Accounting for Past and Future Actions

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ACCOUNTING FOR PAST AND FUTURE ACTIONS*

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Abstract

Explicitly or not, an accounting measurement system must choose whether or not to exclude, from its scope consideration, any economic activities yet to occur. We provide a model where such a scope distinction between measurements has both accounting and economic meanings. In particular, we represent measurements limited to past actions with an assets-in-place (AIP) accounting measurement in contrast to a Full accounting measurement which represents measurements anticipating future actions. We then embed the accounting model into a firm’s accounting choice problem in which the firm rationally recognizes that its accounting choice may change its own investment efficiency as well as the risk premium in its share price. We analyze how the optimal choice between the two measurements depends on the investment environment (e.g., growth opportunities) as well as the inherent measurement characteristics (e.g., measurement noise). We show the optimal choice can be subtle if the firm’s investment is endogenous to the accounting regime itself. For example, Full accounting may be preferable even if the noise in Full accounting is high in some cases. Similarly, AIP accounting may become preferable even if the firm-growth may be sizeable. The underlying driving force is that the endogenous investment makes endogenous the total uncertainty of the firm’s cash flows as well as the resolution of the uncertainty due to the accounting report. This indirect effect of accounting measurement (i.e., the “real effect” via the investment channel) changes the trade-off the firm faces in choosing a preferred accounting measurement.

Key words: accounting measurement systems; measurement scope; accounting choice

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1 Introduction

This paper investigates the economic trade-off of a scope consideration in accounting measurements. Within the accounting structure, a critical scope issue is whether or not accounting measurement excludes, from its measurement consideration, economic activities yet to occur. Traditional measurement devices restrict attention to measuring current or future consequences of past actions. For example, the traditional accounting recognition of assets and liabilities emphasizes past transactions as the drivers for probable future benefits or sacrifices (see, for example, the definition of assets and liabilities in the FASB conceptual statements).\(^1\) Newer measurement devices, such as fair value measurement, implicitly expand their attention to consequences of both past and projected future actions. This scope expansion is particularly pertinent when fair value measurement includes current market prices that presumably reflect information about future cash flows stemming from past actions as well as from projected future actions.\(^2\) The distinction between these two measurement devices can be economically significant when firms face growth prospects (such as investing in assets which contain real options).

We provide a model where such a distinction between measurements has both accounting and economic meanings. In particular, we build two alternative accounting measurements that are designed to measure future consequences emitting from either only the assets in place (AIP) or both the AIP and a future growth opportunity. These alternative measurement choices generate informational difference in the resulting accounting measurements. We then embed the accounting model into a standard economic model in which the accounting measurement choices may change both distributional and allocational efficiency in the economy.

Specifically, we consider a risk-neutral firm making an initial investment (to establish AIP), which is followed by a subsequent growth investment. Both investments generate random cash flows in the future. In between the two investment dates, the firm must take an accounting measurement

\(^1\)In the recent joint FASB/IFRS conceptual framework project, the revised definition under consideration continues to emphasize the presence of economic resources and obligations even though explicit reference to past transactions has been removed.

\(^2\)One can argue that traditional accounting devices do, to some extent, consider future actions. At the initial recognition, the prices of assets and liabilities do anticipate the value of real options. In impairment tests, changes in the value of anticipated future actions would be considered in one form or another. However, the changes in the anticipated future actions are certainly reflected in a slower pace in the traditional accounting system. At the same time, certain fair value accounting treatments (e.g., the Level-3 fair value treatment) may provide only a partial consideration of anticipated future actions. So the distinction may be a matter of degree.
of the future cash flows and report the measured accounting value to risk-averse equity investors who determine competitively the share price of the firm. Economically, the accounting report resolves some uncertainty about the future cash flows and thus increases the share price by reducing the risk premium collectively required by investors (defined as the difference between the expected future cash flows and the price). As a result, the informativeness of the accounting measures affects the risk premium in the firm’s share price. The key feature of the model is the fundamental difference in information properties induced by the two accounting measurement structures: the AIP accounting measure provides information only about the future cash flow from the AIP investment while the Full accounting measure provides information about the future cash flows from both the AIP and future growth investments. Investors understand such a structural difference and make rational inference based on the reported accounting value in pricing the firm. From the firm’s point of view, the two accounting measures affect the risk premium as well as the efficiency of the initial investment differently, thus leading to different economic payoffs to the firm. These modeling choices establish a non-trivial accounting choice problem faced by the firm.

The main results of the model are presented in two steps. First, if the initial investment is set exogenously, the firm’s accounting choice problem boils down to a comparison of the risk premiums in the share price under the two accounting regimes. Both accounting measures are informative of and would resolve some uncertainty about the future cash flows, thus having a direct effect on the risk premium. This direct effect comes from the role of the accounting measures in only reflecting (but not affecting) cash flows generated by the firm’s investment activities. We show that the accounting choice depends on the size of the firm’s growth opportunity. In particular, we show that firms with sufficiently high growth prospects have a preference for Full accounting (for any given accounting noises under the two accounting regimes). This is because the Full accounting measure adds informativeness about the cash flow from the growth investment, reducing more risk premium demanded by investors. When the growth opportunity is important enough, the risk premium under the Full accounting regime is smaller as long as its signal precision is not too low.

When the initial investment is endogenous, the accounting choice is less straightforward because the investment decision is now partially motivated by the interim share price, making it endogenous to the informational property of the share price (or, indirectly, the accounting measure). Unlike the

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3See more discussions and real-world examples about the two accounting measurements in Section 2.2.
exogenous investment setting, the accounting measurements are now evaluated based on both the investment-efficiency and risk-premium concerns. First, we show Full accounting has an advantage in improving the investment efficiency because the Full accounting measure is more value-relevant and provides more aligned investment incentives. Regarding the risk premium, the endogenous initial investment opens another channel through which the accounting measurement affects the risk premium. This is because the endogenous investment makes endogenous the total (ex ante) uncertainty of the firm’s cash flows as well as its resolution due to the accounting report. This indirect effect of the accounting measurement changes the trade-off the firm faces regarding the risk premium. Unlike the exogenous investment setting, we show it is possible that the risk premium is always lower under Full accounting regardless of its measurement noise (i.e., even for extremely large noise) than under AIP accounting. Based on these results, we show it is possible that Full accounting is preferable for any measurement noise it may have as long as the growth opportunity is high enough. In fact, this counter-intuitive result is strong in the sense that the preferred Full accounting measure leads to both higher investment efficiency and lower (share-price) risk premium. In essence, we show that, in evaluating accounting measurements, investment efficiency and risk premium may not necessarily be a trade-off. In addition, the same indirect effect may cause AIP accounting to be preferable for a growth firm even if AIP accounting is as noisy as Full accounting.

Our paper contributes to the disclosure literature by emphasizing the importance of accounting measurement structure. Like earlier work, committing to better accounting disclosure in our model improves the welfare of existing firm owners by raising the share price in the secondary market. However, past disclosure work mostly abstracts away from the structural difference and focuses on higher or lower quality of accounting disclosure (e.g., more or less noise in accounting disclosures). Our paper makes a first attempt to model explicitly the scope dimension of accounting measurements (i.e., inclusion or exclusion of the information about cash flows from future actions) in addition to the noise dimension, thus extending the disclosure literature to capture critical features of the accounting reporting system. Given the foundational role of transactions in accounting, modeling the distinction between past and future actions for the purpose of accounting measurements enhances the disclosure literature.

4For example, although higher accounting noise may resolve a smaller portion of a given total cash-flow uncertainty (i.e., leading to a higher risk premium), it reduces the initial investment and thus reduces the total (ex ante) cash-flow uncertainty (i.e., leading to a lower risk premium).
The results in our paper have implications for policy discussion about the design of the accounting measurement system. This is relevant to the ongoing FASB/IFRS effort to revamp the conceptual framework, which now focuses on the measurement and elements phase. One implication of our results is that the accounting measurement which leads to the highest investment efficiency and/or the lowest risk premium (or cost of capital) depends on firm characteristics (e.g., growth prospects) as well as environmental factors (e.g., measurement noise). This implication may justify the existing mixed-attributes model in GAAP as well as the use of the fair value option under SFAS 159. Another implication of our results is that, in considering accounting measurements, the investment efficiency and risk premium may not be necessarily a trade-off. In our model, Full accounting both improves the investment efficiency and reduces the risk premium in some cases.

The results in our paper also have implications for empirical accounting work on accounting quality and cost of capital. The comparative statics results of our model generate empirical implications relating endogenous cost of capital or investment intensity (to information) to the choice of accounting measures as well as to factors such as growth opportunity. For example, our paper makes the point that the nature and direction of the relation between cost of capital and accounting information quality depends on factors such as firms' growth opportunities, the degree of the market-based accounting measurement use, and the managerial myopia in investment decisions.

In our paper, the presence of a future action (i.e., the growth investment) presents a challenge to measuring an entity's activities, because accounting must deal with the scope choice (i.e., inclusion or exclusion of the growth investment) in addition to other dimensions (e.g., measurement precision). Our paper's central accounting concern follows a broad theme in modeling work on accounting measurement structure.\footnote{There was an older literature on accounting properties such as its axiomatic structure (Mattessich 1964), algebraic representation (Butterworth 1972), objectivity and reliability (Ijiri 1975), and relevance and timeliness (Feltham 1972). This area remains less-explored; Professor Ron Dye attributed the lack of progress partially to Demski's (1973) "General Impossibility" observation "that Blackwell's (1951) theorem, as applied to accounting, indicates that in evaluating two non-comparable accounting information systems, one can always find a pair of decisions problems in which one information system is preferred for one decision problem and the other is preferred for the second decision problem" (page 52 in Dye 2002).}

In the recent strands of this theme closely related to our paper, Dye (2002) views classification as a foundational accounting measurement function and its possible manipulation has implications in equilibrium accounting standards, which he terms, tellingly, "Nash" standards. Dye and Sridhar (2004a) focus on accounting aggregation and the resulting trade-off between relevance and reliability. Along a similar line, Liang and Wen (2007) focus on

Our paper is also related to the literature on the real effect of accounting. The real effect literature, pioneered by Kanodia (1980), develops the notion that disclosure of accounting information has an impact not only upon market prices but also upon corporate production/investment decisions (e.g., Kanodia and Lee 1998, Beyer and Guttman 2011). Our paper follows this line of research and presents both direct and indirect (i.e., real investment) effects of accounting structure on the risk premium in share prices.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 presents the main analyses and results of the model, and Section 4 provides key discussions on the model assumptions, relation to cost-of-capital studies, and policy implications. Section 5 concludes the paper.

2 Model

A risk-neutral entrepreneur owns a technology, referred to as the initial project, which requires an initial investment. Before making the initial investment, on date-1⁻, the entrepreneur chooses between two accounting measurement systems, assets-in-place (AIP) accounting and Full account-
ing, for the firm she establishes. On date-1, the entrepreneur observes a private signal, denoted by \( \theta \in \mathbb{R} \), about the initial project’s profitability and makes a private investment \( I_1(\theta) \) (to establish AIP). The chosen accounting system of the firm generates a public accounting signal \( y \) on date-2\(^-\). On date-2, the entrepreneur sells \( \beta (> 0) \) portion of her ownership in the firm to outside investors (in a secondary market) due to exogenous reasons, and the market price \( P \) is determined based on all publicly available information. On a later date labeled as date-2\(^+\), the firm invests \( I_2 \) into another project, referred to as the future growth project. Both the initial project and the growth project will generate cash flows on date-3. We denote the total cash flows on date-3 as \( x \). Figure 1 summarizes the sequence of events.

\[
\begin{array}{ccccccc}
t & = & 1^- & \quad & t & = & 1 & \quad & t & = & 2^- & \quad & t & = & 2 & \quad & t & = & 2^+ & \quad & t & = & 3 \\
\text{Entrepreneur} & \quad & \text{Entrepreneur} & \quad & \text{Accounting} & \quad & \text{Entrepreneur} & \quad & \text{Entrepreneur} & \quad & \text{Firm makes} & \quad & x \text{ is realized.} \\
\text{selects the} & \quad & \text{privately} & \quad & \text{report} & y & \text{is} & \quad & \text{sells} & \beta & \text{portion} & \quad & \text{the growth} & \quad & \text{The firm is} & \quad & \text{liquidated.} \\
\text{accounting} & \quad & \text{observes} & \theta & \text{and} & \text{announced.} & \quad & \text{of her shares.} & \quad & \text{investment} & I_2 & \quad & \text{investment} & I_2 & \quad & P \text{ is determined.} \\
\text{regime.} & \quad & \text{invests} & I_1(\theta). & \quad & \text{P} & \text{is determined.} & \quad & \text{is realized.} & \quad & \text{is realized.} & \quad & \text{is realized.} & \quad & \text{is realized.} \\
\end{array}
\]

Figure 1. The time line of events

We next provide more details on the model.

