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Who Gains from Growth: A Dynamic Model of Kuznets' Hypothesis*

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Abstract

This paper uses an overlapping generations model to analyze the relationship between economic development and income distribution and the effect of government taxation and transfers on the distribution of income. Individuals differ in their inherited stocks of human capital and therefore in their productivity and income, and their leisure and investment decisions. Individual productivity depends on inheritance, on investment in education and on society's average productivity. Consequently relative incomes and the distribution of income changes as the level of income changes. Government taxes and redistributes income intertemporally and intratemporally based on decisions made by majority voting. These decisions affect productivity but also produce a welfare class.

In his presidential address to the American Economic Association and in subsequent writings, Simon Kuznets (1955, 1966, 1979) suggested that the relation of economic development to income inequality is an inverted U. In the early stages of economic development, inequality increases. As growth proceeds, the spread of the distribution slows, stops and, in the late stages of development, reverses.

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tween groups are relatively large, the disincentive effects of taxes dominate the positive effects of growth on redistribution.

Persson and Tabellini use an overlapping generations model to study the relation of income distribution to the growth rate of income, rather than the level as in Kuznets' hypothesis. Everyone saves at the same rate; individuals with more skill and income invest more per period. Individual investment in education does not directly change the distribution of income. Majority rule redistributes income and, therefore, lowers the growth rate. The authors' model implies, and their tests suggest, that inequality lowers the growth rate.

We also use an overlapping generations model, but we study the effects of the level of income or productivity on the distribution of income as the level of income changes. Productivity (or human capital) is a continuous variable. As in Perotti (1990) there is an externality, but the externality arises here directly from the effects of education on average productivity; individual investment in education increases both individual and average productivity. Both the level and distribution of income change. Voters respond to these changes by setting tax rates to provide redistribution in the second period of life. These political decisions affect incentives thereby changing the level of income and the distribution of income before and after taxes. Since both generations pay taxes, voters' decisions cause both intragenerational and intergenerational redistribution.

Section 1 presents the model of an economy with overlapping generations and investment in education. Section 2 considers the effect of government taxes and transfers. In Section 3, we extend the results in Meltzer and Richard (1981) to a growing economy. The median voter determines the tax rate and amount of redistribution that maximizes his utility and sustains the equilibrium. This section also presents some evidence on the relation of income to income distribution and the equilibrium tax rate. A conclusion summarizes our results.

The Model and Its Implications

The model we develop has two periods with equal numbers of people living in each period. Population is constant. Each period consists of a unit of time. In the first period, individuals allocate time between labor and investment in human capital. Investment increases second period productivity. In the second period, allocation of time is between labor and leisure. There are no bequests, debt or physical capital.

Each person inherits an endowment of human capital from his family. Endowments differ across individuals, so productivity differs across individuals in each generation.

A government collects taxes on the earnings of both generations to finance
lump sum benefits paid to the old generation. At any time the tax rate is a constant independent of income, so tax collections are proportional to income. The lump sum benefit makes net redistribution progressive; upper income earners make a net payment to government, and lower income earners receive a net transfer. The tax rate and the amount redistributed are set by majority rule one period ahead.

All consumers have the same preference function, represented by a Stone-Geary utility function, shown as equation (1).\(^1\) We use the subscripts \(t-1\) and \(t\) to denote the two periods that an individual lives, the symbols \(c\) and \(\ell\) for consumption and leisure, and \(c_{t-1y}\) and \(c_{t0}\) for the consumption of a young and old person.

\[
u(c^\ell_{t-1y}, c^\ell_{t0}, \ell^\ell_{t0}) = \ln(c^\ell_{t-1y} + \theta) + \beta[\ln(c^\ell_{t0} + \gamma) + \alpha \ln(\ell^\ell_{t0} + \lambda)]
\]

(1)

where \(\theta, \gamma, \lambda, \alpha\) and \(\beta\) are positive constants; \(\beta\), the consumer’s time discount factor, is less than one. Both consumption and leisure are assumed to be normal goods. To keep the notation simple, we omit the subscripts \(y\) and \(\ell\) in the remainder of this section and the superscript \(i\) here and in the following sections.

Let \(h_{t-1}\) be the individual’s inherited endowment of human capital. This endowment yields a flow proportional to the stock and is measured in units of productivity or output. \(n_{t-1}\) is the fraction of time spent at work in the first period, so that \(n_{t-1}h_{t-1}\) is the \(t^{th}\) individual’s earned income in the first period, and \(n_th_t\) is his second period earned income. For the community, earned income is the value of output in period \(t\) produced by the current young and old. It is obtained by aggregating the productivity weighted time spent at work over workers in both generations and summing the two generations.

The consumer’s budget constraints are

\[
c_{t-1} = (1 - \tau_{t-1})n_{t-1}h_{t-1}
\]

(2)

\[
c_t = (1 - \tau_t)n_th_t + r_t
\]

(2a)

\[
\ell_t = 1 - n_t
\]

(2b)

where \(1 - n_{t-1}\) is the time allocated to schooling, \(r_t\) is the amount of income redistributed, and \(\tau_{t-1}\) and \(\tau_t\) are the income tax rates. Tax rates are set each period for the next period before allocation decisions are made; \(\tau_{t-1}\) is the tax rate applicable to income earned by young and old in \(t-1\), and

\(^1\)Making utility linear in consumption or leisure does not remove the model's complexity.
\( \tau_t \) is the tax rate for period \( t \) set in period \( t - 1 \) by the current young and old generations. \( \tau_{t-1} \) was set in the previous period, so it is predetermined.\(^2\)

Consumers take tax rates and government spending as given when making consumption, investment (or training) and leisure decisions. The tax rate is always less than 100%.

