A Study of Participation in Dynamic Auctions

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Abstract

We study participation and bidding decisions in repeated Michigan Department of Transportation procurement auctions. The key finding is that dynamic linkages in auctions exist in the participation stage of the game. As a result, with forward looking bidders we find that the sequencing of contracts by size can be used to influence competition within an auction. To fully understand the extent of these effects on auction outcomes, we construct and estimate a dynamic asymmetric auction model with endogenous participation and forward-looking bidders. We then quantify the level of inefficiency under the current auction rules and consider how the sequencing of heterogeneous contracts affects participation.

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1 Introduction

There has been a growing interest in empirically analysing the interaction between auction rules and bidder participation. However, little attention has been paid to the possibility that bidders are forward-looking and how this affects performance of mechanisms. This paper analyses bidding data from repeated Michigan Department of Transportation (MDOT) highway procurement auctions and answers the following questions:

1. Given that auctions occur frequently over time, is there evidence to suggest that bidders consider their participation decisions as inter-temporally linked?

2. If there is evidence to suggest dynamic behaviour, what bidding model could rationalize observed patterns in the data?

3. What implications does this bidding model have for the performance of current auction institutions and the design of auction rules, which would have been overlooked in a static setting?

4. How important are these implications for this market?

To answer the first question, this paper provides descriptive analyses of participation and bidding behaviour. The findings suggest that there are indeed dynamic linkages in participation. In particular, the key patterns that emerge are:

- The presence of above-average sized contracts to be awarded in a subsequent round increases participation probabilities in current auctions.

- A bidder active in a previous round of bidding is more likely to enter an auction in a current round.

- The linkages in participation remain when considering winter periods where no construction can take place but contracts are still awarded. In other words, linkages in participation seem to be related to the participation stage of the game and not necessarily directly to the bidding game.

These effects suggest that a model with dynamic participation synergies and forward-looking bidders could rationalize observed patterns in the data. Specifically, if a bidder is capable of decreasing participation costs in a future round of bidding we would expect there to be "persistence" in participation probabilities. Moreover, if a future contract is large, a bidder would want to take advantage of lower information costs "tomorrow" by participating "today"[1]. The lack of capacity concerns in this

[1] The terms "tomorrow" and "today" are not meant to be taken literally and merely signify the chronology of events. These terms will be more clearly defined when discussing the nature of the dynamics in this market.

market make the dynamics of this model distinct from those considered in Jofre-Bonet and Pesendorfer (2003).

The model allows for dynamic synergies in participation to generate asymmetries between bidders. These are in addition to asymmetries included in the bidding stage.

The introduction of dynamics also opens up the possibility, not previously considered empirically, that the awarding body can alter the level of competition in an auction by changing the order of award of contracts. In particular, bidders in the model consider the size of contracts in future rounds and take into account future participation cost savings. As mentioned previously, if a bidder observes above-average sized contracts in the next round, the probability of participation in a current auction is increased. This suggests that if contracts are clustered by contract size -in other words all below average sized contracts are awarded first followed by larger ones -it is possible for competition in small contract auctions to be lower.

To assess how important these effects are for this market the model primitives, the participation cost distribution and the construction cost distributions, are estimated. Our estimation strategy builds on the approaches of Guerre, Perrigne, and Vuong (2000) and Pesendorfer and Schmidt-Dengler (2008). The parameter estimates suggest that participation synergies do exist and that asymmetries due to differing construction costs are small. With the primitives in hand, it is possible to assess the frequency of misallocation due to bidder asymmetry. It is also then possible, to consider how re-arranging contracts by size can affect auction competition.

In answering the fourth question, we find that in this market asymmetries between bidder types are small and do not have a large influence on misallocations. In addition, bidders face uncertainty over the set of actual competitors in the auction game which attenuates the incentives for strategic bid-shading.

The dynamic synergies, however, do come into play when the order in which contracts are awarded is considered. We conclude that MDOT's current practice of not clustering large and small contracts, ensures that there is still participation in small contracts from long-lived bidders. If all above-average sized contracts are awarded together, regular bidder participation in smaller contract auctions is reduced.

This study highlights that including dynamics in the analysis of auction markets can help develop new tools for influencing bidder behaviour and discover new sources of asymmetry not present in the existing literature.

Our paper contributes to a growing literature on estimating static auctions with endogenous participation[2], for example, Athey, Levin, and Seira (2011), Athey, Coey, and Levin (2010), Bajari and Hortacsu (2003), Krasnokutskaya and Seim (2010), Li and Zheng (2009), and Marmer, Shneyerov, and Xu (2007) [3]. Hendricks, J., and Porter (2003) consider common value first price auction models with endogenous participation and test the predictions of the model. A number of the aforementioned papers trace the effects of different auction rules on participation, for example Athey, Coey, and Levin (2010) and Krasnokutskaya and Seim (2010) consider the effect of preferential treatment of bidders. Haile, Hong, and Shum (2006) consider tests for common value auctions, and also analyse auction participation under unobserved heterogeneity. Silva, Jeitschko, and Kosmopoulou (2005) analyse synergies in bidding using reduced form techniques. Synergies occur when a bidder wins an auction and is able to pass benefits from winning to the next auction.

The only other paper we are aware of that estimates a dynamic auction using highway procurement data is Jofre-Bonet and Pesendorfer (2003). The authors of this paper look at how capacity constraints affect bidding behaviour, and participation is taken to be exogenous. There are also a number of papers that look at entry into markets using dynamic frameworks, these include Collard-Wexler (2006) and Ryan (2006). Auguirregabiria and Mira (2007) also provide an application of their pseudo-maximum likelihood methods to firm entry and exit in local retail markets. Other papers that look at estimating dynamic games are Bajari, Benkard, and Levin (2007) and Pakes et al. (2007). These papers all make use of the insights from Hotz and Miller (1993), Manski (1993) and Rust (1994)[4]. Procurement auctions have been analysed, amongst others, by Porter and Zona (1993), Bajari and Ye (2003), Hong and Shum (2002) and Krasnokutskaya (2011). Krasnokutskaya (2011) considers auctions also run by MDOT, however covering an earlier time period and in a static setting. Paarsch (1992), Laffont, Ossard, and Vuong (1995) and Guerre, Perrigne, and Vuong (2000) have developed empirical methods to estimate private information in static auction environments with exogenous participation.

The remainder of the paper is organised as follows: in Section 2 and Section 3 we provide a summary of the data and descriptive analyses on the role of participation dynamics on auction outcomes. Section 4 outlines the theoretical model. Section 5 outlines identification of the structural model. We then show in Section 6 how the primitive parameters of our model can be estimated. Section 7 outlines the

[2]The theoretical literature on entry in auctions is more developed than the empirical one, with papers from Samuelson (1985), McAfee and McMillan (1987), Levin and Smith (1994)

[3]These papers makes use of static participation games which suffer from multiple equilibria and the typical approach has been to select an equilibrium or to provide conditions that guarantee uniqueness. An alternative is to due to Manski and Tamer (2003) and Tamer (2003). These approaches estimate parameter bounds that are consistent with all equilibria implied by the model.

[4]In particular, Hotz and Miller (1993) establish that conditional choice probabilities at each state can be used in an inversion to infer value functions. Rust (1994) outlines a method for estimating dynamic markovian games, related to current approaches.

main estimation outputs and Section 8 summarises the results of our policy analysis.

2 The Procurement Process in Michigan

In this section the data source and some aspects of the procurement process are described. Descriptive evidence of dynamic linkages between auction rounds is presented. We explore whether dynamic effects exist when we look directly at an individual bidder's participation and bid-level decisions. There is evidence that past participation and specific future contract characteristics have an effect on participation probabilities in current auctions.

The Awarding Process: The auctions under investigation are used to award contracts for highway construction, bridge construction and highway resurfacing. MDOT awards contracts in bi-monthly rounds using a first price sealed bid procurement auction. Rounds are usually timed to be at the middle of the calendar month and at the end with an average of two weeks between bid letting rounds.

A bid contains details on specific costs, such as labour, mobilization and materials, and the required quantities. The winner is determined solely by the level of the final total cost submitted by a bidder. Cost breakdowns are mainly used to ensure that a bid adds up. In particular, breakdowns are used to ensure that a bid is not "materially unbalanced", in order to avoid bid skewing [5].

A pre-qualification process overseen by MDOT prior to bidding seeks to ensure that only competent contractors are allowed to participate[6]. On average 50 contracts are awarded in a round of bidding. The average project size is $1.476 million and the maximum project size in our data set is $165 million. The timing of contract rounds for the year is known in advance, however the contract characteristics are not fully revealed. Prior to the awarding of the contract, bidders can anonymously purchase/download plans for contracts. These detail the location of a project, the nature of the work and the estimated cost. Bids can be submitted in person or electronically. On the letting day, all bids are unsealed and ranked. As mentioned previously, the low bidder wins the auction. Bidders must also provide a bid deposit that is a pre-determined percentage of the contract value, as determined by the engineer's estimate, prior to bidding. Once a contract has been awarded, the primary contract winner is may subcontract up to 60% of the contract value to subcontractors.

Summary Statistics for all Bidders: Table 1 and Table 7 provide summary statistics of the data. Table

[5] Athey and Levin (2001), for example, find that bid skewing is a frequent occurrence in timber auctions.

[6] This entails a check on the financial status of the firm. The information required for qualification includes: The identity of the owners, shareholder and managers of the company, any affiliations with other contractors, recently completed contracts and identity of clients, previous sales, an average of the firm's backlogs over the past three years, activities in other states, connections to other pre-qualified bidders, and firm's balance sheet. MDOT also has a "Disadvantaged Business Programme" to encourage participation from smaller or disadvantaged firms, however this occurs only on a small fraction of contracts.

Table 1: Summary Statistics

	Mean	**Standard Deviation**	**Minimum**	**Maximum**
Number of Bidders	4.943	2.731	1	19
Log of Engineer's Estimate	14.205	1.203	8.517	18.922
$\frac{\text{Ranked2-Ranked1}}{\text{Ranked1}}$	0.083	0.184	2.563e-06	0.880
$\frac{\text{Ranked1-Estimate}}{\text{Estimate}}$	-0.069	0.146	-0.198	0.626

1 reports that on average an auction attracts five bidders, with some contracts having only one bidder and others with 19 participating bidders. The third row of Table 1 presents data on "money left on the table", which can give an indication of the level of uncertainty in the market. This is the percentage difference between the winning bid and the second lowest bid. On average there is a difference of about 7% with a standard deviation of 8%. This suggests that there are substantial informational asymmetries.

In the appendix we also provide summaries of the data by number of bidder.

Participation Summary Statistics: We next turn to the individual bidder's participation decisions. Summary statistics on participation are provided. To analyse an individual bidder's participation decision probit models of the probability of entry into a single auction by an individual bidder are estimated.

We focus on regular bidders since these are firms considered by MDOT to be unconstrained and are able to concurrently complete a number of large projects. Moreover, these regular bidders are active across different project types and are capable of submitting bids on a number of different projects, such as bridge construction and highway resurfacing. Participation rates are 7% of all auctions with the most frequent participating bidder entering 17% of the auctions of which they win on average 1.5% of auctions. On average, regular bidders are plan-holders of around 11% of the contracts. These participation rates, given the absence of binding reserve prices in this market, might suggest that bid submission is a costly process.

Fringe Bidders: However, before conducting a more detailed analysis of regular bidders, we present some statistics on fringe bidder participation in Table 2. On average, there are about 2.760 (standard deviation 2.370) fringe bidders in an auction. There is a potential pool of about 500 fringe bidders. The average participation probability of a single fringe bidder is 0.51% with standard deviation 0.83%. Given that these fringe bidders have such low participation probabilities, it is therefore reasonable to treat them as separate from regular bidders. Moreover, we only observe these fringe bidders entering

Table 2: Summary Statistics for Regular and Fringe Bidders

	Mean	**Standard Deviation**	**Minimum**	**Maximum**
Number of Large Bidders	0.454	0.653	0	3
Number of Small Bidders	1.717	1.760	0	9
Number of Fringe Bidders	2.771	2.368	0	17

sporadically and not participating in a sequence of auctions. Sometimes, a fringe bidder will not re-appear until a year later, whereas a regular bidder participates more frequently.

Participation in Auctions: Given that contracts are awarded in rounds, we also provide some information on participation per round. Regular bidders participate in roughly 70% of the auction rounds. They enter in approximately 8% of auctions in a round, with an average of 50 contracts on offer per round which translates roughly to participation in four auctions in a round. Regular bidders participate in most of the auctions for which they hold plans but there is still uncertainty, being a plan-holder does not guarantee participation and plan-holder lists do not give a complete picture of the potential competition. An average of 0.454 regular large bidders participate in an auction, shown in Table 2. Some auctions attract zero regular large bidders. On average, there are 1.717 small bidders per auction. At this stage it is worth noting that the identity of bidders does not remain fixed. As a result, there is uncertainty over the precise level of competition.

3 Dynamics of Participation

In this section descriptive analyses are used to identify participation linkages across auctions and time. The results presented in this section focus on the behaviour of large and small regular bidders. A regular bidder has participated in more than 100 auctions and a large bidder is one with more than six plants in Michigan State. A plant is a base of operations for the contractor, where equipment is stored as well as materials prepared for construction. We use six plants as a cutoff since plant sizes for bidders are clustered around one and two plants or more than six plants[7]. We would expect regular bidders with a large number of plants, located across Michigan, to be more easily able to mobilize equipment than a regular bidder with only a few plants. This categorization should capture any asymmetries between bidders due to geography. There are four regular large bidders and 26 regular small bidders in this market.

[7] This excludes the possibility that a bidder might have plants in neighbouring states and uses those to mobilise equipment to complete a project. We will be investigating this in future.

