0$ for this proof. Let $V(h) = \max_{\beta} L(\beta, h, \theta)$, where $L(\beta, h, \theta)$ is the Lagrangian defined in Proposition 3. Then,
by the Envelope Theorem, \( \frac{dV(h)}{dh} = \frac{\partial L(\beta^*(h), h, \theta)}{\partial h} = -\frac{\rho \beta^*(h)^2 (h + Q^4 \alpha^2 \mu - Q - \theta \alpha Q^2)}{Q^2 \rho \alpha^2 \beta^*(h)^2 + 1} - \lambda \). Again, we will look at two cases: \( \theta = 0 \) and \( \theta > 0 \).

**Case 1: \( \theta = 0 \).** Notice that the numerator of \( \frac{dV(h)}{dh} \) is a 4th degree polynomial in \( Q \) of the following form: \(-XQ^4 + YQ - Z\), where \( X, Y, \) and \( Z \) are greater than zero and are equal to:

\[
Y = \rho \beta^2, \\
X = \alpha^2 Y (\mu + \lambda), \\
Z = hY + \lambda.
\]

By the Descartes Rule of Signs, this polynomial has either 0 or 2 positive real roots. When \(-X(-\frac{Y}{4Z})^\frac{1}{2} + Y(-\frac{Y}{4Z})^\frac{1}{2} < Z\), the derivative has no positive roots and is always negative, hence \( h^* = 0 \). When \(-X(-\frac{Y}{4Z})^\frac{1}{2} + Y(-\frac{Y}{4Z})^\frac{1}{2} < Z\), \( \frac{dV(\beta,h)}{dh} \) has 2 positive roots \( Q_1 \) and \( Q_2 \) such that \( \frac{dV(\beta,h)}{dh} \) may be greater than 0 for \( Q \in [Q_1, Q_2] \). Hence, when \( Q \in [Q_1, Q_2] \), there exist \( h > 0 \) that makes \( \frac{dV(\beta,h)}{dh} = 0 \).

Since \( \theta = 0 \), the results is feasible only when \( e^* < \frac{1}{\alpha Q^2} \), which adds an additional restriction on \( Q \):

\( Q < \sqrt[3]{\frac{1}{\alpha^2 \mu}} \). The numerator of \( \frac{dV(\beta,h)}{dh} \) evaluated at \( Q = \sqrt[3]{\frac{1}{\alpha^2 \mu}} \) equals \( -\frac{r \beta^2 \lambda^3 \sqrt[3]{\frac{1}{\alpha^2 \mu}}}{\mu} - hr \beta^2 - \lambda \), which is less than zero. Hence, \( Q < \sqrt[3]{\frac{1}{\alpha^2 \mu}} \).

**Case 2. \( \theta > 0 \).** \( e^*(\beta,h) = \frac{1}{\alpha Q^2} \), hence, we can express \( \beta \) as \( \frac{\alpha \mu - \sqrt{Q^3 \alpha^2 \mu^2 - 4h \rho}}{2h \rho a} \) and plug it into \( L(\beta,h,e^*(\beta,h)) \).

\[
\frac{dL(\beta,h,e^*(\beta,h))}{dh} = \frac{1 - \frac{\alpha Q^3/2 \mu}{a^2 \mu^2 - 4h \rho}}{2a^2 Q^3} - \lambda < 0, \text{ hence, } h^* = 0 \text{ and } \beta^* = \frac{1}{\alpha^2 Q^3 \mu}. \text{ \( \theta > 0 \) implies that } Q > \sqrt[3]{\frac{1}{\alpha^2 \mu}} \).