### 2.1 Cash Flows

The prior distribution of the entrepreneur’s private signal, \( \theta \), is normal with mean \( \theta_0 \) and variance \( V_\theta \) (i.e., \( \theta \sim N(\theta_0, V_\theta) \)). Based on \( \theta \), the entrepreneur chooses an investment \( I_1(\theta) \) for the initial project. Following the literature (e.g., Dye 2002; Dye and Sridhar 2004a), we assume the initial investment \( I_1(\theta) \) is made by the entrepreneur privately and is not observable to outside investors. The initial project generates a cash flow \( z(\theta, I_1) \) on date-3. For tractability, we assume its terminal
cash flow is
\[ z(\theta, I_1) = 2\sqrt{\theta I_1} \ (\theta \geq 0). ^{6,7} \]

In addition to the initial investment opportunity, the firm faces another investment opportunity (i.e., the growth project) on date-2+ . Likewise, the firm will observe a private profitability signal, denoted by \( \gamma \in \mathbb{R} \), about the growth project before making the growth investment \( I_2(\gamma) \). The prior distribution of \( \gamma \) is normal with mean \( \gamma_0 \) and variance \( V_\gamma \) (i.e., \( \gamma \sim N[\gamma_0, V_\gamma] \)). For tractability, we assume \( \gamma \) is independent of all other variables including \( \theta \). \(^8\) The net cash return to the growth project is
\[ \pi(\gamma, I_2) = 2\sqrt{g\gamma I_2} - I_2 \ (\gamma \geq 0), \]
realized on date-3, where \( g \) is a commonly known constant reflecting the growth potential of the growth project. \(^9\) That is, the larger the \( g \), the more profitable the growth project is (relative to the initial project). Following Liang and Wen (2007), we assume that the firm will choose the optimal investment for the growth project (i.e., \( I_2^* = g\gamma \)) for simplicity. \(^10\) As a result, the net cash return of the growth project is
\[ \pi(\gamma, I_2^*) = g\gamma. \]

Higher \( g \) implies higher expected return (\( E[\pi(\gamma, I_2^*)] = g\gamma_0 \)) and higher cash-flow volatility from the growth project (\( Var[\pi(\gamma, I_2^*)] = g^2V_\gamma \)). Including the cash flow from the initial project, the firm’s total cash flows on date-3 are
\[ x \equiv z(\theta, I_1) + \pi(\gamma, I_2^*) = 2\sqrt{\theta I_1} + g\gamma. \] \( \tag{1} \)

Notice the growth potential \( g \) also measures the significance of the growth project relative to the initial project. The higher the \( g \), the more important the growth project is to the firm.

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\(^6\) When \( \theta < 0 \), the project is forgone, i.e., \( I_1 = 0 \) and \( z = 0 \).

\(^7\) All of our qualitative results throughout the study still hold if we assume the terminal cash flow is \( z(\theta, I_1) = 2\sqrt{\theta I_1} + \varepsilon_z \), where \( \varepsilon_z \) is a white noise independent of any other variables.

\(^8\) This assumes the profitabilities of the two projects are uncorrelated. However, all qualitative results in the paper remain the same as long as \( \gamma \) is not sufficiently correlated with \( \theta \).

\(^9\) Similarly, when \( \gamma < 0 \), the growth project is forgone, i.e., \( I_2 = 0 \) and \( \pi = 0 \).

\(^10\) Even if the firm’s future investment is not in the optimal level, all qualitative results in the paper remain the same as long as the future investment is exogenously given (i.e., \( I_2 = u_2g\gamma \) where \( u_2 \) is a constant), or endogenously given and independent with the initial investment strategy (i.e., \( I_2 = u_2g\gamma \) where \( u_2 \) is independent with \( I_1 \)).
2.2 Two Accounting Regimes

Before the entrepreneur observes $\theta$ and invests in the initial project, she needs to select between two accounting measurement systems: *assets-in-place (AIP) accounting* and *Full accounting*. The selected accounting system will generate a public accounting signal on date-$2^-$. 

2.2.1 Assets-in-place (AIP) Accounting

If the entrepreneur selects AIP accounting, the accounting signal is a noisy measure of the initial project’s partial cash flow, and generally, we assume the AIP signal is,

$$Y_p = k \sqrt{\theta I_1} + \varepsilon_p.$$ 

Here, the accounting measurement noise $\varepsilon_p \sim N[0, V_p]$ is independent of any other variables. The parameter $k \in [0, 2]$ is a commonly known constant. It reflects certain inter-period revenue/expense-allocation accounting rules in order to compute the date-$2^-$ recognized earnings attributed to the initial project. Given our focus is not on the allocation issues within AIP accounting, we set $k = 2$ for ease of exposition and denote the AIP accounting signal as $y^p$, i.e.,

$$y^p = Y^p|_{k=2} = 2 \sqrt{\theta I_1} + \varepsilon_p. \text{11}$$

Notice that AIP accounting does not reflect the cash flow from the future growth project (i.e., $\pi$), but only provides information about the cash flow from the initial project (i.e., $E[y^p] = z$). This accounting model is consistent with the traditional accounting principle that focuses on the assets in place and ignores any cash flows from future yet-to-occur firm investments.

2.2.2 Full Accounting

Alternatively, if the entrepreneur selects Full accounting, the accounting signal, denoted by $y^f$, is a noisy measure of the total cash flows from both the initial and growth projects, that is

$$y^f = x + \varepsilon_f = 2 \sqrt{\theta I_1} + g \gamma + \varepsilon_f,$$

\text{11} Retaining $k$ (instead of setting $k = 2$) will not change any qualitative results of the paper.
where the accounting measurement noise \( \varepsilon_f \sim N[0, V_f] \) is independent of any other variables. The Full accounting signal provides "comprehensive" information about the cash flows from the future growth project as well as the initial project. However, due to the aggregation, the Full accounting signal does not provide the same precise information about the initial project as the AIP accounting signal does (i.e., \( y_f \) is a more noisy signal of cash flow \( z \) even if \( V_f = V_p \)). This modeling choice is consistent with the market-based accounting, such as fair-value measurements, which considers not only the current and future effects from the assets in place but also the effect from future anticipated but yet-to-be-recognized assets and liabilities.

We view the central scope difference between the two accounting measurement systems as fundamental and envision that it operates at a conceptual level. As such, we emphasize that the model here is one abstract attempt at capturing a key conceptual dimension of accounting measurement design among many important dimensions which practitioners and policy-makers consider. However, in the following, we provide a real-world accounting example to illustrate the two accounting systems in the model. We view the equity method for investments in unconsolidated subsidiaries as a form of AIP accounting while the fair-value-option method (from SFAS 159) for the same investments as a form of Full accounting.\(^{12}\) Generally, Full accounting represents any accounting measurement devices where the market/fair values of specific assets and liabilities or the firm’s own net assets as a whole enter into the accounting measurement of the same firm. This broad category includes examples such as market-based accounting for stock-options and mark-to-market of firms’ own debts (which also correspond to firms’ own equity value).

### 2.3 Entrepreneur’s Objective Function and Interim Share Price

Following prior literature (e.g., Stein 1989; Liang and Wen 2007; Einhorn and Ziv 2007), we assume the entrepreneur is interested in both the firm’s current market price and the future cash flows. In particular, we assume, on date-2, after the accounting report (AIP or Full) is released, the entrepreneur must sell \( \beta \) portion of her shares in the firm (i.e., \( \beta \) portion of the claims on the total future cash flows \( x \)) to outside investors in the secondary market due to exogenous reasons (e.g.,

\(^{12}\)One may argue that the historical price the firm paid for the subsidiaries already contains the expected value of the growth opportunities of the same subsidiaries (not just their assets-in-place). In this case, fair-value-option method would reflect changes in the growth opportunities while the equity method would not. Accordingly, in our model, Full accounting captures the change in the value of the growth opportunities of the subsidiaries as opposed to the total growth-opportunity value.
liquidity) and keep the remaining \((1 - \beta)\) ownership. Accordingly, the entrepreneur’s objective is to maximize a weighted average of the date-2 market price and date-3 total cash flows (net of the initial private investment cost \(I_1\)) conditional on her private signal \(\theta\).\(^{13}\) That is, for a type-\(\theta\) entrepreneur, the objective function on date-1 is

\[
-I_1(\theta) + \beta P + (1 - \beta)x.
\] (2)

Here, \(\beta\) measures the extent to which the entrepreneur’s initial investment is share-price motivated. A higher \(\beta\) indicates a higher share-price motivated incentive in the entrepreneur’s endogenous investment decision.\(^{14}\)

The firm shares are priced in a rational capital market. Investors in the capital market are risk-averse and have CARA utility functions with risk-averse coefficient \(\tau\). That is, the utility function of a typical investor \(i\) is

\[
U(W_i) = -\exp(-\tau W_i),
\]

where \(W_i\) denotes the investor’s wealth or consumption. Given the CARA utility function, following standard results in the literature, the market price \(P\) is equal to the mean of the future cash flows minus a risk premium that is determined by the investors’ perceived cash-flow volatility. We can express the market price in the following mean-variance form:

\[
P = E[x|\Omega] - \beta \tau \text{Var}[x|\Omega],
\] (3)

where \(\Omega\) is the publicly available information set to investors on date-2.\(^{15}\) The first term in (3) represents the market’s expected total future cash flows conditional on all available information, and the second term is the risk premium, which depends on the conditional variance (i.e., the

\(^{13}\)Here we model the stock-price incentive as coming from the liquidity needs of the current owners for simplicity. Another potential stock-price incentive in the literature may come from the presence of managerial stock-based compensation, which is mute in our model.

\(^{14}\)If the weight on the share price in the entrepreneur’s objective function is not the same as the portion of the firm sold (i.e., \(\beta\)) due to any other additional stock-price incentives, all the qualitative results of the paper still hold.

\(^{15}\)Consider a perfectly competitive market. The wealth of a typical investor \(i\) is \(W_i = (x - P)D_i\), where \(D_i\) is the investor’s demand of the firm’s shares given price \(P\). With the CARA utility function, the investor maximizes \(E[W_i|\Omega] - \frac{\tau}{2} \text{Var}[W_i|\Omega] = (E[x|\Omega] - P)D_i - \frac{\tau}{2} D_i^2 \text{Var}[x|\Omega]\). Taking the first order condition, we have \(D_i = \frac{E[x|\Omega] - P}{\tau \text{Var}[x|\Omega]}\). Since \(\beta\) portion of the firm’s shares is available for sale, the market clearing condition gives \(\beta = \int_0^1 D_i \text{di} = \frac{E[x|\Omega] - P}{\tau \text{Var}[x|\Omega]}\). Thus, we have the market price \(P = E[x|\Omega] - \beta \tau \text{Var}[x|\Omega]\).
unresolved cash-flow uncertainty), the risk-averse coefficient \((\tau)\), and the portion of the firm sold \((\beta)\). Combining (2) and (3), we have a setting with a risk-neutral entrepreneur and risk-averse pricing, similar to that of Beyer (2009). In Section 4, we provide some discussion on a setting where both the entrepreneur and investors are risk-averse.

3 Main Analysis

Since the entrepreneur is interested in both the market price and future cash flows, to determine her initial accounting choice, we need to consider both the share-price effect and cash-flow effect of the two accounting regimes. For a given accounting measure \(y \in \{y^p, y^f\}\), the entrepreneur’s ex ante payoff or welfare on date-1\(^-\), denoted by \(W\), can be expressed as

\[
W = \mathbb{E}[I_1(\theta) + \beta P + (1 - \beta)x]
\]

\[
= \mathbb{E}[I_1(\theta) + \beta(\mathbb{E}[x|y] - \beta \mathbb{V}ar[x|y]) + (1 - \beta)x]
\]

\[
= \mathbb{E}[x - I_1(\theta)] - \beta \mathbb{E}[\mathbb{V}ar[x|y]]. \quad (4)
\]

The first term in (4) is the expected total future cash flow net of the initial investment cost. It depends on the efficiency of the initial investment, which may further depend on the accounting measurement choice, especially when the investment is partially motivated by the interim share price. The second term in (4) measures the expected risk premium the entrepreneur needs to compensate outside investors, which depends on the conditional variance of the future cash flows (i.e., the unresolved cash-flow uncertainty). The entrepreneur’s accounting-choice decision depends on both the investment-efficiency and risk-premium concerns.

To facilitate our analysis, we first neutralize the investment-efficiency aspect of the problem by assuming the initial investment is exogenously given and focus on the conditional-variance comparison under the two accounting regimes. Then, we extend our analysis to consider the endogenous investment and examine how the different accounting measures affect the initial investment, the conditional variance, and the entrepreneur’s accounting choice.

