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Abstract. Firms often use both objective/verifiable and subjective/non-verifiable performance measures to provide employees with effort incentives. We study a principal/multi-agent model in which a verifiable team-based performance measure and non-verifiable individual performance measures (one for each agent) are available for contracting. A problem with tying rewards to non-verifiable measures is that the principal has incentives to understate the realization of those measures in order to reduce compensation. We compare two mechanisms for overcoming this credibility problem: bonus pools and reputation. While reputation is fostered by repeated interactions (a low discount rate), repeated interactions create opportunities for agent-agent collusion under bonus pools. These opportunities for collusion can be exacerbated by the team performance measure, to the point that it can be optimal to make the size of the bonus pool independent of the realization of the team measure. In general, strong task interdependencies improve the effectiveness of reputation-based contracting and reduce the effectiveness of bonus pool arrangements.

1. Introduction

Firms often use both objective/verifiable and subjective/non-verifiable performance measures to provide employees with effort incentives (Gibbs, 1995; Gibbs et al., 2004). While verifiable measures usually relate to the output of agents' productive actions—and thus reflect agents' effort only with considerable noise—non-verifiable measures often capture agents' effort input more precisely. At the same time, tying compensation to non-verifiable measures creates a credibility problem as the principal has incentives to understate the measures in order to reduce compensation. The key mechanisms suggested by the prior literature to mitigate this credibility problem are *reputation* (in repeated relationships) and *bonus pools* (in multi-agent settings).[1] In this paper, we study both these mechanisms in a multi-period/multi-agent setting and show that multiple agents make the principal's reputation *more* effective, while repeated interactions make bonus pools *less* effective. This result is in part driven by the fact that technological interdependencies between the agents' efforts facilitate principal-agent relational contracting (the principal's reputation) but also agent-agent relational contracting (collusion against the principal under bonus pools).

Consider a team of agents collaborating to produce a joint output (e.g., firm-wide or divisional earnings, completion of a joint project) repeatedly over an infinite horizon. The agents' actions can be either strategic complements or strategic substitutes in that the marginal productivity of an agent's effort is higher—or lower, respectively—if the other agent also chooses high effort. An example of agents' actions that are strategic complements is a cross-functional team in which effort from each team member is necessary to pull off a success, say, because each team member provides a unique input. An example of strategic substitutes is a team in which agent effort is interchangeable and there are decreasing returns to total effort. Aside from verifiable team output, compensation can also be based on non-verifiable performance measures that are agent-specific.

Under a bonus pool, the principal commits to an amount to be paid out to a group of agents and uses her non-verifiable assessment of the agents' performance only in deciding how to divide the bonus pool among the agents. While, by construction, such an arrangement lends credibility to implicit contracts, it has features of relative performance evaluation and therefore is susceptible to *collusion*. Such agent-agent relational contracting becomes a concern in particular for repeated interactions: the agents then can credibly commit to implicit side contracts to undermine the effort choices desired by the principal. As far as we are aware, ours is the first study to analyze bonus pools under repeated interactions.

In the most commonly encountered form of bonus pools, the total amount to be distributed among the agents is contingent on one or more verifiable performance measures.[2] To begin with, we study bonus pools that are contingent on (verifiable) team output and symmetric in that agents who have been evaluated (non-verifiably) as having chosen the same effort level will receive the same bonus portion. We show that the threat of collusion under such a bonus pool arrangement takes one of two forms. If their actions are strategic complements, the agents will side-contract on always choosing low effort in each period ("*Shirk*"). If their actions are strategic substitutes, the agents will side-contract to have one agent choose high effort only in even periods and the other agent choose high effort only in odd periods ("*Cycle*").

We characterize the contracting cost to the principal associated with a collusion-proof bonus pool and show that this cost is particularly high if the discount rate is small (or,

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[1] See, among others, Baker, Gibbons, and Murphy (1994), Levin (2002, 2003), MacLeod (2003), and Rayo (2007) on the role of reputation; and Baiman and Rajan (1995), Rajan and Reichelstein (2006, 2009), and Ederhof (2007) on the role of bonus pools.

[2] An example is described in Reuters, February 21, 2007: " A maximum bonus pool of $165 million has been established for a group of five senior executives that includes Bear Stearns Chief Executive James Cayne, the company said. Payout will be pegged to the company's return on equity. No executive can get more than 30 percent of the total pool, which can be as little as zero. Bear Stearns' compensation committee also approved the performance goals for a second bonus pool for seven other top executives. The maximum amount will be $140 million, with awards based on pretax return on equity, departmental income and expense controls. Bear

equivalently, the agents' expected collaborative time horizon is long) and if the tasks are either strong complements or strong substitutes. The role of time preferences in this context is straightforward: more patient agents can collude more effectively. To illustrate the more subtle role of task interdependency, consider the case of strongly complementary efforts. In that case, the "bribe" required to induce an agent to deviate (unilaterally) from *Shirk* is very costly because the associated increase in the probability of the bonus pool being paid out is small (the marginal productivity of unilateral effort is small for strong complements). A similar argument implies that the cost to the principal of breaking a collusive agreement of the *Cycle* variation is particularly high if the agents' tasks are strong substitutes.

As an alternative to bonus pools, the principal can write individual (relational) contracts, one with each agent.[3] If the principal ever reneges on her promises, she will lose the agents' trust. As a result, the agents will no longer respond to implicit incentives in any future period, but only to explicit incentives tied to the verifiable performance measure (or they will quit). It is well known that the smaller the discount rate, the more credible are the principal's promises, as future punishments by the agents then would be more costly.

The effect of the production technology on the principal's commitment power is, again, more subtle. Suppose the principal has reneged on the implicit contract and hence, going forward, has to rely on a fallback contract based only on verifiable team output. If the agents' efforts are strategic complements, a naïve fallback contract however would produce both a working and a shirking equilibrium. To avoid having the agents punish her by playing the shirking equilibrium, the principal has to make the working equilibrium the unique one, which in turn requires higher-powered incentives. The stronger the strategic complementarity among efforts, the more costly it is to produce a unique working

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Stearns did not disclose the exact performance targets."

[3] The relational contracting literature has focused for the most part on principal-single-agent models. Two notable exceptions are Rayo (2007), who studies relational contracts in a team setting *without* a principal (i.e., imposing a budget balance constraint), and Levin (2002), who compares bilateral with multilateral contracts.

equilibrium during the punishment phase and, thus, the more credible is the principal's promise to honor the implicit contract. If instead the agents' actions are strategic substitutes, then high effort will be the unique equilibrium under the naïve fallback contract. The optimal contract then is a stark one—and essentially the same as in single-agent settings. It uses either only the non-verifiable individual performance measures (for low discount rates or high degrees of strategic substitutability) or only the verifiable team measure (for high discount rates or low degrees of strategic substitutability).

While the existence of multiple equilibria often aggravates inefficiencies in contracting, in our setting, multiple equilibria in the agents' (fallback) subgame facilitate relational contracting, because they allow the agents to punish a dishonest principal more severely.[4] As a result, the extent to which the principal can commit to implicit contracts is greater under strategic complements than under strategic substitutes. In fact, we show that in finite horizon settings, relational contracts are feasible *only* with strategic complements.

To summarize, individual contracts (principal-agent relational contracts) are the preferred solution for low discount rates (expected long-term relationships) and strong interdependencies among tasks (in particular, strategic complementarities). Bonus pools, on the other hand, perform well for high discount rates (expected short-term relationships) and tasks that are, by and large, technologically independent. While relational contracts between the principal and agents can enhance efficiency and, hence, should be bolstered by cultivating long-term relationships, relational contracts among agents tend to undermine bonus pool arrangements. Job rotation might mitigate such problems as it shortens the agent-agent relationship horizon without affecting the horizon of the principal-agent relationship.[5] Taken

[4] This argument is related to Baker, Gibbons, and Murphy's (1994) observation that a poor-quality (less informative in the sense of Holmstrom, 1979) verifiable performance measure may be desirable. The off-equilibrium threat of having to rely on such a poor quality performance measure in future periods lends credibility to implicit incentives. In our team setting, the production technology (efforts being strategic substitutes or complements) plays a key role in determining the quality of the fallback contract.
[5] A related argument is made by Tirole (1986) in a hierarchical principal-supervisor-agent model.

together, our results imply a non-monotonic relation between discount rates (or contracting horizons) and contracting costs: for small discount rates, individual contracts perform well. As the discount rate increases, so does the principal's contracting cost until a bonus pool becomes the lower-cost mechanism. As the discount rate increases further, the contracting cost under a bonus pool decreases even further as the threat of collusion subsides.

Our results on individual contracts provide a new perspective on the empirical observations of Gibbs et al. (2004). Gibbs et al. (2004) argue that strong task interdependencies create a demand for subjective performance evaluation, because capturing interdependencies in formula-based incentive contracts is difficult. We show that strong task interdependencies, instead, *facilitate* subjective (non-verifiable) performance evaluation by reducing the off-equilibrium payoff to a principal who has lost the agents' trust. On the other hand, earlier studies (e.g., Mookherjee 1984, Shleifer 1985, Ishiguro 2004) have shown that relative performance evaluation schemes are susceptible to collusion. Our results on bonus pools add to this line of literature by demonstrating that task interdependencies aggravate the threat of collusion, again by affecting the agents' off-equilibrium behavior.

