The Brag is Worse than the Fight: Pre-Announcing Competitive Decisions in Oligopoly Markets

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The Brag is Worse than the Fight: Pre-Announcing Competitive Decisions in Oligopoly Markets*

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Abstract

We examine a duopolistic setting in which firms pre-announce their future competitive decisions (e.g., production quantities, capacity investments, etc.) before they actually undertake them. We show that, whether competing with substitute or complementary products, firms overstate their future actions in their pre-announcements, and choose higher real actions than the ones they would choose absent pre-announcements. If products are substitutes, firms overstate their future actions trying to preempt their rivals and, if products are complements, they do so to facilitate collusion. However, in both types of competition, we find that social welfare is maximized when the tolerance for this overstating behavior is at an intermediate level.

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We also examine the equilibrium with discretionary pre-announcements and, in this analysis, we distinguish between sticky and non-sticky disclosure decisions. We find that, if firms compete with substitute products, there is a unique equilibrium in which both firms pre-announce regardless of whether disclosure decisions are sticky or not. If products are complements, however, the equilibrium is contingent on the stickiness of the disclosure decisions. If they are sticky, unless both firms have a low credibility, only the firm with the higher credibility pre-announces and the other remains silent. Nevertheless, when disclosure decisions are not sticky, both firms pre-announce in a unique equilibrium.

1 Introduction

Often firms disclose information about future strategic decisions such as a production plan or a capacity expansion. U.S. automobile makers such as General Motors and Ford, for instance, announce plans for their monthly production of cars through a leading industry trade journal up to six months ahead of actual production (Doyle and Snyder, 1999). Capacity expansions are also frequently pre-announced. For instance, several major firms in the automotive industry recently announced capacity expansions in their facilities in India: Ford announced the construction of a second plant at its Chennai facility in late 2010, and, in early 2011, Hyundai responded with the announcement of an injection of around $325 million in its facility and General Motors disclosed its intention to invest half a billion dollars to expand its multiple productive locations in India. Although current or potential investors constitute an audience for which this kind of disclosures is intended, competitors are another audience that all firms take very much into consideration. Empirical evidence shows that firms take pre-announcements of future competitive decisions made by competitors very seriously, and consider them in making their own competitive decisions (Doyle and Snyder, 1999; Gilbert and Lieberman, 1987; Christensen and Caves, 1997). Our goal in this paper is to shed some light on the incentives that lead firms to make such pre-announcements, and
on how they affect the firms’ real decisions.

The extant research literature has examined exhaustively the disclosure of private information about some state of nature or about some real action taken in the past. However, the pre-announcement of future real actions has not been given much attention. In this paper, we abstract from information about some state of the world to focus instead on the disclosure of future real actions. We analyze the mutual effects that such pre-announcements have on the subsequent real actions of both the disclosing firm and its competitors. Specifically, we examine an oligopolistic Cournot setting in which firms simultaneously preannounce their future competitive actions before they actually undertake them. Pre-announcements do not need to be completely accurate, but it is costly for the firm to perform a real action that differs from what it pre-announced previously. This deviation cost can be interpreted either as the disutility associated to “breaking a promise,” as a reputational cost, as a litigation cost, or as a regulatory punishment. Nevertheless, the more costly it is to deviate from a pre-announced action, the more credible it is the pre-announcement.

The contribution of this paper is threefold. First, we find that the competitive environment leads firms to overstate their future strategic actions. Firms “brag” about their intentions in order to influence their competitors’ subsequent real actions. Since this bragging is costly, firms become compelled to increase their own real actions to mitigate the deviation cost. This result is descriptive of the empirical evidence about overstated production plan pre-announcements in the automotive industry (Doyle and Snyder, 1999) and the documented frequent downward deviation from pre-announced capacity expansions (Gilbert and Lieberman, 1987; Christensen and Caves, 1997). Moreover, our model predicts that this result is robust to the nature of the competition. Whether the products are strategic substitutes or strategic complements, pre-announcements over-

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1 Private information still plays important roles in firms’ disclosure in reality. However, in this paper we focus on whether there are other effects besides those from private information, and exclude private information about some state of nature to streamline our analysis.
state future real actions. Nevertheless, the intuition behind these results differs with the nature of the competition. In a market in which firms compete with strategic substitute products, firms overstate their future actions to intimidate their competitors and drive their competitors’ real actions down. Firms get involved in an overstating contest to become the Stackelberg leader, and that results in higher real equilibrium actions. If products are strategic complements, however, firms brag about their future actions to induce competitors into taking higher actions as well. This way, all firms benefit from higher demands and higher profits. Here pre-announcements are intended to foster collusion.

The second main contribution is of a normative nature, and pertains to the welfare implications of pre-announcements. We find that, regardless of the nature of the competition, real strategic actions are maximized at an intermediate level of tolerance for “bragging.” Furthermore, we find that social welfare is maximized at that same intermediate level of tolerance, regardless of whether firms compete with strategic substitute or complementary products. Intuitively, on one extreme, if bragging is free, pre-announcements are not credible and cannot affect real actions. Hence, the market competition reduces to a simple Cournot duopoly. On the other extreme, if bragging is prohibitively expensive, firms pre-announce their true intentions and, therefore, the setting becomes again a simple Cournot duopoly in which everything is decided at the pre-announcing stage. For intermediate levels of tolerance, however, firms are credible enough to influence their competitor’s future real actions, and are flexible enough to deviate from their pre-announced intentions. As a result, firms have an incentive to overstate their pre-announcements to affect their competitors’ real actions but, later on, feel compelled to increase also their own real actions to mitigate their deviation cost. This upward distortion of quantities/capacity investments increases welfare for consumers who can enjoy higher consumption at lower prices.

Our third main contribution concerns the firms’ discretion to pre-announce or not their future
real actions. We analyze a generalized game in which firms not only decide the content of their pre-announcements but also decide whether to pre-announce at all. Henceforth, we refer to the former decision as a *content* decision and to the latter decision as a *disclosure* decision. We also distinguish between *sticky* and *non-sticky* disclosure decisions. If disclosure decisions are sticky (that is, they persist over time), they become publicly observable before the firms decide the content of their pre-announcements. On the contrary, if disclosure decisions are non-sticky, firms make their disclosure and content decisions concurrently. We show that the effect of the disclosure stickiness on the equilibrium depends on the nature of the competition. When firms compete with substitute products, the stickiness of disclosure decisions does not affect the equilibrium; all firms pre-announce and overstate their future real actions. In a market of complementary products, however, the equilibrium does differ. If disclosure decisions are sticky, the more credible firm (the one with a higher deviation cost) pre-announces while the other remains silent. On the other hand, if disclosure decisions are non-sticky, firms share the collusive effort by pre-announcing together. The relevance of this last result becomes apparent if it is compared with the extant literature on endogenous Stackelberg leadership. While other models obtain multiplicity of equilibria (Van Damme and Hurkens, 1999; Van Damme and Hurkens, 2004), this paper obtains uniqueness by virtue of the partial commitment that pre-announcements provide.

Section 2 provides a review of related studies. We examine a duopolistic setting in which firms pre-announce their future actions in Sections 3 and 4. In Section 3, we investigate the simultaneous pre-announcement setting and analyze the social welfare implications. In Section 4, we expand the model and introduce asymmetric credibilities. In Section 5 we further generalize the model by endogenizing the disclosure decisions. We examine both the case in which disclosure decisions are sticky and the case in which they are non-sticky. Section 6 concludes the paper.
2 Literature Review

This paper is related to the extensive economics and accounting literature that examines the product market effects of discretionary disclosures (among others, Bhattacharya and Ritter, 1983; Darrough and Stoughton, 1990; Wagenhofer, 1990; Ziv, 1993; Gigler, 1994; Dye and Sridhar, 1995; Hwang and Kirby, 2000; Vives, 2006). Several recent papers focus on different aspects of disclosure decisions in a competitive environment. Arya and Mittendorf (2007), for instance, illustrate how firms’ incentives to withhold private information from competitors are undercut by the fact that disclosures also boost analyst following, which provides firms with new information about market conditions. Arya, Frimor, and Mittendorf (2010) study the discretionary disclosure of proprietary information by multi-segment firms and find that the optimal disclosure aggregates segment details. Bagnoli and Watts (2010) examine a Cournot competition setting in which firms can misreport their production costs. Our paper is related to these studies in the sense that we also investigate discretionary disclosure decisions in a product competition setting. Nevertheless, we depart from this literature by assuming that the state of nature is common knowledge and, instead, disclosures contain information about future real competitive actions. Therefore, in our model disclosures do not affect beliefs about some state of nature but, instead, they affect conjectures about future actions. In addition, we also depart from this literature in the way it examines ex-ante incentives to disclose. In some of this literature (Darrough, 1993; Pae, 2002; among others,) it is typical to allow firms to pre-commit to a common disclosure policy. Instead, we do not allow for commitment between competitors. In our model, if disclosure decisions are sticky, firms pre-commit publicly to either disclose or to remain silent, but they do so simultaneously and strategically.

In the extant literature, the analysis of the effect of pre-announcements in a competitive product market setting has been limited to the introduction of a cheap talk stage before the market real action competition. In most cheap talk studies, an informed player uses communication to convey
some private information about some state of nature to an uninformed player. This role of cheap talk was first examined by Crawford and Sobel (1982) and Green and Stokey (1980), and has been applied to a variety of accounting settings (Newman and Sansing, 1993; Gigler, 1994; Stocken, 2000; among others). Absent private information, however, cheap talk communication between players is reduced to a means of coordination among multiple possible equilibria of the underlying game (Farrell, 1987; Rabin, 1994; Park, 2002). Therefore, in games with a unique equilibrium and no private information, cheap talk communication cannot affect real actions. This is the case in the Cournot competition settings studied in this paper. Departing from cheap talk settings, we assume that firms incur a cost if they deviate from a previously pre-announced action. Hence, as opposed to playing a coordination role, communication affects real actions directly even if the underlying game has a unique equilibrium. Moreover, this direct effect allows us to predict empirically consistent characteristics of pre-announcements, such as the fact that they overstate future real actions (Doyle and Snyder, 1999; Gilbert and Lieberman, 1987; Christensen and Caves, 1997).

Our study is also related to the literature on pre-commitment in oligopoly markets. In a seminal paper, Fershtman and Judd (1987) examine the strategic pre-commitment effect of delegation through observable labor contracts. Hughes and Kao (1997) and Hughes, Kao and Williams (2003) extend the study to the pre-commitment provided by forward contracts. More recently, Hughes and Williams (2008) analyze the trade-off between the strategic and non-strategic effects of a broad range of commitment devices. We extend this stream of literature by examining pre-announcements of future competitive actions as strategic partial pre-commitments. Unlike the commitment devices examined so far by this stream of literature, pre-announcements in our model are not exogenously costly. They become costly only if the firm chooses a real action that deviates from the pre-announced one.\footnote{Hughes and Williams (2008) analyze a broad range of commitment devices by assuming that their equilibrium payoff is constant across the domain of commitment device choices. This restriction does not apply to our setting.} Moreover, because pre-announcements are disclosure decisions, we further depart
from the previous literature in extending our focus to discretionary pre-announcements, and distin-
guishing in this analysis between sticky and non-sticky disclosures.