Following Uzawa (1965), Rosen (1976) and Lucas (1988), time devoted to human capital accumulation depends on an individual’s inherited human capital and on society’s average level of human capital, \( \bar{h}_{t-1} \).

\[
\bar{h}_t = \bar{h}_{t-1} + \delta(1 - n_{t-1})\bar{h}_{t-1}
\]

where \( \delta \) denotes the effectiveness of human capital accumulation. As in Kuznets (1966), growth depends on the (transnational) stock of human knowledge.

Each person’s human capital and, thus, his productivity, changes directly with his own investment \( (1 - n_{t-1}) \) and with the general level of human capital or education. Individual productivity is distributed according to the distribution function \( F(h_t, \bar{h}_t) \). \( F \) is continuous and differentiable. The shape of the \( F \) function, measured by the ratio of the mean (\( \bar{h}_t \)) to the median (\( \bar{h}_t^2 \)), changes with \( \bar{h}_t \). Decisions to invest in education can change both the level and distribution of productivity.

Decisions to invest in productivity differ across the income distribution and depend on tax rates, transfer payments and the individual’s productivity and, thus, on inherited human wealth. Since individual productivity and income differ, the opportunity cost of education differs. The higher one’s income, the higher is the value of time devoted to education. The return to education may differ also. If there are diminishing returns to individual investment in education, individual returns decline as income rises. For these reasons, the distribution of income may change, as Kuznets suggested, toward greater equality as the community’s income increases.

Returns to individual investment in education may work in the opposite direction, however. The return to time spent increasing skill and knowledge may increase with the size of the initial endowment, \( \bar{h}_{t-1} \). Increasing returns to investment in education work to widen the spread of the distribution of income.

In addition to the private effects there are spillover effects from individual investment, represented by \( \bar{h}_{t-1} \), the community’s average productivity level, and from the effects of taxes and income redistribution. These government policy decisions modify the changes in income distribution just noted. As Kuznets (1966) suggested, the before tax income distribution will reflect the effects of (distortionary) taxes and transfers. Welfare payments, or in our

\(^2\)We assume that redistribution to start this system occurred long ago.
analysis retirement benefits, encourage leisure. Tax rates also affect labor-leisure choice and decisions to invest in education by altering the return from labor in the second period. The effect of taxes and transfers is not uniform across the income distribution. And taxes and transfers affect the decision to invest in training or education, since the additional income produced by investment in education depends on the decision to remain in the labor force.

A consumer assesses these different effects by maximizing (1) subject to (2), (2a), (2b) and (3). The first order conditions for a maximum are given by

\[ n_{t-1} = \frac{(1 - \tau_t)((h_{t-1} + \delta \tilde{h}_{t-1})k_{t-1} - \theta \delta \beta)n_t + (\gamma + r_t)}{\delta(1 + \beta)(1 - \tau_t)k_{t-1}\tilde{h}_{t-1}n_t} \]  

and

\[ n_t = \frac{(1 + \lambda)(1 - \tau_t)h_t - \alpha(\gamma + r_t)}{(1 + \alpha)(1 - \tau_t)h_t} \]  

where \( k_{t-1} = (1 - \tau_{t-1})h_{t-1}/\tilde{h}_{t-1} \).

Let \( x_t \) be the productivity level at which an individual chooses full time leisure by retiring from the labor force in the second period. Using (4a), we solve for \( n_t = 0 \).

\[ x_t = \frac{\alpha(\gamma + r_t)}{(1 + \lambda)(1 - \tau_t)} \]  

The retirement choice depends positively on the social decisions represented by \( r \) and \( \tau \). The larger the retirement benefit and the higher the tax rate on earned income, the higher is the productivity level at which individual's retire.

We rewrite the first order conditions (4) and (4a) using (5).

\[ n_{t-1} = b_1 + b_2 \frac{x_t}{n_t} \]  

and

\[ n_t = \frac{1 + \lambda}{1 + \alpha}(1 - \frac{x_t}{h_t}), \]  

where

\[ b_1 = \frac{[(h_{t-1} + \delta \tilde{h}_{t-1})k_{t-1} - \theta \delta \beta]/[\delta(1 + \beta)k_{t-1}\tilde{h}_{t-1}]}{(1 + \lambda)/[\delta \alpha(1 + \beta)\tilde{h}_{t-1}]} \]

\[ b_2 = (1 + \lambda)/[\delta \alpha(1 + \beta)\tilde{h}_{t-1}]. \]

By assumption, \( n_t \) and \( n_{t-1} \leq 1 \). From (3), \( n_{t-1} = 1 \) implies \( h_t = h_{t-1} \); there is no investment in productivity in this case. But from (4a) \( h_t \) must be
greater than \( x_t \) to maintain \( n_t \) in the unit interval. Hence those who invest must have \( n_t > 0 \) and \( h_t > x_t \). A positive value of \( n_t \) is required for \( n_{t-1} \leq 1 \) in (4)\( / \).

