Why do Firms Gravitate to Selective Disclosure?

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Abstract

In this paper, we present a theoretical model to explain why firms gravitate to selective disclosure. We consider a setting where a firm chooses its disclosure policy to maximize its price informativeness (i.e., how much information is impounded in the price, measured by the posterior precision of true value conditional on the price) in a market with different types of traders. Through ex-ante acquisition of expertise, traders become sophisticated and improve their ability to better interpret the information disclosed by the firm. We show that firms sometimes prefer to disclose information selectively, providing information only to sophisticated traders, rather than to the public. The primary reason is that selective disclosure promotes the ex-ante expertise acquisition among traders. Consequently, under selective disclosure, the aggregate information quality is overall higher and prices are more informative than under public disclosure. However, selective disclosure may reduce uninformed investor welfare.

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1 Introduction

In October 2000, the U.S. Securities and Exchange Commission (SEC) passed Regulation Fair Disclosure (Reg FD) to prohibit selective disclosure of material nonpublic information by companies to market professionals such as analysts, institutional investment managers, investment companies, and other market professionals. Empirical research has shown that after Reg FD informed trading based on superior information has declined.[1] However, recent research finds that selective disclosure still occurs through other forms of private communication, such as conference presentations (Bushee, Jung, and Miller, 2010).Why do firms gravitate toward selective disclosure practices? After all, it is not necessarily in management's or shareholders' interest to provide a subset of traders or analysts with superior information about the firm.

In this paper, we provide a theoretical explanation for why firms would rationally choose a selective disclosure policy. We do so in a setting where a firm's objective is to maximize price informativeness or efficiency in a market with heterogeneous informed traders. Fishman and Hagerty (1992) show that insiders can improve price informativeness (or efficiency) through insider trading based on more precise private information. Without insider trading, if a firm can publicly disclose its inside information to the market, price informativeness should increase as more traders receive the information. We present a model in which firms may prefer selective disclosure over public disclosure to maximize its price informativeness by encouraging ex-ante expertise acquisition by certain traders.

In our model, the information disclosed by the firm is a noisy signal about the underlying value of the firm.[2] Informed traders in the market differ in their ability to interpret and process the information. Some are *sophisticated traders* and others are *unsophisticated traders*.

[1] See for example: Collver (2007); Ke, Petroni, and Yu (2008), etc.

[2] We assume that firms cannot obtain more precise information than the outside informed traders in our model. However, managers generally know more than outsiders about the true value, and they should not have any incentive to add noise to the information disclosed if their objective is to maximize price informativeness. Our model fits industries operating in complicated and dynamic environments. Firms in these industries may indeed lack information about their true values relative to the experts in the market. For example, a firm discloses some information about its new product development, but is uncertain about the market-wide demand or other macro factors that may influence the outcome of its new development. Outside experts may be able to better access the economic value of this new product based on the information disclosed by the firm. Our model shows that the selective disclosure practice also encourages this type of expertise knowledge acquisition in the market and increases market efficiency.

Both types of informed traders can process the information disclosed by the firm better than the general public and better than uninformed traders. However, the sophisticated informed traders are industry experts or star analysts with superior expertise knowledge in processing the same information as unsophisticated traders.[3] Ex-ante all traders are unsophisticated traders, but they can become sophisticated through some costly training or education before the firm discloses any information. This is similar to the ex-ante information acquisition decision in Verrecchia (1982) in the benchmark setting; here we also allow for a third type of trader labeled as *uninformed traders*. These uninformed traders cannot process the public information, let alone become sophisticated traders through expertise acquisition. These uninformed traders still trade based on rational expectation, and therefore differ from purely noisy traders who submit random orders. We presume that the SEC cares more about the welfare of uninformed traders for the purpose of maintaining integrity and stability of the capital markets and for "fairness" reasons.[4]

There are several reasons why we use maximizing the price informativeness, instead of maximizing the price, as the firm's objective. First of all, prior theoretical studies have linked price efficiency to firms' investment and contracting decisions. Fishman and Hargerty (1989) show that more efficient price can lead to more efficient investment decisions. Gao and Liang (2010) show that increased price informativeness can improve investment efficiency as price reflects information that firms do not possess. Our model, in a similar vein, assumes informed traders generate superior information about firm value. Another benefit of improving price informativeness is related to managerial compensation contracts, as in Holmstrom and Tirole (1993), Baiman and Verrecchia (1995), and Baiman and Verrecchia (1996). In their models the manager's compensation is based on market price as price impounds information about managers' efforts. Improving price efficiency also improves contracting efficiency. Second, in our model the disclosure policy is an ex-ante commitment of the firm to influence the informed traders' expertise acquisition before the information is disclosed. If the firm had

[3] Similarly, Bushman, Gigler and Indjejikian (1996) examine the impact of a two-tiered financial reporting on the expected trading profits of sophisticated and unsophisticated investors, who differ in their ability to process information from a full set of financial statements.

[4] SEC states that *"The regulation is designed to address the core problem of selective disclosure made to those who would reasonably be expected to trade securities on the basis of the information or provide others with advice about securities trading"*. See SEC's document 'Final rule: Selective Disclosure and Insider Trading', available at http://www.sec.gov/rules/final/33-7881.htm.

chosen to maximize the ex-post price, the optimal disclosure strategy would always depend on the actual signal realized. Therefore the firm cannot commit to its disclosure policy, and there would be no impact on the expertise acquisition through its disclosure strategy. As a matter of fact, in our model the ex-ante expected price remains the same in all disclosure regimes. While it is possible that maximizing price informativeness is secondary to the objective of maximizing expected firm value, the latter incentive is muted in our setting.

We compare three disclosure policies: 1) selective disclosure to sophisticated informed traders only; 2) disclosure to both types of informed traders; and 3) public disclosure. In the benchmark case, the proportion of sophisticated informed traders is exogenous (i.e., without the ex-ante choice to acquire expertise), the price informativeness under public disclosure is always higher than the other two types of disclosures. This is intuitive as more traders in the market learns the information and the price formed by rational expectation compounds more information in estimating the true value of the firm. In contrast, with endogenous expertise acquisition, the disclosure policy of the firm will affect the ex-ante incentive of informed traders to become sophisticated. A unique equilibrium exists in which each informed trader is indifferent between becoming sophisticated and remaining unsophisticated.

Our main finding is that firms sometimes prefer selective disclosure over public disclosure in the presence of endogenous expertise acquisition. We measure price informativeness by the posterior precision of estimating the firm's true value conditional on the price. Two conflicting forces contribute to the impact of the disclosure policy on price informativeness: the aggregate trading intensity of informed traders versus the information content of public information. The trading intensity depends on the proportion of sophisticated traders in equilibrium. When selective disclosure is involved, higher trading profit from the superior information induces more informed traders become sophisticated traders. On the other hand, with public disclosure, the trading of uninformed traders -who act as market makers in response to the public information - leads to higher price informativeness. The tradeoff between these two effects on the price informativeness suggests that selective disclosure may at times be preferred to public disclosure.

We characterize several specific conditions under which firms may or may not prefer selective disclosure to public disclosure. We find that firms are more likely to choose selective disclosure when the information is relatively more noisy (where noise can be processed by the informed traders, but not by the general public). This result suggests that in those industries where more expertise knowledge is needed to understand the business model and process information, firms will be more likely to communicate through selective disclosure. Selective disclosure ex-ante promotes the acquisition of expertise among analysts and improves the market efficiency. We also find firms are more likely to prefer selective disclosure when noise trading is large, as sophisticated traders can trade more aggressively in this setting.. On the other hand, we find that firms are more likely to prefer public disclosure when expertise acquisition cost is small. Furthermore, public disclosure is also preferred when unsophisticated informed traders lack significant advantage in their ability to interpret information relative the general public.

A regulator may care not only about price efficiency in the market, but also about ensuring the fairness and transparency of capital markets and protecting investors who have no informational advantage. In our model, we examine two additional metrics that might be of interest to the regulator: market liquidity and the welfare of uninformed traders. It turns out that a conflict of interest may exists between the regulator and the firm. When the firm prefers selective disclosure to maximize price informativeness, the welfare of uninformed traders is lower. However, the market liquidity may be higher under selective disclosure. Whether it is optimal for the regulator to prohibit selective disclosure depends on how these different objectives are balanced.