Our study is also related to some studies on the endogenous order of moves in duopoly games. Indeed, in our model, the decision to pre-announce can be regarded as an attempt to be the first mover. Hamilton and Slutsky (1990) originated this stream of literature examining a two-period duopolistic setting in which players choose to either act in a first period or wait until a second period having observed the competitor’s first period choice. In fact, their model can be regarded as a particular case of our setting in which the firms choose to either pre-announce truthfully or remain quiet. Following Hamilton and Slutsky’s work, Van Damme and Hurkens (1999) and Van Damme and Hurkens (2004) study which firm will emerge as the endogenous leader in a duopoly game in both Cournot competition and Bertrand competition. These studies find multiple equilibria and apply some coordination criteria to obtain uniqueness. Our model differs from this literature in its focus and also in a fundamental assumption: in our model, moving first is not a full commitment; it is simply a pre-announcement that can be followed more or less accurately with the real action. This assumption, surprisingly, produces a unique equilibrium in a setting in which previous studies in this literature need to resort to a complex application of the risk dominance criterion to select a single equilibrium.

Finally, it is worth mentioning that even though this paper analyzes oligopoly competition in a dynamic setting, the nature of the equilibria differs from the ones obtained in the literature on repeated oligopoly games, such as in Friedman (1971) and Rotemberg and Saloner (1986). This line of literature demonstrates that firms can achieve colluding profits by the threat of reverting to competitive strategies in future games if a firm deviates. In our two-stage model, however, there is

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because the equilibrium cost of a pre-announcement varies with the pre-announced action. They also extend their model relaxing the restriction, and assuming that the commitment device payoff varies linearly with the real action. In our paper, however, we have a convex deviation cost from the pre-announced real action.
no future punishment from competitors. In fact, in our substitutes competition, pre-announcements do not induce collusion. On the contrary, they exacerbate competition by involving firms in an overstating contest to obtain a first mover advantage.

3 Pre-Announcement Model

3.1 The Basic Setting

We examine a duopolistic setting in which firms simultaneously take competitive decisions. Nevertheless, before they take those decisions, firms pre-announce them publicly. The competitive decisions can be understood either as production quantities or as capacity investments. For simplicity, we use quantity competition (Cournot) as our main scenario, but our analysis can be interpreted in a broader way.³ In this section and Section 4, we begin the analysis by exogenously imposing that both firms always pre-announce their future real actions. Later in Section 5, we endogenize the decision to pre-announce and examine the equilibrium when pre-announcements are voluntary.

We denote the pre-announcement of firm $i$ by $a_i \in \mathbb{R}_+$, and the corresponding real action by $x_i \in \mathbb{R}_+, i \in \{1, 2\}$. The payoff function of a firm $i$ is given by the following expression:

$$
\Pi_i(x_i, x_j, a_i) = x_i(h - x_i + nx_j) - g_i(a_i, x_i) \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j. \quad (1)
$$

The first term in this expression is the usual profit from product market operations, in which the marginal cost of action has been normalized to zero. If the parameter $n$ is negative (that is, $n \in (-1, 0)$), then $x_i$ and $x_j$ are production quantities of market products that the consumers see as substitutes, and the expression in parenthesis is the inverse demand for the product of firm $i$

³Notice that, by simply interpreting $x_i$ and $x_j$ as prices and the expression in parenthesis as the demand for the product of firm $i$, these same settings can also be understood as a Bertrand game with complementary prices if $n$ is negative (corresponding to the case with substitutes) and a Bertrand game with substitute prices if $n$ is positive (corresponding to the case with complementary products).
(that is, the market price for the product of firm \(i\) as a function of the quantities of both products).

We refer to this setting as \textit{substitutes competition}. Conversely, if \(n\) is positive (that is, \(n \in (0,1)\)), then \(x_i\) and \(x_j\) are production quantities of two firms with complementary products. We refer to this setting as \textit{complements competition}. Regardless of its sign, \(n\)’s absolute value is never greater than 1. That is, the competitor’s influence on the inverse demand is always smaller than the direct influence from the firm’s own action.

The second term in the profit expression is the cost the firm incurs if its real action, \(x_i\), differs from the pre-announced one, \(a_i\) (we will often refer to the difference, \(x_i - a_i\), as a \textit{deviation}, and to the associated cost as a \textit{deviation cost}). The functional form of this deviation cost is assumed to be given by:

\[
g_i(a_i, x_i) = \begin{cases} 
\frac{w}{2} (x_i - a_i)^2 & \text{if firm } i \text{ pre-announces} \\
0 & \text{if firm } i \text{ does not pre-announce}
\end{cases}
\]  

(2)

If a firm \(i\) discloses a pre-announcement but its subsequent real action deviates from the pre-announced one, it incurs a cost that is quadratic in the deviation and is characterized by the convexity parameter, \(w \in (0, \infty)\), that we call \textit{credibility}. If a firm does not pre-announce, the deviation cost is zero. The deviation cost can be interpreted as distaste for “breaking a promise,” a reputational loss, a potential litigation cost, or a punishment enforced by a regulator.

The time line of the model is shown below. This is a dynamic game with two stages. In the first stage, both firms simultaneously pre-announce their future actions and, in the second stage, they choose their real action levels. At date 3, markets clear and firms obtain their profits.
Firms simultaneously pre-announce real actions $x_1, x_2$ and firms future actions, $a_1, a_2$. simultaneously. obtain profits

Time line: Pre-announcement model

In this setting, there is no private information, and firms take decisions simultaneously. In deriving the equilibrium, it is enough to rely on the notion of Subgame Perfect Nash Equilibrium. Therefore, the game can be solved by backward induction. The second stage is essentially a duopoly game with simultaneous competitive decisions. At that stage, however, the pre-announcements made in the first stage are public knowledge. Therefore, each firm $i$ maximizes its profit by choosing its action $x_i$, taking into account the previous pre-announcements, $a_i$ and $a_j$. Since firms choose real actions simultaneously, each firm $i$ must also make a conjecture about its competitor’s action $\hat{x}_j(a_i, a_j)$. Observe that this conjecture depends on the pre-announcements of both firms. The program that each firm $i$ solves is:

$$\max_{x_i} \Pi_i(x_i, \hat{x}_j(a_i, a_j), a_i) \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j.$$ \hspace{1cm} (3)

The corresponding first order conditions for these programs are:

$$h + w a_i + n \hat{x}_j(a_i, a_j) - (2 + w) x_i = 0 \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j.$$ \hspace{1cm} (4)

In equilibrium, conjectures are true. That is, $\hat{x}_i(a_i, a_j) = x_i = x_i^*(a_i, a_j)$. Therefore, the first order conditions in (4) yield a system of equations that we can solve to obtain the subgame equilibrium.
actions:

\[ x^*_i(a_i, a_j) = \frac{(2 + w)(h + wa_i) + n(h + wa_j)}{(2 + w)^2 - n^2} \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j. \] (5)

At date 1, firms choose their pre-announcements simultaneously. Each firm \( i \) chooses its pre-announcement \( a_i \) anticipating the effects on the subsequent market equilibrium actions, and making a conjecture about the other firm’s pre-announcement, \( \widehat{a}_j \). The equilibrium pre-announcements solve the programs,

\[
\max_{a_i} \Pi_i(x^*_i(a_i, \widehat{a}_j), x^*_j(\widehat{a}_j, a_i), a_i) \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j.
\] (6)

By obtaining the first order conditions from firm \( i \)'s program with respect to \( a_i \) for \( i \in \{1, 2\} \), and setting conjectures to their true equilibrium values, \( \widehat{a}_i = a_i = a^*_i \) for \( i \in \{1, 2\} \), we obtain a system of equations that can be solved to obtain the equilibrium pre-announcements, \( a^*_i \). Finally, we use the expressions for \( a^*_i \) to calculate the expressions for the equilibrium real decisions \( x^*_i(a^*_i, a^*_j) \). Proposition 1 states the expressions for the equilibrium real actions and corresponding pre-announcements in a conveniently abridged form:

**Proposition 1** When both firms pre-announce their future real actions, the equilibrium real actions and pre-announcements have the following expressions:

\[
x^*_i \equiv x_o = \frac{h}{2 - n[1 + \lambda_o]},
\]

\[
a^*_i \equiv a_o = \frac{n\lambda_o}{w} x_o,
\]

where \( \lambda_o = \frac{\partial x^*_j(a_j, a_i)}{\partial a_i} = \frac{nw}{(w + 2)^2 - n^2} \) for \( i, j \in \{1, 2\}, i \neq j. \)

The above results are in closed form but they are expressed, for brevity and clarity of intuition,
as functions of a partial derivative, \( \lambda_o = \frac{\partial x^*_j(a_i, a_j)}{\partial a_i} = \frac{\partial x^*_j(a_j, a_i)}{\partial a_j} \). In words, \( \lambda_o \) denotes the marginal effect of one firm’s pre-announcement on its competitor’s real action. We often refer to \( \lambda_o \) as the influence of one firm on the other firm.

From Proposition 1, one can directly derive the expression for the deviation from the pre-announcement, \( x^*_i - a^*_i \). This expression and its comparative statics properties are formally stated in the following proposition:

**Proposition 2** The equilibrium deviation, \( x^*_i - a^*_i = -\frac{n}{w} \lambda_o x^*_i \), is always negative for all \( n \in (-1, 0) \cup (0, 1) \) and all \( w \in (0, \infty) \), where \( i, j \in \{1, 2\}, i \neq j \).

The magnitude of the deviation of firm \( i \), \( |x^*_i - a^*_i| \), is monotonically decreasing in the credibility \( w \), for all \( n \in (-1, 0) \cup (0, 1) \) and all \( w \in (0, \infty) \), where \( i, j \in \{1, 2\}, i \neq j \).

