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Water Spray Evaporative Cooling at System Pressures Below the Triple Point

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Water Spray Evaporative Cooling at System Pressures below the Triple Point

Submitted in partial fulfillment of the requirements for

the degree of

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in

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ABSTRACT

Water Spray Evaporative Cooling at System Pressures below the Triple Point

by

Eric L. Golliher

This research is an investigation of heat transfer from a flat plate using water spray cooling at system pressures that are below standard atmospheric pressure. The focus is on those system pressures below the triple point of water. At the triple point of water, the three phases are in equilibrium: liquid, vapor and solid, at 0.01°C and 612 Pa (4.6 Torr). There are many possible applications of this research, including in space and on earth. In addition to spacecraft applications, this research may apply to vacuum seawater desalination, preservation of biologically active samples, water purification, food storage, and electronics cooling. Experiments were performed to quantify the heat transfer and provide evidence to support the heat transfer mechanism and model. For sprays or any droplet deposition system that can generate discrete small droplets on a surface, this research provides a predictive model to estimate the heat transfer, and experimental data to support the model.
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# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>i</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xii</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>xiii</td>
</tr>
<tr>
<td>1.0 1.0 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.0 2.0 OUTLINE</td>
<td>9</td>
</tr>
<tr>
<td>2.1 LITERATURE SEARCH</td>
<td>10</td>
</tr>
<tr>
<td>2.2 RESEARCH TASKS</td>
<td>11</td>
</tr>
<tr>
<td>2.3 OBJECTIVES</td>
<td>14</td>
</tr>
<tr>
<td>3.0 3.0 EXPERIMENTARY STUDIES</td>
<td>15</td>
</tr>
<tr>
<td>3.1 PROCEDURE FOR DATA ACQUISITION</td>
<td>32</td>
</tr>
<tr>
<td>3.2 DATA REDUCTION PROCESS DETAILS</td>
<td>37</td>
</tr>
<tr>
<td>3.3 SPRAY NOZZLE STARTUP IN VACUUM</td>
<td>41</td>
</tr>
<tr>
<td>3.4 SYSTEM PRESSURE RISE DURING SPRAY PULSE</td>
<td>44</td>
</tr>
<tr>
<td>3.5 SPRAY ANGLE CHANGE IN VACUUM</td>
<td>46</td>
</tr>
<tr>
<td>3.6 PARASITIC HEAT LOSSES FROM THE DISC</td>
<td>52</td>
</tr>
</tbody>
</table>
4.0 DATA OBSERVATIONS ................................................................. 55

4.1 PULSE DURATION ........................................................................ 55

4.2 NOZZLE FLOW RATE ................................................................. 57

4.3 DROPLET COOLING IN FLIGHT ............................................... 59

4.4 WATER DEPOSITION AND WATER UTILIZATION EFFICIENCY ....... 64

4.5 NON-UNIFORM DROPLET SPRAY PATTERN ON THE DISC ........... 68

4.6 STATIC CONTACT ANGLE MEASUREMENT .................................. 70

4.7 OBSERVATIONS OF DROPLETS ON THE DISC ......................... 76

4.8 COOLING OF THE DISC TO DETERMINE THE TIME CONSTANT ....... 84

4.9 VIDEO DATA .............................................................................. 94

4.10 SPHERICAL CAP GEOMETRY TO DETERMINE DROP VOLUME AND VOLUME/AREA PARAMETER .............................................................. 97

4.11 PDPA DATA TO CORROBORATE EXPERIMENTAL MODEL .......... 101

4.12 MULTIPLE PULSES .................................................................... 104

5.0 MODELING AND COMPARISON WITH EXPERIMENTAL RESULTS ...... 110

5.1 BASIC EXPERIMENTAL MODEL .................................................. 114

5.2 SUBLIMATION RATE EVALUATION ........................................... 118

5.3 SPACING PARAMETER MODEL .................................................. 128

5.4 COOLING PROCESS – HEAT TRANSFER PREDICTIVE MODEL ....... 136

5.5 UNCERTAINTY ANALYSIS .......................................................... 139

5.5.1 THE UNCERTAINTY OF ACCOMODATION COEFFICIENT ......... 139

5.5.2 UNCERTAINTY IN THE SPACING PARAMETER MODEL ............. 142
5.5.3 UNCERTAINTY IN THE PREDICTIVE HEAT TRANSFER MODEL .................. 142

5.5.4 UNCERTAINTY IN DATA REDUCTION FOR THE TEMPERATURE HISTORY
.............................................................................................................................. 145

6.0 DESIGN AND APPLICATION ........................................................................ 149

6.1 APPLICATION ............................................................................................. 149

6.2 TEMPERATURE EXCURSION VARIATION .................................................... 151

6.3 HEAT FLUX RESULTS COMPARISON ....................................................... 153

7.0 FUTURE WORK ............................................................................................ 155

8.0 CONCLUSION ............................................................................................... 157

BIBLIOGRAPHY .................................................................................................. 159

APPENDICES
A ....................................................................................................................... 164
B ....................................................................................................................... 170
C ....................................................................................................................... 172
D ....................................................................................................................... 174
E ....................................................................................................................... 178
F ....................................................................................................................... 181
G ....................................................................................................................... 192
H ....................................................................................................................... 198
List of Figures

Figure 1 Space Shuttle Orbiter Diagram Showing Location of Flash Evaporator ..................3
Figure 2 Orbiter FES Cross Section Showing the Spray and Cylindrical Impact Surface which
  is a Single Phase Compact Heat Exchanger [1] .......................................................... 3
Figure 3 Schematic of Test Setup ...................................................................................... 16
Figure 4 Picture of Overall Setup .................................................................................... 16
Figure 5 Close up View of the Gold-coated Copper Disc and Spray Nozzle .................... 18
Figure 6 Typical Data Acquisition Control Screen showing Control Panel and Temperature
  History .......................................................................................................................... 18
Figure 7 Phantom V9 Camera with 200 mm Lens .......................................................... 21
Figure 8 Water Pressure System showing Two Gauges, Setra Pressure Transducer, and
  Plunger ......................................................................................................................... 21
Figure 9 Nozzle and Solenoid Injector ............................................................................. 25
Figure 10 PDPA Laser Beam Complex Refraction Pattern through the Chamber Window ... 29
Figure 11 Experimental Setup Back View ........................................................................ 29
Figure 12 Vacuum Chamber and Slide Mechanism .......................................................... 30
Figure 13 High Speed Camera is Mounted on a Tripod at a 17° Angle ............................ 30
Figure 14 View Inside Vacuum Chamber ......................................................................... 31
Figure 15 View Showing Mounting Structure of Nozzle .................................................. 31
Figure 16 Diagram Illustrating the Pulse (Spray On) Portion of the Spray Event .......... 34
Figure 17 Diagram Illustrating the Cool Down Phase of the Spray Event ..................... 35
Figure 18 First High Speed Video Camera image after Spray Stop for 8 ms Pulse Duration. 38
Figure 19 Black/White Reversed Image to Provide Contrast between Droplets and Disc...... 38
Figure 20 Image Showing Bowtie Cut of the Section that is Useable and In-Focus...........40
Figure 21 Processed Image that is used by ImageJ to Characterize the Droplets..........40
Figure 22 Spray Start Showing Increase Atomization and Unatomized Liquid time = 0 to 5 ms..................................................................................................................................................43
Figure 23 Fully Developed Spray in Air long after Spray Start.....................................43
Figure 24 Spray Start in Vacuum t=0 to 1 ms Showing Immediate Atomization...........43
Figure 25 System Pressure Increase due to Water Spray Mass Entering Chamber..........45
Figure 26 System pressure rise over a temperature range is large in the range of the present research..........................................................................................................................................................49
Figure 27 System Pressure = 0.1 Torr ........................................................................50
Figure 28 System Pressure = 1 Torr ............................................................................50
Figure 29 System Pressure = 3 Torr ............................................................................50
Figure 30 System Pressure = 5 Torr ............................................................................51
Figure 31 Spray Angle versus System Pressure ..........................................................51
Figure 32 Kevlar Threads Suspend Copper Disc to Prevent Parasitic Heat Leaks to the Disc53
Figure 33 Determining Pulse Duration to be between 8 ms and 9 ms (judged as 8 ms) ....56
Figure 34 A Linear Fit From Experimental Data Shows the Mass Flow Rate and Volume Held Up in the Nozzle (Sac Volume)........................................................................................................58
Figure 35 Droplet Cooling In-Flight: Center and Surface Temperature Predictions from Numerical Model for a 50 micron Diameter Drop .........................................................63
Figure 36 Observation of Droplet Streaks bowing Outward, Leading Some Droplets to Miss the Disc.........................................................................................................................................................65
Figure 37 Water Utilization Efficiency Increases with Pulse Duration..........................67
Figure 38 Relatively Dry Area is a Small Percentage of the Disc Total Area...............69
Figure 39 1.1 cm dry area...............................................................................................................69