One critical assumption of our model is that a firm does not change the quality of information disclosed to different parties. This is consistent with the SEC's objective of inducing firms to disclose the same information to the public with the disclosure to analysts or other parties. Opponents argue that forcing companies to disclose the same information to all parties may discourage firms from publicly disclosing information in general. However, empirical evidence looking at voluntary disclosures shows that this concern unwarranted (Heflin, Subramanyam, and Zhang, 2003; Gomes, Gorton, and Madureira, 2007; Bushee, Matsumoto, and Miller, 2004; Francis, Nanda, and Wang, 2006; and Wang, 2007). We provide a scenario where the firm still prefers selective disclosure even if the same information is disclosed under selective disclosure and public disclosure.

Following Verrecchia (1983), the literature has examined the voluntary disclosure decision by the firm to maximize the firm value in a rational expectation market. Many studies in the literature also investigate how a firm's voluntary disclosure affects the private information acquisition activities and private information allocation in a competitive market setting (e.g., Diamond, 1985; Bushman, 1991; Lundholm, 1991; Alles and Lundholm, 1993; Demski and Feltham, 1994; Kim and Verrecchia, 1991) or in a strategic noncompetitive Kyle model setting (e.g., McNichols and Trueman, 1994; Bushman and Indjejikian, 1995; Bagnoli and Watts, 1998). These studies usually compare public disclosure versus no disclosure setting and examine the impact of public signal on the private information market. While we too study how a firm's disclosure policy affects the expertise acquisition among informed traders in the market, we differentiate between the firm's two main disclosure strategies: selective versus public disclosure.

Another strand of the literature looks at selective disclosure directly. Bushman (1991) considers a firm's private disclosure to securities analysts using a model with a monopolist information seller. He shows that firms choose private disclosure of information to additional information sellers in order to create a competitive information sale market and avoid the disclosure costs. In a similar vein, Sabino (1993) considers an insiders sale of information. While our model has no information sellers, we consider a firm's own objective of maximizing the price informativeness through disclosure. Arya, Glover, Mittendorf and Narayanamoorthy (2005) also investigate the impact of Reg FD. They show that prohibiting selective disclosure heightens analysts' herding behavior, which leaves investors worse off. Our paper contributes to this literature by offering a rational explanation for firms' selective disclosure.

The remainder of our paper is organized as follows: Section 2 presents the basic model and equilibrium under three disclosure regimes. Section 3 analyzes the firm's disclosure choice given endogenous expertise acquisition. Section 4 considers different objectives for a regulator. Section 5 concludes our paper.

2 Basic model and equilibrium

2.1 Model setup

The basic framework of the model follows a version of noisy rational expectation model by Vives (1995), which is consistent with other standard noisy rational expectations models such as those in Grossman and Stiglitz (1980), and Diamond and Verrecchia (1981). Consider a market with two securities: a risky asset, with liquidation value of $v \sim N(\bar{v}, h_v^{-1})$, and a riskless asset with unitary return. These securities are traded by risk-averse agents, either informed or uninformed, and noisy traders. The risk-averse traders in the market have the same CARA type utility function, as represented by:

$$U(W_i) = -exp\{-\rho W_i\}$$

where the coefficient of constant absolute risk aversion, $\rho > 0$, is assumed common for all traders. The return of buying x_i units of the risky asset at price p for trader i is

$W_i = (v - p)x_i$. The initial wealth of traders is nonrandom and normalized to zero (without loss of generality with CARA preference).

A continuum of rational traders are uniformly distributed on the interval [0,1]. Among these traders a fixed fraction $(1 - m)$ is innately uninformed traders (type C) who cannot access the firm's information unless the firm chooses to disclose publicly. The remaining m fraction is informed traders who may gain access to the firm's information depending on the firm's disclosure policy. We can think of these informed traders as financial analysts who have the expertise knowledge and ability to interpret any information disclosed by the firm, but they do not have additional sources of information beyond what the firm has disclosed to them. We allow for two types of informed traders, sophisticated (type A) and unsophisticated (type B). We denote the fraction of sophisticated informed traders (type A) as f, and the remaining fraction $m - f$ are type B unsophisticated informed traders. In addition, noise traders trade for exogenous liquidity reasons, resulting in an aggregate order of u , $u \sim N(0, \tau_u^{-1})$, where u is independent of the other random variables.

The firm has information about v that can be disclosed to market participants, denoted as $y = v + \theta + \eta + \varepsilon$. η, θ, ε are all random noise variables with zero mean and normal distributions: $\theta \sim N(0, h_\theta^{-1})$, $\eta \sim N(0, h_\eta^{-1})$ and $\varepsilon \sim N(0, h_\varepsilon^{-1})$. The noise variables are independent of v and of each other. After the firm discloses its information y, each sophisticated informed trader (type A) $i \in [0, f]$ processes the information disclosed by the firm and gets a more precise signal, denoted as $y_i^A = v + \theta_i$, where $\theta_i \sim N(0, h_\theta^{-1})$. Each unsophisticated informed trader (type B) $i \in [f, m]$ processes the information but only gets a less precise signal, denoted as $y_i^B = v + \theta_i + \eta_i$, where $\eta_i \sim N(0, h_\eta^{-1})$. We also assume that all θ_i and η_i are uncorrelated across traders. Each informed trader receives the signal with the same precision. Following the usual convention, we assume that given v, the average signal of a positive mass of informed agents equals v almost surely, i.e., $\int_0^1 \theta_i di = 0$ and $\int_0^1 \eta_i di = 0$. These distributional assumptions are common knowledge among all agents in the economy.

The total fraction of informed traders in the market, m, is exogenous and fixed. In contrast, the fraction of sophisticated informed traders f may be endogenously determined by the expertise acquisition process. All informed traders are ex-ante unsophisticated. Before information is disclosed, each trader can become sophisticated at a cost of k (assumed to be an exogenous and fixed amount).[5] The superior information processing ability could increase the trading profits ex-post because of the more precise information obtained once the firm has made its disclosure. Therefore, the equilibrium fraction of informed traders who incur a cost to become sophisticated makes each trader ex-ante indifferent in becoming sophisticated.[6] Note that although the acquisition of expertise occurs before the firm makes any disclosure, the traders correctly anticipate the equilibrium disclosure policy by the firm at this time.

We focus on symmetric equilibria, i.e., all traders of the same type use the same trading strategy. All traders can condition their trade on the price, but informed traders can also condition trades on the more precise information by processing the firm's disclosed information. Denote each trader i's information set as $\Phi_i^t(d)$, where $t \in \{A, B, C\}$ and d indicates the firm's disclosure policy, which we discuss and develop later. Each type t trader submits

[5] This expertise acquisition cost can be interpreted, for example, as the education or training cost needed to increase the inherent ability of analyzing financial information to become an expert in a certain industry.

[6] This is similar to the private information acquisition in a noisy rational expectation economy as in many of the studies that followed the seminal Verrecchia (1982).

an order of $x_i^t(\Phi_i^A(d), p)$.

A symmetric rational expectations equilibrium is defined by a set of trades contingent on the information each trader has: $x_i^t(\Phi_i^t(d), p)$ for each trader, and a price function p, such that:

i) The market clearing condition holds:

$$\int_0^f x_i^A di + \int_f^m x_i^B di + \int_m^1 x_i^C di + u = 0 \tag{1}$$

ii) Both informed and uninformed traders choose the order to maximize their own utility:

$$x_i^t(\Phi_i^t(d), p) \in arg\max_x E[U_i(W_i^t)|\Phi_i^t(d), p], \quad i \in [0,1] \tag{2}$$

where W_i^t, $t \in \{A, B, C\}$, is the trading profit of each trader i with a position x_i^t, given by $W_i^t = (v - p)x_i^t$.

