7-2010

A Single-Step Maximum A Posteriori Update for Bearing-Only SLAM

Stephen Tully
Carnegie Mellon University

George Kantor
Carnegie Mellon University

Howie Choset
Carnegie Mellon University

Published In

AAAI Conference on Artificial Intelligence.

A Single-Step Maximum A Posteriori Update for Bearing-Only SLAM

Stephen Tully
Department of Electrical and Computer Engineering
Carnegie Mellon University, Pittsburgh, PA
stully@ece.cmu.edu

George Kantor and **Howie Choset**
The Robotics Institute
Carnegie Mellon University, Pittsburgh, PA
{kantor@ri, choset@cs}.cmu.edu

Abstract

This paper presents a novel recursive maximum a posteriori update for the Kalman formulation of undelayed bearing-only SLAM. The estimation update step is cast as an optimization problem for which we can prove the global minimum is reachable via a bidirectional search using Gauss-Newton's method along a one-dimensional manifold. While the filter is designed for mapping just one landmark, it is easily extended to full-scale multiple-landmark SLAM. We provide this extension via a formulation of bearing-only FastSLAM. With experiments, we demonstrate accurate and convergent estimation in situations where an EKF solution would diverge.

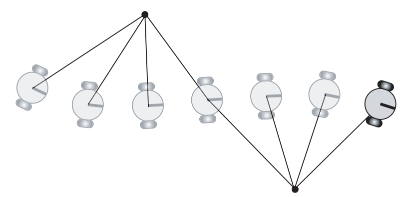

Figure 1: This is a depiction of the SLAM problem. As the mobile robot moves through the environment, it receives exteroceptive measurements to landmarks it observes and must map out their spatial positions.

Introduction

Simultaneous localization and mapping (SLAM) is a mobile robot task that involves incrementally building a map of an unknown environment while simultaneously localizing the robot in that map. The bearing-only variant of SLAM addresses the problem where a robot's exteroceptive sensing is limited to a bearing-only sensor, such as monocular vision. One goal of bearing-only SLAM is to infer the locations of observed landmarks based on noisy measurements from different vantage points, as seen in Fig. 1. Thus, bearing-only SLAM is inherently a filtering problem in which information is collected over time to compute the most likely map.

In (Tully et al. 2008), it is shown that applying the extended Kalman filter (EKF) to bearing-only SLAM will often lead to a divergent state estimate. So far, most researchers have only created workaround solutions to this problem that still choose to rely on the EKF, despite its shortcomings. The majority of these solutions involve delaying the estimation process (Deans and Hebert 2000; Bailey 2003; Costa, Kantor, and Choset 2004) or reparameterizing the state vector (Montiel, Civera, and Davison 2006; Civera, Davison, and Montiel 2007).

Our main contribution in this paper is the introduction of a novel nonlinear filtering algorithm, customized for bearing-only SLAM, that replaces the inherently flawed EKF update with an optimization problem whose goal is to update the mean of the filter with the single-step maximum a posteriori (MAP) estimate. We also make an important insight into the theory of bearing-only mapping: by performing a polar transformation on the landmark coordinates in the Kalman state, the underlying optimization problem for the bearing-only update can be reduced to a one-dimensional search. We then provide a thorough proof that our numerical optimization procedure is guaranteed to find the global minimum of its corresponding cost function. The significance of this proof is that we can be sure that the filter will dependably track the peak of the single-step posterior distribution.

Our proposed filtering scheme is designed to solve a simplified form of the bearing-only SLAM problem in which there is just one landmark to map and the robot has known motion. While this simplification may seem limiting, there is in fact no loss of generality, for our technique is easily extended to full-scale multiple landmark SLAM with a customized implementation of the FastSLAM algorithm, which was first introduced in (Montemerlo et al. 2002).

Related Work

Delaying the initialization of landmarks is a popular method in bearing-only SLAM. A batch update with all of the stored observations is demonstrated in (Deans and Hebert 2000). In (Bailey 2003), initialization is postponed until a pair of measurements are distinguishable enough and the probability density of the corresponding landmark becomes sufficiently Gaussian. In (Costa, Kantor, and Choset 2004), the persistence of landmark pose estimation is tracked without prior knowledge of data association.