We also consider alternative ways of implementing bonus pools. Surprisingly, the principal may be better off committing to pay out a fixed bonus pool, independent of the realization of the verifiable team output ("pay without performance"). This seems to contradict conventional wisdom that the size of bonus pools should be varied with some verifiable measure of performance (such as earnings). The conventional wisdom is based on (1) the informativeness of the verifiable measure (provided it is incrementally informative) and (2) the susceptibility of fixed-prize tournaments to collusion (e.g., Budde 2007). Yet, we show that a fixed bonus pool can reduce the cost of collusion when the agents' actions are strategic complements. (Recall that the bribe to break up *Shirk* collusion is very high with complementary actions when the payout is contingent on the verifiable team measure.)

Last, we show that the threat of collusion can be reduced—albeit not eliminated—by adopting asymmetric payout policies whereby the principal favors one of the agents in the (off-equilibrium) event that both shirk. However, even asymmetric bonus pools are outperformed by individual (reputation-based) incentive contracts under certain conditions.[6]

The remainder of the paper is organized as follows. Section 2 presents the basic model. Sections 3 and 4, respectively, address individual contracts and bonus pools in an infinitely repeated relationship. Section 5 presents results for a finite-horizon (two-period) version of the model, and Section 6 concludes.

2. Model

A principal contracts with two agents, $i = A, B$. Each agent i provides personally costly effort $a_t^i \in \{L, H\}$ in period t, where $L = 0 < H$. In a joint and stochastic fashion, these efforts result in concurrent team output $x_t \in \{0,1\}$. In particular, let

$$\begin{aligned} p_H &= Pr(x_t = 1 \mid a_t^A = a_t^B = H), \\ &> p = Pr(x_t = 1 \mid a_t^i = H, a_t^j = L), \quad i \neq j, \\ &> p_L = Pr(x_t = 1 \mid a_t^A = a_t^B = L). \end{aligned}$$

The team output x_t is commonly observable and *verifiable* (contractible). Aside from output, compensation contracts can also depend on signals, $y_t^i \in \{0,1\}$, about agent i's effort in period t. While these signals are more informative, we assume they are *non-verifiable* metrics.[7] Any contractual obligations based on them therefore need to be self-enforcing. We consider short-term contracts, only. At the beginning of period t, the principal offers agent i the compensation contract $w_t^i(x_t, (\hat{y}_t^i, \hat{y}_t^j))$, where $\hat{y}_t^i \in \{0,1\}$ is the principal's report of the

[6] Ishiguro (2004) also shows that treating agents asymmetrically helps combat collusion under relative performance evaluation. His study however differs from ours in a number of ways. In particular, he assumes stronger side contracting ability on the part of the agents: (i) they can collude on monetary transfers and (ii) explicit side contracts are enforceable by the court. Since Ishiguro's setting is a static one, both these assumptions are critical for collusion to be a credible threat to the principal. In our model, collusion among agents (without monetary transfer) has to be self-enforcing.

[7] The earlier literature has used the labels "non-verifiable" and "subjective" interchangeably for performance measures. We use the term "non-verifiable" throughout since in our model all parties observe all efforts without noise (hence, objectively), yet this information is not contractible because it cannot be verified by the courts.

non-verifiable metric regarding agent i's effort, a_t^i. Agent i either accepts the contract or leaves the employment relationship and receives a reservation utility of zero in perpetuity. To abstract from the threat of dismissal as an alternative source of incentives, we assume that if an agent quits, the firm will shut down or, equivalently, the principal would have to incur prohibitively high search costs in order to find a replacement for the agent. For simplicity, we assume the principal observes a_t^i perfectly, i.e., $y_t^i = 1$ if and only if $a_t^i = H$. The optimal contract can then be expressed in the following additive form:

$$w_t^i = \alpha \hat{y}_t^i H + \beta x_t .$$

We adopt this representation because of its ease in conveying the intuition.

Both agents and the principal are risk-neutral and share a common discount rate of r, capturing the time value of money or the probability the relationship will continue at the conclusion of the current period (or a combination of the two). The agents are protected by limited liability in that $w_t^i \geq 0$ for all i, t. We assume that agent effort is sufficiently important that the principal always finds it worthwhile to elicit high effort from each agent in each period. Agent i's period-t payoff is normalized to $w_t^i - a_t^i$, and the principal's period-t payoff is $x_t - \sum_i w_t^i$.

We evaluate the efficiency of any contractual arrangement by the expected periodic cost, C_t, to the principal of eliciting (H,H) efforts from the agents. As a benchmark, in the first-best solution, agent efforts are contractible, so that the principal would simply direct the agents to take high efforts in each period and reimburse them for their disutility. The resulting first-best expected periodic cost to the principal would equal $C_t^{FB} = 2H$. Another useful benchmark is the case of contracts based on only the verifiable measure (i.e., $\alpha_t^i = 0$), so that $w_t^i(x_t, (\hat{y}_t^i, \hat{y}_t^j)) \equiv w_t^i(x_t)$. It is straightforward to see that playing (H,H) will then be a Nash equilibrium for the two agents provided the principal sets $\beta \geq \beta^{HH} \equiv \dfrac{H}{p_H - p}$. Since making this inequality strict would result in excessive rents earned by the agents, the expected periodic cost to the principal would equal $\overline{C}_t \equiv 2 p_H \beta^{HH} = \dfrac{p_H}{p_H - p} 2H$. Note also that playing (L,L) is *not* a Nash equilibrium (given $\alpha_t^i = 0$) whenever $\beta \geq \beta^{LL} \equiv \dfrac{H}{p - p_L}$.

Throughout the paper, we assume the agents perfectly observe each other's efforts and they will play as the principal intends as long as doing so constitutes a subgame perfect Nash equilibrium in the overall game, which is not Pareto-dominated by any other subgame perfect Nash equilibrium.

We will distinguish between two cases regarding the team production technology: efforts are either *strategic complements* in that $p_H - p > p - p_L$ (i.e., $p < (p_H + p_L)/2 \equiv p_o$), or *strategic substitutes* in that $p - p_L > p_H - p$ (i.e., $p > p_o$). To illustrate, consider again the benchmark case where contracts are based on only the verifiable measure, i.e., $\alpha_t^i = 0$. If efforts are strategic complements, then $\beta^{HH} < \beta^{LL}$ and, therefore, for any $\beta \in \left[\beta^{HH}, \beta^{LL}\right]$ there exist two pure-strategy Nash equilibria, (H,H) and (L,L). For (H,H) to be the unique (in fact, a dominant-strategy) equilibrium, $\beta > \beta^{LL}$ has to hold. If efforts are strategic substitutes, on the other hand, then $\beta^{HH} \geq \beta^{LL}$ and (absent implicit contracts) the effort profile (H,H) constitutes the unique pure-strategy equilibrium for any $\beta > \beta^{HH}$.[8]

We consider an infinitely repeated contractual relationship, first assuming the principal in each period offers the agents individual contracts and then allowing for bonus pool arrangements.

3. Individual Rewards: Principal-Agent Relational Contracting

We first derive the optimal contract offered to the agents individually when both verifiable and non-verifiable measures are available for contracting. Assuming the principal honors the implicit contract, playing (H,H) constitutes a Nash equilibrium for the agents if and only if

$$\alpha H + (p_H - p)\beta \geq H. \tag{1}$$

The agent's periodic expected rent is $U_t^i = \alpha H + p_H \beta - H$. There exists a one-dimensional set of incentive coefficients $\left\{(\alpha, \beta^*(\alpha)) \mid \beta^*(\alpha) \equiv (1-\alpha)\beta^{HH}\right\}$ satisfying (1) with equality. That is, the explicit and implicit performance measures are substitutes in providing effort incentives. Plugging $\beta^*(\alpha)$ into U_t^i yields

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[8] With efforts being strategic substitutes, there exist two asymmetric Nash equilibria (L,H) and (H,L) for $\beta^{LL} < \beta < \beta^{HH}$, and a unique (dominant-strategy) equilibrium (L,L) for $\beta < \beta^{LL}$.

$$U_t^i(\alpha) \equiv U_t^i(\alpha, \beta^*(\alpha)) = (1-\alpha)H \frac{p}{p_H - p},$$

which is decreasing in α. Because in our setting the implicit measure y_t^i is noiseless, the principal wants to set α as high as possible so as to reduce limited-liability related rents. To see this, write out the expected cost to a principal who contracts individually with each agent as a function of α:

$$C_t^{indiv}(\alpha) = 2[\alpha H + p_H \beta^*(\alpha)] = \bar{C}_t - 2H \frac{p}{p_H - p} \alpha .$$

As α increases, such an arrangement becomes increasingly efficient and reaches first-best benchmark performance for $\alpha = 1$. However, since y_t^i is a non-verifiable measure, there are limits to the principal's ability to commit to this measure. Below, we denote by C_t^{comp} and C_t^{sub} the special cases of C_t^{indiv} for strategic complements and substitutes, respectively.