It follows that a minimum level of productivity is required for learning. Let \( z_t \) be that minimum level. If, as assumed above, both consumption and leisure are normal goods,\(^3\)

\[
z_t > \left[ 1 + \left( \frac{1 + \alpha}{\alpha(1 + \beta)} \right)^{\frac{1}{2}} \right] x_t. \tag{6}
\]

\( z_t \) is positively related to \( x_t \) and, therefore, increases with tax rates and redistribution. Higher tax rates and redistribution increase not only the number who choose welfare and full-time leisure but also the number that do not invest in learning when they are young.

There are three distinct groups in the population making decisions to allocate time between work, leisure, and investment. \( F(x_t) \) percent of the population chooses full-time leisure in the second period; they do not invest in \( t-1 \), and they do not work in \( t \). Those with productivity between \( F(x_t) \) and \( F(z_t) \) do not invest in \( t-1 \), but they work in \( t \). They are the working “poor”. Their inherited productivity is above the level at which they choose full time leisure in period \( t \), but it is below the level at which it pays to invest. For these individuals, as for those with lower productivity, \( h_t = h_{t-1} \). The rest of the population, \( 1 - F(z_t) \), chooses to invest in \( t-1 \) and to work in \( t \).

An individual's earned income in the two periods is \( n_{t-1} h_{t-1} \) and \( n_t h_t \) respectively. Let \( Y_t \) be lifetime or permanent income, so the change in an individual's permanent income (neglecting the constant discount rate) is

\[
dY = h_{t-1} dn_{t-1} + h_t dn_t + n_t dh_t. \tag{7}
\]

To learn how the distribution of income changes, we study the changes in \( n_{t-1}, n_t \) and \( h_t \).

Inspection of (4)\( / \) and (4a)\( / \) shows that \( n_t, n_{t-1} \), and \( h_t \) are interdependent; \( n_{t-1} \) depends inversely on \( n_t \), and \( n_t \) depends on \( h_t \). The consumer allocates time to \( n_t \) and \( n_{t-1} \) when he sets his plan, but decisions about these allocations depend on \( h_t \). Solutions for each of these variables in terms of the given \( h_{t-1} \) and \( \bar{h}_{t-1} \) and the policy variables cannot be expressed in an informative way. We proceed by totally differentiating (3), (4)\( / \) and (4a)\( / \) with respect to \( h_{t-1} \) and \( \bar{h}_{t-1} \). Then we evaluate the derivatives for high and low income earners.

\[
\frac{dn_{t-1}}{dh_{t-1}} = \frac{db_1}{dh_{t-1}} - \frac{b_2 x_t}{n_t} \frac{dn_t}{dh_{t-1}} \tag{8}
\]

\(^3\)See the start of section 2 for proof.
\[
\frac{dn_t}{dh_{t-1}} = \frac{(1 + \lambda)x_t}{(1 + \alpha)h^2_{t-1}} dh_{t-1}
\]
(8a)

\[
\frac{dh_t}{dh_{t-1}} = 1 - \delta h_{t-1} \frac{dn_{t-1}}{dh_{t-1}}
\]
(8b)

and

\[
\frac{dn_{t-1}}{dh_{t-1}} = \frac{db_t}{dh_{t-1}} + \frac{x_t}{n_t} \frac{db_2}{dh_{t-1}} - \frac{b_2 x_t}{n_t^2} \frac{dn_t}{dh_{t-1}}
\]
(9)

\[
\frac{dn_t}{dh_{t-1}} = \frac{(1 + \lambda)x_t}{(1 + \alpha)h^2_{t-1}} dh_{t-1}
\]
(9a)

\[
\frac{dh_t}{dh_{t-1}} = \delta(1 - n_{t-1}) - \delta h_{t-1} \frac{dn_{t-1}}{dh_{t-1}}
\]
(9b)

Equations (8) and (9) are two blocs of three equations in three unknowns. We can solve for the derivatives of \(n_{t-1}, n_t, h_t\) with respect to \(h_{t-1}\) and \(h_{t-1}\). For individuals with relatively high income, \(h_t\) is large relative to \(x_t\). At some point in the productivity distribution, \(x_t/h_t\) approaches zero, and from (4a), \(n_t\) approaches a constant. Using this restriction in (8) and (9), we derive the responses of (human capital) investment and labor and leisure choices to changes in private and social productivity for high income earners. These are shown as equations (10)-(10b).

\[
\frac{dn_{t-1}}{dh_{t-1}} = [\theta \delta \beta \frac{h_{t-1}}{h^2_{t-1}} + (1 - \tau_t)h^2_{t-1}]/[\delta(1 + \beta)(1 - \tau_t)h_{t-1} h^2_{t-1}] > 0
\]
(10)

\[
\frac{dn_t}{dh_{t-1}} = -[(1 + \alpha)x_t + \alpha h_{t-1}]/[\delta \alpha(1 + \beta)h^2_{t-1}] < 0
\]
(10a)

\[
\frac{dh_t}{dh_{t-1}} = [1 + \frac{\theta}{(1 - \tau_t)h_{t-1}}] \frac{\delta \beta}{1 + \beta} > 0
\]
(10b)

From (10a) rising average productivity reduces hours of work, so investment in human capital rises in the first period of life for everyone. The size of the responses in (10a) depends on \(h_{t-1}\); the larger is \(h_{t-1}\), the larger the response. Although everyone who invests increases his or her investment in education as income and productivity increase, upper income individuals invest relatively more than individuals with less income. This effect is contrary to Kuznets’ conjecture.