Following prior literature, we employ the following Approximation Assumption for tractability
of the model.\footnote{Dye and Sridhar (2004b) and Stocken and Verrecchia (2004) use approximation assumptions in their analysis to calculating the unconditional mean of an altered normally distributed random variable. Liang and Wen (2007) adopt a similar approximation on the conditional mean calculation.}

**Approximation Assumption:** Let random variables $m$ and $n$ be independently normally distributed. Denote $f(m|am + n)$ the conditional density function for some constant $a \in \mathbb{R}$. We assume, for all realizations of $am + n$,

\[
\int_{0}^{+\infty} m f(m|am + n)dm \equiv \int_{-\infty}^{+\infty} m f(m|am + n)dm \quad \text{and} \quad \int_{0}^{+\infty} m^2 f(m|am + n)dm \equiv \int_{-\infty}^{+\infty} m^2 f(m|am + n)dm.
\]

Clearly, this approximation becomes increasingly accurate as the probability of $m$ less than zero reduces to zero.

### 3.1 Accounting Regime Choice with Exogenous Initial Investment

In this section, we consider a setting where the initial investment is non-strategic and exogenously given. For simplicity, we assume the entrepreneur follows a linear investment strategy given as

\[
I_1 \equiv u^2 \theta \ (\theta \geq 0),
\]

where the investment intensity $u$ is a known positive constant.\footnote{One practical purpose of this assumption is to have the same investment structure across the exogenous and endogenous settings. In the endogenous setting, the investment structure is endogenously determined (see Section 3.2 for details).}

We first consider AIP accounting (i.e., $y^p = 2\sqrt{\theta I_1} + \varepsilon_p$), where the accounting signal $y^p$ provides information about only the initial project and conveys no information about the future growth project. Substituting $I_1$ and $y^p$ into (4), the entrepreneur’s welfare under AIP accounting,
denoted by $W_p$, is as follows (with the Approximation Assumption),

\[
W_p = E[2u\theta - u^2\theta + g\gamma] - \beta^2\tau \cdot E[\text{Var}[x|y^p]]
\]

\[
= (2u - u^2)\theta_0 + g\gamma_0 - \beta^2\tau \cdot \text{Var}[x|y^p]
\]

\[
= (2u - u^2)\theta_0 + g\gamma_0 - \beta^2\tau \left[ \text{Var}(2u\theta) \left(1 - \rho_{z,y^p}^2\right) + g^2V_{\gamma} \right], \tag{5}
\]

where $\text{Var}(2u\theta) = 4u^2V_{\theta}$ and $\rho_{z,y^p} = \sqrt{\frac{1}{1 + \frac{V_f}{4u^2V_{\theta}}}}$.

The first two terms in (5) represent the expected total net cash flows. The last term in (5) represents the risk premium, which depends on the conditional variance comprised of the two terms in the brackets. The first term in the brackets (i.e., $\text{Var}(2u\theta) \left(1 - \rho_{z,y^p}^2\right)$) represents the investors’ perceived cash-flow volatility from the initial project conditional on the accounting signal $y^p$. In particular, $\text{Var}(2u\theta)$ is the unconditional cash-flow volatility from the initial project, and the correlation coefficient $\rho_{z,y^p}$ measures the extent to which the accounting signal $y^p$ resolves the cash-flow uncertainty from the initial project. The correlation decreases in the accounting noise (i.e., $V_p$), which indicates the conditional variance increases in the noise. The second term in the brackets (i.e., $g^2V_{\gamma}$) represents the unconditional cash-flow volatility from the growth project. Because the AIP accounting signal $y^p$ is not informative about the growth project, knowing $y^p$ does not resolve any uncertainty about the cash flow from the growth project.

Next we consider Full accounting (i.e., $y^f = 2\sqrt{\theta_1} + g\gamma + \varepsilon_f$), which is more comprehensive and aggregates the cash flows from both the initial and growth projects. Similarly, by substitution, the entrepreneur’s welfare under Full accounting, denoted by $W_f$, is as follows (with the Approximation Assumption)

\[
W_f = E[2u\theta - u^2\theta + g\gamma] - \beta^2\tau \cdot E[\text{Var}[x|y^f]]
\]

\[
= (2u - u^2)\theta_0 + g\gamma_0 - \beta^2\tau \cdot \text{Var}[x|y^f]
\]

\[
= (2u - u^2)\theta_0 + g\gamma_0 - \beta^2\tau \left[ \text{Var}(2u\theta) + g^2V_{\gamma} \right] \left(1 - \rho_{z,y^f}^2\right), \tag{6}
\]

where $\rho_{x,y^f} = \sqrt{\frac{1}{1 + \frac{V_f}{4u^2V_{\theta} + g^2V_{\gamma}}}}$.

Comparing (6) with (5), the expected total net cash flows are exactly the same under both
accounting regimes, while the conditional variances are different. Under Full accounting, the conditional variance is determined by: (i) the total unconditional cash-flow volatility from both the initial and growth projects (i.e., $\text{Var}(2u\theta + g^2V_\gamma)$), and (ii) the correlation between the accounting signal and the total cash flows (i.e., $\rho_{x,y}$). Unlike the AIP accounting signal, the Full accounting signal $y^f$ is correlated with both the initial and growth projects and helps resolve the cash-flow uncertainty from both projects. Similarly, higher accounting noise (i.e., larger $V_f$) leads to a lower correlation between the accounting signal and the total cash flows and, thus, a higher conditional variance.

Based on the above comparison, with exogenous investment, the entrepreneur’s accounting preference only depends on the conditional variance of the future cash flows (because the expected future net cash flows are the same under both accounting regimes). The structural difference between the two accounting regimes is the inclusion or exclusion of the future growth project in the accounting measurement. This structural difference leads to different conditional variances under the two accounting regimes. The following lemma presents some comparative statics results on the conditional variance.

Lemma 1 When the initial investment is exogenous (i.e., $I_1 = u^2\theta$),

1. the conditional variance under either accounting regime ($\text{Var}[x|y^f]$ or $\text{Var}[x|y^p]$) increases in the accounting noise ($V_f$ or $V_p$), the investment intensity $u$, and the growth potential $g$.

2. The conditional variance under Full accounting ($\text{Var}[x|y^f]$) increases in $g$ at a lower rate than that under AIP accounting ($\text{Var}[x|y^p]$).

Proof. All proofs are in the Appendix. ■

As discussed above, higher accounting noise directly reduces the correlation between the accounting signal and the future cash flow, and hence leads to a higher conditional variance. Besides the accounting noise, higher investment (i.e., higher $u$ or $g$) generates more (ex ante) uncertainty in future cash flows and leads to a higher conditional variance as well.$^{18}$

$^{18}$From (5) and (6), higher $u$ also increases the correlation between the accounting signal and the future cash flow, which may lead to a lower conditional variance. This decreasing effect is always dominated by the main increasing effect under either accounting regime. Similar arguments also apply to $g$ under Full accounting.
Further, given the structural difference, the growth potential $g$ affects the conditional variance in different ways under the two accounting regimes. Under AIP accounting, the accounting signal is independent of the growth project, and the entire cash-flow uncertainty from the growth project (i.e., $g^2V_\gamma$) adds to the conditional variance. On the other hand, the Full accounting signal is informative about the growth project (as well as the initial project) and resolves partial cash-flow uncertainty from the growth project. As a result, the growth potential $g$ increases the conditional variance at a lower rate under Full accounting than under AIP accounting.\(^{19}\)

### 3.1.1 Accounting measurement factor (i.e., $V_p$ or $V_f$)

In this subsection, we analyze how the accounting noise affects the entrepreneur’s accounting regime choice.

**Proposition 1** When the initial investment is exogenous (i.e., $I_1 = u^2\theta$), the necessary condition for the entrepreneur to choose AIP accounting is the AIP accounting measurement noise is lower (i.e., $V_p < V_f$).

When Full accounting contains less measurement noise, Proposition 1 confirms Full accounting is superior. This is generally intuitive because, putting the noise aside, Full accounting already has a natural advantage of including information from future activities and thus resolving more uncertainty (from the future activities) than AIP accounting. To understand this result in the model, first consider a setting where no future growth opportunity exists (i.e., $g = 0$). Under this situation, Full accounting is the same as AIP accounting (in terms of the structure). When their noises are equivalent (i.e., $V_f = V_p$), the entrepreneur is indifferent between the two accounting regimes. Figure 2 illustrates this result. When $g = 0$, the two conditional-variance curves of AIP and Full accounting (indicated by the two dashed curves) intersect at the middle where $V_f = V_p$.

---

\(^{19}\)More specifically, two forces are at work under Full accounting. On the one hand, the Full accounting signal conveys less information about the initial project (than the AIP accounting signal) due to the inclusion of the growth project, which may lead to a higher conditional variance. On the other hand, the Full accounting signal is informative about the growth project and helps resolve the cash flow uncertainty from the growth project, which may lead to a lower conditional variance. The decreasing effect always dominates the increasing effect, and more so when the growth project becomes more significant (i.e., larger $g$) since it becomes more important to resolve the uncertainty from the growth project.
Figure 2. The impact of the accounting noise on the conditional variance (exogenous setting)

Starting from this benchmark, now bring in a future growth project (i.e., $g > 0$). Lemma 1 shows that the presence of a growth project increases the conditional variance at a lower rate under Full accounting than under AIP accounting. Consistently, in Figure 2, the gap between the solid curve ($g > 0$) and the corresponding dashed curve ($g = 0$) is larger under AIP accounting than under Full accounting. Therefore, even when $V_f = V_p$, the conditional variance is lower under Full accounting. In order to lower the conditional variance under AIP accounting, the noise in the AIP accounting signal has to be lower. As Figure 2 shows, the two solid curves intersect at the right hand side of the figure, which indicates that, to have a lower conditional variance, the noise of AIP accounting must be lower than that of Full accounting (i.e., $V_p < V_f$).

### 3.1.2 Growth potential factor $g$

In this subsection, we assume the necessary condition described in Proposition 1 (i.e., $V_p < V_f$) is satisfied and investigate the impact of the growth potential $g$ on the entrepreneur’s accounting regime choice.

**Proposition 2** When the initial investment is exogenous (i.e., $I_1 = \theta^2$) and Full accounting contains more noise ($V_p < V_f$), there exists a cutoff point $g^* > 0$ in the growth potential such that

1. the entrepreneur prefers Full accounting (AIP accounting) if the growth project is more significant (less significant) (i.e., $g > g^*$ ($g < g^*$));

2. the lower (higher) the noise in the Full (AIP) accounting measure, the larger the parame-
ter space under which the entrepreneur prefers Full accounting (i.e., \( g^* \) increases in \( \bar{V}_f \) and decreases in \( \bar{V}_p \)).

Proposition 2 confirms the conventional intuition behind the fair value accounting that, when the future growth is important to a firm, including such information in accounting measures may lead to a lower risk premium. In our model, Proposition 2 is also driven by the second result in Lemma 1. When the growth project is not that important (i.e., \( g < g^* \)), the conditional variance is higher under Full accounting due to its higher measurement noise. To have a lower conditional variance under Full accounting, the growth project has to be significant enough (i.e., \( g > g^* \)) to compensate for the disadvantage of higher noise in the Full accounting signal. As Figure 3 shows, the conditional variance increases in the growth potential \( g \) under both accounting regimes, and it increases in \( g \) more steeply under AIP accounting than under Full accounting (i.e., Lemma 1). The two solid curves intersect at a cutoff point \( g^* \), indicating the entrepreneur prefers Full (AIP) accounting when \( g \) is higher (lower) than \( g^* \).

![Figure 3](image)

Figure 3. The impact of the growth potential \( g \) on the conditional variance (exogenous setting)

If we increase the noise level of AIP accounting (i.e., \( \bar{V}_p \)), the conditional variance goes up to the dotted curve and intersects with the solid curve of Full accounting at a smaller point \( g' \), which indicates the cutoff point \( g^* \) decreases in \( \bar{V}_p \) and Full accounting is preferable over a larger parameter region. Alternatively, if we increase the noise level of Full accounting (i.e., \( \bar{V}_f \)), the intersection goes up to a larger point \( g'' \), which indicates the cutoff point \( g^* \) increases in \( \bar{V}_f \) and Full accounting is preferable in a smaller region.
3.2 Accounting Regime Choice with Endogenous Initial Investment

Now we embed the accounting structure into a setting where the initial investment $I_1(\theta)$ is endogenously chosen. Specifically, we derive the investment $I_1(\theta)$ as a rational choice by the entrepreneur maximizing her objective function (2) (i.e., $-I_1(\theta) + \beta P + (1 - \beta)x$). In equilibrium, the investment depends on exogenous parameters, including the chosen accounting parameters. As a result, changes in these accounting parameters induce not only direct changes in the conditional variance (as shown in the previous exogenous setting), but also indirect changes in the conditional variance due to their impacts on the endogenous investment. In other words, accounting measures play the roles of both reflecting and affecting the cash flows generated by the entrepreneur’s investment.

