Endogenous Precision of Performance Measures and Limited Managerial Attention*

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Abstract

In this paper, we model two drivers which underlie the economic trade-off shareholders face in designing incentives for optimal effort allocation by managers. The first driver is limited managerial attention, by which we mean that performing one task may have an adverse effect on the cost-efficiency of performing another. The second is the presence of a performance reporting task, by which we mean the manager's ability to exert personally costly effort to improve the precision (or quality) of his/her performance measures. We show that the subtle interactions of the two drivers may alter the characteristics of incentive provision. First, we show the interaction may lead to a positive relation between the strength of the incentive and the variance of the performance measures. Second, the interaction may render an otherwise informative performance signal unused in equilibrium incentive contracts. In particular, we show that it is possible that the principal does not use the signal whose precision can be improved by the manager, in order to discourage the manager from diverting attention to the performance reporting task (which makes the productive effort more costly). Finally, we apply the model to a specific project-selection setting and show that in order to induce the agent to choose higher-risk-higher-return projects, the principal may need to raise the bonus rate when the project-selection choice is unobservable.

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1 Introduction

In a modern firm, a well-motivated management team has become a vital source of organizational success. One important component of designing managerial incentives is to properly induce an optimal allocation of managerial effort over multiple tasks (see Roberts 2004, p. 140-153). Among crucial tasks inviting the limited managerial attention, the performance reporting task (both internal and external) stands out as of significant interests by the press, policy makers, and the academic accounting profession. While many wide-publicized cases have been negative (e.g., Enron and Worldcom), most managerial efforts on reporting to outsiders are legitimate and improve the informativeness of reported firm performance. Academic studies have shown that managers may engage in earnings smoothing to improve the informativeness of their earnings about the firms' true performance (Tucker and Zarowin, 2006; Subramanyam, 1996; Hunt, Moyer, and Shevlin, 2000). Managers also work on accruals quality through improving the precision and informativeness of the accounting accrual estimates (Francis, LaFond, Olsson, and Schipper, 2005). In addition, Managers may also influence the variance of the reported outcomes through "real" operating decisions such as hedging. The performance reporting task may compete for managerial attention with other productive tasks. For example, managers are responsible to maintain and improve internal control over financial reporting (ICFR). The Sarbanes-Oxley Act in 2002 (SOX), especially Section 404, imposes significant demand of attention on the management of public companies.[1] Managerial attention has to be diverted from running the companies to ensuring compliance with regulations. In a testimony on Capital Hill in April 2005, SEC Chairman Donaldson commented that complying with SOX Section 404 has been time-consuming and expensive for most companies, as confirmed by surveys evidence (see Stovall 2008). In this light, managers face a trade-off between productive efforts such as identifying real investment opportunities and "non-productive" effort such as performance reporting tasks. This trade-off has received attention in the business press.[2] More broadly, limited attention is a pervasive issue in the management of large organizations. In a classic work, Herb Simon points out

> "... the scarce resource is not information; it is processing capacity to attend to information. Attention is the chief bottleneck in organizational activity, and the bottleneck becomes narrower and narrower as we move to the tops of organizations, where parallel processing capacity become less easy ..." (Simon 1973, page 270.)

[1] In particular, "management should evaluate the design of the controls to determine whether they adequately address the risk that a material misstatement in the financial statements would not be prevented or detected in a timely manner. ... that the evaluation of evidence about the operation of controls should be based on assessments of the controls' associated risk." (KPMG 2007)

[2] In Sayther (2003), it is claimed that compliance demands steal the CFO's focus and leave less time and resources for strategic thinking; and Stone (2005) reports comments by industry insiders that SOX is siphoning away CEO creativity and it's forcing CEOs to be more worried about compliance and losing their jobs than figuring out how to invest in growth for the future.

In this paper, we formally model the two drivers underlying the economic trade-off in managerial effort allocation. The first driver is limited managerial attention, by which we mean that performing one task may have an averse effect on the cost-efficiency of performing another. The second is the presence of a performance reporting task, by which we mean the manager's ability to exert personally costly effort to improve the precision (or quality) of his/her performance measures. We show that the subtle interactions of the two drivers may alter the characteristics of incentive provision. Such alterations shed light on our understanding of some recognizable practices. First, we show that the interaction may lead to a positive relation between the strength of incentive and the exogenous or endogenous variance of the performance measures. This is consistent with many empirically mixed findings of the relation (see Prendergast 2002 and Lafontaine and Bhattacharyya 1995 among others). Second, the interaction may render an otherwise informative performance signal unused in equilibrium incentive contracts. In particular, we show that it is possible that the principal does not use the signal whose precision can be improved by the manager, in order to discourage the manager from diverting attention to the performance reporting task (which makes the productive effort more costly). Finally, we apply the model to a specific project-selection setting and show that in order to induce the agent to choose higher-risk-higher-return projects, the principal may need to raise the bonus rate when the project-selection choice is unobservable.

Specifically, we employed an agency model similar to the single-period multi-task model of Feltham and Xie (1994), which is further examined by Christensen, Sabac and Tian (2010). The main distinguishing features of our model are (1) that the agent may exert personally costly performance-reporting effort to improve the accuracy of the measured performance, which is a noisy signal of future cash flows (and thus the productive effort), and (2) that the two efforts compete for limited managerial attention such that the exertion of higher effort in one leads to a higher marginal cost of exerting effort in another. In other words, two tasks in our setting (labeled productive task and performance reporting task) are hinged together in two respects: They affect the same performance measure, with one affecting the mean and the other the precision, and the performance reporting effort (which reduces the variance) affects the agent's marginal cost of the productive effort. The family of performance signals in such a setting are most likely those generated by a sophisticated information system such as an accounting information system (for internal as well as external use) which requires active managerial attention in order to maintain its accuracy and precision.

When designing the optimal incentive contract in such an environment, the principal must consider subtle interactions induced by the two drivers. Any pay-for-performance scheme using the performance measure will induce the agent to exert performance reporting as well as productive effort, since a risk-averse agent would enjoy a reduced variance in his compensation. The principal also enjoys the reduced variance, as compensation costs (those due to the risk-premium) are lower. Thus, the induced response from the agent is desirable from the principal's perspective. However, such a response may also complicate the problem if the performance reporting effort has a spillover effect on the moral hazard problem involving the productive effort. This would take place if exerting performance reporting effort increases the marginal cost of the agent's productive effort, which indeed makes the moral hazard problem more severe. This is an undesirable aspect of the induced response from the agent. When facing such a problem, the principal must balance the benefits and costs from the desirable as well as the undesirable aspects of multi-tasking.

We use the model to explore several aspects of incentive provision practice. Generally, our analysis points to the subtleties in extending standard LEN agency results to settings in which the agent has an influence over the precision of his own performance measures. The contribution of our study is threefold. First, we investigate the properties of incentive provision when there exists the spillover effect between the agent's two effort choices. We show that, in contrast to standard LEN results, a positive relation between incentive strength and performance variance is possible. That is, high-powered incentives may be associated with high performance variances. The economic intuition is driven by subtle effects with the presence of both the two drivers. With the manager's influence over the variance and the spillover effect between the two effort choices, it may become easier for the principal to motivate more productive effort through a higher bonus rate when the performance reporting effort is less effective as the variance gets larger. As a result, the incentive-risk relation appears positive. This finding offers a multi-task-based rationale to explain the mixed empirical results on the relation between variance of performance measures and equilibrium incentive strength.

Second, we enrich the setting by adding a second performance signal whose precision is not affected by the agent. Within the LEN framework, we derive conditions under which it is efficient for the principal to discard the signal with an endogenous variance, even if the signal is informative. This is because when the second signal is sufficiently informative, placing any incentive weight on the first signal may be too costly due to the spillover effect of increasing the agent's marginal cost of productive effort. As a result, a contractible, informative signal is left unused due to the induced drain of the agent's limited attention, which is different from the prediction of standard agency models. Our finding here offers a novel explanation why informative signals are left unused, complementing other competing reasons (such as incomplete contracts and subjective performance measures).

Finally, we examine in a project-selection setting the effect of endogenous variance. When the manager is able to improve the precision of his performance measure, we find that the principal can motivate a riskier project selection with a higher incentive. This is in contrast to previous prediction by Sung (1995) that the principal would lower the sensitivity of the incentive to motivate a riskier project selection. In Sung's setting the manager cannot influence the risk through his effort. Therefore, when the principal offers a higher incentive, the manager is still reluctant to take a riskier project because the risk premium increases in the incentive. In our paper, however, a higher incentive also induces higher managerial effort to reduce the risk, which may offset the increase in the risk premium.

Previous agency studies of multi-tasking, such as Holmstrom and Milgrom (1991), Feltham and Xie (1994), Zhang (2003) and Christensen, Sabac, and Tian (2010), usually focus on the productive efforts and assume exogenous variance (and covariance) of performance measures. Holmstrom and Milgrom (1991) examine a multi-task setting in which the agent allocates his effort to more than one productive activities. They show that the incentive in one activity should decrease with the difficulty of measuring performance in other activities, so that the agent will not be induced to input his effort only in the activity that is easy to measure. In Holmstrom and Milgrom's model, the principal seeks a more balanced allocation of the agent's effort among productive activities, while in our study the principal restrains the agent's effort from performance reporting activity to avoid a high marginal cost of productive effort. Feltham and Xie (1994) examine a similar model to Holmstrom and Milgrom's setting, but focus more on the congruence of a performance measure with the principal's interest. They show that any informative additional signal can reduce risk and non-congruity (see extensions by Christensen, Sabac and Tian 2010), while in our model it may be efficient to exclude an informative signal from the contract so that the marginal cost of productive effort is reduced. Zhang (2003) studies the multiple tasks that are complements; in our paper the two tasks have a substitutional relationship in the sense that a high variance-reduction effort leads to a higher marginal cost of productive effort. We extend the multi-task literature by enriching the tasks to include those focusing on increasing the precision of performance measures.

Our paper is related to Meth (1996) and Feltham and Wu (2001). Both papers study agencies with the manager's efforts affecting both the mean and the variance of output and both focus on non-linear contract shapes as a consequence of the variance affecting effort by the agent. The key insight from Meth's (1996) analysis is that the variance reduction effort is subject to moral hazard problem of its own and incentive provision for variance reduction effort depends on agent's risk-aversion and the feasibility of non-monotonic contracts (e.g., optimal wage may be decreasing in output over some regions). Feltham and Wu (2001) consider a setting in which both the mean and the variance of a firm's value increase in the manager's effort and show that option-based contracts (i.e., wages are increasing and convex over some region) are more efficient than stock-based contracts when the impact of effort on risk is large. In contrast, we take linear contract as given and focus on the interaction between performance reporting task (i.e, managerial effort affecting variance) and limited attention. Compared to Meth (1996) and Feltham and Wu (2001), our paper extends their models by focusing on the trade-off in performance reporting task when limited managerial attention is the key driving force. Specifically, in our model, there is no conflict of interest in the performance reporting task (variance reduction) without limited attention even if the effort is unobservable. Further, we develop applications of the dual-task model to issues on value of additional signals and project selection, beyond the focus in their papers.

Standard moral hazard models usually predict a negative association between risk and incentives. However, empirical studies show the existence of a positive association between risk and incentives. Recently there have been several theoretical studies that explore this positive association. Using a discrete model which shares the same basic property as the standard model, Hemmer (2006) demonstrates that changes in incentives that affect the optimal effort level also affect the variance of the outcome distribution, thus resulting in a negative or positive relation between risk and incentives. In our model, the positive relation is the result of limited managerial attention and the manager's ability to reduce the variance of his performance measure. Our paper is similar to a study by Dutta (2008) in the sense that both examine the endogeneity of variances in a LEN model. Dutta (2008) introduces an additional information risk from the uncertainty about the manager's expertise, while our paper focuses on the endogenous variance affected by the agent's effort. Also related is a recent paper by Liang, Rajan, and Ray (2008) where variance of performance measurement is endogenous, not because of a performance reporting task but because of the endogeneity of the worker team size. Demski and Dye (1999) consider a principal-agent problem in which the manager privately chooses a project with a certain variance and his effort and project choice decide the mean of the project output, while in our paper the manager exerts two efforts, one affects the mean, and the other affects the variance of the output. Prendergast (2002) and Rajan and Saouma (2006) consider the delegation of responsibilities to an agent and shows that the agent's effort and choice among activities change the risk of output. Hughes (1982), Danielsson, Jorgensen and Vries (2002), Baker and Jorgensen (2005) and Bertomeu (2008) also consider the agent's ability to change the risk profile of the firm output (thus the agent's performance measure). In all these papers, limited managerial attention is not a key research issue.