One concern when dealing with highway construction bidding is the possibility of capacity constraints. Jofre-Bonet and Pesendorfer (2003) provide some evidence of the effects of capacity constraints on bidding. However, we are able to find linkages in participation over time that are not driven by bidder capacity. The analysis in this section is focused on participation decisions of bidders into a single auction. The conditioning variables used throughout these analyses are as follows:

- Contract covariates, including geographic, project type (paving, road construction and the like) and size.
- Covariates of contracts being awarded in the same round as the contract under investigation. In particular, we control for whether a contract being awarded concurrently is in the same region. We also control for the mean size of concurrently awarded contracts.
- Controls for bidder identities.
- Controls for potential auction competition

The first step in our descriptive analyses is to determine whether there is state dependence in participation. To do this, we include the activity status of the bidder in the previous round and of those of his competitors in the participation probabilities. We then examine the relevance of capacity constraints in this market and find them to have no effect. We then introduce forward looking variables to establish the potential for dynamic behaviour.

Isolating Participation Linkages Using Winter Months: During winter months all construction in Michigan stops. However, MDOT still awards contracts during the winter. We therefore have data on participation decisions where capacity constraints have no bearing on behaviour. We first run a probit on a subset of the data only using winter months controlling for auction characteristics and bidder identities. The results are shown in column (i) in Table 3. Previous round activity status has a positive statistically significant effect on participation probabilities. This state dependence is independent of capacity constraints, since no construction is taking place during these time periods. We are therefore finding evidence of state dependence from the participation stage. In particular, this dependence is expressed as a positive relationship between previous round participation and current auction participation. In other words, a bidder active in a previous round of bidding, independent of the outcome of the previous auction, will be more likely to enter a current round auction.

Controlling for Capacity: We now extend our analysis across the rest of the data set, excluding winter months. We now introduce various proxies for capacity constraints to ensure that the effects we are finding are not due to backlog effects. Since we cannot observe the pattern of construction completions and to avoid imposing structure on how contracts are completed, we simply include backward looking variables summarising the size of previously won contracts[8]. We experimented with including contracts won in the previous k rounds, where $k = 1, ..., 20$. In column (ii) we simply sum previous round contracts. This variable is statistically insignificant. We then include separate variables for each round of won contracts in column (iii). Specifically, "Backlog (t-k)" measures the log dollar value of contracts won in the kth previous round . In all specifications, these variables were statistically insignificant and suggests that capacity constraints do not bind. These estimates suggest that regular bidders are not affected by capacity constraints.

Dynamic Behaviour: The previous descriptions establish the existence of state dependence in participation[9]. However, we must still establish that bidders are forward looking and that bidder behaviour is dynamic. To do this we extend our analysis to include forward looking variables that bidders observe. We now include in column (iv) a binary forward looking variable, "Future Large", which measures the presence of above average sized contracts to be awarded in the next round. This variable is statistically significant and positive. This indicates that with large contracts to be awarded in the next round of bidding a bidder is more likely to participate in a current round of bidding. This effect with the state dependence captured by the variable measuring a bidder's own previous round activity status, suggests that there are dynamic linkages in the participation stage of the game.

[8] Jofre-Bonet and Pesendorfer (2003) and Li and Zheng (2009) assume that an equal proportion of the contract is completed over time.

[9] Jofre-Bonet and Pesendorfer (2003)'s analyses establish state dependence, however they do not include forward looking variables to reduce the computational burden of their estimation approach.

Table 3: Probit Estimates

Variable	**Coefficient** (Standard Errors)			
Dependent Variable: Regular Bidder Participation in a Single Auction				
	(i)	(ii)	(iii)	(iv)
Previous Round Activity (0/1)	0.130***	0.180***	0.180***	0.180***
	(0.029)	(0.033)	(0.033)	(0.033)
# Small Bidders	-0.002	-0.007**	-0.007**	-0.008***
Active in Previous Round	(0.002)	(0.002)	(0.002)	(0.002)
# Large Bidders	-0.032**	0.003	0.003	0.008
Active in Previous Round	(0.012)	(0.014)	(0.014)	(0.014)
Engineer	0.058***	0.058***	0.059***	0.057***
	(0.007)	(0.007)	(0.007)	(0.007)
Average Size	0.010	-0.016	-0.016	-0.018
Other Contracts in Round	(0.011)	(0.011)	(0.011)	(0.011)
# Contracts in Round	-0.000	-0.001	-0.001	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)
Geographically Close	0.023	0.251	0.251	0.283*
Contracts In Round (0/1)	(0.139)	(0.143)	(0.143)	(0.144)
Simple Backlog Sum		-0.000		
		(0.001)		
Backlog (t-1)			-0.007	-0.006
			(0.009)	(0.009)
Backlog (t-2)			0.002	0.002
			(0.006)	(0.006)
Backlog (t-3)			-0.003	-0.003
			(0.006)	(0.006)
Backlog (t-4)			-0.008	-0.008
			(0.007)	(0.007)
Backlog (t-5)			0.013	0.012
			(0.007)	(0.007)
Future Large (0/1)				0.048**
				(0.018)
Constant	-3.235***	-2.975***	-2.977***	-3.004***
	(0.117)	(0.105)	(0.105)	(0.105)
Auction Covariates	YES	YES	YES	YES
Bidder Identity	YES	YES	YES	YES
Only Winter Data	YES	NO	NO	NO
# Observations	62396	80040	80040	80040

Table 4: Bid Level Estimates (Heckman corrected)

Variable	**Coefficient**		
Dependent Variable: Log of Bid Level	(Std. Err.)		
	(i)	(ii)	(iii)
Engineer's Estimate	0.9743***	0.9772***	0.9772***
	(0.0017)	(0.0018)	(0.0018)
Participation in Previous Round	0.0110	0.0064	0.0064
	(0.0114)	(0.0115)	(0.0095)
$\mathcal{N}_{\mathcal{S}}$	-0.0001	0.0000	0.0000
	(0.0007)	(0.0006)	(0.0006)
$\mathcal{N}_{\mathcal{L}}$	-0.0015	-0.0015	-0.0015
	(0.0035)	(0.0034)	(0.0034)
Large Bidder	-0.0346***	-0.0356	-0.0356*
	(0.0100)	(0.0185)	(0.0175)
Large Future	0.0073	0.0024	0.0024
	(0.0063)	(0.0062)	(0.0061)
Winter	0.0087*	0.0099*	0.0099*
	(0.0041)	(0.0042)	(0.0042)
Backlog			-0.0000
			(0.0014)
Constant	0.3940***	0.4127***	0.4137***
	(0.0742)	(0.0791)	(0.0773)
Auction Covariates	NO	YES	YES
Bidder Identities	NO	YES	YES

Number of Auctions: 4927
Number of Bidders: 30

3.1 Bid Level Decision

Table 4 summarises results from Heckman estimates of the bid level decision for regular bidders. The engineer's estimate is clearly the strongest influence on the bid level. State variables do not have a statistically significant effect on bid levels. In particular, own participation status has no statistically significant effect on bid levels. The large bidder dummy has a negative sign, which indicates potential asymmetries between large and small bidders. We also consider a specification with bidder fixed effects for each regular type bidder in columns (ii) and (iii).

The Heckit estimates in this section cannot provide any further information on how the participation synergies affect bid levels of regular bidders. A potential source of concern are the fringe bidders. Given that the probability of facing at least one fringe bidder is quite high, the full effect of the dynamics cannot be discerned from bidding behaviour when a regular bidder faces a set of short lived bidders.

3.2 Summary and Explanation of Patterns

The descriptive evidence suggests that dynamic linkages between auctions do exist for regular bidders and can have an impact on procurement costs for MDOT. There is a positive effect of previous round participation on current auction participation as well as of future contract characteristics, which indicates the existence of some form of synergies as well as forward-looking behaviour. The evidence suggest that there are "learning by prepraring" for contracts which can induce higher participation probabilities in future rounds of bidding.

These results merely indicate patterns in the data and are by no means causal relationships. This prevents us from making stronger statements about the causal nature of these outcomes and we also do not have a full sense of the magnitude of these effects. In particular, some of the effects are relatively small, however their full impact on behaviour cannot be completely ascertained in these descriptive analyses.

4 Participation and Auction Game

To understand these patterns more fully, we therefore proceed to construct a structural econometric model that will allow us to analyse the dynamic strategic effects that might be operating in this market and their role in determining auction outcomes. The data suggest that participation is a costly process and that participating in consecutive rounds of bidding is less costly than in non-consecutive rounds. This suggests that there are synergies in participation. Moreover, with synergies and forward looking bidders, we would expect bidders to adjust current behaviour to reap lower participation costs in next round bidding, especially if the value of future contracts is large. This is precisely the pattern we observe in the data.

We now construct a theoretical model of auction participation in a dynamic framework. Participation is decided in every period. Entry into an auction is costly and bidders must pay an information acquisition or bid preparation cost to learn their private completion costs. Bidders compare the expected profit stream from participation and non-participation to determine entry. There are information acquisition synergies between auction rounds. Specifically, a bidder can improve the information cost draws in the next period by participating in an auction in the current round. Once a bidder decides to enter an auction, the bidders engage in a first price sealed bid procurement auction.

4.1 Setup and Assumptions

The focus of the analysis is on risk neutral[10] long-lived bidders who participate in more than 100 auctions in our data set. Bidders who participate in fewer auctions are categorised as short-lived fringe bidders. It is assumed that once private costs are known, a bidder will participate in the auction[11].

Time is discrete with an infinite horizon. There are two types of regular bidders, large bidders $\mathbf{N}_{\mathcal{L}} = \{1, ..., N_{\mathcal{L}}\}$, who have more than six plants in the state and small bidders $\mathbf{N}_{\mathcal{S}} = \{N_{\mathcal{L}}+1, ..., N_{\mathcal{L}}+N_{\mathcal{S}}\}$, who have up to six plants. There is also a fixed set of fringe bidders $\mathbf{N}_{\mathcal{F}} = \{N_{\mathcal{L}} + N_{\mathcal{S}} + 1, ..., N_{\mathcal{F}} + N_{\mathcal{S}} + N_{\mathcal{L}}\}$. We will sometimes denote the total number of bidders as $N = N_{\mathcal{L}} + N_{\mathcal{S}} + N_{\mathcal{F}}$. A typical bidder of either type will be denoted by i. The number of bidders of each type does not vary over time.

The Stage Game: Each time period t is broken down into two stages, the Participation Stage followed by the Auction Stage. The sequence of events is as follows:

1. Participation Stage
   (a) Each regular bidder i receives a draw from a private information acquisition cost distribution at the beginning of period t
   (b) Bidders observe the participation statuses of all bidders, the presence of above-average sized future contracts and the size of the current contract, $c_{0,t}$
   (c) All bidders decide simultaneously whether to enter the auction
2. Auction Stage
   (a) Without observing the outcome of the first stage, each bidder learns its own completion cost privately
   (b) All participating bidders simultaneously submit bids without knowledge of actual competition[12]
   (c) The contract is awarded to the low bidder

[10] The assumption of risk-neutrality can be justified by realising that most regular bidders are large corporations and are active in various states. We can therefore use a portfolio diversification argument.

[11] An alternative participation model is by Samuelson where bidders already know their valuations when they make their entry decision but have to pay a cost to learn their values. Here the set of bidders will be a selected sample of bidders who have valuations above a certain threshold value. It is possible that the true participation decision is a hybrid between the Samuelson (1985) and Levin and Smith (1994) participation game. Li and Zheng (2009) estimate these different auction models in their paper.

[12] Krasnokutskaya (2011) assumes that bidders know the identity of their opponents. In our case this is not a good assumption, since the data is from a period where it was also possible to download plans anonymously. The published plan holder list will therefore not always be complete and there is still uncertainty about the actual level of competition for a contract. However, if we did choose to include the plan holder information, the only difference from our model would be the inclusion of an extra stage prior to the participation stage, which determines plan holder status.

As mentioned before, the procurement auctions under investigation are run in rounds, where up to 100 construction contracts are auctioned off at once. This might introduce scope for possible synergies between contracts offered in the same round or possible exposure problems, where bidders win too many contracts at once. These possibilities are not considered in this model, due to the lack of further information on ex-post costs and MDOT not offering contracts in packages. Moreover, only inter-temporal substitution of auctions is considered. This rules out within-round substitution. The rest of the setup is as follows:

Reserve Price: MDOT requires that the winning bid be lower than 110% of the engineer's estimate. If the Department wishes to accept a bid higher than this threshold, it is required to write a justification for doing so. The data include a number of projects being awarded for more than the threshold. This suggests that these restrictions do not come into effect very often. We follow Krasnokutskaya (2011) and assume there is no binding reserve price. To avoid the possibility that bidders submit infinite bids, we follow Li and Zheng (2009). The authors suggest that when a bidder is the only entrant he must compete with the Department. We, therefore, assume that when a bidder is the sole participant, he will then face the DOT that draws a completion cost from a regular large bidder's cost distribution.

Contract Characteristics at time t are denoted by $c_{0,t} \in \mathbf{C}_0$ and are drawn from the known exogenous distribution $F_0(.)$. Future contract characteristics are unknown to all bidders other than information encoded in the state variable z, defined below. The contract characteristics are the physical attributes of the contract. Our analysis restricts attention to the engineer's estimate of the project size. MDOT does not systematically order contracts by size, therefore bidders cannot precisely determine what type of contracts will be awarded next.

Future Contracts: Bidders at time t observe the presence of above average-sized contracts in the next round $(t+1)$. Let $z_t \in \mathbf{Z} = \{0, 1\}$ be a variable that equals 1 if there are above-average sized contracts in the next round of bidding. They, however, do not observe the full set of contracts to be awarded in the next round and do not know the number of large contracts. These assumptions are in line with the descriptive evidence found in the previous section.