Figure 40 Diagram for Relating Surface Tension Forces and Contact Angle at the Contact
Line [36] ...........................................................................................................................................71

Figure 41 Contact angle vs Temperature measurements of a 1.5 L droplet of.........................74

Figure 42 Clean As-delivered Gold Coated Copper Disc shows a Static Contact Angle of 70°
.........................................................................................................................................................75

Figure 43 Alcohol Wiped Gold Coated Copper Disc shows a Static Angle of 47 °. ............75

Figure 44 Look Angle from Camera Enables Only a Projection of Droplet Base Area........77

Figure 45 Geometry of a Spherical Cap Showing 47 ° Contact Angle .................................77

Figure 46. Relationship between Light Source and Disc as Seen from the Camera’s Point of
View....................................................................................................................................................79

Figure 47 ImageJ Outline Plot of “Bowtie” Section.................................................................81

Figure 48 ImageJ Shows Droplet Area Distribution has a Double Exponential Form..........81

Figure 49. Temperature History Data for Pulse Durations of 8 ms, 28 ms, and 49 ms.........89

Figure 50 Temperature Scatterplot Data with a Least Squares Curve Fit for the 8 ms Pulse
Duration .............................................................................................................................................89

Figure 51 Temperature Scatterplot Data with a Least Squares Curve Fit for the 28 ms Pulse
Duration .............................................................................................................................................90

Figure 52 Temperature Scatterplot Data with a Least Squares Curve Fit for the 49 ms Pulse
Duration .............................................................................................................................................90

Figure 53 Time Constant Derived from Experimental Temperature Data with 95 % Prediction
Bands based on Dispersion of Data Points ......................................................................................92

Figure 54 Time constant versus pulse duration shows additional data points which support the
uncertainty analysis and repeatability of data .................................................................................93
Figure 55 Time lapse pictures for the 8 ms case show more rapid change of deposited mass in the early time frames ................................................................. 95

Figure 56 Time lapse pictures for the 8 ms case show a slower change of deposited mass in the later time frames ................................................................. 96

Figure 57 Definition of geometry parameters for a spherical cap used in Mathematica [46] .............................................................................................................. 100

Figure 58 Contact angle definition of contact angle typically used by the thermal-fluids community ................................................................................................. 100

Figure 59 PDPA Laser Particle Sizing Results for the Spray in Air. ........................................ 103

Figure 60. Vapor Pressure over Ice as a Function of Temperature ........................................ 105

Figure 61 Disc Temperature and Chamber Pressure When Ice is Resident on the Disc (Case 1 Pump Off) ........................................................................ 106

Figure 62 Disc Temperature and Chamber Pressure When Ice is Resident on the Disc (Case 2 Pump On) ........................................................................ 108

Figure 63 Summary of Modeling .......................................................................................... 112

Figure 64 The Accommodation Coefficient Literature Survey of Marek [50] ......................... 122

Figure 65 Droplet shaper for ideal (left) and possible realistic larger surface area (right) will lead to underestimated effective surface area if a perfect spherical cap geometry is assumed when the real surface area has ridges, peaks, and valleys.............. 127

Figure 66 Spacing Parameter "n" .......................................................................................... 130

Figure 67 ImageJ Droplet Outlines 8 ms, n= 1.9.................................................................. 134

Figure 68 ImageJ Droplet Outline 28 ms, n= 1.45 ................................................................. 134

Figure 69 ImageJ Droplet Outline 49 ms, n =1.25 ................................................................. 134
Figure 70 Plot of the spacing parameter "n" versus the equivalent film thickness “δ”, where δ is derived from the nozzle flow rate, the pulse duration, the deposition efficiency, and the wetted area. ................................................................. 135

Figure 71 Uncertainty Analysis in the Predictive Model for the Time Constant .............. 143

Figure 72 Time constant versus pulse duration shows additional data points which support uncertainty analysis and repeatability of data................................................................. 144

Figure 73 Application of Spray Cooling Shows Trade-Off between Higher Heat Flux and Smaller Temperature Swing ................................................................. 153

Figure 74 Trial-averaged nozzle flowrate data plotted as: blue diamonds are short pulses: 0.008, 0.028, 0.049 seconds and brown squares are the 10 second spray pulse (only 0.01 second duration shown here for compactness)............................. 173

Figure 75 Thermal math model droplet temperature during flight for outermost location and center location shows that for a 10 micron diameter droplet, complete freezing could possibly occur. ......................................................................................... 177

Figure 76 Plot shows that most drops are within free molecular flow regime and all are very far from continuum flow criteria. ................................................................. 179

Figure 77 The example of the ImageJ output from the video data shows the listing of frame number (946) and area measurement of each droplet [m²]............................. 182

Figure 78 Complete ImageJ listing of all droplet areas [m²] for the first frame for the 8 ms pulse [m²] part 1........................................................................................................ 183

Figure 79 ImageJ provides a listing of all droplet areas [m²] for the first frame for the 8 ms pulse [m²] part 2........................................................................................................ 184

Figure 80 A Mathematica illustration of the surface profile assuming the smooth spherical cap surface is replaced with a sine wave surface............................................ 189
Figure 81 The largest puddle (frozen) in the 49 ms case appears to have some surface irregularities that may explain the under-prediction of the $w$ parameter when assuming a smooth spherical cap geometry.

Figure 82 The full view of the 49 ms pulse. Figure 81 is a close up view of the large puddle on the right side of this picture.
List of Tables

Table 1 Droplet Mass Derived from Spherical Cap Geometry (Theory) versus Mass Derived from Experimental Disc Temperature Change Data (Measured) .......................... 83
Table 2 Comparison of the Volume/Area Ratio or $w_0$ Parameter ........................................ 126
Table 3 Raw Data for Nozzle Flow Rate Determination ......................................................... 172
Nomenclature

\( A_{surface,j} \) Surface area of a drop in a summation using \( j \) as the summation variable

\( V_{drop,j} \) Volume of a drop in a summation using \( j \) as the summation variable

\( a \) Radius of spherical cap

\( A_{base} \) Base area of a droplet on a surface

\( A_{base} \) Base area of the drop, base are a spherical cap; area that contacts the disc.