Value of additional signals has also been a focus of agency work since its early years. Holmstrom (1979) pioneered this inquiry and established the early standard result called the *Informativeness Criterion.* In accounting, this work is followed by Antle and Demski (1988), Demski (1994), Feltham and Xie (1994), Feltham and Wu (2000), Arya, Glover, and Radhakrishnan (2005), and Christensen, Sabac, and Tian (2010), among others. The focus has been on the conditional nature of the informativeness criterion, or on the different informational roles in valuation versus control settings, or on signal aggregation over time. We extend this literature by bringing into focus the role of limited managerial attention on the value of additional information.

Limited managerial attention has been examined in the prior literature, but from different perspectives. Geanakoplos and Milgrom (1991) focus on how to allocate different tasks among managers with different ability and the optimal organization structure of a firm, while our paper looks at how a manager allocates his effort between different tasks. Darrough and Melumad (1995) examine a setting in which a principal motivates a manager with unknown ability to allocate his effort between his own division and other division, and illustrate that sometimes it is optimal to motivate the manager to concentrate on his own division. Unlike the setting in our paper in which the cost of efforts spillover into each other, in their study the manager's effort is costless. Peng and Roell (2008) study earnings manipulation to mislead stock market prices within a setting where productive and manipulation tasks compete for the limited managerial attention.[3]

There is a literature in project selection, including Lambert (1986) and Sung (1995), among many others. Our paper contribute to this literature by introducing endogenous variance into the model. We illustrate how the managerial effort to reduce risk affects a specific project selection problem. Lambert (1986) considers a setting in which the principal motivates the agent to get information about the profitability of projects and select the best project based on that information. Lambert shows that the principal and the agent may not agree on which is the "best" project. In our paper, we also look at a setting with interest conflict between the principal and the agent on project selection, but we do not consider the agency problem to motivate the agent to acquire information. Instead, we focus on how the agent's ability to reduce the risk of the project influences the project selection problem. Similar to Sung (1995), we examine a setting in which the agent's project selection choice is unobservable, and study how the principal motivates the agent to choose a riskier project with higher return. However, in our model the agent is able to reduce the compensation risk through his performance reporting effort, in addition to his ability to affect project risk through project selection choice. This additional tension leads to a stronger incentive needed to motivate selecting the riskier project, in contrast to result in Sung's model.[4]

Finally, to the extent the performance measure precision is related to the accounting measures' predictive power (which underlies the notion of accounting quality in many empirical accounting investigations), our model points to the endogenous nature of such an empirical notion. That is, the precision of performance measures (thus its predictive power) is a result of both exogenous environmental conditions as well as the performance reporting effort exerted by the managers induced by equilibrium incentive contracts. Environmental

[3] Other studies include an experimental study in which Bruggen and Moers (2007) examine a setting in which the agent makes an effort-level choice and effort-allocation choice. The agent's effort is allocated between two tasks, A and B. However, only Task A has an observable and verifiable performance measure and thus the agent has incentive to input effort on Task A only. Introducing social incentives congruent with the principal's interest helps mitigate the distortion in the agent's effort allocation, but may lead to lower total effort. Their paper, though also look at effort allocation problem, has a different focus. In their setting the agent chooses the *total* effort level, and the proportion of total effort allocated to Task B. Therefore the focus is the trade-off between higher total effort and more congruent effort allocation. In our paper, however, the agent makes two effort level decisions on two tasks, and the focus is the interaction between these two efforts.

[4] Notice that our model is distinct from the literature on precision reporting. Penno (1996) considers a setting where management privately chooses a precision level of an estimate of the firm's market value after observing a public signal about the value. Hughes and Pae (2004) study a setting in which the manager privately learns her cost to acquire precision information and decides whether to acquire the information. In addition, if she is informed, she may voluntarily disclose the precision information to the market. Different from these papers, in our model the manager is able to directly influence the precision of the performance measure.

changes not only will have a direct effect on accounting measures' predictive power (or accounting quality), but also will have an indirect effect via the equilibrium performance reporting tasks especially when the managers have limited attention.

The rest of the paper is organized as follows: Section 2 lays out the basic model and analyzes the key economic tension caused by the introduction of the two drivers. Section 3 analyzes the relation between incentive strength and performance variance and shows forces that cause a positive relation. Sections 4 and 5 analyze two variations of the basic model in order to study how performance reporting task and limited attention affect the role of additional performance signals and project selection. Section 6 concludes the paper.

2 Basic Model

We consider a single-period two-task LEN agency setting where a risk-neutral principal contracting with a risk-averse agent. The agent provides two-dimensional effort, denoted $\{e_1, e_2\}$, where $e_i \in \mathbb{R}^+$, at a personal cost $C(e_1, e_2)$.[5] The agent's productive effort, denoted e_1, raises expected output, denoted x. We assume a constant return to scale $x'(e_1) = q > 0$, and the noise of x follows a zero-mean normal distribution, $\varepsilon_x \sim N(0, \sigma_x^2)$. We also assume that the output x is realized too late for contracting, but there is a contractible signal y which is a noisy signal of x:

$$y = x + \varepsilon_y,$$

where ε_y is a zero-mean normally distributed random variable with variance $V(e_2, \sigma^2)$. That is,

$$\varepsilon_y \sim N\left(0, V(e_2, \sigma^2)\right).$$

We regard e_2 as the agent's performance reporting effort to reduce the error in his performance measures.[6] Generally these activities may include any choices or decisions of managers to make their performance measure more accurate on

[5] We focus on the single agent setting in the model. However, the key assumption is that the agent's unobservable managerial efforts contain these two dimensions. Even if the principal assigns the two efforts to two agents separately, one agent may still choose to execute both efforts since the efforts are unobservable and the agent still has an incentive to improve the precision of performance measures as well as improving production. See related work on teams in Huddart and Liang (2005) and in Liang, Rajan and Ray (2008).

[6] Dye and Sridhar (2007) and Stocken and Verrecchia (2004) also look at the case in which the precision of a disclosed estimate or that of a firm's accounting reporting system is a choice variable. In Dye and Sridhar's study, a risk-averse initial owner discloses an estimate of future cash flow mean to risk-neutral investors. The study shows that whether the initial owner's precision choice is private or public and whether his disclosure is voluntary or mandatary lead to different equilibria of allocating risk between the owner and the investors. Their paper focuses on the allocational effects while our paper focuses on the interaction between the agent's productive effort and precision choice. Stocken and Verrecchia's study examines the interaction between the manager's choice of the precision of a firm's accounting reporting system and his disclosure management decision. It shows that the manager may not choose the most precise reporting system when he has the option to manipulate the financial report. Again, their study does not consider the precision choice's effect on productive effort.

their managerial abilities/efforts. For example, the managers may influence the performance measure through "real" operating decisions such as hedging. The financial derivatives market has been developing rapidly since the 1990s and offers managers greater availability and accessibility of hedging instruments, and managers are now able to modify the variances of the reported outcomes. We assume that a higher e_2 leads to a more accurate performance measure (i.e., $V_1 \equiv \frac{\partial}{\partial e_2}V(e_2,\sigma^2) < 0$).[7] In addition, it also satisfies typical regularity conditions:$V_{11} \geq 0$, $V_1|_{e_2=0} = -\infty$ and $V_1|_{e_2=+\infty} = 0$. Parameter σ^2 is a known constant and can be regarded as the exogenous variance of the performance measure. This parameter captures the idea that output (and thus managerial productive effort) may be harder or easier to measure for a given amount of performance reporting effort for different firms in different industries during different periods of time. Let $V_2 \equiv \frac{\partial}{\partial \sigma^2}V(e_2,\sigma^2) > 0$. We further assume e_2 does not influence σ_x^2.

As usual, we assume the agent's personal efforts cost $C(e_1,e_2)$ is increasing and (weakly) convex in both e_1 and e_2. Further, we assume $C_{12}(e_1,e_2) \equiv \frac{\partial}{\partial e_1 \partial e_2}C(e_1,e_2) \geq 0$ to highlight the interaction between the two actions. In particular, a positive cross-partial derivative implies a limited managerial attention where higher level of one effort increases the marginal cost of performing the other effort.[8] When $C_{12}(e_1,e_2) > 0$, a spillover is present between the cost of

[7] In this paper, to foucs on agent's legitimate effort to improve the precision of performance measures, we do not model earnings managment. However, earnings management and behavior of garbling performance measures are always concerns in reality, and have been examined by many studies.

If we allow the agent to garble the preformance measures in this model through e_2, the results in the analysis of incentive-risk relation and additional signals may still remain, or may be even strengthened. In the analysis of incentive-risk relation, as the preformance reporting effort brings less benefit (potential garbling in addition to inducing a higher marginal cost of productive effort), the principal would have a stronger motivation to induce less performance reporting effort and more productive effort, which may lead to the positive relation between the variance σ^2 and incentive. In the analysis of additional signal, with potential garbling through e_2, it may be more likely that the principal ignores the signal whose precision can be manipulated.

[8] Peng and Roell (2008) record a recent example of limited managerial attention, that "in the real world, the time constraint is one of the most important constraints faced by managers. And they do complain of the significant amount of time and attention they are forced to devote to public relations and reassuring the stock market (in Europe, prominent business leaders have pointed out that the threat of a takeover, now that corporate control is more contestable than it used to be, is having the unfortunate side effect of distracting management from running the underlying business). This time cost comes out clearly in the London Stock Exchange's A Practical Guide to Listing: 'Both the flotation process itself and the continuing obligations – particularly the vital investor relations activities ... - use up significant amounts of management time which might otherwise be directed to running the business ... It is vital that you maintain your company's profile, and stimulate interest in its shares on a continuing basis. Many listed companies, even relatively small ones, employ specialist financial public relations and investor relations advisors on a retainer basis to keep the business on the financial pages and in the minds of investors. ... However, you cannot leave press or investor relations to your advisers. Top executives will commonly devote at least a couple of days a month to developing and nurturing such contacts. ...This must be regarded as time well-spend. ... As a publicly-quoted company, it is a core element of running your business properly and responsibly.'(pp. 11, 47-48)"

two actions; and when $C_{12}(e_1, e_2) = 0$, the cost is separable between the two efforts and there is no spillover.[9]

The principal offers a linear contract on y, with a fixed wage α and a bonus rate β on the performance measure y.

$$w = \alpha + \beta y$$

The time line of the events is:

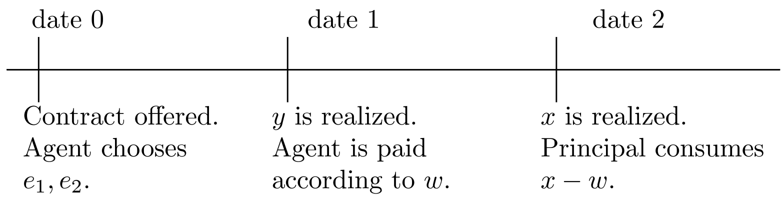

Figure 1: Time line

The agent's preference is represented by a negative exponential utility function $(-e^{r(w - C(e_1,e_2))})$ with Arrow-Pratt measure r. This allows the standard transformation of the agent's problem into

$$\max_{e_1,\ e_2} \alpha + \beta E[y] - \frac{r}{2}\beta^2[V(e_2, \sigma^2) + \sigma_x^2] - C(e_1, e_2),$$

which yields a standard incentive constraint on the equilibrium choice of e_1 (in equilibrium this constraint always binds):

$$C_1(e_1, e_2) = q\beta. \tag{1}$$

In addition, it yields an additional incentive constraint on the equilibrium choice of e_2:[10]

$$-\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - C_2(e_1, e_2) = 0. \tag{2}$$

Notice from (2), if $\beta > 0$, then the optimal e_2 is positive. Intuitively, when performance measure y is used in contract, the agent always has an incentive

[9] Formally, we assume $C(e_1, e_2)$ is continuous and differentiable over $\left(\mathbb{R}^+\right)^2$, where $C_1(), C_{11} \geq 0$,and $C_2(), C_{22} \geq 0$. In some examples, we may consider specific cost function to illustrate economic intuition using closed-form solutions. In these examples, we consider $C(e_1, e_2) = f(e_2)e_1^2$, where $f(e_2) > 0$. In this case, condition $C_{12} = f'(e_2)2e_1 > 0$ reflects the limited managerial attention. In the example for separable costs, we consider $C(e_1, e_2) = L(e_1) + K(e_2)$, which has the property $C_{12} = 0$. Finally, we assume $C_{11}C_{22} - (C_{12})^2 \geq 0$ to satisfy the second-order condition.

