Public Supply of Private Goods

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by

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Prepared for the Interlaken Seminar, May 1980. To assist the reader who does not wish to follow the formal argument, we have written an extended introduction that contains a summary and states the principal conclusions.
From the time of Locke (1967), theoretical analysis of the role of government distinguished those services that can be provided at lowest cost if provided collectively. These goods and services, usually called public goods, are part of a potentially larger class of goods that are subject to external economies or diseconomies in production or consumption.

Modern governments do not limit their activities to the provision of public goods and the elimination of external effects. Governments supply services that can be produced privately. At times, public and private supply coexist. Housing services, medical care and education are examples, but so too are consumer protection, safety and many of the so-called regulatory activities.

In a previous paper, Meltzer and Richard (1979), we identified the size of government with the share of government spending to total output and showed the conditions under which a decisive voter determines the size of government by choosing the tax rate and amount of income redistribution that maximize his utility. The choice of a decisive voter depends on the voting rule that a society adopts. In a society operating under strict majority rule, the median voter dominates the choice of tax rate and redistribution.

In our earlier work and in much of the literature on the theory of redistribution, it is optimal to redistribute general purchasing power or the composite consumption good. Friedman's argument for a negative income tax is probably the most frequently cited statement of the case for cash
transfers and against distribution of specific goods and services. He uses three main arguments to make his case. First, the recipient of cash transfers is permitted to choose the goods and services he prefers, so he obtains maximum utility from the transfer. Second, the recipient's incentive to work is reduced by a negative income tax, but it is not eliminated. A dollar of earned income permits the recipient to increase consumption. Third, the cost of administering the system is reduced. (Friedman, 1962, pp. 192-3)

Nowhere do we find redistribution using only negative income taxes. Several attempts to introduce a negative income tax as a substitute for other types of welfare have been rejected by the Congress. This record suggests that even if a negative income tax is the least distorting means of redistributing income, other systems of redistributing income are chosen and may be preferred by voters and their representatives.

Friedman's arguments, and other arguments for cash transfers, ignore the political process. The system of redistribution that is preferred by the recipients will not be adopted unless the recipients are in a position to impose their choice on the society or some non-recipients favor cash transfers also.

To find circumstances under which cash transfers are likely to be made, we combine political and economic processes in a model of optimizing behavior. The decisive voter maximizes a concave utility function subject to a budget constraint. We assume that all individuals have the same utility function. This avoids making the conclusion depend on a non-observable -- differences in tastes. Budget constraints differ because there are differences in productivity, and income depends on productivity. Productivity is not observable, but
income is, so taxes are levied on income. A proportional tax on earned income balances the government budget and pays for redistribution. All redistribution takes the form of lump sum transfers, but redistribution may be in-kind or in cash. People are not permitted to resell the goods or services they receive as redistribution; housing allowances or medical services cannot be resold; they can be wasted.

The economy produces two goods using constant returns to scale production functions with labor as the only input. Individuals value both goods but also value leisure. In the absence of taxes and redistributive transfers, each individual equates the ratios of the marginal utilities to the price ratios. Everyone works. In the presence of taxes and transfers, people with relatively low productivity find that they value the additional consumption they can obtain by working less than they value (additional) leisure. As in our previous paper, Meltzer and Richard (1979), some people choose full-time leisure and consume only the basket of goods they can obtain with their transfer payments.

Suppose the decisive voter reasons as follows: If I allow each recipient to choose the bundle of goods and services that maximize his utility, those below some level of income choose not to work. By giving income in kind, I can encourage people who are net beneficiaries of the tax-transfer system to make different labor-leisure choices. For example, if I allow the government to distribute only one good, people with relatively low productivity will find that all of their disposable income is in one consumption good; they are surfeited with the good that the government offers and have none of the other good(s). Since they cannot resell the good they receive, they will be willing to trade leisure for income to purchase the good(s) they do not have. By
working, they trade leisure, valued at the marginal product of labor, for consumption of a good that has relatively high marginal utility. They will continue to trade leisure for consumption of the good they purchase until they reach a constrained optimum. At the constrained optimum, they prefer to sell some of the good they receive as a transfer and increase leisure, but they cannot. They may grumble about the welfare system, but they will prefer the constrained outcome to any other feasible point. Since they work and produce more goods and services, everyone else, including me, is better off. It is in my interest to get them to work more than they would choose to work if I permit redistribution of cash.

By reasoning in this way, the decisive voter reaches a more general conclusion about redistribution. The welfare system can increase the amount of work, and the amount of earned income at a given tax rate, by making transfers in kind. If both goods are normal, people will work to increase the good for which marginal utility is greater than price.

How does the decisive voter choose the size and composition of the bundle of goods redistributed? Our answer to both questions is the same: he finds the outcome he prefers. To reach this conclusion, we require two additional assumptions. First, we assume that the choices of size and composition are independent in the electoral process. This prevents coalitions and avoids indeterminacy. People vote on one issue at a time -- size or composition -- and do not change their vote when they learn the outcome of the previous vote. (They correctly anticipate the outcome of the second vote when casting their joint ballot.) Second, we assume that as taxes and the redistribution of one (but not both) of the goods increases, the amount of redistribution required to maintain indifference does not decrease as the wage (or productivity)
increases. This assumption implies that if a tax increase takes one dollar from a person who earns ten dollars and five dollars from a person who earns fifty dollars, the absolute value of redistribution required to compensate for the loss is not smaller at the higher income.