\( A_{disc} \) Area of the 1 inch (.0254 m) diameter disc = 0.000506707 m\(^2\)

\( A_o \) Total disc temperature change as calculated by the curve fit software Igor

\( A_{surface} \) Surface area of the drop on the disc, not including the \( A_{base} \)

\( C_{p,d} \) Specific heat of the disc

\( C_{p,w} \) Specific heat of water

\( D_{10} \) Average Diameter of all droplets

\( g \) Slope of the line for \( w \) versus \( r_o \)

\( h \) Heat transfer coefficient, or height of a spherical cap

\( h_{fg} \) Latent heat of evaporation of water

\( M \) Mass of copper disc
$m$ Mass of water droplet

$m_p$ Mass of water during one pulse

$ms$ Milliseconds

$M_w$ Molecular weight of water

$N$ Total number of drops on the surface; a summation limit used in calculating $w$

$ns$ Nanoseconds

$PDPA$ Phase Doppler Particle Analyzer

$PLC$ Programmable Logic Controller

$P_{sink}$ Vapor pressure of the system that accepts the evaporated vapor

$P_{source}$ Vapor pressure of the evaporating surface

$P_{vapor}$ Vapor pressure

$P_{wett}$ Percentage of the disc that is wetted by water droplets

$R$ Universal Gas Constant = 8.3144 J/g mol ; Radius of a sphere; integration limit for maximum radius of the disc

$r_o$ Original radius of a spherical droplet in a free spray

$SMD$ Sauter Mean Diameter

$t$ Time
T  Temperature of the disc

$T^*$  Disc temperature scaled by the initial and final disc temperatures

$T_{\text{droplet}}$  Droplet temperature at the beginning of the cooldown

$T_f$  Final disc temperature

$T_i$  Initial disc temperature

$t_{\text{pulse}}$  Pulse duration in seconds

$V_{\text{disc droplet}}$  Volume of a droplet or spherical cap on the disc

$V_{\text{drop}}$  Volume of a deposited drop on the disc

$V_p$  Volume of water during one pulse

$V_{\text{SMD}}$  Volume of a spherical droplet in a free spray

$w$  The ratio of the total drop volumes to the total drop surface areas

$w_0$  The initial ratio of the total drop volumes to the total drop surface areas

$Y_0$  Final disc temperature as calculated by the curve fit software Igor

$\alpha$  Accommodation coefficient

$\Delta P_L$  Differential pressure across the nozzle; nozzle pressure

$\eta$  Water Utilization Deposition Efficiency

$\Delta t_p$  Pulse Duration [seconds]
Contact Angle as traditionally defined in the fluid mechanics field, [degrees]

\[ \theta_M = \frac{\pi}{180}(90 - \theta_C) \]

Spherical Cap internal angle as defined by Mathematica Spherical Cap Geometry [radians]

Density of Ice = 920 kg/m\(^3\)

Density of Water = 1000 kg/m\(^3\)

Surface Tension of Solid/Air

Surface Tension of Solid/Liquid

Surface Tension of Liquid/Air

Contact Angle as described in deGenne’s notation
1.0 INTRODUCTION

This research is an investigation of heat transfer from a flat plate using water spray cooling at system pressures that are below the triple point of water. There has been limited research in this area. At the triple point of water, the three phases are in equilibrium: liquid, vapor and solid, at 0.01°C and 612 Pa (4.6 Torr). An understanding of the physical processes of operating below the triple point is needed for thermal control systems of future space missions. There are many possible additional applications of this research, including in space and on earth.

The original interest in this research comes from a piece of Shuttle hardware called the Flash Evaporator System (FES). This hardware cools the Orbiter main coolant loop to 4 °C during certain mission phases. Figure 1 shows the location of the FES relative to the other thermal control hardware. Although the FES has been mostly successful, some past shuttle missions have had failures of this hardware [1]. It is thought that unexpected icing caused the failure. These failures have resulted in mission delays and undesirable warm crew cabin temperatures: “During the mission, Discovery's cabin Flash Evaporator System iced up, causing cabin temperatures to rise to the mid-80’s for much of the flight” [2]. As the ice sublimated slowly, some mission delays were on the order of one hour, which impacted some mission operations and could have impacted re-entry. Little understanding exists for the icing mechanism of failure or possible mitigation. The FES used a restriction in the downstream steam outlet to create “backpressure” that would cause the system pressure within the FES to nominally rise above the triple point and avoid freezing. This passive device relied upon a minimum vapor mass flowrate to guarantee choking at the restriction. It
is thought that the failures of the FES were caused by some blockage or other inability of the passive device to create the proper system pressure within the FES. The occasional and unexplainable failure of the FES has had negative consequences for the future replacement of the Shuttle: a less efficient type of evaporative cooler for the follow-on human spaceflight capsule “Orion” has been selected. This device is called a sublimator, and is relatively inefficient on a mass basis in its use of water, for operational conditions of rapid on/off or short duration cooling mission phase. If a good understanding of the icing mechanism for a spray evaporative cooling system can be achieved, it is possible that the Shuttle replacement, whatever that will be, may indeed have the option of using a more efficient spray evaporative cooling system, thereby saving vehicle weight for more payload items. This research may provide the ability to design a better Flash Evaporator System as an option for NASA’s next human space transportation system.
Figure 1 Space Shuttle Orbiter Diagram Showing Location of Flash Evaporator

Figure 2 Orbiter FES Cross Section Showing the Spray and Cylindrical Impact Surface which is a Single Phase Compact Heat Exchanger [1]
The Shuttle spray evaporative system, known as the Flash Evaporator System (FES) was developed in the late 1970’s by Hamilton Standard of Windsor Locks, Connecticut. Since very little was known about spray cooling at that time, the design process consisted of much testing and re-testing, but seems to have involved little published fundamental understanding of the processes involved. As shown in Figure 2, the FES is a cylindrical shaped custom heat exchanger with the hot single phase liquid flowing through compact heat exchanger-type passages. The hot liquid is Freon-21 that has acquired heat from the various heat dissipating items within the rest of the Orbiter. The water spray evaporates on the inner side of the heat exchanger, thereby cooling the liquid as it flows through the heat exchanger. The spray vapor then exits the FES through a steam outlet. For most of the Orbiter mission, the hot Freon-21 is routed through a fin-tube heat exchanger located on the inside of the Orbiter bay doors. The Orbiter bay doors are normally open so that this fin-tube heat exchanger can reject the heat via thermal radiation to space. However, for a short time prior to re-entry, the bay doors must be closed to prepare for re-entry. During this mission period, the FES is used to reject the heat from the Freon-21 coolant loop.