Another popular method, which is related to the extended Kalman filter (EKF), is the Gaussian sum filter (GSF). The GSF approximates arbitrary probability density functions by a weighted combination of many multivariate Gaussians. Since the GSF requires maintaining a large set of EKFs, its computational complexity can become problematic when a large number of landmarks are initialized simultaneously. An approximated Gaussian sum method is proposed in (Solà et al. 2005), where a set of parametrized cascaded Gaussian distributions is managed and updated by federated information sharing (FIS). Although this method is an improvement over the standard GSF, a larger state is still required to initialize landmarks.

Particle Filters (PFs), which incorporate non-Gaussian distributions, are widely used in SLAM research. In (Kwok and Rad 2006), particle filters are adopted for bearing measurements by associating hypothesized pseudo-ranges with each measurement and by implementing a re-sampling procedure to eliminate improbable particles. In (Davison 2003), a set of particles are maintained along the viewing ray of a landmark and initialization is delayed until the range distribution is roughly Gaussian. In (Eade and Drummond 2006), a FastSLAM particle filter is used for monocular SLAM with a partial initialization strategy that estimates the inverse-depth of new landmarks rather than depth. Unfortunately, particle filter methods often require a large number of particles for initialization.

Direct parametrization of inverse-depth is used for monocular SLAM in (Montiel, Civera, and Davison 2006) and a method of inverse-depth and depth conversion is proposed by the same authors in (Civera, Davison, and Montiel 2007). It is well known that the inverse-depth representation can handle distant landmarks more efficiently than conventional parameterizations because it incorporates a measurement equation with low linearization error. Unfortunately, though, this method is still susceptible to a diverging state, which we show in our experiments.

Simplified Bearing-Only Update Problem

Before introducing our full-scale SLAM solution, we will first analyze a simplified form of bearing-only SLAM, which we will refer to as the *simplified bearing-only update problem*, see Fig. 2. Although this formulation is contrived by design, the forthcoming analysis is a prerequisite to our more general full-scale SLAM solution.

For the simplified bearing-only update problem, we assume the pose of the robot X_k^r at time step k is known and therefore not included in the state. The robot is mapping just one landmark with a bearing sensor and has a prior estimate of the landmark location: its mean $\hat{X}_{k-1}^l$ and covariance P_{k-1}. To simplify the problem even further, we assume the following convenient initial values,

$$X_k^r = \begin{bmatrix} x_k^r \\ y_k^r \\ \theta_k^r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \;\; \hat{X}_{k-1}^l = \begin{bmatrix} \hat{x}_{k-1}^l \\ \hat{y}_{k-1}^l \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Thus, the robot is sitting at the origin with an estimate that places the landmark on the x-axis one meter away.

We assume the robot obtains, at time step k, a noisy bearing measurement z_k to the landmark. The task that we need to solve is how to properly compute the new mean $\hat{X}_k^l$ and the new covariance P_k for our landmark distribution given the information provided by the sensor measurement.

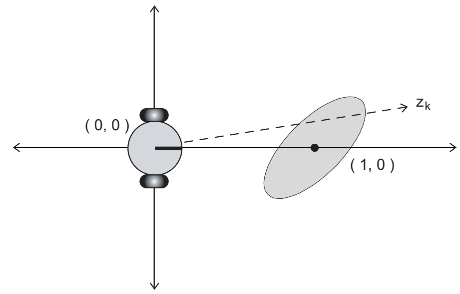

Figure 2: For the *simplified bearing-only update problem*, the robot is located on the origin, the prior distribution for the landmark is shown with a gray ellipse, and a bearing measurement z_k is shown as a ray emanating from the robot.