The power of the decisive voter to alter the outcome is strictly limited by the voting rule, tastes and the distribution of productivity. He knows that everyone maximizes subject to constraints. If he imposes a tax rate, after-tax wages fall. Everyone works less, but income is available for redistribution. He continues to raise taxes until the loss from the disincentive effect of higher tax rates reduces the amount he gains from additional redistribution and leisure is less than the amount he gains by working and buying consumption goods in the market. If he raises taxes, beyond his optimum, he -- and all those who work -- choose less labor and more leisure. The additional consumption obtained from his lower earned income and the higher level of redistribution is worth less to him than the gain from additional leisure. Conversely, if he lowers taxes he moves to a position that he can improve upon by working less and receiving a larger transfer.

Once the decisive voter finds an optimum, he finds a majority of voters who vote for the optimum. Voters with income above the decisive voter's income (productivity) prefer lower taxes and less redistribution. They cannot find a majority to support their choice of lower taxes, so they vote for the lowest tax rate they can obtain under the prevailing rule. Voters below the decisive voter's income prefer higher taxes, but they cannot obtain majority support.

Obviously, the position of the decisive voter depends on the voting rule. A two-thirds majority lowers the equilibrium tax rate. Extensions of the franchise that permit more low income recipients to vote raise taxes and redistribution.
Given the tax rate, the decisive voter can vary the amount of income, aggregate tax revenue and redistribution by varying the composition of redistribution. His choice depends on his income -- where he lies on the distribution of productivity -- and, thus, on the voting rule. There are three main outcomes.

First, the decisive voter does not work at the chosen tax rate. The decisive voter is part of a majority, with identical tastes, who prefer to consume the amount redistributed and enjoy full-time leisure. The decisive voter chooses the tax rate that maximizes the amount available for redistribution and is indifferent between a distribution of cash and a distribution of the goods he buys with the transfer he receives.

Second, at the opposite extreme, the decisive voter works and consumes more of each good than the maximum he can receive under any feasible transfer. The composition of his consumption, is unaffected by the composition of redistribution, but his choices change the incentives others face. The decisive voter distributes only one of the two goods to assure that the marginal utility of one good is substantially larger than the marginal disutility of labor. By forcing people with low productivity into a corner solution, redistribution in kind increases participation in the labor force. If only one good is redistributed, everyone works.

Third, the distribution of either good exceeds the decisive voter's desired consumption of that good. If redistribution is restricted to a single good, the decisive voter is in a corner. It is in his interest to distribute both goods. As the distribution of the second good increases, he moves toward his individual optimum; but by redistributing both goods, he
lowers the marginal utility of the second good and reduces incentives to
work. The choice he makes reflects the loss he bears as the aggregate amount
of labor declines and the benefit he obtains by moving toward a position at
which the ratio of the (private) marginal utilities in consumption equals
the ratio of the market prices.

The analysis shows why the equilibrium reached in a political-economic
system differs from the equilibrium reached in a market economy. The decisive
voter does not choose redistribution to maximize the welfare of the recipients;
he maximizes the utility of the decisive voter. The choice reflects the
constraints in the political-economic environment. These include the effects
on everyone's incentives.

Much of the work on political and economic systems discusses the rents
collected by bureaucrats and politicians. Few, perhaps none, of these studies
explain why maximizing voters do not learn enough about the political process
to reduce the rents to a minimum or to eliminate rents.

The increase in aggregate income resulting from redistribution in kind
is a source of rent. Government employees, bureaucrats, can capture part of
the difference between the aggregate income in a society with cash transfers
and the larger aggregate income in a society that distributes income in kind.
As long as the decisive voter is no worse off than in a system with cash
transfers, the bureaucracy may be able to capture the rent.

The qualified conclusion about the distribution of the rent reflects the
present state of our work. We have established the existence of the rent,
but we have not developed a rule for sharing the rent or for determining the
size of the bureaucracy.

The remainder of the paper establishes the main propositions summarized
in this section. Section 2 sets out the model of maximizing behavior and
the assumptions about taxing and spending. Redistribution can take the form of money or goods, so we compare the allocations an individual chooses when redistribution is in kind and in cash. In section 3, we extend Roberts' (1977) result to two consumption goods and show that the median voter remains decisive in a Condorcet (1785) process. Voting is by majority rule. Section 4 derives the median voter's choice of taxes and redistribution. We show that if the median voter works, he prefers redistribution in kind to redistribution of cash. Conversely, if the median voter does not work, he prefers redistribution in cash.
2. The Economic Environment

In the economy we consider there are a large number of individuals. Each individual is endowed with a unit of time which he allocates to leisure, \( \ell \), and to labor, \( n = 1-\ell \), which he sells in a competitive labor market. The productivity of labor varies among the individuals in the economy so each does not receive the same wage. An individual with productivity \( x \) receives a pre-tax wage of \( x \) and earns a pre-tax income, \( y \).

\[
y(x) = x n(x)
\]

For convenience wages and income are measured in an arbitrary unit of account we call money.

There are two consumption goods each produced by a separate constant returns to scale technology with labor as the only factor of production. There is no capital and no uncertainty. The consumption goods are sold in competitive markets at money prices \( p_1 \) and \( p_2 \), respectively. These prices do not vary with the supply of labor and we assume they are constant throughout this paper.

Each individual has an identical strictly increasing and strictly concave utility function, \( u(c_1, c_2, \ell) \), for the two consumption goods, \( c_1 \) and \( c_2 \), and leisure. The two consumption goods and leisure are normal goods. The marginal utility for each good at the zero level is infinite. Each person seeks to maximize his utility subject to his budget constraint. The individual's budget constraint is determined by his income, the taxes he pays and the redistribution he receives.