The trade between the different types of heat rejection options is complicated and requires the consideration of many factors. In Figure 1, the “Forward Deployed Radiator Panels” and “Aft Radiator Panels” are shown as located on the inside of the orbiter bay doors. During normal on-orbit operation, the single phase Freon-21 fluid in the thermal control system loop acquires heat from the various heat sources within the shuttle such as electronic boxes and the cabin air heat exchanger that cools the cabin air. This hot fluid of the thermal control system is normally routed to these radiators to be cooled, rather than to the flash evaporator system. By using a radiator, the heat acquired by the thermal control system fluid loop is rejected to
space by thermal radiation. The fluid is then recirculated to re-acquire heat from the various heat sources. Since rejecting heat to space via thermal radiation requires no loss of valuable water or any expendable, this method is of course preferred. However, many spaceflight applications and hardware configurations do not allow the use of a radiator due to volume or operational constraints. With the shuttle orbiter, the bay doors must close several minutes before re-entry, such that the radiator is unavailable for rejecting heat during a portion of the flight operation. As mentioned earlier, this operational constraint means that some other temporary way of rejecting heat, such as the FES, must be used. A different spray evaporator system is also used to cool the hydraulic system that controls the shuttle orbiter’s flight control surfaces that steer the shuttle during re-entry. Yet another spray boiling system using ammonia [3] is available to cool the thermal control loop below altitudes where water evaporation is ineffective due to the higher atmospheric pressure or post landing, prior to the arrival of ground cooling systems.

In addition to the space shuttle orbiter, an astronaut’s space suit is cooled by evaporating water. In this case, a radiator would be possible, but would require too much volume to make application to a space suit practical. Several trade studies throughout the decades have examined concepts for a space suit radiator, but none have led to the replacement of the current method. The current hardware used to evaporate water for an astronaut’s suit is called a sublimator. This device allows water to freeze within a porous plate that is exposed to vacuum. The operational disadvantage over spray cooling is that the currently available sublimator requires about 4 minutes to chill down to the freezing temperature required to provide cooling, while a pulsing spray requires only seconds to reach operational freezing temperature. Also, a spray cooling system is able to adjust rapidly to time varying heat loads,
while a sublimator is slower to react. For relatively steady heat loads, such as the approximately 150 W metabolic heat load of the human body, the sublimator is adequate. For the time-varying heat loads of the flight control surface hydraulic systems and re-entry environmental control systems, a spray cooling evaporator shows advantages.

Other relatively constant heat load applications of the sublimator include the instrument ring of the Saturn rocket and the thermal control system of the lunar module. Both of these examples were short-term cooling requirements. The Saturn rocket’s cooling requirement was no more than a few minutes, while the lunar module’s cooling requirement was less than three days. The trade between a radiator, sublimator or flash evaporator spray cooler requires consideration of the heat rejection environment, the duration of operation, the volume constraints upon radiator hardware, the availability of water, the human safety consideration (versus other working fluids such as toxic ammonia), and the immediacy of need for cooling given a transient heat load. As such, the present research expands the operational knowledge trade space of a flash evaporation spray cooling option to operation below the water triple point, such that this hardware option may now possibly compete better against a sublimator and radiator for future spaceflight thermal control systems.

In addition to spacecraft applications, this research may apply to infrared detectors, vacuum seawater desalination, preservation of biologically active samples, water purification, food storage, and electronics cooling.

In the application described in [4] is a high altitude missile, whereby an infrared detector within the missile must be maintained at a cold temperature for a few minutes, only until impact with the target. No detailed modeling or test data has been reported in conjunction
with this patent [4]. Therefore, this research may allow a greater understanding of the type of application described in this patent [4].

The concept of vacuum desalination competes with the reverse osmosis system. Some concepts generate a vacuum by using the hydrostatic pressure of a tall column of seawater [5]. The relatively warm seawater is sprayed into a vacuum, frozen, and then sublimated. The solids are left behind, and re-introduced to the ocean. The salt-free water vapor is then condensed using the energy of cooler seawater extracted at several feet below the ocean surface. Other concepts use flashing sprays to effect desalination [6].

Removing water from biological entities can ensure viability during long term storage. However, drying biological samples at high temperatures, e.g. at the temperature of boiling water, may destroy or alter the sample. Vacuum sublimation for the preservation of viruses has been an area of research in the past and is an active area of research [7]. Also, a recent technology approach using ultrasonic spray freeze drying is able to produce “significantly more viable” probiotics because the “thermal and oxidative stresses” are less than those occurring during a high temperature drying process [8]. Recent popular news stories have found that viruses do not die in ice, but can be resurrected even after 30,000 years in ice [9].

Recently, freeze crystallization by vacuum freezing and flash evaporation has been studied as a low power and potentially less expensive way to filter water and remove solids [10], [11]. When water freezes, solid impurities precipitate. If melting happens later, those impurities will be removed by the liquid water runoff, leaving the ice as pure water ice, which can be melted and used as clean water [12].
Water spray freezing of food competes with other methods of freezing food, such as forced air. For some vegetables, it is desired to limit water loss during freezing and packaging. Rather than removing water and drying the food, the goal is to quickly freeze the food to avoid water crystal growth and the subsequent destruction of the cells as the crystals grow. Spray freezing is a high heat flux and therefore faster method, as opposed to conventional atmospheric pressure convection freezing methods [13].

Water is a non-flammable and non-toxic coolant. Therefore, electronics cooling that can be accomplished with water is inherently safer to humans in the area, considering the potential for accidental release of the coolant. Water also has a high latent heat, and in this respect, has advantages over competing common refrigerants such as R-134a, for removing heat at ever increasing heat fluxes and overall heat loads. Combined with recent past interest in operating electronics at temperatures far below ambient in order to increase electronics reliability [14], these aspects of water cooling, and possibly water spray cooling at temperatures below the triple point, may lead to a robust use of water as the electronics coolant in the near future.
2.0 OUTLINE

This research builds upon past investigations of water spray cooling in a vacuum system pressure environment. The literature is sparse in this area. The literature that has been found has been published long ago. In order to move this area of research forward and re-establish and improve the understanding of the heat transfer mechanism involved, new and modern analytical and data reduction tools are used to provide new insights and pose additional questions. The major tasks of the research that have been carried out are briefly described here in this chapter. Also, since this area of research is large and complicated, the limited objectives of this particular research are listed here in this chapter as well.

Chapter 3 describes the experimental studies and the rationale for choosing the particular hardware items. Although all the hardware was built specifically for this research, nothing was exceptional or difficult to procure. Since a heat transfer mechanism and quantitative values for key parameters such a heat transfer coefficient were not known at the time of experiment design, some approximations had to be made in order to select the hardware.

Chapter 4 provides the model development, experimental results, and a discussion of the data reduction method. Open-source and easily-purchased software was used.

Chapter 5 discusses the modeling and comparison with results which formulates the analytical model. The model and method is easy to use, and should be useful for many future engineering applications.

Chapter 6 discusses the application of the results to a design process, and a summary how the results may be used to predict heat transfer rates of a surface to be cooled.

Chapter 7 and 8 provide suggestions for future work, the conclusions and final recommendations for use of the model as a predictive tool.
2.1 LITERATURE SEARCH

In 1971, Steddum [15] was the first to study water sprays in vacuum conditions. He used a large vacuum chamber in which water was sprayed at an ambient pressure of \( 2 \times 10^{-5} \) Torr. The pressurization method for the nozzle consisted of a pressurized gas cylinder forcing water through the nozzle. Therefore, there was some uncertainty as to the amount of dissolved gas. A high speed camera at 4700 frames per second was mounted at approximately 30° from the spray axis and recorded the freezing of the droplets and the velocity. A plate was placed inside the chamber at a distance two to six feet from the spray. The plate was used to determine if the droplets had frozen. If they bounced off the plate, they were assumed frozen, if they stuck to the plate and led to ice accumulation on the plate, the droplets were assumed to have been liquid. Steddum did not measure droplet size, but did reason that there existed a range of droplet sizes in a given spray, and that smaller droplets would freeze faster. In the analysis section, he assumed the droplets were on the order of 1000 micron diameter and proceeded with thermal analysis. The freezing times reported by Steddum, for lower pressure sprays, were from 110 ms to 130 ms. No attempt at resolving the heat transfer of the impacting droplets was made.

