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Taxes, Votes and the Distribution of Income:
A Dynamic Model of the Growth of Government

by

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and
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Is majority rule compatible with a market economy in which incomes differ? Why do the poor not use their votes to confiscate and redistribute wealth? Is there a limit to the size of government in a growing economy? Will the political process eventually equalize after-tax incomes?

These questions have occupied philosophers and social scientists for more than a century. Four approaches to the relation of individual and collective choice have yielded some answers in recent years. One, starting from principles of justice or equity, derives normative rules of behavior. Recent books by Rawls (1971) and Nozick (1974) are well-known examples. A second analyzes public action as the result of agreements to supply public goods. Buchanan and Tullock (1962), Olson (1965) and Buchanan (1975) are representative of the diversity of this large, growing literature. The third approach treats the managers of collective enterprises as entrepreneurs who seek to achieve their personal aims at the expense of particular groups or society as a whole. Niskanen (1971) and Romer and Rosenthal (1977) differ about the constituents of the manager or bureaucrat, but both treat the managers' entrepreneurial activity as central to the explanation of why agencies and governments grow or decline. Buchanan (1975, 156-61) uses this argument also. Fourth, the literature on coalition formation emphasizes the activities of groups that impose their tastes on the community. For example, see Riker and Ordeshook (1973)

The collective that we analyze is "government." Our intent is to develop a positive theory of the growth of government¹ and to find whether majority rule is compatible with a market economy and differences in individual income. The recent approaches we have cited may add considerably to social scientists' ability to explain some features of the evolution of institutions, laws and regulations. They do not offer a useful starting point for analyses of the questions that concern us.

The services provided by modern governments and contributing to their growth include activities that do not fit easily into the class of goods, called public goods, that become available to members of a group or society once they are supplied to any member of the group. Defense, law, courts, and clean air or water are examples of public goods. For each of these goods or services it is plausible to claim that collective action increases utility. Collective action enables individual to obtain a supply of goods or services that all desire but which few citizens can purchase in optimal quantities. Extension of the analysis to treat income redistributions, the creation of public monopolies for health care, medical insurance and myriad other services as public goods emphasizes the returns to cooperation and neglects the loss from coercion.

¹
The limits to cooperation is a major theme of Nozick's (1974) book, but Nozick analyzes the limits of voluntary agreement, not the effect of coercion. Buchanan and Tullock (1962) and Rawls (1971) analyze idealized states in which observed institutions and practices reflect agreements or contracts acceptable to citizens. Rawls's analysis is normative; Buchanan and Tullock's is positive. Both treat constitutions, however, as contracts voluntarily arrived at under conditions of uncertainty about the outcome of the political and economic processes. Olson (1965, pp. 15,16) defines a state as an "organization that provides public goods for its members, the citizens" and treats other services of government as the inducements that "give potential members an incentive to join."

Models of public choice take a "public interest" approach to government. If collective action permits individuals to remove external diseconomies, the growth of government must be explained by the discovery of new or larger diseconomies. Individual members may be coerced or enticed to accept an outcome that is in the public interest, but, in the public choice literature, most of the observed increase in the size of governments in countries with majority voting increases utility for the individual members of the communities.

Some writers recognize that collective action may be coercive and may reduce utility for many voters. Niskanen (1971) analyzes the behavior of utility maximizing bureaucrats who transfer income or utility from the citizenry to themselves by increasing their budgets and span of control. Barro (1973) analyzes the gains to politicians from increasing income or power. Buchanan (1975) discusses these entrepreneurial theories and suggests some additional reasons why majority rule, practicing politicians and entrepreneurial bureaucrats may produce a government of larger than optimal size.

Entrepreneurial theories and coalition formation are unlikely starting points for a rational explanation of the growth of government. We have no problem accepting that politicians and bureaucrats or coalitions desire to increase their income or utility. The problem is to explain why rational voters do not learn about, or anticipate, the outcome. Most of the models of entrepreneurial behavior and public choice are static; learning, rational expectations about the future, and the search for a new optimum are not considered.

We have chosen a different approach. Households, the units of our analysis, behave in the polling place as they do in the market place. Just as their individual purchases have no effect on price and quantity in the market, their votes have no effect on taxes or the redistribution of income. They are price takers and takers of political outcomes who maximize the expected utility of present and future consumption subject to constraints. We use this model to derive the household's most preferred tax rate, to show the relation between the tax rates chosen by households at different levels of income and the tax rate chosen by the society, and to answer the questions we asked at the outset.
Our approach is closer to the spirit of Schumpeter's (1947) classic analysis of the decline of capitalism than to the modern approaches based on public goods. Schumpeter believed that large government bureaucracies would be voted into positions of power as the institutions supporting capitalism crumbled. He called the resulting system "socialism" and discussed the importance of rising income and the narrowing of the income distribution for the evolution he expected. Instead of Schumpeter's historical-institutional vision, however, we rely on maximizing behavior to determine the evolution of the interacting political and economic processes.

In the following section, we develop a model of an individual voter who chooses tax rates knowing that tax rates affect the growth of output. The voter is uncertain about the rate of growth. We solve for tax rates and the after-tax distribution of income with constant and changing before-tax distributions of income and under different assumptions about the utility function. Similar conclusions are obtained from each of the cases we analyze.

To incorporate growth of output and uncertainty, our model is dynamic and stochastic. We have attempted to reduce the burden on the reader who is willing to accept our results by limiting the derivations and putting some of the formal theory into an appendix.
A Dynamic Model of Voting for Taxes

The unit of our analysis is a rational household that maximizes the utility of consumption. Consumption depends on expected income, and increases with income. Each household expects disposable income to grow at a rate that depends on the growth of aggregate income, on the taxes paid, and on transfers from other households. Taxes are known to affect growth by changing effort or incentives, so future as well as current income and consumption are altered by taxation and redistribution.