Notice that the size of the deviation is proportional to both the size of the real action, \( x^*_i \), and the influence of one firm on the other firm, \( \lambda_o \). The influence is multiplied by \( n \), making the product \( n \lambda_o \) always positive and, thus, making the deviation always negative. Since the deviation is always negative, that is, the pre-announcement is always “overstating,” we will often refer to the act of releasing an overstating pre-announcement as “overstating” or “bragging.” One can think of the product \( n \lambda_o \) as the incentive of one firm to overstate its future action, which always increases with \( w \). Nevertheless, an increase in credibility also implies a higher cost of overstating, and that makes the overall effect of a credibility increase on the deviation negative. More specifically, the deviation is always negative but its magnitude decreases towards zero as \( w \) goes to infinity. It is also worth mentioning that as \( w \) approaches 0, the deviation is still finite and positive. This is intuitive because, even though the cost of overstating goes to zero, the influence of one firm on the other firm and, therefore, the incentive to brag, also goes to zero.

The incentives for bragging are intuitively different between substitutes and complements com-
petitions. In substitutes competition, a firm overstates its future action to intimidate its rival. It threatens to choose a high action level with the purpose of inducing the rival to choose a low action level. Thus, one could see the overstatement as an attempt to become the Stackelberg leader. Both firms try to condition each other’s future actions to the extend that their credibilities allow them. In complements competition, however, each firm overstates its future action to induce the competitor to choose a higher action level. This, in turn, favors the overstating firm because the competitor’s production of a complementary product increases the demand of its own products. One could see the addition of a pre-announcing first stage to a substitutes competition setting as exacerbating competition and, its addition to a complements competition as taming competition, or facilitating collusion.

### 3.2 Analysis of Social Welfare

In this section, we examine the welfare implications of pre-announcements. With this purpose in mind, it is useful to analyze how the real action, $x_0$, varies with the credibility, $w$. From Proposition 1, we can derive that the effect of the credibility in the equilibrium market decisions adopts a non-monotonic pattern. Notice first that when $w$ tends to zero, the influence, $\lambda_o$, also tends to zero and, hence, $x_o$ tends to $\frac{h}{2-n}$. This expression is just the Nash equilibrium quantity in a pure duopoly setting with simultaneous play (henceforth, referred to as the pure simultaneous duopoly). Indeed, with zero credibility, pre-announcements become irrelevant, and the game is reduced to the second stage, which becomes a pure simultaneous duopoly. At the opposite extreme, in a market with infinitely credible firms (i.e. $w \rightarrow \infty$), the influence, $\lambda_o$, also becomes zero. In this limit, both firms are forced to announce their true future actions. A pre-announcement fully commits a firm to a...

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4 This also applies to a Bertrand game with complementary products. In such a setting, product prices are substitutes. Thus, firms will overstate their future prices attempting to be the Stackelberg leader.

5 This can be applied to a Bertrand game with substitute products. In such setting, prices are complements and, therefore, by overstating the future price, one firm tries to induce the other firm to set a high price too, so that both can reap collusion benefit.
Figure 1: $\lambda_o$ as a function of $w$ in complements competition.

specific action and, therefore, that action cannot be affected by the competitor’s pre-announcement. The game is then reduced to the first stage, the pre-announcing stage. After that, there is no other decision to make. Thus, as $w$ approaches infinity, the equilibrium also converges to the pure simultaneous duopoly equilibrium. For any intermediate credibility level, however, firms have some credibility, and are not fully constrained by their pre-announcements. Their pre-announcements can influence each other’s future actions (the influence, $\lambda_o$, is non-zero). Therefore, firms have an incentive to overstate their pre-announcements and, as a result, they become compelled to increase their own actions to mitigate their deviation costs. Figure 1 plots the influence $\lambda_o$ as a function of $w$, and Figure 2 plots the pre-announcement $a_o$ and the real action $x_o$ in complements competition as a function of $w$ (the analogous graph for substitutes competition is very similar.)

In complements competition, the influence that firms have on each other, $\lambda_o$, is a single-peaked function of the credibility, $w$, with its maximum at an intermediate level of credibility, $w_o = \sqrt{4 - n^2}$. Thus, $\lambda_o$ ranges between zero and $\lambda_o(w_o)$. For these values, the equilibrium real action $x_o$ is a monotonic function of $\lambda_o$. Therefore, $x_o$ is also a single-peaked function of $w$ with its maximum

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at \( w_o \). In substitutes competition, \( \lambda_o \) is negative and has a minimum at \( w_o \). However, since in the expression for \( x_o \), \( \lambda_o \) is multiplied by a negative \( n \), \( x_o \) is also single-peaked and maximized at \( w_o \).

These facts are summarized in the following lemma:

**Lemma 1** If firms pre-announce their future real actions, the resulting equilibrium real actions are higher than the ones in a pure simultaneous duopoly equilibrium for any \( w \in (0, \infty) \). Moreover, the real actions of both firms are single-peaked functions of \( w \) with a maximum at \( w_o = \sqrt{\frac{4}{n^2}} \) and tend to the pure simultaneous duopoly equilibrium strategy as \( w \to 0 \) and as \( w \to \infty \).

For a small \( w \), bragging is almost free. Firms overstate their pre-announcements a lot, and they only feel compelled to increase their real future actions slightly above those in a pure simultaneous duopoly equilibrium. As \( w \) gets larger, a firm has a stronger incentive to overstate because a more credible pre-announcement has a larger impact on the competitor’s real action. However, a higher \( w \) also makes overstating more costly and, therefore, the firm increases its real action level to reduce the overstating cost. Beyond \( w_o \), the benefit from overstating declines because the pre-announcement becomes a more rigid commitment and, therefore, it is harder to affect the
competitor’s real action. Consequently, firms overstate their pre-announcements less and, thus, keep their action levels lower.

We have illustrated that the real action level follows an inverse U-shape pattern, and now we provide a social welfare analysis by examining how the credibility of the pre-announcements affects two measures of welfare: the total surplus and the consumer surplus. Following the classical work by Singh and Vives (1984), we assume a representative consumer that maximizes a utility of the form:

$$ U(x_1, x_2) = \alpha_1 x_1 + \alpha_2 x_2 - \frac{1}{2}(\beta_1 x_1^2 + 2\gamma x_1 x_2 + \beta_2 x_2^2). $$

Here, $x_i$ is the quantity produced by firm $i$, and $\alpha_i, \beta_i$ and $\gamma$ are positive utility parameters. With this assumptions, $U(x_1, x_2)$ coincides with the total surplus,\(^6\) and $U(x_1, x_2) - p_1 x_1 - p_2 x_2$ is the consumer surplus, where $p_i$ is the price of the product of firm $i$. Henceforth, we denote total surplus and consumer surplus by $ts$ and $cs$ respectively.

For a quantity (Cournot) competition, conforming the utility parameters in $U(x_1, x_2)$ to those in our model yields the values: $\alpha_1 = \alpha_2 = h$, $\beta_1 = \beta_2 = 1$ and $\gamma = -n$ ($n < 0$). With this parametrization, in both substitutes and complements competitions, the welfare measures are:

$$ ts = 2hx_o - (1 - n)x_o^2, $$
$$ cs = (1 - n)x_o^2. $$

In the range of values that $x_o$ can adopt, all the welfare measures stated above are monotonic transformations of $x_o$. The surplus expressions are both increasing and, hence, they inherit the...\(^6\)When the cost of bias is interpreted as a litigation cost or a regulatory penalty, the cost is ultimately a transfer to other parties in the economy (as opposed to a dead-weight-loss) and, therefore, does not need to be considered in the calculations.
single-peaked shape of $x_0$ as a function of $w$ with a maximum at the same level of credibility, $w_o$.

The similar single-peaked shape of both surplus measures suggests that imposing truthfulness in pre-announcements does not maximize welfare. Instead, welfare is maximized at an intermediate level of credibility, in both substitutes and complements competitions. These results are summarized in the following proposition:

**Proposition 3** In both substitutes competition and complements competition ($n \in (-1, 0) \cup (0, 1)$), total surplus and consumer surplus are maximized at an interior credibility point $w_o = \sqrt{4 - n^2}$.

In markets characterized by substitutes competition, some tolerance for overstatement in pre-announcements induces firms into a Stackelberg warfare. This results in increased production quantities and, as a result, it increases social welfare. Moreover, even in a market with complements competition where firms overstate future actions to facilitate collusion, social welfare maximizes again with some tolerance for “bragging.” In such a market, overstatement in pre-announcements also results in higher quantities produced and, consequently, lower prices for consumers. However, while in a substitutes market overstated pre-announcements exacerbate competition and drive firms’ profits down, in a complements market, they foster collusion and drive profits up. Figure 3 illustrates how the consumer surplus changes with $w$ in a quantity competition with complementary products.\(^7\)

### 4 Asymmetric Credibility

So far, we have assumed that firms are completely symmetric. To relax this assumption, we generalize the model to the case in which firms have different credibilities in their pre-announcements.\(^7\)

\(^7\)The total surplus of complements competition and the corresponding welfare measures for substitutes competition are similar.

\(^8\)The social welfare implication is different for a Bertrand game. In a Bertrand game, total surplus and consumer surplus are minimized at an interior credibility point, $w_o = \sqrt{4 - n^2}$. Detailed analysis is available upon request.
In the real world, the credibility of a firm may be affected by multiple firm-specific factors and may be contingent on the firm’s history. However, in this paper we introduce this asymmetry in a parsimonious way by assuming that firms incur a different cost in deviating from a pre-announced action. That is, we exogenously assume that the credibility of firm $i$, $w_i$, is different from the credibility of firm $j$, $w_j$. Following an analogous derivation to the one in Section 3, but introducing the asymmetry through the deviation cost $g_i(a_i, x_i)$,

$$
g_i(a_i, x_i) = \begin{cases} 
\frac{w_i}{2}(x_i - a_i)^2 & \text{if firm } i \text{ pre-announces} \\
0 & \text{if firm } i \text{ does not pre-announce}
\end{cases}
$$

we can obtain the expressions for the equilibrium pre-announced and real actions, and the equilibrium payoffs, for firms with asymmetric credibilities. We use the superscript $T$ to identify the equilibrium results in this setting, which are summarized in the following proposition:9

---

9If we introduce uncertainty into the model, then pre-announcing future production quantities may limit firms’ production flexibility ex post. Consider, for instance, a setting in which the demand parameter $h$ is uncertain, and all firms share common prior beliefs about its distribution. In that case, the resulting expressions for the equilibrium pre-announced and real actions are:
Proposition 4 When both firms pre-announce their future actions with credibility levels \( w_i \) and \( w_j \), the equilibrium pre-announcements and real actions have the following expressions:

\[
x_i^T = h \frac{n+(2-n\lambda_{ji}^T)}{(2-n\lambda_{ij}^T)(2-n\lambda_{ji}^T)-n^2}, \quad a_i^T = \frac{n\lambda_{ij}^T+w_i}{w_i}x_i^T,
\]

where \( \lambda_{ji}^T = \frac{\partial x_i^*(a_i, a_j)}{\partial a_i} = \frac{nw_j}{(2+w_j)(2+w_i)-n^2} \)

for \( i, j \in \{1, 2\}, i \neq j \).