Rising average productivity also raises human capital and productivity directly, (10b).\(^4\) From (10b), we see that the effect of \(h_{t-1}\) on \(h_t\) declines

\(^4\) There has been considerable interest in the convergence of income or productivity. Equation (10b) is independent of \(h_{t-1}\). This implies that the response of individual productivity to average productivity does not converge as average productivity rises.
with inherited productivity, $h_{t-1}$. Private productivity works in the opposite direction. Equation (10) shows that those with higher productive endowment ($h_{t-1}^i$) work more in $t - 1$ and invest less to increase their productivity. The effect is to narrow the distribution of productivity and income among those with relatively large inherited productivity.

We now consider the other end of the productivity distribution where productivity is $F(z_t)$ or less. A worker with low productivity who works in period $t$ has $x_t/h_t$ close to unity; from (4a) $n_t$ is close to zero, and $n_{t-1}$ approaches one. There is little incentive to invest in productivity improvement, so

$$h_t \approx h_{t-1}.$$  

Using this approximation in equations (8) and (9), we derive the effects of changes in $h_{t-1}$ and $\bar{h}_{t-1}$ on low income earners' decisions to work and invest.

$$\frac{dn_{t-1}}{dh_{t-1}} = -\frac{(h_{t-1} - x_t)[\alpha(h_{t-1} - x_t) + (1 + \alpha)x_t]}{\alpha(1 + \beta)(h_{t-1} - x_t)^2 - (1 + \alpha)x_t^2} \frac{h_{t-1}}{\delta \alpha h_{t-1}^2} < 0 \quad (10c)$$

$$\frac{dn_{t-1}}{dh_{t-1}} = \frac{[\alpha(h_{t-1} - x_t)^2 - (1 + \alpha)x_t^2]/\delta \bar{h}_{t-1} + \alpha \theta \beta/(1 - r_{t-1})h_{t-1}^2}{\alpha(1 + \beta)(h_{t-1} - x_t)^2 - (1 + \alpha)x_t^2} > 0 \quad (10d)$$

From (6), the denominators of (10c) and (10d) are positive, so the signs are determined unambiguously.

As average productivity rises, investment in human capital rises for low as for high income individuals. As in (10a), the size of the response in (10c) to average productivity is negative and rises with $h_{t-1}$; increases in average productivity tend to spread the distribution of income by reducing first period work and increasing investment, contrary to the Kuznets' hypothesis.

The combined effects of rising average productivity on the income distribution resulting from the decisions by upper and lower income groups is uncertain. Given the lower propensity to invest by those with lowest incomes, the combined effect may spread the distribution of income.

From (10d), hours of work increase and investment declines with increases in individual productivity. The signs of (10d) and (10) are the same, so the direction of the effect of higher inherited productivity is the same for upper and lower income groups. The size of the derivative in (10d) rises at a decreasing rate, so the effect of private investment is to narrow the distribution of income for the low income as for the high income group.

The preliminary conclusion from the model is consistent with Kuznets' hypothesis about an inverted U shape if two conditions are met. First, at low levels of income and average productivity, the effect of average productivity on investment in education and training dominates the response to individual
productivity. Inequality increases. Second, as income rises, the relative size of the responses reverses; the effect of individual investment decisions dominate, reducing inequality.

Using productivity as a measure of income, inspection of (10a) and (10c) shows that, as the economy grows (average income and productivity increase) the responses in (10a) and (10c) become smaller. Average productivity works to spread the distribution of income but at a slower rate.

We can now bring together the effects of rising productivity on lifetime incomes. Partial derivatives of (7) with respect to $h_{t-1}$ and $\bar{h}_{t-1}$ are given by

$$\frac{dY_t}{dh_{t-1}} = \frac{1 + \lambda}{1 + \alpha} + n_{t-1}(h_{t-1} - \frac{1 + \lambda}{1 + \alpha} \frac{dn_{t-1}}{dh_{t-1}}$$

$$\frac{dY_t}{d\bar{h}_{t-1}} = \frac{\delta(1 + \lambda)}{1 + \alpha} (1 - n_{t-1}) + (\bar{h}_{t-1} - \frac{1 + \lambda}{1 + \alpha} \frac{dn_{t-1}}{dh_{t-1}}$$

The truth or falsity of Kuznets' hypotheses depend on two effects that work in opposite directions. First is the response of $dY_t/dh_{t-1}$ to changes in productivity at different levels of individual income. This is shown by $d^2Y/dh_{t-1}$. Second is the response of $dY/d\bar{h}_{t-1}$ to changes in individual productivity at different levels of individual productivity. This is shown by the cross partial derivative $\frac{d^2Y_t}{dh_{t-1} \, d\bar{h}_{t-1}}$. The signs of the relevant derivatives are:

$\frac{d^2Y_t}{dh_{t-1} \, d\bar{h}_{t-1}}$ is positive for upper, and lower income individuals, and

$\frac{d^2Y_t}{dh_{t-1} \, d\bar{h}_{t-1}}$ is negative for upper and lower income individuals.

Kuznets' proposition requires that the second derivatives be smaller than the cross partials after $Y$ reaches some level. The former spread the income distribution, while the latter compress the distribution. In the opposite case, data relating income distribution to levels of income would reject the proposition, as some of the papers cited earlier have found.