MAP Simplified Bearing-Only Update Filter

A popular method for updating the mean and covariance is to adopt the well known extended Kalman filter (EKF) equations, which is used for SLAM in (Dissanayake et al. 2001). But in (Tully et al. 2008), it is shown that for bearing-only SLAM, the EKF update can lead to unrecoverable divergence of the state estimate. This can be attributed to the nonlinearity of the measurement function and the deficiency of the EKF.

In this section, we will introduce a new filter update step that is specific to the *simplified bearing-only update problem* and that will be used in place of the EKF update.

SLAM as an Optimization Problem

We argue that a more sophisticated solution than the EKF would be to update the mean of the landmark distribution with the single-step maximum a posteriori (MAP) estimate. This represents the peak of the posterior distribution defined for the most recent measurement and, in our opinion, a better guess at the new mean. By definition, the solution is as follows,

$$\hat{X}_k^l = \arg\max_X f_z(z_k|X) f_X(X)$$

where f_z is the density function that defines the probability of measuring z_k when the landmark is positioned at $X = [\,x \;\; y\,]^T$ and f_X is the density function that corresponds to the prior distribution for the landmark location from the previous time step.

When assuming Gaussian models, solving for the single-step MAP estimate for the simplified bearing-only update problem is equivalent to finding, via optimization, the global minimum over x and y of the following cost function,

$$\begin{aligned} C_1(X) &= \frac{(z_k - h(X))^2}{\sigma_z^2} + \begin{bmatrix} x-1 \\ y \end{bmatrix}^T P_{k-1}^{-1} \begin{bmatrix} x-1 \\ y \end{bmatrix} \quad (1) \\ \hat{X}_k^l &= \arg\min_X C_1(X). \end{aligned}$$

where $h(X)$ is the measurement function,

$$h = \text{atan2}\,(y - y_k^r, x - x_k^r) = \text{atan2}\,(y, x)\,.$$

In addition to updating the mean via the optimization of Eq. 1, we must compute an update to the covariance matrix for our landmark estimate. We believe that the conventional EKF covariance update, while just an approximation to the true covariance due to linearization, is sufficient in this context and performs well in our experiments,

$$P_k = P_{k-1} - P_{k-1}H^T \left[HP_{k-1}H^T + \sigma_z^2\right]^{-1} HP_{k-1}, \quad (2)$$

where H is the Jacobian of the measurement function, which we evaluate at the updated mean location $\hat{X}_k^l$.

Reparameterization

As discussed above, we need to find the global minimum of Eq. 1 in order to find the MAP-optimal state for the simplified bearing-only update problem, which will then become the new mean for our filter (rather than using the flawed EKF equations). But if one were to numerically optimize Eq. 1, it is unclear whether the converged solution would be a local minimum or if it would be the desired global minimum.

Instead, we will perform a polar transformation to obtain a cost function that is easier to analyze. Let us define ϕ and r as the angle and range to the updated landmark mean,

$$\begin{aligned} \phi &= \text{atan2}\left(y - y_k^r, x - x_k^r\right) = \text{atan2}\left(y, x\right) && (3) \\ r &= \sqrt{(y - y_k^r)^2 + (x - x_k^r)^2} = \sqrt{y^2 + x^2}\ , && (4) \end{aligned}$$

and let us impose the constraints $-\pi < \phi \leq \pi$ and $r \geq 0$ to ensure that the transformation is one-to-one.

Thus, instead of optimizing over x and y, we can optimize over ϕ and r and then undo the polar transformation[1]. The cost function in Eq. 1 can now be rewritten in terms of ϕ and r as follows,

$$\begin{aligned} C_2(\phi, r) &= \frac{(z-\phi)^2}{\sigma_z^2} + \left[W_\phi r - d\right]^T P_{k-1}^{-1} \left[W_\phi r - d\right] \quad (5) \\ W_\phi &= \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} \qquad d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \end{aligned}$$

Interestingly, there is an analytically optimal solution for r, as a function of ϕ, that minimizes Eq. 5 and obeys the imposed positivity constraint,

$$r_\phi^* = \max\left(\left[W_\phi^T P_{k-1}^{-1} W_\phi\right]^{-1} W_\phi^T P_{k-1}^{-1} d\ ,\ 0\right). \quad (6)$$