Furthermore, firms obtain the following payoffs:

\[
T_i = \frac{2+w_i}{2}(a_i^T)^2 - \frac{w_i}{2}(a_i^T)^2,
\]

for \( i \in \{1, 2\} \).

Notice that we denote as \( T_i \) the firm \( i \)'s equilibrium payoff when both firms “talk” about their future plans. The above results are in closed form but they are expressed, for brevity and clarity of intuition, as functions of two partial derivatives, \( \lambda_{ji}^T = \frac{\partial x_i^*(a_i, a_j)}{\partial a_i} \) and \( \lambda_{ij}^T = \frac{\partial x_i^*(a_j, a_i)}{\partial a_i} \). Similar to Section 3, \( \lambda_{ji}^T \) denotes the marginal effect of firm \( i \)'s pre-announcement on its competitor’s real production, which we refer to as the influence of firm \( i \) on firm \( j \). Observe that a firm \( i \)'s real action, \( x_i \), is only contingent on the credibility parameters \( w_i \) and \( w_j \) through the two influences, \( \lambda_{ji}^T \) and \( \lambda_{ij}^T \). Therefore, credibilities only affect real productions to the extend that firms can influence each other (i.e. \( \lambda_{ji}^T \) and \( \lambda_{ij}^T \) are non-zero). It is apparent that the absolute value of the influence of firm \( i \) on firm \( j \), \( \lambda_{ji}^T \), is increasing in firm \( i \)'s credibility, \( w_i \), and monotonically decreasing in firm \( j \)'s credibility, \( w_j \). Intuitively, as \( w_i \) increases, the pre-announcement of firm \( i \) becomes a stronger commitment to actually perform the declared production and, therefore, it has a larger impact on the competitor’s real production. In contrast, as \( w_j \) increases, it is the competitor’s pre-announcement that becomes a stronger commitment, making its real production harder to be
affected by firm $i$’s pre-announcement.

To examine the effect of the credibilities on the equilibrium real actions, it is useful to take the derivative of $x_i^T$ with respect to $w_i$:

$$
\frac{\partial x_i}{\partial w_i} = \frac{\partial x_i}{\partial \lambda_{ij}} \frac{\partial \lambda_{ij}}{\partial w_i} + \frac{\partial x_i}{\partial \lambda_{ij}} \frac{\partial \lambda_{ij}}{\partial w_i}.
$$

(8)

We can see that this derivative is composed of two terms. These two terms reflect the effect of firm $i$’s credibility $w_i$ on its own real action $x_i$ through two different influences. In the first term, this effect is channeled though the influence that firm $i$’s pre-announcement has on the competing firm $j$’ real action. In the second term, the effect is conducted through the influence that the competing firm $j$’s pre-announcement has on firm $i$’s own real action. In substitutes competition, both terms are positive. Indeed, the first term tells us that, with a higher credibility, firm $i$ has more influence on its competitor. Therefore, firm $i$ has a higher incentive to pre-announce a high production, which, subsequently, compels firm $i$ to actually produce more to mitigate the deviation cost. The second term indicates that, when firm $i$ enjoys a higher credibility, firm $j$ has a smaller influence on firm $i$’s real action. Therefore, firm $i$ will be less intimidated by firm $j$’s pre-announcement and, thus, will be able to produce a higher quantity. Overall, the two terms go in the same direction in substitutes competition. The more credible a firm is, the more it will produce in equilibrium.

The analysis is not as straightforward in complements competition. In this case, the first term is positive, but the second term is negative. The intuitive interpretation of the first term is similar to that in substitutes competition. That is, the more credible firm $i$ is, the more it can influence firm $j$’s production. Therefore, firm $i$ is willing to overstate more its pre-announcement and, thus, it increases its own production to mitigate its deviation cost. However, the second term intuition is now different. A more credible firm $i$’s real action is less influenced by firm $j$’s pre-announcement. Therefore, firm $j$ is less willing to overstate its pre-announced production and, since now firms
compete with complementary products, firm $i$ reacts by reducing its real production. The trade-off between these two terms determines the overall effect of a credibility increase on the real production. The relative size of the two terms is, in fact, related to the competitor’s credibility, $w_j$. When the competitor, firm $j$, is very credible, its real production is less flexible and, thus, it can not be affected much by firm $i$’s pre-announcement. In this case, the positive effect reflected in the first term is relatively small, and is dominated by the negative effect represented by the second term. On the other hand, if firm $j$ has a sufficiently low credibility, the positive effect of the first term dominates and firm $i$’s action level increases with its credibility $w_i$. The following corollary formally states these comparative statics results:

**Corollary 1** In substitutes competition ($n \in (-1,0)$), the equilibrium production of firm $i$, $x_i^T$, is monotonically increasing in firm $i$’s credibility $w_i$.

In complements competition ($n \in (0,1)$), there exist thresholds $\underline{w}, \overline{w} \in (0, \infty)$ such that,

- for $w_j < \underline{w}$, the production of firm $i$, $x_i^T$, is monotonically increasing
  - in firm $i$’s credibility $w_i$ and,

- for $w_j > \overline{w}$, the production of firm $i$, $x_i^T$, is monotonically decreasing
  - in firm $i$’s credibility $w_i$.

5 Discretionary Disclosure

Up until this point, we have analyzed the model by exogenously imposing that both firms pre-announce. In this section, we generalize the model by allowing firms to choose whether to pre-announce or not. We distinguish between the decision to pre-announce or not and the decision of what to pre-announce. Henceforth, we refer to the former as a disclosure decision, and to the latter as a content decision.

The stickiness of the disclosure decisions is important. If disclosure decisions are sticky, past dis-
closure decisions constrain current disclosure decisions. Thus, when firms reach the pre-announcing stage, they know with some degree of certainty whether other firms will pre-announce or not. That is, the disclosure decisions are observable to competitors. On the contrary, if disclosure decisions are not sticky, every time a firm pre-announces a future action, it must decide whether to pre-announce or not and what action level to pre-announce. In this case, the disclosure decisions are not observable. As we will show in this section, making this distinction is important because it affects the resulting equilibria.

To account for the stickiness of disclosure decisions, we examine two extended games. We call the first game the *Sticky Disclosure Game*. This game has three stages and, at each stage, firms make simultaneous strategic decisions. In the first stage, both firms publicly declare their disclosure decisions. That is, they declare whether they intend to pre-announce or not. They do not yet specify what action level they will pre-announce, but they publicly commit either to make a pre-announcement or to remain silent. In the second stage, both firms simultaneously follow their disclosure decisions by pre-announcing or not. That is, if they committed to pre-announce, they decide the content and disclose that content, but if they committed not to pre-announce, they just remain silent. Finally, in the third stage, both firms simultaneously choose their real actions.

The second game, which we refer to as the *Non-Sticky Disclosure Game*, has only two stages. In the first stage, both firms simultaneously make both the disclosure decision and the content decision. In contrast with the sticky disclosure game, since now both decisions are made at the same time, the content decisions are made ignoring whether the competitor will disclose or not. In the second stage, both firms decide their real actions simultaneously.\(^\text{10}\)

\(^{10}\)One could consider an intermediate case between the two games by allowing firms to breach its commitment to a declared disclosure decision at a cost. The analysis of such a setting, though complex, does not seem to provide any further insight. Therefore, we limit our scope to the two extreme cases.
5.1 Sticky Disclosure Game

We first examine a game in which disclosure decisions are sticky. The decision tree in Figure 4 illustrates the Sticky Disclosure Game in its extended form. In the first stage of the game, firms simultaneously and publicly declare disclosure decisions $p_i \in \{ND, D\}$ with $i \in \{1, 2\}$. A firm can declare its intent to remain silent, $ND$, or to pre-announce, $D$. In the second stage, knowing its competitor’s disclosure decision, each firm $i$ decides its pre-announcement content. If the firm declared a disclosure decision $p_i = D$ in the first stage, it chooses the pre-announcement content, $a_i \in \mathbb{R}_+$, and then discloses it. However, if the firm declared a disclosure decision $p_i = ND$, the firm simply remains silent, $a_i = \phi$. If both firms pre-announce, they do it simultaneously. Finally, in the third stage, firms simultaneously choose their real actions knowing the pre-announcements made in the second stage.

There are four possible disclosure decision pure strategy profiles: both firms pre-announce, $(D, D)$, both keep silent, $(ND, ND)$, and one firm pre-announces while the other keeps quiet, $(ND, D)$ and $(D, ND)$. Since disclosure decisions are sticky, in the second stage firms choose
the content of their pre-announcements knowing whether the competitor pre-announces or not. Consequently, each continuation game starting at the second stage constitutes a different subgame. Therefore, it is possible to reduce the whole game to the disclosure decision stage by replacing each one of these subgames by its subgame equilibrium payoffs. We now proceed to characterize the equilibrium payoffs of each subgame.

If both firms choose to pre-announce, the firms’ payoffs are equivalent to those in Section 4. That is, if the disclosure decision profile is \((D, D)\), the payoffs for the two firms are \(T_1\) and \(T_2\), as derived in Section 4. In the case in which no firm pre-announces, \((ND, ND)\), both firms get the same payoffs as in a pure simultaneous duopoly equilibrium. The equilibrium actions have the expression \(x_i^M = \frac{h}{2-n}\), and the payoffs are given by \(M_i = (x_i^M)^2 = \left(\frac{h}{2-n}\right)^2, i \in \{1, 2\}\) (here the \(M\) stands for “mute”).

If the disclosure decision profile is either \((ND, D)\) or \((D, ND)\), the payoffs can be obtained from the setting analyzed in Section 4. Indeed, once the disclosure policies are set, a firm with no credibility is equivalent to a non-pre-announcing firm. Therefore, if firm \(j\) is the non-disclosing firm, one can obtain the equilibrium payoffs of this subgame by taking the limit of the payoffs in the \((D, D)\) equilibrium as \(w_j \to 0\). In particular, the payoff of the pre-announcing firm, which we denote by \(S_i\) for “sender,” is obtained as the limit of \(T_i\) as \(w_j \to 0\), and the payoff of the quiet firm, which we denote by \(R_j\) for “receiver,” is obtained by taking the limit of \(T_j\) as \(w_j \to 0\). It is worth noticing that that the equilibrium payoffs in this subgame, in contrast to the results stated in Proposition 1, transition monotonically from those in a pure simultaneous duopoly equilibrium to those in a Stackelberg equilibrium as \(w_i\) goes from 0 to \(\infty\).