In 1979, Grissom [16] studied spray cooling in vacuum conditions. Using a “careful trial and error” method, he found the maximum heat flux for a given set of conditions that related to the design of the Shuttle Flash Evaporator System (FES). There have been no publications revealing the exact design process for the Shuttle FES. The publication by Grissom appears to be the only one that makes public at least some of the process. Grissom studied spray cooling at a system pressure of 6.76 Torr, which is slightly above water’s triple point (4.6 Torr). Above the triple point of water, water can exist as a liquid. Below the triple point, water can exist at equilibrium in only two phases: solid (ice) and water vapor. His
limited data can therefore not be used to evaluate the failure mechanisms of freezing caused by very low system pressures on the order of 0.01 Torr or lower. Also, the Phase Doppler Particle Analyzer (PDPA) had not been invented [17] and detailed characteristic of the spray such as droplet size distribution and velocity were not available.

Also in 1979, Rizza [18] studied a water spray in vacuum at the triple point, but not below the triple point. He found that splash and stick droplet dynamics did not apply near the triple point, since partial solidification (freezing) of the droplets affected the impact.

In 1985, Yanosy [19], who earlier worked for Hamilton Standard on the design team of the FES, developed a relationship to predict heat transfer for spray cooling in a vacuum. He only considered system pressures above the triple point pressure of water, which is the nominal operating condition of the FES. Thus, any freezing or failure modes of the FES were not a part of his PhD dissertation.

The past research has not truly investigated the operational mode well below the triple point to understand the mechanism of heat transfer in this range. This understanding is needed to decide whether it is possible to actually operate normally in this range. If so, then recovery from a failure mode would be possible. It is also possible that operation in this range may allow water to be used as a working fluid for cooling electronics. There are other issues to consider for this application, but basic knowledge of the spray cooling in a vacuum would be part of the answer. A conference presentation of the early results of this research is available in Golliher & Yao [20].

2.2 RESEARCH TASKS

This research investigates the mechanism of freezing and heat transfer of an impacting water spray at system pressures below the triple point of water. Experiments were
performed to quantify the heat transfer and provide evidence to support the heat transfer mechanism and model. This data provides the first-ever understanding of the mechanism, and helps outline possible future research. Using the transient temperature response of a small 1 inch diameter, 1mm thick copper disc impacted by a water spray, the method determines a heat transfer model that has general application to problems of this type. The characteristics of this transient temperature curve provide evidence of the previously unknown heat transfer mechanism.

A small chamber suitable for vacuum levels below the triple point was designed and built for this research. Inside the chamber is a spray nozzle that impinges water onto the small copper disc. The chamber was designed to have both power and data feedthroughs, visible window ports, and a water line feed-through. This nozzle is composed of an automotive fuel injector, whereby the original atomizer is replaced with a very low flowrate Parker swirl nozzle [21]. The spray pulse duration is controlled by a programmable logic controller (PLC) which can switch the injector solenoid on and off with a pulse duration as short as 8 milliseconds (ms). A laboratory vacuum pump is used to achieve system pressures as low as 0.04 Torr. The equilibrium water temperature corresponding to this system pressure is about -47 C.

First, the spray itself is investigated to determine how cone angle varies with system pressure. This is important to know, in order to choose the proper nozzle-to-disc spacing prior to closing and pumping down the chamber. Although an explanation is offered for this cone angle variation, the actual mechanism of why the cone angle varies is complicated and not a subject of this research.

Second, the Phase Doppler Particle Analyzer (PDPA) is used to determine the drop spectrum, size and number, in air. The droplet sizing of a spray is a detailed and complicated
process. As such, the many issues involved with determining a mean diameter, or characterizing the spray are not discussed in this research, because this is not the focus of this research. The PDPA data called the Sauter Mean Diameter (SMD) is used for reference to a Volume/Area parameter that is developed as part of the modeling and experimental data reduction.

Third, the heat transfer resulting from the spray impacting on the disc is investigated to determine the engineering parameters that affect the cooling. Two thermocouples mounted on the back side, one in the center and one very near the edge, provide a temperature history of the cooling at the two extreme locations on the disc. A high speed video camera records at 1000 frames per second (fps) to resolve what is happening during the spray pulse, as well as afterwards. This video data provides some important information about the droplet size, number of droplets, and their appearance. Of course, a standard video camera at 60 fps would be too slow to capture the spray dynamics for this research. Video from this camera provides pictures of the size and shape of the water droplets on the disc. A software program called ImageJ is used to analyze the droplets on the disc just after the pulse ends. Then, the physical characteristics of the water spray are related to the temperature history of the disc, and the models are developed. A predictive heat transfer model developed here might have broad application to all sprays.

Fourth, an analytical model is developed in order to have a way to predict heat transfer for spray cooling in a vacuum. A physical model and associated a reasonable analytical equation reproduces the temperature history within some reasonable error. With this model, an engineer can then estimate heat transfer for future design problems. Also, the
insight gained from the model points to possible future research and improvements in hardware that might cause more effective heat transfer.

2.3 OBJECTIVES

1. Assemble a test chamber and spray assembly in order to acquire experimental data for spray cooling at low system pressures below the triple point of water. This chamber and associated equipment have been designed with the capability to perform microgravity testing in a drop tower at some later date. However, microgravity testing is not being proposed here.

2. Use a Phase Doppler Particle Analyzer (PDPA) to provide a supportive reference for the model parameter called “w”, which is the ratio of the total droplet’s volume to the total droplet’s surface area of the droplet ensemble. The PDPA data is not used in the data reduction, or in the modeling.

3. Conduct tests of a single pulse and continuous pulses to find the behavior of various pulse conditions.

4. Develop a physical model that describes the heat transfer of an impacting water spray at system pressures below the triple point of water, and validate with the experimental results.

5. Provide a comprehensive understanding of the process and predictive capability.
3.0 EXPERIMENTAL STUDIES

The diagram in Figure 3 shows the overall setup and components. Two types of data are acquired: high speed video data and temperature history data. The system pressure is reduced in the vacuum chamber by a small lab vacuum pump. There are two different references to “pressure”: 1) the “nozzle pressure”, which is the differential pressure across the nozzle that forces the water through the nozzle, and 2) the “system pressure”, which is the measure of vacuum pressure in the vacuum tank. The water is supplied to the nozzle by a plunger type system whereby air is pressurized on one side of the plunger and monitored with a pressure gauge. Water is filled on the other side of the plunger and thus, separate from the pressurized air. The spray nozzle solenoid controller is programmable to provide the variation of pulse duration and start time. The digital data acquisition system is a high speed system capable of 6400 measurements per second for 5 channels. Figure 4 shows some additional details, such as a digital voltmeter that acts as second method to measure system pressure by measuring the voltage from the pressure transducer directly.
Figure 3 Schematic of Test Setup