We restrict attention to the linear Bayesian equilibria. Due to the presence of noise traders (u), the equilibrium will not be fully revealing. The informed traders optimize their trades by taking into account the equilibrium relation between prices and the random variables of v and u. The maximization of CARA utility, $U(W_i^t) = -exp\{-\rho W_i^t\}$, yields the optimal trade order for type t informed trader as below,

$$x_i^t(\Phi_i^t(d), p) = \frac{E[(v-p)|\Phi_i^t(d), p]}{\rho Var[(v-p)|\Phi_i^t(d), p]}, \quad t \in \{A, B, C\}. \tag{3}$$

The characterization of the equilibrium price and trading strategies depends on the firm's disclosure policy, which determines the information available to all agents in the market. We consider three cases of disclosure: 1) selective disclosure to sophisticated informed traders only, 2) disclosure to all informed traders, and 3) public disclosure. We characterize and compare equilibrium for all three cases. The disclosure assumptions imply that firms can in fact distinguish between the sophisticated and unsophisticated informed traders.[7] The firm commits to its disclosure policy and then traders acquire expertise before the information is available. All aspects of the model is common knowledge.

[7]The assumption is reasonable as sophisticated traders are often industry experts or star analysts who can be easily identified by firms.

Figure 1 summarizes the timeline of the model.

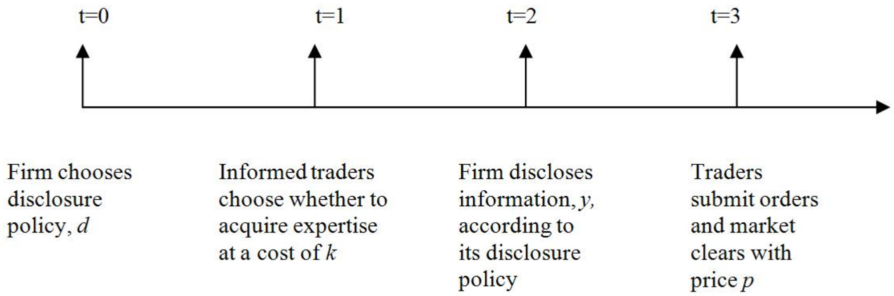

Figure 1: Timeline

2.2 Disclosure policy and equilibrium

In this section we assume that the fraction of sophisticated informed traders, f, is exogenous and known to all agents in the market. We characterize the equilibrium price and trading orders for each disclosure policy.

In the first disclosure regime, the firm chooses to disclose its information y only to the sophisticated informed traders in the market. The sophisticated informed traders (type A) can process the information using their expertise and obtain more precise information, y_i^A. The type B unsophisticated informed trader becomes the same as the uninformed type C traders, who trade based on their rational expectation without any information about the firm. In the second case, the firm chooses to disclose its information to both types of informed traders, but not to the public. The sophisticated informed traders (type A) can process the information similarly to the first regime and each receives a signal y_i^A. The unsophisticated informed traders (type B) also process the information disclosed by the firm, but receive a less precise signal y_i^B. The uninformed type C traders have no information except the price and will trade based on their rational expectations without any information. In the third regime of public disclosure, the firm discloses the information y to everyone in the market. The informed traders can still process the public information and generate more precise signals, y_i^A and y_i^B, respectively. The uninformed traders and market can only interpret the information as disclosed. In contrast to the previous two cases, uninformed traders receive the public information and also observe the price. Table 1 summarizes the traders and public's information sets in three disclosure regimes.

Table 1: Information sets of interested parties

Information sets \ Disclosure Policy (d)	$d=1$	$d=2$	$d=3$
$\Phi_{i,d}^A$	y_i^A, y	y_i^A, y	y_i^A, y
$\Phi_{i,d}^B$	ϕ	y_i^B, y	y_i^B, y
$\Phi_{i,d}^C$	ϕ	ϕ	y
Φ^P (Public)	ϕ	ϕ	y

d=1 represents the selective disclosure regime,
d=2 represents the interim regime with disclosure to both types of informed traders,
d=3 represents the public disclosure regime.

In equilibrium, the trading order submitted takes into account each trader's own information set and the rational expectation of price based on the knowledge about other traders' information sets and trading strategies. Lemma 1 characterizes the linear Bayesian equilibrium in each disclosure regime:

Lemma 1 *When the firm chooses the disclosure policy d, there exists a unique linear equilibrium in the competitive market, such that:*

$$x_i^A(\Phi_{i,d}^A) = \beta_d^A(y_i^A - p_d) + \gamma_d(E[v|\Phi^P] - p_d)$$

$$x_i^B(\Phi_{i,d}^B) = \beta_d^B(y_i^B - p_d) + \gamma_d(E[v|\Phi^P] - p_d)$$

$$x_i^C(\Phi_d^C) = \gamma_d(E[v|\Phi^P] - p_d)$$

$$p_d = \lambda_d z_d + E[v|\Phi^P]$$

where $\beta_d^A = \rho^{-1}h_\theta$ $(d = 1, 2, 3)$, $\beta_1^B = 0$, $\beta_2^B = \beta_3^B = \rho^{-1}(h_\theta^{-1} + h_\eta^{-1})^{-1}$, $\gamma_d = h_v(\rho + \beta_d h_u)^{-1}$, $\lambda_d = (\beta_d + \gamma_d)^{-1}$, $z_d = \beta_d(v - E[v|\Phi^P]) + u$, *and* $\beta_d = f\beta_d^A + (m - f)\beta_d^B$.

Proof. See Appendix. ■

The equilibria characterized in Lemma 1 in three disclosure regimes have parallel. Each agent trades based on one's own rational expectation. For the uninformed traders, their trading intensity is measured by γ_d, $d \in 1, 2, 3$ which is the sensitivity of trade to the difference between the price and the prior expectation of uninformed traders. Uninformed traders act as market makers and cannot determine whether a trade is made by informed or noise traders. Since $\gamma_d > 0$, the uninformed traders will sell (buy) when the price is above (below) the prior expectation of the liquidating value of the firm. Therefore they trade against the wind. The prior expectation of the liquidating value is different in the case of selective disclosure and public disclosure. In the disclosure regimes (1) and (2), the prior expectation is simply $\bar{v}$. In the case of public disclosure, the prior expectation is $E[v|y]$ because the uninformed traders and public also receive the information disclosed by the firm. However, in all three cases, the trading intensity of uninformed traders has the same form, $\gamma_d = h_v(\rho + \beta_j h_u)^{-1}$, where $\beta_d = f\beta_d^A + (m - f)\beta_d^B$. Therefore uninformed traders' trading intensity increases with the amount of noise trading (depends on h_u^{-1}) and decreases with the aggregate trading aggressiveness of informed traders, β_d. When the fraction of sophisticated informed traders f is exogenous and fixed, the trading intensities of uninformed traders in disclosure regimes (2) and (3) are the same; but the trading intensity in Case I is greater than the other two cases, i.e., $\gamma_1 > \gamma_2 = \gamma_3$. This is because in the first case there are fewer informed traders in the market. Later on we examine the equilibrium when the fraction f is endogenously determined.

For the informed traders, their trades include two parts as explained in Vives (2008). First, each informed trader i speculates on the precise signal obtained from the firm's disclosure. They either buy or sell, depending on whether the price is larger or smaller than their own signal, $y_i^t - p$. β_d^t represents the responsiveness to the signal for this type of speculating trading by each type and is determined by the traders' risk aversion ρ and the precision of their signal. Increasing the risk aversion ρ decreases the aggressiveness of the trading. For type A and B traders, their trading responsiveness increases with the precision h_θ and $(h_\theta^{-1} + h_\eta^{-1})^{-1}$ respectively, which reflects their differential ability to filter noise in the information disclosed to them. $\rho^{-1}h_\theta$ and $\rho^{-1}(h_\theta^{-1} + h_\eta^{-1})^{-1}$ are viewed as the risk-adjusted information advantage of each type of informed traders, respectively. The responsiveness to the informed traders' better signals does not depend on the noise trading or prior variance (h_v^{-1}) about v. $\beta_d^A > \beta_d^B$, i.e., the sophisticated informed traders trade more aggressively than the unsophisticated informed traders due to their superior information advantage. The second part of the trade of informed traders is related to their market-making capacity, similar to uninformed traders. Since uninformed traders and informed traders have the same degree of risk-aversion, each type t informed trader also has a trading intensity of γ_d for the market-making motivated trading.[8] This part of trading is again based on the difference between the price and prior expectation of the liquidating value, $\bar{v} - p$ or $E[v|y] - p$, depending on the disclosure policy. In the selective disclosure regime when the type B traders do not receive any information from the firm's disclosure, they act exactly the same as uninformed type C traders and do not engage in speculative trading.