Having obtained the payoffs in all four subgame equilibria, we can now summarize the Sticky Disclosure Game as a normal form game in a two by two matrix:
Comparing the payoffs across cells, we obtain the following results:

**Lemma 2** *In the Sticky Disclosure Game,*

- For all \(w_1, w_2 \in (0, \infty)\), and \(n \in (-1, 0) \cup (0, 1)\), \(S_i > M_i\).
- For substitutes competition \((n \in (-1, 0))\), \(T_i > R_i\);
- For complements competition \((n \in (0, 1))\):
  - there exists a threshold \(\overline{w}\) such that \(T_i < R_i\) for \(w_j \in (\overline{w}, \infty)\).
  - there exists a sufficiently small \(\hat{w}\) such that \(T_i > R_i\) for \(w_j \in (0, \hat{w})\) with \(\overline{w} > \hat{w}\).

From Lemma 2, it is apparent that \((ND, ND)\) is never an equilibrium. Since \(S_i > M_i\) is always true, both players want to deviate by pre-announcing. For \(n < 0\), that is, in substitutes competition, \((D, D)\) is the only equilibrium. Indeed, Lemma 2 states that \(T_i > R_i\) when \(n \in (-1, 0)\). This implies that neither \((ND, D)\) nor \((D, ND)\) is an equilibrium because the quiet firm would rather deviate by pre-announcing. Conversely, the strategy profile \((D, D)\) constitutes an equilibrium because both firms would rather pre-announce when the other firm is also pre-announcing than being the only quiet firm. Intuitively, in substitutes competition, both firms prefer to disclose their future action plans to preempt their rivals. As long as the firm has some credibility, remaining silent means surrendering in the fight to be the Stackelberg leader, that is, giving the first-mover advantage to the competing firm. Therefore, both firms pre-announce in the unique equilibrium, \((D, D)\).

In complements competition, the credibility space is split into four disjoint regions. When both
$w_1$ and $w_2$ are sufficiently low, $T_i > R_i$ is true for both firms, $i \in \{1, 2\}$. Therefore, in this region, as in substitutes competition, $(D, D)$ is the unique equilibrium. However, if the credibility of one firm is sufficiently low but the other firm has a sufficiently high credibility (beyond a certain threshold, $\overline{w}$) we have that for the low credibility firm, say firm 2, $T_2 < R_2$, and for the high credibility firm, $T_1 > R_1$. In this region, there is a unique equilibrium in which only firm 1, the firm with a higher credibility, pre-announces (i.e. $(D, ND)$). There exists also an analogous third region where the sizes of the credibilities, and therefore, the equilibrium strategies are inverted. Finally, there is a fourth region for large enough credibilities (both credibilities are larger than the threshold, $\overline{w}$) in which $T_1 < R_1$ and $T_2 < R_2$ are both true. In this region, two pure strategy equilibria, $(ND, D)$ and $(D, ND)$, coexist. Indeed, in both of these equilibria, the pre-announcing firm does not want to deviate because $S_i > M_i$ is always true, and the quiet firm does not want to deviate because $R_i > T_i$. The description of the disclosure decision equilibria is summarized in the following proposition:

**Proposition 5** In the Sticky Disclosure Game,

· In substitutes competition ($n \in (-1, 0)$), there exists a unique equilibrium for all $w_1, w_2 > 0$ in which both firms pre-announce their future real actions, $(D, D)$.

· In complements competition ($n \in (0, 1)$):

  (i) For a sufficiently low $w_j \neq 0$, there exists a $\hat{w} > 0$ such that, for $i, j \in \{1, 2\}, i \neq j$:

  · for any $w_i > \hat{w}$, in the unique equilibrium, firm $i$ pre-announces and firm $j$ remains mute.

  · for any $w_i < \hat{w}$, in the unique equilibrium, both firms pre-announce.

(ii) There exists a threshold $\overline{w}$, with $\overline{w} > \hat{w}$, such that for any $w_1 > \overline{w}$ and $w_2 > \overline{w}$, there are two equilibria, $(ND, D)$ and $(D, ND)$.

Figure 5 plots the regions in which the different equilibria exist in a complements competition. In the lower left region, region 1, where both firms have very low credibility, both firms disclose.
In the upper right region, region 4, two equilibria coexist. In one equilibrium firm 1 pre-announces and firm 2 remains quiet. In the other equilibrium the roles switch. In the remaining two regions, one firm has a high credibility while the other firm’s credibility is low. In those regions, only the firm with a high credibility pre-announces. In the figure, the thresholds between regions look like straight lines but, in fact, the thresholds are curves and it is technically difficult to characterize them. Nevertheless, we are able to show numerically that the four regions are delimited by sharp boundaries.

The payoffs of the equilibria in region 4 satisfy $R_i > S_i$ for $i \in \{1, 2\}$. Therefore, each firm prefers to be in the equilibrium in which it remains silent and the other firm pre-announces. In other words, no equilibrium Pareto dominates the other. Consequently, we apply the well-known criterion of risk dominance to select the equilibrium to which the firms will coordinate. In a game with two players and binary choices, the risk dominance criterion can be characterized with a simple inequality. Equilibrium $(D, ND)$ risk-dominates equilibrium $(ND, D)$ if
\[(S_1 - M_1)(R_2 - T_2) > (S_2 - M_2)(R_1 - T_1).\]  

That is, the equilibrium with a larger product of deviation losses dominates the other.\(^{11}\)

The application of the risk dominance criterion to the region of credibilities, in which complements competition obtains multiplicity of equilibria, selects the equilibrium in which the firm with the higher credibility pre-announces and the firm with the lower credibility remains silent. This result is summarized in the following corollary:

**Corollary 2** In complements competition, when the equilibria \((D, ND)\) and \((ND, D)\) coexist, \((D, ND)\) risk dominates \((ND, D)\) if \(w_1 > w_2\) and \((ND, D)\) risk dominates \((D, ND)\) if \(w_1 < w_2\). If \(w_1 = w_2\), the mixed strategy equilibrium risk dominates the two pure strategy equilibria.

Applying the risk dominance criterion numerically, we can obtain a graphical representation of Corollary 2. Figure 6 illustrates how applying the risk dominance criterion partitions the space of credibilities in three regions, each of which represents the set of credibility pairs in which a single equilibrium prevails.

### 5.2 Non-Sticky Disclosure Game

Departing from the previous subsection, we assume now that disclosure decisions are not declared before the pre-announcement stage. That is, both firms choose whether to pre-announce or not and what to pre-announce in the first stage. Moreover, since firms decide simultaneously, they do that ignoring whether the other firm pre-announces or not. This variation of the previous game seems to be a minor one and, indeed, it has no effect on the resulting equilibrium for the substitutes competition. However, it changes the equilibrium dramatically for complements competition.

\(^{11}\)Detailed illustration of risk dominance application is available in Appendix I.
Figure 6: Equilibrium regions applying the risk dominance criterion

Specifically, now there is a unique equilibrium in which both firms pre-announce for any credibility pair \((w_i, w_j) \in \mathbb{R}_+ \times \mathbb{R}_+\) and for any type of market competition \(n \in (-1, 0) \cup (0, 1)\).

The Non-Sticky Disclosure Game has two stages. In the first stage, each firm chooses to pre-announce or remain silent, and if it decides to pre-announce, it chooses also what content to disclose. This first stage is equivalent to the first two stages of the Sticky Disclosure Game if one assumes that the disclosure decision is not publicly observable. Essentially, each firm makes the two decisions before the other firm can observe any of them. In the second stage, both firms take the real action decisions simultaneously having observed any pre-announcements made in the first stage. Figure 7 shows a graphical representation of the game in extended form.

Formally, each firm \(i\) chooses a strategy of the form \(\{d_i, x(d_i, d_j)\}\). In the first stage, firm \(i\) chooses \(d_i \in \{\phi\} \cup \mathbb{R}_+\). That is, it chooses either to remain silent, \(d_i = \phi\), or to make a pre-announcement, \(d_i = a_i \in \mathbb{R}_+\). The continuation game starting at each node after the first stage constitutes a subgame in which each firm \(i\) chooses its real action level contingent on the previous
disclosures, $x_i(d_i, d_j)$. Notice that, in Figure 7 we symbolize the strategy choice from a continuum of strategies (the positive real numbers) by drawing a triangle as opposed to a line. In such cases, there is a continuum of subsequent nodes, and therefore a continuum of continuation games after that decision. If that decision is not observable to the following player, all continuation games are identical. For instance, this is the case in Figure 7 when player 2 chooses its $a_2$ after player 1 has chosen $a_1$. However, if the decision is observable, then there is a continuum of different continuation games. This is the case, for instance, when player 1 chooses $x_1$ after player 2 has chosen $a_2$. Therefore, any change of strategy in the first stage will lead to a different subgame in the second stage, as the real actions of both firms are contingent on the disclosure decisions taken in the first stage. This is true not only for equilibrium strategies but also for out-of-equilibrium strategies. The analysis of this game, therefore, is a bit more subtle than the analysis of the Sticky Disclosure Game.

In the Sticky Disclosure Game analyzed in the previous subsection, disclosure decisions are observable after they are declared. Therefore, in analyzing a candidate equilibrium, any deviation
at the disclosure decision stage leads to one of four possible subgame equilibria. This makes it possible to reduce the disclosure decision stage to a normal form game that can be summarized in a $2 \times 2$ matrix, as shown in Table 1. This simplification, however, cannot be made in the Non-Sticky Disclosure Game. In this game, a deviation from a candidate equilibrium at the first stage is not observable until the second stage, when pre-announcements have already been made. Therefore, the other firm cannot adapt its pre-announcement to the deviation. Take, for instance, a candidate equilibrium in which a firm pre-announces and the other remains silent. If we consider that the quiet firm deviates by pre-announcing, we must keep the other firm’s disclosure choice fixed. This means that the deviation leads to a continuation game that is not described by any of the cells in the $2 \times 2$ matrix of the Sticky Disclosure Game. In fact, since the deviating firm can choose any pre-announcement from the positive real numbers, there are uncountably many possible deviations.

The following proposition summarizes the analysis of the game:

**Proposition 6** In the Non-Sticky Disclosure Game, for both substitutes and complements competitions, there exists a unique equilibrium for all $w_i \in (0, \infty), i \in \{1, 2\}$, in which both firms pre-announce their future actions.

The intuition behind this result can be conveyed by sketching the proof of the proposition in intuitive terms. Since, in equilibrium, each firm’s disclosure strategy must be a best-response to the other firm’s disclosure strategy, any equilibrium of this game must still have the same payoffs and disclosure choices as one of the four subgame equilibria described in Section 5.1.

Let’s take each of the four candidate equilibria at a time starting with the simplest one, the situation in which no firm pre-announces, $(d_i, d_j) = (\phi, \phi)$. If a firm $i$ deviates, it must pre-announce, and the best this firm can do is to best respond the non-disclosure of firm $j$ by pre-announcing $a_i^j$. Since firm $j$ remains quiet, the deviating firm obtains a payoff of $S_i$. We know from the previous section that $S_i > M_i$ for any $w_i \in (0, \infty)$ and $n \in (-1, 0) \cup (0, 1)$. Therefore, the
deviation would always be profitable and, thus, $(\phi, \phi)$ can never be an equilibrium.