[10] To ensure that the two first-order conditions characterize the maximum, we compute and verify that the Hessian,

$$\begin{bmatrix} -C_{11} & -C_{12} \\ -C_{12} & -\frac{r}{2}\beta^2 V_{11}(.) - C_{22} \end{bmatrix}$$

is indeed negative-definite given we had assumed that $C_{11}C_{22} > (C_{12})^2$. In the specific examples we used later in the paper, second-order conditions are satisfied with details given in the appendix.

to exert performance effort (e_2) to reduce the variance of that measure.[11] Conditions (1) and (2) implicitly define the agent's best response (e_1 and e_2) to a given choice β by the principal.

Without loss of generality, the reservation wage for the agent is set at zero. The principal will set the fixed wage α so that the agent's individual rationality constraint binds.[12] The principal's problem, labeled (PP), is

$$\max_{\beta} x\,(e_1) - \frac{r}{2}\beta^2[V(e_2,\sigma^2)+\sigma_x^2] - C(e_1,e_2), \tag{PP}$$

yielding a first-order condition for optimal choice of incentive β:

$$[q - C_1(e_1,e_2)]\,\frac{de_1}{d\beta} - r\beta[V(e_2,\sigma^2)+\sigma_x^2] + \left[-\frac{r}{2}\beta^2 V_1(e_2,\sigma^2) - C_2(e_1,e_2)\right]\frac{de_2}{d\beta} = 0, \tag{3}$$

with the associated second-order-condition, denoted SOC_β, being negative.[13] From (1), we deduce that $\frac{de_1}{d\beta} = \frac{q - C_{12}\frac{de_2}{d\beta}}{C_{11}(e_1,e_2)}$. If (2) is satisfied in equilibrium (thus eliminating the third term in equation 3), when substituting $q\beta$ for $C_1(e_1,e_2)$ using equation (1), (3) can be written into an implicit function

$$\beta = \frac{q^2}{q^2 + r[V(e_2,\sigma^2)+\sigma_x^2]C_{11}/[1 - C_{12}\frac{de_2}{d\beta}\frac{1}{q}]}. \tag{4}$$

Our model differs from traditional multi-task models in several ways. First, the performance reporting task e_2 is endogenous to the moral hazard problem of the productive task e_1. Notice it is easy to see that the first best action combination is $\langle e_1^{FB} > 0, e_2^{FB} = 0\rangle$ while in the second-best, $\langle e_1^{FB} > e_1^{SB} > 0, e_2^{SB} > 0\rangle$. In other words, without the moral hazard problem with respect to e_1 (e.g., if the principal could contract directly on e_1), the principal would not demand any agent's effort to reduce the error in his performance metric. Second, managerial attention (e.g., $C_{12}(e_1,e_2)$) is a key factor in determining the optimal choice of performance reporting effort. If no such indirect effect is in place, the agent will increase both the productive effort and the performance reporting effort as

[11] Formally, for any given positive bonus weight β, a manager choosing $e_2 = 0$ is not optimal because at $e_2 = 0$, the marginal benefit is proportional to $-V_1(e_2,\sigma^2) = +\infty$ and the marginal cost is $C_2(e_1,e_2) < +\infty$. By continuity, the manager can always find an $e_2 > 0$ to equate the marginal benefit and the marginal costs.

[12] This is because the principal can always adjust the fixed wage α, without affecting any incentive constraints, to make sure the agent takes the contract by setting $\alpha = -\beta E[y] + \frac{r}{2}\beta^2[V(e_2,\sigma^2)+\sigma_x^2] + C(e_1,e_2)$.

[13] Specifically, substituing agent's first- order-conditions (equations 1 and 2) into 3 and differentiate 3 with respect to β again, we have

$$SOC_\beta \equiv -q\frac{de_1}{d\beta} - r[V(e_2,\sigma^2)+\sigma_x^2] + q(1-\beta)\frac{d^2e_1}{d\beta^2} - r\beta[V_1(e_2,\sigma^2)\frac{de_2}{d\beta}.$$

To ensure the first-order-condition characterizes a global maximum, we assume

$$SOC_\beta < 0.$$

This condition is verified for specific examples we use later in the paper.

the principal offers a higher incentive. That is, a higher performance reporting effort can only help lessen the agency problem.[14]

However, if marginal cost of the productive effort, $C_1(e_1, e_2)$, is an increasing function of e_2, the issue becomes more complicated. In particular, because of the limited managerial attention, inducing the agent to provide performance reporting effort leads to an interaction (or a spillover) effect on the agent's choice of productive effort. From the agent's perspective, one obvious effect is that inducing a higher e_2-choice makes the agent lower his e_1-choice for a given bonus rate (such that condition 1 holds). From the principal's perspective, inducing a higher e_2-choice makes e_1 marginally more costly (i.e., a higher $C_1(e_1, e_2)$). On one hand, the principal would like to increase the bonus rate β to motivate a higher e_2 to obtain a more precise performance measure (i.e., a lower $V(e_2, \sigma^2)$), which amounts to a "less severe" moral hazard problem. On the other hand, a higher e_2 leads to higher marginal cost of motivating e_1, which results in a "more severe" moral hazard problem and would press the principal to lower the optimal bonus rate β. This two-way interaction is a result due to the combination of (i) induced demand for the performance reporting task and (ii) limited managerial attention.

We use this two-task model to address three long-standing issues on management control, and we show that there are subtleties in extending standard results to settings in which the agent has influence over the variance of his own performance measures. In Section 3, we investigate how the presence of the performance reporting task and the limited attention (spillover effect) affect the characteristics of the optimal incentive provision. We show that unlike the setting in which performance variance is exogenous, it is possible the relation between incentive strength (β) and performance variance ($\sigma^2 + \sigma_x^2$) is positive. In Sections 4, we introduce an additional performance signal whose variance is not affected by e_2 and derive conditions under which it is efficient for the principal to discard the signal with endogenous variance, even if the signal is informative. Finally, section 5 applies the model to a specific project-selection setting and show that in order to induce the agent to choose higher-risk-higher-return projects, the principal may need to raise the bonus rate when the project-selection choice is unobservable.

[14] Further, it is straightforward to verify that the optimal e_2 supplied by the agent at the solution to (PP) is identical to the solution of a slightly modified problem (PP') where e_2 is supplied by the principal (at the same cost, separate from cost of e_1). In other words, without spillover costs, there is no conflict of interest with respect to the provision of e_2.

However, our assumption that the manager instead of the principal provides e_2 is realistic. In practice, some preformance reporting efforts, such as improving the precision of accruals estimates and smoothing earnings based on managerial predictions of future profitability to improve the informativeness of financial reports, are difficult to be centralized to the principal because these activities require managerial expertise.

3 Incentive-Variance Relation

In this section we examine the relation between incentive and variance with the presence of the two-way interaction. Recall that in standard LEN moral hazard models, the variance of the performance measures is typically unaffected by the agent's effort. In these settings, a typical prediction is that risk and incentive are negatively related. That is, the principal offers a lower bonus rate when the agent's performance is measured with high variance (risk). Our model nests such a special case and such a prediction. Consider the case where e_2 is a known constant denoted by E (and thus not a choice of the agent). Then, the principal's trade-off is captured by the following special case of equation (4):

$$\beta = \frac{q^2}{q^2 + r[V(E, \sigma^2) + \sigma_x^2]C_{11}(e_1, E)}. \tag{5}$$

The negative relation between incentive and signal variance is intuitive: principal lowers incentive rates in response to a higher variance in the performance measure imposed on a risk-averse agent. Indeed, from equation (5), an increase in σ^2 leads to a decrease in β.[15] The key is that such an increase in σ^2 does not generate a response in the agent's choice of e_2, which would have affected β indirectly.

Outside this special case, an increase in σ^2 would induce a response from the agent's performance reporting effort (e_2). This is because the σ^2-parameter affects the agent's trade-off in choosing e_2. From equation (2), a higher σ^2 increases the agent's marginal benefit of providing e_2, holding incentive rate β constant, pressing the agent to provide more reporting effort. Anticipating this change in the agent's effort calculus, the principal would react by adjusting the incentive provision (i.e., bonus rate β). In the principal's calculus, an increase in σ^2 induces changes in equilibrium β. First, holding e_2 and V unchanged (as in the special case above), a higher σ^2 makes the performance measure noisier, leading to a lower incentive rate. Second, as discussed earlier, the presence of e_2 has a subtle, two-way interaction effect on the incentive rate. The overall impact of an exogenous change in σ^2 on incentive rate β is far more complicated than it is in the standard setting.

3.1 Endogenous Variance with No Spillover Effect

We first consider the case in which the agent is able to influence performance measure variance through e_2, but there is no spillover effect of e_2 on the marginal cost of e_1. For example, if the effort cost function is additively separable, that is, in the form of $C = L(e_1) + K(e_2)$, then there is no spillover ($C_{12} = 0$) and the agent would consider each task separately because the benefit and cost of each task are separable in his choice problem.

[15] It can be shown that this intuition holds even if C_{11} is a function of e_1 (thus β), making equation (5) implicit in β. In the standard LEN model, cost of effort is usually quadratic so C_{11} is a constant, making equation (5) explicit in β.

Following the basic set up, when we totally differentiate (1) and (2) and divide them by $d\beta$, we get

$$\begin{aligned} q - C_{11}\frac{de_1}{d\beta} - C_{12}\frac{de_2}{d\beta} &= 0, \\ -rV_1\beta - C_{12}\frac{de_1}{d\beta} - (\frac{r}{2}V_{11}\beta^2 + C_{22})\frac{de_2}{d\beta} &= 0. \end{aligned} \tag{6}$$

To reduce the mathematical complexity while maintaining the basic economic intuition, we assume $V_{11} = 0$ to better illustrate the main point of the analysis.[16] With $V_{11} = 0$, (6) gives us

$$\begin{aligned} \frac{de_1}{d\beta} &= \frac{qC_{22} + rC_{12}V_1\beta}{\Delta}, \\ \frac{de_2}{d\beta} &= -\frac{qC_{12} + rC_{11}V_1\beta}{\Delta}, \text{ where } \Delta \equiv C_{11}C_{22} - C_{12}^2. \end{aligned} \tag{7}$$

With (1), (2), (7), and $C_{12} = 0$, the first-order condition for optimal choice of incentive β (equation 3) can be transformed into

$$(1-\beta)\frac{q^2}{C_{11}} - r\beta[V(e_2, \sigma^2) + \sigma_x^2] = 0. \tag{8}$$

When σ^2 increases, the principal needs to adjust β accordingly so that the new marginal benefit equals to the new marginal cost of changing β. That is, $SOC_\beta \frac{\partial\beta}{\partial\sigma^2} - r\beta V_2 = 0$, where SOC_β is the second-order condition discussed in Footnote 12. Since the second order condition of β must be negative and $V_2 > 0$, the principal has to decrease the bonus rate β as σ^2 increases. Therefore the incentive-risk relationship remains negative.

Proposition 1 *With endogenous variance but no spillover effect ($V_1 < 0$ and $C_{12} = 0$), the incentive-variance relation is negative.*

Proof. All proofs appear in the appendix. ■

Intuitively, as σ^2 increases, the agent's working environment is harder to measure (for example, a technology change makes the firm's competitive environment harder to predict), imposing more risk on the agent; to respond, the principal lowers bonus rate. This intuition is captured by equation (8). That is, the marginal cost gets larger because the performance measure gets noisier and the principal has to pay a higher risk premium. The only way to reduce the risk premium is to offer a lower β. With a lower incentive, the marginal

[16] We thank an anonymous referee for suggestions on simplifying the math of our analysis. This assumption is not the driving assumption for our results, but helps simplify the mathematical complexity in our analysis of incentive-variance relation. All results in Section 3 remain without this assumption. In addtion, Sections 4 and 5 are not restricted by this assumption. The more general analysis without this simplying assumption was in an earlier version of our paper and is available upon request.

benefit, $(1-\beta)\frac{q^2}{C_{11}}$, also increases, and the new optimal β matches the marginal benefit with the marginal cost. Notice the effects on performance reporting effort (e_2) is zero by Envelop theorem.[17] The trade-off here is similar to that in a standard agency setting without endogenous variance, and the incentive-risk relation remains negative.