We consider the following trigger strategy to support the implicit contract. As long as the principal honors the implicit contract, the agents are willing to play the desired (H,H) strategy (provided it constitutes a subgame perfect equilibrium that is not Pareto-dominated by any other subgame perfect equilibrium). If however the principal reneges on the implicit contract by claiming that $\hat{y}_t^i = 0$ for some i and t, whereas in fact $a_t^i = H$, then both agents will not respond to implicit incentives any longer. Instead, they will punish the principal by playing (L,L) indefinitely, unless (H,H) is a unique equilibrium under the continuation contract which is solely dependent on the verifiable measure.[9]

Strategic Complements

Recall that with efforts being strategic complements ($p < p_o$), by setting $\beta = \beta^{HH}$ the principal makes (H,H) a Nash equilibrium in the stage game under the fallback contract ($\alpha = 0$), but not a unique one. Uniqueness requires $\beta \geq \beta^{LL} > \beta^{HH}$. With that in mind, the relational contract offered by the principal has to satisfy the following *reneging constraint*:

$$\alpha H \leq \frac{1}{r}\left[p_H \beta^{LL} - (\alpha H + p_H \beta^*(\alpha))\right]. \tag{2}$$

The left-hand side of this constraint gives the principal's benefit of reneging on the implicit contract, in which case she would save αH for each agent. The right-hand side states the

[9] Throughout the paper, we confine attention to pure-strategy equilibria. Given all players perfectly observe the agents' efforts, albeit non-verifiably, there will never be a situation where the principal gets falsely punished.

principal's cost of reneging: to prevent the agents from playing (L,L) in all future periods, the principal will have to raise the explicit bonus from $\beta^{HH} = \dfrac{H}{p_H - p}$ to $\beta^{LL} = \dfrac{H}{p - p_L}$ in order to ensure the (H,H) equilibrium is unique in the stage game, while at the same time avoiding the expected "status quo" compensation of $\alpha H + p_H \beta^*(\alpha)$. By revealed preference, the term in square brackets on the right-hand side is strictly positive for any $\alpha > 0$ (otherwise, the principal would not have set a positive α to begin with).

As one would expect, the reneging constraint is easier to satisfy if the discount rate r is small. But how does task interdependency affect this constraint? The main driving force behind our first main result is that the principal finds it easier to commit to honoring implicit contracts if the agents' tasks are strategic complements.

Proposition 1. If the agents' actions are strategic complements (i.e., $p < p_o$), then the weight placed on the non-verifiable measure equals:

$$\alpha^* = \begin{cases} \bar{\alpha} = \dfrac{p_H\left(\dfrac{1}{p-p_L} - \dfrac{1}{p_H - p}\right)}{r - \dfrac{p}{p_H - p}} \in (0,1), & \text{if } r > \dfrac{p_H - (p - p_L)}{p - p_L}, \\ 1, & \text{otherwise.} \end{cases}$$

Proof: See Appendix.

Proposition 1 demonstrates that individual contracts achieve first-best performance if the parties are sufficiently patient, i.e., $C_t^{comp} = C_t^{FB}$ for $r \le \dfrac{p_H - (p - p_L)}{p - p_L}$. To illustrate the role of task interdependency, suppose we increase the degree of complementarity by decreasing p while holding p_L and p_H constant. By examining the principal's contracting cost C_t^{comp}, one can see that it is decreasing in the degree of the complementarity. The reason is that as p becomes smaller, the fallback contract, which has to ensure the (H,H) equilibrium is unique while relying on only the verifiable measure, becomes costlier.

Corollary 1. If the agents' actions are strategic complements, the use of implicit incentives is:

(i) decreasing in the discount rate *r* and

(ii) increasing in the degree of strategic complementarity (captured by decreasing *p* while holding p_H and p_L constant).

Counting on the agents punishing the principal in a way that also punishes themselves is nothing new. In a repeated Prisoners' Dilemma, one subgame perfect equilibrium is "tit-for-tat." Each agent threatens to revert to the (Pareto-dominated) stage game equilibrium if the other ever defects from cooperating.[10] (A difference between our setting and the repeated Prisoners' Dilemma is that the stage game equilibrium is unique in the repeated Prisoners' Dilemma.)

Strategic Substitutes

If instead the agents' actions are strategic substitutes ($p > p_o$), then the same (IC) constraint (1) applies in that (H,H) will be a Nash equilibrium for any $\beta \geq \beta^*(\alpha)$. In the strategic substitutes case, this equilibrium can be made unique by increasing the bonus payment by any arbitrarily small positive amount. For ease of exposition, we ignore this small additional cost throughout the paper. The principal's reneging constraint for efforts that are strategic substitutes reads

$$\alpha H \leq \frac{1}{r}\left[p_H \beta^{HH} - (\alpha H + p_H \beta^*(\alpha))\right],$$

which simplifies to

$$r \leq \frac{p}{p_H - p}. \tag{3}$$

Note that condition (3) is independent of α. If it is violated, then implicit incentives will not be sustainable, i.e., $\alpha^* = 0$ and $\beta^*(0) = \beta^{HH}$. If (3) is satisfied, then $\alpha^* = 1$ and $\beta^*(1) = 0$, i.e., the first-best solution obtains. As a result, there is a discontinuity at $r = \dfrac{p}{p_H - p}$, in that

[10] See also Levin (2002, 2003). In contrast, Bernheim and Whinston (1998) assume that only Pareto-unranked equilibria can be used as punishments. Under that alternative approach, the following analysis for the strategic

the first-best solution is realized if and only if implicit contracts are feasible: $C_t^{sub} = C_t^{FB}$ if (3) holds, and $C_t^{sub} = \overline{C}_t$ otherwise. As we show below in connection with Figure 1, in comparison with the reneging constraint, (2), for the case of strategic complements, (3) is a stronger condition.

Suppose we increase the degree of substitutability by increasing p while holding p_L and p_H constant. By examining C_t^{sub}, one can see the expected compensation cost is discontinuously decreasing in the degree of the substitutability. When $r < \frac{p}{p_H - p}$, the first-best can be achieved; otherwise only the verifiable measure can be used. The reason for both the discontinuity and the non-monotonicity (when we put the strategic complements and strategic substitutes cases together) is that, under strategic complements, the off-equilibrium fallback contract uses the verifiable measure differently ($p - p_L$ comes into play) than under the equilibrium contract ($p_H - p$ comes into play). Under strategic substitutes, both the on- and off-equilibrium contracts use the verifiable measure in the same way (only $p_H - p$ comes into play). Proposition 2 and its corollary summarize the strategic substitutes case.

Proposition 2. If the agents' actions are strategic substitutes (i.e., $p > p_o$), then the weight placed on the non-verifiable measure equals:

$$\alpha^* = \begin{cases} 0, & \text{if } r > \dfrac{p}{p_H - p}, \\ 1, & \text{otherwise.} \end{cases}$$

Corollary 2. If the agents' actions are strategic substitutes, the use of implicit incentives is:

(i) decreasing in the discount rate r and

(ii) *discontinuously* increasing in the degree of substitutability (captured by increasing p while holding p_H and p_L constant).

Propositions 1 and 2 are illustrated in Figure 1. For strong strategic complements and substitutes, the principal can rely fully on implicit incentives, i.e., $\alpha = 1$. Since the non-verifiable individual measures are noiseless, first-best performance obtains in those cases. Denote by p^{comp} and p^{sub} the respective thresholds for p below (above) which first-best

subsitutes case also applies to strategic complements.

performance obtains for agent efforts that are strategic complements (substitutes). It is easy to show that $p_o - p^{comp} = p^{sub} - p_o$. Put differently, the range of parameter values for which the performance of individual contracts falls short of first-best is the same for complements as for substitutes. Yet, for complements the principal can always put some positive incentive weight on the non-verifiable individual measures, even if the degree of complementarity is small; not so for efforts that are strategic substitutes where the solution is "bang-bang." Thus, as mentioned above, the scope for implicit contracts is greater in the case of complementary efforts.

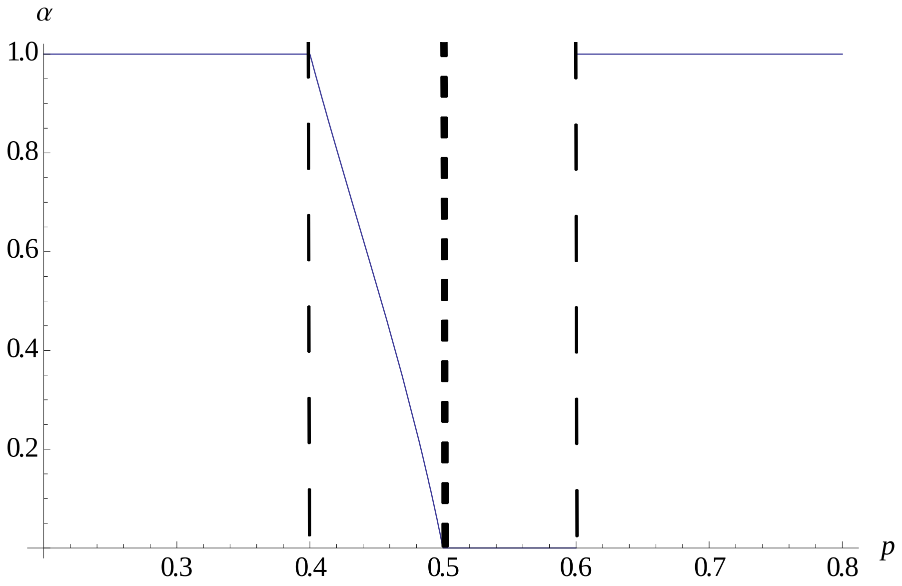

Figure 1: Individual Contracts for ($p_L = .2,\ p_H = .8, r = 3$)

The preceding findings throw a different light on Gibbs et al. (2004) who document a positive relation between firms' use of implicit incentives (subjective performance measures) and the extent of organizational interdependencies. They argue that verifiable/objective performance measures often insufficiently capture such interdependencies. Hence, they describe subjective performance measures as supplementary to objective ones, and used only to the extent that objective measures fail to be comprehensive. Our findings imply similar empirical associations, yet the underlying logic (and causality) is different. In our model, the principal always prefers using the non-verifiable metrics, because they are inherently more informative by assumption. Yet, commitment problems on the part of the principal limit the use of such metrics. These commitment problems are mitigated by task interdependencies. Hence, in our setting such interdependencies *facilitate* the use of implicit contracts.