Figure 4 Picture of Overall Setup
Two Omega 40-gauge type T thermocouples with insulated wires and bare junctions are attached with epoxy to the back side of a 2.54 cm diameter, 1 mm thick, pure copper disc that is Gold/Nickel coated. The disc can be seen in Figure 5. The smallest available thermocouples were chosen to minimize the lag in temperature response, and assure the transient temperature history of the copper disc was being measured. The epoxy that bonds the thermocouples to the back side of the copper disc, Tra-Bond 2151, has a thermal conductivity of 1 W/mK, is suitable for vacuum applications, and is electrically insulating. The annealed temper oxygen free copper discs (UNS-C10100) were manufactured by Midwest Metals of Westlake, Ohio following ASTM-F-68 standard. The gold plating was performed by Aetna Plating of Cleveland, Ohio. The initial coating is nickel 0.0003 to 0.0005 inches thick. The top coating is 24 Karat gold 50 microinches thick. In comparison to the 1 mm thickness of the copper disc, the very thin coatings had a negligible contribution to thermal diffusivity. The copper disc rests on two thin strings of Kevlar to eliminate parasitic thermal conduction from the support structure. Kevlar conductivity is very low, approximately 0.04 W/mK at room temperature. The weight of the copper discs was measured by a Denver Instrument Company A-160 analytical balance with readability of 0.1 mg. This balance was also used to weigh a 10 ml graduated cylinder which was used to determine the amount of water sprayed by the nozzle during the various pulses. Data and discussion from this “catch and carry” method is provided in Chapter 4.
Figure 5 Close up View of the Gold-coated Copper Disc and Spray Nozzle

Figure 6 Typical Data Acquisition Control Screen showing Control Panel and Temperature History
The thermocouples are recorded and processed with a Kinetic Systems DAQ 532 data acquisition system capable of 6400 measurements per second with 5 channels operational: two experiment temperatures, one reference temperature for the thermocouple calibration, one pressure reading in terms of voltage, and one solenoid on/off power signal. A typical computer screen of the DAQ is shown in Figure 6. The pressure channel had no electrical filter. However, the unit was returned to the manufacturer for removal of the 100 Hz filter from the experiment temperature channels at the time of the second calibration. The intention of such a filter is to eliminate excessive “white noise” that may be present from environmental sources. The thermocouple wires act as antennae to receive these unwanted signals and introduce them into the temperature data acquisition system. They are cyclic in nature and have a mean that is zero. This noise comes from radios waves, electromagnetic interference from the main 60 HZ power lines in and around the lab, and from other instruments in and around the lab such a power supplies and even possibly the A/D converter sub-assembly within the data acquisition itself. In steady state temperature measurement, such filters are highly desirable, since there is an expectation that the measured temperature does not fluctuate. However, in transient temperature measurement such as in this research, any filter might alter a real temperature transient. Although the frequency cutoff of 100 Hz is relatively high, such a filter might create lag in the temperature history data and produce erroneous transient temperature profiles. In this research however, the need was for fast and true temperature response to the spray cooling event. There was no need for an electrical filter in the data acquisition system. Software provided the true temperature transient, within some error band. The data is not smoothed, but rather reduced to a curve via a traditional least squares curve fit and traditional error analysis. Although some controls analysis of the entire system could have been performed in principle [22], in order to account for this noise,
the analysis would only have been an estimate, since modeling the details of the data acquisition system were proprietary.

The high speed camera is a Phantom V9 set at a resolution of 1632 X 1200, 1000 frames per second (fps), 990 microseconds exposure. This camera was state-of-the art when purchased in 2004. The on-board memory allowed for about 3 seconds of recording time. This recording length was an estimate of the disc cooldown duration. In fact, the disc cooldown event is much shorter than 3 seconds, such that a complete video recording is available for the entire sublimation event. Figure 7 shows a Nikon 200 mm short-working-distance lens was attached to the camera. The short working distance is a feature that allows very close focus, which was needed for this research. Normally, lenses with long focal lengths, such as 200mm, are desired for focusing on distance subjects.
Figure 7 Phantom V9 Camera with 200 mm Lens

Figure 8 Water Pressure System showing Two Gauges, Setra Pressure Transducer, and Plunger
The water is delivered to the injector by a Clippard brand “Minimatic” 6498K4 stainless steel plunger-type piston, whereby the air side of the piston is pressurized by filtered facility “shop” air, and the water side of the piston is manually filled with distilled water. By separating the water from the pressurized air, this avoids excessive gas absorption in the water during the experiment. An alternate method of pressurizing the water would have been to store the water in a pressure tank and supply pressurized nitrogen inside the tank over the water. This would have introduced additional unquantifiable gas into the water, depending on the duration of pressurization, etc.

The nozzle solenoid controller is an Automation Direct DL06 programmable logic controller (PLC) programmed to allow variable open and close times for the nozzle spray. The solenoid required 12 Volts to actuate, typically of the voltage available on a car, but also low enough to be safe in vacuum. At higher voltages, the Paschen curve predicts the possibility of electrical arcing at low system pressures [23]. If electrical arcing occurs, this would effectively be an electrical “short circuit” and could possibly destroy the hardware. The PLC allowed for pulse durations as low as a 10 ms setting. However, at 10 ms, the nozzle would not open at all. The minimum pulse setting was 20 ms nominal in order to get the solenoid to actuate. In addition, the high speed video observed that the spray duration was actually only 8 ms, when the nominal spray pulse duration was set to 20 ms. This difference may be due to the delay in the electrical chamber feed-through lines and/or the electrical inductance of the solenoid magnet. It was the case that nominal settings of 40 and 60 ms resulted in actual pulse durations, observed by the high speed camera, as 28 and 48 ms respectively. In all cases, the actual pulse duration was used in the calculations that required knowledge of pulse duration.
The camera and data acquisition system are not connected and no hard wired synchronization exists between the two. The synchronization takes place because a voltage signal sent to the valve to open is recorded along with the temperature. For example, for the 8 ms pulse duration case, the data acquisition system records that the voltage signal is “open” for a duration of 0.01078125 seconds. Since the increment of time for the data acquisition systems is 1/6400 seconds, or 0.000156250 seconds, the minimum time to be open is $0.01078125 - 2(0.000156250) = 0.01045875$ seconds. Since the camera visually recorded the actual open time to be 0.008 seconds, the error is 0.00246875 seconds. Therefore, the actual time the valve opens and the actual time the water first hits the disc; that is the synchronization accuracy, is 0.00246875 seconds. The accuracy is actually of little consequence, since the disc temperature changes less than 0.01 °C during a 0.0025 second time interval. For example, during the portion of the temperature history with greatest temperature change with time, which occurs at about the 100 ms mark for the 8 ms pulse duration case, the temperature of thermocouple 2, which is the outer thermocouple, changes from 26.2588 °C to 26.2531 °C. This 0.0057° change is very much smaller than the error assumed in the uncertainty analysis. Therefore, hard wiring the camera and data acquisition system would achieve only slightly better accuracy, but that increased accuracy would be of no use in the research, since other error sources are much greater.

As Figure 8 shows, the nozzle pressure is controlled by a manual valve and measured with two Bourdan pressure gauges, an Ashcroft brand with range 0-30 psig and a Wikah brand with range 0-160 psig, and a SETRA model 240 pressure gauge with 0-5 volt output over a range of 0-25 psia. This redundant capability was sequentially added over time in an
effort to clarify data results. For the SETRA output voltage was measured by an Agilent 3440A 6½ digital multimeter and converted to psia manually via pressure/voltage tables.