Comparing the results in regimes (2) and (3), we observe that public disclosure does not affect the trading intensity or responsiveness to the signals and price for each agent when the fraction of sophisticated informed traders is fixed. Instead it only affects the prior expectation of all market participants. The price fully reflects the expectation of all traders given the public disclosure by the firm. The trading intensity β_d^t and γ_d are unaffected by public disclosure, because the relative information advantage of informed traders does not change. In contrast, the information content of price does change.

We are going to focus on the following three matrices in our main analysis: price informativeness, market liquidity, and ex-ante expected utility of traders.

Price informativeness. Price reveals the average expectation of investors about the liquidating value and the risk premium required for risk-averse traders to absorb noise traders' demands. Following the literature (e.g., Grossman and Stigliz, 1980; Verrecchia, 1982; and Vives, 1995), we also define the price informativeness as $\tau_d \equiv Var[v|p_d]^{-1}$, which is the precision of price in estimating v in the disclosure regime d. We can show that in the first two disclosure regimes,

$$\tau_d = h_v + \beta_d^2 h_u, \quad d \in \{1, 2\}. \tag{4}$$

[8] If they have different degree of risk-aversion, the trading intensity will be determined by the ratio of their risk-aversion, see Vives (2008).

Intuitively price is more informative when either the liquidating value is less volatile (higher h_v), or noise trading is less prevalent (higher h_u). The price informativeness also increases with the aggregate trading intensity of informed traders (β_d), which depends on the risk aversion and the quality of information received by the informed traders. This is consistent with prior literature's finding that informed trading contributes to the price informativeness. All else equal, when there are more sophisticated informed traders (f increases), the weighted average trading intensity of informed traders increases, and the price becomes more informative.

In the case of public disclosure, we have the following:

$$\tau_3 = h_v + h_u[\beta_3 + \gamma_3 H]^2, \quad H \equiv \frac{h_v^{-1}}{h_v^{-1} + h_\theta^{-1} + h_\eta^{-1} + h_\varepsilon^{-1}} \tag{5}$$

Without public disclosure, uninformed traders' orders are irrelevant to the price informativeness. With public disclosure, their trading intensity contributes to the price informativeness as they also reveal the information contained in the public information. H is a measure of information content of public information y about the liquidating value v since $E[v|y] = Hy + (1-H)\bar{v}$. Increasing the precision of public information increases price informativeness. It is easy to show the following for the case of fixed fraction of sophisticated informed traders:

Lemma 2 *When the fraction of sophisticated informed traders (f) is fixed, public disclosure results in higher price informativeness compared to selective disclosure.*

But when there is ex-ante expertise acquisition among informed traders, disclosure to more agents may reduce the incentive of informed traders to acquire expertise.

Market liquidity. λ_d^{-1} measures market liquidity or market depth. When λ_d is low, a shock in noise trading does not have much of an impact on the price, which means the market is deep. $\lambda_d^{-1} = \beta_d + \gamma_d$, is the aggregate responsiveness of all traders to the price. Substituting γ_d into λ_d^{-1}, we get

$$\lambda_d^{-1} = \beta_d + h_v(\rho + \beta_d h_u)^{-1}. \tag{6}$$

It is easy to verify a higher volatility of fundamentals (h_v^{-1}) decreases the market liquidity, as the uninformed traders who act as market makers protect themselves by being more responsive to the information contained in price when the liquidating value is more difficult to predict. On the other hand, more noise trading increases the market liquidity, because the uninformed traders are now less responsive to price because they know that they are less likely to trade against informed traders. The trading intensity of informed traders, β_d, has a mixed effect on market liquidity. It directly increases market liquidity because of higher price informativeness, yet it also indirectly decreases market liquidity as the uninformed traders protect themselves when informed traders trade more aggressively in the presence of adverse selection.[9]

Ex-ante expected utility of traders. Ex-ante expected utility of each trader is the expectation of the utility from the terminal wealth (W_i) each trader receives based on his trading strategy. Given the CARA utility function, the ex-ante expected utility of each type t trader i in regime d is:

$$E[U(W_{i,d}^t)] = E[-exp\{-\rho x_i^t(v - p_d)\}], \quad t \in \{A, B, C\},$$

where p_d represents the equilibrium price in each disclosure case.

The conditional expected utility plays an important role in our analysis. Using the result in (3), an informed trader's conditional expected utility given the price and his information set is:

$$E[U(W_{i,d}^t)|\Phi_{i,d}^t, p_d] = -exp\{-\frac{(E[v|\Phi_{i,d}^t, p_d] - p_d)^2}{2Var[v|\Phi_{i,d}^t, p_d]}\}. \tag{7}$$

Let $\Sigma_d^t = E[U(W_{i,d}^t)|\Phi_{i,d}^t, p_d]$. Following Vives (2008), we can also derive the following conditional expectation:[10]

$$E[\Sigma_d^A|p_d] = -exp\{-\frac{(E[v|p_d] - p_d)^2}{2Var[v|p_d]}\}\sqrt{\frac{h_v + \beta_d^2 h_u}{h_v + h_\theta + \beta_d^2 h_u}}, \quad d \in \{1, 2, 3\}, \tag{8}$$

$$E[\Sigma_d^B|p_d] = -exp\{-\frac{(E[v|p_d] - p_d)^2}{2Var[v|p_d]}\}\sqrt{\frac{h_v + \beta_d^2 h_u}{h_v + (h_\theta^{-1} + h_\eta^{-1})^{-1} + \beta_d^2 h_u}}, \quad d \in \{2, 3\}.$$

[9]For more detailed discussion of market liquidity, see Vives (2008).

[10]See Appendix for the derivation of the equation (8).

where p_d refers to the equilibrium price in Lemma 1 in each disclosure regime d. For the unsophisticated informed trader, the above conditional expected utility only applies to the disclosure regimes (2) and (3), when they are able to process the disclosed information. In the disclosure regime (1), the type B traders remain uninformed, and their expected utility conditional on the price is given by:

$$E[\Sigma_1^B|p_1] = -exp\{-\frac{(E[v|p_1] - p_1)^2}{2Var[v|p_1]}\}. \tag{9}$$

A direct calculation of the ex-ante expected utility for each trader is difficult because of the presence of a product of two random variables in the exponential utility function. Therefore, in our analysis later, we use these conditional expected utility functions in comparing different traders' utilities.

For all type C uninformed traders, their ex-ante expected utility is similar to the unsophisticated traders as in (9). We have the following ex-ante expected utility for all type C traders:

$$E[U(W_{i,d}^C)] = E[-exp\{-\frac{(E[v|p_d] - p_d)^2}{2Var[v|p_d]}\}], \quad d \in \{1, 2, 3\}. \tag{10}$$

3 Expertise acquisition and firm's disclosure choice

In the equilibrium of Section 2, we assume that the fraction of sophisticated informed traders is exogenously fixed and examine the equilibrium under different disclosure policies. In our model, informed traders ex-ante can incur a cost to become sophisticated type and improve their ability to interpret and analyze the information disclosed by the firm. The sophisticated informed trader will have a higher ex-post utility from the trading; however as the proportion of sophisticated informed traders increases, the price informativeness increases, which will in turn reduce the benefit of becoming sophisticated informed traders. Therefore ex-ante there exists a unique equilibrium such that for each informed trader, the ex-ante expected utility of becoming a sophisticated trader after incurring the expertise acquisition cost is exactly the same as the ex-ante expected utility of unsophisticated traders under each disclosure policy chosen by the firm. This endogenous expertise acquisition is similar to information acquisition in prior literature (Verrecchia, 1982; Diamond, 1985; etc). An implicit assumption of such endogenous expertise acquisition equilibrium is that: 1) before informed traders pay the expertise acquisition cost, they know firm's disclosure policy and make the expertise acquisition decision accordingly; 2) the firm commits to its disclosure policy as expected by everyone in the market.