Private Completion Costs: Regular Bidder i of type $j = \mathcal{L}, \mathcal{S}, \mathcal{F}$ draws private costs, $c_{i,t} \in \mathbf{C}_j$, independently and identically from the cost distribution $F_j(c_{i,t}|c_{0,t})$ on $[\underline{c}, \overline{c}]$, conditional on $c_{0,t}$. The assumption of independent private values can be justified by assuming that differences in cost estimates are due to firm-specific factors such as differing opportunity costs and input prices. Notice that in this specification, the completion cost distribution does not depend on state variables. The estimation does not rely on this assumption. In fact, we estimate completion costs point-wise from a first order condition. This is done for every bid separately and for different state variable configurations. We can then compare cost distributions for different states, i.e. where $s_{i,t} = 0$ or $s_{i,t} = 1$. The estimation results do not, however, suggest that there are differences in the cost distribution for bidders with different states. We therefore maintain the assumption that dynamic linkages occurs only through participation costs and their associated synergies.

An action for bidder i in period t is given by the participation decision and the bid submitted at the auction stage, $a_{i,t} \in A_i = \{0,1\} \cup [0,\infty)$. The participation decision will be denoted separately by $d_{i,t} \in \{0,1\}$ and the bid by $b_{i,t} \in [0,\infty)$.

Public States for Regular Bidder i: Bidder i is characterised by a publicly observable state variable $s_{i,t} \in \mathbf{S}_i \equiv \{0,1\}$ that affect its actions. The state is the participation status of a bidder in the previous round of bidding, i.e. $s_{i,t} = d_{i,t-1}$. The vector of all bidders' state variables is given by $\mathbf{s}_t = (s_{1,t},...,s_{N,t}) \in \mathbf{S} = \times_{k=1}^{N}\mathbf{S}_k$. We will sometimes use the notation $\mathbf{s}_{-i,t} = (s_{1,t}, s_{2,t},..., s_{i-1,t}, s_{i+1,t},...s_{N,t}) \in \mathbf{S}_{-i} = \times_{l\neq i}\mathbf{S}_l$ to denote the vector of state variables excluding bidder i. The cardinality of the state space $\mathbf{S}$ equals $m_s = 2^N$.

Private States for Regular Bidder i of type j: Information Costs, $\phi_{i,t} \in \Phi_j = [0,\infty)$, are drawn independently and identically from the conditional distribution $H_j(\phi_{i,t}|s_{i,t})$ with associated density $h_j(\phi_{i,t}|s_{i,t})$, and are unobserved by other bidders and the econometrician. Motivated by the descriptive analysis of participation behaviour in the previous section, information costs are assumed to have a Markov structure and have following transition probability:

$$h_j(\phi_{i,t}|s_{i,t}) = \lambda_j(s_{i,t})e^{-\lambda_j(s_{i,t})\phi_{i,t}} \tag{1}$$

where $\log[\lambda_j(s_{i,t})] = \lambda_{0,j} + \lambda_{1,j}s_{i,t}$[13]. Both parameters are unknown to the econometrician but known to the bidders. Rivals' actions and states do not affect the private costs of information acquisition. We have experimented with alternative specifications of the information cost distributions and estimated the model with these, inter alia the pareto[14], normal and half-logistic, however none of these distributions comes close to matching the estimated choice probabilities. We therefore opted for the exponential distribution which provided a better fit. Notice that the structure of the information cost distribution is such that participating in an auction in a current period will allow bidders to draw information costs

[13]Our estimator is also able to handle an information cost distribution that also depends on c_0. However, we have omitted this in our current model

[14]The Pareto also has the added disadvantage that estimates might imply an undefined first moment, which is required for Proposition 1 of Hotz and Miller (1993) to hold.

from a more advantageous distribution, i.e. a distribution with a lower mean, in the next period. It is possible to make the information cost depend on more than one previous period and to allow for cumulative cost advantages. This has been excluded in the current analysis but could be explored at a later date[15]. This decision was motivated by the descriptive analyses in the previous section. There it became clear that only consecutive rounds have a bearing on participation probabilities and non-consecutive rounds did not yield any statistically significant change to entry probabilities.

Information Costs for Fringe Bidders: Fringe bidder i needs to pay information cost K to learn her private completion costs.

Discounting: Bidders discount the future with common discount factor $\beta \in (0,1)$ fixed over time, known to the econometrician and the bidders. The annual discount factor equals $\beta = 0.8$. Notice that this imposes forward-looking behaviour of regular bidders. Hendricks and Porter (2007) discuss possible strategies for identifying the discount factor using exogenous variation in the bidding environment. Arcidiacono and Miller (2010) also discuss situtations where β can be estimated.

Conditional Independence: As in Rust (1987), it is assumed that the unobserved information costs are conditionally independent of observable states. The structure of the problem already embodies the usual assumption that private "shocks", here the information costs, are additively separable. For further discussion of these assumptions see Rust (1994).

Regular Bidder Strategies: Strategies for bidder i of type j are restricted to be Markovian for the entry game. The strategy for bidder i of type $j = \mathcal{L}, \mathcal{S}, \mathcal{F}$ consists of a participation strategy $d^{\sigma}_{i,j}(\mathbf{s}, \phi_i, c_0, z)$ and a bidding strategy $b^{\sigma}_{i,j}(\mathbf{s}, c_i, c_0, z)$ and will be denoted $\sigma_{i,j} = (d^{\sigma}_{i,j}(\mathbf{s}, \phi_i, c_0, z), b^{\sigma}_{i,j}(\mathbf{s}, c_i, c_0, z))$. Formally, a Markov strategy is a map, $\sigma_{i,j} : \mathbf{S} \times \Phi_j \times \mathbf{C}_j \times \mathbf{C}_0 \times \mathbf{Z} \rightarrow A_i$. Fringe bidder strategies are denoted separately by $\sigma_{i,\mathcal{F}}$ and the set of all fringe strategies is denoted $\sigma_{\mathcal{F}} = \{\sigma_{i,\mathcal{F}} : i = 1, ..., N_{\mathcal{F}}\}$. Strategies consist of a bidding strategy and an entry strategy $d^{\sigma_{\mathcal{F}}}$.

Beliefs on the Probability of Participation: To form the necessary expectations and to compute the probability of a bidder winning an auction, bidders' beliefs of the likely number of bidders based on the decision rules of bidders must be defined. Beliefs are

$$q_{i,j}(\mathbf{s}_t, c_{0,t}, z_t) \equiv \Pr(i \text{ of type } j \text{ enters} | s_{i,t}, \mathbf{s}_{-i,t}, c_{0,t}, z_t) = \int_0^{\infty} \mathbf{1}\{d^{\sigma}_{i,j}(\mathbf{s}_t, \phi, c_{0,t}, z_t) = 1\} h_j(\phi | s_{i,t}) d\phi \quad (2)$$

[15] For example, it is possible to maintain a similar structure by redefining $s_{i,t}$ to equal participation from T previous rounds, i.e. $s_{i,t} = \mathbf{1}\left\{\sum_{j=1}^{T} d_{i,t-j} > 0\right\}$. With this setup, $s_{i,t} = 1$ if a bidder was active in at least one auction in T of the previous rounds of bidding. However, this would require us to discard a number of auction observations due to an initial conditions problem. An alternative would also be to redefine the state as simply $s_{it+1} = s_{it} + (\mathbf{1}_{\{d_{i,t}=1\}} - (1 - \mathbf{1}_{\{d_{i,t}=1\}}))$.

The above is the expected behaviour of regular bidder i of type j when i follows its participation strategy in σ. The integration is over private information costs ϕ.

Characterisation of Payoffs for Regular Bidders: A bidder decides whether to enter an auction and incur the information cost by comparing the value of participation and non-participation. Let:

- $W_{i0}^{j}(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}})$ be the value of not participating at state $(\mathbf{s}_t, c_{0,t}, z_t)$ with all opponents following their strategies prescribed in $(\sigma, \sigma_{\mathcal{F}})$
- $W_{i1}^{j}(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}})$ be the value of participation.

We will define these values more carefully after introducing some more notation. The Bellman equation for bidder i of type j is then:

$$W_i^j(\mathbf{s}_t, \phi_{i,t}, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) = \max\left\{W_{i1}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) - \phi_{i,t}, W_{i0}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}})\right\} \tag{3}$$

Define the *ex-ante* value function as the integrated version of the above Bellman equation, where all private information is integrated out:

$$V_i^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) = \int_0^\infty W_i^j(\mathbf{s}_t, \phi, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) h_j(\phi|s_{i,t}) d\phi, \ \forall \mathbf{s}_t \tag{4}$$

Choice specific values are then:

$$\begin{aligned} W_{i1}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) = E_c &\left[\max_b [b-c] \Pr(i \text{ wins}|s_{i,t}, \mathbf{s}_{-i,t}, b, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}})\right] \\ &+ \beta E_{c'_{0,t+1}, z'_{t+1}} \sum_{\mathbf{s'}_{t+1} \in \mathbf{S}} \Pr(\mathbf{s'}_{t+1}|\mathbf{s}_t, d_{i,t}=1; \sigma, \sigma_{\mathcal{F}}) V_i^j(\mathbf{s'}_{t+1}, c'_{0,t+1}, z'_{t+1}; \sigma, \sigma_{\mathcal{F}}) \end{aligned} \tag{5}$$

and the value of not participating

$$W_{i0}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) = 0 + \beta E_{c'_{0,t+1}, z'_{t+1}} \sum_{\mathbf{s'}_{t+1} \in \mathbf{S}} \Pr(\mathbf{s'}_{t+1}|\mathbf{s}_t, d_{i,t}=0; \sigma, \sigma_{\mathcal{F}}) V_i^j(\mathbf{s'}_{t+1}, c'_{0,t+1}, z'_{t+1}; \sigma, \sigma_{\mathcal{F}}) \tag{6}$$

The value function can equivalently be written as

$$\begin{aligned} V_i^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) = q_{i,j}(\mathbf{s}_t, c_{0,t}, z_t)(W_{i1}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) - E_\phi[\phi|\phi \le \zeta^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}})]) \\ + [1 - q_{i,j}(\mathbf{s}_t, c_{0,t}, z_t)] W_{i0}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) \end{aligned} \tag{7}$$

where

$$\zeta^j(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}) = W_{i1}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) - W_{i0}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) \tag{8}$$

and we have made use of an individual bidder's decision rule, as defined in (2). This formulation will be useful for estimation.

The probability of a bidder winning an auction is given by,

$$\Pr(i \text{ wins}) = \Pr\begin{pmatrix} i \text{ wins against} \\ N_{\mathcal{L}} \text{ potential } \mathcal{L} \\ \text{type bidders} \end{pmatrix} \times \Pr\begin{pmatrix} i \text{ wins against} \\ N_{\mathcal{S}} \text{ potential } \mathcal{S} \\ \text{type bidders} \end{pmatrix} \times \Pr\begin{pmatrix} i \text{ wins against} \\ N_{\mathcal{F}} \text{ potential } \mathcal{F} \\ \text{type bidders} \end{pmatrix} \tag{9}$$

To compute the probability of winning an auction against N_j potential bidders of type j , let us first define the following objects:

- Let $G_j(.|c_{0,t}, z_t, \mathbf{s}_{k,t})$ be the equilibrium bid distribution at state $\mathbf{s}_{k,t} = (k, \mathbf{s}_{-i,t})$ where $k = 0, 1$ is the bidder's own participation status, $\mathbf{s}_{0,t}$ and $\mathbf{s}_{1,t}$ are states for a bidder who did not participate in a previous round and a bidder that did participate, respectively.
- Define $N_{k,j}$ to be the total number of bidders of type j who have activity status $k = 0, 1$.
- Let $\mathbf{1}_{\{s_{i,t}=k\}}$ be an indicator that equals one if the bidder has participation status k.
- $\mathbf{C}_{n_{k,j}}^{N_{k,j}-\mathbf{1}_{\{s_{i,t}=1\}}} = \begin{pmatrix} N_{k,j} - \mathbf{1}_{\{s_{i,t}=k\}} \\ n_{k,j} \end{pmatrix}$ are the usual binomial coefficients.
- Let $\mathbf{1}_{\{j=\mathcal{L}\}}$ be an indicator that captures the aforementioned assumption that a bidder will always face at least one large bidder, either an actual large bidder or MDOT[16].