We can plug this solution back into Eq. 5 to obtain a reduced cost function to optimize, which is now a function of only one variable (the angle ϕ to the updated landmark mean),

$$\begin{aligned} C_3(\phi) &= \begin{cases} \frac{(z-\phi)^2}{\sigma_z^2} + \sigma_x^2\,, & r_\phi^* = 0 \\ \frac{(z-\phi)^2}{\sigma_z^2} + d^T \Lambda_\phi d\,, & \text{otherwise} \end{cases} \quad (7) \\ \Lambda_\phi &= P_{k-1}^{-1} - P_{k-1}^{-1} W_\phi \left[W_\phi^T P_{k-1}^{-1} W_\phi\right]^{-1} W_\phi^T P_{k-1}^{-1}. \end{aligned}$$

For every ϕ value to consider in Eq. 7, there is an optimal r value and therefore a corresponding x and y landmark location. Thus, by optimizing over ϕ in Eq. 7, we are effectively searching over a one-dimensional manifold in the space of possible updated landmark positions. This manifold is shown in Fig. 3.

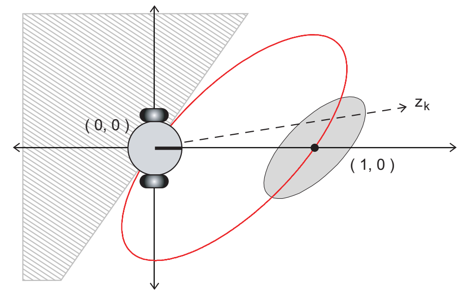

Figure 3: Possible solutions for the landmark mean update lie on the (red) ellipse.

Our resulting filter can be summarized by the following algorithmic steps: find the global minimum of Eq. 7 with respect to ϕ, compute the optimal range r_ϕ^* with Eq. 6, reverse the polar transformation in Eqs. 3 and 4 to obtain the maximum a posteriori x and y coordinates to replace the landmark mean, and lastly update the covariance using Eq. 2.

Cost Function Minimization

So far, we have introduced the simplified bearing-only update problem along with a filtering scheme that aims to update the landmark mean via the optimization of Eq. 7. In this section, we will provide a proof that we can always find the global minimum of Eq. 7 with numerical optimization.

Global Optimality

First, we must define bounds that will become useful when analyzing the optimization of the reduced cost function.

$$\begin{aligned} \phi_{MIN} &= \text{atan2}\left(-\alpha\,,\,\rho\beta\right) \\ \phi_{MAX} &= \text{atan2}\left(-\alpha\,,\,\rho\beta\right) + \pi, \end{aligned}$$

where α, β, and ρ are parameters in the pre-update covariance matrix, as follows,

$$P_{k-1} = \begin{bmatrix} \alpha^2 & \rho\alpha\beta \\ \rho\alpha\beta & \beta^2 \end{bmatrix}.$$

The following conservative assumption is necessary for the forthcoming proof.

Assumption 1. *The bearing measurement z_k is assumed to be in the range $\phi_{MIN} < z_k < \phi_{MAX}$ with $z_k \neq 0$.*

The portion of the polar space that is outside of this range can be seen in Fig. 3 as the gray shaded area and is defined as the region for which the optimal range value r_ϕ^* (Eq. 6) equates to zero. For any measurement in this region, the corresponding bearing ray would point *away* from the prior landmark uncertainty ellipse. We believe Assumption 1 is valid because receiving such a measurement is unlikely and can only be explained by an incorrect association or an unmodeled disturbance. In our implementation, we simply enforce this assumption by discarding measurements outside of the defined bounds.

[1] One might ask why we would not choose to use a polar representation in the first place when parameterizing the landmark. The reason is that the representation of uncertainty would be different and the forthcoming derivations would not be possible.