If only one firm discloses, $(d_i, d_j) = (a_i^s, \phi)$, the disclosing firm $i$ obtains a payoff of $S_i$ and the quiet firm $j$ a payoff of $R_j$. The disclosing firm never deviates because $S_i > M_i$. If firm $j$ deviates by pre-announcing, the best it can do is to disclose $a^{BR}(a_i^S)$ that satisfies:

$$a^{BR}(a_i^S) = \argmax_a \pi_j(a, a_i^S, n, w_j).$$

That is, firm $j$ pre-announces the best response to $a_i^S$. It turns out that, $\pi_j(a^{BR}(a_i^S), a_i^S, n, w_j) > S_j = \pi_j(\phi, a_i^S, n, w_j)$ for all $w_j \in (0, \infty)$ and $n \in (-1, 0) \cup (0, 1)$. Therefore this is never an equilibrium either. The intuition behind the last inequality can be illustrated by examining the extreme scenario in which the disclosing firm $i$ has an infinite credibility. Observe that as $w_i$ tends to infinity, the candidate equilibrium converges to a canonical Stackelberg equilibrium in which firm $i$ pre-announces the leader’s action in the first stage, and firm $j$ chooses the follower’s reaction in the second stage. (The infinite credibility of the disclosing firm makes its pre-announcement a full commitment to an action choice. Therefore, the quiet firm’s best response in the second stage is to choose the Stackelberg follower’s reaction.) In this situation, firm $j$ is indifferent between pre-announcing or not. Indeed, since it cannot affect the leader’s action, disclosing has no effect on the final payoffs. It is then intuitive that when the disclosing firm’s credibility is finite, the quiet firm is better off by deviating, since that allows it to affect firm $i$’s action in the second stage.

Finally, we must check whether $(d_i, d_j) = (a_i^T, a_j^T)$ is an equilibrium. If firm $i$ deviates, it must do so by remaining quiet. In such a deviation, firm $i$ obtains a payoff of $\pi_i(\phi, a_i^T, n, w_i)$. However, it can be proven that $T_i > \pi_i(\phi, a_i^T, n, w_i)$ for all $(w_i, w_j) \in \mathbb{R}^+ \times \mathbb{R}^+$ and $n \in (-1, 0) \cup (0, 1)$. Therefore, this is always an equilibrium.\(^{12}\) In a substitutes competition setting it is intuitive that remaining quiet always makes a firm worse off, because it entails to leave the other firm’s pre-

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\(^{12}\) In the real world, the future is uncertain and, therefore, pre-committing to a future action entails the cost of making a choice that is possibly sub-optimal ex post. Adding uncertainty to the settings in this paper, however, does not add any unexpected insight. The study of such an extension obtains that with sufficiently high uncertainty about the future, firms prefer not to pre-announce.
commitment to a higher action level without retaliation. In complements competition setting, however, the deviation must be interpreted as a defection from collusion. Recall that in the Sticky Disclosure Game firm $i$ prefers to remain quiet for a big enough $w_j$. In that game, however, when only one firm discloses, it chooses a pre-announcement that best-responds to the quiet firm. This is not the case here. When firm $i$ deviates by remaining quiet, firm $j$ still pre-announces $a^T_j$ since firm $j$ conjectures that firm $i$ would disclose and take its share of collusive work. This implies that the collusive role which was supposed to be exerted by firm $i$ is now abandoned without firm $j$ being able to compensate for it. As a consequence, the collusion benefit shrinks and both firms get worse off.

The uniqueness of the equilibrium for both substitutes and complements competitions in this game is even more surprising when we compare this result with previous studies. Hamilton and Slutsky (1990) examine a two-period duopolistic setting in which players choose to act either at the first period or to wait until a second period, in which they already observed the competitor’s first period choice. Following Hamilton and Slutsky’s work, Van Damme and Hurkens (1999) and Van Damme and Hurkens (2004) study which firm will emerge as the endogenous leader in a duopoly game in both Cournot competition and Bertrand competition (which are comparable to our substitutes and complements competitions, respectively). Although these studies focus on the endogenous order of moves, their models can be regarded as a special case of our setting in which the firms either choose to pre-announce truthfully or choose not to pre-announce. In other words, their models are equivalent to the limit case of our model in which $w_i = w_j = \infty$. In all these studies, there always exist multiple equilibria and various criteria have to be applied to eliminate unwanted equilibria. In our paper, however, simply because $w_i, w_j \in (0, \infty)$ and the pre-announcement is a partial commitment, we steer clear of multiple equilibria problems and both firms pre-announcing turns out to be the only equilibrium.
6 Conclusions

Our paper examines an oligopolistic setting in which firms pre-announce their strategic decisions. We analyze the incentives firms have to pre-announce in two types of product markets: markets in which firms compete with substitute products and markets in which firms compete with complementary products. We show that, if products are substitutes, firms are driven by the incentive to obtain the first-mover advantage. Firms overstate their pre-announced action levels in an attempt to threaten their competitors and induce them to reduce their actual action levels. In this overstating contest, all firms end up with higher real action levels than the ones they would have chosen had they not pre-announced. If instead firms compete with complementary products, they have an incentive to pre-announce high action levels in order to induce their competitors to increase their real actions as well. That is, pre-announcements foster collusion. We show, in addition, that social welfare is maximized at an intermediate level of tolerance for “bragging,” even when bragging facilitates collusion. We further analyze a setting in which pre-announcements are discretionary, and show that the resulting equilibrium is sensitive to the nature of the competition and the stickiness of the disclosure decisions. In particular, if firms compete with substitute products, there is a unique equilibrium in which both firms pre-announce. This same equilibrium is also the unique one if products are strategic complements and disclosure decisions are non-sticky. However, when the disclosure decisions are sticky, both firms pre-announce when firms have a low credibility, but only the most credible firm discloses if its credibility level is high enough.

The study of pre-announcements of competitive decisions is an analytically under-explored area in accounting. This paper is a first step in understanding the incentives that induce firms to make such disclosures from an analytical perspective, but it leaves many questions still unanswered. Indeed, our work could actually be extended in many different ways. For instance, the simultaneity of decisions in our model is obviously an approximation. In the real world firms rarely
pre-announce at the same time. Thus, one could examine the endogenous ordering of a sequence of pre-announcements. Alternatively, one could extend the study of pre-announcements under other sources of asymmetry between firms other than a difference in credibilities. The study of these and many other possible extensions goes beyond the analysis that can be contained in a single paper and is, therefore, left for future work.

The extant empirical literature on pre-announcements of future competitive decisions is also scarce. Nevertheless, there are a few related papers that examine these types of disclosures. For instance, Christensen and Caves (1997) study the pre-announcement of new capacity expansions in the pulp and paper industry. They study the factors that affect the likelihood that capacity expansions are abandoned and provide evidence of the strategic effects of pre-announcements: a competitor’s subsequent announcement of a capacity expansion increases the likelihood that a firm abandons its previously announced project. On a related paper, Doyle and Snyder (1999) analyze U.S. automobile makers’ pre-announcements and revisions of their production plans and show that a firm’s plan pre-announcement influences competitors’ later revisions. However, this is still an under-explored area in accounting and future empirical research may be fruitful in testing the predictions of this paper about the pre-announcement of future competitive decisions.

References


Appendix I

The concept of risk dominance portrays the intuitive idea that, when players are uncertain about which equilibrium to play, they will consider the risk involved in playing each of these equilibria and coordinate on the less risky one. This concept can be illustrated in a graphical way (see Figure 8).

Suppose that both players anticipate that they are in one of the two equilibria, \((ND, D)\) or \((D, ND)\), and suppose firm \(i\) assigns a probability \(\alpha_j\) to firm \(j\) playing strategy \(ND\). Firm \(i\) is then indifferent between playing strategy \(D\) or \(ND\) if \(\alpha_j = \alpha_j^*\) where \(\alpha_j^*\) satisfies:

\[
\alpha_j^* S_i + (1 - \alpha_j^*) T_i = \alpha_j^* M_i + (1 - \alpha_j^*) R_i.
\]

Moreover, firm \(i\) prefers to play strategy \(D\) whenever it believes \(\alpha_j \in (\alpha_j^*, 1]\). From these equalities for \(i \in \{1, 2\}\), we can derive the strategy profile \((\alpha_1^*, \alpha_2^*)\), where

\[
\alpha_i^* = \frac{1}{1 + \frac{M_j - M_i}{R_j - R_i}} \quad \text{for} \; i \in \{1, 2\}.
\]

This strategy profile, in fact, constitutes the mixed strategy equilibrium of the game and, in Figure 8, is located at the intersection of the two lines that divide the mixed strategy space in four areas. Now, if the event that firm \(i\) has a specific belief \(\alpha_j\) is equally likely for all \(\alpha_j \in [0, 1]\) and for \(i, j \in \{1, 2\}, i \neq j\), then the equilibrium \((D, ND)\) will be played with probability \((1 - \alpha_2^*)\alpha_1^*\) and the equilibrium \((ND, D)\) will be played with probability \((1 - \alpha_1^*)\alpha_2^*\). This is illustrated in Figure 8 as the two shaded areas in the graph. The risk dominance criterion then can be understood as a comparison between these two areas. In particular, equilibrium \((D, ND)\) dominates equilibrium \((ND, D)\) if \((1 - \alpha_2^*)\alpha_1^* > (1 - \alpha_1^*)\alpha_2^*\). It is easy to see that, this inequality reduces to the inequality in expression (9).
Appendix II

Proposition 1 and Proposition 4

Proof. We present the proof using the setting with asymmetric credibilities, since the result in Proposition 1 is nested by the result in the setting with asymmetric credibilities.

In the second stage, the first order conditions for firm $i$’s program

$$\max_{x_i} \Pi_i(x_i, \bar{x}_j(a_i, a_j), a_i) \quad \text{for } i \in \{1, 2\}$$

are $h + w_i a_i + n \hat{x}_j(a_i, a_j) - (2 + w_i) x_i = 0 \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j$. From the first order conditions, we get the subgame optimal real action, $x_i^*(a_i, a_j) = \frac{(2+w_j)(h+w_i a_i)+n(h+w_j a_j)}{(2+w_j)(2+w_i)-n^2} \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j$. In the first stage of the game firms choose the pre-announcements solving the following program:

$$\max_{\hat{a}_i} \Pi_i(x_i^*(a_i, \hat{a}_j), x_j^*(\hat{a}_j, a_i), a_i) \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j$$
Taking first order conditions with respect to \( \gamma \) and solving the system of equations, we obtain the expressions provided in the Proposition 4.