Elections are simple, costless, and continuous. Bureaucrats and legislators remain in the background and do not affect the outcome of elections or the size of government. The relative size of government is measured by the fraction of before tax income collected as taxes. These assumptions are arbitrary but seem preferable to the more common assumptions of a once-and-for-all choice of the size of government.  

Proportional taxes and lump sum redistributions assure that income is redistributed from upper to lower income earners. Let $y(t)$ be the mean earned income before taxes. Each household receives $\tau y(t)$ from the government and pays $\tau h(t)$ in taxes, where $h(t)$ is the earned income of the household. Disposable income $g(t)$ is given by

\begin{equation}
 g(t) = (1 - \tau) h(t) + \tau y(t)
\end{equation}

The fraction of mean aggregate income earned by each household is $\phi(t)$ defined as

\begin{equation}
 \phi(t) = h(t) / y(t).
\end{equation}

Rewriting $g(t)$ in terms of $y$ and $\phi$ makes

\begin{equation}
 g(t) = [(1 - \tau) \phi(t) + \tau] y(t).
\end{equation}
To determine the growth rate of income, we adapt the findings of the balanced growth literature. In the absence of elections mean income grows at a constant long run average rate $a$ per unit time.

\begin{equation}
    y(t) = y(0) e^{at}
\end{equation}

Elections produce external shocks by changing tax rates. The effect of an election shock is to cause $y(t)$ to remove from one balanced growth path to another. We assume that the change in $y(t)$ at election time is directly proportional.
to the length of time, $\Delta t$, between elections. This is reasonable if voters make adjustments that last until the next election occurs, or until their anticipation of the outcome changes, $\Delta t$ later. Hence, if an election is held at time $t$,

$$y(t) = y(t^*) e^{-b(r(t))\Delta t}$$

The function $b(r)$ represents the rate of loss in mean income resulting from the disincentive effects of taxation. There are three reasons. First, pure public goods and external diseconomies are excluded by previous assumption. (See footnote 3). Second, the household, as a small part of a large society, regards aggregate and mean income as independent of the household's effort. Third, all transfers are lump sum transfers, independent of earned income and employment, but taxes are not. As tax rates rise, after-tax income per hour of labor or per unit of capital falls, so the household chooses to supply less labor, to consume more leisure, to take less risk and to supply less saving. These decisions reduce the growth of mean before-tax income. Higher tax rates increase the loss; $b'(r) > 0$.

A typical path for $y(t)$ is shown in Figure 1. As can be seen in the figure,

[INSERT FIGURE 1 ABOUT HERE.]
Figure 1

A Typical Path for $y(t)$
If elections or, equivalently, adjustments to anticipated election outcomes occur continuously, then the process for $y(t)$ can be derived from (6) by taking the limit as $\Delta t \to 0$:

$$y(t) = y(0) \exp \left[ \int_0^t \gamma(s) \, ds \right]$$

where

$$\gamma(t) = a - b \, (t). \quad a, b > 0$$

Writing the process for mean income in differential form allows us to easily interpret $\gamma(t)$. From (7) we see that

$$\frac{\dot{y}}{y} = \gamma(t)$$

so that $\gamma(t)$ is the instantaneous rate of growth of mean income. Hence, the effect of taxation in our model is to change the instantaneous rate of growth of the economy.

The median income voter plays an important role in our model. We denote by $\hat{\phi}$ the median income voter's fraction of average income. Throughout we assume that $\hat{\phi} < 1$ so that the median income household earns less than the mean income. The median income household's fraction of the mean earnings may change over time so that $\hat{\phi}$ has dynamics.
The median income household, however, remains for all time at the median and realizes this to be the case. Households above the median remain above and those below remain below.

Taxes are determined by majority rule. In equilibrium the tax rate is chosen as \( \hat{y} \) the median income voter, hereafter called the median voter, were dictator. Hotelling (1929) just established the dominance of the median voter under majority rule in a single period, once-and-for-all election. We prove below that the median voter result extends in our model to many periods and in fact to continuous elections. Hence, the median voter selects the tax rate by maximizing his lifetime utility.

To establish the dictatorship of the median voter we apply the equilibrium concept of rational expectations, just suggested by Muth [ ]. We show that if all voters expect (or know) that the median income voter will prevail in setting future tax rates, then they vote in such a way that the median income voter sets the current tax rate. As time passes the voters' anticipation of the dominance of the median voter is self-fulfilling or rational.

\[
(10) \quad \phi(t) = a(\hat{\phi}(t)).
\]
Each household's position in the distribution of earned income is given by \( \phi(t) \) and depends only on the parameter \( \sigma^2 \). If \( \sigma^2 \) is not a random variable, \( \phi \) is not a random variable. Any particular household, say \( i \), may be certain forever about its position, \( \phi_i(t) \), in the distribution of income. The household is uncertain, however, about disposable income since household income depends on the mean aggregate income. For example if \( \sigma^2(t) = \sigma^2 t + \sigma_0^2 \), the median income household would know for certain that they will earn a fraction

\[
\phi(t) = e^{-\frac{1}{2} \sigma^2 t - \frac{1}{2} \sigma_0^2}
\]

of the average income, even though the time \( t \) average income \( y(t) \) is a random variable.