4. Bonus Pools: Agent-Agent Relational Contracting

As the preceding analysis has shown, a key impediment to relational contracting is the principal's limited ability to commit to honoring implicit contracts. Bonus pools avoid any such commitment issues. A bonus pool is a contractual agreement by which the principal commits to allocate a certain dollar amount among a set of agents, where the total does not depend on any non-verifiable signals observed. That way, the principal is indifferent as to how to split the total bonus as it is a sunk cost anyway. Earlier literature (e.g., Baiman and Rajan 1995, Rajan and Reichelstein 2006, 2009) has shown that bonus pools can be powerful contracting tools in static settings, but has remained silent on how these arrangements perform in dynamic settings. In this section, we show that the repeated nature of transactions impedes the effectiveness of bonus pools as a result of *agent-agent relational* contracting.

Within the class of bonus pool arrangements various payout policies are conceivable. We refer to a payout policy as *symmetric* if, given a total bonus pool amount B to be distributed in period *t*, each agent receives $B/2$, if $y_t^i = y_t^j$; whereas the entire amount B goes to that agent with the higher y_t^i measure, if $y_t^i \neq y_t^j$. The payout policy is labeled *conditional* if is it contingent on the realization of the verifiable team measure x_t. We will begin our analysis with the most commonly-encountered form of bonus pools in which the payout is *symmetric* and *conditional* in that neither agent receives any bonus based on y_t^i if $x_t = 0$, whereas the full amount *B* is paid out (to one agent or split evenly, depending on the non-verifiable measures) if $x_t = 1$. Below we will consider alternative payout policies.[11]

4.1 Bonus Pools with Conditional, Symmetric Payout

We denote by $U_{kl}^{C,S}$ an agent's expected period-*t* utility from choosing effort $a_t^i = k$ under a bonus pool arrangement with *conditional, symmetric* payout when the other agent chooses effort $a_t^j = l$:

[11] In particular, we show below that asymmetric payout policies can help reduce collusion costs, in line with results in Ishiguro (2004). While we confine attention to these discrete payout policies, it would be desirable for future research to characterize *optimal* payout policies in a more general contracting framework.

		Agent B: L	Agent B: H
Agent A	L	$U_{LL}^{C,S} = p_L \frac{B}{2}$, $U_{LL}^{C,S} = p_L \frac{B}{2}$	$U_{LH}^{C,S} = 0$, $U_{HL}^{C,S} = pB - H$
	H	$U_{HL}^{C,S} = pB - H$, $U_{LH}^{C,S} = 0$	$U_{HH}^{C,S} = p_H \frac{B}{2} - H$, $U_{HH}^{C,S} = p_H \frac{B}{2} - H$

Table 1: Payoffs under Bonus Pool with Conditional, Symmetric Payout

Under such an arrangement, playing (H,H) will constitute a Nash equilibrium for the agents' stage subgame, if and only if the following incentive compatibility constraint is satisfied:

$$U_{HH}^{C,S}(B) \geq U_{LH}^{C,S}(B). \tag{4}$$

The incentive constraint (4) imposes a lower bound on *B*, denoted $B^{HH} = 2H / p_H$. Yet, other equilibria may exist for a designated bonus pool amount of $B = B^{HH}$ and, worse, they may Pareto-dominate (H,H) for the two agents. We therefore now turn to the issue of *collusion* — i.e., agent-agent relational contracting. To derive the optimal collusion-proof bonus pool (with conditional-symmetric payout), we will abstract from monetary transfers between agents, as those will be difficult to enforce, but instead allow the agents to only coordinate on their action choices.

Given the infinitely repeated nature of the game, even without monetary transfers among the agents, the side contracting space for the two agents is very rich. In general, the agents can agree to play any strategy profile $\mathbf{a}_0 = \{a_t^A, a_t^B\}_{t=0}^{\infty}$, $a_t^A \in \{L, H\}$, provided $\mathbf{a}_0$ constitutes a subgame perfect equilibrium. To support such a side contract, the agents would adopt a trigger strategy whereby each agent abides by the side contract until some agent *j* defects, in which case agent i would retaliate by choosing high effort indefinitely thereafter. That is, the agents would return to the (H,H) effort profile (as desired by the principal) in each subsequent period. While the agents' side contracting space is infinitely rich, our next result shows that attention can be confined to two specific and intuitive collusive strategies:

Lemma 1. In designing a renegotiation-proof bonus pool with symmetric, conditional payout it is without loss of generality to consider only the following two collusive strategies:

(i) "*Shirk*": $\mathbf{a}_0^{SHK} = \{a_t^A = L, a_t^B = L\}_{t=0}^{\infty}$, i.e., each agent chooses low effort in each period.

(ii) "*Cycle*": $\mathbf{a}_0^{CYC} = \left\{ \left\{ a_t^A = H, a_t^B = L \right\}_{t=0,2,4,\ldots} \cup \left\{ a_t^A = L, a_t^B = H \right\}_{t=1,3,5,\ldots} \right\}$, i.e., the agents alternate choosing high and low effort.[12]

Proof: See Appendix.

Why would the *Cycle* strategy ever be the "binding" collusion constraint? After all, it involves agents incurring disutility of high effort every other period, whereas under *Shirk* effort cost is avoided altogether. However, which of the two collusion constraints is the binding one depends not on the agents' payoffs under the respective strategies, but on the cost incurred by the principal in order to *break them up*. If the agents' actions are strategic substitutes, then the *B* amount required to prevent side contracting of the *Cycle* type will be high because the probability of realizing a high outcome increases only marginally (from p to p_H) when the agents' action profile changes to (H,H) from (L,H). With complementary actions, on the other hand, it is relatively cheap for the principal to prevent collusion via *Cycle* because an agent who defects from colluding by choosing high effort benefits from a steep increase in the probability that $x_t = 1$ given that the other agent also chooses high effort. In that case, the binding collusion constraint will be to prevent *Shirk*.

In light of Lemma 1, we need only consider the following two collusion-proofness constraints: to prevent *Shirk, B* needs to be set high enough to ensure

$$U_{HL}^{C,S}(B) + \frac{U_{HH}^{C,S}(B)}{r} \geq \frac{1+r}{r} U_{LL}^{C,S}(B) \tag{5}$$

At the same time, preventing *Cycle* requires the following inequality to hold:

$$\frac{1+r}{r} \frac{U_{HH}^{C,S}(B)}{r} \geq \bar{U}_{0,i}^{CYC}(B), \quad i = A, B, \tag{6}$$

where $\bar{U}_{0,A}^{CYC}(B) = \sum_{t=0,2,4,\ldots} \frac{pB-H}{(1+r)^t}$ and $\bar{U}_{0,B}^{CYC}(B) = \sum_{t=1,3,5,\ldots} \frac{pB-H}{(1+r)^t}$ are the respective agents' expected utilities in present value terms when abiding to the *Cycle* side contract.

In summary, to derive the cost-minimizing collusion-proof bonus pool arrangement, the principal needs to ensure that:

- (H,H) is a Nash equilibrium of the stage game (requires $B \geq B^{HH}$);

[12] We arbitrarily label Agent A the one to choose high effort in the first period. Alternatively, the agents could toss a coin to determine who chooses high effort first. This variant would not alter our results qualitatively.

- (L,L) is not an equilibrium of the stage game (requires $B \geq B^{LL}$); or if it is, it must be Pareto-dominated by the (H,H) equilibrium from the point of view of the agents (requires $B \geq B^{Pareto}$; both B^{Pareto} and B^{LL} are derived in the Appendix);
- The bonus pool must be collusion-proof. Let B^{SHK} and B^{CYC}, respectively, denote the lower bounds for *B* defined by (5) and (6). By the above logic, it is easy to show that $B^{SHK} > B^{CYC}$ holds for complementary actions, whereas $B^{CYC} > B^{SHK}$ for substitute actions.

Our next result summarizes the contracting cost under this arrangement:

Proposition 3. The expected per-period compensation cost using a bonus pool with conditional, symmetric payout equals $C^{C,S} = p_H B^{C,S}$, where:

$$B^{C,S} = \begin{cases} B^{Pareto} = \dfrac{2H}{p_H - p_L}, & p < \dfrac{p_H}{2} \\ B^{SHK} = \dfrac{2(1+r)H}{p_H + 2rp - (1+r)p_L}, & p \in \left[\dfrac{p_H}{2}, p_o\right] \\ B^{CYC} = \dfrac{2(1+r)H}{(2+r)p_H - 2p}, & p > p_o \end{cases}$$

Proof: See Appendix.