3.2 Endogenous Variance and Spillover Effect

Now we consider the case with the spillover effect. That is, the manager can affect the performance measure variance through his effort e_2, but the performance reporting effort increases the marginal cost of his productive effort e_1. With $C_{12} > 0$, first-order condition for optimal choice of incentive β (equation 3) becomes

$$q(1-\beta)\frac{qC_{22} + rC_{12}V_1\beta}{\Delta} - r\beta[V(e_2, \sigma^2) + \sigma_x^2] = 0.$$

As σ^2 increases, again, the principal needs to adjust β accordingly so that the new marginal benefit equals to the new marginal cost of changing β. Therefore, we have $SOC_\beta \frac{\partial \beta}{\partial \sigma^2} + \frac{q(1-\beta)rC_{12}V_{12}}{\Delta} - r\beta V_2 = 0$. Different from the no spillover case in the last subsection, with the spillover between the two efforts, $\frac{\partial \beta}{\partial \sigma^2}$ can turn positive if $V_{12} > 0$ and is sufficiently large while remains negative otherwise.

What makes the result different from the no-spillover case is the critical role of limited attention: C_{12}. In the presence of spillover effects ($C_{12} > 0$), the incentive-risk relation depends on the sign and magnitude of V_{12}, that is, the relative effectiveness of performance reporting task at reducing measurement error (V_1) when environment is hard to measure (high σ^2) versus when it is easier to measure (low σ^2). Suppose the performance reporting effort becomes less effective in reducing the variance as σ^2 becomes larger (that is, V_1 becomes less negative for larger σ^2), the manager is less motivated to provide performance reporting effort e_2, and less e_2 reduces the marginal cost of productive effort e_1.[18] As a result, it becomes easier for the principal to motivate more productive effort by a higher β. Therefore, the principal will offer a higher incentive as σ^2 increases, and the relation between σ^2 and the incentive β appears positive.

To summarize, with both endogenous variance and spillover effect, we find that the relation between risk and incentive becomes ambiguous. Two mutually exclusive equilibrium outcomes are possible. First, in response to an increase in measurement risk in the environment (σ^2), the principal induces the agent to re-allocate efforts toward more performance reporting task (e_2) and thus lowers the incentive rate β. This happens when the performance reporting effort

[17] That is, the agent chooses e_2 endogenously for any given parameter to satisfy equation (2), therefore there is no e_2-related effect on the principal's choice of β.

[18] In practice, it is possible that the manager's e_2 effort becomes less effective when facing high risk in business. When addressing the risk management in industries that rely on R&D and innovations, Elsum (2008) comments that "one size does not fit all – distinctly different management frameworks are required for success in research, development and/or innovation with high compared with low uncertainty. Most organizations find this difficult to cope with."

becomes more effective in reducing the performance variance as σ^2 becomes larger (V_{12} is either negative or positive but sufficiently small). Second, when the performance reporting effort becomes sufficiently ineffective in reducing the performance variance, the principal induces the agent to re-allocate efforts toward more productive task (e_1) and thus raises the incentive rate β. The interaction between spillover and performance reporting task is the driving force of the results, which is in contrast to the economic intuition in the standard agency model.

The following proposition summarizes our finding on positive incentive-risk relation:[19]

Proposition 2 *With endogenous variance and spillover effect ($V_1 < 0$ and $C_{12} > 0$),*

$$\frac{\partial \beta^*}{\partial \sigma^2} > 0 \text{ if } V_{12} \text{ is positive and sufficiently large.}$$

The preceding analysis has implications on empirical analysis in managerial accounting research. Our model indicates that the relation between the exogenous precision of the performance metrics and the strength of managerial incentives depends on the limited attention effect. Our analysis shed new lights on the reason underlying the mixed finding about the relation between risk and incentive. Further, our paper shows that in the empirical research design, controlling for cross-sectional differences about the spillover (C_{12}) may be important and empirically measuring the endogenous variables such as performance reporting task (e_2) may also be helpful.

In addition, sometimes the performance measurement variance captured by empirical data may be more likely the endogenous variance, $V(e_2, \sigma^2) + \sigma_x^2$, instead of the exogenous variance, $\sigma^2 + \sigma_x^2$. Based on our above analysis, the relation between the endogenous variance and incentive can be positive as well. For example, in the above case of positive incentive-to-exogenous-variance, as σ^2 increases, the principal may offer a higher β to motivate more productive effort while the manager may input less performance reporting effort. Less performance reporting effort, together with a higher σ^2, lead to a higher $V(e_2, \sigma^2)$. As a result, the relation between the incentive and the endogenous variance can be positive.

4 Additional Signal

This far, we have limited the way with which the principal can address the incentive problem caused by limited attention. That is, the only way to promote more attention to production is for the principal to offer a higher incentive β. In this section, we consider an alternative method of redirecting managerial attention, which is to employ an additional performance signal whose variance is

[19] We can show that an explicit sufficient condition for $\frac{\partial \beta^*}{\partial \sigma^2} > 0$ is $V_{12} > \max\{0, \frac{r\Delta(-V_1)V_2}{q^2 C_{22} - r\Delta(\sigma^2 + \sigma_x^2)}\}$. Detailed analysis is in the proof in Appendix.

unaffected by the agent's performance reporting effort. Compared with signals generated by a sophisticated accounting information system, the precision of certain other signals (such as hours worked, output quantities, cash flows, or stock price) are affected by managers' performance reporting task to a lesser degree. Here we abstract away from the richness in the different sensitivities to managerial reporting efforts and explore the extreme case of signals with precision unaffected by the management. This exploration allows us to compare, qualitatively, the optimal use of two different signals with such a distinctive difference and offers new insights into the value of an additional signal, a vital theoretical interest in agency theory since Holmstrom (1979). In particular, it may be efficient to exclude a signal with endogenous variance from contracting in the presence of a signal with exogenous variance.

To begin, we modify the model to include an additional performance measure z. Both z and y are noisy measures of x.[20]

$$
\begin{aligned}
y &= x + \varepsilon_y \\
z &= x + \varepsilon_z
\end{aligned}
$$

However, unlike y, the additional signal z's variance σ_z^2 cannot be reduced through the agent's effort. That is,

$$
\begin{bmatrix} \varepsilon_y \\ \varepsilon_z \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} V(e_2, \sigma^2) & 0 \\ 0 & \sigma_z^2 \end{bmatrix} \right).
$$

The principal offers a linear contract on y and z. As in previous setting, α is a fixed wage and β is the bonus rate on y. In addition, the contract also assigns a bonus rate δ on z.

$$
w = \alpha + \beta y + \delta z
$$

We first examine the optimal use of the two signals in two benchmarks. In the first benchmark, both variances of performance measure y and z are exogenous, as in most standard agency models. In the second benchmark, the variance of y can be reduced by e_2, but the costs of e_1 and e_2 are separable. In these two benchmarks, we find that both measures are useful (that is, the principal is better off by including both measures into the contract) as long as their variances are non-degenerate. Then we consider a setting in which the variance of y is endogenous and e_2 has a spillover effect on the marginal cost of e_1. In this setting, we show it may be efficient to exclude measure y from the contract even if the variance of y is non-degenerate. The reason is, again, that using y would, via e_2, induce a higher marginal cost of productive effort e_1 and y's incentive benefit cannot offset this cost increase in the presence of another performance signal.

[20]If the additional signal (z) is informative about e_2 (e.g., $z = e_2 + \varepsilon_z$), we show that z is not used in optimal contract if in equilibrium e_2 is interior.

4.1 Benchmark settings

Consider the following two settings:

- In the first benchmark, we return to a simpler setting in which the agent's effort does not affect the variance of performance measures. This setting is consistent with standard agency studies such as Holmstrom (1979) and Feltham and Xie (1994). Without loss of generality, we parameterize this benchmark by setting $V(e_2, \sigma^2) = \sigma^2$. We label this setting *exogenous variance.*

- In the second benchmark, the agent is able to exert e_2 to reduce variance of the performance measure y. However, the personal cost of the agent's effort is separable in e_1 and e_2 ($C_{12} = 0$). Without loss of generality, we parameterize this benchmark by setting $C(e_1, e_2) = L(e_1) + K(e_2)$. We label this setting *separable costs.*

Lemma 3 summarizes the optimal use of the two performance measures in these two benchmark settings.

Lemma 3 *Under either exogenous variance setting $V(e_2, \sigma^2) = \sigma^2$ or the separable cost setting $C(e_1, e_2) = L(e_1) + K(e_2)$,*

$$\beta^*,\ \delta^* > 0 \Longleftrightarrow V(e_2, \sigma^2),\ \sigma_z^2 < +\infty \textit{ for all } e_2 \tag{9}$$

In Lemma 3, the result of the first benchmark is a reproduction of the standard agency conclusion from Holmstrom (1979), Banker and Datar (1989), and Feltham and Xie (1994). The standard agency models with exogenous variances show that any informative signal about the agent's productive effort, no matter how imperfect, can be used in contracting to improve the principal's welfare. The key argument is that the principal will always use a signal as long as its variance is finite, because the principal can always place a sufficiently small weight on the signal to balance the marginal cost from a higher risk premium against the marginal benefit from a higher productive effort.

The result of the second benchmark shows that the standard agency conclusion still holds with an endogenous variance, as long as the cost of performance reporting effort is separable from the cost of productive effort (no spillover). Again, the principal can always choose a proper weight on the signal to balance the marginal cost and benefit. However, there is one more marginal cost in the second benchmark: the cost of performance reporting effort. Nevertheless, when the bonus weight is close to zero, the marginal benefit always outweighs the marginal cost; thus the principal can always benefit from slightly increasing the bonus weight from zero.

4.2 Additional Signal Setting with Spillover

Now we examine a setting with the spillover effect. That is, where exerting performance reporting effort e_2 may affect the marginal cost of productive effort e_1. For simplicity, we use a specific example, supposing $V(e_2, \sigma^2) = \frac{\sigma^2}{e_2}$

and $C(e_1, e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$. This specification helps us to provide explicit analysis and clearly illustrate the intuition. A general analysis is available in the Appendix.

For this specific example, we define $\widehat{\beta}$, $\widehat{\delta}$ as the optimal incentives on y and z, and $\widehat{e_1}$, $\widehat{e_2}$ as the optimal effort by the agent. Assuming an interior solution, the optimal bonus coefficients β and δ can be shown to be

$$\widehat{\beta} = \frac{q(q - \sigma\sqrt{kr}) - q^2\widehat{\delta} - r\widehat{\delta}\sigma_x^2 c_1}{q^2 - \sigma^2 kr + r\sigma_x^2 c_1}, \quad \text{and } \widehat{\delta} = \frac{q^2(1 - \widehat{\beta}) - r\widehat{\beta}\sigma_x^2 c_1}{q^2 + c_1 r(\sigma_z^2 + \sigma_x^2)}.$$

Even with a spillover effect between the two effort choices, the signal with an exogenous precision, z, is always used in the optimal contract. To see this, suppose $\widehat{\delta} = 0$, then we have $\widehat{\beta} = \frac{q(q-\sigma\sqrt{kr})}{(q+\sigma\sqrt{kr})(q-\sigma\sqrt{kr})+r\sigma_x^2 c_1}$, and it can be easily verified that the marginal benefit of increasing δ is higher than the marginal cost at $\delta = 0$. Therefore the principal can improve her payoff by increasing δ from zero. In other words, signal z is always used in the contract, consistent with the intuition in Lemma 3.

However, $\widehat{\beta}$ is no longer guaranteed to be positive with the non-separable costs of e_1 and e_2. Suppose $\widehat{\beta} = 0$, then $\widehat{\delta} = \frac{q^2}{q^2+c_1 r(\sigma_z^2+\sigma_x^2)}$. we can show that, when the precision of signal z is high enough, the marginal benefit of using signal y is lower than its marginal cost, and a corner solution $\widehat{\beta} = 0$ is indeed optimal.[21] That indicates that the signal y, although informative, may be ignored in the contracting. This is different from conclusions in standard agency models and the separable costs case without spillover effect (shown in Lemma 3).

Notice the key different assumption is the spillover effect. In the separable costs case (i.e., no spillover), although the marginal cost does not approach zero as the bonus weight β approaches zero, it is always outweighed by the marginal cost. However, with the spillover effect, the marginal cost of increasing β does not approach zero as β approaches zero, and it can outweigh the marginal benefit because spillover effect makes using signal y more expensive (i.e., the magnitude of marginal cost of productive effort e_1 is made higher). As a result, the principal may find it efficient to drop the signal y whose variance can be affected by the manager's e_2 effort.[22] We summarize this result in the following proposition.