By differentiating (19), we can characterize the dynamics of \( \phi(t) \):

\[
d\phi(t) = -\frac{1}{2} \sigma^2 \phi(t) dt.
\]

More generally, we investigate the case where

\[
d\phi(t) = a(\phi(t)) dt
\]

for any smooth function \( a \). To each function \( a \) corresponds a function \( \sigma^2(t) \) that generates (21) and characterizes the distribution of \( \phi(t) \). The typical distribution of earned income is approximately log normal but has a positive lower bound. Let \( p \) be the ratio of the minimum earned income to the mean. The minimum earned income for a household is \( p y(t) \) and the distribution of income is a displaced log normal distribution, as in (22). See Aitchison and Brown (1957).

\[
\phi - p \sim \Lambda(-\frac{1}{2} \sigma^2(t) + \phi(1-p), \sigma^2(t)) \text{ for } \phi > p.
\]

The average value of \( \phi \) is 1 as required, and the median is

\[
\hat{\phi} = p + e^{\frac{1}{2} \sigma^2(t)} (1-p).
\]

If, for example, \( \sigma^2(t) = \sigma^2 t + \sigma_0^2 \), then

\[
d\hat{\phi}(t) = -\frac{1}{2} \sigma^2 e^{-\frac{1}{2}(\sigma^2 t + \sigma_0^2)} (1-p) dt = -\frac{1}{2} \sigma^2 (\hat{\phi} - p) dt,
\]
and the relative rate of change of \( \phi \) is
\[
a(x) = \frac{1}{2} \sigma^2 (x - p).
\]
Our previous example is a special case of (24) when \( p = 0 \).

To complete the model, we make explicit three assumptions about the behavior of voters stated earlier. First, no voter believes that his vote has any effect on the tax rate. The tax rate is treated as the price per unit of earned income paid to redistribute income. Each voter treats the price as given. Second, the amount of income redistributed is determined by majority rule. Some voters gain disposable income and utility; others lose. Third, each voter chooses the tax rate that maximizes the utility of consumption. Consumption is proportional to disposable income, so we assume each voter maximizes

\[
\max_{\tau \in [0,1]} \mathbb{E} \int_0^\infty e^{-rt} u(g(t))dt,
\]

where \( u(\cdot) \) is the common utility function for disposable income or consumption in each household. Each household's vote is determined by the state variables \( y(t) \) and \( \phi(t) \), which characterize the state of the economy and the state of the household respectively.

For voters with (household) income greater than or equal to the mean, \( \phi \geq 1 \), the optimal vote is a negative tax, but we restrict them to vote for zero. The reason is that transfers reduce their current disposable income, as shown by (3), and taxation reduces the growth of earned income by our assumption, \( \gamma'(r) < 0 \). Voters with incomes below the mean receive transfers that raise disposable income but lose future income as growth declines. The lower their before-tax incomes, the larger is the transfer and the higher is the desired tax. The voter with median before-tax (earned) income determines the current tax rate. This result, first established by Hotelling (1929), implies that the choice of optimal tax rate under majority rule depends on the behavior of the voter with the median income hereafter called the median voter.

To find the optimal vote, define the value function

\[
V(y, \phi) = \max_{\tau \in [0,1]} \mathbb{E} \int_0^\infty e^{-rt} u(g(t))dt,
\]
where the dynamics for $y$ are given by (13) and the dynamics for $\phi$ are given by (21).

We use optimal stochastic control theory, Fleming and Rishel (1976), to formulate the Bellman equation for our problem:

$$
(27): \quad 0 = \max_{\tau \in [0, 1]} \left\{ u[(\phi - \tau \phi + \tau)y] - rV + V_y \gamma'(\tau) + V_{\phi} \alpha(\phi) + \frac{1}{2} \eta^2 V_{yy} \right\}
$$

where $V_y \equiv \frac{\partial V}{\partial y}$, etc. If there is an internal maximum, the first order condition for (27) is

$$
(28): \quad 0 = u'[\phi - (\tau \phi + \tau)y] (1 - \phi)y + V_y \gamma'(\tau).
$$

The second order condition is

$$
(29): \quad u''[\phi - (\tau \phi + \tau)y] (1 - \phi)^2 y^2 + V_y \gamma''(\tau) < 0.
$$

To proceed further we must know more about the functions $u(\cdot), \alpha(\cdot)$ and $\gamma(\tau)$. If taxes have no effect on growth, voters with income below the mean gain from income redistribution. The rational solution to this case is trivial. A fortiori, a positive relation between tax rates and growth makes non-confiscatory taxation puzzling. Earlier, we assumed a simple, negative relation.

$$
(10): \quad \gamma(\tau) = a - b\tau, \quad a, b > 0.
$$

Substituting (10) into (29) we see that the second order conditions are satisfied if $u$ is merely concave.

The effects of $u$ and $\alpha$ on voting require more discussion. The following sections consider these effects and answer the questions posed in the introduction using different assumptions about the distribution of income and different classes of utility functions.
The Optimal Tax Rate when the Income Distribution is Fixed

The choice of an optimal tax rate in a world of "tax takers" depends on the preferences of the voters, the distribution of earned income and the costs and benefits of redistribution. We recognize that differences in attitudes toward risk may contribute to the growth of government, but we exclude interpersonal differences in risk aversion. This section analyzes voting using a general class of utility functions and a specific assumption about the distribution of income. Voters choose an optimal tax rate in a society where the distribution of income does not change; \( \alpha = 0 \) (in eq. (27)). The position of the distribution shifts as mean income changes, but the shape of the distribution remains unchanged.

Each household has a utility function for consumption (or disposable income) characterized by constant relative risk-aversion.

\[
\text{(30)}: \quad u(g) = \begin{cases} 
\frac{g^\lambda}{\lambda} & \lambda < 1, \, \lambda \neq 0 \\
\ln g & \lambda = 0
\end{cases}
\]

Substituting (10) and (30) into (28), we get

\[
\text{(31)}: \quad 0 = [ (\phi - \tau \phi + \tau) y ]^{(1-\lambda)} (1 - \phi) - V_y b,
\]

which can be solved for the optimal tax rate.

\[
\text{(32)}: \quad \tau = \frac{-\phi}{1-\phi} + \frac{1}{(1-\phi) y} \left( \frac{V_y b}{1-\phi} \right)^{\frac{-1}{1-\lambda}}
\]

Substituting (10), (30), (32), and \( \alpha = 0 \) into (27) yields, for the distribution of income and class of utility functions considered in this section, the equation

\[
\text{(33)}: \quad 0 = \frac{1}{\lambda} \left( \frac{V_y b}{1-\phi} \right)^{\frac{-1}{1-\lambda}} - rV + ayV_y + \frac{b\phi}{1-\phi} yV_y + \frac{1}{2} \eta y^2 V_{yy}.
\]
A solution to (33) is

\[ V(y) = \frac{(1-\phi)}{\lambda b} \left[ \frac{\gamma - \alpha \lambda}{1-\lambda} + \frac{1}{2} \eta^2 \lambda \right] \left( \frac{1-\phi}{b} \right) - \frac{\lambda \phi}{1-\lambda} \cdot y^\lambda. \]

Finally, substituting (34) into (32) gives the optimal tax rate as

\[ \tau(\phi) = \frac{\lambda \phi}{b (1-\lambda)} + \frac{1}{2} \eta^2 \lambda b - \frac{\phi}{(1-\phi)(1-\lambda)}. \]

By previous assumption, we ignore tax rates outside the range zero to one.