For strongly complementary actions ($p < p_H / 2$) the threat of the agents colluding on *Shirk* is very costly to the principal. In response, the principal has to raise *B* to the point where (H,H) becomes the Pareto-dominant equilibrium for the agents. This yields the threshold B^{Pareto} in the proposition, as defined by $U_{HH}^{C,S}\left(B^{Pareto}\right) \equiv U_{LL}^{C,S}\left(B^{Pareto}\right)$. There is no need to increase *B* beyond that level, and hence B^{Pareto} constitutes an upper bound on the contracting cost for complementary actions. If efforts are mild complements ($p \in [p_H / 2, p_o]$), the binding constraint is to prevent the agents from colluding on *Shirk*. For efforts that are strategic substitutes ($p > p_o$), on the other hand, the relevant collusive strategy for the principal to combat is *Cycle*. Note that B^{SHK} is decreasing in *p*, whereas

B^{CYC} is increasing in *p*.[13] Therefore:

Corollary 3.

(i) The expected per-period compensation cost using a bonus pool with conditional, symmetric payout is non-monotonic in *p*: (weakly) decreasing in *p* for complementary efforts and strictly increasing in *p* for substitute efforts.

(ii) Bonus pools with conditional, symmetric payout never achieve first-best performance.

(iii) $C^{C,S}$ is weakly decreasing in *r*.

Proof: Parts (i) and (ii) are trivial. As for part (iii), B^{Pareto} is independent of *r*, whereas differentiating B^{SHK} and B^{CYC} shows each term to be decreasing in *r*, if and only if $p \geq p_H / 2$. Given the cutoffs for *p* in Proposition 3, part (iii) of the Corollary follows. □

For a bonus pool to achieve first-best performance, the binding constraint has to be that (H,H) be a Nash equilibrium (because $p_H B^{HH} = C_t^{FB}$). Proposition 3 and its corollary however show that the specter of side contracting always prevents the principal from attaining this benchmark. Depending on the production technology, the principal always has to worry about the agents colluding via one of the two strategies defined in Lemma 1. From the viewpoint of the agents, the effectiveness of colluding via *Shirk* decreases as effort complementarity becomes smaller (i.e., as p approaches p_o from below). Similarly, collusion via *Cycle* becomes less effective as the degree of effort substitutability decreases (i.e., as p approaches p_o from above). As a result, the principal's contracting cost will reach its lowest level when tasks are "technologically independent," i.e., when $p = p_o$, as the agents' ability to collude will then be severely limited. See Figure 2 for illustration (the effective bonus pool amount $B^{C,S}$ is depicted in boldface).[14]

[13] With symmetric payout and substitute tasks ($p > p_o$), the payoffs for the agents of the stage game when colluding via *Cycle* cannot be Pareto-ranked with their respective payoffs from obeying the principal's intention by playing (H,H). The agent whose turn under *Cycle* it is to exert high effort always receives a higher payoff than under (H,H), but the reverse holds for the agent who is supposed to lie low. Hence, for substitute tasks there does not exist an upper bound on *B* akin to the upper bound B^{Pareto} in the case of complements.

[14] It is easy to show that preventing (L,L) is never a binding constraint. For $p < p_H / 2$, $B^{LL} > B^{SHK}$, but at the same time, $B^{LL} > B^{Pareto}$.

Lastly, recall that the preceding section has confirmed the intuition that when the principal contracts individually with each agent, efficiency will improve as the discount rate goes down, because the principal will then find it easier to commit to honor his promises. In the case of bonus pools, by the same logic, implicit contracts again perform better as *r* decreases. Now, however, the implications for the principal's welfare are reversed: the more patient all players, the more effectively the agents can side-contract and thereby obstruct the principal's intentions — this gives rise to part (iii) of Corollary 3.

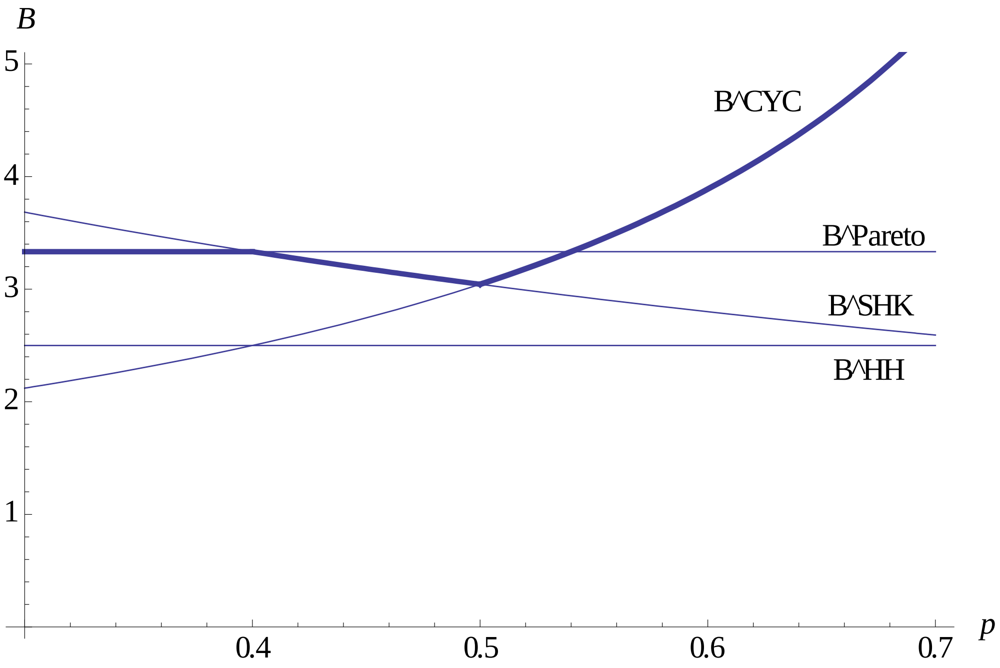

Figure 1: Bonus Pool with Conditional, Symmetric Payout for
$(p_L = .2,\ p_H = .8,\ H = 1,\ r = .4)$

We are now in a position to compare the performance of individual contracting and bonus pools (with conditional and symmetric payout). Individual contracts perform particularly well for low discount rates and for settings that exhibit either strong complementarity or strong substitutability among the agents' actions. The reverse holds for bonus pools where high discount rates and actions that are technologically largely independent (*p* close to p_o) make it harder for the agents to collude. Recall that $C_t^{indiv} \in \{C_t^{sub}, C_t^{comp}\}$ denotes the principal's per-period cost under individual contracting and $C_t^{C,S}$ is the per-period cost under conditional-symmetric bonus pools.

Corollary 4.

(i) If efforts are substitutes ($p > p_o$), then $C_t^{indiv} < C^{C,S}$ if and only if $r < \dfrac{p}{p_H - p} \equiv r_{(i)}$.

(ii) If efforts are strong complements ($p < \dfrac{p_H}{2}$), then $C_t^{indiv} < C^{C,S}$ if and only if

$r < \dfrac{p(p_H - p)}{(p - p_L)^2} \equiv r_{(ii)}$.

(iii) If efforts are weak complements ($p \in \left[\dfrac{p_H}{2}, p_o\right]$), then $C_t^{indiv} < C^{C,S}$ if and only if

$r < r_{(iii)}$, where $r_{(iii)}$ is derived in the proof.

(iv) Furthermore, if the technological interdependence between the agents' tasks becomes stronger, the relative performance of individual contracting improves—i.e., $r_{(i)}$ is increasing in p, whereas $r_{(ii)}$ and $r_{(iii)}$ are decreasing in p.

Proof: See Appendix.

Our results imply a non-monotonic relation between discount rates (or contracting horizons) and contracting costs. For small discount rates, individual relational contracts outperform bonus pools. As *r* increases, so does the principal's contracting cost until a bonus pool becomes the lower-cost mechanism (at the thresholds identified in parts (i)-(iii) of Corollary 4). As the discount rate increases further, the contracting cost under a bonus pool decreases even further as the threat of collusion subsides. Stepping outside the model (where all players share the same discount rate *r*), our results lend support for the common practice of job rotation as a way to "decouple" the time horizons of the principal-agent and the agent-agent relationships. That way, the principal can build up reputation by dealing with the same set of agents over time. At the same time, agents will find it hard to collude given that they are matched with different agents over time. As a result, both principal-agent relational contracting as well as bonus pools are fostered.

4.2 Alternative Payout Policies

Unconditional Payout Policy

While in most firms the total financial reward to be distributed among employees by means of a bonus pool is contingent on some verifiable outcome such as income, EPS, or sales,[15] we now allow for the possibility that the principal commits to paying out B *irrespective of the realization of* x_t. *A priori,* from an agency perspective, this may seem implausible as fixed-sum tournaments are subject to severe collusion problems (e.g., Budde, 2007). On the other hand, recall that we assume that the principal can observe the agents' efforts perfectly, while the verifiable output measure x_t is observed with noise. Hence, from an informativeness perspective, the principal would want to weigh the individual observations as heavily as possible in the contract, subject to the constraint regarding their non-verifiability.