When \( w_i = w_j \), we get the results in Proposition 1. ■

**Proposition 2**

**Proof.** Directly from Proposition 1. ■

**Lemma 1**

**Proof.** When there is no pre-announcement, the setting reduces to a simultaneous Cournot duopoly equilibrium. The equilibrium quantity is such a setting is \( \frac{h}{2-n} \).

In the common credibility case, from Proposition 1 we have \( x_i^* = x_o = \frac{h}{2-n[1+\lambda_o]} \). We can see that, since \( \lambda_o > 0 \), \( x_o > \frac{h}{2-n} \).

From Proposition 1, \( x_o = \frac{h}{2-n[1+\lambda_o]} = \frac{h[-n^2+(2+w)^2]}{n^2-n^2(2+w)+(2-n)(2+w)^2} \). Taking the first order condition with respect to \( w \), we obtain the unique positive root, \( w_o = \sqrt{4-n^2} \). The second order condition is also satisfied. Therefore, both firms exert a maximum action level at \( w_o = \sqrt{4-n^2} \). ■

**Proposition 3**

**Proof.** A representative consumer maximizes a utility \( U(x_1, x_2) = \alpha_1 x_1 + \alpha_2 x_2 - \frac{1}{2}(\beta_1 x_1^2 + 2\gamma x_1 x_2 + \beta_2 x_2^2) \). In Cournot competition, the market price will be \( \alpha_i - \gamma x_j - \beta_i x_i \). Therefore, we need to assign \( \alpha_1 = \alpha_2 = h, \beta_1 = \beta_2 = 1 \) and \( \gamma = -n \) \((n > 0)\). The total surplus is then \( h(x_1 + x_2) - \frac{1}{2}(x_1^2 + x_2^2 - 2nx_1 x_2) \). Substitute \( x_1 = x_2 = x_o = \frac{h}{2-n[1+\lambda_o]} \), to get \( ts = 2hx_o - (1-n)x_o^2 = \frac{h^2(3-n-2n\lambda_o)}{[2-n(1+\lambda_o)]^2} \).

Take derivative with respect to \( w \), we have \( \frac{\partial ts}{\partial w} = \frac{2h(n\lambda_o-1)}{n\lambda_o+n-2} \frac{\partial x_o}{\partial w} \), which is zero either when \( \frac{\partial x_o}{\partial w} = 0 \) or when \( \lambda_o = \frac{1}{n} \). Since \( \lambda_o = \frac{1}{n} \) has no real root for \( w \) for \(-1 < n < 1 \), it has to be the case that \( \frac{\partial ts}{\partial w} = 0 \) when \( \frac{\partial x_o}{\partial w} = 0 \). In other words, the total surplus maximizes at \( w_o = \sqrt{4-n^2} \).

The consumer surplus in Cournot competition is \( cs = (1-n)x_o^2 \). Obviously, it maximizes at \( w_o = \sqrt{4-n^2} \) too, which is the \( w \) level that maximizes \( x_o \). ■

**Corollary 1**
Proof. From the expression of $x_i^T$, we derive \[
\frac{\partial x_i^T}{\partial w_i} = \frac{\partial x_i^T}{\partial \lambda_{ij}} \frac{\partial \lambda_{ij}}{\partial w_i} + \frac{\partial x_i^T}{\partial \lambda_{ij}} \frac{\partial \lambda_{ij}}{\partial w_i} = \frac{h_n(2-n\lambda_{ij})(2+n-n\lambda_{ij})}{([2-n\lambda_{ij})(2-n\lambda_{ij})-n^2]^2} \frac{n(4-n^2+2w_i)}{[(2+w_i)(2+w_i)-n^2]^2} - \frac{h_n(2-n\lambda_{ij})(2+n-n\lambda_{ij})}{([2-n\lambda_{ij})(2-n\lambda_{ij})-n^2]^2} \frac{nw_j(2+w_i)}{[(2+w_i)(2+w_i)-n^2]^2}.
\] The sign of $\frac{\partial x_i^T}{\partial w_i}$, therefore, depends on the sign of $(4-n^2+2w_j)(2-n\lambda_{ij})(2+n-n\lambda_{ij}) - nw_j(2+n)(2+n-n\lambda_{ij})$. Using the expression for the influences, the above expression can be rewritten as,

\[
(4-n^2+2w_j)(2-n\lambda_{ij})(2+n-n\lambda_{ij}) - nw_j(2+n)(2+n-n\lambda_{ij}) = (4-n^2+2w_j)(2+n)(2+n-n\lambda_{ij}) - n^2w_j(2+n)(2+n-n\lambda_{ij}).
\]

When $-1 < n < 0$, the above expression is positive. That is, in a substitute competition, $x_i^T$ is monotonically increasing in $w_i$. \[\frac{\partial x_i^T}{\partial w_i} > 0.\]

If $0 < n < 1$, when $w_j \leq \frac{(2-n)(2-n^2) - \sqrt{(2-n)^2(2-n^2)+2(4-n^2)^2}}{2n} \equiv w$, we can show that the above expression is positive. That is, in complements competition, there exists a threshold $w \in (0, \infty)$ such that for $w_j \leq w$ the action of firm $i$, $x_i^T$, is monotonically increasing in firm $i$’s credibility $w_i$.

In addition, when $0 < n < 1$ and $w_j \geq \frac{8-(2-n)n^2+\sqrt{[8-(2-n)n^2]^2+8(2-n)n(2+n)^3}}{2n(2+n)} \equiv \bar{w}$, the above expression is negative. That is, in complements competition, there exists a threshold $\bar{w} \in (0, \infty)$ such that for $w_j \geq \bar{w}$ the quantity of firm $i$, $x_i^T$, is monotonically decreasing in firm $i$’s credibility $w_i$. \[\]

Henceforth, in all proofs we use $S$ and $R$ as superscripts to identify the optimal decisions in an equilibrium in which one firm pre-announces (the “Sender”, $S$) and the other firm does not pre-announce (the “Receiver”, $R$) respectively.

**Lemma 2**

Proof. For convenience, we first list all payoffs:

1. When neither firm discloses, each firm’s payoff is $M_i = (x_i^M)^2 = \frac{h^2}{(2-n)^2}$, $i \in \{1, 2\}$.

2. When only firm $i$ discloses, the disclosing firm’s payoff is $S_i = \frac{1}{2}[(2+w_i) (x_i^S)^2 - w_i (a_i^S)^2] = \frac{1}{2}[2 + w_i - \frac{1}{w_i} (n \frac{\partial x_i^R}{\partial a_i} + w_i)^2] (x_i^S)^2$ where $x_i^S = \frac{h(2+n)}{4-n^2-2n \frac{\partial x_i^R}{\partial a_i}}$, and $\frac{\partial x_i^R}{\partial a_i} = \frac{nw_i}{2(2+w_i)-n^2}$. Explicitly,
\[ S_i = \frac{h^2(2+n)^2(2+w_i)}{2(4-n^2)^2+8(2-n^2)w_i} \]

The non-disclosing firm’s payoff is

\[ R_j = \left( x_j^R \right)^2 = \left( \frac{h(2+n-\frac{\partial x_j^R}{\partial w_j})}{4-n^2-2n^2} \right)^2 \]

where

\[ \frac{\partial x_j^R}{\partial w_j} = \frac{nw_j}{2(2+w_j)-n^2}. \]

Explicitly,

\[ R_j = \frac{[h(-2+n)(2+n)^2+n(4-(-2+n)n)w_i]^2}{[(4-n^2)^2-4(-2+n^2)w_i]^2} \]

(3) When both firms disclose, each firm’s payoff is

\[ T_i = \frac{1}{2}[(2+w_i)T_i + w_i(2T_i^2)] = \left( x_i^T \right)^2 - w_i(a_i^T)^2 \]

with

\[ x_i^T = h \frac{n+2-n\lambda_i^T}{(2-n\lambda_i)(2-n\lambda_j)^{-n^2}} \]

and

\[ \lambda_j^T = \frac{n w_j}{(2+w_j)(2+w_j)-n^2}. \]

Explicitly,

\[ T_i = \Phi \Psi \]

for all \( 0 < n < 1 \), we can rewrite \( T_i \) to be

\[ T_i = C(w_i)^{\frac{2}{3}} \]

where

\[ C(w_i) \equiv (2 + w_i - w_i(\frac{n \lambda_i^T}{w_i} + w_i)^2 \]

with

\[ \lambda_j^T = \frac{n w_j}{(2+w_j)(2+w_j)^{-n^2}} \]

Taking derivative of \( T_i \) with respect to \( w_i \), we have

\[ \frac{\partial T_i}{\partial w_i} = x_i \frac{\partial x_i^T}{\partial w_i} C(w_i) + x_i^T \frac{\partial C(w_i)}{\partial w_i} \]

It can be shown that

\[ C(w_i) = (2 + w_i) - w_i(\frac{n w_j}{(2+w_j)^{-n^2}})^2 \]

and

\[ \frac{\partial C(w_i)}{\partial w_i} = \frac{n^2((-8+n^2-4w_j)(-4+n^2-2w_j)-w_j(-8+3n^2-4w_j)w_j)}{(-4+n^2-2w_j)^2} < 0 \]

if \( n \neq 0 \). Therefore, if the sign of \( \frac{\partial x_i^T}{\partial w_i} \) is negative, then \( \frac{\partial T_i}{\partial w_i} < 0 \). According to Corollary 2, when

\[ 0 < n < 1 \]

and

\[ w_j \geq \frac{\sqrt{5244+n(n-2)^2}-32}{(2+n)-n} \]

\[ \equiv \Pi \]

\[ \frac{\partial x_i^T}{\partial w_i} \]

is negative. Therefore, for complements competition \((n \in (0,1))\), there exists a threshold \( \Pi \) such that \( T_i < R_i \) for \( w_j \in (\Pi, \infty) \). Firm \( i \)'s payoff, \( T_i \), decreases with its own credibility, \( w_i \), when the competitor’s credibility, \( w_j \), is sufficiently high.

In addition, for complements competition, we have

\[ \lim_{w_i \to 0, w_j \to 0} \frac{\partial T_i}{\partial w_i} = \frac{h^2 n^4}{2(2-n)^n(2+n)^2} > 0 \]

\[ \text{in the} \]

\[ \text{The expression of } \frac{\partial x_i^T}{\partial w_i} \text{ and the proof that is positive are not included for brevity. They are available upon request.} \]
neighborhood of \((w_i = 0, w_j = 0)\) and \(\frac{\partial T_i}{\partial w_i}\) is continuous. Therefore, there must exist a sufficiently small \(\hat{w}\) such that \(T_i > R_i\) for \(w_j \in (0, \hat{w})\). Firm \(i\)'s payoff, \(T_i\), increases with its own credibility, \(w_i\), when the competitor’s credibility, \(w_j\), is sufficiently low. ■

**Proposition 5**

**Proof.** \((ND, ND)\) is never sustainable in the equilibrium since \(S_i > M_i\) for all \(w_1, w_2 \in (0, \infty)\), according to Lemma 2.