Given the realization of $\theta$, the first-best investment strategy $I_{FB}^I(\theta)$ (defined as when the investment is entirely cash-flow motivated or $\beta = 0$) maximizes the initial project’s net return $z(\theta, I_1) - I_1 = 2\sqrt{\theta I_1} - I_1$, leading to $I_{FB}^I = \theta$ when $\theta \geq 0$; and $I_{FB}^I = 0$ when $\theta < 0$. That is, the first-best investment intensity $u$ is one, which is denoted by $u_{FB} = 1$.

Below we define the equilibrium where the investment decision is made in the self-interest of the entrepreneur.

**Definition 1** An equilibrium relative to $\Omega$ consists of an investment function $I_1^*(\cdot)$ and a perfectly competitive market pricing function $P(\cdot)$ such that,

(i) given the pricing function $P(\cdot)$, the optimal investment $I_1^*(\cdot)$ maximizes $E[-I_1 + \beta P(\cdot) + (1 - \beta)x|\theta]$; and

(ii) given the investment function $I_1^*(\cdot)$, the market pricing function $P(\cdot)$ satisfies $P = E[x|\Omega] - \beta\tau Var[x|\Omega]$.

Proposition 8 in the Appendix shows and characterizes a linear equilibrium (i.e., the price is linear in the accounting signal) for both AIP accounting and Full accounting. We denote the equilibrium investment intensity under AIP accounting and under Full accounting as $u_p$ and $u_f$, respectively (i.e., $I_1 = u_p^2\theta$ under AIP accounting and $I_1 = u_f^2\theta$ under Full accounting). The following proposition presents some comparative statics results regarding the equilibrium.

**Proposition 3** When the initial investment is endogenous,

1. under both accounting regimes, the equilibrium investment intensity $u_p$ ($u_f$)
i) approaches the first-best (i.e., \( u_p(u_f) \to 1 \)) as \( V_p(V_f) \to 0 \) and approaches \( 1 - \beta \) as \( V_p(V_f) \to +\infty \), and

ii) is higher when the accounting signal is less noisy and when the entrepreneur’s share-price motivation is lower (i.e., \( u_p(u_f) \) decreases in \( V_p(V_f) \) and \( \beta \)).

2. Under AIP accounting, the equilibrium investment intensity \( u_p \) is independent of the firm’s growth potential \( g \); while under Full accounting, the equilibrium investment intensity \( u_f \) increases in the growth potential \( g \) and approaches the first-best as \( g \to +\infty \).

Under both accounting regimes, the equilibrium investment intensity \( u_p \) or \( u_f \) is always lower than one, which indicates that the entrepreneur under-invests in equilibrium. This is a standard result in the literature.\(^{20}\) Due to the noise in the accounting signal, investors discount the accounting signal in the pricing (i.e., the pricing coefficient of the accounting signal is lower), which reduces the entrepreneur’s initial investment incentive. The under-investment problem is alleviated as the accounting signal becomes less noisy. Similarly, higher \( \beta \) indicates that the entrepreneur focuses more on the interim stock price and less on the future cash flow. Because the stock price provides under-investment incentives as argued above, higher \( \beta \) induces lower equilibrium investment.

Given the structural difference between the two accounting regimes, the growth potential \( g \) affects the equilibrium investment in different ways. Under AIP accounting, the induced investment \( u_p \) is independent of the growth potential \( g \), because the pricing coefficient of the accounting signal is not affected by \( g \) (given the AIP accounting signal is independent of the growth project). However, under Full accounting, higher growth potential induces higher investment intensity \( u_f \), because the Full accounting signal becomes more value-relevant with higher growth potential and thus the pricing coefficient on the signal becomes higher. In the limit, the equilibrium investment \( u_f \) approaches the first-best level.

3.2.1 Equilibrium Investment Analysis

Comparing (5) with (6), we can see the entrepreneur’s accounting choice decision boils down to a comparison of the equilibrium initial investments (i.e., \( u_p \) vs. \( u_f \)) and the conditional variances

\(^{20}\)For example, in Dye and Sridhar (2004a), the equilibrium investment level is always below first best. Liang and Wen (2007) find the similar result that output-based accounting (similar to the two accounting measures in this paper) always induces under-investment decisions by the firm.
(i.e., $\text{Var}[x|y^p]$ vs. $\text{Var}[x|y^f]$) under the two accounting regimes. In this subsection, we explore which accounting regime induces a more efficient initial investment (i.e., closer to the first-best level of one). In the next subsection, we explore which accounting regime leads to a smaller conditional variance and provides some interesting results on the entrepreneur’s accounting choice.

To compare the equilibrium investments under the two accounting regimes, first consider a setting where no future growth project exists (i.e., $g = 0$). Under this setting, Full accounting is the same as AIP accounting (in terms of the structure). Given that the initial investment decreases in the noise under AIP accounting, if $V_p > V_f$ ($V_p < V_f$), the investment is more (less) efficient under Full accounting than under AIP accounting. Now bring in the growth project (i.e., $g > 0$). From Proposition 3, the growth project does not affect the initial investment under AIP accounting, but it helps improve the investment efficiency under Full accounting. Therefore, even if $V_p < V_f$, once the growth project is sufficiently significant (i.e., $g$ is large enough), Full accounting induces more efficient investment. The following proposition summarizes the results.

**Proposition 4** There exists a cutoff point $g^{**} \geq 0$ in the growth potential such that Full accounting induces more (less) efficient investment for firms with higher (lower) growth potential (i.e., $u_f \geq u_p$ if and only if $g \geq g^{**}$). The cutoff point is zero ($g^{**} = 0$) when AIP accounting contains more noise ($V_p \geq V_f$), and is strictly positive ($g^{**} > 0$) otherwise.

**Proof.** The proof is omitted. ■

Similar to the earlier results, Proposition 4 further enhances the benefit of bringing future actions into accounting measures, that is, it better aligns market pricing with firm activities (e.g., investments). When the initial investment is endogenous, its efficiency is made higher by Full accounting when the growth potential is high enough.

### 3.2.2 Conditional Variance and Entrepreneur’s Welfare Analysis

With endogenous investment, the relationship between exogenous factors (such as $V_p$, $V_f$, and $g$) and the conditional variance will become more complicated, because these exogenous factors not only affect the conditional variance directly as in the exogenous setting, but also affect it indirectly through their impact on the endogenous initial investment. In this subsection, we first investigate
the impact of the accounting noise \((V_p \text{ or } V_f)\) on the conditional variance and on the entrepreneur’s accounting preference. Then, we employ similar analysis based on the growth potential \(g\).

**Accounting measurement factor (i.e., \(V_p \text{ or } V_f\))**

With endogenous investment, the endogenous investment opens an *indirect* channel through which the accounting noise affects the conditional variance. In particular, the relationship between the accounting noise and the conditional variance is determined by two countervailing forces: (i) higher measurement noise increases the conditional variance by resolving less cash-flow uncertainty (referred to as the direct conditional-variance increasing effect); and (ii) higher measurement noise also decreases the conditional variance by inducing lower endogenous initial investment (referred to as the indirect conditional-variance decreasing effect), because the lower endogenous investment leads to lower (ex ante) cash-flow uncertainty perceived by investors. As a result, we may not see a simple monotonic relationship between the accounting noise and the conditional variance as in the exogenous setting.

Lemma 4 in the Appendix characterizes the impact of the accounting noise on the conditional variance under both accounting regimes. Consistent with the above idea, the conditional variance may increase or decrease in the accounting noise. Figures 4 and 5 visualize the results for AIP accounting and Full accounting, respectively.\(^{21}\)

<table>
<thead>
<tr>
<th>(P1: \text{ if } \beta \leq \frac{1}{3})</th>
<th>(P2: \text{ if } \beta &gt; \frac{1}{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="" /> (\text{Var}[x</td>
<td>y])</td>
</tr>
<tr>
<td>(L^*)</td>
<td>(L^*)</td>
</tr>
<tr>
<td>(g^*V_p)</td>
<td>(g^*V_p)</td>
</tr>
<tr>
<td>(V_p)</td>
<td>(V_p, V_p^*, V_f)</td>
</tr>
</tbody>
</table>

Figure 4. The impact of the accounting noise on the conditional variance under AIP accounting

Note: \(L^* = 4(1 - \beta)^2V_\theta + g^2V_\gamma\)

\(^{21}\)See the proof in the Appendix for detailed descriptions of the undefined variables in the figures.
Under AIP accounting (see Figure 4), when $\beta$ is small (i.e., $\beta \leq \frac{1}{3}$), the equilibrium investment $u_p$ is large, and its marginal impact on the conditional variance is small,\textsuperscript{22} which indicates that the indirect conditional-variance decreasing effect is small. The direct conditional-variance increasing effect is dominant, and the conditional variance increases in $V_p$, as shown in Figure 4_P1. In contrast, when both $\beta$ and $V_p$ are large (i.e., $\beta > \frac{1}{3}$ and $V_p > V_p^*(\beta)$), the equilibrium investment $u_p$ is small, and its marginal impact on the conditional variance is large, which indicates that the indirect effect is large. At the same time, large accounting noise $V_p$ makes the direct effect of $V_p$ on the conditional variance small.\textsuperscript{23} As a result, the indirect effect is dominant and the conditional variance decreases in $V_p$, as shown in Figure 4_P2.

Similar arguments also apply to the Full accounting regime (see Figure 5). As shown in Proposition 3, when $g$ is large, the investment intensity $u_f$ is large as well, which indicates that the indirect effect is small. The direct effect is dominant and the conditional variance increases in $V_f$ when $g$ is large, as shown in Figure 5_F1. Alternatively, when $g$ is small, the conditional variance may decrease in the accounting noise $V_f$. In particular, when both $\beta$ and $V_f$ are large (i.e., $\beta > 1 - u_f^2$ and $V_f > V_f^* (\beta)$), the equilibrium investment $u_f$ is small, leading to a large and dominant indirect effect. Therefore, the conditional variance decreases in $V_f$, as shown in Figures 5_F2 and 5_F3.\textsuperscript{24}

Under both accounting regimes, when the accounting noise goes to infinity, the accounting signal provides no information and the conditional variance becomes the unconditional variance (i.e., $L^* = 4(1 - \beta)^2V_\theta + g^2V_\gamma$), which is the same for both accounting regimes, as shown in both Figure 4 and Figure 5.

\textsuperscript{22}Treating all parameters independently, differentiating $\text{Var}[x|y^p]$ with respect to $u_p^2$ gives $(\text{Var}[x|y^p])'_{u_p^2} = \frac{4V_\theta V_p^2}{(4u_p^2V_\theta + V_p)^2}$, which decreases in $u_p^2$. In other words, the independent marginal effect from $u_p$ decreases in $u_p$.

\textsuperscript{23}Similarly, we have $(\text{Var}[x|y^p])'_{V_p} = \frac{4k^2V_p^2}{(4u_p^2V_\theta + V_p)^2}$, which decreases in $V_p$. In other words, the independent marginal effect from $V_p$ decreases in $V_p$.

\textsuperscript{24}However, when both $\beta$ and $V_f$ are extremely large (i.e., $\beta > 1 - u_f^2$ and $V_f > V_f^* (\beta)$), the indirect effect becomes weaker and the conditional variance increases in $V_f$, as shown in Figure 5_F3. The reason is that, in such an extreme situation, although the marginal conditional-variance effect from $u_f$ is large due to small $u_f$, the marginal decrease in $u_f$ from higher noise is minimal when the noise is large. The overall indirect effect from the higher noise becomes relatively weaker and the conditional variance increases in $V_f$. 

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Using the above results, we develop intuition needed when comparing the conditional variances under the two accounting regimes. In the exogenous setting, the conditional variance always monotonically increases in the accounting noise and goes to the same limit value under both accounting regimes. Therefore, if the accounting signal is sufficiently noisier under Full accounting than under AIP accounting, the conditional variance is higher under Full accounting, which conforms to the conventional wisdom. However, with endogenous investment, a comparison of the graphs in Figures 4 and 5 produces a couple of different/counter-intuitive results. First, comparing Figure 4_\(P2\) with Figure 5_\(F1\), over a common parameter region\(^{25}\), once AIP accounting is noisy enough (i.e., \(V_p > \bar{V}_p\)), Full accounting \emph{always} produces a lower conditional variance than AIP accounting. Second, comparing Figure 4_\(P2\) with Figure 5_\(F2\), over another common parameter region\(^{26}\), for any \(V_p > \bar{V}_p\), the conditional variance is lower under Full accounting for any sufficiently large accounting noise than under AIP accounting. Both results are in contrast to the result in the exogenous setting in that higher noise under Full accounting does not necessarily lead to higher conditional variance (or risk premium). These results are due to the indirect impact of the accounting noise on the conditional variance (through the endogenous initial investment) and the resulting non-monotonic relationship between the conditional variance and the accounting noise.