Each voter chooses a tax rate that depends on his (fixed) position in the distribution of income but is independent of mean per capita income, \( y \).

Majority rule implies that the actual tax rate is determined by the median voter whose share of earned income \( \phi = \bar{\phi} \).

Our model explains why a majority of the voters do not vote to confiscate income. The cost of redistributing income increases with \( b \). If the disincentive effect of taxes on growth, \( b \), is large, voters below the median vote against confiscation to avoid the costs they bear individually. The present value of future income falls as the discount rate, \( r \), increases. Societies in which voters discount the future at high rates, have lower perceived costs of redistribution, so redistribution increases.\(^9\)

The distribution of income is constant in this section, so our analysis of changes in the distribution of income is limited to comparative statics.

\[ \frac{\partial \tau}{\partial \phi} = - \frac{1}{(1-\lambda)(1-\bar{\phi})^2} < 0. \]

Our result for a growing economy under uncertainty is similar to Romer's (1975) finding for a single-period static economy under certainty.

The behavior of the tax rate as a function of the variance, \( \eta^2 \), of mean income depends on risk aversion, \( \lambda \). The Pratt-Arrow measure of risk aversion for the class of utility functions given by (30) is

\[ R(\lambda) = \frac{1-\lambda}{g}, \lambda < 1, \]
so risk-aversion increases as \( \lambda \) decreases. We can see from (35) that a more risk-averse society, \( \lambda < 0 \), lowers tax rates as risk, \( \eta^2 \), increases whereas a less risk-averse society, \( \lambda > 0 \), raises tax rates to compensate for higher risk. Similarly, more risk-averse individuals choose not to share risks through redistribution; less risk-averse individuals vote to increase taxes as risk increases.

This result appears anomalous to anyone who believes that government grows because the most risk averse vote to shift their risks to others by voting for programs like social security, medical care or disaster insurance. Such reasoning is incorrect, where taxes reduce growth. Consider the instantaneous coefficient of variation for \( dy(t) \), which is \( \eta/\gamma(\tau) \). When selecting a tax rate voters, among other things, determine \( \eta/\gamma(\tau) \). More risk-averse voters tolerate lower values of \( \eta/\gamma \); less risk-averse voters tolerate larger values. The choice of tax rate determines in part the nature of the income gamble each voter faces throughout his lifetime. More risk-averse voters want a higher mean growth rate relative to a fixed standard deviation for this gamble than less risk-averse voters, so they vote for lower taxes and higher growth at every level of \( \eta \).

The distribution of disposable income that results from decisions to tax and redistribute income is obtained by substituting \( \hat{\tau} \) in (3), where \( \hat{\tau} \) denotes the median voter's choice of tax rate.

\[
(38) \quad g(t) = ((1-\hat{\tau})\phi + \hat{\tau})y(t)
\]

Since \( \hat{\tau} \) is a constant and \( \phi \) has a lognormal distribution, we see from (38) that, conditional upon \( y(t) \), the distribution of income is a displaced lognormal distribution. The average per capita disposable income is, of course, equal to the average per capita earned income. Not surprisingly, the voters' median disposable income \( \hat{g} \) is larger than the median earned income. Redistribution and taxation raise the median and reduce the difference between the median and the mean.

\[
(39) \quad \hat{g} = \hat{\tau}y + (1-\tau)\hat{h},
\]
where \( \hat{h} = \hat{\phi}y \) is the median before tax income. The entire distribution of income has become tighter. This can be seen by taking the difference between disposable income and earned income as a function of \( \phi \):

\[
g(t) - h(t) = \hat{\gamma}(1-\phi)y(t)
\]

For \( \phi < 1 \), \( g - h > 0 \), while for \( \phi \geq 1 \) we have \( g - h \leq 0 \). Voters below the mean gain through the political process; voters above the mean lose.

Voters in a growing economy subject to uncertainty do not equalize after tax incomes even if the distribution of income is fixed, as in this section. The result depends on the cost borne by the voters and is independent of the presence of uncertainty. Equation (35) can be used to develop the conditions for confiscation in a world of uncertainty by setting \( \lambda = 0 \) and solving for the special case of confiscatory taxes, \( r = 1 \).

\[
\phi = 1 - \frac{b}{r}
\]

Again, the mean and the median disposable income differ if future consumption is reduced by taxation. If mean income grows and taxes reduce growth, a necessary condition for confiscation is \( r > b > 0 \).

The model analyzed in this section does not imply that government grows in countries with majority rule. The reason is that mean income, the only dynamic variable in our model, does not affect the voter's choice of tax rates, eq (35). The reason \( \tau \) does not depend on \( y(t) \) is the assumption, made in (7) and (13) that there is no effect of \( y \) on \( \tau \). To explain the observed growth of government, we must rely on changes that are not fully described by the model, such as the reduction in median income of voters through the spread of the franchise. In the following sections, we develop more complete explanations.
The Maximum Size of Government and the Effect of Redistribution

This section relaxes one previous restriction but retains another. Tax rates remain independent of $y$. The distribution of income changes, however, and the changing distribution introduces a dynamic element in the determination of tax rates. We derive the steady state tax rate resulting from majority rule, and explore some effects of taxation on the distribution of disposable income. To obtain the results, we restrict analysis to a particular form of (30), the logarithmic utility function, $u(g) = \ln g$. Our previous results suggest that differences in attitudes toward risk are not critical for the problem, but we are not able to confirm this result.  