[21] Recall that β must be non-negative, which can be regarded as an implicit constraint $\beta \geq 0$. If we incorporate this constraint in the program and examine the Kuhn-Tucker conditions, we see in most cases this condition is not binding. However, when $\sigma_z^2 < \frac{\sigma\sqrt{rk}(q^2+rc_1\sigma_x^2)}{rc_1(q-\sigma\sqrt{rk})}$, the first order condition with respect to β is not zero and its Kuhn-Tucker multiplier is zero, while the condition $\beta \geq 0$ is binding with its Kuhn-Tucker multiplier being positive. β must be zero and will not deviate from zero.

Given $\beta = 0$, we only need to check the second order condition with respect to δ. The second order derivative of principal's objective function with respect to δ is $-\frac{q^2}{c_1} - r\left(\sigma_z^2 + \sigma_x^2\right) < 0$. Therefore, the second order condition is satisfied, and $\{\widehat{\delta} = \frac{q^2}{q^2+c_1 r(\sigma_z^2+\sigma_x^2)}, \widehat{\beta} = 0\}$ is indeed global maximum when $\sigma_z^2 < \frac{\sigma\sqrt{rk}(q^2+rc_1\sigma_x^2)}{rc_1(q-\sigma\sqrt{rk})}$.

[22] In standard agency settings, when the bonus weight is close to zero, the marginal benefit is positive while the marginal cost is zero because both the risk premium and the manager's

Proposition 4 *In the case of endogenous variance with additional signal, suppose* $V(e_2, \sigma^2) = \frac{\sigma^2}{e_2}$ *and* $C(e_1, e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$*, then the optimal solution has* $\{\widehat{\beta} = 0, \widehat{\delta} = \frac{q^2}{q^2 + c_1 r(\sigma_z^2 + \sigma_x^2)} > 0\}$ *iff* $\sigma_z^2 < \frac{\sigma\sqrt{rk}(q^2 + rc_1\sigma_x^2)}{rc_1(q - \sigma\sqrt{rk})}$.

On the surface, ignoring the signal y may seem to be an undesirable move, since an informative signal is unused and the manager may be less motivated. For example, a firm's information system collects many financial and non-financial metrics, not all of which are used in the top managers' compensation even if they are all, presumably, informative about their productive effort. There may be many sound reasons for which they are not used in equilibrium managerial contracts. In our model, ignoring signal y also has a desirable consequence of drawing the managers' attention away from performance reporting tasks, and towards productive tasks instead. When signal z is precise enough, the desirable effect dominates the undesirable effect in the trade-off, and the principal finds it efficient not to use signal y.

Allocation between the two effect choices is the key underlying tension here, we see that by introducing an additional performance measure z, the principal is able to redirect the agent's attention from performance reporting to production (it can be shown that $\widehat{e_1} > e_1^*$ and $\widehat{e_2} < e_2^*$). When the performance reporting effort has the spillover effect on the cost of productive effort, including y in contracting draws the agent's attention to performance reporting, thus making the productive tasks more costly. An additional performance measure that cannot be modified by the agent may help the principal alleviate the tension in managerial attention. Further, we see sometimes it is efficient for the principal to exclude the performance measure y from contracting ($\widehat{\beta} = 0$).[23]

5 Project Selection and Endogenous Variance

We now apply the baseline model of endogenous variance to study project selection. We show that the manager's performance reporting task may affect the principal's trade-off in inducing the project selection choice by the manager. More specifically, different from predictions from previous studies in our model the principal may motivate riskier projects by a higher incentive when the performance variance can be reduced through the manager's effort, .

personal cost of efforts are quadratic. In the separable costs case with no spillover, the marginal cost of increasing the incentive on signal z approaches zero as the bonus weight β approaches zero due to the quadratic form of personal effort, but the total marginal cost does not go to zero as the term $r\delta\sigma_x^2$ due to the covariance between the signals (see equation 14 in appendix). However, the total marginal cost is always outweighed by the marginal benefit, and it is still efficient to include signal y in the contract. With the spillover effect between effort choices, not only the total marginal cost of increasing β does not approach zero as β approaches zero, it can also outweigh the marginal benefit (see equation 23 in appendix).

[23] This result is in contrast to Lemma 3 where any signal with a bounded variance will be used in contracting. With the spillover effect, the standard agency conclusion applies to the bonus rate on z (i.e., the optimal δ is always positive) but may not necessarily hold for the bonus rate on y (i.e., β may become zero).

To facilitate the project-selection choice, we enrich the model by assuming there are two mutually exclusive types of projects, H- and L-project. The H-project has a higher profitability than the L-project, but also higher risk. The expected return of a project also depends on the manager's productive effort e_1. Formally,

$$x_i = q_i e_1 + \varepsilon_{xi}, i \in \{H, L\},$$

where $q_H > q_L, \varepsilon_{xi} \sim N(0, \sigma_{xi}^2)$, and $\sigma_{xH}^2 > \sigma_{xL}^2$. Again, we assume the output of the project is realized too late for contracting, but there is a contractible signal y which is a noisy signal of the output,[24]

$$y_i = x_i + \varepsilon_{y_i}, i \in \{H, L\}.$$

In addition, we assume that the original performance measure for the outcome of a high-risk project (H-project) is also noisier, but the original performance measure noise can be reduced through the manager's e_2 effort. That is,

$$\begin{aligned}
\varepsilon_{yH} &\sim N\left(0, \frac{\sigma_{yH}^2}{e_2}\right), \\
\varepsilon_{yL} &\sim N\left(0, \frac{\sigma_{yL}^2}{e_2}\right), \text{ where } \sigma_{yH}^2 > \sigma_{yL}^2.
\end{aligned}$$

We further assume the cost function of effort is $C(e_1, e_2) = \frac{c}{2} e_1^2 + k e_2$.[25] In addition, the manager's project selection is costless.

We use these assumptions to capture the potential industry variations in the underlying risk of cash flows (σ_x^2) and in performance measures intended to capture these risks (σ_y^2). Some empirical evidence may suggest that firms undertaking high-risk projects are motivated to improve the informativeness of their performance measurements. For example, the current financial reporting model is alleged to be particularly ill-suited for high-tech industries such as pharmaceuticals, computers, and telecommunications. In other words, earnings for these industries are much noisier performance measures than those in traditional industries. However, Francis and Schipper (1999) find that compared with traditional industries, high-tech industries do not show lower value relevance of their financial information. Similarly, Collins, Maydew and Weiss (1997) show that the combined value-relevance of earnings and book values has not declined over the past 40 years as we shift from an industrialized economy to a high-tech, service-oriented economy. Instead, the value relevance appears to have increased slightly. These evidences may indicate that firms with high-risk projects are engaged in improving the informativeness/precision of their financial information.

[24] Our result does not change if $y_i = q_i e_1 + \varepsilon_{yi}$.

[25] The cost function is separable in the two efforts and there is no spillover. The key for the different prediction on project selection is the endogenous variance instead of spillover effect. Introducing spillover effect may weaken the result that the principal is able to motivate riskier project with a higher incentive, as a higher incentive may induce lower e_2 effort with the spillover effect.

5.1 Observable Project Selection Benchmark

We first look at a benchmark case in which the manager's project selection decision is observable. Here project selection can be dictated by the principal. Incentive contracts are used only to induce the agent's effort choices for a project already chosen.

As a simple benchmark, we assume $\frac{\sigma_{yH}}{q_H^2} = \frac{\sigma_{yL}}{q_L^2} \equiv \phi$ and $\frac{\sigma_{xH}}{q_H} = \frac{\sigma_{xL}}{q_L} \equiv \psi$. With these simplifying assumptions, we show the principal places the same incentive weight the performance signal for either the H-project or the L-project (i.e., $\beta_H^* = \beta_L^* = \frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}$.)

In this setting in which the principal observes the manager's project selection decision, her payoff when she motivates H-project is

$$PP_H = \frac{1-c\sqrt{2rk}\phi}{2c(1+rc\psi^2)}(q_H^2 - 2c\sigma_{yH}\sqrt{2rk} + c\sqrt{2rk}\phi q_H^2),$$

and her payoff when she motivates L-project is

$$PP_L = \frac{1-c\sqrt{2rk}\phi}{2c(1+rc\psi^2)}(q_L^2 - 2c\sigma_{yL}\sqrt{2rk} + c\sqrt{2rk}\phi q_L^2).$$

The principal prefers H-project if and only if $PP_H > PP_L$, which implies $\phi < \frac{1}{c\sqrt{2rk}}$. Intuitively, the principal finds the risk-premium too high relative to the expected return when ϕ is too high. Even though the principal is risk-neutral, confronting a risk-averse agent makes the principal act as if she is risk-averse when it comes to project selections.

5.2 Unobservable Project Selection

Now we suppose the manager's project selection decision is unobservable. In this setting, if the principal still offers $\beta_H^* = \frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}$, the manager may not choose the principal's desirable project. The manager's certainty equivalent when choosing H-project is

$$\begin{aligned} CE_H &= \alpha^* + \beta_H^* q_H e_{1H}^* - \frac{r}{2}\beta_H^{*2}(\sigma_{xH}^2 + \frac{\sigma_{yH}^2}{e_{2H}^*}) - \frac{c}{2}e_{1H}^{*2} - ke_{2H}^* \\ &= \alpha^* + \frac{q_H^2}{2c}\beta_H^{*2} - \frac{r\sigma_{xH}^2}{2}\beta_H^{*2} - \sqrt{2rk}\sigma_{yH}\beta_H^*, \end{aligned}$$

and his certainty equivalent when choosing L-project is

$$\begin{aligned} CE_L &= \alpha^* + \beta_H^* q_L e_{1L}^* - \frac{r}{2}\beta_H^{*2}(\sigma_{xL}^2 + \frac{\sigma_{yL}^2}{e_{2L}^*}) - \frac{c}{2}e_{1L}^{*2} - ke_{2L}^* \\ &= \alpha^* + \frac{q_L^2}{2c}\beta_H^{*2} - \frac{r\sigma_{xL}^2}{2}\beta_H^{*2} - \sqrt{2rk}\sigma_{yL}\beta_H^*. \end{aligned}$$

When $CE_H < CE_L$, the manager will choose L-project, even if the principal desires the riskier project. The fundamental tension introduced by unobservable project selection (by the agent) is the potential conflict of interests between the agent and the principal regarding the choice of projects. Even if the act of choosing project is not personally costly, there may exist a moral hazard problem on the project-choice, which is termed induced moral hazard by Baiman and Demski (1980).

A sufficient condition for the manager to choose L-project ($CE_H - CE_L < 0$) is $\phi > \frac{1}{3c\sqrt{2rk}}$.[26] Therefore, when $\frac{1}{3c\sqrt{2rk}} < \phi < \frac{1}{c\sqrt{2rk}}$, the manager will choose the safer L-project when offered the same contract as in the observable setting, while the principal desires the riskier H-project.

If the principal would like to motivate the manager to choose H-project in this unobservable setting, she has to offer a different β_H. Solving the principal's program gives us $\beta_H^{'*} = \frac{2c\sqrt{2rk}\phi}{1-rc\psi^2}$ when $\frac{1}{3c\sqrt{2rk}} < \phi < \frac{1}{c\sqrt{2rk}}$. Obviously, $\beta_H^{'*} > \beta_H^*$. That is, to induce the manager to choose the desirable riskier project, the principal must offer a higher incentive. With this higher incentive, the manager's performance reporting effort, $e_{2H}^{'*} = \beta_H^{'*}\sigma_{yH}\sqrt{\frac{r}{2k}}$, is higher too.

Proposition 5 *With endogenous variance, when* $\frac{1}{3c\sqrt{2rk}} < \phi < \frac{1}{c\sqrt{2rk}}$*, the principal has to offer a higher incentive* $\beta_H^{'*} = \frac{2c\sqrt{2rk}\phi}{1-rc\psi^2}$ *than that in the observable project selection benchmark to motivate the manager to choose riskier project. Further, the induced performance reporting effort is higher* $e_{2H}^{'*} > e_{2H}^*$.

This result is in contrast to Sung (1995)'s prediction that the principal would lower the sensitivity of the incentive to motivate the manager to take a riskier project. The reason for this difference is the manager's ability to reduce the performance measurement risk in our model, which is absent in Sung (1995). In Sung's setting, as the incentive increases, the manager is still reluctant to take a riskier project with a higher return since the risk premium increases in the incentive. In our paper, however, the increase in incentive β also induces a higher managerial effort to reduce the risk, which may offset the increase in the risk premium due to a higher β. Therefore, it is more likely that the principal can motivate a riskier project selection with a higher incentive in our setting.