The chamber volume is 0.08367 m³ with the following feedthroughs: copper/constantan for thermocouples, power, pressure measurement voltage, vacuum pump duct, and water entrance. The chamber has two quartz glass windows at 150° angle to accommodate refractive measurements with the PDPA.

The Dantec Flowlite PDPA can measure velocity in 2 directions and droplet size. The associated Dantec BSA software provides “turnkey” diagnostics and resolves mean diameters such as Sauter Mean Diameter (SMD).

The nozzle is a low flow swirl type nozzle “button” supplied by Parker Hannefin Co. (Mentor, Ohio) and normally used in a different application for various Parker Hannefin customers. This nozzle is attached to a modified Delco FJ10066-11B1 solenoid fuel injector with blue Tra-Bond 2151 epoxy as shown in Figure 9.
Figure 9 Nozzle and Solenoid Injector
A 600 W Smith-Victor model 700 tungsten halogen lamp is located outside the chamber but is directed to illuminate the spray at a 150° angle from the viewing window, as can be seen in Figure 10. The high speed camera requires unusually high levels of light, depending on exposure duration and resolution.

The system pressure sensor is an MKS 629B1TQCJ2B capacitance manometer calibrated from 0 to 100 Torr over the output voltage range of 0-10 V. The system pressure transducer voltage was converted to pressure [Torr] with an MKS PR4000 electronics unit. Also, the system pressure transducer raw voltage was displayed by a Keithly 2001 7½ digit Multimeter. A capacitance manometer was chosen over other options, because it is ideal for this pressure range of near 0.01 Torr, and offers the highest degree of accuracy for this pressure range [24].

A Pirani gauge was ruled out, because it is sensitive to the constituent gas. In our case, we would be mostly water vapor, but calibrations for water vapor are not common. Also, our case would possibly have a mixture of gas such as nitrogen and water vapor. Although we can calculate from the data the expected percentage of water vapor and nitrogen from the data, there is no assurance that such a mix is present in the Pirani gauge when pressure measurements are recorded.

A capacitance manometer was decided upon, because of several reasons. The corrosion resistance was one. Inside the unit, a diaphragm made of Inconel deflects in response to pressure changes. This material, Inconel, is well known to be corrosion resistant, especially to water. It is used in high pressure steam generator tubes of power generators, for example. The pressure range was correct for our requirements. The high accuracy was needed. The response time was adequate. The response time depends on the conductance of the device to
remove gas from, or have gas enter, the small central area where the diaphragm is located. This determines the ability of the device to react to rapid changes in pressure. The design of a generic manometer of the type used in our research is outlined [25]. In this paper, the response time is estimated to be not larger than 10 ms. The general response time quoted in the spec sheet of the particular manometer used for this research is less than 20 ms.

The vacuum pump is a 120 V Varian 600 Tri-Scroll dry pump that is capable of handling small amounts of water vapor such as anticipated in this experiment. The vacuum pump is not large enough to remove the water at the same rate that it is introduced to the chamber. A larger pump may have accomplished this, but would not have been portable. Portability was needed for ease in moving and re-arranging the hardware as needed. Therefore, there is some pressure rise during the spray pulse, but the pressure rise is small. Effectively, for the short pulses studied in this research, the effective sink temperature, defined as that equilibrium temperature corresponding to the system pressure, had only negligible change.

For the PDPA measurement, the spray and chamber move, while the PDPA is stationary. The windows and cover of the tank were removed to make the measurement. There was not a PDPS measurement in vacuum. The in air measurement was made in the vacuum chamber so that all the electrical connections could be as they would be in operation in the vacuum chamber. The very small (1mm) step movement is accomplished by a VELMEX PK296-03A-A6 stepper motor and controller that operates in conjunction with the PDPA. It is worth noting that, it was discovered that the VELMEX controllers are not electrically shielded, and emit an odd electro-magnetic interference (EMI) pattern that adds a considerable amount of noise with the thermocouple temperature acquisition, even when the motors are not moving. Therefore, the controllers were turned off when temperature measurements occurred.
Without the chamber widow installed, the VELMEX stepper motor moved the spray in 1 mm increments to resolve spray characteristics for approximately 40 locations. The measurements took place without the copper disc present, but at a nozzle-to-plate distance that corresponded to where the disc would have been. The nozzle to plate spacing was 1.8 cm.

Making PDPA measurements through glass windows and in a vacuum is a very difficult exercise from an optics point of view and therefore was not performed for this research. The laser beams would have to shine through thick vacuum-rated silica windows. If the two windows had been present, one for the laser beams entering the vacuum tank and one for the laser beams exiting the vacuum tank, the alignment would have to have changed for every movement of the tank. In fact, for each time the spray and tank were moved, a new alignment would have been necessary. It happens that each re-alignment has a slightly new error associated with the measurement. Some consultation with an optical diagnostic expert at NASA revealed that this type of measurement would be possible, but would need some additional PDPA expertise, a knowledge of optics with concentration on polarization and window refraction, and careful re-alignment at every single movement of the spray [26]. This measurement of a spray-in-vacuum was not needed for any modeling or data reduction, however, since the ImageJ and high speed camera measurement was able to provide number and size data of droplets on the disc. The PDPA data in air, with the windows removed, was useful as a sanity check for the model and other experimental data reduction. Figure 10 shows the vertical velocity (green) and horizontal velocity (red) lasers shining through the chamber windows. Three to five apparent separate and perhaps problematic reflections or refractions can be seen in the windows for each beam.
Figure 10 PDPA Laser Beam Complex Refraction Pattern through the Chamber Window

Figure 11 Experimental Setup  Back View
Figure 12 Vacuum Chamber and Slide Mechanism

Figure 13 High Speed Camera is Mounted on a Tripod at a 17° Angle
Figure 14 View Inside Vacuum Chamber

Figure 15 View Showing Mounting Structure of Nozzle
For low system pressures, the spray cone angle increases when compared to atmospheric pressure. Therefore, the nozzle to plate distance had to be adjusted by trial and error accordingly to ensure that the spray impact circle matches the disc area in vacuum. The investigation of how spray cone angle changes with system pressure was not a research goal, but was necessary and useful for pre-positioning the nozzle prior to vacuum tank sealing and pumping down to vacuum. This saved time in the trial and error sequence of matching the spray impacting area to the surface area of the copper disc.

Measuring contact angle between the water and gold plated copper disc was accomplished with an Attension Theta Lite TL100 Optical Tensiometer. The measurement was made at standard atmospheric pressure and room temperature. Water was taken from the spray itself, after the water had passed through the water supply system and nozzle.

3.1 PROCEDURE FOR DATA ACQUISITION

1. Pumpdown from atmospheric pressure to 0.04 Torr requires about 5 minutes after the pump is turned on. The lowest pressure capable is 0.04 Torr, because of the small unavoidable leaks in the system and the size of the vacuum pump.

2. The nozzle pressure is set to the desired range by first overfilling the pressure cylinder and then manually venting the pressure cylinder down to the desired pressure point.

3. The data acquisition system, laptop and solenoid control system (PLC) are powered on.
4. The high speed camera is turned on and set to start recording once the Enter key is pressed on the attached laptop.

5. In order to record one pulse, the solenoid, data acquisition system and camera are all started. Figure 16 illustrates the spraying portion of the spray event.