Solving (27) and (28) in a manner parallel to the procedure in the previous section we find

\begin{equation}
(42) \quad V(y, \phi) = \frac{1}{r} \ln y + c(\phi),
\end{equation}

where $c(\phi)$ is a solution to

\begin{equation}
(43) \quad \theta = \ln \left[ \frac{1 - (1 - \phi) b}{b \phi} \right] = r c + \frac{1}{r} \left[ a + \frac{b \phi}{1 - \phi} \right] - 1 + \alpha(\phi) c' - \frac{1}{2} \eta^2 \phi.
\end{equation}

The tax rate chosen by majority vote is independent of $y$ and is given by

\begin{equation}
(44) \quad \tau(t) = \frac{r}{b} - \frac{\hat{\phi}(t)}{1 - \hat{\phi}(t)}.
\end{equation}

Comparing (44) with (35), we see that the two equations are identical for $\lambda = 0$.

The only effect of changes in the distribution of income on the optimal tax rate is that the median value of $\phi$ changes. As before, the tax rate chosen by majority vote is never confiscatory for $b > r > 0$. The result shown as eq. (41) applies equally to the changing and to the static distribution of income.

The movement of $\tau(t)$ through time depends upon the movement of $\phi(t)$ through time. If $\phi(t)$ is moving away from 1, the voters' median income is falling as a percentage of the average income. Equation (44) implies that the tax rate rises as the median voter's income falls relative to the mean. A decline in median income relative to mean income is sufficient, therefore, to explain the growth of the government.
Our results imply that the growth of government depends on the difference between the median and the mean income. We have shown that changes in the distribution of income that shift the median away from the mean, or extensions of the franchise that lower the income of the median voter, are sufficient to explain the growth of government in countries with majority rule. Further, the model implies that if tax rates reduce growth, the mean and the median do not coincide.

To show that the political-economic system we analyze has a steady state solution, we use the model of the displaced log-normal distribution, given by (22). The dynamics of the median income in the model, shown as (24) are consistent with the model we have solved. Substituting (23) and $\sigma^2(t) = \sigma^2_1 + \sigma^2_0$ into (44) yields

$$\tau(t) = \frac{r - \frac{p}{1-p} + \frac{1}{2}(\sigma^2 + \sigma_0^2)}{(1-p) \left(1 - \frac{1}{2}(\sigma^2 + \sigma_0^2)\right)}.$$  \hspace{1cm} (45)

As all incomes become arbitrarily close to the mean income, $p$ approaches one, tax rates are always at a lower bound, zero in our analysis, as (45) shows.

The time rate of change $\tau$ is given by

$$\tau'(t) = \frac{1}{2} \sigma e^2 - \frac{1}{2}(\sigma^2 + \sigma_0^2) > 0.$$  \hspace{1cm} (46)

Hence the movement of $\phi$ as specified in (24) causes the government to grow until it accounts for a fraction of earned income

$$\tau(\infty) = \frac{r}{b} - \frac{p}{1-p}.$$  \hspace{1cm} (47)

The steady state tax rate depends on three parameters, the discount rate $r$, the effect of incentives on growth, $b$, in the first term of (47) and $p$, the ratio of minimum to mean income in the second term. The second term is a fraction that cannot exceed the first term at any positive tax rate. The sufficient condition for a positive tax rate less than unity is, therefore,

$$b > r.$$  \hspace{1cm} Under this condition for $1 > p > \frac{1}{2}$ the steady state tax rate converges to zero. As long as the franchise is restricted to voters with relatively high incomes, tax rates do not increase and can decrease.
The conditions for high and low values of $\tau$ have intuitive appeal. If voters expect to reduce growth of income more than they discount future earnings, the tax rate is bounded below unity. If all incomes are the same, no one gains from redistribution. If the franchise is restricted to property owners or households with relatively high income, there is less redistribution.\(^{13}\)

The conclusions just drawn depend on the movement of $\sigma^2(t)$, the variance parameter of voters' earned income. For growth of government to result from changes in the distribution of voters' income, $\sigma^2(t)$ must increase over time. The spread of the franchise is a plausible explanation of a decline in voters' median income, but once property qualifications and other restrictions on the franchise are removed, a principal force reducing the median voters' income, relative to mean earned income, disappears. Moreover, it is widely believed that incomes have become more homogeneous—that the interquartile range of the earned income distribution, or any other fractile range narrows in advanced industrial countries. The rise in $\sigma^2(t)$ seems inconsistent with this belief.

For a log normal distribution of income, such as (22), an increasing $\sigma^2(t)$ results in a greater homogeneity of income. The failure of intuition is the neglect of the effect of the long right tail of skewed distribution such as the lognormal. This is shown by analyzing (22).

\[
\omega_n(\phi - p) - \omega_n(1 - p) + \frac{1}{2} \sigma^2(t) \sim N(0,1).
\]

Let $f$ denote the $n$-fractile of the unit-normal distribution; e.g., $f_{.25} = -.67$ is the lower quartile, and $f_{.75} = .67$ is the upper quartile. Similarly denote $\phi_n$ as the $n$-fractile of the distribution of $\phi$. Then from (48) we get

\[
\phi_n = p + (1 - p) \exp[\sigma(t)f_n - \frac{1}{2} \sigma^2(t)].
\]