Finally, we use a numerical example to show the above result. Suppose $\phi = 1, \psi = \frac{5}{2}, c = \frac{1}{8}, r = 1, k = 18, q_L^2 = 1$ and $q_H^2 = 2$. In the observable setting, the principal offers $\beta_H^* = 0.1905$. She prefers H-project since the riskier project brings higher payoff, with $PP_H - PP_L = 0.86 > 0$. However, if the project selection decision is unobservable and the manager is still offered the same contract, the manager will choose L-project since the safer project gives him higher certainty equivalent ($CE_H - CE_L = -0.347 < 0$). To induce H-project, the principal has to offer $\beta_H^{'*} = 2.1818$, which is higher than β_H^*.

[26] When $\phi > \frac{1}{3c\sqrt{2rk}}$, we have $1 - c\sqrt{2rk}\phi < 2c\sqrt{2rk}\phi$, and thus $1 - c\sqrt{2rk}\phi < 2c\sqrt{2rk}\phi + 2rc^2\psi^2\sqrt{2rk}\phi$, which can be rewritten as $\frac{\beta_H^{*2}}{2c}(q_H^2 - q_L^2) < \sqrt{2rk}\beta_H^*(\sigma_{yH} - \sigma_{yL})$. Therefore we have $CE_H < CE_L$.

6 Conclusion

The paper focuses on the trade-off between two competing demands on managerial attention. One is the productive effort which increases the expected output of the firm and the other is the performance reporting effort which increases the quality of the manager's own performance measure. This research identifies a complication in the manager's effort allocation. Our analysis illustrates that the incentive contract shows a mixed risk-incentive relation. Further, we find that, with the spillover effect, sometimes an informative signal is discarded to avoid high marginal cost of productive effort. We also apply our model to a specific project-selection setting and show that when the project-selection choice is unobservable, the principal may raise the incentive to motivate the manager to choose a riskier project.

The main analysis is carried out in a tractable LEN framework. It would be interesting to see if the result regarding additional signals holds in generalized non-linear contracts. In addition, a multiple-period version of this model that allows for an intertemporal performance reporting effort may elicit additional features. These are potential extensions of the current setting.

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Appendix

Proof of Proposition 1

With (1), (2) and (7), (3) can be rewritten into a quadratic equation of β :

$$-rqC_{12}V_1(e_2,\sigma^2)\beta^2-\{q^2C_{22}+r[(V(e_2,\sigma^2)+\sigma_x^2)\Delta-qC_{12}V_1(e_2,\sigma^2)]\}\beta+q^2C_{22}=0. \tag{10}$$

With $C_{12}=0$, from (10) we see the optimal solution for β is$\leq$

$$\beta^* = \frac{q^2}{q^2+r[V(e_2,\sigma^2)+\sigma_x^2]C_{11}(e_1)}.$$

It is easy to verify that

$$\frac{\partial\beta^*}{\partial\sigma^2} = -\frac{rq^2C_{11}V_2}{[q^2+r(V+\sigma_x^2)C_{11}]^2} < 0.$$

Proof of Proposition 2

With $V_1<0$ and $C_{12}>0$, from (10) the optimal solution for β is

$$\beta^* = \frac{q^2C_{22}+r[(V+\sigma_x^2)\Delta-qC_{12}V_1]-\sqrt{\{q^2C_{22}+r[(V+\sigma_x^2)\Delta-qC_{12}V_1]\}^2+4rq^3C_{12}C_{22}V_1}}{-2rqC_{12}V_1}.$$

Take derivative of β^* with respect to σ^2, we get $\frac{\partial\beta^*}{\partial\sigma^2} = \frac{q^2C_{22}V_{12}(1-\beta^*)-r\Delta[(-V_1)V_2+V_{12}(V+\sigma_x^2)]\beta^*}{(-V_1)\sqrt{\{q^2C_{22}+r[(V+\sigma_x^2)\Delta-qC_{12}V_1]\}^2+4rq^3C_{12}C_{22}V_1}}$. The denominator of $\frac{\partial\beta^*}{\partial\sigma^2}$ is positive, therefore the sign of $\frac{\partial\beta^*}{\partial\sigma^2}$ depends on the numerator, $q^2C_{22}V_{12}(1-\beta^*)-r\Delta[(-V_1)V_2+V_{12}(V+\sigma_x^2)]\beta^*$. It is easy to verify that the numerator is positive if $V_{12} > \frac{r\Delta(-V_1)V_2\beta^*}{(1-\beta^*)q^2C_{22}-r\Delta(V+\sigma_x^2)\beta^*}$.

In addition, from the analysis in Section 3.2, V_{12} must be positive to have $\frac{\partial\beta^*}{\partial\sigma^2}>0$.

To get a sufficient condition for $\frac{\partial\beta^*}{\partial\sigma^2}>0$, notice that $\Gamma \equiv \frac{r\Delta(-V_1)V_2\beta^*}{(1-\beta^*)q^2C_{22}-r\Delta(V+\sigma_x^2)\beta^*} = 0$ when $\beta=0$, and $\Gamma = \frac{(-V_1)V_2}{-(V+\sigma_x^2)}<0$ when $\beta=1$. In addition, $\frac{\partial\Gamma}{\partial\beta}>0$ if $\beta<\frac{1}{2}$, and $\frac{\partial\Gamma}{\partial\beta}\leq 0$ if $\beta\geq\frac{1}{2}$. This indicates that Γ maximizes at $\beta=\frac{1}{2}$ and its maximum value is $\frac{r\Delta(-V_1)V_2}{q^2C_{22}-r\Delta(V+\sigma_x^2)}$. Further, $V\leq\sigma^2$. Therefore, an explicit sufficient condition for $\frac{\partial\beta^*}{\partial\sigma^2}>0$ is $V_{12}>\max\{0,\frac{r\Delta(-V_1)V_2}{q^2C_{22}-r\Delta(\sigma^2+\sigma_x^2)}\}$.

Proof of Lemma 3

(1) Benchmark 1: Exogenous Variance ($V(e_2,\sigma^2)=\sigma^2$)

Define $V(e_2^0,\sigma^2)=\sigma^2$. In Benchmark 1, the agent can only choose e_2^0 so that the variance is not modified.

The agent chooses his productive effort e_1 to maximize his payoff $\alpha+\beta E[y]+\delta E[z]-\frac{r}{2}\beta^2(V+\sigma_x^2)-\frac{r}{2}\delta^2(\sigma_z^2+\sigma_x^2)-r\beta\delta\sigma_x^2-C(e_1,e_2^0)$. From the first order condition with respect to e_1, we have $C_1(e_1, e_2) = q(\beta + \delta)$, $\frac{de_1}{d\beta} = \frac{q}{C_{11}(e_1,e_2)}$, and $\frac{de_1}{d\delta} = \frac{q}{C_{11}(e_1,e_2)}$.

The principal's problem is:

$$\max_{\beta,\delta} q e_1^* - \frac{r}{2}\beta^2(\sigma^2 + \sigma_x^2) - \frac{r}{2}\delta^2(\sigma_z^2 + \sigma_x^2) - r\beta\delta\sigma_x^2 - C(e_1^*, e_2^0)$$

The principal's first order conditions show that:

$$\begin{aligned}
\frac{q^2}{C_{11}(e_1, e_2^0)} - r\beta(\sigma^2 + \sigma_x^2) - r\delta\sigma_x^2 - \frac{q^2(\beta+\delta)}{C_{11}(e_1, e_2^0)} &= 0;\\
\frac{q^2}{C_{11}(e_1, e_2^0)} - r\delta(\sigma_z^2 + \sigma_x^2) - r\beta\sigma_x^2 - \frac{q^2(\beta+\delta)}{C_{11}(e_1, e_2^0)} &= 0.
\end{aligned}$$

From the principal's first order conditions, we have:

$$\begin{aligned}
\beta^* &= \frac{q^2/C_{11}}{r(\sigma^2+\sigma_x^2)\frac{\sigma_z^2}{\sigma^2} + r\sigma_x^2 + q^2(\frac{\sigma_z^2}{\sigma^2}+1)/C_{11}} > 0,\\
\delta^* &= \frac{(q^2\frac{\sigma_z^2}{\sigma^2})/C_{11}}{r(\sigma^2+\sigma_x^2)\frac{\sigma_z^2}{\sigma^2} + r\sigma_x^2 + q^2(\frac{\sigma_z^2}{\sigma^2}+1)/C_{11}} > 0.
\end{aligned}$$

Therefore, in Benchmark 1, β^* and δ^* are both positive, as long as $\sigma_z^2, \sigma^2, \sigma_x^2 < +\infty$. In addition, β^* and δ^* have a relation that $\beta^*\sigma^2 = \delta^*\sigma_z^2$.

(2) Benchmark 2: Separable Cost ($C(e_1, e_2) = L(e_1) + K(e_2)$ and following convention, the $'$ symbol indicates partial derivatives such as $L'(e_1)$ and $K'(e_2)$)

The agent's problem with the additional signal is

$$\max_{e_1,\ e_2} \alpha + \beta E[y] + \delta E[z] - \frac{r}{2}\beta^2[V(e_2,\sigma^2) + \sigma_x^2] - \frac{r}{2}\delta^2(\sigma_z^2 + \sigma_x^2) - r\beta\delta\sigma_x^2 - C(e_1, e_2) \tag{11}$$

The first order condition with respect to e_1 show that

$$L'(e_1) = q(\beta + \delta).$$

Totally differentiate this first order condition, we get $\frac{de_1}{d\beta} = \frac{q}{L''(e_1)}$ and $\frac{de_1}{d\delta} = \frac{q}{L''(e_1)}$.

The first order condition with respect to e_2 is

$$-\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - K'(e_2) = 0 \tag{12}$$

(12) implies that if $\beta > 0$, then $e_2^* > 0$ must be true since $V_1 = -\infty$ at $\beta = 0$.

The principal's problem is

$$\max_{\beta,\delta} E[x(e_1)] - \frac{r}{2}\beta^2[V(e_2,\sigma^2) + \sigma_x^2] - \frac{r}{2}\delta^2(\sigma_z^2 + \sigma_x^2) - r\beta\delta\sigma_x^2 - L(e_1) - K(e_2) \tag{13}$$

The problem yields a first-order condition for optimal choice of incentive β:

$$\frac{q^2}{L''}-\frac{q^2(\beta+\delta)}{L''}-r\beta[V(e_2,\sigma^2)+\sigma_x^2]-r\delta\sigma_x^2+\left[-\frac{r}{2}\beta^2V_1(e_2,\sigma^2)-K'(e_2)\right]\frac{de_2}{d\beta}=0 \tag{14}$$

$\frac{q^2}{L''}$ is the marginal benefit of increasing β, and $\frac{q^2(\beta+\delta)}{L''}+r\beta[V(e_2,\sigma^2)+\sigma_x^2]+r\delta\sigma_x^2+\left[-\frac{r}{2}\beta^2V_1(e_2,\sigma^2)-K'(e_2)\right]\frac{de_2}{d\beta}$ is the marginal cost. If marginal benefit is lower than marginal cost, then the optimal β will be zero, which is a corner solution.

If (12) is satisfied (that is, if $e_2^*>0$), according to (12), $-\frac{r}{2}\beta^2V_1(e_2,\sigma^2)-K'(e_2)=0$, we have

$$\beta\ =\beta\left(e_2,\delta,\sigma^2\right)\equiv\frac{q^2(1-\delta)-r\delta\sigma_x^2L''}{r[V(e_2,\sigma^2)+\sigma_x^2]L''+q^2}. \tag{15}$$

The first order condition with respect to δ yields

$$\frac{q^2}{L''}-\frac{q^2(\beta+\delta)}{L''}-r\delta(\sigma_z^2+\sigma_x^2)-r\beta\sigma_x^2=0. \tag{16}$$

$$\delta=\Delta(e_2,\beta,\sigma_z^2)\equiv\frac{q^2(1-\beta)-r\beta\sigma_x^2L''}{r(\sigma_z^2+\sigma_x^2)L''+q^2}$$

There are four possible cases to consider:

1. $\beta^*=0,\delta^*=0$: It leads to $e_1^*=0,e_2^*=0$. This cannot be the optimal solution, since (16) shows a marginal benefit of $\frac{q^2}{L''}>0$ and zero marginal cost. It can be improved by increase δ slightly.