Individual productivity cannot be observed directly so taxes are levied against earned income. The tax rate, \( t \), is a constant fraction of earned income but a falling fraction of disposable income. Tax revenues finance lump-sum redistribution of money, \( r_0 \), good one, \( r_1 \), and good two,
The publicly provided goods $r_1$ and $r_2$ must be consumed and may not be resold, i.e.,

$$c_1 \geq r_1$$  \hspace{1cm} (2)

$$c_2 \geq r_2$$  \hspace{1cm} (3)

We denote by $r$ the total money value of the redistribution.

$$r = r_o + p_1 r_1 + p_2 r_2$$  \hspace{1cm} (4)

There is no savings; spending equals disposable income.

$$p_1 a_1 + p_2 a_2 = (1-t) x n + r_o = I(x)$$  \hspace{1cm} (5)

where $a_1 \geq 0$ and $a_2 \geq 0$ are the amounts of good one and good two, respectively, purchased from disposable income, $I(x)$. The individual's consumption is

$$c_1 = r_1 + a_1$$  \hspace{1cm} (6A)

and

$$c_2 = r_2 + a_2$$  \hspace{1cm} (6B)

Each individual acts as a price taker in the labor market, treats $z = (r_o, r_1, r_2, t)$ as given and chooses $n, a_1 \geq 0$ and $a_2 \geq 0$ to maximize utility, i.e.,

$$\max u(r_1 + a_1, r_2 + a_2, 1-n) \hspace{1cm} (7)$$

subject to the budget constraint (5). An individual with productivity $x$ has a unique optimal allocation $a_1(x, z), a_2(x, z)$ and $n(x, z)$ satisfying (7)
since u is concave and the constraints are a convex set.\(^1\)

We make the first two of three assumptions about individual tastes. The first is that for any \( z \), an increase in the wage, \( x \), does not decrease \( y \).

\[
\frac{\partial y}{\partial x} > 0 . \tag{8}
\]

Hence disposable income rises as the wage rises

\[
\frac{\partial I}{\partial x} > 0 . \tag{9}
\]

Our second assumption is that for any \( z \) both consumption goods are normal goods, i.e.,

\[
\frac{\partial c_1}{\partial x} > 0 \quad \text{and} \quad \frac{\partial c_2}{\partial x} > 0 \tag{10}
\]

with at least one of the inequalities holding strictly. Combining our assumptions we have

\[
\frac{\partial c_1}{\partial x} > 0 \quad \text{and} \quad \frac{\partial c_2}{\partial x} > 0 . \tag{11}
\]

These assumptions permit us to simplify considerably the social choice set.

Because both goods are normal, for any redistribution with money, there is an equivalent redistribution without money. This means that if the redistribution with money is \( (r_0, r_1, r_2) \) then there exists a redistribution \( (0, \hat{r}_1, \hat{r}_2) \) without money at which the total amount redistributed, \( r \), is unchanged and every individual makes an identical

\(^1\)We will suppress the dependence on \( z \) for notational convenience, except when needed for clarity.
Figure 1
The Equivalent Redistributions
\((r_0, r_1, r_2)\) and \((0, c_1(0), c_2(0))\)
consumption and labor decision. To see this consider the opportunity set available from \((r_0, r_1, r_2)\):

\[
\begin{align*}
  c_1 &\geq r_1 \\
  c_2 &\geq r_2 \\
  p_1c_1 + p_2c_2 &\geq r_0 + r_1p_1 + r_2p_2 = r_0 
\end{align*}
\]  

(2) \hspace{1cm} (3) \hspace{1cm} (12)

This opportunity set is shown as the shaded area in Figure 1. Now consider an individual with no productivity, i.e., \(x = 0\). The constraint (12) must be an equality since this individual is unable to earn any income and lives only on redistribution. Let his optimal allocation of consumption, subject to (2) and (3) be \((c_1(0), c_2(0)); c_1(0) - r_1\) and \(c_2(0) - r_2\) are the allocations of \(r_0\) by an individual who consumes \(c_1(0)\) and \(c_2(0)\). All individuals with \(x > 0\) will choose to consume at least as much as the zero productivity individual by the normality of both goods, i.e., \(c_1(x) \geq c_1(0)\) and \(c_2(x) \geq c_2(0)\) since \(I(x) \geq I(0)\) for all \(x > 0\). This is illustrated in Figure 1 by the line \((c_1(x), c_2(x))\).

Now consider the redistribution \((0, c_1(0), c_2(0))\). The new opportunity set is shown as the crosshatched area in Figure 1. Clearly all the optimal allocations \((c_1(x), c_2(x))\) are still feasible and hence any optimal allocation is unchanged. Without loss of generality we may set \(r_0 = 0\) so that all redistribution takes the form of grants of specific goods rather than money.
We can now simplify the individual's maximization problem. Define the after-tax wage

\[ w(x, t) = x(1-t) . \]  

(13)

Let \( r_0 = 0 \) in (5), solve for \( n \) and substitute into (7) to get

\[ v(r_1, r_2, w) = \max_{a_1, a_2 \geq 0} u(r_1 + a_1, r_2 + a_2, 1 - \frac{p_1 a_1 + p_2 a_2}{w}), w > 0 , \]  

(14)

where \( v \) is the indirect utility function of an individual with wage \( w \).

The first order conditions for (14) are

\[ \frac{\partial u}{\partial a_1} = u_1 - \frac{p_1 u_3}{w} \leq 0 \]  

(15)

with equality for \( a_1 > 0 \) and

\[ \frac{\partial u}{\partial a_2} = u_2 - \frac{p_2 u_3}{w} \leq 0 \]  

(16)

with equality for \( a_2 > 0 \). The assumption of a strictly concave utility function assures that the first order conditions define a unique maximum.