In substitutes competition \((-1 < n < 0)\), according to Lemma 2 we have \(T_i \to R_i\) as \(w_i \to 0\) and \(\frac{\partial T_i}{\partial w_i} > 0\) for all all \(w_i, w_j \in (0, \infty)\). Therefore \(T_i > R_i\) for all \(w_1, w_2 > 0\) and \((D, D)\) is the only equilibrium.

For complements competition \((0 < n < 1)\), we first prove that in a neighborhood of \((w_1, w_2) = (0, 0)\), there is a unique equilibrium \((D, D)\). Since \(T_i \to R_i\) as \(w_i \to 0\) and \(\lim_{w_i \to 0, w_j \to 0} \frac{\partial T_i}{\partial w_i} = \frac{h^2 n^4}{2((-2+n)^2 - 4(-2+n)w_i)} > 0\), it is always better to pre-announce. Therefore \((D, D)\) is the unique equilibrium when \(w_1\) and \(w_2\) are both sufficiently low.

Next, we prove that for a sufficiently small \(w_j\), there exists a sufficiently small \(\hat{w}\) such that for any \(w_i < \hat{w}\) the unique equilibrium is \((D, D)\), while for any \(w_i > \hat{w}\) the unique equilibrium is \((D, ND)\) in which only firm \(i\) discloses:

As \(w_j \to 0\), \(\lim_{w_j \to 0} \frac{\partial T_i}{\partial w_i} = \frac{h^2 n^4 (2+n)^2}{2((-4+n^2)^2 - 4(-2+n)w_i)^2} > 0\). That is, firm \(i\) will always disclose.

For firm \(j\), as \(w_j \to 0\), \(\lim_{w_j \to 0} \frac{\partial T_j}{\partial w_j} = \frac{A}{2((-4+n^2-2w_i)^2 ([(-4+n^2)^2 - 4(n^2-2)w_i]^3, where \(A \equiv h^2 n^3((n-2)(2+n)^2 + (-4 + (n-2)n)w_i)((n-2)^3n(n+2)^4 + w_i((n-2)(n+2)^2(-32 + (n-4)n(-8 + (n-4)n)) + \frac{8w_i((-4 + (n-1)n(-12 + n(2 + 3n)) + w_i(24 - n(-8 + n(n+6) + 2(2+n)w_i))]}{2})].

The denominator of \(\lim_{w_j \to 0} \frac{\partial T_j}{\partial w_j} 2((-4+n^2-2w_i)^2 ([(-4+n^2)^2 - 4(n^2-2)w_i]^3\), is always positive.

For the nominator, \(A\), to be positive, we need either \(n < 0\) (substitutes competition) or \(n > 0\) (complements competition) and \(w_i\) is sufficiently small. That is, for a sufficiently small \(w_j\), both
firms disclose in substitutes competition (as we already proved), and in complements competition both firms disclose when \( w_i < \bar{w} \).

Last, we prove that there is a threshold \( \bar{w} \) such that for any \( w_i > \bar{w} \) and \( w_j > \bar{w} \) there are two equilibria \((D, ND)\) and \((ND, D)\). To prove this, we need to show that there is a \( \bar{w} \) such that whenever \( w_i > \bar{w} \) and \( w_j > \bar{w} \), we have \( T_i < R_i \) and \( T_j < R_j \). According to the proof of Lemma 2, when \( 0 < n < 1 \) and \( w_j \geq \frac{1}{8(n-2)\pi(n+2)(-32+24n+n(n-12))-n}{(2+n)} \equiv \bar{w} \), we have \( T_i < R_i \), \( i, j \in \{1, 2\} \). Therefore, for any \( w_i > \bar{w} \) and \( w_j > \bar{w} \) there are two equilibria \((D, ND)\) and \((ND, D)\).

\[ \text{Corollary 2} \]

\textbf{Proof.} We need to prove that for \( w_2 > w_1 \) the following inequality is satisfied:

\[ (S_1 - M_1)(R_2 - T_2) < (S_2 - M_2)(R_1 - T_1) \]

The proof for the case in which \( w_1 > w_2 \) is analogous and, therefore, we omit it. When both \((D, ND)\) and \((ND, D)\) are equilibria, we have that \( S_i > M_i \) and \( R_i > T_i \) for \( i \in \{1, 2\} \). Therefore, all expressions in the above inequality are positive and we can rearrange it as,

\[ \frac{R_2 - T_2}{R_1 - T_1} < \frac{S_2 - M_2}{S_1 - M_1}. \]

To prove that this last inequality is true, we take two intermediate steps. In particular, we prove the following sequence of inequalities:

\[ \frac{R_2 - T_2}{R_1 - T_1} < \frac{T_2}{T_1} < 1 < \frac{S_2 - M_2}{S_1 - M_1}. \]

Proving the inequality on the right is immediate if one realizes that \( S_i \) is increasing in \( w_i \). Indeed,
the derivative can be simplified to

\[
\frac{\partial S_i}{\partial w_i} = \left( \frac{hn^2(2 + n)}{2(n^4 + 8(2 + w_i) - 4n^2(2 + w_i)^2)} \right)^2,
\]

which is always positive. Therefore, for \( w_2 > w_1 \) we have that \( S_2 > S_1 \), and since \( M_1 = M_2 \) the last inequality is satisfied.

To prove the inequality on the left, we rewrite it as \( \frac{R_2 - T_2}{T_2} < \frac{R_1 - T_1}{T_1} \), which can be further simplified to \( \frac{T_1}{T_2} < \frac{R_1}{R_2} \). Therefore, to prove the first to inequalities it is enough to prove: \( 1 < \frac{T_2}{T_1} < \frac{R_1}{R_2} \) for \( w_2 > w_1 \). Proving these inequalities is a tedious exercise of basic algebra and it is omitted for brevity. The reader can obtain the proof upon request from the authors. ■

**Proposition 6**

**Proof.** The proof is sketched in the main text after the proposition. It only remains to prove some of the steps not detailed in the main text.

If only one firm discloses, \((d_i, d_j) = (a_i^S, \phi)\), the disclosing firm \( i \) obtains a payoff of \( S_i \) and the quiet firm \( j \) a payoff of \( R_j \). If firm \( j \) deviates by pre-announcing, the best it can do is to disclose \( a^{BR}(a_j^S) \) that satisfies:

\[
a^{BR}(a_j^S) = \arg\max_a \pi_j(a, a_j^S, n, w_j).
\]

That is, firm \( j \) pre-announces the best response to \( a_j^S \). To show that this is never an equilibrium we need to prove that such a deviation is profitable:

\[
\pi_j(a^{BR}(a_j^S), a_i^S, n, w_j) > S_j = \pi_j(\phi, a_i^S, n, w_j) \text{ for all } w_j \in (0, \infty) \text{ and } n \in (-1, 0) \cup (0, 1).
\]

To calculate \( \pi_j(a^{BR}(a_j^S), a_i^S, n, w_j) \) we can use the subgame equilibrium real actions \( x^*_i(a_i, a_j) \) and \( x^*_j(a_i, a_j) \) as well as the first order condition for firm \( j \) with respect to its pre-announcement \( a_j \) derived in Section 4. However, at the pre-announcing stage, firm \( i \) acts as in the candidate equilibrium. That is, it pre-announces \( a_i^S \) as if it was the only firm pre-announcing. The resulting
profit has the following expression:

$$\pi_j(a^{BR}(a^S_i), a^S_i, n, w_j) = \frac{1}{T}[2 + w_j(1 - (1 + \frac{n}{w_j} \frac{\partial x_j}{\partial a_j})^2)]x_j^2,$$

where, $x_j = \frac{h(2+n)+(h+n\alpha)}{(2-n\frac{\partial x}{\partial a_j})(2+w_j)-n^2}$.

With some algebra one can show that the ratio between the deviating profit and the candidate equilibrium profit for firm $j$ is:

$$\frac{\pi_j(a^{BR}(a^S_i), a^S_i, n, w_j)}{R_j} = \frac{1}{T}[2 + w_j(1 - (1 + \frac{n}{w_j} \frac{\partial x_j}{\partial a_j})^2)]\frac{2(2+w_j)-n^2}{(2-n\frac{\partial x}{\partial a_j})(2+w_j)-n^2}.$$

This expression is larger than 1 for any $n \in (-1, 0) \cup (0, 1)$. Therefore, $\pi_j(a^{BR}(a^S_i), a^S_i, n, w_j) > R_j$.

That is, firm $j$ always wants to deviate.

Finally, to prove that $(d_i, d_j) = (a^T_i, a^T_j)$ is an equilibrium, we must show that no firm wants to deviate by remaining quiet. In such a deviation, a firm $i$ obtains a payoff of $\pi_i(\phi, a^T_j, n, w_i)$. Thus we need to prove that $T_i > \pi_i(\phi, a^T_j, n, w_i)$ for all $(w_i, w_j) \in \mathbb{R}_+ \times \mathbb{R}_+$ and $n \in (-1, 0) \cup (0, 1)$.

We first obtain the expression for the profit firm $i$ obtains by deviating, $\pi_i(\phi, a^T_j, n, w_i)$. Taking the limit as $w_i \to 0$ of the subgame equilibrium real actions $x^*_i(a_i, a_j)$ and $x^*_i(a_i, a_j)$ derived in Section 4, we obtain the expressions for the subgame equilibrium real actions when only firm $j$ pre-announces, $x^*_i(\phi, a_j)$ and $x^*_i(\phi, a_j)$. The profit of firm $i$ is then obtained by plugging in the pre-announcement $a^T_j$ in the expression for the profit obtaining $\pi_i(\phi, a^T_j, n, w_i) = x^*_i(\phi, a^T_j)^2$. The ratio between the deviation profit and the candidate equilibrium profit for firm $i$ is

$$\frac{\pi_i(\phi, a^T_j, n, w_i)}{T_j} = \frac{(2+w_j)(2(2+w_j)-n^2)}{2((2+w_j) - n^2)^2 + 2w_i(2+w_j)-n^2)(2+w_j)}.$$

This ratio is smaller than one for $n \in (-1, 0) \cup (0, 1)$. Therefore, $T_j > \pi_i(\phi, a^T_j, n, w_i)$. That is, $(D, D)$, is an equilibrium for $n \in (-1, 0) \cup (0, 1)$. ■