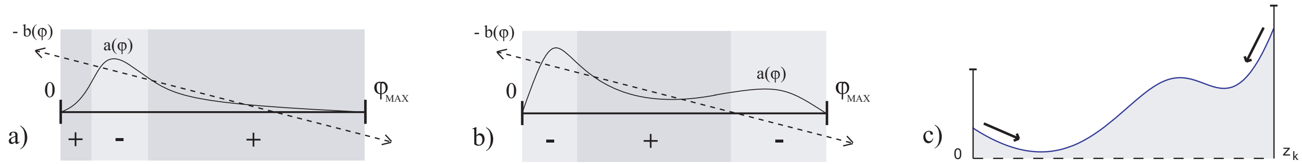

Figure 4: In (a) and (b), two examples are shown in which the derivative of the reduced cost function has three roots. The cost function would then have two minima as in (c).

We also assume z_k is unequal to zero. A bearing of zero for the simplified bearing-only update problem means that the measurement completely agrees with the prior estimate and thus updating the filter is unnecessary. In our implementation, we simply skip the update step in this case.

The next few lemmas will be important for proving that our optimization converges to the global minimum and tracks the peak of the posterior distribution.

Lemma 1. *The global minimum of the reduced cost function given in Eq. 7 must lie between $\phi = 0$ and $\phi = z_k$.*

Proof. First, the global minimum of Eq. 7 must lie between $\phi = \phi_{MIN}$ and $\phi = \phi_{MAX}$ inclusive. To see this, we point out that the optimal range (Eq. 6) for all ϕ outside of these bounds will evaluate to 0 and therefore Eq. 7 will produce a cost greater than the corresponding costs for both ϕ_{MIN} and ϕ_{MAX}. Second, the derivative of Eq. 7 is as follows,

$$\begin{aligned}
\frac{\partial C_3(\phi)}{\partial \phi} &= a(\phi) + b(\phi) \qquad (8)\\
a(\phi) &= \frac{8\alpha^2\beta^2(1-\rho^2)\sin(\phi)\tau(\phi)}{(\alpha^2+\beta^2+(\alpha^2-\beta^2)\cos(2\phi)+2\rho\alpha\beta\sin(2\phi))^2}\\
b(\phi) &= \frac{-2(z-\phi)}{\sigma_z^2}\\
\tau(\phi) &= \alpha^2\cos(\phi) + \rho\alpha\beta\sin(\phi).
\end{aligned}$$

Zeros of Eq. 8 correspond to extrema in the cost function. For positive z_k and $z_k < \phi \leq \phi_{MAX}$, Eq. 8 is strictly positive, though. For positive z_k and $\phi_{MIN} \leq \phi < 0$, Eq. 8 is strictly negative. Thus, there can be no optima outside of the range $\phi = 0$ to $\phi = z_k$. The same argument is valid in the case of $z_k < 0$. □

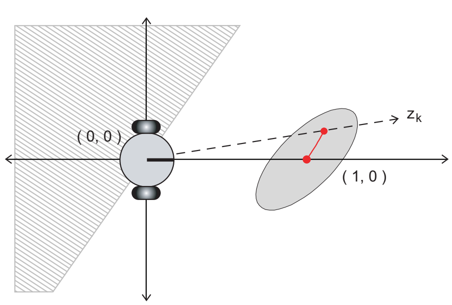

Figure 5: Possible solutions for the landmark mean update lie on the (red) curve.

The result of Lemma 1 is that we need only optimize a portion of the manifold, shown in Fig. 5, that corresponds to ϕ between 0 and z_k. The global minimum must lie in this region.

Lemma 2. *$C_3(\phi)$ is decreasing at $\phi = 0$ in the direction of $\phi = z_k$, and $C_3(\phi)$ is decreasing at $\phi = z_k$ in the direction of $\phi = 0$.*

Proof. For positive z_k, Eq. 8 evaluated at 0 is negative and Eq. 8 evaluated at z_k is positive. For negative z_k, Eq. 8 evaluated at 0 is positive and Eq. 8 evaluated at z_k is negative. □

Lemma 3. *There are at most 2 minima of the reduced cost function given in Eq. 7 between $\phi = 0$ and $\phi = z_k$.*