The probability of a bidder winning against N_j potential bidders of type j can then be written as:

$$\Pr\begin{pmatrix} i \text{ wins against } N_j \\ \text{potential } j \text{ type} \\ \text{bidders} \end{pmatrix} = \tag{10}$$

$$\sum_{\substack{n_{0,j}=0 \\ n_{1,j}=\mathbf{1}_{\{j=\mathcal{L}\}}}}^{\substack{N_{0,j}-\mathbf{1}_{\{s_{i,t}=0\}} \\ N_{1,j}-\mathbf{1}_{\{s_{i,t}=0\}}}} \prod_{k=0,1} \left[\mathbf{C}_{n_{k,j}}^{N_{k,j}-\mathbf{1}_{\{s_{i,t}=k\}}}\right] q_j(\mathbf{s}_{k,t}, c_{0,t}, z_t)^{n_{k,j}} (1 - q_j(\mathbf{s}_{k,t}, c_{0,t}, z_t))^{N_{k,j}-\mathbf{1}_{\{s_{i,t}=k\}}-n_{k,j}}$$

$$\times[1 - G_j(b_{i,t}|c_{0,t}, z_t, \mathbf{s}_{k,t})]^{n_{k,j}}$$

If a bidder i is not of type j the above is simply:

$$\Pr\begin{pmatrix} i \text{ wins against } N_j \\ \text{potential } j \text{ type} \\ \text{bidders} \end{pmatrix} = \tag{11}$$

[16]We in fact assume that the bidder they will face was active in a previous round. The estimates are not sensitive to changes in this.

$$\sum_{\substack{n_{0,j}=0 \\ n_{1,j}=\mathbf{1}_{\{j=\mathcal{L}\}}}}^{N_{0,j},N_{1,j}} \prod_{k=0,1} \left[\mathbf{C}^{N_{k,j}}_{n_{k,j}}\right] q_j(\mathbf{s}_{k,t}, c_{0,t}, z_t)^{n_{k,j}} (1 - q_j(\mathbf{s}_{k,t}, c_{0,t}, z_t))^{N_{k,j}-n_{k,j}} [1 - G_j(b_{i,t}|c_{0,t}, z_t, \mathbf{s}_{k,t})]^{n_{k,j}}$$

The main difference between (10) and (12), is that in (10) we are correcting for the number of opponent players that are of the same type as player i. For fringe bidders the equivalent expression is:

$$\Pr\left(\begin{array}{c} i \text{ wins against } N_{\mathcal{F}} \\ \text{potential } \mathcal{F} \text{ type} \\ \text{bidders} \end{array}\right) = \sum_{n_{\mathcal{F}}=0}^{N_{\mathcal{F}}} \Pr(n_{\mathcal{F}}|\mathbf{s}_t)[1 - G_{\mathcal{F}}(b_{i,t}|c_{0,t}, z_t, \mathbf{s}_t)]^{n_{\mathcal{F}}} \tag{12}$$

We discuss in detail how we specify the term $\Pr(n_{\mathcal{F}}|\mathbf{s}_t)$ in the next section.

Markov Perfect Equilibria: A MPE in this game is a set of strategy functions σ^*, such that for any i of type j and for any $(\mathbf{s}, \phi_i, c_i, c_0) \in \mathbf{S} \times \Phi_j \times \mathbf{C}_j \times \mathbf{C}_0$,

$$d^{\sigma^*}_{i,j}(\mathbf{s}, \phi_i, c_0, z) = \arg\max_{d_i\in\{0,1\}} \left\{d_i(W^j_{i1}(\mathbf{s}, c_0, z; \sigma^*, p^*_{\mathcal{F}}) - \phi_i) + (1 - d_i)(W^j_{i0}(\mathbf{s}, c_0, z; \sigma^*, p^*_{\mathcal{F}}))\right\} \tag{13}$$

and

$$b^{\sigma^*}_{i,j}(\mathbf{s}, c_i, c_0, z) \in \arg\max_{b_i\in\mathbf{B}} [b_i - c_i] \Pr(i \text{ wins}|\mathbf{s}, b_i, c_0, z; \sigma^*, \sigma^*_{\mathcal{F}}) \tag{14}$$

where $\mathbf{B} = [0, \infty)$.

Equilibrium Existence: The auction game resembles existing static auctions considered in the theoretical literature. In the auction game considered here:

1. The completion cost space $\mathbf{C}_j$ is a separable metric space with measurable partial order.

2. The joint density of types is bounded and atomless.

3. Action space is compact.

4. Payoffs are continuous for every $c \in [\underline{c}, \overline{c}]$.

5. Interim payoffs are log supermodular and therefore single crossing holds.

Following Reny (2008), the auction game we consider has an equilibrium in monotone pure strategies, given the optimal participation strategies of players.

Following Auguirregabiria and Mira (2007) and Pesendorfer and Schmidt-Dengler (2008), the existence of equilibria of the participation game is analysed in probability space. A regular bidder will enter an auction if

$$W^j_{i1}(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) - \phi_{i,t} \geq W^j_{i0}(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) \tag{15}$$

(15) characterises the optimal decision rule. The above can be evaluated before the information acquisition costs are observed which yields the ex-ante optimal choice probabilities for regular bidders, induced by σ^*, given perceptions of opponents' entry strategies, σ and $\sigma_{\mathcal{F}}$.

$$p_{i,j}(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) = \tag{16}$$

$$\int_0^\infty \mathbf{1}\{W_{i1}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) - W_{i0}^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}}) \geq \phi\} dH_j(\phi|s_{i,t}) \equiv \Lambda_{i,j}(\mathbf{s}_t, c_{0,t}, z_t; \sigma, \sigma_{\mathcal{F}})$$

The optimal conditional choice probability $p_{i,j}$ is induced by the MPE σ^* defined previously. Equilibrium existence of the participation game is easily shown by looking at the ex-ante optimal choice probabilities defined previously. Let the set of ex-ante choice probabilities be given by $\Lambda(\mathbf{q}) = \{\Lambda_{i,j}(\mathbf{s}_t, c_{0,t}, z_t; \sigma) : i = 1, ..., N \ \& \ j = \mathcal{L}, \mathcal{S}\}$, these are the choice probabilities induced by the strategies in σ, where $\mathbf{q} = \{q_{i,j}(\mathbf{s}', c_0', z') : i = 1, ..., N \ \& \ j = \mathcal{L}, \mathcal{S} \ \& \ \mathbf{s}' \in \mathbf{S} \ \& \ z' \in \mathbf{Z} \ \& \ c_0' \in \mathbf{C_0}\}$ is the set of entry decision rules for all regular bidders, as defined in (2). Equilibrium points are therefore fixed points, i.e. let $\mathbf{p}$ be the set of optimal choice probabilities for every state and every bidder then

$$\mathbf{p} = \Lambda(\mathbf{p}), \tag{17}$$

since beliefs in equilibrium are consistent. The choice probabilities $\mathbf{p}$ are contained in the unit interval. The function Λ is continuous in $\mathbf{p}$. Brouwer's fixed point theorem implies that there exists a fixed point $\mathbf{p}$ of the function Λ.

4.2 Discussion of the Model

The key features of the model presented previously are:

1. Construction cost asymmetries between large ($\mathcal{L}$) and small ($\mathcal{S}$) regular bidders and fringe bidders ($\mathcal{F}$) through separate construction cost distributions $F_j(c|c_0)$: These asymmetries will have a bearing on the efficient allocation of contracts. However, given that bidders do not observe the actual level of competition they will face in the bidding game, the effects of these asymmetries might be dampened.

2. Information cost synergies captured through the dependence of λ_j (the parameter of information costs) on $s_{i,t}$.

3. Information cost asymmetries, introduced through participation synergies, generate different beliefs on participation probabilities: Regular bidders (of the same type) will have different beliefs on the level of actual competition in an auction based on their own activity status $s_{i,t}$. This can generate differences in bid-shading behaviour.

4. Forward looking bidders who account for large future contracts (through state variable z) and participation cost savings in the subsequent bidding round: This element allows for the arrangement of contracts to have a bearing on participation probabilities. For example, clustering small contracts together might reduce the incentive to participate in current small contract auctions, relative to a situation where a small contract is followed by a large contract. Moreover, z affects a bidder's belief of opponent participation behaviour. If there are synergies in participation, we expect a bidder to believe that the level of actual competition will be higher if a large contract is to be awarded in the next round. As a result, strategic bid shading will be affected.

5. Uncertainty over the actual level of competition in the auction game: This will potentially attenuate the effects of the aforementioned asymmetries, in particular the completion cost asymmetries. The standard argument for strategic bid-shading in asymmetric auctions requires a bidder to know the actual level of competition. However, in this setting a bidder will only have an expectation of the number of large and small bidders he will face.

6. Presence of myopic fringe bidders: These bidders affect the extent to which policies influencing regular bidders translate into changes in transaction costs.

In the next section we consider the identification of the model.

5 Identification

Identification of the latent values of regular bidders follows directly from the conditions of Guerre et al. (2000). In particular, as pointed out by Athey and Haile (2007), the identification result from Guerre et al. (2000) can be re-interpreted as being conditional on the realisation of auction specific covariates and state variables. For identification we require monotonicity of the markup term in (26) conditional on auction covariates and state variables.

The parameters of the dynamic game are over-identified. This can be established following Pesendorfer and Schmidt-Dengler (2008) which follows a similar approach as Magnac and Thesmar (2002). For identification, the discount factor β and the functional form of $H_j(.)$ have to be fixed. There then exists an equilibrium characterization linear in the unknown parameters for a bidder who is just indifferent between entering and not. This equation system will have more equations than unknowns. In our case, period payoffs are known, except for the bid distribution and the optimal choice probabilities. The only unknown parameters of the dynamic game are therefore those of the information cost distribution. The best estimator for our problem is the asymptotic least squares estimator to be outlined next. Details of the identification argument can be found in the appendix.

6 Estimation

Our data consist of repeated observations of bids, participation decisions for all players and contract characteristics for T periods.

$$\text{data} = \{b_{i,t}, d_{i,t}, c_{0,t} : i = 1, ..., N;\ t = 1, 2, ..., T\} \tag{18}$$

In this section it is shown how private costs can be inferred from observed bids. The participation model is shown to be estimable using an asymptotic least squares estimator. The first step requires the estimation of auxiliary parameters of the model, i.e. the conditional choice probabilities of participation $p_{i,j}(\mathbf{s}_t, c_{0,t}, z_t)$ and the equilibrium bid distributions. The first order condition for an optimal bid is used to compute an expression for privately known costs as a function of the submitted bid, the equilibrium bid distribution and the conditional choice probabilities. An optimal minimum distance estimator is shown that finds parameters which minimise the distance between the non-parametrically estimated conditional choice probabilities and the choice probabilities implied by our model. To summarise, the two steps of estimation are:

1. In the first stage estimate strategies (conditional choice probabilities, bid distributions and transition matrices) as flexibly as possible.

2. In the second stage make use of equilibrium conditions implied by the model to estimate primitives of interest.

6.1 Conditional Choice Probabilities

Regular Bidders: Per period profits do not depend on the identity of the regular bidder but merely on the number of each type of regular bidder, i.e. the number of large and small bidders. As a result, all relevant information in $\{d_{i,t-1} : i = 1, 2, ..., N\}$ can be captured in a bidder's own participation status $d_{i,t-1}$ and the number of competitors of each type, defined as $\mathcal{N}_{\mathcal{L},t} = \sum_{k \in \mathbf{N}_{\mathcal{L}} \setminus i}^{N_{\mathcal{L}}} d_{k,t-1}$ & $\mathcal{N}_{\mathcal{S},t} = \sum_{k' \in \mathbf{N}_{\mathcal{S}} \setminus i}^{N_{\mathcal{S}}} d_{k',t-1}$. Following Auguirregabiria and Mira (2007), the conditional choice probabilities at some state $(\mathbf{s}^m, \widetilde{c}_0, , \widetilde{z}) = (\widetilde{s}, \widetilde{\mathcal{N}_{\mathcal{L}}}, \widetilde{\mathcal{N}_{\mathcal{S}}}, \widetilde{c}_0, \widetilde{z})$ are estimated by simple frequency estimators. The estimator has the following form:

$$\widehat{p}_j(\widetilde{s}, \widetilde{\mathcal{N}_{\mathcal{L}}}, \widetilde{\mathcal{N}_{\mathcal{S}}}, \widetilde{c}_0, \widetilde{z}) = \frac{\sum_{i \in \mathbf{N}_j} \sum_t^T \mathbf{1}\{d_{i,t} = 1, s_{i,t} = \widetilde{s},\ \mathcal{N}_{\mathcal{L},t} = \widetilde{\mathcal{N}_{\mathcal{L}}},\ \mathcal{N}_{\mathcal{S},t} = \widetilde{\mathcal{N}_{\mathcal{S}}}, c_{0,t} = \widetilde{c}_0, z_t = \widetilde{z}\}}{\sum_{i \in \mathbf{N}_j} \sum_{k=0,1} \sum_t^T \mathbf{1}\{d_{i,t} = k, s_t = \widetilde{s}_i,\ \mathcal{N}_{\mathcal{L},t} = \widetilde{\mathcal{N}_{\mathcal{L}}},\ \mathcal{N}_{\mathcal{S},t} = \widetilde{\mathcal{N}_{\mathcal{S}}}, c_{0,t} = \widetilde{c}_0, z_t = \widetilde{z}\}} \tag{19}$$

To compute the above we discretize the grid of engineer's estimates c_0 into 35 bins. There are a number of approaches to dealing with choice probability estimates that are based on few observations and non-observed states, such as kernel smoothers. Hotz et al. (1994) compare different methods in a Monte-Carlo study. One conclusion from Hotz et al. (1994) is that dropping states that have few observations can improve estimates. We follow this procedure and drop states that are not observed frequently[17].

State transitions are estimated following Pesendorfer and Schmidt-Dengler (2008) using frequency estimators:

$$\widehat{\Pr}(\mathbf{s}', c_0', z'|\mathbf{s}, d) = \frac{\sum_t \mathbf{1}\{\mathbf{s}_{t+1} = \mathbf{s}', c_{0,t+1} = c_0', z_{t+1} = z', \mathbf{s}_t = \mathbf{s}, d_t = d\}}{\sum_{s'' \in \mathbf{S}, c_0'' \in \mathbf{C_0}, z'' \in \mathbf{Z}} \sum_t \mathbf{1}\{\mathbf{s}_{t+1} = \mathbf{s}'', c_{0,t+1} = c_0'', z_{t+1} = z'', \mathbf{s}_t = \mathbf{s}, d_t = d\}} \tag{20}$$

Fringe Bidders: Fringe bidders enter with same probability p^* conditional on state variables, as specified by their entry strategy in $\sigma^*_{\mathcal{F}}$. With a large number of potential bidders, we model the number of fringe bidders in an auction as a Poisson process with parameter δ depending on $\mathbf{s}$, similar to Bajari and Hortacsu (2003). In other words, the probability of observing $n_{\mathcal{F}}$ fringe bidders is given by:

$$\Pr(n_{\mathcal{F}}|\widetilde{\mathbf{s}}, \widetilde{z}) = \frac{e^{-\delta(\widetilde{\mathbf{s}}, \widetilde{z})} \delta(\widetilde{\mathbf{s}}, \widetilde{z})^{n_{\mathcal{F}}}}{n_{\mathcal{F}}!} \tag{21}$$

where $\log[\delta(\widetilde{\mathbf{s}}, \widetilde{z})] = \delta_0 + \delta_1 \widetilde{\mathcal{N}_{\mathcal{S}}} + \delta_2 \widetilde{\mathcal{N}_{\mathcal{L}}} + \delta_3 \widetilde{z}$.