Henceforth \( a_1(r_1, r_2, w) \) and \( a_2(r_1, r_2, w) \) refer to the optimal allocations of an individual with after-tax wage \( w(x, t) \). Letting \( z = (r_1, r_2, t) \) we find the optimal pre-tax income \( y(x, z) \), of an individual with productivity \( x \) is

\[ y(x, z) = \frac{p_1 a_1 + p_2 a_2}{1-t} . \]  

(17)

\[ ^2/ \] The assumption that \( u_3(c_1, c_2, 0) = \infty \) assures that \( p_1 a_1 + p_2 a_2 < w \) so that \( n < 1 \) is a natural constraint.
The first order conditions divide individuals into two groups. Those with $x \leq x_0(z)$, where

$$x_0(z) = \min \left[ \frac{p_1u_3(r_1, r_2, 1)}{(1-t)u_1(r_1, r_2, 1)}, \frac{p_2u_3(r_1, r_2, 1)}{(1-t)u_2(r_1, r_2, 1)} \right], \quad (18)$$

choose not to work and earn no income, i.e., $y(x) = 0$ for $x \leq x_0$. Notice that if either $r_1 = 0$ or $r_2 = 0$, then $x_0 = 0$, i.e., all who can earn will earn. Those with productivity greater than $x_0$ work. For $x > x_0$ either (15) or (16) or both hold as an equality.

We now derive the relationship between $t$, $r_1$, and $r_2$ required to balance the government's budget while maintaining an equilibrium in the labor market. Let $F(\cdot)$ denote the distribution function for individual productivity, i.e., $F(x)$ is the fraction of the population with productivity less than $x$. Per capita or average income is then

$$\bar{y}(z) = \int_{x_0(z)}^{\infty} y(x, z) \, dF(x). \quad (19)$$

To balance the government's budget we must have tax revenues equal to redistribution expenditures.

$$t\bar{y}(r_1, r_2, t) = p_1r_1 + p_2r_2. \quad (20)$$

We denote by $Z$ the set of all triples $z = (r_1, r_2, t)$ satisfying (20) and call $Z$ the social choice set.
3. The Political Process

The function of the political process is to determine the size of the budget and the allocation of spending between $r_1$ and $r_2$. In this section we introduce a specific voting process which results in the selection of an element $z = (r_1, r_2, t)$ of $Z$ as the social choice.

The voting process is a Condorcet [1785] process that allows each fully informed voter to vote separately for $r_1$ and $r_2$. For example we consider elections between $z^1 = (r_1^1, r_2, t^1)$ and $z^2 = (r_1^2, r_2, t^2)$ or between $z^3 = (r_1, r_2^1, t^1)$ and $z^4 = (r_1, r_2^2, t^2)$. We do not permit elections between $z$'s where both $r_1$ and $r_2$ are changed. Elections are repeated until a majority agrees on stable values for $t$, $r_1$ and $r_2$. We refer to these values as the equilibrium values.

By requiring separate elections for $r_1$ and $r_2$, we assure that among the enfranchised individuals the voter with median productivity, $x_m$, is decisive. Furthermore, we prevent the formation of coalitions that link the choices of $r_1$ and $r_2$. By requiring voters to be fully informed we allow them to know the result we have not yet established: that the median voter is decisive for each level of redistribution.

To establish the decisiveness of the median voter we make another assumption about tastes. Consider the effect of lowering an individual's after-tax wage rate, $w$, by one percent and compensating him by increasing $r_1$ or $r_2$, but not both, so that he is indifferent between the higher grant-lower wage and lower grant-higher wage. We assume that the compensation required for indifference does not decrease as $w$ increases.

Analytically our assumption is that

$$\frac{\partial A}{\partial w} > 0 \text{ and } \frac{\partial B}{\partial w} > 0$$

(21)
where

\[ A(r_1, r_2, w) = \frac{u_3 (a_1 p_1 + a_2 p_2)}{u_1 w} \]  \hspace{1cm} (22)

and

\[ B(r_1, r_2, w) = \frac{u_3 (a_1 p_1 + a_2 p_2)}{u_2 w} . \]  \hspace{1cm} (23)

To show this consider the optimal utility of an individual with wage \( w \).

\[ v(r_1, r_2, w) = u(r_1 + a_1, r_2 + a_2, 1 - \frac{p_1 a_1 + p_2 a_2}{w}) . \]  \hspace{1cm} (24)

Totally differentiating (24) we get

\[ dv = da_1 \left[ u_1 \frac{p_1 u_3}{w} \right] + da_2 \left[ u_2 \frac{p_2 u_3}{w} \right] + u_1 dr_1 + u_2 dr_2 + \frac{u_3 (p_1 a_1 + p_2 a_2)}{w} dw . \]  \hspace{1cm} (25)

Using the first order conditions (15) and (16) we can reduce (25) to

\[ dv = u_1 dr_1 + u_2 dr_2 + \frac{u_3 (p_1 a_1 + p_2 a_2)}{w} dw . \]  \hspace{1cm} (26)

Holding \( r_2 \) fixed, moving along an indifference curve requires

\[ dr_1 = \frac{u_3 (p_1 a_1 + p_2 a_2)}{u_1 w} \left( \frac{-dw}{w} \right) . \]  \hspace{1cm} (27)

Eq. (27) says that a decrease (increase) in the wage of \( \frac{dw}{w} \) percent requires an increase (decrease) in the good one redistribution of \( dr_1 \) for indifference. If \( A(r_1, r_2, w) \) as given in (22) is an non-decreasing function of \( w \) then \( dr_1 \) does not decrease as \( w \) increases, holding fixed the percentage wage decrease \( \frac{dw}{w} \). Similarly holding \( r_1 \) fixed and moving along an indifference curve requires

\[ dr_2 = \frac{u_3 (p_1 a_1 + p_2 a_2)}{u_2 w} \left( \frac{-dw}{w} \right) . \]  \hspace{1cm} (28)
This shows that \( \frac{\partial B}{\partial w} > 0 \) implies that \( dr_2 \) increases as \( w \) increases, holding fixed the percentage wage decrease \( dw/w \).