Under a bonus pool arrangement with *unconditional, symmetric payout ("U,S"),* the agents' payoffs are independent of the success probabilities for high output:

		Agent B: L	Agent B: H
Agent A	L	$U_{LL}^{U,S} = \frac{B}{2}$, $U_{LL}^{U,S} = \frac{B}{2}$	$U_{LH}^{U,S} = 0$, $U_{HL}^{U,S} = B - H$
	H	$U_{HL}^{U,S} = B - H$, $U_{LH}^{U,S} = 0$	$U_{HH}^{U,S} = \frac{B}{2} - H$, $U_{HH}^{U,S} = \frac{B}{2} - H$

Table 2: Payoffs under Bonus Pool with Unconditional, Symmetric Payout

The requirement that (H,H) be a Nash equilibrium now reduces to $B \geq 2H$, which also rules out (L,L) as an equilibrium. But the principal again faces a collusion problem. Since now the payout is independent of the project success probabilities, the only relevant collusive strategy is *Shirk*. (The *Cycle* strategy now is dominated from the agents' point of view by *Shirk* as the former involves positive effort costs without altering the total monetary

[15] In addition to the Bear Stearns example given in footnote 3, see also Genentech's DEF 14A filing with the SEC: "Our bonus pool funding is based on an analysis of bonus funding levels as a percent of net income at our comparator group, as well as broader biotechnology and pharmaceutical companies (the "bonus pool comparator group"). Our bonus pool funding is composed of two parts — a base bonus pool and an incremental bonus pool. The base bonus pool, which is linked to performance of specific annual corporate objectives, targets the 50th percentile of net income percentage bonus pool contribution used by the bonus pool comparator group. The incremental bonus pool, which is linked to earnings per share ("EPS") and operating revenue growth relative to

reward.) To prevent the agents from colluding on *Shirk*, the principal needs to set *B* high enough so that the collusion-proofness condition (5) holds (with superscript "U,S" substituted for "C,S"). This requirements boils down to $B \geq \frac{1+r}{r}2H$. Thus, with unconditional payout, the collusion-proofness constraint is always the binding one. Since the bonus is now paid out with probability one, the resulting cost to the principal of securing high effort from the agents is $C_t^{U,S} = \frac{1+r}{r}2H$. A comparison with the contracting cost under a bonus pool with conditional (symmetric) payout $C_t^{C,S}$ — characterized in Proposition 3 — yields our next result (the proof follows from straightforward algebra and is hence omitted).

Proposition 4.

(i) If efforts are strategic complements, an unconditional-symmetric payout policy dominates conditional-symmetric payout if and only if *r* is sufficiently high.

(ii) For substitute efforts, conditional-symmetric payout dominates unconditional-symmetric payout for any *r*.

Part (i) may come as a surprise given our earlier intuition that conditioning the payout on the verifiable team measure should alleviate the threat of collusion. To illustrate, write out the collusion-proofness conditions in (5) for unconditional and conditional payout, respectively:

$$U_{HL}^{U,S} + \frac{U_{HH}^{U,S}}{r} = B - H + \frac{\frac{B}{2} - H}{r} \quad > \quad \frac{1+r}{r}U_{LL}^{U,S} = \frac{1+r}{r}\frac{B}{2},$$

$$U_{HL}^{C,S} + \frac{U_{HH}^{C,S}}{r} = pB - H + \frac{p_H\frac{B}{2} - H}{r} \quad > \quad \frac{1+r}{r}U_{LL}^{C,S} = \frac{1+r}{r}p_L\frac{B}{2}.$$

Rearranging terms shows how breaking away from *Shirk* in some period *T* affects the deviating agent's payoff in the *current* and in all *future* periods:

the comparator group, targets up to the 75th percentile of net income percentage bonus pool contribution used by the bonus pool comparator group."

	$t = T$	$t = T+1, T+2, \ldots$
Unconditional Payout ("*U,S*")	$\frac{B}{2} - H$	$-H$
Conditional Payout ("*C,S*")	$pB - p_L \frac{B}{2} - H$	$-H + (p_H - p_L)\frac{B}{2}$

Table 3: Effect on the Payoff of an Agent who Deviates from *Shirk*

With efforts being strategic complements, the contemporaneous reward from defecting is smaller with conditional than with unconditional payout because there is a positive probability that the bonus pool will not be paid out due to $x_t = 0$ (note that $2p - p_L < 1$ for complementary efforts). At the same time, the future punishments from triggering a reversal to (H, H), indefinitely, are reduced under conditional payout by the fact that the probability of the bonus pool being paid out increases by $(p_H - p_L)$ in each period. Thus, if agents are sufficiently impatient, then it will be cheaper for the principal to entice them to deviate from *Shirk* by using an unconditional payout policy. This is the intuition behind the first part of Proposition 4.

For the second part of the proposition, when efforts are substitutes, the relevant collusion-proofness constraint under the conditional payout policy is to prevent *Cycle*, while it remains *Shirk* for the unconditional policy. Since *Cycle* forces each agent to incur the disutility of high effort every other period, it is less costly for the principal to induce them to break away from this side contract.[16] Figure 3 illustrates Proposition 4, with part a) depicting the case of *r* sufficiently high so that for strong complements (*p* small) the contracting cost under unconditional payout (dashed horizontal line) is less than with conditional payout. Part b) depicts the case of *r* low enough so that conditional payout is always the dominant policy.

[16] An interesting open question is what the optimal payout policy would look like. Proposition 4 indicates that it will not always pay out zero if $x_t = 0$ (as does our "conditional" payout policy), but also not always the same amount regardless of the realization of x_t (as does our "unconditional" payout policy). In a more general formulation, there are two bonus pool amounts, $B_k, k = 0,1,$ to be distributed between the agents conditional on $x_t = k$. Such a formulation method would nest the two symmetric payout regimes considered here. A conceptual challenge to the modeler, however, is that the agents' collusive strategies depend endogenously on the payout policy.

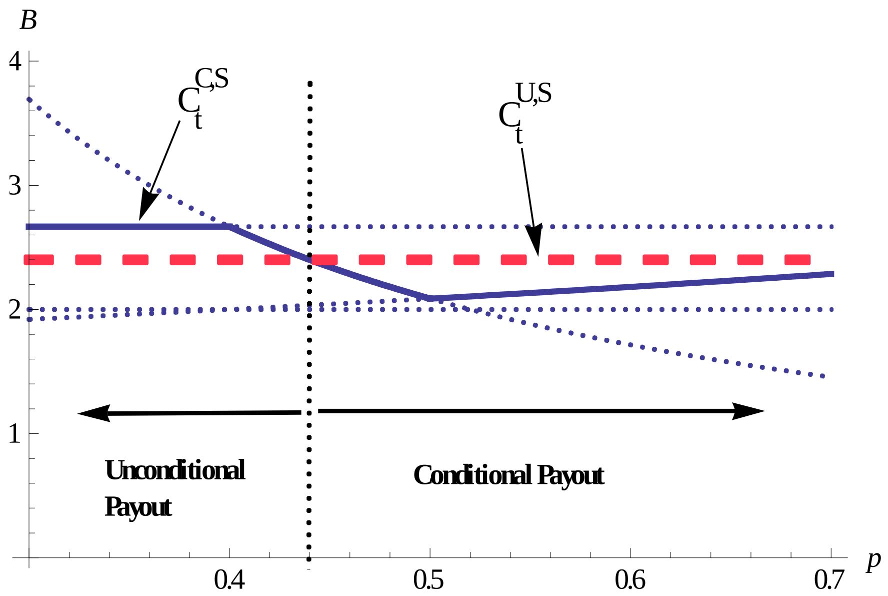

Figure 3a: High *r*

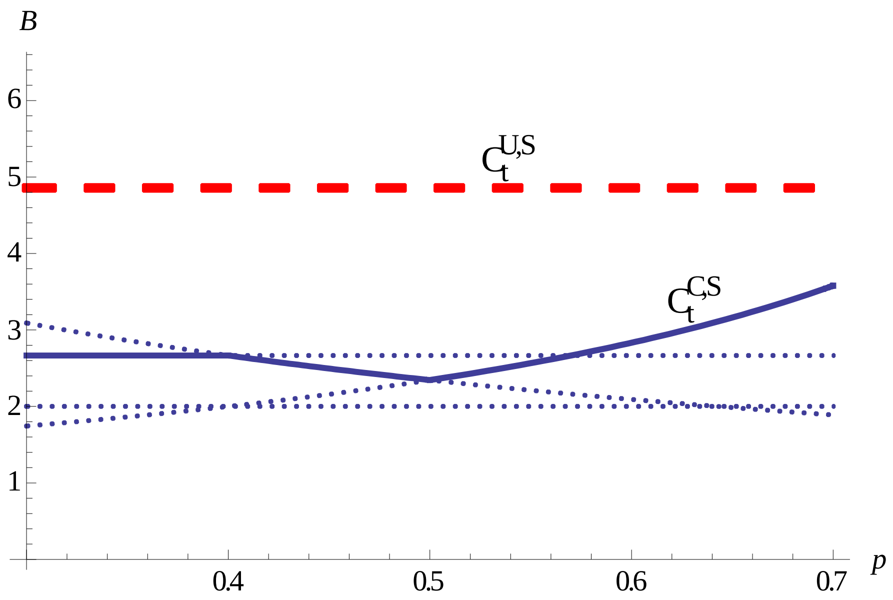

Figure 3b: Low *r*

Figure 3: Compare Conditional and Unconditional Payout Policies (Proposition 4)

Asymmetric Payout Policy

As we have seen above, bonus pools with conditional, symmetric payout suffer from two qualitatively different threats of collusion, *Shirk* and *Cycle*. The *Shirk* strategy in particular capitalizes on the fact that with complementary effort there may be multiple equilibria in the stage game, specifically (H,H) and (L,L). In different settings, Demski and Sappington (1984) and Ishiguro (2004) have shown that (some) undesirable equilibria can be eliminated by treating agents asymmetrically. Discriminating against one of the agents creates dominant-strategy incentives for that player to comply with the principal's preferred action. The other player then chooses his best response. Similar logic can be applied to our setting.[17]