2. $\beta^*>0,\delta^*=0$: (14) becomes $(1-\beta)\frac{q^2}{L''}-r\beta[V(e_2,\sigma^2)+\sigma_x^2]=0$, which gives $\beta^*=\frac{q^2}{r(V+\sigma_x^2)L''+q^2}$. Substitute β^* to (16), we see the net between the marginal benefit and the marginal cost, $[1-\frac{q^2}{r(V+\sigma_x^2)L''+q^2}]\frac{q^2}{L''}-\frac{r\sigma_x^2q^2}{r(V+\sigma_x^2)L''+q^2}=\frac{rVq^2}{r(V+\sigma_x^2)L''+q^2}$, is always positive. That is, the marginal benefit is always higher than the marginal cost and the principal can improve by increasing δ. Therefore $\beta^*>0,\delta^*=0$ cannot be the optimal solution.

3. $\beta^*=0,\delta^*>0$: This leads to $e_2^*=0$. From (16), when $\beta^*=0$ we have $\delta^*=\frac{q^2}{r(\sigma_z^2+\sigma_x^2)L''+q^2}>0$. Substitute δ^* into (14), we have that, evaluated at $e_2^*=0$, $\beta^*=0$ holds when

$$\frac{rq^2(\sigma_z^2+\sigma_x^2)}{r(\sigma_z^2+\sigma_x^2)L''+q^2}\le\frac{rq^2\sigma_x^2}{r(\sigma_z^2+\sigma_x^2)L''+q^2}+K'(e_2)\frac{de_2}{d\beta}\mid_{\beta=0} \tag{17}$$

   Because $K'(e_2)|_{\beta=0}=0$ when costs are separable, inequality (17) doesn't hold, and it is impossible to have $\beta^*=0,\delta^*>0$.

4. $\beta^* > 0, \delta^* > 0$ leading to $e_1^* > 0, e_2^* \geq 0$. When (12) holds ($e_2^* > 0$), the optimal β^* and δ^* are:

$$\beta^* = \frac{q^2/L''}{r(V + \sigma_x^2)\frac{\sigma_z^2}{V} + r\sigma_x^2 + q^2(\frac{\sigma_z^2}{V} + 1)/L''} \tag{18}$$

$$\delta^* = \frac{(q^2\frac{\sigma_z^2}{V})/L''}{r(V + \sigma_x^2)\frac{\sigma_z^2}{V} + r\sigma_x^2 + q^2(\frac{\sigma_z^2}{V} + 1)/L''} \tag{19}$$

When (12) shows a greater marginal cost of e_2 than its marginal benefit ($e_2^* = 0$), we have the optimal β^* and δ^* decided by (18) and (19).

General Analysis of Additional Signal Setting with Spillover

The agent's problem with the additional signal is

$$\max_{e_1,\ e_2} \alpha + \beta E[y] + \delta E[z] - \frac{r}{2}\beta^2[V(e_2, \sigma^2) + \sigma_x^2] - \frac{r}{2}\delta^2(\sigma_z^2 + \sigma_x^2) - C(e_1, e_2)$$

The first order condition with respect to $e_1$shows that the optimal e_1 satisfies

$$q\,(\beta + \delta) = C_1(e_1, e_2).$$

In addition, it yields an additional incentive constraint on the equilibrium choice of e_2:

$$-\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - C_2(e_1, e_2) = 0 \tag{20}$$

The principal's problem is now

$$\max_{\beta,\delta} E[x\,(e_1)] - \frac{r}{2}\beta^2[V(e_2, \sigma^2) + \sigma_x^2] - \frac{r}{2}\delta^2(\sigma_z^2 + \sigma_x^2) - r\beta\delta\sigma_x^2 - C(e_1, e_2) \tag{PP2}$$

yielding a first-order condition for optimal choice of incentive δ, which after substituting $q\,(\beta + \delta) = C_1(e_1, e_2)$, the incentive constraint for e_1, can be written as

$$q\frac{de_1}{d\delta} - r\delta(\sigma_z^2 + \sigma_x^2) - q\,(\beta + \delta)\,\frac{de_1}{d\delta} - r\beta\sigma_x^2 = 0, \tag{21}$$

From $q\,(\beta + \delta) = C_1(e_1, e_2)$, we have $qd\delta = C_{11}de_1 + C_{12}de_2$ which implies $q = C_{11}\frac{de_1}{d\delta} + C_{12}\frac{de_2}{d\delta}$. $-\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - C_2(e_1, e_2) = 0$ implies $\frac{de_2}{d\delta} = 0$. Therefore $\frac{de_1}{d\delta} = \frac{q}{C_{11}(e_1,e_2)}$. Substituting $\frac{de_1}{d\delta}$, the first-order condition for optimal choice of incentive δ becomes

$$\delta = \Delta(e_2, \beta, \sigma_z^2) \equiv \frac{q^2(1 - \beta) - r\beta\sigma_x^2 C_{11}(e_1, e_2)}{r(\sigma_z^2 + \sigma_x^2)C_{11}(e_1, e_2) + q^2}.$$

Additionally, the first order condition with respect to β, which after substituting $q(\beta+\delta)=C_1(e_1,e_2)$ and $\frac{de_1}{d\beta}$, can be written as

$$\frac{q(q-C_{12}\frac{de_2}{d\beta})}{C_{11}}-\frac{q(\beta+\delta)(q-C_{12}\frac{de_2}{d\beta})}{C_{11}}-r\beta(V+\sigma_x^2)-r\delta\sigma_x^2+\left[-\frac{r}{2}\beta^2V_1-C_2\right]\frac{de_2}{d\beta}=0. \tag{22}$$

If we assume $C(e_1,e_2)=e_1^2f(e_2)$, $\frac{de_2}{d\beta}=0$, and if in equilibrium $-\frac{r}{2}\beta^2V_1(e_2,\sigma^2)-C_2(e_1,e_2)=0$ is satisfied, the above equation becomes

$$\frac{q^2}{C_{11}(e_1,e_2)}-\frac{q^2(\beta+\delta)}{C_{11}(e_1,e_2)}-r\beta[V(e_2,\sigma^2)+\sigma_x^2]-r\delta\sigma_x^2+0=0$$

where the $\frac{q^2}{C_{11}(e_1,e_2)}$ term is the marginal benefit of increasing β. The rest of terms are the marginal costs. The above equation is reduced to

$$\beta\ =\frac{q^2(1-\delta)-r\delta\sigma_x^2C_{11}(e_1,e_2)}{r[V(e_2,\sigma^2)+\sigma_x^2]C_{11}(e_1,e_2)+q^2}.$$

In this case, it is easy to see both signals are used in contracts, similar to the benchmark cases. However, if in equilibrium, (20) is not satisfied (that is, if $e_2^*=0$, a corner solution), we show it is possible that $\beta^*=0$ and $\delta^*>0$. Suppose the solution of $\beta=0$ and $\delta>0$ is contemplated. From (21), we learn $\delta=\Delta(e_2,\beta=0,\sigma_z^2)\equiv\frac{q^2}{r(\sigma_z^2+\sigma_x^2)C_{11}(e_1,e_2)+q^2}<1$. From (22), we learn that, at $\beta=0$, the marginal net benefit (i.e., marginal benefit minus all marginal costs) is

$$\frac{q^2(1-\delta)}{C_{11}(e_1,e_2)}-\left[\frac{q(1-\delta)C_{12}(e_1,e_2)}{C_{11}(e_1,e_2)}+C_2(e_1,e_2)\right]\frac{de_2}{d\beta}-r\delta\sigma_x^2 \tag{23}$$

Let us consider each term in the marginal net benefit evaluated at the contemplated solution $\beta=0$ and $\delta>0$,

- the $\frac{q^2(1-\delta)}{C_{11}(e_1,e_2)}$ term is always positive since $\delta<1$.
- At the contemplated $\beta=0$, e_2 is a corner solution ($e_2=0$) but $C_{12}(.)$ and C_2 are both positive because e_1 is positive (because a positive δ is inducing some productive effort). Further, notice $\frac{de_2}{d\beta}|_{\beta=0,\delta>0}>0$.[27] Therefore, $\left[\frac{q(1-\delta)C_{12}(e_1,e_2)}{C_{11}(e_1,e_2)}+C_2(e_1,e_2)\right]\frac{de_2}{d\beta}$ is positive.
- $r\delta\sigma_x^2$ is always positive.

If the second and the third terms dominate, $\beta=0$ is indeed optimal because the marginal net benefit (equation (23)) is negative. As a result, $\beta=0$

[27] Notices when the first order condition with respect to e_2 ($-\frac{r}{2}\beta^2V_1(e_2,\sigma^2)-C_2(e_1,e_2)=0$) is not satisfied at the corner solution $e_2^*=0$, $\frac{de_2}{d\beta}\neq 0$ even with the assumption $C(e_1,e_2)=e_1^2f(e_2)$.

and $\delta > 0$ can be sustained as an equilibrium.[28] As long as $\frac{q^2(1-\delta)}{C_{11}(e_1,e_2)} < \left[\frac{q(1-\delta)C_{12}(e_1,e_2)}{C_{11}(e_1,e_2)} + C_2(e_1,e_2)\right]\frac{de_2}{d\beta} + r\delta\sigma_x^2$, signal y will be ignored. The key, again, is the spillover effect between the two efforts.

Intuitively, the principal would always use z. No matter whether signal y is used or not ($\beta > 0$ or $\beta = 0$), the marginal benefit of increasing δ is always greater than the marginal cost. However, at $\beta = 0$ the marginal benefit of increasing β may be less than the marginal cost because of the spillover effect. That is, if the principal uses y in the contract ever so slightly, the marginal cost from risk-sharing is zero but the marginal cost from limited attention is positive, which may overweigh the positive marginal benefit. Therefore, sometimes it is efficient for the principal to ignore signal y ($\beta^* = 0$), though signal y is informative.

Similar to the second part of the proof of Lemma 3, there are four possible cases. Cases 1, 2 and 4 are follows nearly identical arguments. We only provide case-3 with details.

If $\beta^* = 0, \delta^* > 0$, then $e_2^* = 0$. From (21), when $\beta^* = 0$ we have $\delta^* = \frac{q^2}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} > 0$. Substitute δ^* into (23), we have

$$\frac{qr(\sigma_z^2+\sigma_x^2)}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} - \left[\frac{qr(\sigma_z^2+\sigma_x^2)C_{12}}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} + C_2\right]\frac{de_2}{d\beta}\ |_{\beta=0,\delta=\frac{q^2}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2}}$$

As long as

$$\frac{qr(\sigma_z^2+\sigma_x^2)}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} \leq \left[\frac{qr(\sigma_z^2+\sigma_x^2)C_{12}}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2} + C_2\right]\frac{de_2}{d\beta}\ |_{\beta=0,\delta=\frac{q^2}{r(\sigma_z^2+\sigma_x^2)C_{11}+q^2}}, \tag{24}$$

$\beta^* = 0, \delta^* > 0$ is sustained as an equilibrium. Notice that this condition requires a non-negative $\frac{de_2}{d\beta}$ at $\beta = 0$. In (20), we see that a slight increase of β from $\beta = 0$ will increase the marginal benefit of e_2 tremendously (from zero to positive infinity). Therefore we have $\frac{\partial e_2}{\partial \beta}\ |_{\beta=0\to 0^+} > 0$.

In addition, we see that a slight decrease of β from $\beta = 0$ will increase the marginal benefit of e_2 from zero to positive infinity. That is, we have $\frac{\partial e_2}{\partial \beta}\ |_{\beta=0\to 0^-} < 0$. Therefore, a decrease of β from 0 to 0^- will only increase the marginal cost (which includes a higher risk premium and a higher marginal cost of e_1), and the principal would not choose a negative β. In other words, β must be non-negative.

Proof of Proposition 4

Now we show the existence of the case that signal y can be ignored through a specific example. In the example with $V(e_2, \sigma^2) = \frac{\sigma^2}{e_2}$ and $C(e_1, e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$, the agent's problem is:

[28] See proof of Proposition 4 for discussion on second order conditions.