An immediate consequence of (21) is our previous assumption (8) that pre-tax income increases with productivity. If income is being earned either (15) or (16) holds as an equality; otherwise earned income is zero. If \( a_1 > 0 \) then from (15) and (17)

\[
y(x, z) = \frac{a_1 p_1 + a_2 p_2}{1-t} = \frac{p_1 (a_1 p_1 + a_2 p_2)}{(1-t)} \frac{u_3}{u_1 w}
\]

\[
= \frac{A(r_1, r_2, x(1-t))}{(1-t)}
\]

Hence

\[
\frac{\partial y}{\partial x} = \frac{\partial A}{\partial w} > 0
\]

A similar analysis holds if \( a_2 > 0 \). Hence we may replace the assumption (8) with (21).

We now show that productivity orders the voter's choices for \( r_1, r_2 \) and \( t \). Let \( z^1 = (r_1, r_2, t^1) \) and \( z^2 = (r_1, r_2, t^2) \) with \( t^2 > t^1 \).

Let \( \hat{x} \) be the productivity of an arbitrary individual. Then for all \( \hat{x} \):

1. If \( \hat{x} \) is indifferent between \( z^1 \) and \( z^2 \) (denoted \( z^1 \approx_{\hat{x}} z^2 \)), then all individuals with \( x > \hat{x} \) weakly prefer \( z^1 \) to \( z^2 \) (denoted \( z^1 \succeq_x z^2 \)) and \( z^2 \succeq_x z^1 \) for all \( x < \hat{x} \).

2. If \( \hat{x} \) strictly prefers \( z^1 \) to \( z^2 \) (denoted \( z^1 >_{\hat{x}} z^2 \)) then \( z^1 >_x z^2 \) for all \( x > \hat{x} \).

3. If \( z^2 >_{\hat{x}} z^1 \), then \( z^2 >_x z^1 \) for all \( x < \hat{x} \).

We prove each of these statements in turn.
To prove\(^3\) (1) consider \(\hat{x}\)'s indifference map in \(z\) space which is derived from his indirect utility function \(v(r_1, r_2, \hat{x}(l-t))\). An indifference curve connects \(z^1\) and \(z^2\) and, from (27), holding \(r_2\) fixed its slope is

\[
\frac{dr_1}{dt} = \frac{A(r_1, r_2, \hat{x}(l-t))}{1-t},
\]

where we have substituted (22), \(w = \hat{x}(l-t)\) and \(dw = -\hat{x}dt\) into (27). Consider \(x > \hat{x}\). Moving along the indifference curve, (26) gives

\[
\frac{dv}{dt}(r_1, r_2, x(l-t)) = \frac{u_x(r_1, r_2, x(l-t))}{1-t} \left[A(r_1, r_2, \hat{x}(l-t)) - A(r_1, r_2, x(l-t))\right] \leq 0
\]

by (21). Hence

\[
v(r_1^2, r_2, x(l-t_2)) - v(r_1^1, r_2, x(l-t_1)) = \int_{t_1}^{t_2} \frac{dv}{dt}(r_1, r_2, x(l-t)) dt \leq 0
\]

which shows that \(z^1 \succeq_x z^2\). When \(x < \hat{x}\), the inequalities are reversed and \(z^2 \succeq_x z^1\).

The proof of (2) relies on the proof of (1). Since \(z^1 \succeq_x z^2\) there exists an \(\epsilon > 0\) such that \(z^1_\epsilon = (r_1^1 - \epsilon, r_2, t_1) \succeq_x z^2\). For \(x > \hat{x}\) we have from (1) that \(z^1_\epsilon \succeq_x z^2\). Hence \(z^1 \succeq_x z^1_\epsilon \succeq_x z^2\) so that \(x\) prefers \(z^1\) to \(z^2\). The proof of (3) is symmetrical to the proof of (2).

We can now establish the decisiveness of the median voter. From (1)-(3) we know that holding \(r_2\) fixed the median voter is decisive for \(r_1\). By interchanging the roles of \(r_1\) and \(r_2\) in (1)-(3) we can also show that the median voter is decisive for \(r_2\), holding \(r_1\) fixed. Suppose we vote for \(r_1\) first. In casting their ballots for \(r_1\), the fully informed rational voters in our model anticipate that the median voter

\(^3\)This proof is a generalization of Lemma 1 of Roberts [1977].
will be decisive for \( r_2 \) and hence set \( r_2 = r_2^m \), the median voter's optimal choice. The balloting for \( r_1 \) then results in \( r_1^m \) and \( t^m \), the median voter's optimal choices. If we vote for \( r_2 \) first the reasoning is reversed. In both cases the median voter's optimal choice \( z^m = (r_1^m, r_2^m, t^m) \) in \( Z \) is the outcome of the political process and is a stable equilibrium.
4. The Median Voter's Optimal Choices

In the model we have developed the government's relative size, as measured by the share of national income spent or taken in taxes, and the mix of goods provided by the government is completely determined by the median voter's choices for \( r_1, r_2, \) and \( t. \) In this section we show that if the median voter chooses to work, then he does not choose to redistribute cash and, conversely, if he chooses not to work, then he redistributes cash.