6.2 Bid Distributions

Following Jofre-Bonet and Pesendorfer (2003) and Athey et al. (2011), bid distributions are estimated parametrically[18]. Both large and small bidder distributions are assumed to be log-normally distributed[19]. The mean parameter, μ_j and variance parameter, σ_j, are assumed to depend on the contract size and the state variables:

$$\log(b_{i,t}) \sim \mathbf{N}\left[\mu_j(c_0, s_{i,t}, \mathcal{N}_{\mathcal{S}t}, \mathcal{N}_{\mathcal{L}t}, z_t), \sigma_j(c_0, s_{i,t}, \mathcal{N}_{\mathcal{S}t}, \mathcal{N}_{\mathcal{L}t}, z_t)\right] \tag{22}$$

where

$$\mu_j(c_0, s_{i,t}, \mathcal{N}_{\mathcal{S}t}, \mathcal{N}_{\mathcal{L}t}, z_t) = \mu_{j,0} + \mu_{j,1} c_{0,t} + \mu_{j,2} s_{i,t} + \mu_{j,3} \mathcal{N}_{\mathcal{S}t} + \mu_{j,4} \mathcal{N}_{\mathcal{L}t} + \mu_{j,5} z_t \tag{23}$$

and

$$\sigma_j(c_0, s_{i,t}, \mathcal{N}_{\mathcal{S}t}, \mathcal{N}_{\mathcal{L}t}, z_t) = \sigma_{j,0} + \sigma_{j,1} c_{0,t} + \sigma_{j,2} s_{i,t} + \sigma_{j,3} \mathcal{N}_{\mathcal{S}t} + \sigma_{j,4} \mathcal{N}_{\mathcal{L}t} + \sigma_{j,5} z_t \tag{24}$$

[17] In particular, we drop states with fewer than thirty observations. This can be interpreted as smoothing a tail condition.

[18] Jofre-Bonet and Pesendorfer (2003) discuss when chosen parametric structures on bid distributions are consistent with the underlying cost distributions and strategies.

[19] We experimented with Weibull distributions for the regular bidders but found that the log Normal was a better fit. An advantage of using the Weibull assumption is that parametric unobserved heterogeneity can be introduced. If we assume that bids are a Gamma-Weibull mixture estimation is still tractable and allows for potentially unobserved contract characteristics to influence bidding. This approach is equivalent to approaches already used in labour economics when estimating hazard rates. We have experimented with including unobserved heterogeneity and found that markups were reduced and that period payoffs would also be reduced slightly. However, the Gamma-Weibull mix did not provide as good a fit as the assumptions presented in the main text.

Fringe Bidders: It is assumed that the fringe bid distribution is described by a Weibull distribution.

$$G_{\mathcal{F}}(b_{i,t}|c_{0,t},\mathbf{s}_t,z_t) = 1 - \exp\left[-\psi_{2\mathcal{F}}(c_{0,t},\mathbf{s}_t,z_t)\log(b_{i,t}+1)^{\psi_{1\mathcal{F}}(c_{0,t},\mathbf{s}_t,z_t)}\right] \tag{25}$$

where $\log\psi_{k,\mathcal{F}}(c_{0,t},\mathbf{s}_t,z_t) = \psi_{k,\mathcal{F},0} + \psi_{k,\mathcal{F},1}c_{0,t} + \psi_{k,\mathcal{F},2}\mathcal{N}_{\mathcal{S}} + \psi_{k,\mathcal{F},3}\mathcal{N}_{\mathcal{L}} + \psi_{k,\mathcal{F},4}z_t$ for $k = 1, 2$, where the last two terms in the expression are the number of large and small bidders who participated in an auction in the previous round. Bid distributions are then estimated using maximum likelihood.

6.3 Private Costs for Regular Bidders

The first order condition for an optimal bid can be re-written to yield an expression for private information $c_{i,t}$, in terms of observables:

$$c_{i,t} = b_{i,t} - [\eta_{\mathcal{L}}(b_{i,t},\mathbf{s}_t,c_{0,t},z_t;\sigma^*) + \eta_{\mathcal{S}}(b_{i,t},\mathbf{s}_t,c_{0,t},z_t;\sigma^*) + \eta_{\mathcal{F}}(b_{i,t},\mathbf{s}_t,c_{0,t},z_t;\sigma^*_{\mathcal{F}})]^{-1} \tag{26}$$

where

$$\eta_j(b_{i,t},\mathbf{s}_t,c_{0,t},z_t;\sigma^*) = \left[\frac{\partial \Pr\left(\begin{array}{c} i \text{ wins against } N_j \\ \text{potential } j \text{ type bidders}\end{array}\middle| b_{i,t},\mathbf{s}_t,c_{0,t},z_t;\sigma^*\right)/\partial b_{i,t}}{\Pr\left(\begin{array}{c} i \text{ wins against } N_j \\ \text{potential } j \text{ type bidders}\end{array}\middle| b_{i,t},\mathbf{s}_t,c_{0,t},z_t;\sigma^*\right)}\right] \tag{27}$$

The second term in (26) is the markup term. The form of (27) can be found in the appendix. The above closely follows Guerre et al. (2000), except that expectations are taken over the number of actual competitors with the number of potential competitors constant over time. Quasi-valuations can be computed by estimating the equilibrium bid distributions $G_j(.|c_{0,t},s_{i,t},\mathbf{s}_{-i,t},z_t)$ and substituting these into expression (26). This allows for point-wise estimation of the private cost distribution. Note that the auction game estimation only relies on the specification of the dynamic game through the non-parametrically estimated entry probabilities. It does not rely on the parametric structure of the dynamic game.

6.4 Parameters of the Participation Game

The primitives are estimated by finding parameters that minimise the distance between the non parametrically estimated conditional choice probabilities and the choice probabilities implied by the model. Values can be computed from the data conditional on the structural parameters. These can then be used to compute optimal choice probabilities for our model. Let $\theta = (\theta_I, \lambda_{0,\mathcal{L}}, \lambda_{1,\mathcal{L}}, \lambda_{0,\mathcal{S}}, \lambda_{1,\mathcal{S}}) \equiv (\theta_I, \theta_{II})$, where $\theta_I = (\Psi, \mathbf{p}, \Psi_{M_0,M_1})$ are the parameters from the first stage of estimation, i.e. the optimal choice probabilities, transition matrix elements (Ψ_{M_0,M_1}) and bid distribution parameters and $\theta_{II} = (\lambda_{0,\mathcal{L}}, \lambda_{1,\mathcal{L}}, \lambda_{0,\mathcal{S}}, \lambda_{1,\mathcal{S}})$. We can use equation (7) together with the assumption on the information cost distribution to re-write the value function for bidder of type j as:

$$V^j(s_{i,t}, \mathbf{s}_{-i,t}, c_{0,t}, z_t; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) = p^j(\mathbf{s}_t, c_{0,t}, z_t) W_1^j(s_{i,t}, \mathbf{s}_{-i,t}, c_{0,t}, z_t; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) \tag{28}$$

$$+[1 - p^j(\mathbf{s}_t, c_{0,t}, z_t)] W_0^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) - \xi_j(\mathbf{s}_{i,t}, c_{0,t}, z_t, \sigma^*)$$

where

$$\xi_j(\mathbf{s}_t, c_{0,t}, z_t, \sigma^*) = \frac{1}{\lambda_j(s_{i,t})}[(1 - p^j(\mathbf{s}_t, c_{0,t}, z_t)) \log(1 - p^j(\mathbf{s}_t), c_{0,t}, z_t) + p^j(\mathbf{s}_t, c_{0,t}, z_t)] \tag{29}$$

where $\xi_j(\mathbf{s}_t, c_{0,t}, z_t, \sigma^*) = p^j(\mathbf{s}_t, c_{0,t}, z_t) E(\phi | \phi \leq \zeta(\mathbf{s}_t; \sigma, \sigma_{\mathcal{F}}))$ and we made use of the inversion $H_j^{-1}(p^j(\mathbf{s}_t)) = \zeta(\mathbf{s}_t; \sigma^*, \sigma^*_{\mathcal{F}})$[20]. The next step involves substituting the expression for an optimal bid into the expected period payoff function. Specifically, changing the variable of integration from cost (c) to bids (b) yields

$$\pi^j(s_{i,t}, \mathbf{s}_{-i,t}, c_{0,t}, z_t; \sigma^*, \sigma^*_{\mathcal{F}}) = \left[\int_0^{\bar{b}} \frac{\Pr(i \text{ wins} | s_{i,t}, \mathbf{s}_{-i,t}, b, c_{0,t}, z_t; \sigma^*, \sigma^*_{\mathcal{F}})}{\eta_{\mathcal{L}} + \eta_{\mathcal{S}} + \eta_{\mathcal{F}}} g_j(b | s_{i,t}, \mathbf{s}_{-i,t}, c_{0,t}, z_t) db \right] \tag{30}$$

where η_L, η_S and $\eta_{\mathcal{F}}$ are as defined in (27). The above can be used in the ex-ante value function which is then given by

$$V^j(s_{i,t}, \mathbf{s}_{-i,t}, c_{0,t}, z_t; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) = p^j(\mathbf{s}_t, c_{0,t}, z_t) \pi^j(s_{i,t}, \mathbf{s}_{-i,t}, c_{0,t}, z_t; \sigma^*, \sigma^*_{\mathcal{F}}) \tag{31}$$

$$+\beta \left[\sum_{\mathbf{s}'_{t+1} \in \mathbf{S}} p^j(\mathbf{s}_t, c_{0,t}, z_t) \Pr(\mathbf{s}'_{t+1} | \mathbf{s}_t, d_{i,t} = 1; \sigma^*) + [1 - p^j(\mathbf{s}_t, c_{0,t}, z_t)] \Pr(\mathbf{s}'_{t+1} | \mathbf{s}_t, d_{i,t} = 0; \sigma^*) \right]$$

$$\times E_{c'_{0,t+1}, z'_{t+1}} V^j(\mathbf{s}'_{t+1}, c'_{0,t+1}, z'_{t+1}; \sigma^*, \sigma^*_{\mathcal{F}}, \theta) - \xi_j(\mathbf{s}_t, \sigma^*)$$

The next step is to write the value functions for all states in matrix form.

- Let $\Pi^j(\sigma^*, \sigma^*_{\mathcal{F}}) = [\pi^j(\mathbf{s}', c'_0, z'; \sigma^*, \sigma^*_{\mathcal{F}})]_{\mathbf{s}' \in \mathbf{S},\ c'_0 \in \mathbf{C_0},\ z' \in \mathbf{Z}}$ be the vector of expected period payoffs.
- Denote $V^j(\sigma^*, \sigma^*_{\mathcal{F}}, \theta) = [V^j(\mathbf{s}', c'_0, z'; \sigma^*, \sigma^*_{\mathcal{F}}, \theta)]_{\mathbf{s}' \in \mathbf{S},\ c'_0 \in \mathbf{C_0},\ z' \in \mathbf{Z}}$ as the vector of values.
- Let $\mathbf{P}^j(\sigma^*) = \text{diag}\left([p_j(d_{i,t} = 1 | \mathbf{s}', c'_0, z'; \sigma^*)]_{\mathbf{s}' \in \mathbf{S},\ c'_0 \in \mathbf{C_0},\ z' \in \mathbf{Z}}\right)$ be the vector of choice probabilities.
- Denote $\Xi^j(\theta_{II}) = [\xi_j(\mathbf{s}', c'_0, z'; \sigma^*, \theta)]_{\mathbf{s}' \in \mathbf{S},\ c'_0 \in \mathbf{C_0},\ z' \in \mathbf{Z}}$ as the vector of expected information costs.
- $M_1^j(\sigma^*)$ is the transition matrix induced by participation in the current round of auctions . In other words, row $s \in \mathbf{S}$ of the transition matrix M_1 is given by $[\Pr(s', c'_0, z' | s, d = 1; \sigma^*)_{\mathbf{s}' \in \mathbf{S},\ c'_0 \in \mathbf{C_0},\ z' \in \mathbf{Z}}]$.

[20]This is the inversion technique developed by Hotz and Miller (1993). Conditions for this approach to hold are summarised by Proposition 1 in their paper.

- $M_0^j(\sigma^*)$ is the transition matrix induced by non-participation in a current auction round.