We summarize the results to this point in Figure 1. Rising individual productivity increases the response to individual productivity, shown in the top panel, and lowers the response to average productivity, shown in the lower panel. The effect on income is the sum of the two changes. The broken segments show that we have obtained derivatives only for the upper and lower income groups.

None of the responses to this point take account of the effects on the income distribution of changes in the size of taxes and transfers. The following section considers these effects. The responses we have discussed also do not consider changes in demand as the economy grows. Demand for professional services increases with growth of income and longevity, which itself
Figure 1
increases with income. Satisfying these demands for medical, legal, financial, accounting and educational services requires more of the population to invest in skills and training, thereby compressing the income distribution. Kuznets does not discuss this channel, and we have not introduced this additional complexity. It would have the same effect as a positive response to rising $h_{t-1}$ on the derivative in (6). The result would be relatively more investment in human capital by those with income below the mean (or other point of the distribution) than by those above.

The Effect of Taxes and Transfers

Kuznets recognized that the (before tax) distribution of income that we observe reflects individual responses to taxes and transfers. In our model the income distribution may change in response to taxes and transfers as a result of labor and leisure choices and decisions to invest in productivity.

The government’s budget is always balanced; tax revenues finance equal lump sum transfers to everyone. The tax rate on income earned by the young and old is the same at any time. Let $\tau_t$ be the actual tax rate.

$$\tau_t(\bar{y}_{t0} + \bar{y}_{ty}) = r_t = \tau_t\bar{Y}_t$$

(11)

where $\bar{y}_{t0}$ and $\bar{y}_{ty}$ are the mean income of the old and young generation in period $t$ and $\bar{Y}_t$ is society’s per capita income at time $t$. The share of disposable income taxed at time $t$ rises as disposable income increases.

$\tau$ and $r$ affect the allocation of time to labor and investment in education through their effect on retirement decisions, given by $x_t$, as shown in (5). As tax rates increase, the marginal utility of second period consumption declines and, as $r$ increases, more of desired consumption is satisfied by transfers. Individuals choose more leisure, as shown by $(4a)t$. By raising tax rates and transfer payments, policy raises the level of earned income and productivity at which individual’s retire: more people retire. Lower tax rates and transfers reduce retirement.

Equations (12) to (12b) express $n_{t-1y}, n_{to}$ and $h_{to}$ as functions of $x_t$, where $n_{t-1y}$ are the young in period $t - 1$ and $n_{to}$ are old in period $t$.

$$n_{t-1y} = n_{t-1y}(x_t)$$

(12)

$$n_{to} = n_{to}(x_t)$$

(12a)

$$h_{to} = h_{to}(x_t)$$

(12b)

Totally differentiating (3), (4)/ and $(4a)/$ and using (12)–(12b) we obtain the responses of $n_{t-1y}, n_{to}$, and $h_{to}$ to the current tax rate and transfer summarized in $x_t$. The derivatives are shown as equations (9) to (9b).
\[
\frac{dn_{t-1y}}{dx_t} = -\frac{b_2}{\pi_t} \left( \frac{1}{n_t} + \frac{(1 + \lambda)x_t}{(1 + \alpha)h_{to} n_{to}^2} \right) > 0 \quad (13)
\]

\[
\frac{dn_{to}}{dx_t} = \frac{(1 + \lambda)}{(1 + \alpha)\pi_t} \left( \frac{1}{h_{to}} \frac{1}{n_{to}^2} + \frac{\delta b_2 h_{t-1y} x_t}{n_{to} h_{to}^2} \right) < 0 \quad (13a)
\]

\[
\frac{dh_{to}}{dx_t} = \frac{\delta b_2 h_{t-1y}}{\pi_t} \left( \frac{1}{n_t} + \frac{(1 + \lambda)x_t}{(1 + \alpha)h_{to} n_{to}^2} \right) < 0 \quad (13b)
\]

where

\[
\pi_t = \frac{1 + \alpha}{\alpha(1 + \beta)} \left( \frac{x_t}{h_t - x_t} \right)^2 - 1 \quad (13c)
\]

The signs in (13) depend on the sign of \( \pi_t \), and the sign of \( \pi \) depends on the response of consumption and leisure to transfer payments. Both goods are normal goods, so the partial derivative are positive. Hence \( \frac{\partial d_{to}}{\partial r_t} > 0 \).

Since \( \ell_t = 1 - n_t \)

\[
\frac{\partial n_{to}}{\partial r_t} = -\frac{\partial n_{to}}{\partial x_t} \frac{\partial x_t}{\partial r_t}.
\]

From (5) \( \frac{\partial x_t}{\partial r_t} < 0 \). The sign of (13a) is negative. Therefore, \( \pi < 0 \). The signs of (13) and (13b) follow.

The signs in equations (13) imply that increases in transfer payments increase effort in the first period, thereby reducing investment in education, and reduce second period productivity and effort. More people choose retirement. Reductions in transfer payments have the opposite effect; investment in education and second period productivity increase. From (5) the response of retirement to tax rate changes is in the same direction as for changes in transfers.