To obtain these results we must restate an assumption about tastes. We assume that leisure is a normal good for all \( r_1, r_2 \) and \( w \). That is, we assume that if redistribution of income increases, holding the wage fixed, then labor decreases in a specified manner. We specify our assumption in detail for each of four cases: (1) an individual purchasing both good one and good two; (2) an individual buying only good one; (3) an individual buying only good two; and (4) a non-worker.

We assume that for given \( r_1, r_2 \) and \( w \), if \( a_1 > 0 \) and \( a_2 > 0 \), then

\[
\frac{\partial n}{\partial r_1} (r_1, r_2, w) < 0 \quad \text{and} \quad \frac{\partial n}{\partial r_2} < 0 ,
\]

(32)

where

\[
n(r_1, r_2, w) = \frac{p_1 a_1 + p_2 a_2}{w} .
\]

(33)

Since this individual is purchasing both goods, the form of the incremental redistribution - either \( dr_1 \) or \( dr_2 \) - does not affect his labor decision since he may privately substitute \( a_1 \) for \( a_2 \). Hence it is easily...
shown that

\[ \frac{1}{p_1} \frac{\partial n}{\partial r_1} = \frac{1}{p_2} \frac{\partial n}{\partial r_2} < 0 \]  

(34)

i.e., the effect on labor of an increased dollar of \( r_1 \) is equal to the effect of an increased dollar of \( r_2 \) for those individuals with \( a_1 > 0 \) and \( a_2 > 0 \).

In the second case we assume that for given \( r_1, r_2 \) and \( w \), if \( a_1 > 0 \) and \( a_2 = 0 \), then

\[ \frac{1}{p_1} \frac{\partial n}{\partial r_1} < \frac{1}{p_2} \frac{\partial n}{\partial r_2} < 0, \]  

(35)

where

\[ n = \frac{p_1 a_1}{w}. \]  

(36)

This assumption says an increase of one dollar in either \( r_1 \) or \( r_2 \) causes the individual to work less, but the effect of increasing \( r_1 \) is a greater reduction of labor. It implies that good one and good two are weak substitutes in consumption for individuals with \( a_2 = 0 \), since an increase in \( r_2 \) causes \( c_2 = r_2 \) to increase, but causes \( c_1 = r_1 + a_1 \) to either decrease or remain constant. Differentiating the first order condition (15) gives

\[ \text{It is a straightforward, but tedious task to differentiate the two first order conditions (15) and (16) to get four linear equations in the four unknowns } \frac{\partial a_1}{\partial r_1}, \frac{\partial a_2}{\partial r_1}, \frac{\partial a_1}{\partial r_2} \text{ and } \frac{\partial a_2}{\partial r_2}. \]  

Solving these four equations establishes (34).
\[ \frac{1}{p_2} \frac{\partial m}{\partial x_2} = \frac{p_1}{p_2 w} \frac{\partial a_1}{\partial x_2} = \frac{1}{w D_1} \left[ \frac{u_{12}}{p_1 p_2} - \frac{u_{23}}{p_2 w^2} \right] \]  

(37)

and

\[ \frac{1}{p_1} \frac{\partial m}{\partial x_1} = \frac{1}{w} \frac{\partial a_1}{\partial x_1} = \frac{1}{w D_1} \left[ \frac{u_{11}}{p_1^2} - \frac{u_{13}}{p_1^2 w} \right], \]  

(38)

where

\[ D_1 = \frac{-u_{11}}{p_1^2} + \frac{2u_{13}}{p_1 w} - \frac{u_{33}}{w^2} > 0 \]  

(39)

is positive by the concavity of \( u \). Our assumption (35) requires that for \( a_1 > 0, a_2 = 0 \) we have

\[ \frac{u_{11}}{p_1^2} - \frac{u_{13}}{p_1 w} < \frac{u_{12}}{p_1 p_2} - \frac{u_{23}}{p_2 w} < 0. \]  

(40)

This condition is satisfied, for example, by a concave additively separable utility function. The intuitive meaning of the result is that the redistribution of the good that is purchased (\( c_1 \)) has a greater disincentive effect than the redistribution of the good that is consumed but not purchased (\( c_2 \)).

The third case is completely symmetrical to the second, so we state the assumption without further discussion. For given \( r_1, r_2, \) and \( w \), if \( a_1 = 0 \) and \( a_2 > 0 \), then

\[ \frac{1}{p_2} \frac{\partial m}{\partial x_2} < \frac{1}{p_1} \frac{\partial m}{\partial x_1} < 0, \]  

(41)

where

\[ n = \frac{p_2 a_2}{w}. \]  

(42)
This is equivalent to assuming that
\[ \frac{u_{22}}{p_2^2} - \frac{u_{23}}{p_2 w} < \frac{u_{12}}{p_1 p_2} - \frac{u_{13}}{p_1 w} \leq 0 \]  
(43)

for \( a_1 = 0 \) and \( a_2 > 0 \).

In the fourth case an individual chooses not to work and an increase in either \( r_1 \) or \( r_2 \) cannot induce him to enter the labor force.