The matrix equation for the value function can then be written as follows:

$$V^j(\sigma^*, \sigma_{\mathcal{F}}^*; \theta) = \mathbf{P}^j(\sigma^*)\Pi^j(\sigma^*, \sigma_{\mathcal{F}}) + [\beta\mathbf{P}^j(\sigma^*)M_1^j(\sigma^*) + \beta[I - \mathbf{P}^j(\sigma^*)]M_0^j(\sigma^*)]V^j(\sigma^*, \sigma_{\mathcal{F}}^*; \theta) - \Xi^j(\theta_{II}) \quad (32)$$

The above can then be re-written as

$$V^j(\sigma^*, \sigma_{\mathcal{F}}^*; \theta) = [I - \beta\mathbf{P}^j(\sigma^*)M_1^j(\sigma^*) - \beta[I - \mathbf{P}^j(\sigma^*)]M_0^j(\sigma^*)]^{-1}[\mathbf{P}^j(\sigma^*)\Pi^j(\sigma^*, \sigma_{\mathcal{F}}^*) - \Xi^j(\theta_{II})] \quad (33)$$

To compute the value function estimates of the choice probabilities, defined above, and the period payoffs are required. The period payoffs can be computed by numerically integrating the expression in (30). To compute the expectation with respect to b in (30) Gaussian quadrature methods are applied as outlined in Judd (1998). Given the above we can then compute the optimal choice probabilities implied by our model. The probability that a bidder enters an auction, given her participation in the previous round of bidding, is given by

$$p_j(s_{i,t}, \mathbf{s}_{-i,t}, c_{0,t}, z_t; \sigma^*, \sigma_{\mathcal{F}}^*) = \quad (34)$$

$$H_j\Bigg\{\Bigg[\pi^j(\mathbf{s}_t, c_{0,t}, z_t; \sigma^*, \sigma_{\mathcal{F}}^*) + \beta \sum_{\mathbf{s}'_{t+1}\in\mathbf{S}} [\Pr(\mathbf{s}'_{t+1}|\mathbf{s}_t, d_{i,t} = 1; \sigma^*) - \Pr(\mathbf{s}'_{t+1}|\mathbf{s}_t, d_{i,t} = 0; \sigma^*)]$$

$$\times E_{,c'_{0,t+1},z'_{t+1}} V^j(\mathbf{s}'_{t+1}, c'_{0,t+1}, z'_{t+1}; \sigma^*, \sigma_{\mathcal{F}}^*)\Bigg]\Bigg| s_{i,t}\Bigg\}$$

Stacking the above expression over states and bidder types yields $\mathbf{p} = \Lambda(\mathbf{p}; \theta)$. The estimator forces the equality constraints $\mathbf{p} - \Lambda(\mathbf{p}; \theta) = 0$ to estimate the structural parameters. In other words the estimator given first stage estimates is given by

$$\min_{\theta_{II}} \left(\widehat{\mathbf{p}} - \Lambda(\mathbf{p}; \widehat{\theta}_I, \theta_{II})\right)' \widehat{W}(\widehat{\theta}_I, \theta_{II}) \left(\widehat{\mathbf{p}} - \Lambda(\mathbf{p}; \widehat{\theta}_I, \theta_{II})\right) \quad (35)$$

where W is the optimal weight matrix. This matrix depends on the covariance matrix of auxiliary parameters and the bid distributions and the derivatives of the estimating equations with respect to the auxiliary parameters and the parameters of the bid distribution functions[21]. Specifically, the optimal W is given by

$$W(\theta_I, \theta_{II}) = \left(\left[(\mathbf{I} : \mathbf{0}) - \nabla_{\theta'_I}\Lambda(\mathbf{p}; \theta_I, \theta_{II})\right] \Sigma \left[(\mathbf{I} : \mathbf{0}) - \nabla_{\theta'_I}\Lambda(\mathbf{p}; \theta_I, \theta_{II})\right]'\right)^{-1} \quad (36)$$

[21]Note that the special feature of auction participation games as opposed to standard market entry games, is that the payoffs do not contain any unknown parameters. As a result, for certain information cost specifications we can directly compute the information cost parameters, for example this is the case with the exponential distribution.

$\mathbf{0}$ is a $(2 * m_s) \times (m_\psi + 2 * m_s^2)$ matrix of zeros, where m_ψ is the number of parameters in the bid distribution function, $2 * m_s^2$ is the number of estimates for the state transition matrices, Ψ is the vector of bid distribution parameters, and Σ is the variance covariance matrix of the choice probability estimator and of the bid distribution parameters. The optimality of this weight matrix follows from the conditions presented in Gourieroux and Monfort (1995). Asymptotic normality of this estimator is also established there. Our estimator is different from Pesendorfer and Schmidt-Dengler (2008) since payoffs are known and computed in the first stage of estimation. As a result, our weight matrix will not only depend on the variance covariance matrix of the optimal choice probabilities and transition matrices but also on the bid distribution estimates. Given that our first stage estimates are consistent and asymptotically normal we can directly apply the results in Gourieroux and Monfort (1995).

7 Results

This section presents results of the estimation. We begin by summarising the estimation of the auxiliary parameters followed by estimation results on period payoffs and the structural parameters. In each section we provide evidence on the goodness of fit of each estimator.

7.1 Conditional Choice Probabilities Estimates

We estimate choice probabilities by frequency estimators, as shown in (19), and use these to compute the transition matrices. *Goodness of Fit*: To test the goodness of fit we compare the average number of large and small bidders across all auctions in our data with simulated numbers computed using our choice probability estimates. For each realisation of the state variables observed in the data, we select the associated choice probability and draw a uniform random variable on $[0, 1]$. If the choice probability is greater than the uniform variable the bidder enters. The procedure is completed for large and small bidders separately. The mean is computed for the simulated number of large and small participants across all auctions and compared with the data. The observed mean number of bidders is 0.4583 and the simulated mean is 0.4670, with standard deviations given by 0.6577 and 0.6561, respectively. The means are not statistically different at 99% confidence. The average number of small bidders is 1.7048 with standard deviation 1.7402. The simulated number is 1.7048 with standard deviation 1.3821. The means are not statistically different at 99% confidence.

7.2 Bid Distribution Estimates

The parameters of the bid distributions are shown in Table 5. *Goodness of Fit*: To test the goodness of fit of the bid distributions, we follow Jofre-Bonet and Pesendorfer (2003) and Athey et al. (2011) and compute the mean and standard deviation of the observed bids across all auctions and compare with means and standard deviations of bids generated by the estimated distribution. We focus attention on simulation of the minimum bid, i.e. the winning bid. This test is appropriate since the minimum bid determines the procurement costs of MDOT. Means are computed as follows.

1. First we extract bids, the number of bids submitted in each auction and the associated auction covariates, including state variables, from the data.

2. For each auction in the sample, the lowest bid submitted by large bidders and the lowest bid submitted by small bidders is separately extracted. In addition, the number of large and small bids submitted and the associated auction covariates are collected.

3. The data on the number of bids submitted in an auction and associated covariates are used to draw bids from the estimated distribution.

4. The minimum bid for each auction is computed.

5. The mean and standard deviation across all drawn auctions is then computed.

This is done separately for large and small bidders.

For large bidders the mean of the minimum observed log bids is 14.1852 and the simulated mean is 14.1669, with standard deviations given by 14.8824 and 14.8382. The difference between the two means is statistically not significant at 99% confidence. We also compute normalised versions of the above tests by taking the simulated bids and dividing by the engineer's estimate. The mean of the normalised bids is 1.0186 and 1.0293 for the simulated and observed, respectively. The difference between the two means is not statistically different at 99%. Similar results are found for the small bidder distributions, with an observed mean of 13.512 and a simulated mean of 13.596, with standard deviations 1.1232 and 1.2287, respectively. The two means are not statistically different at 99% confidence. We conduct the same tests for fringe bidder distributions and find similar results.

Effect of Individual Variables on Large Bid Distribution: The effect on the mean bid of individual variables can be seen by noting that the mean of the distribution of bids is given by $E(b) = \exp(\mu + 1/2\sigma^2)$.

Log of Engineer's Estimate: Increasing the engineer's estimate by 0.01 increases the mean by 1%.

Large Future Contracts: The presence of large future contracts decreases the mean bid by 0.35%

State Variables: Increasing $\mathcal{N}_{\mathcal{L}}$ by one increases the mean of the distribution by 1.8%. Increasing $\mathcal{N}_{\mathcal{S}}$ by one decreases the mean by 0.34%. Very similar results are found for the small bidders distribution. Having been active in a previous round of bidding leads to a higher mean by about 2.1%.

Table 5: Bid Distribution Estimates Whole Sample

Variable	**Coefficient** (Std. Err.)		
	Large	Small	Fringe
	μ		ψ_1
Engineer	0.9907	0.9797	0.0201
	(0.0028)	(0.0015)	(0.0005)
Bidder i's activity status, s_i	0.0021	0.0150	
	(0.0160)	(0.0087)	
$\mathcal{N}_{\mathcal{S}}$	-0.0022	-0.0010	-0.0032
	(0.0010)	(0.0006)	(0.0002)
$\mathcal{N}_{\mathcal{L}}$	0.0196	0.0036	0.0031
	(0.0080)	(0.0032)	(0.0001)
Large Future Contracts, z	0.0085	0.0140	-0.0157
	(0.0102)	(0.0057)	(0.0048)
Constant	0.1361	0.3138	3.1870
	(0.0397)	(0.0211)	(0.0065)
	σ		ψ_2
Engineer	-0.0175	-0.0203	-4.0381
	(0.0017)	(0.0009)	(0.0007)
Bidder i's activity status s_i	0.0255	0.0085	
	(0.0124)	(0.0057)	
$\mathcal{N}_{\mathcal{S}}$	-0.0012	-0.0015	-0.2441
	(0.0008)	(0.0004)	(0.0169)
$\mathcal{N}_{\mathcal{L}}$	-0.0019	0.0067	0.2458
	(0.0054)	(0.0022)	(0.0002)
Large Future Contracts, z	-0.0120	0.0255	0.0098
	(0.0073)	(0.0039)	(0.0034)
Constant	0.4096	0.4348	-28.5296
	(0.0241)	(0.0131)	(0.5356)
Number of Observations	2191	8296	13389

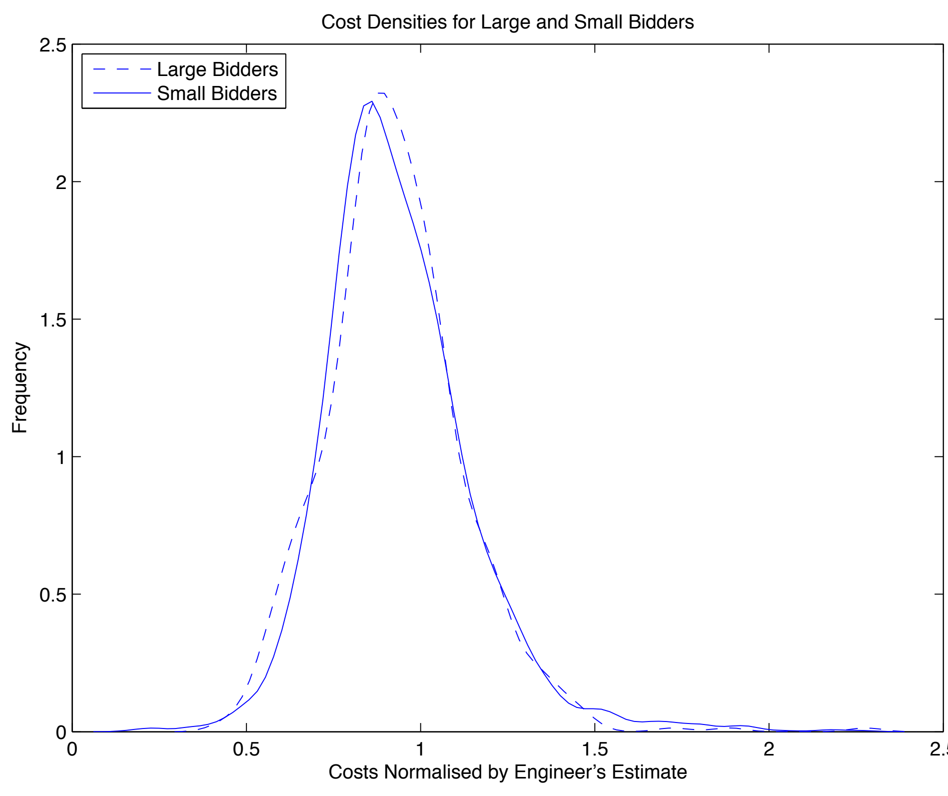
7.3 Estimation of Private Costs and Markups

Markups over costs are defined in (26). We substitute the estimated density and bid distribution into this expression evaluated at the observed bid and its associated covariates. On average, the observed markup for large bidders is equal to 13.45% , averaged across all bidders and all auctions. The average markup for a winning bid is equal to 16.58%. The average markup for a small bidder is 10.13%. The average markup for a winning small bid is 14.29%.

Krasnokutskaya (2011) estimates her model using MDOT data as well, however for a different time period, and finds markups in the order of 8%; for a winning bid the markup is 16%. The main difference between these results and ours is the inclusion of unobserved heterogeneity which reduces the amount of variation due to private information. It is possible, as in Athey et al. (2011), to include parametric unobserved heterogeneity into the bid distributions here and estimate a Gamma-Weibull mixture. Jofre-Bonet and Pesendorfer (2003) find markups that exclude dynamic effects on average roughly equal to 20%. Bajari et al. (2006) and Bajari et al. (2004) estimate markups that are equal to 6%.

In Figure 1 we plot the cost distributions for the sample average of the engineer's estimate and at the most frequently observed state configuration, which occurs in 12.56% of the auctions. The large bidder distribution has a lower mean, which is to be expected given that a large bidder is more able to mobilise equipment. This was not imposed during the estimation. The average cost for a large bidder normalised by the engineer's estimate is 92.69% with standard deviation 19.82% and for a small bidder the mean is 118.97% with standard deviation 12.72%. Moreover, the difference in costs is statistically significant at 99% confidence.