3.1 Endogenous expertise acquisition ex-ante

We now determine the equilibrium with endogenous expertise acquisition. We assume the cost of expertise acquisition is k. The willingness to pay the cost k is the amount that makes the ex-ante expected utility of sophisticated traders equal to that of unsophisticated traders, i.e., the following equations should hold in equilibrium:

$$\frac{E[U(W_{i,d}^A - k)]}{E[U(W_{i,d}^B)]} = exp\{\rho k\}\frac{E[\Sigma_d^A]}{E[\Sigma_d^B]} = exp\{\rho k\}\frac{E[\Sigma_d^A|p_d}{E[\Sigma_d^B|p_d]} = 1 \tag{11}$$

Using the results in equation (8) and (9), we derive the following lemmas about the equilibriums with endogenous expertise acquisitions by informed traders under each of the disclosure policies:

Lemma 3 *When the informed traders can pay k to become sophisticated traders, the equilibrium fraction of sophisticated traders, f_d^*, is determined by the following equations in each regime:*

- *(1):* $k = \rho^{-1} ln\sqrt{\dfrac{h_v + h_\theta + [\beta_1(f_1^*)]^2 h_u}{h_v + [\beta_1(f_1^*)]^2 h_u}}$
- *(2):* $k = \rho^{-1} ln\sqrt{\dfrac{h_v + h_\theta + [\beta_2(f_2^*)]^2 h_u}{h_v + (h_\theta^{-1} + h_\eta^{-1})^{-1} + [\beta_2(f_2^*)]^2 h_u}}$
- *(3):* $k = \rho^{-1} ln\sqrt{\dfrac{h_v + h_\theta + [\beta_3(f_3^*)]^2 h_u}{h_v + (h_\theta^{-1} + h_\eta^{-1})^{-1} + [\beta_3(f_3^*)]^2 h_u}}$

  where $\beta_d(f_d^*) = f_d^* \beta_d^A + (m - f_d^*)\beta_d^B$ *as defined in Lemma 1.*

Proof. See Appendix. ■

Since the expertise acquisition cost c is the same in all three cases, we can compare the equilibrium fraction of sophisticated informed traders by setting the right-hand sides of equations in Lemma 3 equal to each other. This implies:

$$\beta_1(f_1^*) > \beta_2(f_2^*) = \beta_3(f_3^*.) \tag{12}$$

With endogenous acquisition of expertise, the aggregate trading intensity of informed traders in the first regime (selective disclosure) is the largest. Because the sophisticated informed traders in this case benefit most from their superior information advantage, a larger fraction of traders are ex-ante willing to incur the cost to become sophisticated ($f_1^* > f_2^*$). This overall increases the aggregate trading intensity due to the speculation based on the superior information. In the regimes (2) and (3), the aggregate trading intensity is the same as the equilibrium fraction of sophisticated informed traders is the same, $f_2^* = f_3^*$. In both cases, both types of informed traders receive the same information and their relative information advantage remains unchange. The incentive for the informed trader to become sophisticated is mainly driven by the relative information advantage compared with unsophisticated informed traders, rather than compared with uninformed traders.

3.2 Disclosure choice

In this section we examine the firm's disclosure choice in anticipation of the equilibrium expertise acquisition by informed traders. Prior studies that examined the disclosure decisions of firms in similar models usually assume the firm's objective either coincides with the traders' demand for disclosure to maximize the their trading profits (Bushman, 1991), or to maximize the utility of the firm's insider who makes the disclosure decisions (Bushman and Indjejikian, 1995). However, these assumptions are often not descriptive of firms behavior; in particular, the second assumption implies management undertakes illegal action, which arguably is not the norm in practice. Instead, we assume that the firm's objective is to maximize price informativeness. Greater price informativeness is important to firms because it improves investment efficiency and reduces the firms' cost of capital. Fishman and Hagerty (1989) show that a more efficient (informative) price better reflects the investment decisions made by the firm, and consequently improve investment efficiency. A more informative price can also reduce the information asymmetry between firms and prospective investors, thereby reducing reduce the cost of capital. However, the optimal disclosure policy for the firm may not be the one that is regulators prefer. In this section we look at the firm's optimal disclosure practices, and in the next section we focus on the regulator's objectives.

Proposition 1 *For any firm, disclosure to all informed traders (i.e., regime (2)) is never optimal, i.e.,*

$$\tau_2(f_2^*) \leq \{\tau_1(f_1^*), \tau_3(f_3^*)\}.$$

The proof and intuition behind Proposition 1 are straightforward. In the first two disclosure regimes, the difference in price informativeness (See (4)) depends only on the aggregate trading intensity or aggregate information advantage of informed traders $\beta_d(f_d^*)$. Because exclusive selective disclosure to sophisticated traders encourages the informed traders to engage in ex-ante acquisition of expertise, the aggregate information advantage to all informed traders increases. As a result, in the selective disclosure regime the price informativeness increases as a result of higher overall information quality contained in aggregate informed trading. In general, $\tau_2(f_2^*)$ is smaller than $\tau_1(f_1^*)$ as long as $h_\eta > 0$. A special case arises when $h_\eta = 0$. In that case, the information disclosed by the firm contains an extremely noisy component that cannot be interpreted by unsophisticated traders, and they are unable to trade based. In equilibrium only the sophisticated traders trade based on their superior information. Therefore when $h_\eta = 0$, we have $\tau_2(f_2^*) = \tau_1(f_1^*)$.

In regimes (2) and (3), the price informativeness takes on different forms: $\tau_2 = h_v + \beta_2^2 h_u$ and $\tau_3 = h_v + [\beta_3 + \gamma_3 H]^2 h_u$. The price informativeness of public disclosure is always greater or equal to that in regime (2) because more information is contained in price through the trading in response to public disclosure (γ_3) by uninformed traders who act as market makers. Therefore $\tau_3 \geq \tau_2$ for any given f. In equilibrium, the aggregate trading intensity of all informed traders is the same in both cases because the incentive to acquire expertise is the same and therefore we have $\tau_2(f_2^*) \leq \tau_3(f_3^*)$.

The results in Proposition 1 suggest that firms will prefer to either make selective disclosure to sophisticated traders only, or to make public disclosure. However, it is not immediately clear which one of these two policies is preferred. The rest of the analysis will focus on comparing selective disclosure and public disclosure, and ignore regime (2). The preferred disclosure policy depends on the relative magnitude of $\beta_1^2(f_1^*)$ and $[\beta_3(f_3^*)+\gamma_3(f_3^*)H]^2$. The net effect is determined by two relevant forces: from (12), we know that $\beta_1(f_1^*) > \beta_3(f_3^*)$ because of the stronger expertise acquisition incentive induced by the selective disclosure policy. On the other hand, public disclosure has a direct effect on increasing price informativeness through γ_3. Therefore, the firm's optimal disclosure choice given the endogenously determined f_d^* (in Lemma 2) is characterized in Proposition 2:

Proposition 2 *Firms prefer selective disclosure to sophisticated traders to public disclosure if and only if:*

$$(\beta_1^*)^2 > (\beta_3^* + \gamma_3^* H)^2 \tag{13}$$

where $\gamma_3^* = h_v(\rho + \beta_3^* h_u)^{-1}$, *and* $H \equiv \dfrac{h_v^{-1}}{h_v^{-1} + h_\theta^{-1} + h_\eta^{-1} + h_\varepsilon^{-1}}$

To establish that the condition in (13) is non empty, we characterize several special cases as shown in Corollary 1 below:

Corollary 1 *Firms prefer selective disclosure to sophisticated traders over public disclosure when the information noise component that can be filtered by informed traders but not by the public is very high,* $h_\varepsilon \to 0$*; or the noise trading amount is very large,* $h_u \to 0$.

Firms prefer public disclosure to selective disclosure when the information disclosed by the firm contains a highly noisy component that cannot be interpreted by unsophisticated traders, $h_\eta \to 0$.