As a preliminary to solving the median voter's problem, we investigate the effects on per capita income of changes in the composition of redistribution. For a given \( t \), let \( \rho \in [0, 1] \) be the fraction of tax revenues spent for redistribution of good one and \( 1-\rho \) be the fraction spent for redistribution of good two.

\[ r_1 = \rho \bar{y}/p_1 \]  
(44)

and

\[ r_2 = (1-\rho)\bar{y}/p_2 \]  
(45)

Eqs. (44) and (45) imply the government's budget constraint (20) is satisfied. Average income \( \bar{y}(\rho, t) \) is then the solution to (19):

\[ \bar{y}(\rho, t) = \int_{x_0}^{\infty} \frac{a \bar{y}}{p_1} \left[ \frac{\rho \bar{y}}{p_1}, \frac{(1-\rho)\bar{y}}{p_2}, x(1-t) \right] \, dF(x), \]  
(46)

where \( n \) is given by (33), (36) or (42) and \( x_o \left[ \frac{\rho \bar{y}}{p_1}, \frac{(1-\rho)\bar{y}}{p_2}, t \right] \) is given by (18). Hence we see that choosing \( \rho \) and \( t \) and requiring the government's budget to balance, determines \( \bar{y} \) and thus \( r_1 \) and \( r_2 \).

For a given tax rate, \( t \), we first consider the redistribution mix, \( \rho_o(t) \), which is equivalent to redistributing cash. The identifying feature of a cash redistribution is that all individuals are able to equate their marginal
utility of consumption for good one with their marginal utility of consumption for good two, i.e., \( \frac{u_1}{p_1} = \frac{u_2}{p_2} \). But as we showed in section two if we choose \( \rho \) so that non-workers can equate \( \frac{u_1}{p_1} = \frac{u_2}{p_2} \) then the constraints (2) and (3) hold as equalities. \(^5\) Hence \( \rho_o(t) \) must be the solution to

\[
\frac{1}{p_1} u_1 (\frac{\rho o t \vec{y}(p_o,t)}{p_1}, \frac{(1-\rho_o) t \vec{y}(p_o,t)}{p_2}, 1) = \frac{1}{p_2} u_2 (\frac{\rho o t \vec{y}(p_o,t)}{p_1}, \frac{(1-\rho_o) t \vec{y}(p_o,t)}{p_2}, 1) \quad (47)
\]

where \( \vec{y} \) is given by (46).

We now show that per capita income increases as \( \rho \) increases for \( \rho > \rho_o \), i.e., \( \frac{\partial y}{\partial \rho} > 0 \). Since \( \rho > \rho_o \) the government is supplying a mix of redistribution which contains more good one than some workers desire given their wage. One constraint is binding; there is a set of workers who purchase good two, but not good one. \(^6\) Let \( x_1(\rho, t) > x_o \) be the productivity of the last worker not to purchase good one, i.e.,

\[
u_1(x, \rho, t) - \frac{u_3(x, \rho, t)p_1}{x(1-t)} < 0 \quad \text{for} \quad x < x_1.
\]

Hence for \( x \in (x_o, x_1) \) \( a_2 > 0, a_1 = 0 \) and using (1) and (17) \( n(x) = p_2 a_2 / x(1-t) \).

For \( x > x_1, a_1 > 0, a_2 > 0 \), and \( n(x) = (p_1 a_1 + p_2 a_2) / x(1-t) \). Per capita

\[
\overline{y} = \int_{x_0}^{x_1} \frac{p_2}{1-t} a_2 \left( \frac{\rho o t \vec{y}}{p_1}, \frac{(1-\rho_o) t \vec{y}}{p_2}, x(1-t) \right) dF(x)
+ \int_{x_1}^{\infty} \frac{p_1 a_1 + p_2 a_2}{1-t} dF(x) \quad (48)
\]

\(^5\) This follows from the assumption that goods are normal and that income does not decrease with productivity.

\(^6\) We are implicitly assuming that there are individuals at every level of productivity, i.e., \( F(x) \) is strictly increasing.
Therefore

\[ \frac{\partial y}{\partial \rho} = - \frac{\partial x}{\partial \rho} \frac{p_2 a_2(x_0)}{1-t} + \frac{\partial x}{\partial \rho} \left[ p_2 a_2(x_1) - p_2 a_2(x_1) - p_1 a_1(x_1) \right] \left( \frac{1}{1-t} \right) \\
+ \int_{x_0}^{x_1} \frac{p_2}{1-t} \frac{\partial a_2}{\partial \rho} dF(x) + \int_{x_1}^{\infty} \frac{p_1}{1-t} + \frac{p_2}{1-t} \frac{\partial a_2}{\partial \rho} dF(x). \] (49)

Since \( a_2(x_0) = 0 \) and \( a_1(x_1) = 0 \), the first two terms of (49) are zero.

Now for \( x \in (x_0, x_1] \) we have from (42) that

\[ \frac{p_2}{1-t} \frac{\partial a_2}{\partial \rho} = x \frac{\partial m}{\partial \rho} \left( \frac{\rho \bar{y}}{p_1}, \frac{(1-\rho) \bar{y}}{p_2}, x(1-t) \right) \]

\[ = x t \left[ \frac{1}{p_1} \frac{\partial m}{\partial x_1} (\bar{y} + \rho \bar{y}') + \frac{1}{p_2} \frac{\partial m}{\partial x_2} (-\bar{y} + (1-\rho) \frac{x}{\partial \rho} \bar{y}) \right] \]

\[ = x t \bar{y} \left[ \frac{1}{p_1} \frac{\partial m}{\partial x_1} - \frac{1}{p_2} \frac{\partial m}{\partial x_2} \right] \\
- x t \frac{\partial \bar{y}}{\partial \rho} \left[ (1-\rho) \left( \frac{1}{p_1} \frac{\partial m}{\partial x_1} - \frac{1}{p_2} \frac{\partial m}{\partial x_2} \right) - \frac{1}{p_1} \frac{\partial m}{\partial x_1} \right]. \] (50)