Proof. In Fig. 4 (a) and (b), we show two example cases for $a(\phi)$ and for $-b(\phi)$, which are the terms in Eq. 8. Any intersection of $a(\phi)$ and $-b(\phi)$ drives Eq. 8 to zero which implies an extremum in the reduced cost function (Eq. 7). We also note that there are at most two zeros of the *second derivative* of $a(\phi)$ in the range $\phi = 0$ to $\phi = \phi_{MAX}$. There is also a constraint that $a(\phi)$ has to be positive for $\phi = z_k$. These two facts restrict the number of times the function $a(\phi)$ can cross over the line defined by $-b(\phi)$ between $\phi = 0$ and $\phi = z_k$ to at most three, see Fig. 4 (a) and (b) for examples. At most three intersections implies at most three extrema of the reduced cost function in the range $\phi = 0$ to $\phi = z_k$, two of which must be minima (based on Lemma 2 and as seen in Fig. 4 (c)). It is therefore impossible to have more than two minima in the range 0 to z_k. The same argument is valid in the case of $z_k < 0$. □

Optimization Procedure

The objective is to find the global minimum of Eq. 7 in order to replace the mean of the filter with the MAP estimate.

According to Lemma 1, the global minimum must lie between $\phi = 0$ and $\phi = z_k$. According to Lemma 3, there are at most *two* minima in that range, one of which is the desired global minima. This means that a bidirectional search with a numerical optimization algorithm, such as Gauss Newton's method, will find both minima. We start one search from $\phi = 0$ and another search from $\phi = z_k$. The searches are guaranteed to "fall" into the two minima in the desired range because of Lemma 2. See Fig. 4 (c) for a depiction of the optimization procedure.

After running both searches until convergence, the result with the lower cost (Eq. 7) is the desired global minimum.

We use this MAP-optimal ϕ value to then compute the optimal x and y updated landmark coordinates, which become the new mean for the filter.

Bearing-Only FastSLAM

In the previous two sections, we have thoroughly explored a novel nonlinear filtering method to solve the simplified bearing-only update problem. We also have proven that the algorithm is guaranteed to replace the mean of the filter with the maximum a posteriori estimate. But the simplified bearing-only update problem is limited by several assumptions, e.g. the robot position must be on the origin and the previous estimate must lie on the x-axis.

To relax these assumptions, we adopt a FastSLAM approach, which was introduced in (Montemerlo et al. 2002). FastSLAM factors the problem by using a rao-blackwellized particle filter to estimate the map and the pose.

For each particle in the filter, a candidate map is estimated. And because each candidate map is estimated *given* the trajectory of the robot, each landmark estimate is independent. This means we can perform a measurement update to a single landmark without considering correlations between any of the landmarks. Lastly, when the robot measures a landmark, we can simply rotate, translate, and scale the coordinate frame accordingly so that the FastSLAM landmark update fits the form we presented for the *simplified bearing-only update problem*, see Fig. 6. Also, any heading offset we need to subtract to fit the correct form can be added to the relative measurement z_k. After solving for the MAP-optimal estimate, we can reverse the transformation for the solution, back to the coordinate frame for the particle.

Therefore, FastSLAM is a way to take the optimality of our novel nonlinear optimization method and apply it to multiple landmark SLAM with motion uncertainty. This is done without breaking any of the original assumptions.

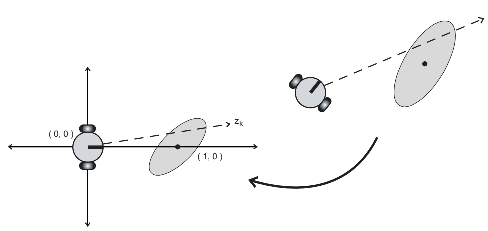

Figure 6: Any filtering update in the FastSLAM formulation can be transformed into the correct form for the *simplified bearing-only update problem*, and then transformed back.