To analyze the responses of individuals with different income levels, we substitute (4a) into (13) and (13a).

\[
\frac{dn_{t-1y}}{dx_t} = \frac{(1 + \alpha)b_2}{1 + \lambda} \left( \frac{h_{to}/x_t}{h_{to}/x_t - 1} \right)^2 \left[ 1 - \frac{1 + \alpha}{\alpha(1 + \beta)} \left( \frac{1}{h_{to}/x_t - 1} \right)^2 \right]^{-1} > 0 \quad (13)'
\]

\[
\frac{dn_{to}}{dx_t} = \frac{(1 + \lambda)x_t}{(1 + \alpha)(h_{to}/x_t)} \left[ 1 + \frac{1 + \alpha}{\alpha(1 + \beta)(h_{to}/x_t - 1)} \right] \left[ 1 - \frac{1 + \alpha}{\alpha(1 + \beta)} \left( \frac{1}{h_{to}/x_t - 1} \right)^2 \right]^{-1} < 0 \quad (13a)'
\]

The absolute values of both (13)' and (13a)' decline with an increase in \( h_{to}/x_t \). As productivity increases, individuals respond less to the increase in either the tax rate or transfers. Hence, rising tax rates and transfer payments widen the spread of the distribution of productivity and income, while a declining tax rate and transfer payment compress the distribution.
Our conclusion about tax and transfers accords with Kuznets’ conjecture. Kuznets argued that increased transfers spread the before-tax income distribution by reducing the wages paid to workers with lower skills. In our model, the effects on human capital and retirement (or leisure), are the principal means by which the distribution changes.

Higher taxes and transfers impose a cost on future generations by discouraging investment. This cost falls particularly on the offspring of those with current low incomes. Since the low income groups vote for the transfers, the choice is voluntary. But future generations do not vote in the current election and, because they inherit lower productivity as a result of the current decision, they are likely to vote for increased transfers and retirement in their turn. In this way, benefits paid today can induce behavior which creates a welfare class. The welfare class in our model, however, must work at least one period.

The Equilibrium Tax Rate and Size of Government

The tax rate is set by majority rule voting. All voters, young and old, at time \( t - 1 \) are eligible to vote. Their votes determine the tax rate applicable to income earned in period \( t \). As shown in (11), all tax receipts are paid as transfers to the generation that is old in \( t \).

The choice of tax rate and redistribution creates both intragenerational and intergenerational transfers. The reason is that three generations are involved. The interests and influence of the three generations differ, however.

The current old are indifferent; they vote randomly and have no effect on the outcome. The tax rate they pay and the transfer payment they receive were set earlier. Since there is no capital and no debt, the only bequest the old leave is the human capital that they passed on to the current young when the young were born.

Since the tax-transfer system is progressive, the current young transfer income within their generation. The tax rate they choose is paid also by the next generation of young on their earnings in \( t \). This provides an intergenerational transfer to finance part of the redistribution paid next period to the current young, as shown in (11).

The generation that will be young in \( t \) is not born when the vote is taken. Hence they do not vote on the tax rate they will pay in \( t \). It might seem that the current young (in \( t - 1 \)) would have an incentive to tax future income heavily, but they are permitted to do so only if they tax themselves at the same rate. Moreover, the current generation recognizes the disincentive imposed by higher taxes on the next generation. There are no bequests, so the model does not directly link the utility of current and future generations. But the income of the generation that will be young in \( t \) enters through the
government budget equation; in this way the current generation is forced to recognize the effects of the taxes they impose on their offspring.

Next period's tax rate is the only issue to be decided. In a single issue election, the voter with median income is the decisive voter. We denote the median or decisive voter by \( d \), and use the decisive voter's first order condition to obtain

\[
\frac{dx_t}{d\tau_t} = \frac{1}{1 - \tau_t} (x_t + \frac{\alpha}{1 + \lambda} y_{to}^d) \quad (14)
\]

where \( y_{to}^d \) is the decisive voter’s income in \( t \). Substituting (11) into (5) and differentiating \( x_t \) totally with respect to \( \tau_t \), we have

\[
0 = (1 + \lambda)(1 - \tau_t) \frac{dx_t}{d\tau_t} - (1 + \lambda) x_t - \frac{\alpha}{2} [\bar{y}_t + \bar{y}_t + \tau_t(\frac{dy_{ty}}{d\tau_t} + \frac{dy_{to}}{d\tau_t})] \quad (15)
\]

where the mean income of the two generations are defined by

\[
\bar{y}_{ty} = \int_0^{\infty} h_{ty} dF(h_{ty}) +
\]

and

\[
\int_{-\infty}^{\infty} n_{to} h_{to} dF(h_{to}),
\]

where \( F(h) \) is the distribution function of individual productivity. \( F(h) \) is continuous and differentiable. Substituting (15) into (14) gives

\[
y_t^d - (\bar{y}_t + \bar{y}_{to}) \frac{\tau_t}{2} \left( \frac{dy_{ty}}{d\tau_t} + \frac{dy_{to}}{d\tau_t} \right) = 0 \quad (16)
\]

Equation (16) resembles the equation derived in Meltzer and Richard (1981, equation 13). The only difference is that the mean income of two generations (and changes in income) replace the single generation they consider. Hence, as in their model, the choice of tax rate depends on the spread of the income distribution, represented by the difference between mean and median, and on the response of mean income to the tax rate. We can, again, use Roberts (1977) lemma to order individual incomes and choose the tax rate.