\(^{25}\)That is, when either \(\{g^2 > 4V_\theta /3V_\gamma\text{ and } \beta > \frac{1}{3}\}\) or \(\{g^2 < 4V_\theta /3V_\gamma\text{ and } \frac{1}{3} < \beta < 1 - u_2\}\). Notice that \(0 < u_{f1} < u_{f2} < \frac{2}{3}\) as shown in the proof.

\(^{26}\)That is, when \(g^2 < 4V_\theta /3V_\gamma\) and \(1 - u_2 < \beta < 1 - u_{f1}\).
The following proposition summarizes the results.

**Proposition 5** We have the following results regarding the conditional-variance comparison:

1. when the initial investment is exogenous, for any \( V_p \) and \( g \), there exists a \( \tilde{V}_f \) such that, when \( V_f > \tilde{V}_f \), the conditional variance under Full accounting is larger than that under AIP accounting.

2. When the initial investment is endogenous,
   - i) if \( \{ g^2 > 4V_\theta/3V_\gamma \text{ and } \beta > \frac{1}{3} \} \) or \( \{ g^2 < 4V_\theta/3V_\gamma \text{ and } \frac{1}{3} < \beta < 1 - u_{f2} \} \), then for any \( V_p > \tilde{V}_p \), the conditional variance under Full accounting is smaller than that under AIP accounting for any \( V_f \); and
   - ii) if \( g^2 < 4V_\theta/3V_\gamma \text{ and } 1 - u_{f2} < \beta < 1 - u_{f1} \), then for any \( V_p > \tilde{V}_p \), there exists a \( \tilde{V}_f \) such that the conditional variance under Full accounting is smaller than that under AIP accounting for any \( V_f > \tilde{V}_f \).

**Proof.** The proof is omitted because it immediately follows from the previous results.

These counter-intuitive results regarding the conditional-variance comparison can lead to a corresponding result on the entrepreneur’s welfare comparison.\(^{27}\) First, as Proposition 4 shows, given any \( V_p \) and \( V_f \), when the growth potential is sufficiently large (i.e., \( g > g^{**} \)), the initial investment is more efficient under Full accounting. Second, as Proposition 5 shows, given a large growth potential and a large \( \beta \) (i.e., \( g^2 > 4V_\theta/3V_\gamma \text{ and } \beta > \frac{1}{3} \)), if \( V_p > \tilde{V}_p \), the conditional variance under Full accounting is smaller for any \( V_f \). Combining both results, over the common parameter region, regardless of the values of \( \theta_0 \) and \( \tau \), the entrepreneur would prefer Full accounting with sufficiently large growth potential, because the initial investment is more efficient and the conditional variance is smaller under Full accounting (refer to the welfare expressions (5) and (6)).

The following proposition summarizes the results.

**Proposition 6** When the initial investment is endogenous, for any \( \theta_0 \) and \( \tau \), if \( \beta > \frac{1}{3} \), then for any \( V_p > \tilde{V}_p \) and \( V_f \), there exists a \( g^{***} = \max\{g^{**}, 4V_\theta/3V_\gamma\} \) such that, when \( g > g^{***} \), the initial

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\(^{27}\) Here we do not plan to present comprehensive results on the welfare comparison or the accounting choice problem. Instead, we focus on the results that are more counterintuitive or insightful. Other comprehensive results are trivial and would not bring in any more insights.
investment is more efficient and the conditional variance is smaller under Full accounting, and therefore the entrepreneur always prefers Full accounting.

**Proof.** The proof is omitted because it immediately follows from the previous results. ■

In the accounting policy debates, the higher measurement noise in the fair value measures (relative to the historical cost measures) is the major criticism on the fair value accounting. To the extent Full accounting in the model captures the key scope-inclusion feature of the fair value accounting, Proposition 6 shows that firms with great growth potentials may still prefer fair value accounting regardless of the larger noise in the fair value measures, because fair value accounting not only induces more efficient investment but also leads to a lower risk premium. This is an instance where the trade-off regarding fair value is not between the measurement noise and investment efficiency.

**Growth potential factor \( g \)**

Now we investigate how the growth potential \( g \) affects the conditional variance and the entrepreneur’s welfare under both accounting regimes. Rewrite the conditional variance terms in (5) and (6) under the two accounting regimes as follows,

\[
\begin{align*}
Var[x|y^p] &= \frac{4u_p^2V_pV_\theta}{4u_p^2V_\theta + V_p} + g^2V_\gamma, \quad \text{and} \\
Var[x|y^f] &= \frac{(4u_f^2V_\theta + g^2V_\gamma)V_f}{4u_f^2V_\theta + g^2V_\gamma + V_f} = C + g^2V_\gamma, \quad \text{where } C = \frac{4u_f^2V_\theta(V_f - g^2V_\gamma) - g^2V_\gamma^2}{4u_f^2V_\theta + g^2V_\gamma + V_f}. \quad (7)\end{align*}
\]

From (7), under AIP accounting, given the initial investment \( u_p \) is independent of the growth potential \( g \), the conditional variance increases in \( g \) because the AIP accounting signal does not resolve any cash-flow uncertainty from the growth project and the entire cash-flow uncertainty from the growth project contributes to the conditional variance. Under Full accounting, the growth potential \( g \) increases the initial endogenous investment \( u_f \). Therefore, the conditional variance increases in \( g \) for two reasons: (i) higher \( g \) leads to more cash-flow uncertainty from the growth project, though it is partially resolved by the accounting signal; and (ii) higher \( g \) induces higher initial investment, which results in more cash-flow uncertainty from the initial investment. The following lemma summarizes the result.
Lemma 2 When the initial investment is endogenous, the conditional variance increases in the future growth potential $g$ under both accounting regimes.

Proof. The proof is omitted.

Even though the growth project increases the conditional variance under both accounting regimes, the rate of increase is different. From (7) and (8), one can see that comparing the conditional variances under the two accounting regimes boils down to a comparison between $\frac{4u_f^2V_0V_p}{4u_f^2V_0+V_p}$ and $C$. If the focal point is how the growth potential $g$ affects the comparison, we only need to investigate the relationship between $C$ and $g$ since $\frac{4u_f^2V_0V_p}{4u_f^2V_0+V_p}$ is independent of $g$. If $C$ increases (decreases) in $g$, the growth project increases the conditional variance at a higher (lower) rate under Full accounting than under AIP accounting. The following lemma presents the result.

Lemma 3 When the initial investment is endogenous,

1. if $\beta \leq \frac{2\sqrt{10}-2}{9}$ or $(\beta > \frac{2\sqrt{10}-2}{9}$ and $V_f \leq \frac{16(1-\beta)V_0}{9\beta^2+4\beta-4}$), the conditional variance under Full accounting increases in $g$ at a lower rate than that under AIP accounting (i.e., $\frac{\partial C}{\partial g} < 0$).

2. If $\frac{1}{2} < \beta < 1$ and $V_f$ is sufficiently high, there exists a cutoff point $\tilde{g}$ such that, when the growth potential $g$ is lower (higher) than $\tilde{g}$, the conditional variance under Full accounting increases in $g$ at a higher (lower) rate than that under AIP accounting (i.e., if $g \geq \tilde{g}$, then $\frac{\partial C}{\partial g} \leq 0$).

The above lemma shows that, under certain conditions, the growth investment may increase the conditional variance at a higher rate under Full accounting than under AIP accounting. This result contrasts with that in the exogenous investment setting where the growth investment always increases the conditional variance at a lower rate under Full accounting (see Lemma 1).

The intuition of this result relies on the impact of the endogenous investment $u_f$ on the conditional variance. On one hand, the Full accounting signal helps resolve some cash-flow uncertainty from the growth project, and the growth potential $g$ increases the conditional variance at a lower rate under Full accounting (referred to as the direct effect). On the other hand, with endogenous investment, the growth potential $g$ also increases the equilibrium initial investment $u_f$, which increases the conditional variance at a higher rate under Full accounting (referred to as the indirect
effect) because, under AIP accounting, $u_p$ is independent of $g$. Therefore, although Full accounting benefits from resolving partial cash-flow uncertainty from the growth project, it may lead to a higher conditional variance due to the induced higher initial investment.\footnote{Specifically, when $\beta$ and $V_f$ are relatively large and $g$ is small, the investment $u_f$ is relatively small and its marginal impact on the conditional variance is large (i.e., large indirect effect). In addition, when $g$ is small, it becomes relatively less important to resolve the cash flow uncertainty from the growth project (i.e., small direct effect). As a result, the indirect effect outweighs the direct effect, which leads to a higher increase in the conditional variance under Full accounting than under AIP accounting.}

As an interesting example to illustrate the difference discussed above, consider the case where $V_f = V_p$, $\frac{1}{2} < \beta < 1$, and $V_f$ is sufficiently large. When the growth project does not exist ($g = 0$), Full accounting is the same as AIP accounting (in terms of the structure). Given $V_f = V_p$, the two accounting regimes lead to equivalent initial investments and conditional variances. If the initial investment is exogenous, the conditional variance is lower under Full accounting for any positive $g$ because the growth investment increases the conditional variance at a lower rate under Full accounting for any positive $g$. However, if the initial investment is endogenous, because the growth investment increases the conditional variance at a higher rate under Full accounting for small $g < \bar{g}$, there exists a cutoff point $\tilde{g} > \bar{g}$ such that the conditional variance is lower under Full accounting if and only if $g > \tilde{g}$ (see Figure 6).

![Figure 6. The impact of $g$ on the conditional-variance comparison](image)

These results on the conditional variance can easily extend to the entrepreneur’s welfare comparison as long as the impact on the welfare from the conditional variance dominates that from the investment efficiency (e.g., $\theta_0$ is sufficiently smaller than $\tau$).

**Proposition 7** When $\theta_0$ is sufficiently smaller than $\tau$, $V_f = V_p$, $\frac{1}{2} < \beta < 1$, and $V_f$ is sufficiently large,
1. if the initial investment is exogenous, the conditional variance is always lower under Full accounting than under AIP accounting, and the entrepreneur always prefers Full accounting for any \( g > 0 \) (i.e., \( \text{Var}[x|y^f] < \text{Var}[x|y^p] \) and \( W_f > W_p \) for any \( g > 0 \)); and

2. if the initial investment is endogenous, there exists a cutoff point \( \tilde{g} > 0 \) such that the conditional variance is lower under Full accounting than under AIP accounting, and the entrepreneur prefers Full accounting if and only if \( g > \tilde{g} \) (i.e., \( \text{Var}[x|y^f] < \text{Var}[x|y^p] \) and \( W_f > W_p \) if and only if \( g > \tilde{g} \)).

**Proof.** The proof is omitted. □

Proposition 7 shows that, when the initial investment is relatively less important or the entrepreneur cares much more about the risk premium, we may have contrasting results between the exogenous and endogenous settings. When the noise level is the same, the entrepreneur always prefers Full accounting as long as there exists a growth opportunity in the exogenous setting. However, in the endogenous setting, the entrepreneur prefers Full accounting only when the growth potential is relatively high, because when the growth potential is low, the cash-flow uncertainty from the initial investment becomes relatively more important and, under Full accounting, the growth potential induces higher initial investment and accordingly higher cash-flow uncertainty from the initial project.

### 4 Discussion

#### 4.1 Risk-averse Entrepreneur

In our model, we assume that the entrepreneur is risk-neutral. The entrepreneur’s accounting preference depends on the efficiency of the initial investment and the risk premium she needs to compensate investors.

Alternatively, one could assume that the entrepreneur is risk-averse. Suppose the entrepreneur has a CARA utility function with risk-averse coefficient \( \tau_e \). We can express the entrepreneur’s certainty equivalent welfare on date-1^- under AIP accounting, denoted by \( CE_p \), as follows (with
the Approximation Assumption),

\[ CE_p = E[-I_1(\theta) + \beta P + (1 - \beta)x] - \frac{\tau_e}{2} \text{Var}[-I_1(\theta) + \beta P + (1 - \beta)x], \]  \hspace{1cm} (9)

where \( P = a_p + b_p y^p \), \( E[-I_1(\theta) + \beta P + (1 - \beta)x] = W_p \), and

\[ \text{Var}[-I_1(\theta) + \beta P + (1 - \beta)x] = \beta^2 b_p^2 V_p + 4(1 - \beta + \beta b_p)^2 u_p^2 V_\theta + (1 - \beta)^2 g^2 V_\gamma. \]

Similarly, the entrepreneur’s certainty equivalent welfare on date-1 under Full accounting, denoted by \( CE_f \), is as follows (with the Approximation Assumption),

\[ CE_f = E[-I_1(\theta) + \beta P + (1 - \beta)x] - \frac{\tau_e}{2} \text{Var}[-I_1(\theta) + \beta P + (1 - \beta)x], \]  \hspace{1cm} (10)

where \( P = a_f + b_f y^f \), \( E[-I_1(\theta) + \beta P + (1 - \beta)x] = W_f \), and

\[ \text{Var}[-I_1(\theta) + \beta P + (1 - \beta)x] = \beta^2 b_f^2 V_f + 4(1 - \beta + \beta b_f)^2 u_f^2 V_\theta + (1 - \beta + \beta b_f)^2 g^2 V_\gamma. \]

From (9) and (10), we can see the entrepreneur’s certainty equivalent welfare under both accounting regimes equals her corresponding welfare in the risk-neutral setting (i.e., \( W_p \) as defined in (5) or \( W_f \) as in (6)) minus her own risk premium. Notice that the risk-aversion does not change the equilibrium initial investment (i.e., \( u_p \) and \( u_f \) remain the same as in the risk-neutral setting), because the entrepreneur observes \( \theta \) before making the initial investment and the only noise in the price (coming from the noise in the accounting signal) would not affect the entrepreneur’s investment decision.\(^{30}\) Therefore, the difference between the risk-averse and risk-neutral settings is the additional entrepreneur’s risk premium term in (9) and (10).