Proof. See Appendix. ■

The results in Corollary 1 are intuitive. When the noise component that can be filtered by the informed traders but not by the public is large (h_ε is small), the firm wants to restrict its disclosure to the party that can better understand the information so that more precise information can be communicated to the market through informed trading. This suggests that empirically we would observe specialized industries -where information processing is more costly – choosing selective disclosure to sophisticated analysts. When the variance in noise trading is sufficiently large, the contribution to price informativeness through the information content of public information is relatively negligible. Incentives for sophisticated traders to acquire expertise increases when there is more noise trading; this incentive is amplified with selective disclosure. As a result, these firms prefer selective disclosure over public disclosure.

4 The regulator's preference

In Section 3, assuming firms attempt to improve price informativeness, we provide a rational for the continuing practice of selective disclosure despite the goal of Reg FD to limit such practices. While the regulator is also interested in improving market efficiency, his objective may be broader than price informativeness. In this section, we investigate the impact of firms' disclosure policy on the different objectives the regulator may have.

The first metric of interest for the regulator is market liquidity, which measures the information asymmetry between informed and uninformed traders in the market. The lower the market liquidity, the more likely uninformed traders are trading against informed traders.

Market liquidity is given by $\lambda_d^{-1} = \beta_d + h_v(\rho + \beta_d h_u)^{-1}$ as in (6), which depends on the aggregate trading intensity (aggregate risk-adjusted informational advantage of informed traders), the noise trading, and the volatility. To compare the market liquidity for selective disclosure and public disclosure, we also focus on the case where firms prefer selective disclosure, as in the following proposition:

Proposition 3 *When firms prefer selective disclosure as the noise trading becomes large, i.e.,* $h_u \to 0$*, the market liquidity for selective disclosure is greater than that for public disclosure:* $\lambda_1^{-1} > \lambda_3^{-1}$.

Proof. See Appendix. ∎

In other words, when noise trading is sufficiently large, the regulator's objective of maximizing liquidity and the firm's objective of maximizing price efficiency meet.

Another aspect we consider is the welfare of uninformed investors. The purpose of Reg FD is to increase the fairness and transparency of capital market and the confidence of financial markets by reducing information asymmetry between market participants. Many who support Reg FD, including SEC, are concerned about the welfare of uninformed investors when those with superior information make a profit by trading against them. We take the regulator's perspective and focus on type C uninformed traders: those who can never become informed traders. The welfare of type B traders is not relevant to the regulator, as these traders have the (ex-ante) option of becoming sophisticated traders.[11] This assumption is more consistent with the fact that small and uninformed investors often are those who do not have the resources to become informed traders, and it is the regulator's role to protect such investors through regulation.

In our model the uninformed type C traders' welfare is given by (10). For the case of selective disclosure and that of public disclosure, we can write out the utility function in (10), as:[12]

$$E[U(W_{i,1}^C)] = -\left[1 + \frac{\lambda_1^2(h_v - \beta_1\gamma_1 h_u)^2}{h_v h_u}\right]^{-\frac{1}{2}} \tag{14}$$

$$E[U(W_{i,3}^C)] = -\left[1 + \frac{\lambda_3^2(h_v - (\beta_3 + \gamma_3 H)(1-H)\gamma_3 h_u)^2}{h_v h_u}\right]^{-\frac{1}{2}} \tag{15}$$

Since $\lambda_d(h_v - \beta_d\gamma_d h_u) = \rho\lambda_d\gamma_d = \dfrac{\rho h_v}{\rho\beta_d + \beta_d^2 h_u + h_v}$, which is decreasing in β_d, the comparison of the type C traders' utility under selective disclosure and public disclosure is ambiguous. Thus we focus on the special cases where firms prefer selective disclosure, and examine whether this has a negative impact on the uninformed traders' welfare. We find that it indeed does, as summarized in the following proposition:

Proposition 4 *When firms prefer selective disclosure as $h_\varepsilon \to 0$, type C uninformed traders' welfare under selective disclosure is smaller than under public disclosure:*

$$E[U(W_{i,1}^C)] > E[U(W_{i,3}^C)].$$

Proof. See Appendix. ■

[11]Further, in equilibrium sophisticated and unsophisticated informed traders are indifferent in becoming sophisticated or remaining unsophisticated.

[12]See Appendix for details.

From proposition 4, a potential conflict of interest between the regulator and the firm arises, if the regulator cares more about the welfare of uninformed investors than price efficiency and market liquidity. Whether the regulator prefers to abandon the selective disclosure practice depends on how the regulator balances the two different objectives.

5 Conclusion

In this paper, we examine firms' incentives to choose selective disclosure in a noisy rational expectation model. We show that when the firm's objective is to maximize its price informativeness, it may be optimal for the firm to selectively disclose the information. The value of selective disclosure depends on the traders' sophistication. Informed traders who become sophisticated through the acquisition of expertise can better process the information disclosed by the firm. Selective disclosure by the firm may encourage more acquisition of expertise ex-ante and increase the proportion of sophisticated traders in the market. Thus, the overall price informativeness is higher under selective disclosure because of the higher aggregate information quality conveyed through informed trading.

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Appendix

Proof. Lemma 1: selective disclosure

From the market clearing condition in (1), we have the following:

$$\int_0^f \{\beta^A(y_i^A - p_1) + \gamma_1(\bar{v} - p_1)\}di + \int_f^m \gamma_1(\bar{v} - p_1)di + \int_m^1 \gamma_1(\bar{v} - p_1)di + u = 0 \tag{16}$$

Since $\int_0^f y_i^A di = v$, and let $\beta_1 = f\beta^A$, we can get the following price function from the market clearing condition:

$$p_1 = \frac{1}{\beta_1 + \gamma_1}(\beta_1 v + u + \gamma_1 \bar{v}) = \lambda_1 z_1 + \bar{v} \tag{17}$$

where $z_1 = \beta_1(v - \bar{v}) + u$. From this condition, we can get $\lambda_1 = \dfrac{1}{\beta_1 + \gamma_1}$

Given the price function above, the random variable z_1 is informationally equivalent to the price. It follows that

$$\begin{aligned} E[v|p_1] &= \frac{h_v \bar{v} + \lambda_1^{-1}\beta_1 h_u(p - \lambda_1\gamma_1\bar{v})}{h_v + \beta_1^2 h_u} \\ Var[v|p_1] &= (h_v + \beta_1^2 h_u)^{-1}. \end{aligned} \tag{18}$$

The optimization of CARA utility for the type B traders and type C uninformed trader gives us the following:

$$x_i^B(p_1) = x_i^C(p_1) = \frac{E[v|p_1] - p_1}{\rho Var[v|p_1]}.$$

Substitute (18), we can obtain

$$x_i^B(p_1) = x_i^C(p_1) = \frac{1}{\rho}(h_v - \beta_1\gamma_1 h_u)(\bar{v} - p_1) = \gamma_1(\bar{v} - p_1) \tag{19}$$

which implies $\gamma_1 = \dfrac{h_v}{\rho + \beta_1 h_u}$.

Similarly we can also get:

$$E[v|p_1, y_i^A] = \frac{h_v \bar{v} + h_\theta y_i^A + \lambda_1^{-1}\beta_1 h_u(p - \lambda_1\gamma_1\bar{v})}{h_v + h_\theta + \beta_1^2 h_u} \tag{20}$$
$$Var[v|p_1, y_i^A] = (h_v + h_\theta + \beta_1^2 h_u)^{-1}$$

From the optimization of CARA utility of type A informed traders, we can get

$$x_i^A(y_i^A, p_1) = \frac{E[v|y_i^A, p_1] - p_1}{\rho Var[v|y_i^A, p_1]}.$$

Substitute (25), we obtain the following:

$$x_i^A(y_i^A, p_1) = \frac{1}{\rho}[h_\theta(y_i^A - p_1) + (h_v - \beta_1\gamma_1 h_u)(\bar{v} - p_1)] = \beta^A(y_i^A - p_1) + \gamma_1(\bar{v} - p_1), \tag{21}$$

which implies $\beta^A = \dfrac{h_\theta}{\rho}$ and $\gamma_1 = \dfrac{h_v}{\rho + \beta_1 h_u}$. γ_1 is the same as above.