For $n > \frac{1}{2}$, the symmetry of the unit normal distribution gives $f_{1-n} = -f_n$. Hence for $n > \frac{1}{2}$ the interfractile range $I(n)$ of the $\phi$ distribution is

\[
I(n) \equiv \phi_n - \phi_{1-n} = (1 - p)e^{-\frac{1}{2} \sigma^2(t)} \sigma(t)f_n - e^{-\sigma(t)f_n}, \text{ for } n > \frac{1}{2}.
\]
Therefore,

\[
\frac{\partial I(n)}{\partial t} = -(1 - p)\sigma'(t)e^{-\frac{1}{2}\sigma^2(t)} \left[ (\sigma(t) - f_n)e^{n\sigma(t)} - (\sigma(t) + f_n)e^{-n\sigma(t)} \right].
\]

Clearly for \(\sigma'(t) > 0\) and large enough \(\sigma(t)\), \(\partial I(n)/\partial t < 0\). In fact if \(\sigma(t) \to +\infty\) as \(t \to +\infty\) then \(I(n) \to 0\). For example if \(\sigma^2(t) = \sigma^2 + \sigma_0^2\) as we assumed, so that taxes are given by (45), then \(I(n)\) will collapse as \(t\) increases. For sufficiently small values of \(\sigma(0)\), however, \(I(n)\) will initially expand. To see this we approximate the right hand side of (51) to a first order in \(\sigma\) for small \(\sigma\):

\[
\frac{\partial I(n)}{\partial t} \approx 2(1 - p)\sigma'(t)f_n > 0 \text{ for } \sigma'(t) > 0.
\]

Equation (51) and (52) show that the interfractile range shrinks as \(\sigma^2\) increases. Earned income become more homogeneous through time as the median income declines relative to the mean. Our result does not deny that the interfractile range may increase for a time but, beyond some point, the range decreases.

In Figure 2 we illustrate the shift in distribution of relative income. Figure 2a shows the distribution of \(\phi\) for \(p = 0\) and two values of \(\sigma^2\); \(\sigma^2 = .2\) and \(\sigma^2 = 1\). Notice that the higher \(\sigma^2\) concentrates the distribution towards the left while raising the right hand tail. In Figure 2b we set \(p = 1/3\) and examine the distribution for the same values of \(\sigma^2\). Again, increasing \(\sigma^2\) concentrates the distribution to the left while raising the right hand tail.

An intuitive explanation of the changing shape of the income distribution is not hard to find. Division of labor increases in a growing economy. There are more opportunities for specialized skills to be recognized and rewarded. The long-tail of the right side of the log normal distribution stretches off to very high incomes relative to the mean income, raising \(\sigma^2(t)\) while the mean of \(\phi\) remains at 1. At the same time, the mass of incomes are being more tightly compressed into a narrow range by the spread of education, the reduction in unskilled and semi-skilled occupations relative to the wider range of specialized occupations that have similar, but not identical, returns. Economists should not be surprised to find that division of labor increases with the size of the market or to learn that the returns to many different types of human wealth are pushed toward equality.
Figure 2a: The distribution of $\phi$ for $p=0$ and $\sigma^2=.2$ or $\sigma^2=1$. 
Figure 2b: The distribution of $\phi$ for $p = 1/3$ and $\sigma^2 = .2$ or $\sigma^2 = 1$. 
As income rises, \( \frac{1}{y_k} \) approaches zero, so the tax rate approaches a steady state value for all \( \phi < 1 \), shown as (58)

\[
(58) \quad \tau(\infty, \phi) = \frac{c - \phi}{1 - \phi}
\]

The speed of approach depends on the degree of risk aversion, \( \kappa \), the discount rate, \( r \), and other parameters, but a nonconfiscatory steady state tax rate in a growing economy depends only on \( c < 1 \) which is to say that

\[
(59) \quad b > a - \frac{1}{2} \eta^2 \kappa > 0
\]

is a necessary and sufficient condition for nonconfiscation. Equation (59) requires the disincentive parameter, \( b \), to exceed the risk adjusted maximal growth rate. From inspection of (58) and (59) it is clear that the steady state tax rate is positive if (59) holds and the tax rate increases as \( b \) declines relative to \( a \). The tax rate falls as the cost, in foregone income, rises.

Rising income raises tax rates, as shown in (57), provided

\[
(60) \quad \frac{1}{1 - \phi} > r_{bc}(\phi)
\]

A sufficient condition for (60) is most readily established for \( \eta^2 = 0 \). For this case, (61) is a sufficient condition.

\[
(61) \quad a > r
\]

The discount rate must be less than the maximal attainable growth rate in a world without taxes. Combining (59) and (61) for \( \eta^2 = 0 \), we have a sufficient condition for tax rates to rise with income and approach an asymptote at \( 0 < r < 1 \). The condition is

\[
(62) \quad b > a > r
\]
As in all previous cases, the voter compares the disincentives effect of higher tax rates to the discount rate applicable to future income and votes accordingly.

The effect of income on tax rates is not entirely unambiguous. For low values of $\phi$ and $r > a$, rising income, for a time, lowers tax rates. As $\phi$ rises, however, the left side of (60) rises also, and tax rates rise with income. If the poorest voter has a marketable skill, $\phi$ is bounded from below, and under the assumed conditions, tax rates rise as income rises unambiguously.

These conditions for Schumpeter's conjecture do not require any assumptions about diminishing investment opportunities, obsolescence of the entrepreneurial function, high discount rates on the future, or the hostility of the intellectuals. Schumpeter (1947, Chaps. 10-13). The discount rate on private projects, $r$, may be large. If the disincentive effect of government programs is larger than $r$, rational voters do not vote higher taxes. Neither uncertainty nor growing risk aversion is required for our conclusions. We have, instead, assumed constant risk aversion and $\eta^2 = 0$ to reach (61) and (62).

We can relax the restriction on $\eta^2$ and derive the condition for positive, non-confiscatory tax rates in a world of uncertainty and show that our conclusions hold.

\[
(63) \quad \frac{ba}{c} > b + 1/2 \eta^2 k > a > 1/2 \eta^2 k
\]

The first inequality in (63) satisfies eq. (60) and, therefore, assures that rising income is a sufficient condition for the growth of government. A necessary condition for the first inequality is $a > r$, as in (61). The second inequality requires the disincentive effect of taxes to exceed the risk adjusted maximal growth rate. This is condition (59) and we know that (59) keeps the steady state tax rate below unity. The third inequality assures that tax rates are positive. If (62) holds there is always a value of $\eta^2 k$ small enough to satisfy (63).