Intuitively, if the entrepreneur’s risk-averse coefficient \( \tau_e \) is sufficiently small compared to the investor’s risk-averse coefficient \( \tau \), all prior qualitative results on the entrepreneur’s accounting preference remain the same in this risk-averse setting, because the investor’s risk premium will be the dominant determinant of the accounting choice decision.\(^{31}\) However, if the entrepreneur’s risk-averse coefficient \( \tau_e \) is sufficiently large compared to the investor’s risk-averse coefficient \( \tau \), the entrepreneur’s risk premium will be the dominant determinant and some results from the

\(^{29}\)Please refer to Proposition 8 for the detailed expressions of the unknown variables.

\(^{30}\)Please see the proof of Proposition 8 for details.

\(^{31}\)Some economics literature (e.g., Puri and Robinson 2009) has shown that typical entrepreneurs are less risk-averse or more risk-loving.
risk-neutral setting may change.

For example, part of the entrepreneur’s risk premium arises from the volatility in the share price $P$, which comes from both the noise in the accounting signal and the volatility in the cash flow the signal measures. Because the Full accounting signal measures cash flows from both the future growth and initial projects, the share price is more volatile under Full accounting even if the noise level is the same under the two accounting regimes. When the growth project is significant enough (or the cash-flow volatility from the growth project, $g^2V$, is large enough), the higher share-price volatility under Full accounting will dominate other factors, and the entrepreneur will prefer AIP accounting to Full accounting.\textsuperscript{32} This result is in contrast to that in the risk-neutral setting, where when the growth project is significant enough, the entrepreneur prefers Full accounting to AIP accounting, because Full accounting induces not only more efficient investment but also lower investors’ risk premium.

### 4.2 Link to the Cost-of-capital Studies

Our results on the conditional variance under both accounting regimes can also be linked to the study on the relation between the accounting quality and (firm-equity) cost of capital. Rearranging (3), we define the cost of capital of the firm as $E[x|y] - P$, which is the ex post return of the firm’s stock on date-2 after the accounting report $y \in \{y^p, y^f\}$ is released.\textsuperscript{33}

**Definition 2** The ex post cost of capital of the firm on date-2 is defined as

$$COC = E[x|y] - P = \beta \tau Var[x|y],$$

where $y \in \{y^p, y^f\}$.

Notice here the cost of capital equals a known constant $\beta \tau$ multiplied by the conditional variance $Var[x|y]$. Thus, our above analysis on the conditional variance in both the exogenous and endogenous settings can be interpreted as an analysis on the relationship between the accounting

\textsuperscript{32}Mathematically, comparing (9) with (10), in the expression of $Var[-I_1(\theta) + \beta P + (1 - \beta)x]$, we have $(1 - \beta)^2 g^2 V$, under AIP accounting and $(1 - \beta + \beta b_p)^2 g^2 V$, under Full accounting. The difference is due to the additional volatility in the price coming from the volatility in the cash flow from the growth project under Full accounting. Since $u_p, u_f, b_p, \text{ and } b_f$ are all bounded (i.e., smaller than one), when $g$ is large enough, we must have higher entrepreneur’s risk premium and lower certainty equivalent welfare under Full accounting.

\textsuperscript{33}Easley and O’Hara (2004), Lambert and Verrecchia (2010), and Bertoneu et al. (2011) use the same way to define the cost of capital. Nevertheless, we have explicitly modeled the cost of capital scaled by the ex ante expectation of the total future cash flows $E[x] = 2u\theta_0 + g\gamma_0$ (which expresses the cost of capital as a percentage), and the related comparative statics results (e.g., Lemma 1 and Lemma 4) still hold qualitatively.
measurement structure and cost of capital. The results in our paper have implications on how the accounting information quality and investment factors affect the cost of capital under both accounting regimes. Particularly, the comparative statics results of our model (e.g., Lemma 1 and Lemma 4) indicate that the nature and direction of the relation between the cost of capital and accounting information quality (i.e., $V_f$ or $V_p$) depends on factors such as firms’ growth opportunities ($g$), the degree of the market-based accounting measurement use, and the managerial myopia in investment decisions ($\beta$), which seems to be less explored in the empirical cost-of-capital literature.

4.3 Policy Implications

Our paper’s central accounting concern is the scope dimension of accounting measurements. This is a pervasive issue in the accounting standard-setting and practice. At the conceptual level, the definition of accounting assets, for example, must confront the question of whether to include future yet-to-occur transactions. At a practical level, hedge accounting, for example, must decide whether to bring onto the balance sheet forecasted future transactions when a firm enters into a derivative transaction to hedge the cash flow risk in a yet-to-occur transaction.

Our analyses bring two research perspectives to the accounting standard setting regarding the scope issue. First, we show specifically how uniform rules regarding future transactions may harm some firms while benefiting others. The precision in measuring cash flows from future transactions of different firms can vary, and so can the importance or relative size of future investment opportunities. Ignoring firm-specific issues such as these may lead to inefficient investment as well as high risk premium in share prices. In this light, standards which allow some discretion, such as the fair value option, may be better suited, allowing different firms to adapt appropriate accounting measurements given their own specific information and investment environment.

Second, a typical and sometimes intuitive reaction to fair value accounting is that while it may align firms’ investment incentives better, it brings more noise into the accounting measurement (either intentionally or unintentionally). In our model, this trade-off does exist. For example, if Full accounting is associated with more measurement noise, then for a growth firm, switching to Full accounting from AIP accounting could mean more efficient investment decision but less uncertainty resolution (leading to higher risk premium). However, our paper also shows that investment efficiency and risk premium may not necessarily be a trade-off. Proposition 6 identifies
cases where firms are better-off in both fronts under its preferred Full accounting measurement. Positively, this implies the policy tension or trade-off must lie somewhere else and due to forces not modeled by our admittedly partial-equilibrium setting.

5 Conclusion

In this paper, we provide an economic model where the conceptual scope issue with every accounting measurement has an economic meaning. In particular, we build an accounting model to highlight one important scope dimension: inclusion or exclusion of future actions. We embed the accounting model into a standard economic model in which the accounting measurement choice induces distributional and allocational efficiency. We conclude that the accounting scope issue creates complexity into the accounting choice problem at the firm-level as it may impact real variables such as investment intensity (to information) and financial variables such as risk premium in share prices. Growth opportunity of the firm appears to be an important determinant of the relative importance and direction of the impacts.

While we believe the paper opens the question on a key scope issue in accounting measurement, we view the paper is limited on a few fronts. We have limited our attention at the formal accounting measurement and abstract away from other forms of disclosure such as corporate voluntary disclosure, which also have impact on risk premium as well as investment. Similarly, we have not considered the issue of accounting or disclosure manipulation by the firm and its welfare implications. Further, outside investors are silent in collecting their own information in our model. These are all fruitful avenues to explore in future studies.
Appendix

Proof. of Lemma 1.

From (5), $\text{Var}[x|y^p]$ can be expressed as

$$
\text{Var}[x|y^p] = \frac{4u^2V \theta V_p}{4u^2V \theta + V_p} + g^2V_\gamma.
$$

It is obvious that $\text{Var}[x|y^p]$ increases in $V_p$, $u$ and $g$. In the same way, we have $\text{Var}[x|y^f]$ increases in $V_f$, $u$ and $g$.

From (5) and (6), we also have $\frac{\partial \text{Var}[x|y^p]}{\partial (g^2)} = V_\gamma$ and $\frac{\partial \text{Var}[x|y^f]}{\partial (g^2)} = V_\gamma + \frac{\partial C}{\partial (g^2)}$, where $C \equiv \frac{4u_\theta^2V_\theta(V_f-g^2V_\gamma)+g^4V_\theta^2}{4u_\theta^2V_\theta+g^2V_\gamma+V_f}$. Since $\frac{\partial C}{\partial (g^2)} < 0$, $\text{Var}[x|y^f]$ increases in $g$ at a lower rate than $\text{Var}[x|y^p]$.

Proof. of Proposition 1.

From (5) and (6), it is easy to find that the conditional variance difference $\text{Var}[x|y^f] - \text{Var}[x|y^p]$ is:

$$
\begin{align*}
\text{Var}[x|y^f] - \text{Var}[x|y^p] &= A[4u^4 \cdot 16V_\theta^2 (V_f - V_p - g^2V_\gamma) - u^4 \cdot (4V_\theta g^4V_\gamma^2 + 8V_\theta V_p g^2V_\gamma) \\
&\quad - g^4V_\gamma^2V_p] \\
&\text{where } A = \frac{1}{(4u^2V_\theta + V_p)(4u^2V_\theta + g^2V_\gamma + V_f)} > 0.
\end{align*}
$$

(11)

The sign of the difference depends on the sign of the function in the bracket, which is a quadratic function of $u^2$. If $\text{Var}[x|y^f] > \text{Var}[x|y^p]$, it must be the case that the parameter of $u^4$ is positive. That is $V_f - V_p - g^2V_\gamma > 0$. For any $g \geq 0$, the necessary condition is $V_f - V_p > 0$ or $V_f > V_p$. ■

Proof. of Proposition 2.

Rewrite (11) in the following way:

$$
\begin{align*}
\text{Var}[x|y^f] - \text{Var}[x|y^p] &= A[-g^4 \cdot V_\gamma^2 (4u^2V_\theta + V_p) - g^2 \cdot V_\gamma (8u^2V_\theta V_p + 16u^4V_\theta^2) \\
&\quad + 16u^4V_\theta^2 (V_f - V_p)].
\end{align*}
$$

(12)

The function in the brackets is a quadratic function of $g^2$. It is obvious that, when $V_f - V_p < 0$,
the above function is always negative, which is \( Var[x|y^f] < Var[x|y^p] \). When \( V_f - V_p > 0 \), \( Var[x|y^f] - Var[x|y^p] \) is strictly decreasing in \( g \), since \( g^2 > 0 \). The positive root of the equation \( Var[x|y^f] - Var[x|y^p] = 0 \) is the cutoff point \( g^* \), which is

\[
(g^*)^2 = 4u^2V_\theta \frac{\sqrt{(2V_p + 4u^2V_\theta)^2 + 4(4u^2V_\theta + V_p)(V_f - V_p) - (2V_p + 4u^2V_\theta)}}{V_\gamma (4u^2V_\theta + V_p)}. \tag{13}
\]

It is obvious that \( g^* \) increases in \( V_f \), since \( V_f \) only increases the numerator and doesn’t change the denominator. To find the relationship between \( g^* \) and \( V_p \), taking the derivative of (13) w.r.t. \( V_p \) gives

\[
\frac{\partial (g^*)^2}{\partial V_p} = \frac{4u^2V_\theta}{V_\gamma (4u^2V_\theta + V_p)^2 \sqrt{(2V_p + 4u^2V_\theta)^2 + 4(4u^2V_\theta + V_p)(V_f - V_p)}}.
\]

\[- \left[ - (4u^2V_\theta + V_p)^2 - V_p^2 - 2(4u^2V_\theta + V_p)(V_f - V_p) \right] - 4u^2V_\theta \sqrt{(2V_p + 4u^2V_\theta)^2 + 4(4u^2V_\theta + V_p)(V_f - V_p)} \right] < 0
\]

Thus, \( g^* \) decreases in \( V_p \). ■

**Proposition 8** Given the Approximation Assumption,

1. for AIP accounting where \( y^p = 2\sqrt{\theta I_1} + \varepsilon_p \), there exists a linear equilibrium characterized by

\[
P = a_p + b_p \cdot y^p = E[x|y^p] - \beta \tau \text{VAR}[x|y^p] \text{ and } I_1 = \begin{cases} u_\theta^2, & \text{if } \theta \geq 0, \text{ and} \\ 0, & \text{if } \theta < 0 \end{cases}, \text{ where}
\]

\[
a_p = \frac{2u_\theta \theta t_p - 4\beta u_\theta^2V_\theta t_p}{u_\theta^2 + t_p} + g\gamma_0 - \beta \tau g^2V_\gamma,
\]

\[
b_p = \frac{u_\theta}{u_\theta^2 + t_p},
\]

\[
u_p = \beta b_p + (1 - \beta) \in (1 - \beta, 1),
\]

\[
E[x|y^p] = \frac{2u_\theta \theta t_p + u_\theta^2 y^p}{u_\theta^2 + t_p} + g\gamma_0, \text{ and}
\]

\[
\text{VAR}[x|y^p] = \frac{4u_\theta^2V_\theta t_p}{u_\theta^2 + t_p} + g^2V_\gamma, \text{ where } t_p = \frac{V_p}{4V_\theta}.
\]