Note that both terms in brackets in (50) are positive. Similarly for \( x > x_1 \)

we have from (33) that

\[ \frac{1}{1-t} \frac{p_1}{p_1} \frac{\partial a_1}{\partial \rho} + \frac{p_2}{\partial \rho} \frac{\partial a_2}{\partial \rho} = x \frac{\partial m}{\partial \rho} = \frac{x t}{p_1} \]

\[ = \frac{x t}{p_1} \frac{\partial m}{\partial x_1} \frac{\partial y}{\partial \rho}. \] (51)

In (51) the coefficient of \( \frac{\partial y}{\partial \rho} \) is negative. Substituting (50) and (51) into

(49) gives

\[ \frac{\partial y}{\partial \rho} = \frac{t y \int_{x_0}^{x_1} x \left[ \frac{1}{p_1} \frac{\partial m}{\partial x_1} - \frac{1}{p_2} \frac{\partial m}{\partial x_2} \right] dF(x)}{1 + t \int_{x_0}^{x_1} (1-\rho) \left( \frac{1}{p_1} \frac{\partial m}{\partial x_1} - \frac{1}{p_2} \frac{\partial m}{\partial x_2} \right) dF(x) - t \int_{x_0}^{\infty} \frac{x}{p_1} \frac{\partial m}{\partial x_1} dF(x)} > 0. \] (52)

Therefore \( \frac{\partial y}{\partial \rho} > 0 \) for \( \rho > \rho_0 \).
By reversing the roles of good one and good two in the above proof, we can show that \( \frac{\partial \bar{y}}{\partial \rho} < 0 \) for \( \rho < \rho_o \). Furthermore when \( \rho = \rho_o \), no workers are in the corner so \( x_1 = x_o \) and \( \frac{\partial \bar{y}}{\partial \rho} = 0 \) at \( \rho = \rho_o \). Therefore we have shown that the cash redistribution \( (\rho = \rho_o) \) results in the minimum per capita income for any given \( t \)

\[
\frac{\partial \bar{y}}{\partial \rho} < 0 \quad \text{for} \quad \rho = \rho_o .
\]

(53)

We can now prove the main result of our paper. Assume that each voter, if decisive, has unique optimal choices \( \rho(x) \) and \( t(x) \). Then \( \rho(x) = \rho_o(x) \) if and only if the decisive voter chooses not to work.

To prove our proposition we formulate the decisive voter's problem:

\[
\max_{\rho, t} u\left( a_1 + \frac{\rho \bar{y}}{p_1}, a_2 + \frac{(1-\rho)t \bar{y}}{p_2}, 1 - \frac{p_1 a_1 + p_2 a_2}{x(1-t)} \right)
\]

(54)

where \( \rho \in [0,1], t \in [0,1] \). The first order conditions define a unique optimum. They are (15), (16) and

\[
\frac{\partial u}{\partial \rho} = \left( \frac{u_1}{p_1} - \frac{u_2}{p_2} \right) \left( \bar{y} + \rho \frac{\partial \bar{y}}{\partial \rho} \right) + \frac{u_2}{p_2} \frac{\partial \bar{y}}{\partial \rho} < 0 \quad \text{if} \quad \rho = 1
\]

(55)

and

\[
\frac{\partial u}{\partial t} = \left[ \left( \frac{u_1}{p_1} - \frac{u_2}{p_2} \right) \rho + \frac{u_2}{p_2} \right] \left( \bar{y} + t \frac{\partial \bar{y}}{\partial t} \right) - \frac{u_3 p_1 a_1 + p_2 a_2}{x(1-t)^2} < 0 \quad \text{if} \quad t = 0
\]

Suppose the decisive voter chooses not to work. Then \( \rho = \rho_o \) satisfies (55) since \( \frac{\partial \bar{y}}{\partial \rho} |_{\rho=\rho_o} = 0 \) and from (47) \( \frac{u_1}{p_1} = \frac{u_2}{p_2} \) for non-workers. Furthermore because \( a_1 = a_2 = 0 \), (56) gives the optimal tax rate for a non-worker as

\[
\frac{\text{tax}}{\text{income}} = -1
\]

(57)
That is, it is optimal for a non-worker to set the elasticity of average income with respect to the tax rate equal to minus one. Given $p = p_o$ this maximizes the cash equivalent redistribution.

Now suppose the decisive voter chooses to work. If he chooses to purchase only one good, but not both, then from (15) and (16) we know that

$$\frac{u_1}{p_1} \neq \frac{u_2}{p_2} \text{ so that } \frac{\delta Y}{\delta p} = 0 \text{ cannot satisfy (55); the optimal choice is not } p = p_o.$$  

If he chooses to purchase both goods then from (15) and (16) we know that

$$\frac{u_1}{p_1} = \frac{u_2}{p_2} \text{ so that (55) reduces to}$$

$$\frac{\delta u}{\delta p} = \frac{u_2}{p_2} \frac{\delta Y}{\delta p}$$  

(58)

From (53) and (58) we see that if the decisive voter chooses to purchase both goods then he chooses either $p = 0$ or $p = 1$, whichever gives the highest per capita income. Furthermore substituting (15) and (16) into (56) gives the optimal $t$ as zero or the solution to

$$\ddot{y} + t \frac{\delta Y}{\delta t} = y_m,$$  

(59)

where $y_m$ is the decisive voter's optimal income. From (59) we see that if $x_m > x$, the decisive voter's productivity is greater than or equal to the average productivity, then $t = 0$ is the optimal choice. If $x_m < x$ then $t > 0$ is optimal.

In summary we have shown that if the median voter, (whom we have shown to be decisive in the previous section,) has great enough productivity, then only one good is redistributed. If his productivity is somewhat lower, but he still works, both goods are redistributed, but not in the proportions equivalent to cash. If the median voter is a non-worker both goods are redistributed in proportions equivalent to cash.
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