■

Proof. lemma 2: regime (2)

The proof is very similar to the proof of the selective disclosure above. The market clearing condition gives us the price function:

$$\int_0^f \{\beta^A(y_i^A - p_2) + \gamma_2(\bar{v} - p_2)\}di + \int_f^m \beta^B(y_i^B - p_2) + \gamma_2(\bar{v} - p_2)di + \int_m^1 \gamma_2(\bar{v} - p_2)di + u = 0 \tag{22}$$

Let $\beta_2 = f\beta^A + (m - f)\beta^B$, we can get

$$p_2 = \frac{1}{\beta_2 + \gamma_2}(\beta_2 v + u + \gamma_2\bar{v}) = \lambda_2 z_2 + \bar{v} \tag{23}$$

where $z_2 = \beta_2(v - \bar{v}) + u$. From this condition, we can get $\lambda_2 = \dfrac{1}{\beta_2 + \gamma_2}$

The optimal trade orders for each type A and type C trader are still the same as in Lemma 1. Similar to Lemma 1, we get $\beta^A = \rho^{-1}h_\theta$ and $\gamma_2 = \dfrac{h_v}{\rho + \beta_2 h_u}$.

For the type B informed trader, each of them also gets information y_i^B. From the CARA utility optimization, we have the following:

$$x_i^B(y_i^B, p_2) = \frac{E[v|y_i^B, p_2] - p_2}{\rho Var[v|y_i^B, p_2]}. \tag{24}$$

Given that

$$\begin{aligned} E[v|p_2, y_i^B] &= \frac{h_v\bar{v} + (h_\theta^{-1} + h_\eta^{-1})^{-1} y_i^B + \lambda_2^{-1}\beta_2 h_u(p - \lambda_1\gamma_1\bar{v})}{h_v + (h_\theta^{-1} + h_\eta^{-1})^{-1} + \beta_2^2 h_u} \\ Var[v|p_2, y_i^B] &= (h_v + (h_\theta^{-1} + h_\eta^{-1})^{-1} + \beta_2^2 h_u)^{-1} \end{aligned} \tag{25}$$

Substitute into (24), we can get $\beta^B = \dfrac{(h_\theta^{-1} + h_\eta^{-1})^{-1}}{\rho}$. ■

Proof. Derivation of the equation (8)

We are going to show the type A traders' expected utility function and the other two types's utility can be derived in a similar way.

From (7), denote $\Lambda_d^A = \dfrac{E[v|y_i^A, p_d] - p_d}{\sqrt{2Var[v|y_i^A, p_d]}} = \dfrac{E[v|y_i^A, p_d] - p_d}{\sqrt{2(h_v + h_\theta + \beta_d^2 h_u)^{-1}}}$, i.e., the expected utility for type A trader becomes

$$\Sigma_d^A = E[U(W_d^A|y_i^A, p_d)] = -exp\{-(\Lambda_d^A)^2\}.$$

Given the equilibrium price function under each disclosure policy, we can write out Λ_d^A as the following:

$$E[v|y_i^A, p_d] - p_d = \frac{(E[v|p_d] - p_d)(h_v + \beta_d^2 h_u) + h_\theta(y_i^A - p_d)}{h_v + h_\theta + \beta_d^2 h_u} = E[v|p_d] - p_d.$$

Therefore the expectation of Λ_d conditional on the price p_d is:

$$E[\Lambda_d|p_d] = \frac{E[v|p_d] - p_d}{\sqrt{2(h_v + h_\theta + \beta_d^2 h_u)^{-1}}}. \tag{26}$$

From this we can also derive:

$$\begin{aligned} & E[v|y_i^A, p_d] - p_j - E[(E[v|y_i^A, p_d] - p_d)|p_d] \\ = \; & \frac{(E[v|p_d] - p_d)(h_v + \beta_d^2 h_u) + h_\theta(y_i^A - p_d)}{h_v + h_\theta + \beta_d^2 h_u} - (E[v|p_d] - p_d) \\ = \; & \frac{h_\theta}{h_v + h_\theta + \beta_d^2 h_u}(y_i^A - E[v|p_d]). \end{aligned}$$

Then we can get the conditional variance of Λ_d on the price p_d as

$$\begin{aligned} Var[E[v|y_i^A, p_d] - p_d|p_d] &= E[(E[v|y_i^A, p_d] - p_d - E[(E[v|y_i^A, p_d] - p_d)|p_d])^2|p_d] \\ &= \frac{h_\theta^2}{(h_v + h_\theta + \beta_d^2 h_u)^2} E[(y_i^A - E[v|p_d])^2|p_d] \\ &= \frac{h_\theta^2}{(h_v + h_\theta + \beta_d^2 h_u)^2}(Var[v|p_d] + h_\theta^{-1}) \\ &= \frac{h_\theta}{h_v + h_\theta + \beta_d^2 h_u} Var[v|p_d]. \end{aligned} \tag{27}$$

Therefore we have:

$$Var[\Lambda_d^A|p_d] = \frac{h_\theta}{2} Var[v|p_d]. \tag{28}$$

Λ_d^A conditional on p_d is normal distribution with mean $E[\Lambda_d|p_d]$ and variance $Var[\Lambda_d^A|p_d]$, then

$$E[-exp\{-(\Lambda_d^A)^2\}|p_d] = -\frac{1}{\sqrt{1 + 2Var[\Lambda_d^A|p_d]}} exp\{-\frac{(E[\Lambda_d|p_d])^2}{1 + 2Var[\Lambda_d^A|p]}\}. \tag{29}$$

Substitute (26) and (28) to the above equation, we get

$$E[\Sigma_d^A] = E[-exp\{-(\Lambda_d^A)^2\}|p_d] = -\frac{\sqrt{h_v + \beta_d^2 h_u}}{\sqrt{h_v + h_\theta + \beta_d^2 h_u}} \cdot exp\{-\frac{(E[v|p_d] - p_d)^2}{2Var[v|p_d]}\}. \tag{30}$$

■

Proof. Lemma 3

Regime (1): only sophisticated traders A receive the information. From (8), for type A

trader, we have:

$$E[\Sigma_1^A|p_1] = -\frac{\sqrt{h_v + \beta_1^2 h_u}}{\sqrt{h_v + h_\theta + \beta_1^2 h_u}} \cdot exp\{-\frac{(E[v|p_1] - p_1)^2}{2Var[v|p_1]}$$

.

For type B trader, we have:

$$E[U(W_1^B)|p_1] = -exp\{-\frac{(E[v|p_1] - p_1)^2}{2Var[v|p_1]}$$

.

In equilibrium, the following condition must hold:

$$exp\{\rho k\} \frac{E[\Sigma_1^A|p_1]}{E[U(W_1^B)|p_1]} = 1. \tag{31}$$

Hence we have the equilibrium fraction f_1^* of sophisticated traders will satisfy:

$$k = \rho^{-1} ln \sqrt{\frac{h_v + h_\theta + [\beta_1(f_1^*)]^2 h_u}{h_v + [\beta_1(f_1^*)]^2 h_u}}.$$

Regime (2): For the type A trader, we still have:

$$E[\Sigma_2^A|p_2] = -\frac{\sqrt{h_v + \beta_2^2 h_u}}{\sqrt{h_v + h_\theta + \beta_2^2 h_u}} \cdot exp\{-\frac{(E[v|p_2] - p_2)^2}{2Var[v|p_2]}.$$

For the type B trader, the expected utility conditional on the price becomes:

$$E[\Sigma_2^B|p_2] = -\frac{\sqrt{h_v + \beta_2^2 h_u}}{\sqrt{h_v + (h_\theta^{-1} + h_\eta^{-1})^{-1} + \beta_2^2 h_u}} \cdot exp\{-\frac{(E[v|p_2] - p_2)^2}{2Var[v|p_2]}.$$

Therefore in equilibrium we have:

$$k = \rho^{-1} ln \sqrt{\frac{h_v + h_\theta + [\beta_2(f_2^*)]^2 h_u}{h_v + (h_\theta^{-1} + h_\eta^{-1})^{-1} + [\beta_2(f_2^*)]^2 h_u}}.$$

In the disclosure regime (3), the information set for type A and type B traders are the same, i.e.,

$$E[v|y_i^t, y, p_3] = E[v|y_i^t, p_3]$$

$$Var[v|y_i^t, y, p_j] = Var[v|y_i^t, p_3]$$

Therefore the ex-ante expertise acquisition incentive will be the same, and in equilibrium,

$$k = \rho^{-1} ln \sqrt{\frac{h_v + h_\theta + [\beta_3(f_3^*)]^2 h_u}{h_v + (h_\theta^{-1} + h_\eta^{-1})^{-1} + [\beta_3(f_3^*)]^2 h_u}}.$$

■

Proof. Corollary 1

Comparing the price informativeness under selective disclosure and public disclosure is equivalent to comparing $\beta_1^2(f_1^*)$ and $[\beta_3(f_3^*) + \gamma_3(f_3^*)H]^2$.