$\max_{e_1,\ e_2} \alpha + \beta E[y] + \delta E[z] - \frac{r}{2}\beta^2[V(e_2) + \sigma_x^2] - \frac{r}{2}\delta^2(\sigma_z^2 + \sigma_x^2) - \frac{1}{2}(c_1 + ke_2)e_1^2.$

The agent's first order conditions, in closed form, are:

$$\widehat{e_1} = \frac{\beta(q-\sigma\sqrt{rk})+q\delta}{c_1},$$
$$\widehat{e_2} = \frac{c_1\sigma\beta}{\beta\left(q-\sigma\sqrt{rk}\right)+q\delta}\sqrt{\frac{r}{k}}.$$

The principal's problem is:

$$\max_{\beta,\delta}\ q\widehat{e_1} - \frac{1}{2}\left(c_1 + k\widehat{e_2}\right)\widehat{e_1}^2 - \frac{r}{2}\beta^2\frac{\sigma^2}{\widehat{e_2}} - \frac{r}{2}\delta^2\sigma_z^2 - \frac{r}{2}\beta^2\sigma_x^2 - \frac{r}{2}\delta^2\sigma_x^2.$$

After substituting the agent's FOCs, the problem becomes

$$\max_{\beta,\delta}\ q\frac{\beta(q-\sigma\sqrt{rk})+q\delta}{c_1} - \frac{\left[\beta(q-\sigma\sqrt{rk})+q\delta\right]^2}{2c_1} - \frac{\beta\sigma\sqrt{rk}\left[\beta(q-\sigma\sqrt{rk})+q\delta\right]}{c_1} - \frac{r}{2}\delta^2\sigma_z^2 - \frac{r}{2}\beta^2\sigma_x^2 - \frac{r}{2}\delta^2\sigma_x^2.$$

It leads to a first-order condition and an interior solution (if binding) of β

$$\begin{aligned}
0 &= \frac{q(q-\sigma\sqrt{rk})}{c_1} - \frac{\left[\beta(q-\sigma\sqrt{rk})+q\delta\right]q}{c_1} - \frac{(q-\sigma\sqrt{rk})\beta\sigma\sqrt{rk}}{c_1} - r\beta\sigma_x^2 \\
\widehat{\beta} &= \frac{q(q-\sigma\sqrt{kr})-q^2\widehat{\delta}}{q^2-\sigma^2kr+r\sigma_x^2c_1}
\end{aligned}$$

and a first-order condition with respect to δ *(FOC-δ)

$$\begin{aligned}
0 &= \frac{q^2}{c_1} - \frac{\beta q(q-\sigma\sqrt{rk})+q^2\delta}{c_1} - \frac{\beta\sigma\sqrt{kr}q\delta}{c_1} - r\delta\sigma_z^2 - r\delta\sigma_x^2 \\
\widehat{\delta} &= \frac{q^2(1-\widehat{\beta})}{q^2+c_1r(\sigma_z^2+\sigma_x^2)}
\end{aligned}$$

Suppose $\widehat{\delta}=0$, then we have $\widehat{\beta} = \frac{q(q-\sigma\sqrt{kr})-q^2\widehat{\delta}}{q^2-\sigma^2kr+r\sigma_x^2c_1} = \frac{q(q-\sigma\sqrt{kr})}{\left(q+\sigma\sqrt{kr}\right)(q-\sigma\sqrt{kr})+r\sigma_x^2c_1} < 1$. This implies that FOC-$\delta$ becomes $\frac{q^2}{c_1} - \frac{\widehat{\beta}q(q-\sigma\sqrt{rk})}{c_1} = \frac{q}{c_1}(q-\widehat{\beta}(q-\sigma\sqrt{rk})) > 0$, a contradiction of the hypothesis $\widehat{\delta}=0$; so it must be $\widehat{\delta}>0$ in equilibrium.

However, $\widehat{\beta}$ is no longer guaranteed to be positive with the non-separable costs of e_1 and e_2. Suppose $\widehat{\beta}=0$, then $\widehat{\delta} = \frac{q^2(1-\beta)}{q^2+c_1r(\sigma_z^2+\sigma_x^2)}$ gives $\widehat{\delta} = \frac{q^2}{q^2+c_1r(\sigma_z^2+\sigma_x^2)}$. Substituting into the first order condition with respect to β, we have the marginal benefit net of marginal cost, evaluated at the proposed solution, equal to:

$$\begin{aligned}
& \frac{q(q-\sigma\sqrt{rk})}{c_1} - \frac{0+q\widehat{\delta}}{c_1}[q-\sigma\sqrt{rk}] - \frac{0+q\widehat{\delta}}{c_1}\sigma\sqrt{rk} \\
= \quad & \frac{q}{c_1}\left[(q-\sigma\sqrt{rk}) - \widehat{\delta}\left(q-\sigma\sqrt{rk}\right) - \widehat{\delta}\sigma\sqrt{rk}\right] \\
= \quad & \frac{q}{c_1}\left[(q-\sigma\sqrt{rk}) - q\widehat{\delta}\right] \\
= \quad & \frac{q}{c_1[q+c_1r(\sigma_z^2+\sigma_x^2)]}\{[q^2+c_1r(\sigma_z^2+\sigma_x^2)](q-\sigma\sqrt{rk}) - q^3\}
\end{aligned}$$

Thus, if $[q^2+c_1r(\sigma_z^2+\sigma_x^2)](q-\sigma\sqrt{rk})-q^3 < 0$, or equivalently, $c_1r(\sigma_z^2+\sigma_x^2) < \frac{q^2\sigma\sqrt{rk}}{q-\sigma\sqrt{rk}}$, then marginal benefit is less than marginal cost, and $\widehat{\beta} = 0$ can be optimal. Recall that β must be non-negative, which can be regarded as an implicit constraint $\beta \geq 0$. If we incorporate this constraint in the program and examine the Kuhn-Tucker conditions, we see in most cases this condition is not binding. However, when $c_1r(\sigma_z^2+\sigma_x^2) < \frac{q^2\sigma\sqrt{rk}}{q-\sigma\sqrt{rk}}$, the first order condition with respect to β is not zero and its Kuhn-Tucker multiplier is zero, while the condition $\beta \geq 0$ is binding with its Kuhn-Tucker multiplier being positive. β must be zero and will not deviate from zero.

Given $\beta = 0$, we only need to check the second order condition with respect to δ. The second order derivative of principal's objective function with respect to δ is $-\frac{q^2}{c_1} - r\left(\sigma_z^2+\sigma_x^2\right) < 0$. Therefore, the second order condition is satisfied, and $\{\widehat{\delta} = \frac{q^2(1-\widehat{\beta})}{q^2+c_1r(\sigma_z^2+\sigma_x^2)}, \widehat{\beta} = 0\}$ is indeed global maximum when $c_1r(\sigma_z^2+\sigma_x^2) < \frac{q^2\sigma\sqrt{rk}}{q-\sigma\sqrt{rk}}$.

Proof of Proposition 4

In the benchmark case in which the manager's project selection decision is observable, if the manager chooses H-project, his certainty equivalent is $\alpha + \beta q_H e_{1H} - \frac{r}{2}\beta^2(\sigma_{xH}^2 + \frac{\sigma_{yH}^2}{e_{2H}}) - \frac{c}{2}e_{1H}^2 - ke_{2H}$, and his optimal effort choices are $e_{1H}^* = \frac{\beta q_H}{c}$ and $e_{2H}^* = \beta\sigma_{yH}\sqrt{\frac{r}{2k}}$. Similarly, if the manager chooses L-project, we have $e_{1L}^* = \frac{\beta q_L}{c}$ and $e_{2L}^* = \beta\sigma_{yL}\sqrt{\frac{r}{2k}}$.

If the principal would like the manager to choose the H-project, her design program is

$$\max_{\beta_H} q_H e_{1H}^* - \frac{r}{2}\beta^2(\sigma_{xH}^2 + \frac{\sigma_{yH}^2}{e_{2H}^*}) - \frac{c}{2}e_{1H}^{*2} - ke_{2H}^*.$$

Substitute e_{1H}^* and e_{1L}^* into the program, we have

$$\max_{\beta_H} \frac{q_H^2\beta_H}{c} - \frac{r}{2}\beta_H^2\sigma_{xH}^2 - \beta_H\sigma_{yH}\sqrt{2rk} - \frac{q_H^2\beta_H^2}{2c},$$

and the principal's optimal β_H is $\beta_H^* = \frac{1-c\sigma_{yH}\sqrt{2rk}/q_H^2}{1+rc\sigma_{xH}^2/q_H^2}$. Similarly, if the principal would like the manager to choose L-project, she will offer $\beta_L^* = \frac{1-c\sigma_{yL}\sqrt{2rk}/q_L^2}{1+rc\sigma_{xL}^2/q_L^2}$. As we assume $\frac{\sigma_{yH}}{q_H^2} = \frac{\sigma_{yL}}{q_L^2} \equiv \phi$ and $\frac{\sigma_{xH}}{q_H} = \frac{\sigma_{xL}}{q_L} \equiv \psi$, we have $\beta_H^* = \beta_L^* = \frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}$.

In this setting in which the principal observes the manager's project selection decision, her payoffs when the manager choosing H-project and L-project are, separately:

$$\begin{aligned} PP_H &= \frac{1-c\sqrt{2rk}\phi}{2c(1+rc\psi^2)}(q_H^2 - 2c\sigma_{yH}\sqrt{2rk} + c\sqrt{2rk}\phi q_H^2), \\ PP_L &= \frac{1-c\sqrt{2rk}\phi}{2c(1+rc\psi^2)}(q_L^2 - 2c\sigma_{yL}\sqrt{2rk} + c\sqrt{2rk}\phi q_L^2). \end{aligned}$$

The principal prefers H-project if and only if $PP_H > PP_L$, which implies $\phi < \frac{1}{c\sqrt{2rk}}$.

Now we suppose the manager's project selection decision is unobservable. In this setting, if the principal still offers $\beta_H^* = \frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}$, the manager's certainty equivalent when choosing H-project is

$$\begin{aligned} CE_H &= \alpha^* + \beta_H^* q_H e_{1H}^* - \frac{r}{2}\beta_H^{*2}(\sigma_{xH}^2 + \frac{\sigma_{yH}^2}{e_{2H}^*}) - \frac{c}{2}e_{1H}^{*2} - ke_{2H}^* \\ &= \alpha^* + \frac{q_H^2}{2c}\beta_H^{*2} - \frac{r\sigma_{xH}^2}{2}\beta_H^{*2} - \sqrt{2rk}\sigma_{yH}\beta_H^*, \end{aligned}$$

and his certainty equivalent when choosing L-project is

$$\begin{aligned} CE_L &= \alpha^* + \beta_H^* q_L e_{1L}^* - \frac{r}{2}\beta_H^{*2}(\sigma_{xL}^2 + \frac{\sigma_{yL}^2}{e_{2L}^*}) - \frac{c}{2}e_{1L}^{*2} - ke_{2L}^* \\ &= \alpha^* + \frac{q_L^2}{2c}\beta_H^{*2} - \frac{r\sigma_{xL}^2}{2}\beta_H^{*2} - \sqrt{2rk}\sigma_{yL}\beta_H^*. \end{aligned}$$

When $CE_H < CE_L$, the manager will choose L-project, even if the principal desires the riskier project.

A sufficient condition for $CE_H - CE_L < 0$ is $\phi > \frac{1}{3c\sqrt{2rk}}$. Therefore, when $\frac{1}{3c\sqrt{2rk}} < \phi < \frac{1}{c\sqrt{2rk}}$, the manager will choose the safer L-project when offered the same contract as in the observable setting, while the principal desires the riskier H-project.

If the principal would like to motivate the manager to choose H-project in this unobservable setting, she has to offer a different β. Her design program becomes

$$\max_{\beta_H} \frac{q_H^2\beta_H}{c} - \frac{r}{2}\beta_H^2\sigma_{xH}^2 - \beta_H\sigma_{yH}\sqrt{2rk} - \frac{q_H^2\beta_H^2}{2c},$$

$$s.t. \frac{\beta_H^2}{2c}(q_H^2 - q_L^2) \geq \frac{r\beta_H^2}{2}(\sigma_{xH}^2 - \sigma_{xL}^2) + \sqrt{2rk}\beta_H(\sigma_{yH} - \sigma_{yL}).$$

Solving this program gives us $\beta_H^{'*} = \frac{2c\sqrt{2rk}\phi}{1-rc\psi^2}$ when $\frac{1}{3c\sqrt{2rk}} < \phi < \frac{1}{c\sqrt{2rk}}$. $\beta_H^{'*} = \frac{2c\sqrt{2rk}\phi}{1-rc\psi^2} > \beta_H^{*} = \frac{1-c\sqrt{2rk}\phi}{1+rc\psi^2}$.