Suppose without loss of generality that the principal discriminates against Agent B by allocating the entire bonus pool amount to Agent A if he observes (non-verifiably but without noise) that both agents have chosen low effort. The agents' expected payoffs under such a conditional, asymmetric payout policy are the same as in Table 1, except for the (L, L) cell:

		Agent B: L	Agent B: H
Agent A	L	$U_{LL,A}^{C,A} = p_L B$, $U_{LL,B}^{C,A} = 0$	$U_{LH}^{C,A} = 0$, $U_{HL}^{C,A} = pB - H$
	H	$U_{HL}^{C,A} = pB - H$, $U_{LH}^{C,A} = 0$	$U_{HH}^{C,A} = p_H \frac{B}{2} - H$, $U_{HH}^{C,A} = p_H \frac{B}{2} - H$

Table 4: Payoffs under Bonus Pool with Conditional, Asymmetric Payout

It is easy to see that Agent A now has dominant-strategy incentives to choose high effort, provided $B \geq B^{HH}$. Agent B will respond by choosing high effort also, because (H, H) is a Nash equilibrium. Hence, such an asymmetric payout policy effectively deters collusion by the agents using the *Shirk* strategy. At the same time, the threat of agent side contracting via *Cycle* remains unmitigated, as this strategy is unaffected by the manipulation of the agents' payoffs in the (L, L) cell. Therefore, the bonus pool amount to be paid out in case $x_t = 1$ equals $B^{C,A} = \max\{B^{HH}, B^{CYC}\}$. Solving for the cutoff *p*-value that equates B^{HH} and B^{CYC} yields (here we use the fact that unconditional-asymmetric payout is economically equivalent to unconditional-symmetric payout):[18]

[17] While Demski and Sappington (1984) consider a multi-agent adverse selection setting, Ishiguro (2004), like us, models relative performance evaluation in a moral hazard setting. His model, however, is static. Therefore, for collusion to be feasible, side contracts can include monetary transfers among agents and can be enforced by courts. We place far weaker (and arguably more descriptive) assumptions on the agents' collusive ability, namely that explicit side contracts have to be self-enforced and do not include transfers.

[18] As for conditional payout, introducing asymmetric payouts in case the principal observes (L, L) eliminates the *Shirk* strategy for agent side contracting. Yet the agents again can collude via *Cycle*. Now, for unconditional payout it is easy to show that the bonus pool amount required to prevent *Cycle* again just equals $[(1+r)/r]2H$.

Corollary 5. Bonus pools with conditional, asymmetric payout policy result in periodic contracting cost of $C^{C,A} = p_H B^{C,A}$, where:

$$B^{C,A} = \begin{cases} B^{HH}, & p < \dfrac{p_H}{2} \\ B^{CYC}, & p > \dfrac{p_H}{2} \end{cases}.$$

Therefore, $C^{C,A} \leq \min\left\{C^{C,S}, C^{U,A} = C^{U,S}\right\}$, i.e., this payout policy dominates all other bonus pool arrangements considered.

An asymmetric payout policy dominates symmetric ones as it costlessly eliminates the undesired (L, L) equilibrium. It also removes the indeterminacy regarding the ranking of conditional and unconditional payout. It is beyond the scope of this paper to address the question why asymmetric contracts favoring some agents over others are rarely seen in practice. Aside from obvious fairness problems, such contracts could suffer from excessive risk premiums in case agents are risk averse and non-verifiable measures are observed with noise. Note, however, that for efforts that are strategic substitutes, even bonus pools with asymmetric payout are still dominated by individual (reputation-based) relational contracts between the principal and the individual agents if the discount rate is sufficiently small.

5. Finite Horizon Model

To illustrate the robustness of our findings, we now consider a two-period model.[19] This is clearly an extreme situation because in practice, while planning horizons are finite, they are usually subject to an uncertain ending date, which is conceptually equivalent to the above infinite horizon specification. For simplicity, we set the discount rate, *r*, equal to zero as it plays less of a role now.

The scope for relational contracts is severely impaired in finite horizon settings due to the well-known unraveling problem. As a result, bonus pools become very effective because the agents will find it hard or impossible to collude. In fact, it is easy to show that bonus pools can then achieve first-best performance, under certain conditions, even in their simplest and most common form with conditional and symmetric payout. The reason is that, by backward induction, the two-period contracting problem collapses to a twofold repetition of a static bonus pool, in which the principal only needs to ensure (H,H) is a Nash equilibrium and, as such, is not Pareto-dominated from the agents' point of view by (L,L). Thus, conditional-symmetric bonus pools achieve first-best if and only if the agents' efforts are strategic substitutes, because in that case, whenever (H,H) constitutes an equilibrium, (L,L) does not.[20]

By the same logic, one would also expect that relational principal-agent contracting will become infeasible, again by backward induction. Surprisingly, however, this intuition turns out to be incomplete. Suppose that principal contracts with each agent individually. Then, in the second period, implicit contracting is not credible since there are no future cooperation benefits to the relationship. Hence, the principal pays each agent a bonus of $\beta_2 = \beta^*(\alpha_2 = 0) = \beta^{HH}$ conditional on $x_2 = 1$. Under strategic substitutes, the (H,H) equilibrium is unique in Period 2, so the agents have no threat to use against any promise by the principal in Period 1. Hence, under strategic substitutes, the principal uses only the verifiable team-based performance measure in both periods, with a total (over two periods and two agents) expected compensation cost of $2\overline{C}_t$. That is, the above intuition holds for efforts that are strategic substitutes: introducing a definitive end to the relationship at some future point makes implicit incentives altogether unsustainable.

Under strategic complements and the above contract, if the principal pays out $\beta_2 = \beta^*(\alpha_2 = 0) = \beta^{HH}$ conditional on $x_2 = 1$, then (L,L) will also be an equilibrium in Period 2. Hence, the agents can threaten to play this equilibrium if the principal does not honor the implicit promise she made for Period 1. Thus, the principal would need to increase the bonus to ensure that the (H,H) equilibrium is unique in case she has reneged on the implicit contract. This lends credibility to rewards tied to non-verifiable performance measures in Period 1 that are bounded by the following reneging constraint:

$$\alpha H \le p_H\left(\frac{H}{p - p_L} - \frac{H}{p_H - p}\right),$$

in analogy with the infinite horizon model. Denote by $\overline{\alpha}_1$ that value of α at which this Period-1 reneging constraint is binding. Then the optimal Period-1 weight on the non-

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[19] Generalizing the model to a (finite) T periods would not qualitatively alter the results in this section.

[20] Bonus pools coupled with either of the alternative payout policies analyzed in Section 4.2 achieve first-best performance for any p. Under either of these alternative policies, by setting $B \ge B^{HH}$ the principal not only makes (H,H) a Nash equilibrium in each period, but also precludes (L,L) as an equilibrium.

verifiable measure equals $\alpha_1^* = \min\{\bar{\alpha}_1, 1\}$ and the total expected compensation cost (over two periods and two agents) equals $\left(\frac{p_H - \alpha_1^* p}{p_H - p} + \frac{p_H}{p_H - p}\right) 2H < 2\bar{C}_t$, as α_1^* will be strictly positive. Moreover, as p decreases, α_1^* will increase and eventually reach one, in which case Period-1 contracting cost achieve the first-best level and total contracting cost (across two agents and two periods) equals $C_t^{FB} + \bar{C}_t$. Hence, once again, if the agents' actions are strategic complements, increasing the complementarity facilitates implicit contracting.

These findings are summarized in our next result.

Proposition 4. In the two-period setting:

(i) If the agents' actions are strategic substitutes, the non-verifiable individual measures will not be used.

(ii) If the agents' actions are strategic complements, the non-verifiable individual measures will be used in Period 1, and the use of implicit incentives is increasing in the degree of the complementarity.

To summarize, under strategic substitutes, the finitely-repeated contractual relationship behaves largely as a single-agent one because of the uniqueness of the equilibrium: with a finite horizon, relational contracts between the principal and a single agent unravel by backwards induction. Under strategic complements, the multiple equilibria that emerge in the last period facilitate implicit contracting in earlier periods.[21] Introducing a definitive end to the contracting relationship thus sharpens our predictions of Section 3 that effort complementarity is a key factor in facilitating implicit contracts.

6. Conclusion

Contracting relationships within firms are dynamic in nature, and they often involve technological interdependencies in that the firm (or division) output depends on actions taken by a number of productive agents. The expectation that the employment relationship endures, creates scope for implicit incentives. While the prior literature has focused mostly on the bright side of relational contracts, this paper argues that there is a dark side too: agent-

[21] Arya, Fellingham, and Glover (1997) study implicit side contracting and mutual monitoring among agents in a two-period model. The principal intentionally creates multiple equilibria in the agents' second-period

agent side contracting. As we have shown, bonus pools are hampered by the specter by collusion. If all parties are sufficiently patient and, thus, expect significant future gains to be had from current cooperation, then this threat of collusion will be severe enough that bonus pools are dominated by individual contracting arrangements whereby the principal contracts with each agent unilaterally and relies on her reputation to pay out rewards tied to non-verifiable performance measures. By focusing on static models, thus, the earlier literature has arrived at an overly optimistic outlook regarding the performance of bonus pools.