All of the conditions have been obtained for $\phi < 1$, so together they answer the questions posed at the start. The answers differ only in detail from the conclusions reached previously. Rising real income appears to be sufficient to explain the growth of government observed in most countries during this century.
Limitations

Our explanation of the growth of government is based on rational behavior by voters but suffers from several limitations. Three limitations are sources of concern.

First, our model is not a general equilibrium model. The principal deficiency is the failure to treat the effect of taxation on saving and consumption choice and on the distribution of time between labor and leisure. Disincentives to labor and to investment have a large role in our analysis. A general equilibrium analysis would make the rate of growth dependent on the properties of the system by determining the supply of productive services. As the model stands, the level of income and the growth rate are affected by disincentives, but $\phi$ is unaffected.

Second, a problem of a different kind arises if we attempt to find the empirical counterparts for the distribution of income. The distribution of earned income is not easily observed. Not only are there the usual effects of age and permanent income that Paglin (1975) has shown to be important, but there are also effects of tax rates on effort. The distribution of income that we observe incorporates the influence of expected tax rates on the household's choice of labor and leisure. The problem can be reduced by analyzing the after-tax distribution, $g$, instead of $h$ or $\phi$, but simple comparisons of median and mean income may not provide a proper test of the conclusions we reached about the relation of tax rates and the distribution of income. Compounding this problem is the poor quality of the data on voter's income.

Third, neither the political process nor many of the variables alleged to affect choice of candidate have a role in the analysis. A world in which two candidates strive for a majority seems compatible with our analysis but has not been incorporated. If all candidates and voters knew the median voter's income, there would be more certainty about the outcome of elections and little incentive for voters in the left or right tail to vote. Uncertainty about who votes or lack of information about voters' incomes would remove this limitation. Since all voters do not vote in each election, uncertainty about the median voter's plays a role in elections.

Despite these limitations, our model offers an explanation, based on rational behavior of two phenomena observed in many countries. Growth rates of the government and the private sector differ, and the government grows faster than the private sector for long periods of time. Also, the growth of private sector output is not a constant but depends on institutions and the structure of incentives that a society—in our case a society of rational voters—permits.
Conclusion

Four questions are posed at the start of the paper. The answers are obtained from a model in which voters are rational, the distribution of income is lognormal, the growth of income is uncertain, and the majority rules. Contrary to most models of public choice, voters believe they have no effect on the tax rate or on other political decisions. They are "tax takers," who maximize the utility of current and future disposable income and respond to incentives and the disincentives in their environment.

Rising mean income or the spread of the franchise is sufficient to explain the growth of government observed in many countries during this century. Changes in the distribution of earned income that reduce the median relative to the mean are sufficient also.

We show that in a growing economy, there is a limit to the size of government. The limit depends, in each of the cases we consider, on the relation between the disincentives to growth from taxes and redistribution and the rate at which income recipients discount the future. Higher tax rates, or other disincentives that slow the growth of income, reduce the growth of government.

The economy we consider has a steady state solution at which taxes are not confiscatory and disposable incomes differ. The poor do not confiscate the rich once they recognize that the rich, like the poor, contribute to future incomes by supplying labor and capital. If growth ceases, and there is no further loss to the poor from redistributing income equally, all income is confiscated and redistributed. Of course, there may be other large costs of redistribution—such as the costs of maintaining the redistribution agency—that limit the size of government in an economy without growth. These costs are difficult to reconcile with maximizing behavior and the long-run character of our model and are not discussed.
FOOTNOTES

* We have benefitted from helpful discussion with Thomas Romer, William Poole, Gordon Tullock and Karl Brunner. Helpful comments were made by Peter Aranson and Morris Fiorina at the 1977 Public Choice Society meeting.

1 Collective action is not the same as government, so our approach differs from other approaches in this respect also Olson (1965) emphasizes this point and it is central in Nozick's (1974) discussion of anarchy.

2 See the references in the introduction. The assumption that elections are costless can be relaxed without affecting the main conclusions. A fixed cost of holding an election would determine an optimal spacing of elections and avoid continuous recontracting. Continuous recontracting permits anticipations of the outcome of future elections to change continuously.

3 Public goods and externalities can be introduced by setting a minimum tax rate that covers the cost of these programs. The principal changes would be the effect on output achieved by increasing efficiency and on the minimum tax rate chosen by voters whose incomes are above the mean. Nothing would change if a fraction of the lump sum transfer is paid to bureaucrats. Bureaucrats are voters and taxpayers whose vote depends on the solution of their household maximization problem.

4 The term \(-\frac{1}{2}\gamma^2t\) in (4) is required to get the expected result shown as eq. (5) of the text. If \(-\frac{1}{2}\gamma^2t\) were absent, the average growth of \(y(t)\) would be \(a+1/2\gamma^2t\) instead of \(a\). This is easily shown. Let \(\tilde{x}\) be a lognormal random variable with parameters \(\mu\) and \(\sigma^2\), i.e., \(\tilde{x} \sim \Lambda(\mu, \sigma^2)\). Then

\[
E(\tilde{x}) = e^{\mu + 1/2 \sigma^2}
\]

and

\[
Var(\tilde{x}) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1).
\]
The effect of income redistribution on aggregate consumption is analyzed in Blinder (1975). Using a permanent income consumption function, Blinder shows that redistribution from upper to lower income groups changes the marginal propensity to consume and reduces aggregate consumption. This result is obtained from a model in which all redistribution is voluntary. There are no taxes and no effects on the supply of labor. Romer (1975) has shown the disincentive effect on labor supply in a static one-period model.