2. For Full accounting where \( y^f = 2\sqrt{\theta I_1} + g\gamma + \varepsilon_f \), there exists a linear equilibrium characterized
by $P = a_f + b_f \cdot y^f = E[x|y^f] - \beta \tau \text{VAR}[x|y^f]$ and $I_1 = \begin{cases} u_p^2 \theta, & \text{if } \theta \geq 0, \text{ and} \\ 0, & \text{if } \theta < 0 \end{cases}$, where

\begin{align}
a_f &= \frac{(2u_f \theta_0 + g \gamma_0)V_f - \beta \tau V_f(4u_f^2V_0 + g^2V_\gamma)}{4u_f^2V_0 + g^2V_\gamma + V_f}, \\
b_f &= \frac{4u_f^2V_0 + g^2V_\gamma}{4u_f^2V_0 + g^2V_\gamma + V_f}, \\
u_f &= \beta b_f + (1 - \beta) \in \left(\frac{(1 - \beta)V_f + g^2V_\gamma}{V_f + g^2V_\gamma}, 1\right), \\
E[x|y^f] &= \frac{(2u_f \theta_0 + g \gamma_0)V_f + (4u_f^2V_0 + g^2V_\gamma)y^f}{4u_f^2V_0 + g^2V_\gamma + V_f}, \text{ and} \\
\text{VAR}[x|y^f] &= \frac{(4u_f^2V_0 + g^2V_\gamma)V_f}{4u_f^2V_0 + g^2V_\gamma + V_f}.
\end{align}

Proof. of Proposition 8 and Proposition 3.

1) The entrepreneur’s ex ante expected payoff is $E[-I_1 + \beta P + (1 - \beta) x]$. Given the linear pricing conjecture $P = a_p + b_p \cdot y^p$, the entrepreneur selects the optimal investment $I_1(\theta)$ to maximize its expected payoff:

$$\max_{I_1(\theta)} E[-I_1 + \beta P + (1 - \beta) x] \text{ or } \max_{I_1(\theta)} \{\beta a_p + (1 - \beta)g \gamma_0 + (2\beta b_p + 2(1 - \beta))\sqrt{\theta I_1} - I_1\}.$$ 

Thus, $I_1 = 0$ for $\theta < 0$. When $\theta \geq 0$, FOC gives

$$I_1 = u_p^2 \theta, \text{ where } u_p = \beta b_p + (1 - \beta).$$

Then, for $\theta \geq 0$, $x = 2\sqrt{\theta I_1} + g \gamma = 2u_p \theta + g \gamma$ and $y^p = 2\sqrt{\theta I_1} + \varepsilon_p = 2u_p \theta + \varepsilon_p$. We know

$$\begin{bmatrix} \theta \\ 2u_p \theta + \varepsilon_p \end{bmatrix} \sim N\left( \begin{bmatrix} \theta_0 \\ 2u_p \theta_0 \end{bmatrix}, \begin{bmatrix} V_\theta & 2u_p V_\theta \\ 2u_p V_\theta & 4u_p^2V_\theta + V_p \end{bmatrix} \right).$$

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From pricing function (3) and the Approximation Assumption,

\[ P = E[x | y^p] - \tau \beta VAR[x | y^p] \]

\[ \cong E[2u_p \theta | 2u_p \theta + \epsilon_p] - \tau \beta \cdot VAR[2u_p \theta | 2u_p \theta + \epsilon_p] + g \gamma_0 - \tau \beta g^2 V_\gamma \]

\[ = \frac{2u_p \theta t_p - 4\tau \beta u_p^2 V \theta t_p}{u_p^2 + t_p} + g \gamma_0 - \tau \beta g^2 V_\gamma + \frac{u_p^2}{(u_p^2 + t_p)} y^p, \text{ where } t_p = \frac{V_p}{4V_\theta}. \]

Thus, it must be that

\[ a_p = \frac{2u_p \theta t_p - 4\tau \beta u_p^2 V \theta t_p}{u_p^2 + t_p} + g \gamma_0 - \tau \beta g^2 V_\gamma \text{ and } b_p = \frac{u_p^2}{(u_p^2 + t_p)}. \]

We below show the existence of \( u_p \). From (14) and (15), we have

\[ b_p = \frac{u_p^2}{u_p^2 + t_p} = \frac{u_p - (1 - \beta)}{\beta}, \text{ or } \]

\[ f(u_p) \equiv u_p^2(u_p - 1) + t_p(u_p - (1 - \beta)) = 0. \]

Since \( t_p > 0 \), we have

\[ f(u_p) < 0 \text{ if } u_p \leq 1 - \beta, \text{ and } f(u_p) > 0 \text{ if } u_p \geq 1. \]

From the property of continuity, there exists at least one root between \( 1 - \beta \) and 1 and, for any root \( u_p \), it must be between \( 1 - \beta \) and 1. We always pick the root closest to one if there are multiple roots.

We below show the comparative statistics.

From the expression of \( f(u_p) \), it is easy to see that the root \( u_p \rightarrow 1 \) as \( t_p \) (or \( V_p \)) \( \rightarrow 0 \), \( u_p \rightarrow 1 - \beta \) as \( t_p \) (or \( V_p \)) \( \rightarrow +\infty \), and \( g \) has no impact on \( u_p \). By Implicit Function Theorem, we have

\[ \frac{\partial u_p}{\partial t_p} = \frac{(1-\beta) - u_p}{f'(u_p)} \text{ where } f'(u_p) = 3u_p^2 - 2u_p + t_p. \]

Now we show \( f'(u_p) > 0 \) where \( u_p \) is the root closest to one. If \( f'(u_p) < 0 \), since \( f(u_p) = 0 \), there exists a \( u_p^* \in (u_p, 1) \) such that \( f(u_p^*) < 0 \).

Given \( f(1) > 0 \), there exists a \( u_p^{**} \in (u_p^*, 1) \) such that \( f(u_p^{**}) = 0 \), which is a contradiction with the assumption that \( u_p \) is the root closest to one. Thus, \( f'(u_p) > 0 \).

Therefore, \( \frac{\partial u_p}{\partial t_p} = \frac{(1-\beta) - u_p}{f'(u_p)} < 0 \). Given \( t_p = \frac{V_p}{4V_\theta} \), \( \frac{\partial u_p}{\partial V_\theta} < 0 \) and \( \frac{\partial u_p}{\partial \beta} > 0 \). Similarly, \( \frac{\partial u_p}{\partial \gamma} = \frac{-t_p}{f'(u_p)} < 0 \).

2) With similar arguments, under this case, the entrepreneur maximizes \( \{\beta a_f + (\beta b_f + 1 - \)
\( \beta g\gamma_0 + 2(\beta b_f + 1 - \beta)\sqrt{\delta I_1 - I_1} \). Thus, \( I_1 = 0 \) for \( \theta < 0 \). When \( \theta \geq 0 \), FOC gives

\[
I_1 = u_f^2\theta, \text{ where } u_f = \beta b_f + 1 - \beta.
\]

Then, for \( \theta \geq 0 \), \( x = 2\sqrt{\delta I_1} + g\gamma = 2u_f\theta + g\gamma \) and \( y^f = 2\sqrt{\delta I_1} + g\gamma + \varepsilon_f = 2u_f\theta + g\gamma + \varepsilon_f \). We know

\[
\begin{bmatrix}
2u_f\theta + g\gamma \\
2u_f\theta + g\gamma + \varepsilon_f
\end{bmatrix}
\sim N
\begin{bmatrix}
2u_f\theta_0 + g\gamma_0 \\
2u_f\theta_0 + g\gamma_0
\end{bmatrix},
\begin{bmatrix}
4u_f^2V\theta + g^2V\gamma & 4u_f^2V\theta + g^2V\gamma \\
4u_f^2V\theta + g^2V\gamma & 4u_f^2V\theta + g^2V\gamma + V_f
\end{bmatrix}
\]

Therefore,

\[
P = E[x|y^f] - \tau V VAR[x|y^f]
\]

\[
\approx E[2u_f\theta + g\gamma|2u_f\theta + g\gamma + \varepsilon_f] - \tau V VAR[2u_f\theta + g\gamma|2u_f\theta + g\gamma + \varepsilon_f]
\]

\[
= \frac{(2u_f\theta_0 + g\gamma_0)V_f - \tau\beta V_f(4u_f^2V\theta + g^2V\gamma)}{4u_f^2V\theta + g^2V\gamma + V_f} + \frac{4u_f^2V\theta + g^2V\gamma}{4u_f^2V\theta + g^2V\gamma + V_f}y^f.
\]

Then, it must be that

\[
a_f = \frac{(2u_f\theta_0 + g\gamma_0)V_f - \tau\beta V_f(4u_f^2V\theta + g^2V\gamma)}{4u_f^2V\theta + g^2V\gamma + V_f}
\]

and

\[
b_f = \frac{4u_f^2V\theta + g^2V\gamma}{4u_f^2V\theta + g^2V\gamma + V_f}.
\]

Below we show the existence of \( u_f \). From (16) and (17), we have

\[
b_f = \frac{4u_f^2V\theta + g^2V\gamma}{4u_f^2V\theta + g^2V\gamma + V_f} = \frac{u_f - (1 - \beta)}{\beta}, \text{ or}
\]

\[
g(u_f) \equiv 4V\theta u_f^2(u_f - 1) + (V_f + g^2V\gamma)(u_f - \frac{(1 - \beta)V_f + g^2V\gamma}{V_f + g^2V\gamma}) = 0.
\]

Note that

\[
g(u_f) < 0 \text{ if } u_f \leq \frac{(1 - \beta)V_f + g^2V\gamma}{V_f + g^2V\gamma} (> 1 - \beta), \text{ and}
\]

\[
g(u_f) > 0 \text{ if } u_f \geq 1.
\]

Therefore, there exists at least one root between \( \frac{(1 - \beta)V_f + g^2V\gamma}{V_f + g^2V\gamma} \) and 1. For any root \( u_f \), it has to be between \( \frac{(1 - \beta)V_f + g^2V\gamma}{V_f + g^2V\gamma} \) and 1. We always pick the root closest to one if there are multiple roots.

We below show the comparative statics. From the expression of \( g(u_f) \), it is straightforward that the root \( u_f \to 1 \) as \( V_f \to 0 \) or \( g \to +\infty \), and \( u_f \to 1 - \beta \) as \( V_f \to +\infty \). From \( g(u_f) = 0 \), by Implicit
Function Theorem, we have \( \frac{\partial u_f}{\partial V_f} = \frac{(1-\beta)-u_f}{g'(u_f)} \) where \( g'(u_f) = 12\theta u_f^2 - 8\theta u_f + V_f + g^2V_\gamma \). With similar argument, \( g'(u_f) > 0 \) where \( u_f \) is the root closest to one. Therefore, \( \frac{\partial u_f}{\partial V_f} < 0 \). Similarly,

\[
\frac{\partial u_f}{\partial g^2} = \frac{-V_\gamma(u_f-1)}{g'(u_f)} > 0, \quad \frac{\partial u_f}{\partial \theta} = \frac{-4u_f^2(u_f-1)}{g'(u_f)} > 0, \quad \text{and} \quad \frac{\partial u_f}{\partial \beta} = \frac{-V_f}{g'(u_f)} < 0. \]

**Lemma 4** When the initial investment is endogenous, we have the following relationship between the conditional variance and the accounting noise.

1. **Under the AIP accounting regime,**
   
   i) if \( \beta \leq \frac{1}{3} \), \( \text{Var}[x|y^p] \) increases in \( V_p \), and
   
   ii) if \( \beta > \frac{1}{3} \), \( \text{Var}[x|y^p] \) increases (decreases) in \( V_p \) when \( V_p < V_p^*(\beta) \) (\( V_p > V_p^*(\beta) \)), where \( V_p^*(\beta) \) is the value of \( V_p \) such that \( u_p = \frac{2}{3} \).
   
   iii) \( \text{Var}[x|y^p] \rightarrow g^2V_\gamma \) as \( V_p \rightarrow 0 \), and \( \text{Var}[x|y^p] \rightarrow 4(1-\beta)^2V_\theta + g^2V_\gamma \) as \( V_p \rightarrow +\infty \).