We have also shown that the greater the technological interdependency among agents' productive efforts, the stronger the principal's preference for individual contracts, all else equal. Such individual contracts are particularly effective if the agents' actions are strategic complements because then the agents can credibly threaten to play the shirking equilibrium in case the principal reneges on her promises. The cost this punishment would impose on the principal serves as a useful commitment device for the latter to remain truthful.

A simplifying assumption throughout this paper is that all players observe the non-verifiable performance measure equally and without noise. Arguably, in many settings the principal will observe the agents' effort only with noise and subjectively (i.e., her observation may not be commonly known), even though the agents can observe each other's effort perfectly. While a complete analysis of this extension is beyond the scope of the paper, it turns out that such observation noise reduces the relative performance of bonus pools even further. The reason is that the additional noise will strengthen the agents' incentive compatibility constraints without at the same time relaxing their collusion-proofness constraint (provided they observe each other's efforts perfectly). A formal analysis of imperfect measurement, both for bonus pools and individual contracts, seems an interesting avenue for future work.

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subgame to facilitate the implicit side contracting, whether the actions are strategic complements or strategic substitutes.

Appendix

Proof of Proposition 1: Recall the principal's reneging constraint for asks that are strategic complements, as stated in the main text:

$$\alpha H \leq \frac{1}{r}\left[p_H \beta^{LL} - (\alpha H + p_H \beta^*(\alpha))\right].$$

Plugging in $\beta^*(\alpha)$ and rearranging yields:

$$\alpha\left(r - \frac{p}{p_H - p}\right) \leq p_H\left(\frac{1}{p - p_L} - \frac{1}{p_H - p}\right). \tag{7}$$

The right-hand side of (7) is always positive for strategic complements, whereas the left-hand side is negative for $r < \dfrac{p}{p_H - p}$. In the latter case, the optimal incentive weights are $\alpha^* = 1$ and $\beta^* = 0$. For $r > \dfrac{p}{p_H - p}$ the left-hand side is also positive and we can rewrite the reneging constraint as follows:

$$\alpha \leq \bar{\alpha} \equiv \frac{p_H\left(\frac{1}{p - p_L} - \frac{1}{p_H - p}\right)}{r - \frac{p}{p_H - p}}.$$

The optimal α^* in the case of strategic complements therefore case is given by $\alpha^* = \min\{1, \bar{\alpha}\}$ and $\beta^* = (1 - \alpha^*)\beta^{HH}$. Lastly, it is easy to show that $\bar{\alpha} \geq 1$ if and only if $r \leq \dfrac{p_H - (p - p_L)}{p - p_L}$. □

Proof of Lemma 1: Consider any generic collusive strategy $\mathbf{a}_0$. Denote by $\mathbf{a}_t = \left\{a_\tau^A, a_\tau^B\right\}_{\tau=t}^{\infty}$ the continuation strategy at date *t*, and by $U_t^{cont}\left(B\middle|\mathbf{a}_t\right)$ the attendant continuation payoff for a representative agent in present value terms at date *t*. Under the collusive strategy $\mathbf{a}_0$, one of two cases can arise in any period *t:* (i) $a_t^A = a_t^B = L$ or (ii) $a_t^i = L$ and $a_t^j = H$, $j \neq i$. In case (i), collusion will be prevented, if and only if

$$U_{HL}^{C,S}(B) + \frac{U_{HH}^{C,S}(B)}{r} \geq U_{LL}^{C,S}(B) + U_{t+1}^{cont}\left(B\middle|\mathbf{a}_{t+1}\right). \tag{8}$$

Denote by $B_{(i)}$ the value of *B* at which this requirement holds with equality.

In case (ii), collusion-proofness requires *B* to be high enough such that

$$U_{HH}^{C,S}(B)+\frac{U_{HH}^{C,S}(B)}{r}\geq U_{LH}^{C,S}(B)+U_{t+1}^{cont}\left(B\middle|\mathbf{a}_{t+1}\right). \tag{9}$$

Evaluating this condition at $B=B_{(i)}$, we find that (9) will be satisfied at $B_{(i)}$, if and only if

$$U_{HH}^{C,S}(B_{(i)})-U_{LH}^{C,S}(B_{(i)})\geq U_{HL}^{C,S}(B_{(i)})-U_{LL}^{C,S}(B_{(i)}).$$

Simple algebra shows that this condition is equivalent to $p\leq p_o$. That is, if efforts are strategic complements, then the bonus pool amount required to prevent collusion in period *t* is higher if the agents collude on (L,L); whereas for $p>p_o$ it is higher if the agents in that period collude asymmetrically on (L,H) (or (H,L)).

In a last step, proceed recursively by applying similar arguments to period *t*-1 with $\mathbf{a}_t=\left\{\left(a_t^A,a_t^B\right)\cup\mathbf{a}_{t+1}\right\}$ where, by the preceding arguments, $\left(a_t^A,a_t^B\right)=\left(L,L\right)$ if $p\leq p_o$, and $\left(a_t^A,a_t^B\right)=\left(L,H\right)$ (or $\left(H,L\right)$) if $p>p_o$. Thus, for efforts that are strategic complements the binding collusion-proofness constraint is (8) (i.e., the agents adopt the *Shirk* strategy $\mathbf{a}_0^{SHK}$ as defined in the Lemma). For efforts that are strategic substitutes, on the other hand, the binding collusion-proofness constraint is (9). The last step required to establish that the agents will adopt the *Cycle* strategy $\mathbf{a}_0^{CYC}$ for $p>p_o$ is to note that all (infinitely many) collusive strategies in which $a_t^i=L$ and $a_t^j=H,\quad j\neq i$, yield the same aggregate payoff to the agents in present value terms. To break this collusive arrangement, the principal needs to set *B* high enough to induce that agent to break away who has the lowest payoff in present value terms, the agents will settle on that collusive strategy which yields the most symmetrical payoffs in present value terms, among those described by $a_t^i=L$ and $a_t^j=H,\quad j\neq i$, i.e., they choose *Cycle*. □

Proof of Proposition 3: As shown in the main text, for (H,H) to be an equilibrium *B* has to exceed $B^{HH}=\frac{2H}{p_H}$. At the same time (L,L) will *not* be an equilibrium, if and only if $U_{HL}^{C,S}\geq U_{LL}^{C,S}$, which is equivalent to $B\geq B^{LL}=\frac{2H}{2p-p_L}$. If $B<B^{LL}$, so that (L,L) is an equilibrium, then this equilibrium has to be Pareto-dominated by (H,H). Given the inherent symmetry among the agents, this amounts to:

$$U_{HH}^{C,S}\geq U_{LL}^{C,S}\quad\Longleftrightarrow\quad B\geq\frac{2H}{p_H-p_L}\equiv B^{Pareto}.$$

Collusion-proofness with regard to the *Shirk* strategy in Lemma 1 requires that

$$U_{HL}^{C,S}(B)+\frac{U_{HH}^{C,S}(B)}{r}\geq\frac{1+r}{r}U_{LL}^{C,S}(B) \quad\Longleftrightarrow\quad B\geq\frac{2(1+r)H}{p_H+2rp-(1+r)p_L}\equiv B^{SHK}.$$

Lastly, consider agent-agent side contracting via the *Cycle* strategy in Lemma 1. Taking Agent A to be the one to choose high effort in period 0 (without loss of generality), the respective agents' expected utilities in present value terms read:

$$\bar{U}_{0,A}^{cyc}(B) = \sum_{t=0,2,4,...}\frac{pB-H}{(1+r)^t} = \frac{(1+r)^2}{r(2+r)}(pB-H),$$

$$\bar{U}_{0,B}^{cyc}(B) = \sum_{t=1,3,5,...}\frac{pB-H}{(1+r)^t} = \frac{\bar{U}_{0,A}^{cyc}(B)}{1+r}.$$

Agent B realizes a lower payoff than Agent A due to time value of money-reasons. To prevent such *Cycle* collusion, the principal has to entice the "weak link" — i.e., Agent B — to break away from the side contract:

$$\frac{1+r}{r}U_{HH}(B)\geq\bar{U}_{0,B}^{cyc}(B) \quad\Longleftrightarrow\quad B\geq\frac{2(1+r)H}{(2+r)p_H-2p}\equiv B^{cyc}.$$

In the last step, it is a matter of straightforward algebra, holding constant (p_H, p_L), to derive cutoffs for *p* that permit a ranking of the relevant *B*-values; the three cases in Proposition 3 then follow. □

Proof of Corollary 4: Parts (i)-(iii) follow from straightforward comparisons of the per-period costs given in Propositions 1-3. Equating C_t^{sub} with $p_H B^{CYC}$ yields $r_{(i)}$ for substitutes; equating C_t^{comp} with $p_H B^{Pareto}$ yields $r_{(ii)}$ for strong complements; and equating C_t^{comp} with $p_H B^{SHK}$ yields $r_{(iii)}$ for weak complements, where

$$r_{(iii)}\equiv\frac{2pp_L-p_L^2+\sqrt{16p^3(p_H+p_L)-12p^4+p_L^4-4p^2p_H(p_H+5p_L)+4pp_L(p_H^2+p_Hp_L-p_L^2)}}{2(p-p_L)(3p-p_H-p_L)}.$$

Part (iv) is established by taking derivatives of $r_{(i)}$ and $r_{(ii)}$ with respect to p; the result that $r_{(iii)}$ is decreasing in p follows from the facts that C_t^{comp} is increasing p (Corollary 1) while B^{SHK} is decreasing in p. □

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