Differentiating \( \bar{y}_{ty} \) and \( \bar{y}_{to} \) with respect to \( \tau_t \) and letting \( \bar{Y}_t = \bar{y}_{ty} + \bar{y}_{to} \), we have

\[
y_t^d - \bar{Y}_t + \frac{\alpha \tau_t(1 - F)}{(1 + \alpha)(1 - \tau_t)^2} \left[ \varphi + \tau_t \bar{Y}_t + (1 - \tau_t) y_{to}^d \right] = 0 \quad (17)
\]

where

\[
\varphi = \gamma - \frac{\theta \beta (1 + \alpha)}{\alpha (1 + \beta)(1 - F)} \int_{\tau_t}^{\infty} \frac{h_{ty}}{\pi_{t+1}} dF(h_{ty})
\]

(18)
where \( \pi \) is given in (13c), and \( \pi < 0.5 \).

Dividing (17) by \( y^d_{t0} \) and letting \( m = Y_{t0}/y^d_{t0} \) and \( e = \varphi/y^d_{t0} \), we have

\[
\frac{1 + \alpha F}{\alpha(1 - F)}(1 - \tau_t)^2 + (2m - 1 + e)(1 - \tau_t) - (m + e) = 0. \tag{19}
\]

Solving for \( \tau_t \) gives the following approximation.\(^6\)

\[
\tau_t \approx \frac{(1 + \alpha)(m - 1)}{\alpha(1 - F)(1 + e)}. \tag{20}
\]

As in Meltzer and Richard (1981), the median voter chooses a tax rate that increases with (1) \( m \), the ratio of mean to median income, (2) \( F \), the proportion of the population that voluntary retires, and (3) the income of the median or decisive voter, \( y^d \). The last effect enters through \( e \). In addition, the tax rate chosen by the decisive voter in \( t - 1 \) depends on the numerator of \( \varphi \). In the static case, \( \varphi = \gamma \). In the dynamic case, \( \varphi \) is positive and larger than in the static case.\(^7\) The tax rate is therefore lower in the dynamic model, ceteris paribus.

In a growing economy with rising income the current generation of young uses its vote on the tax rate to share in the growth of income. The amount by which the tax rate increases in the dynamic model depends on the response of effort to the tax rate. A low response raises the tax rate; a large response lowers the rate.

The current model has neither debt nor capital, so the only way the current generation can share in the higher future income is by taxing future income to increase current consumption. If there were debt and real capital and an unending sequence of future generations, as in Cukierman and Meltzer (1989), the current generation of voters could smooth income by incurring deficits and leaving government debt to their offspring. Unlike taxation, deficit finance would not require the current generation to impose higher taxes on themselves to share in future income.

Equation (20) relates the tax rate to the ratio of mean to median income and to the level of median income. All three are endogenous variables in our model, and the equation relating them is non-linear. Kuznets’ hypotheses are difficult to test directly also. The first hypothesis neglects the effects

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\(^5\)We take future tax rates as given. Our solution is a Nash equilibrium for today’s voters.

\(^6\)We use a second-order Taylor’s expansion.

\(^7\)The expression inside the integral on the right of (18) is the response to the tax rate of the labor supply by the young generation in \( t \) (who were unborn and did not vote to set \( \tau_t \)). The integral shows the young generation’s aggregate response of work to the tax rate. This response is negative but is multiplied by a minus sign, so \( \varphi \) rises relative to the static case.
of taxation, which are undoubtedly present in the data. We consider the combined effects of taxation, redistribution and the level of income, as in his second hypothesis.

Table 1 shows a modest effort to consider the relation between the principal variables in countries with relatively high ("rich") income and low and middle income as classified by the World Bank. Data for median income are the mean of the second and third quintiles from World Bank (1991, Table 30) expressed in the standard real dollars used in the table. These data are for different years and are subject to many qualifications.\(^8\) We have interpreted tax rate as equal to the share of government spending on housing, welfare, and retirement from World Bank (1991, Table 11).\(^9\) This measure is most closely related to our model.

The data suggest that inequality declines with per capita income. The ratio of median to mean income in the samples is 0.72 for the "rich" countries and 0.60 for poor to middle income countries; the ratio of median to mean income rises as income increases.

The regressions provide additional support. They suggest that each $1000 of additional mean income raises the median by $740 in the rich countries and $640 in the poor and middle countries if median income is used as the dependent variable. When mean income is the dependent variable, the comparable values (the reciprocal of the coefficient of median income) are 0.82 and 0.75. These values are shown below mean income in the last three rows of the table. In all cases, the marginal effect of rising income is above the average effect, so the ratio of median to mean income increases with income.

The change in inequality is relatively small. If per capita income is $12,000 median income is $8640 at the mean value (0.72). A $1000 increase in per capita income adds $740 or $820 to the median "rich" income depending on whether median or mean income is the dependent variable. The ratio of median to mean income rises from 0.72 to 0.7215 or 0.7277. A doubling of per capita income could raise the ratio as much as 5 percentage points. This estimate neglects any non-linear effects, however.

The last two rows of the table introduce "the tax rate", measured here by the share of government spending on welfare and retirement. The coefficient of this variable is positive for median income and negative for mean income given the level of mean (median) income. Increased welfare spending appears to be associated with increased equality, contrary to Kuznets' hypothesis. The World Bank describes the data on income distribution as disposable income, so our estimate is not a direct test of our model or Kuznets' hypothesis. The principal result is unchanged.

\(^8\)Our conclusions are unchanged if we use the income distribution data from Papanek and Kyn (1986). We thank Torsten Persson and Guido Tabellini for providing these data.