From Lemma 5, we have

$$\frac{h_v + h_\theta + (\beta_1^*)^2 h_u}{h_v + (\beta_1^*)^2 h_u} = \frac{h_v + h_\theta + (\beta_3^*)^2 h_u}{h_v + (h_\theta^{-1} + h_\eta^{-1})^{-1} + (\beta_3^*)^2 h_u}$$

$$\Rightarrow \quad (\beta_1^*)^2 - (\beta_3^*)^2 = \frac{h_\eta}{h_\theta h_u}[h_v + h_\theta + (\beta_3^*)^2 h_u] \tag{32}$$

Hence we have

$$[\beta_3^* + \gamma_3^* H]^2 - (\beta_1^*)^2 = [\beta_3^* + \gamma_3^* H]^2 - \frac{h_\theta + h_\eta}{h_\theta}(\beta_3^*)^2 - \frac{h_\eta(h_\theta + h_v)}{h_\theta h_u} \tag{33}$$

Then we can show the following:

- If $h_\varepsilon \to 0$, then $H \to 0$, we have $[\beta_3^* + \gamma_3^* H]^2 - (\beta_1^*)^2 < 0$.
- From (33), we get

$$[\beta_3^* + \gamma_3^* H]^2 - (\beta_1^*)^2 = -\beta_3^*[\beta_3^* \frac{h_\eta}{h_\theta} - 2H\gamma_3^*] + (\gamma_3^* H)^2 - \frac{h_\eta(h_\theta + h_v)}{h_\theta h_u}$$

  where $\gamma_3^* H > 0, \forall h_u$. $\beta_3^* \to \infty$ when $h_u \to 0$. Hence $[\beta_3^* + \gamma_3^* H]^2 - (\beta_1^*)^2 < 0$, if $h_u \to 0$.

- If $h_\eta \to 0$, we have $\gamma_3^* H > 0$, thereby $[\beta_3^* + \gamma_3^* k]^2 - (\beta_1^*)^2 > 0$

■

Proof. Proposition 3

$$\begin{aligned} & \lambda_1^{*-1} - \lambda_3^{*-1} \\ = & \frac{1}{h_u}(\frac{h_\theta h_\eta}{(h_\theta + h_\eta)(\rho^2 k^2 - 1)}) + \frac{h_v}{\rho + \frac{h_\theta}{\rho^2 k^2 - 1} - h_v} - \frac{h_v}{\rho + \frac{h_\theta^2}{(h_\theta + h_\eta)(\rho^2 k^2 - 1)} - h_v} \end{aligned}$$

As $h_u \to 0$, $\frac{1}{h_u}(\frac{h_\theta h_\eta}{(h_\theta + h_\eta)(\rho^2 k^2 - 1)}) \to \infty$. Hence, $\lambda_1^{*-1} > \lambda_3^{*-1}$.

■

Proof. Equation (14) and (15)

$E[U(W_{i,d}^C)] = E[-exp\{-\frac{(E[v|p_d] - p_d)^2}{2Var[v|p_d]}\}]$

To derive these utility function, we need to use the following property of the expectation of exponential:

$$E[e^{-mxy}] = [(1 + m\sigma_{xy})^2 - m^2 \sigma_x^2 \sigma_y^2]^{-\frac{1}{2}}, \quad if \quad E[x] = 0 \quad and \quad E[y] = 0$$

.

Let $g_d = E[v|p_d] - p_d$, and $m_d = \frac{1}{2Var[v|p_d]}$, then we have $E[U(W_{i,d}^C)] = -[1 + 2m_d \sigma_{g_d}^2]^{-\frac{1}{2}}$.

1) for selective disclosure,

$$\begin{aligned} g_1 &= E[v|p_1] - p_1 = \frac{(h_v - \beta_1 \gamma_1 h_u)(\bar{v} - p)}{h_v + \beta_1^2 h_u} \\ \sigma_{g_1}^2 &= (\frac{h_v - \beta_1 \gamma_1 h_u}{h_v + \beta_1^2 h_u})^2 (\lambda_1^2 \beta_1^2 h_v^{-1} + \lambda_1^2 h_u^{-1}) \\ m_1 &= \frac{h_v + \beta_1^2 h_u}{2} \end{aligned}$$

Substitute the above values into $E[U(W_{i,1}^C)]$, we get

$$E[U(W_{i,1}^C)] = -\left[1 + \frac{{\lambda_1}^2 (h_v - \beta_1 \gamma_1 h_u)^2}{h_v h_u}\right]^{-\frac{1}{2}}$$

2) Similarly, for the public disclosure, we have

$$
\begin{aligned}
g_3 &= E[v|p_3] - p_3 = \frac{(h_v - (\beta_3 + \gamma_3 H)(1-H)\gamma_3 h_u)(\bar{v} - p)}{h_v + (\beta_3 + \gamma_3 H)^2 h_u} \\
\sigma_{g_3}^2 &= (\frac{h_v - (\beta_3 + \gamma_3 H)(1-H)\gamma_3 h_u}{h_v + (\beta_3 + \gamma_3 H)^2 h_u})^2 \lambda_3^2 [(\beta_3 + \gamma_3 H)^2 h_v^{-1} + h_u^{-1}] \\
m_3 &= \frac{h_v + (\beta_3^2 + \gamma_3 H)^2 h_u}{2}
\end{aligned}
$$

Substitute into $E[U(W_{i,3}^C)]$, we get

$$
E[U(W_{i,3}^C)] = -\left[1 + \frac{\lambda_3^2 (h_v - (\beta_3 + \gamma_3 H)(1-H)\gamma_3 h_u)^2}{h_v h_u}\right]^{-\frac{1}{2}}
$$

■

Proof. Proposition 4

From (15), when $h_\varepsilon \to 0$, we have $k \to 0$. Then the comparison becomes to compare $-\left[1 + \frac{\lambda_1^{*2} (h_v - \beta_1^* \gamma_1^* h_u)^2}{h_v h_u}\right]^{-\frac{1}{2}}$ and $-\left[1 + \frac{\lambda_3^{*2} (h_v - \beta_3^* \gamma_3^* h_u)^2}{h_v h_u}\right]^{-\frac{1}{2}}$.

And since $\lambda_j^2 (h_v - \beta_j \gamma_j h_u)^2 = \lambda_j^2 \gamma_j^2 = \frac{\rho^2 h_v^2}{(\rho \beta_j + \beta_j^2 h_u + h_v)^2}$, and $\beta_1^* > \beta_3^*$,

we get $-\left[1 + \frac{\lambda_1^{*2} (h_v - \beta_1^* \gamma_1^* h_u)^2}{h_v h_u}\right]^{-\frac{1}{2}} < -\left[1 + \frac{\lambda_3^{*2} (h_v - \beta_3^* \gamma_3^* h_u)^2}{h_v h_u}\right]^{-\frac{1}{2}}$.

Hence as $h_\varepsilon \to 0$, we have

$$
E[U(W_{i,1}^C)] > E[U(W_{i,3}^C)]
$$

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