Two other growth and election models lead to (9). First, suppose as in eq. (5) that \( \ln y(t) \) increases at the certain rate \( a \) between elections, but now assume election results are uncertain. An election at time \( t \) results in an uncertain change

\[
\ln y(t) - \ln y(t') \sim N([1/2\eta^2 - b(\gamma(t))], \eta^2\Delta t).
\]

Taking the limit as \( \Delta t \to 0 \) yields (9). Second, we can combine the scenario given in the text—stochastic growth with certain election outcomes—with the model given above; we get a model with stochastic growth and stochastic election outcomes. It is easily shown that (9) can be derived by choosing the stochastic growth and election processes to be independent.

Observed distributions of income have been found to be approximately lognormal in several studies. See the references in Battalio, Kagel and Reynolds (1977). Note, however, that measured income distributions incorporate the disincentives resulting from current and anticipated tax rates on the use of labor and capital, so they are not entirely accurate measures of the true distribution.

The introduction of public goods changes the votes above the mean. Instead of zero, households above the mean vote for a tax to pay for the services of Nozick's (1974) minimum state.

If there are a few rich and many poor, the cost of not redistributing income includes the prospect that others will redistribute your income to themselves. This may explain why majority rules has never evolved or has been replaced by dictatorship in many countries.
Under the special assumption \( \lambda = 0 \), we obtain \( \tau = 0 \) when
\[
\phi \geq \frac{r}{r+b}.
\]

We have solved our model for \( \lambda \neq 0 \) and \( a(\phi) = a \phi \), but have found the result to be an uninterpretable and complicated equation containing indefinite integrals. We will supply the solution to interested parties upon request.

In particular \( a(\phi) = -\frac{1}{2} (\phi - p) \sigma^2 \) is not an explicit function of time, so the infinite horizon, steady-state solution of the problem is an acceptable solution.

Restrictions on the franchise help to explain a puzzling phenomenon—repeal of restrictions on trade and exchange and the spread of classical liberalism in the late 18th and early 19th centuries. The extension of the franchise in the nineteenth century increased the proportion of low income voters and reduced the voters' median income.

The diffusion process given by (53) was originally studied by Feller [1951]. Cox and Ross [1976] introduced the process in the economics literature to analyze common stock prices. They show that the distribution of \( y(t) \) is complex. Cox and Ross [1976, p. 161]. For \( \gamma(\tau) \) constant the mean of \( y(t) \) is
\[
E[y(t)] = y_0 e^{\gamma t}
\]
and the variance is
\[
Var [y(t)] = y_0 \eta^2 e^{\gamma t} (e^{\gamma \tau} - 1) / \gamma.
\]

Constant risk aversion implies that each individual is indifferent between buying and selling risky ventures at the market price. If risk aversion were permitted to change for each individual and for society as income grows, the dynamic effect of risk could be separated from the effect of rising income.
APPENDIX

Stochastic differential equations (SDEs) are similar to ordinary differential equations in some respects, but their solutions are stochastic processes instead of ordinary functions of time. Their usefulness derives from the fact that the set of solutions to all possible SDEs is essentially the set of all diffusion processes. A diffusion process is simply a continuous time Markov process which changes continuously in the space coordinate; jumps, such as in a Poisson process, are not permitted. See the appendix to Fischer (1975) for a brief treatment, and see Arnold (1974) or Gihman and Skorohod (1972) for a thorough treatment.

It can be shown that all diffusion processes are transformations of the Wiener process which plays a central role in SDEs. The Wiener process \( w(t) \), also known as Brownian motion, has the following properties.

1. \( w(t) \) is a normal random variable with mean zero and variance \( t \).
   \( w(0) = 0 \).
2. For any period \( t_0 < t_1 < t_2 < t_3 \), the increment \( w(t_3) - w(t_2) \) is independent of previous increments \( w(t_1) - w(t_0) \).
3. \( w(t) \) is continuous with probability one.
4. \( w(t) \) is nowhere differentiable with probability one. The fourth property makes the terms \( dw(t) \) that appears below (and in other works using SDEs) seem rather mysterious.

The \( dw(t) \) terms refer to integration with respect to the Wiener process. Just as the ordinary differential equation
\[
df(t) = dt
\]
can be integrated to find a solution function
\[
f(s) - f(0) = \int_0^s dt = s,
\]
the stochastic differential equation
\[
df(t) = dw(t)
\]
can be integrated to find a solution stochastic process
\[
f(s) - f(0) = \int_0^s dw(t) = w(s).
\]
But here the similarity ends. For example

\[ \int_0^s w(t) \, dw(t) = \frac{1}{2} s w^2(s) - \frac{1}{2} s. \]

The extra term in the right-hand-side (RHS) of (A1) appears so that the solution make sense probabilistically. In fact Ito (see Arnold (1974), Fischer (1975)) has developed a stochastic calculus that gives rules for correctly integrating and manipulating stochastic integrals or, equivalently, SDEs.

The fundamental theorem of the stochastic calculus is known as Ito's Lemma or Formula.

If certain regularity conditions are imposed on the mean, \( \mu \) and standard deviation, \( \sigma \), [see Gihman and Skorohod (1972)] then the SDE

\[ \text{(A2)} \quad dx(t) = \mu(x(t), t) \, dt + \sigma(x(t), t) \, dw(t), \]

has a solution, \( x(t) \), which is a diffusion process.

Ito's Lemma says that if the diffusion process \( f(x(t), t) \) is differentiable, it has a stochastic differential given by

\[ \text{(A3)} \quad df = \left[ \frac{\partial f}{\partial x} (x, t) \mu(x(t), t) + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} (x, t) \sigma^2(x(t), t) \right] dt + \frac{\partial f}{\partial x} (x, t) \sigma(x(t), t) \, dw(t). \]

The extra term, \( \frac{1}{2} \frac{\partial^2 f}{\partial x^2} \sigma^2 \), distinguishes the SDE from an ordinary differential equation.
REFERENCES


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