2. **Under the Full accounting regime,**

   i) if \( g^2 > 4\theta /3V_\gamma \) or \( \{g^2 < 4\theta /3V_\gamma \text{ and } \beta < 1-u_{f2}\} \), \( \text{Var}[x|y^f] \) increases in \( V_f \),

   ii) if \( g^2 < 4\theta /3V_\gamma \) and \( 1-u_{f2} < \beta < 1-u_{f1} \), \( \text{Var}[x|y^f] \) increases (decreases) in \( V_f \) when \( V_f < V_f^*(\beta) \) (\( V_f > V_f^*(\beta) \)), and

   iii) if \( g^2 < 4\theta /3V_\gamma \) and \( \beta > 1-u_{f1} \), \( \text{Var}[x|y^f] \) increases (decreases) in \( V_f \) when \( V_f < V_f^*(\beta) \) or \( V_f > V_f^*(\beta) \) (\( V_f^*(\beta) < V_f < V_f^**(\beta) \)). (see the proof for the descriptions of the unknown variables.)

   iv) \( \text{Var}[x|y^f] \rightarrow 0 \) as \( V_f \rightarrow 0 \), and \( \text{Var}[x|y^f] \rightarrow 4(1-\beta)^2V_\theta + g^2V_\gamma \) as \( V_f \rightarrow +\infty \).

**Proof.** of Lemma 4.
1) Differentiating (7) with respect to $V_p$, we have

$$
\frac{\partial \text{Var}[x|y_p]}{\partial V_p} = \frac{\partial \text{Var}[x|y_p]}{4\partial t_p} \cdot \frac{1}{V_\theta} \\
= \frac{u_p}{(u_p^2 + t_p)^2}\left[u_p^3 + 2t_p^2 \frac{\partial u_p}{\partial t_p}\right] \\
= \frac{u_p}{(u_p^2 + t_p)^2}\left[u_p^3 + 2t_p^2\left(\frac{(1 - \beta) - u_p}{3u_p^2 - 2u_p + t_p}\right)\right] \\
= \frac{u_p}{(u_p^2 + t_p)^2}\left[u_p^3 + 2t_p(\frac{u_p^3 - u_p^2}{3u_p^2 - 2u_p + t_p})\right] \text{ (from } f(u_p) = 0) \\
= \frac{u_p^3}{(u_p^2 + t_p)(3u_p^2 - 2u_p + t_p)}(3u_p - 2).
$$

We have shown $f'(u_p) = 3u_p^2 - 2u_p + t_p > 0$ before. If $\beta \leq \frac{1}{3}$, then $u_p > 1 - \beta \geq \frac{2}{3}$. Thus, $\frac{\partial \text{Var}[x|y_p]}{\partial V_p} > 0$. In contrast, if $\beta > \frac{1}{3}$, then there exists a $V_p = V_p^*(\beta)$ such that $u_p = \frac{2}{3}$. When $V_p \geq V_p^*(\beta)$, $u_p \leq \frac{2}{3}$ and, thus, $\frac{\partial \text{Var}[x|y_p]}{\partial V_p} \leq 0$. From (7), we can see $\text{Var}[x|y] \rightarrow g^2V_\gamma$ as $V_p \rightarrow 0$, and $\text{Var}[x|y] \rightarrow 4(1 - \beta)^2V_\theta + g^2V_\gamma$ as $V_p \rightarrow +\infty$.

2) Differentiating (8) with respect to $V_f$ (where $g'(u_f) = 12V_\theta u_f^2 - 8V_\theta u_f + V_f + g^2V_\gamma > 0$), we have

$$
\frac{\partial \text{Var}[x|y_f]}{\partial V_f} = \frac{[(4u_f^2V_\theta + g^2V_\gamma)^2 + 8u_fV_\theta V_f^2(1 - \beta - u_f)/g'(u_f)]}{(4u_f^2V_\theta + g^2V_\gamma + V_f)^2},
$$

$$
= \frac{(4u_f^2V_\theta + g^2V_\gamma)[(4u_f^2V_\theta + g^2V_\gamma)g'(u_f) + 8u_fV_\theta V_f(u_f - 1)]}{(4u_f^2V_\theta + g^2V_\gamma + V_f)^2g'(u_f)} \text{ (from } g(u_f) = 0) \\
= \frac{(4u_f^2V_\theta + g^2V_\gamma)V_f[(1 - \beta - u_f)g'(u_f) + 8u_fV_\theta(u_f - 1)^2]}{(4u_f^2V_\theta + g^2V_\gamma + V_f)^2g'(u_f)(u_f - 1)} \text{ (from } g(u_f) = 0) \\
= \frac{\beta(4u_f^2V_\theta + g^2V_\gamma)V_f}{(4u_f^2V_\theta + g^2V_\gamma + V_f)^2g'(u_f)(1 - u_f)}(12V_\theta u_f^2 - 8V_\theta u_f + g^2V_\gamma) \quad (18)
$$

From (18), if $g^2 > 4V_\theta/3V_\gamma$, then $12V_\theta u_f^2 - 8V_\theta u_f + g^2V_\gamma > 0$ and $\frac{\partial \text{Var}[x|y_f]}{\partial V_f} > 0$. If $g^2 < 4V_\theta/3V_\gamma$, then when $u_f \in (u_{f1}, u_{f2}) \equiv \left(\frac{-2\sqrt{4 - 3g^2V_\gamma/V_\theta}}{6}, \frac{2\sqrt{4 - 3g^2V_\gamma/V_\theta}}{6}\right)$, $12V_\theta u_f^2 - 8V_\theta u_f + g^2V_\gamma < 0$, and otherwise, $12V_\theta u_f^2 - 8V_\theta u_f + g^2V_\gamma \geq 0$. Notice that, $u_{f1} > 0$ and $u_{f2} < \frac{2}{3}$. As shown before, $u_f$ is decreasing in $V_f$, $u_f \rightarrow 1$ as $V_f \rightarrow 0$, and $u_f \rightarrow 1 - \beta$ as $V_f \rightarrow +\infty$. Then,

a) if $1 - \beta > u_{f2}$, $u_f > 1 - \beta > u_{f2}$ and $\frac{\partial \text{Var}[x|y_f]}{\partial V_f} > 0$;

b) if $u_{f1} < 1 - \beta < u_{f2}$, then $u_f > u_{f1}$ and there exists a cutoff point $V_f^*(\beta)$ such that, when
\( V_f \geq V_f^*(\beta), u_f \leq u_{f2} \) and \( \frac{\partial \text{Var}[x|y']}{\partial V_f} \leq 0 \); and

c) if \( 1 - \beta < u_{f1} \), there exists another cutoff points \( V_f^{**}(\beta) (> V_f^*(\beta)) \) such that, when \( V_f \geq V_f^{**}(\beta), u_f \leq u_{f1} \). Then when \( V_f > V_f^{**}(\beta) \) or \( V_f < V_f^*(\beta), \frac{\partial \text{Var}[x|y']}{\partial V_f} > 0 \), and when \( V_f^*(\beta) < V_f < V_f^{**}(\beta), \frac{\partial \text{Var}[x|y']}{\partial V_f} < 0 \).

Last, from (8), we can see \( \text{Var}[x|y'] \to 0 \) as \( V_f \to 0 \), and \( \text{Var}[x|y'] \to 4(1 - \beta)^2 V_\theta + g^2 V_\gamma \) as \( V_f \to +\infty \).

**Proof.** of Lemma 3.

Differentiating \( C \) with respect to \( g^2 V_\gamma \) yields

\[
\frac{\partial C}{\partial (g^2 V_\gamma)} = \frac{(-16u_f^4 V_\theta^2 - 8u_f^2 V_\theta V_f - 8u_f^2 g^2 V_\gamma - 2V_f g^2 V_\gamma - g^4 V_\gamma^2) + 8u_f V_0 V_f(1 - u_f)/g'(u_f)}{(4u_f^2 V_\theta + g^2 V_\gamma + V_f)^2}
\]

\[
= \frac{-[48u_f^4 V_\theta^2 - 32u_f^3 g^2 V_\gamma + 16u_f^2 V_\theta (V_f + g^2 V_\gamma) - 8u_f V_0 (V_f + g^2 V_\gamma) + 2V_f g^2 V_\gamma + g^4 V_\gamma^2]}{g'(u_f)(4u_f^2 V_\theta + g^2 V_\gamma + V_f)}
\]

\[
= \frac{-T}{g'(u_f)(4u_f^2 V_\theta + g^2 V_\gamma + V_f)} \quad \text{(from } g(u_f) = 0) \text{),}
\]

where

\[
T = (16V_\theta^2 + 4V_0(V_f + g^2 V_\gamma))u_f^2 - 12\beta V_0 V_f u_f + 2V_f g^2 V_\gamma + g^4 V_\gamma^2 + 4((1 - \beta) V_f + g^2 V_\gamma)V_\theta.
\]

Thus, if \( T \geq 0 \), then \( \frac{\partial C}{\partial (g^2 V_\gamma)} \leq 0 \). Notice \( T \) is a quadratic function of \( u_f \). Let \( \Delta = 144\beta^2 V_\theta V_f^2 - (64V_\theta^2 + 16V_0(V_f + g^2 V_\gamma))(2V_f g^2 V_\gamma + g^4 V_\gamma^2 + 4((1 - \beta) V_f + g^2 V_\gamma)V_\theta) \). From the property of quadratic function, if \( \Delta < 0 \), then \( T > 0 \); and if \( \Delta > 0 \), then \( T = 0 \) has two roots, \( u_{f1} \) and \( u_{f2} \), such that \( T > 0 \) as \( u_f < u_{f1} \) or \( u_f > u_{f2} \) and \( T < 0 \) as \( u_{f1} < u_f < u_{f2} \). It is easy to see \( \Delta \) is decreasing in \( g \). When \( g = 0 \), we have

\[
\Delta|_{g=0} = 16V_\theta^2 V_f [(9\beta^2 + 4\beta - 4)V_f - 16(1 - \beta)V_\theta].
\]

Thus, if \( \beta \leq \frac{2\sqrt{10} - 2}{9} \) or \( \{ \beta > \frac{2\sqrt{10} - 2}{9} \text{ and } V_f \leq \frac{16(1 - \beta)V_\theta}{9\beta^2 + 4\beta - 4} \} \), then \( \Delta|_{g=0} \leq 0 \), which implies \( \Delta < 0, T > 0, \) and \( \frac{\partial C}{\partial (g^2 V_\gamma)} < 0 \) for any \( g \). If \( \beta > \frac{2\sqrt{10} - 2}{9} \) and \( V_f > \frac{16(1 - \beta)V_\theta}{9\beta^2 + 4\beta - 4} \), then \( \Delta|_{g=0} > 0 \).

Since \( \Delta \to -\infty \) as \( g \to +\infty \) and \( \Delta \) is decreasing in \( g \), there exists a \( \hat{g} \) such that \( g \leq \hat{g} \), \( \Delta \leq 0 \).

So \( \frac{\partial C}{\partial (g^2 V_\gamma)} < 0 \) for any \( g > \hat{g} \). Now consider the case where \( g < \hat{g} \). When \( g = 0 \), \( T = 0 \) has two roots, \( u_{f1}, u_{f2} = \frac{12\beta V_\theta V_f \pm \sqrt{\Delta|_{g=0}}}{2V_\theta + 8V_0 V_f} \). Below we compare \( u_f \) with \( u_{f1} \) and \( u_{f2} \), where \( u_f \) is the root of \( g(u_f)|_{g=0} = 4V_\theta u_f^2 (u_f - 1) + V_f (u_f - (1 - \beta)) = 0 \).

Notice that as \( V_f \to \infty, u_f \to 1 - \beta \) and \( u_{f1}, u_{f2} \to \frac{3\beta \pm \sqrt{9\beta^2 + 4\beta - 4}}{2} \). When \( \frac{1}{2} < \beta < 1 \),
$u_{f1} < 1 - \beta$ and $u_{f2} > 1 - \beta$. So as $V_f \to \infty$, $u_{f1} < u_f < u_{f2}$, which implies $T < 0$ and $\frac{\partial C}{\partial (g^2 V_\gamma)} > 0$. By virtue of continuity, when $\frac{1}{2} < \beta < 1$, $V_f$ is relatively large, and $g$ is relatively small, $T < 0$ and $\frac{\partial C}{\partial (g^2 V_\gamma)} > 0$. Notice that as $g$ increases, $u_f$ increases and $u_{f2}$ decreases. Thus, there may exist a point $\tilde{g'} < \tilde{g}$ such that $u_f = u_{f2}$. Then, when $g > \tilde{g'}$, $T > 0$ and $\frac{\partial C}{\partial (g^2 V_\gamma)} < 0$. To sum up, there always exists a point $\tilde{g} > 0$ ($\tilde{g} = \tilde{g'}$ or $\tilde{g}$) such that when $g \geq \tilde{g}$, $\frac{\partial C}{\partial (g^2 V_\gamma)} \leq 0$ given $\frac{1}{2} < \beta < 1$ and $V_f$ is relatively large. □
References


REFERENCES


