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A VARIATIONAL PROBLEM ARISING FROM A MODEL IN THERMODYNAMICS

by

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Report on "A variational problem arising from a model in thermodynamics"
by M. Marcus

We study a model for the thermodynamics of equilibrium of materials for which the free energy density depends not only on the concentration u but also on its first and second gradients. Specifically, we assume that the free energy associated with a concentration field u is of the form,

$$\psi_u(x) = u''^2 - bu'^2 + c\tilde{\psi}_0(u)$$

where $\tilde{\psi}_0(u)$ is a two-well function such as $(u^2-1)^2$. We restrict our attention to uni-dimensional bodies so that our variational problem is

$$(*) \qquad \inf\left\{J[u]:\ u \in H_2(\alpha,\beta),\ \langle u\rangle = a\right\}$$

where

$$J[u] = \int_\alpha^\beta \psi_u(x)dx$$

$$\langle u\rangle = \frac{1}{\beta-\alpha}\int_\alpha^\beta u dx$$

and α is a specified number. From the physical point of view u should be nonnegative, but for technical reasons it is more convenient not to impose this restriction. However, our results remain valid also under this condition.

The variational problem mentioned above was studied by Coleman, Marcus and Mizel in the case of bodies of infinite extent. In this case (after an appropriate modification of the definition of J[u] and $\langle u\rangle$) it is possible to obtain rather detailed information. Let us mention just two main results.

(1) If $\phi(a)$ denotes the value of the infimum described before, then ϕ is a convex function.

(2) If a is an "exposed" point of ϕ then the variational problem has a periodic minimizer. Otherwise, there exists a minimizer which can be described as a composite of two periodic functions.

In order to apply this information to the study of bodies of finite extent one is naturally interested in the following questions. Consider the sequence of minimizers u_n of the variational problem (*) for $(\alpha,\beta) = (-n,n)$. Does the sequence $\{u_n\}$, or at least a subsequence, converge in some sense to a minimizer on the whole line? What is the nature of the limiting minimizer? In particular, is this minimizer periodic in some generalized sense?

The first step in this investigation is to establish uniform estimates for the sequence $\{u_n\}$ which will imply its compactness in some appropriate sense. This problem is currently under study by Marcus and Mizel. The results (not yet complete) indicate that the sequence $\{u_n\}$ is uniformly bounded in $C^k_{loc}(R)$ for every $k \geq 1$.

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