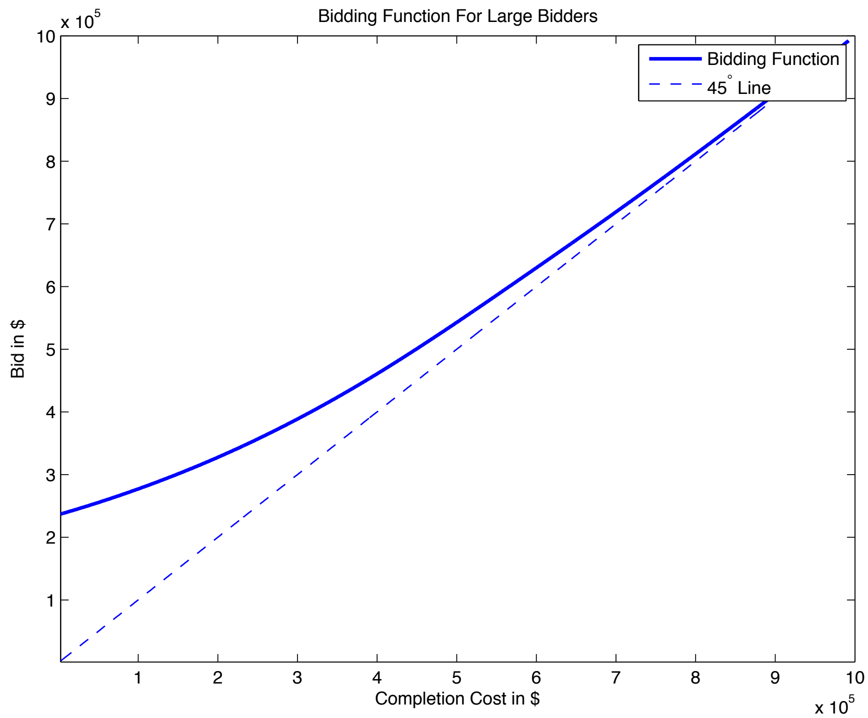
Figure 1: Completion Cost Distributions For Most Frequently Realised State



Bidding Function: Given our estimates, we can compute the bidding function. We use (26) to compute an optimal bid for different completion costs. The bid function is plotted by holding the state variables fixed and varying the completion cost. The x-axis is the completion cost and the y-axis is the optimal bid. The results are shown in Figure 2 for a large bidder. The bid function approaches the

45° line as costs increase, i.e. markups are reduced. At the lower end of the bid support, we compute negative costs which we find implausible. Negative costs are set equal to zero, as in Jofre-Bonet and Pesendorfer (2003).

Figure 2: Bid Function for Large Bidders

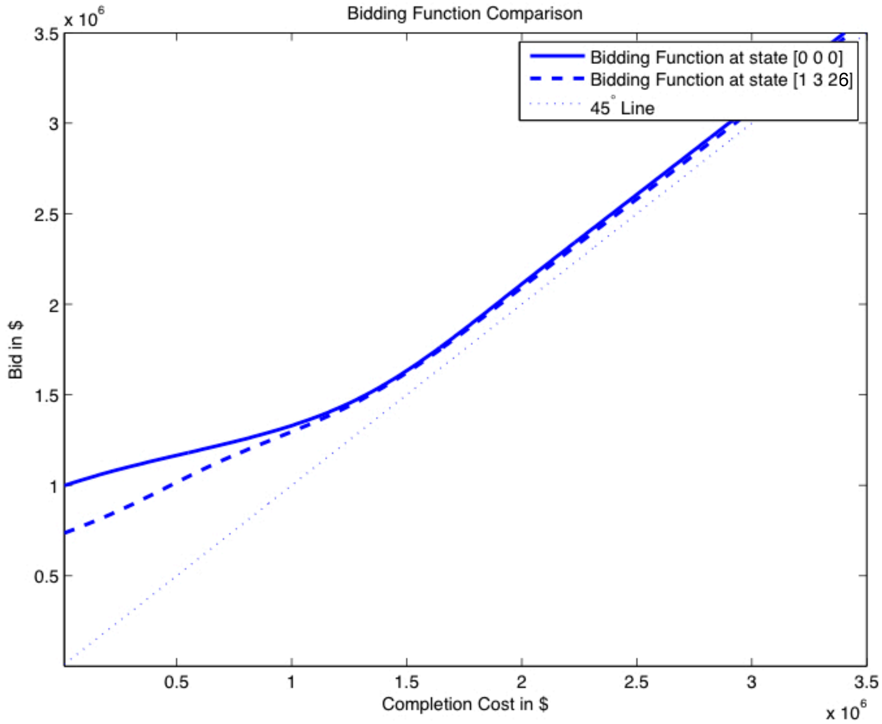


In Figure 3 we compare bidding functions, holding contract size fixed and $z = 1$, when the potential competitiveness of the auction is changed. This is done by moving from a state where few bidders were active in the previous round, i.e. $s_t = [s_{i,t}, \mathcal{N}_{\mathcal{L}t}, \mathcal{N}_{\mathcal{S}t}] = [0, 0, 0]$, to one where the maximum number of large and small bidders were active in a previous round, i.e. $s_t = [1, 3, 26]$. In the latter case beliefs are that there will be a high number of actual competitors. With more active bidders, the probability that they enter a current auction is increased, which indicates that the level of actual competition in a current auction is going to be higher. Bidding should become more aggressive and compress markups. In Figure 3 the bid function associated with the potentially more competitive state dictates lower markups for the same cost levels shown on the x-axis.

7.4 Period Payoffs

We compute ex-ante expected period payoffs using (30) yielding payoffs as a function of public states. These results will be used as inputs in the estimation of the structural parameters. For large players the average contract size is \$1.1233 Million. The average payoff (over all states) is then \$2,415. For

Figure 3: Bid Function for Large Bidders: Altering the State



small player's state spaces average contract size is \$2.3303 Million. The average normalised payoff for small players is then equal to \$19,957. The difference between large and small bidders here is being driven by the fact that small bidders are observed to have participated in a number of contracts over \$4 billion in size which generates high average payoffs. However, there is substantial variation with profits ranging from \$759 to \$1.303 million. Similar variation has also been found in Jofre-Bonet and Pesendorfer (2003).

7.5 Information Costs

The parameters for the participation game are presented in Table 6. The estimated parameters suggest that average information costs for large bidders are \$23,071 and for small bidders \$101,267 when $s_{i,t} = 0$ which can be found by taking $\exp(-\lambda_{j,0})$ for $j = \mathcal{L}, \mathcal{S}$. When $s_{i,t} = 1$, then average information costs are given by $\exp(-(\lambda_{j,0} + \lambda_{j,1}))$ which are \$19,138 for large bidders and for small bidders \$52,070.

Goodness of Fit: To test the goodness of fit of our model we simulate the number of entrants for a random sample of auctions and compare the average number of large and small bidders actually observed in those auctions with averaged simulated numbers. The average number of large bidders is 0.4451 with standard deviation 0.6504. The model predicts an average number of 0.2785 with standard deviation 0.6383. The model under-predicts the number of large entrants on average. On average there are 1.6843 small bidders with standard deviation 1.7553. The model predicts an average number of

Table 6: Structural Parameter Estimates

Variable	Coefficient (Ltd. Err.)
$\lambda_{\mathcal{L},0}$	-10.0463
	(0.0013)
$\lambda_{\mathcal{L},1}$	0.1869
	(0.0020)
$\lambda_{\mathcal{S},0}$	-11.5255
	(0.0073)
$\lambda_{\mathcal{S},1}$	0.6552
	(0.0019)

entrants equal to 1.807 with standard deviation 6.1271 and over-predict the number of small bidders.

8 Policy Simulations

In this section we consider the performance of the auction mechanism using the estimated primitives. With the primitives in hand, it is possible to assess potential allocative inefficiencies and to simulate the effects of addressing these inefficiencies. The first step is to determine whether the presence of asymmetries between bidders leads to misallocations due to the use of a first price auction. The second section deals with the timing of contracts, in particular addresses the current MDOT practice of distributing large contracts throughout the year. We consider the alternative policy of grouping contracts by size and seek to analyse the effect this policy has on participation, procurement costs and the allocation of the contract.

8.1 Inefficiencies due to Auction Format

When bidders are asymmetric it is possible that a first price sealed bid auction will lead to inefficient outcomes. In our model there are two potential sources of asymmetry. The first is through the size of the bidder, which affects the completion costs, and the other is through the dynamic synergies in participation. To determine the frequency with which misallocations occur, the primitives are used to compute how often the low bidder loses an auction, holding the entry process fixed.

We take a random sample of 1000 contract characteristics and compute the simulations for the same set of contracts 1000 times. The steps of the simulation are as follows:

1. Draw information costs from our estimated information cost distribution.

2. Use the estimated strategies of large and small bidders to determine entry. For fringe bidders the estimated Poisson model is used to determine the number of entrants.

3. After entrants have been determined, bids are drawn.
4. The inverse bidding strategy is used to compute costs.
5. The fraction of auctions where the low bid does not correspond to the lowest cost is then computed.

We find that, on average, misallocations occurs in 0.5% of auctions. The average difference between the low bidder and the winner's cost is 14.70% of the engineer's estimate. Krasnokutskaya (2011) estimates the average probability of inefficient outcomes in her MDOT data at 5%. Jofre-Bonet and Pesendorfer (2003) find a 32% average probability of the low bidder not winning.

These low numbers should not be surprising. In particular Figure 1 highlights that cost differences are not that significant between small and large bidders and hence will not contribute to misallocation of the contract. This implies that the asymmetries between large and small players are small.

Secondly and more importantly, even in the presence of large asymmetries between large and small players the misallocation problem should still not be so great, since the auction game involves bidders having imperfect knowledge of actual competition. Uncertainty over actual bidder numbers should attenuate the effect of strategic bid-shading normally found in asymmetric auctions. A bidder no longer knows for certain how many large and small bidders he will be facing in the auction game.

8.2 Timing of Auctions

Given the finding that large (above-average sized) contracts in future rounds can have an effect on participation in the current round of bidding, we consider changing the timing of auctions. Currently, MDOT does not systematically arrange contracts by size- some rounds will involve only below average contracts some rounds will have some larger than average contracts. We consider simulations where we first randomly distribute contracts over time, current MDOT practice, and a second set of simulations where we arrange contracts by size. In particular, larger auctions are clustered at the end of the sequence of auction rounds. This change to the environment should not change the equilibrium found in the data, since bidders took the sequencing of contracts as exogenous. However, bidders did make use of next round information which we maintain in these counterfactuals. We can therefore avoid having to solve for potentially multiple new participation equilibria.

We expect that changing the sequencing of auctions by size might induce changes in participation patterns of bidders. In particular, if all large auctions are held off until the end of a year, the number of bidders in smaller auctions might be lower than in a regime where a large auction might follow a small one quite frequently- since a bidder might want to participate in a small auction today to reap the benefits of lower participation costs in a large auction tomorrow.

We generate a random sample of contract sizes which we will hold constant across simulations. In the first set of simulations we maintain the MDOT regime and mix large and small contracts across time. In the second regime we re-arrange contracts by size and award all the small contracts first and then award above-average sized contracts in the subsequent rounds. The simulations are then run as follows:

1. Start the simulations with initial state of no bidders active in a previous set of auctions.
2. Draw information costs
3. Determine participants
4. Draw bids for the entrants and compute costs.
5. Given the participants in the auction, compute the next round's state and continue simulating forwards.
6. Repeat these simulations 1000 times.

Effect on the Number of Regular Participants: The effects of re-arranging contracts can be seen in Figure 4 and Figure 5. The average number of large bidder participants and the average number of small participants for different contract sizes (over simulations and over contract types). The number of large bidders varies only between 0.2 and 0.5. However, we can see that for some of the smaller contracts the number of large bidders is lower when we cluster small contracts. This is the case for contracts between a log dollar value of 11.8 and 12.7. However, for contracts between with a log Dollar value of 15 and 16 we can see that the number of bidders is higher when we cluster large contracts.

This effect is more pronounced when we look at Figure 5. The contracts below a log dollar value of 14 have a much lower number of bidders. However, for contracts larger than 15 the number of small bidders is higher than when contracts are not clustered together.

These effects should not be surprising given that we have forward-looking bidders and participation synergies. When small contracts are followed by large contracts, incentives to participate in a (large or small) contract today, in order to reap the benefits of lower participation costs on large contracts tomorrow, are greater than in an environment where small contracts are clustered together. In the latter case, future values of playing the game tomorrow are lower and incentives to participate today are reduced. The next step is to determine whether this clustering can generate procurement cost differences given that participation patterns change as well.