With this final solution, we have a set of particles, each of which is building a map. For each measurement, each particle is performing two one-dimensional optimization procedures to optimize Eq. 7. The complexity is therefore comparable to a conventional FastSLAM approach (a small and finite number of iterations are required for convergence of each optimization procedure).

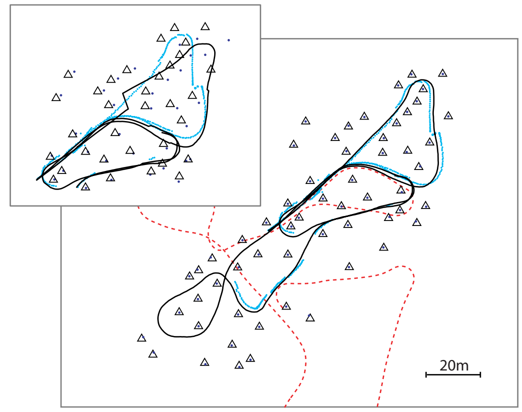

Figure 7: This is an experiment based on a subset of the Victoria Park dataset (Guivant, Nieto, and Nebot). The landmarks depicted with triangles are the range/bearing result. The dots are the result of our proposed filter. The partial map (upper left of image) shows the inverse-depth result.

Experiments

To validate our approach, we turned to a well-studied SLAM benchmark, the Victoria Park dataset (Guivant, Nieto, and Nebot). We implemented our FastSLAM approach, which uses the MAP-optimal landmark update, and tested the algorithm versus (1) an EKF solution, (2) an inverse-depth parameterization (Montiel, Civera, and Davison 2006), and (3) a range/bearing FastSLAM implementation. For all of the bearing-only implementations, the range information provided in the Victoria Park dataset was simply ignored. We initialized landmarks arbitrarily along the initial bearing ray with a large initial range uncertainty. In our experiments we conservatively assume Gaussian noise on all bearing measurements with a standard deviation of 4 degrees.

The following map in Fig. 7 shows our MAP-optimal bearing-only FastSLAM result versus a range/bearing FastSLAM solution. We should expect the range/bearing outcome to vastly outperform all of the bearing-only solutions because range is particularly informative when filtering the spatial positions of landmarks. Remarkably, though, without any range information, our bearing-only SLAM method nearly matches the performance of the range/bearing filter. In Fig. 7, both maps are shown and align very closely. The average euclidean error for our method was 0.2502 meters per landmark when compared to the range/bearing result.

For this Victoria Park experiment, the EKF solution diverged almost immediately. Interestingly, the inverse-depth implementation also diverged because an inverse-depth value for one or more of the landmarks was pushed below zero by the filter. We then re-ran the same experiment while ignoring any measurements that caused this problem. While this prevented divergence, it caused the estimate to drift, as seen in the upper left corner of Fig. 7.

We also performed another experiment to further test our proposed filter. This experiment was performed with an outdoor mobile robot with omnidirectional vision and compares our method to the the inverse-depth parameterization

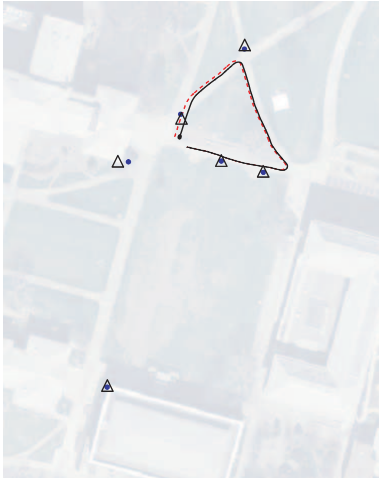

Figure 8: This is an outdoor experiment. The ground truth landmark locations are shown with triangles and the estimated locations are shown with dots.