\(^9\)We also used the share of GDP spent by government separately and in addition to the welfare spending ratio. The principal result is unchanged.
Table 1:

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Real Per Capita Income</th>
<th>Welfare Spending&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Constant</th>
<th>$R^2$</th>
<th>N</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>median income</td>
<td>0.74</td>
<td>0.1 (12.94)</td>
<td>-71.5</td>
<td>.90</td>
<td>19</td>
<td>&quot;rich&quot; countries</td>
</tr>
<tr>
<td>median income</td>
<td>0.64</td>
<td>0.4 (9.72)</td>
<td>-92.7</td>
<td>.85</td>
<td>18</td>
<td>&quot;poor and middle&quot; income</td>
</tr>
<tr>
<td>median income</td>
<td>0.76</td>
<td>0.4 (45.35)</td>
<td>-392.4</td>
<td>.98</td>
<td>37</td>
<td>all</td>
</tr>
<tr>
<td>median income</td>
<td>0.74</td>
<td>0.23 (15.59)</td>
<td>-1062.1</td>
<td>.94</td>
<td>19</td>
<td>&quot;rich&quot;</td>
</tr>
<tr>
<td>median income</td>
<td>0.74</td>
<td>0.11 (24.99)</td>
<td>-490.7</td>
<td>.98</td>
<td>26</td>
<td>all</td>
</tr>
</tbody>
</table>

| Median Income      | 1.22                   | 1.22 (12.93)               | 1225.1   | .90  | 19 | "rich" |
| mean income        | 1.33                   | 0.75 (9.72)                | 530.3    | .84  | 18 | "poor and middle" |
| mean income        | 1.28                   | 0.78 (15.59)               | 1954.5   | .94  | 17 | "rich" |

<sup>a</sup>share of government spending on welfare in 1989.

$t$ statistics in parentheses.

hypothesis. We do not know of any data on before tax income distribution. We suggest caution, also, about inferring the direction of causality among these interdependent variables.

Conclusion

In the 1950s and 1960s, Simon Kuznets offered two hypotheses about the effects of economic development, taxes and transfers on the distribution of income. First, he suggested that initially economic development increased inequality in income distribution, but continued development reversed the effect, reducing inequality. The relation between the level of income and the distribution of income is an inverted U, on this hypothesis.

Kuznets' second hypothesis states that increased taxes and transfers increased inequality by widening the (before tax) distribution of income. Thus, the effect of higher income may be offset by the effect of higher taxes and transfers if taxes and transfers rise with income.

This paper develops a dynamic version of the Meltzer-Richard (1981) hypothesis that relates the level of income, the distribution of income, the tax rate and income transfers in a growing economy where income changes. As before, individuals differ in productivity and, therefore, in income. Productivity can be increased by investment in education or training. As in recent growth theory, investment in education is the driving force in growth but, in our model, the effects are not uniform across the income distribution. Everyone benefits from rising average productivity, but individuals can benefit also from investment that changes individual productivity.

The tax rate in our model is proportional to income, but transfers are lump sum, so the tax-transfer system is progressive. This provides for redistribution within the working population. Taxes are paid by young and old workers to finance benefits to the current old. Tax rates and transfers are set each period for the next period ahead, so the current young and old generations vote on the tax rate to be paid by the generation that is not yet born. Hence, redistribution is intergenerational as well as intragenerational. There is no capital and no debt. Budgets are always balanced.

To derive Kuznets' propositions, we first analyze the effects of changes in individual and average productivity on individuals with high and low incomes, neglecting taxes and transfers. We find that changes in average and individual productivity have opposite effects on the distribution of income. Higher individual productivity spreads the distribution, while higher average productivity compresses the distribution. The truth or falsity of Kuznets' first proposition depends on the relative strength of the two effects. This seems consistent with evidence from a number of cross-section and time series studies. Some of these studies support, and others reject, Kuznets' hypoth-
esis. The evidence we present suggests that on balance inequality declines modestly as the level of income rises.

Kuznets' second hypothesis suggests that the observed relation between the level and distribution of before tax income depends on the tax-transfer system. Studies that neglect these fiscal effects do not properly test Kuznets' propositions. Our analysis provides a foundation for Kuznets' second hypothesis, but the data suggest that spending for redistribution increases equality in the distribution of income. However, this is not a direct test, since the data on which we rely are for the distribution of disposable income, not before tax income.

Our analysis suggests that the tax-transfer system can produce a "poverty class." The reason is that transfer payments encourage low productivity workers to leave the labor force in the second period of life. Knowing that they will retire, these workers invest less in their own productivity. Since children inherit their productivity level from their parents, the children also choose early retirement and invest relatively little in education. Thus, the progressive tax-transfer system tends to discourage generations of the poorest individuals from investing in education.

To close the model, we determine the tax rate (and the level of spending) by majority rule. As in the static Meltzer and Richard (1981) model, the tax rate increases with the ratio of mean to medium income, the proportion of the population that subsists on transfers in the second period of life, and the level of (median) income. The dynamic model implies that the tax rate depends, also, on the income of the next (future) generation. With rising income and productivity, the current generation levies taxes on future income. These taxes pay for some of the transfers that the current generation receives. Since the current young generation must pay the same tax rate they levy on their children, there are limits to the amount of intergenerational redistribution they choose.
References


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