Figure 4: Number of Large Bidders

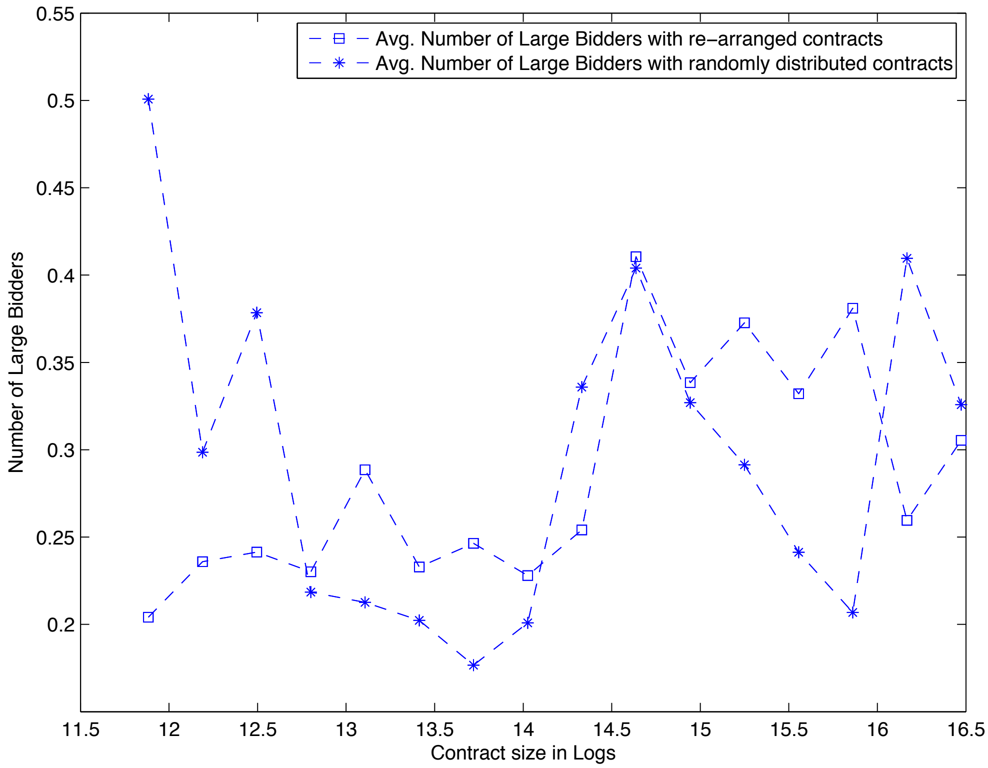


Figure 5: Number of Small Bidders

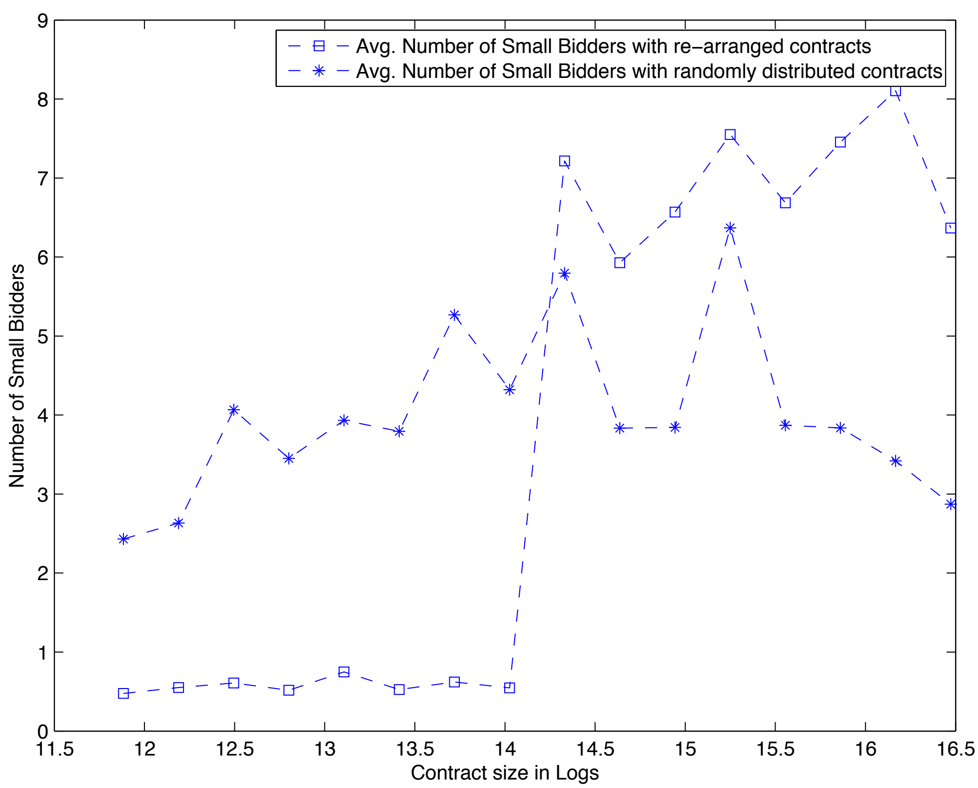
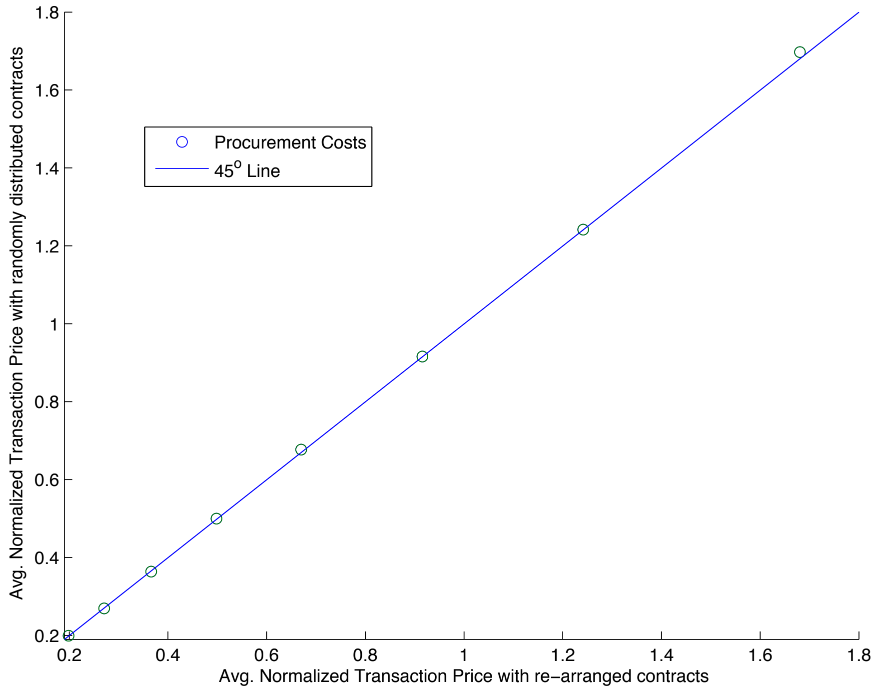


Figure 6: Procurement Costs



Effect on Procurement Costs: We present a scatter plot between the average normalized procurement costs when we re-arrange contracts by size and when we follow MDOT's policy. The results are presented in Figure 6. On the x-axis are the normalized procurement costs when re-arranging by size and on the y-axis we have the normalized procurement cost when not clustering contracts by size. Each circle represents the transaction price for a contract of the same size under the different regimes.

The difference between the two regimes are slight. We can see that occasionally the transaction price for contracts under the regime of no clustering is higher and sometimes the same. The presence of fringe bidders can explain the reason the pronounced effects on participation from before are not passing through to the transaction price. Fringe bidders are myopic and not affected by the future arrangement of contracts. Therefore, the number of fringe bidders will not be affected greatly by the re-arrangement of contracts[22] and the presence of these bidders keeps transaction prices looking the same on average. If there was no set of fringe bidders we would expect there to be shifts in the transaction prices according to the different regimes, since participation patterns would be quite different. However, given that fully characterising that change would require us to solve the game again with no fringe bidders, we cannot say directly which way the effect will go.

Effect on Inefficiencies: We also check how the re-arrangement of contracts affects inefficiencies. Since transaction prices are not changing dramatically, we do not expect there to be a large change in the

[22] Other than through their beliefs about the actual competition in the auction game.

level of misallocations due to the first price auction rule. On average the probability of misallocation when re-arranging contracts is 14.90% and when not re-arranging it is 15.41%. The means are not statistically different at 99% confidence.

It therefore seems to be the case that in this market with a set of myopic bidders engaging in auctions as well as a set of regular forward-looking bidders, that re-arranging contracts by size has no significant effect on the outcomes of auctions.

9 Conclusion

This paper analyses procurement auctions run by MDOT. We find in this market, that bidders treat auctions as intertemporally linked and behave in a forward looking manner. To understand these patterns more fully we posit a structural model of bidder participation with inter-temporal participation synergies in a dynamic bidding game.

The structural model opens up the possibility for further sources of asymmetry and new tools for influencing participation. In particular the model suggests that:

- participation synergies can generate asymmetries between players and affect bidding behaviour through beliefs

- the sequence with which contracts are awarded can be used to influence participation

The paper shows the feasibility of estimating this auction game. With the estimates in hand we can then assess the importance of the aforementioned features of the model in this market. We find that given the estimated participation game, misallocations due to bidder asymmetries and the first-price rule are not frequent. This is due to the fact that uncertainty over the actual competitors in the auction attenuates the strategic bid-shading normally associated with asymmetric auctions.

We then change the sequencing of contracts over time and consider clustering large contracts together. This has the effect of enhancing participation in larger auctions and reducing the number of regular bidders in smaller contract auctions. However, given the presence of fringe bidders this has no further consequences for transaction prices and the probability of misallocation under a first-price auction rule. It is clear that in the absence of fringe bidders this need not be the case and there might be effects on procurement costs if we only considered environments with only forward-looking bidders.

A Appendix: Summary Statistics by Number of Bidders

Table 7 summarises the data by number of bidders. The "Observations" row of the table reveals that 91 auctions attracted one bidder, 742 auctions attracted two bidders and so on. Table 7 also reproduces "money left on the table" data by number of bidders. It can be seen that "money left on the table" decreases with the number of bidders. The average amount of "money left on the table" is still substantial but is in line with other studies, see for example Krasnokutskaya (2011).

Table 7 also sheds some light on participation behaviour. In particular, the first column of the table shows roughly a hundred auctions with only one bidder present. It is possible that if bidders were aware they faced no other competitors they would systematically bid higher and further above the engineer's estimate relative to auctions with more than one bidder. However, the table shows that this is not the case. There is also no significant pattern in the percentage difference between the transaction price as we increase the number of bidders. In particular, moving from 9-10 bidders to 11-19 does not dramatically change transaction prices on average. This suggests that bidders might not have perfect information on actual competition in an auction. However, auction heterogeneity prevents us from making stronger statements. Further analyses, presented in the following section, are required to more fully understand the nature of participation patterns in the data.

Table 7: Summary Statistics by Number of Bidders

	Number of Bidders:	1	2	3	4	5	6	7	8	9-10	11-19
	Obs.	91	742	999	781	618	534	376	272	286	228
Estimate											
	Mean	12.838	13.208	13.334	13.108	13.290	13.372	13.411	13.459	13.339	13.336
	Standard Deviation	1.030	1.093	1.176	1.297	1.286	1.262	1.221	1.087	1.157	1.053
$\frac{\text{Ranked1-Est.}}{\text{Est.}}$											
	Mean	0.006	-0.015	-0.045	-0.052	-0.062	-0.089	-0.084	-0.101	-0.112	-0.130
	Standard Deviation	0.128	0.133	0.152	0.161	0.163	0.1323	0.138	0.120	0.113	0.119
$\frac{\text{Ranked2-Ranked1}}{\text{Ranked1}}$											
	Mean		0.113	0.082	0.100	0.063	0.059	0.053	0.048	0.043	0.041
	Standard Deviation		0.121	0.078	0.088	0.059	0.060	0.051	0.048	0.041	0.049

B Appendix: Identification

Identification in our context is different from Pesendorfer and Schmidt-Dengler (2008), since we do not have to estimate payoff parameters, but the parameters of the information cost distribution, i.e. the parameters of the distribution of payoff "shocks". We take the same approach as Pesendorfer and Schmidt-Dengler (2008) and focus on the equation system that characterises a bidder of type j that is indifferent between participating and non-participation, fixing β and making use of the functional form assumption on the distribution of information costs. Writing this out yields

$$\Pi^j(\sigma^*, \sigma, \sigma_{\mathcal{F}}) + \beta(M_1^j(\sigma^*) - M_0^j(\sigma^*))V^j(\sigma, \sigma_{\mathcal{F}}) = \tilde{\tilde{\Xi}} \tag{A-1}$$

where $\tilde{\tilde{\Xi}}^j = [-\log(1-p_j(s'))/\lambda_j(s')]_{s' \in \mathbf{S}}$ is a vector of the level of information costs that make a bidder indifferent between entering and not. To compute this expression we have made use of the inversion from choice probabilities to value functions. As originally shown by Hotz and Miller (1993) and again shown by Pesendorfer and Schmidt-Dengler (2003), there is a mapping from choice probabilities to a unique vector of indifferent types. Let $\Gamma^j(\sigma^*) = \beta(M_1^j(\sigma^*) - M_0^j(\sigma^*))[I - \beta \mathbf{P}^j(\sigma^*)M_1^j(\sigma^*) - \beta[I - \mathbf{P}^j(\sigma^*)]M_0^j(\sigma^*)]^{-1}$. Substituting the expression for the value function and re-arranging yields:

$$[I + \Gamma^j(\sigma^*)\mathbf{P}^j(\sigma^*)]\Pi^j(\sigma^*, \sigma, \sigma_{\mathcal{F}}) = \Gamma^j(\sigma^*)\Xi^j(\sigma^*; \theta) + \tilde{\tilde{\Xi}} \tag{A-2}$$

All elements on the left hand side of A-2 are known. The right hand side involves the choice probabilities and the parameters of interest. We can re-write the entire equation system as

$$[I + \Gamma^j(\sigma^*)\mathbf{P}^j(\sigma^*)]\mathrm{diag}[\Pi^j(\sigma^*, \sigma, \sigma_{\mathcal{F}})]A^j \begin{pmatrix} \lambda_0^j \\ \lambda_1^j \end{pmatrix} = \Gamma^j B^j(\sigma^*) + C^j(\sigma^*) \tag{A-3}$$

where $A^j(\sigma^*; \theta) = \begin{bmatrix} 1 & s_1 \\ 1 & s_2 \\ \vdots & \vdots \end{bmatrix}$ is a $m_s \times 2$ matrix, $B^j(\sigma^*) = [(1 - p_j(s'))\log(1 - p_j(s')) + p_j(s')]_{s'\mathbf{S}}$ and $C^j(\sigma^*) = [\log(1 - p_j(s'))]_{s'\mathbf{S}}$. We have two unknown parameters with m_s equations for each player. The system is overidentified. The least squares estimator presented previously is the best estimator for our parameter vector.

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