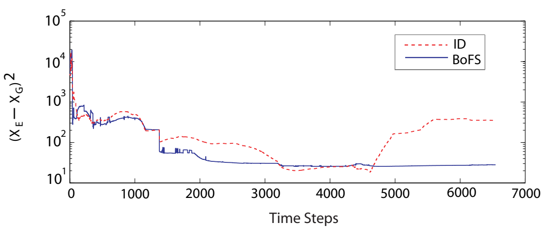

Figure 9: This plot depicts the filtering performance for the outdoor mobile robot experiment. The squared-error for our MAP-optimal Bearing-Only FastSLAM filter (BoFS) is shown with a (blue) solid line. The squared-error for the inverse-depth method (ID) is shown with a (red) dotted line.

method. The result for our MAP-optimal FastSLAM solution is shown in Fig. 8. An error comparison between our filter and the inverse-depth method is shown in Fig. 9.

The inverse-depth parameterization is designed to reduce linearization error so that distant landmarks can be mapped accurately. We have shown here, though, that our algorithm can perform better (see Fig. 9) *and* keep the conventional euclidean landmark parameterization.

Conclusion

In this paper, we have introduced a novel nonlinear filtering method for bearing-only SLAM that replaces the commonly used EKF update with an optimization procedure that is guaranteed to replace the filter mean with the maximum a posteriori estimate. While the conventional EKF update causes divergence, our filter offers convergent estimation that tracks the peak of the single-step posterior distribution.

In our experiments, we show our method outperform an EKF solution and an implementation of the inverse-depth parameterization. We also show that our algorithm works well enough to nearly match the performance of a range/bearing experiment that is mapping with much more informative measurements than in the bearing-only case.

For the most part, this paper has ignored the problem of data association and instead focuses on the underlying filtering theory for SLAM. We acknowledge that identifying landmarks is an important problem, but in the case of bearing-only sensing, data association may be best solved with visual information. Thus, in our minds, the bigger issue is how to create a nonlinear filter that provides convergent estimation for bearing-only SLAM. We believe we have provided a favorable solution to this problem.

References

Bailey, T. 2003. Constrained initialisation for bearing-only SLAM. In *Proceedings of the 2003 IEEE International Conference on Robotics and Automation*.

Civera, J.; Davison, A. J.; and Montiel, J. M. M. 2007. Inverse depth to depth conversion for monocular SLAM. In *Proceedings of the 2007 IEEE International Conference on Robotics and Automation*.

Costa, A.; Kantor, G.; and Choset, H. 2004. Bearing-only landmark initialization with unknown data association. In *Proceedings of the 2004 IEEE International Conference on Robotics and Automation*.

Davison, A. 2003. Real time simultaneous localisation and mapping with a single camera. In *International Conference on Computer Vision*.

Deans, M., and Hebert, M. 2000. Experimental comparison of techniques for localization and mapping using a bearings only sensor. In *Proceedings of the 7th International Symposium on Experimental Robotics*.

Dissanayake, M.; Newman, P.; Durrant-Whyte, H.; Clark, S.; and Csorba, M. 2001. A solution to the simultaneous localisation and map building (SLAM) problem. *IEEE Transactions of Robotics and Automation* 17(3):229–241.

Eade, E., and Drummond, T. 2006. Scalable monocular SLAM. In *IEEE Computer Society Conference on Computer Vision and Pattern Recognition*.

Guivant, J.; Nieto, J.; and Nebot, E. Victoria park dataset.

Kwok, N. M., and Rad, A. B. 2006. A modified particle filter for simultaneous localization and mapping. *Journal of Intelligent and Robotic Systems* 46(4):365–382.

Montemerlo, M.; Thrun, S.; Koller, D.; and Wegbreit, B. 2002. Fastslam: A factored solution to the simultaneous localization and mapping problem. In *Proceedings of the 2002 AAAI National Conf. Artificial Intelligence*.

Montiel, J. M. M.; Civera, J.; and Davison, A. J. 2006. Unified inverse depth parametrizaiton for monocular SLAM. In *Robotics Science and Systems*.

Solà, J.; Monin, A.; Devy, M.; and Lemaire, T. 2005. Undelayed initialization in bearing only SLAM. In *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*.

Tully, S.; Moon, H.; Kantor, G.; and Choset, H. 2008. Iterated filters for bearing-only SLAM. In *Proceedings of the 2008 IEEE International Conference on Robotics and Automation*.