6. The data acquisition system has to be manually stopped after it is observed the disc temperature stops changing, but the camera and solenoid control time out according to the limited memory in the camera and the preset pulse duration on the solenoid control, respectively.

7. Once the pulse event is finished, the video data takes several minutes to download from the camera’s onboard memory to an external hard drive or laptop hard drive.
9. **Figure 17** illustrates the completion of the pulse and the beginning of the cool down.

10. Under vacuum conditions, it takes some time for the nozzle to return to room temperature, because cooling of the nozzle occurs during a spray. For the 8 ms spray pulse duration, the recovery time was on the order of minutes, but for the 49 ms pulse durations, the recovery time appeared to be hours. A decision was made to take the data presented in this research as the first spray in the early morning, after all the hardware in the experimental system had the time of several hours to reach a uniform temperature.

![Diagram Illustrating the Pulse (Spray On) Portion of the Spray Event](image-url)
Figure 17 Diagram Illustrating the Cool Down Phase of the Spray Event
A 12 V DC signal is sent by the Programmable Logic Controller (PLC) to the solenoid to open the solenoid and start the spray. The actual measured voltage is somewhat less, 10.4 V, and is time stamped along with all the other data, 6.4 measurements per millisecond. The 10.4 V signal is sustained until the Programmable Logic Controller (PLC) stops the signal, based on the desired pulse duration. Since there is no synchronization between the high speed video and the data acquisition system, the assumption is made that the end of the pulse as observed by the high speed camera corresponds to the end of the pulse as shown by the voltage dropping off the 10.4 V value. For example, a nominal pulse duration of 20 ms is shown to have a voltage of 10.4 volts for a duration of 69 time increments, which is 10.8 ms with a resolution of 1/6400, or 0.156 ms. The high speed video, however, shows the duration of the spray to be about 8 ms, with a resolution of 1 ms (at 1000 frames per second), which corresponds to the data acquisition resolution of 6.4 time increments. The possible error of 1 ms at both the opening and closing of the solenoid for the voltage observed pulse duration can be subtracted; the voltage observed pulse duration is actually is 10.8-1.0-1.0, or 8.8 ms, which agrees with the high speed camera observed pulse duration of 8 ms. Therefore, the assumption that the cool-down event starts at the end of the pulse as observed by the voltage signal, is a reasonable assumption.

In an attempt to improve accuracy of this measurement, a small solar cell was placed inside the chamber, and a green laser pointer was shone through the window from outside the chamber, that crossed the path of the spray. The solar cell voltage was recorded. When the spray started, the spray would block the laser light and the solar cell voltage would drop. However, this method did not work, because the lag in the voltage response of the solar cell was too large compared to the timescale needed to provide verification. However, this is
not a problem, because, as will be shown later in the model development section, the error with this assumption of the start of the spray being the point at which the voltage departs from 10.4 volts is very small compared to other uncertainties in the model/experiment agreement.

3.2 DATA REDUCTION PROCESS DETAILS

The procedure to analyze the disc temperature data is as follows. The data acquisition system generates a text file of two disc temperatures and one thermocouple reference junction temperature, one voltage measurement for the solenoid valve which controls the spray, and one voltage measurement for the system pressure transducer. These are imported into an Excel worksheet with a number of macros that further process the data. Appendix A has details of the code written to process the data.

1. The data acquisition system acquires time and temperature data as a text file which is imported to an Excel worksheet.

2. The Excel worksheet then processes the data using macros (see Appendix A):

   a. Sets the point at which the spray pulse stops as time = zero by locating the voltage drop.

   b. Calculates and plots the area-weighted disc average thermocouple temperature history.

3. A data analysis software package called Igor performs a least squares curve fit to the scatterplot temperature data. The form is an exponential decay.
Figure 18 First High Speed Video Camera image after Spray Stop for 8 ms Pulse Duration

Figure 19 Black/White Reversed Image to Provide Contrast between Droplets and Disc
The procedure to analyze the video data is as follows:

1. The first image of the spray just after the spray pulse ends is captured from the high speed video, as shown in Figure 18.

2. This image is reversed black/white to provide contrast and make the droplets appear black as shown in Figure 19. Note that only the mid-section is in focus and useable.

3. This image is imported into ImageJ.

4. Since the useable portion of the image is only that portion which is in focus, a bowtie-shaped cut is made and analyzed using ImageJ’s particle analyzer option as shown in Figure 20. The depth of field is common photography term for the area that is in focus. By shrinking the aperture, a large depth of field is possible, but then, the amount of light getting to the CCD in the high speed camera is limited. So the desirable depth of field is traded against a desirable brighter and clearer image. The area that is in focus provides adequate statistical sample that represents the droplets on the entire disc.

5. ImageJ provides an estimate of the number and area each droplet occupies from the information provided in Figure 21.
Figure 20 Image Showing Bowtie Cut of the Section that is Useable and In-Focus

Figure 21 Processed Image that is used by ImageJ to Characterize the Droplets.
For each area, the following calculations are made:

1. ImageJ uses a “Moments” thresholding algorithm to separate the dark droplets from the light background (the copper disc) and draw outlines of each droplet.

2. The surface area of a spherical cap sitting on top of the area in (1) is calculated.

3. The volume of a spherical cap sitting on top of the area in (1) is calculated

Then, the ratio of the sum of each droplet’s volume and the sum of each droplet’s surface area is calculated. This is labeled the parameter “w”. The “Moments” thresholding algorithm, which is a part of ImageJ, was chosen because the volume produced by this method very closely agrees with the volume that is necessary to cause the disc temperature change observed from the beginning of time zero to the final time. More of these details are explained in the next chapter under Data Observations. Details of the calculations are in Appendix B. The original journal articles describing the thresholding techniques are cited by ImageJ as reference material for the ImageJ user [27], [28].

3.3 SPRAY NOZZLE STARTUP IN VACUUM

The high speed video shows time of flight for the droplets to travel from the nozzle to the disc is between 1 and 2 ms. Further, the high speed video in vacuum shows that the spray appears to be fully developed at the very first frame recorded that can be seen after spray start. This is in sharp contrast to spray behavior in air. For spray startup in air, there is some residual water held up in the nozzle that takes time to accelerate for good atomization. This is pronounced for all types of spray nozzles operating at nozzle pressures that are below their optimum design point, such as in this research, 25 psid. The nozzle manufacturer has specified 100 to 1000 psid nozzle pressure difference for normal operation [29]. As such,
the nozzle used in the present research underperforms in atomization, and is not representative of the capability of the manufacturer. The below-design-point operation is desirable for the present research, in order to minimize the mass flow rate. This smaller mass flow rate results in smaller disc temperature changes. It also results in smaller droplet velocities that lead to less bounce and greater water deposition. Smaller velocities also lead to clearer video. A small mass flow rate also creates a smaller pressure rise within the vacuum tank as the pulse occurs. All these effects simplify the research data acquisition and data reduction, thereby allowing a better focus on the qualitative and quantitative investigation of the cooling mechanism.

This startup effect is not important for most sprays because most sprays are steady sprays. Startup effects are a small portion of overall focus of this research, but worth mentioning at this point. Panao [30] is one of the few researchers that have recently studied the “leading front” and “tail” of a spray by using an advanced Phase Doppler Particle Analyzer (PDPA). He was able to quantify this effect and show its importance for the emerging field of high frequency pulsed spray cooling at atmospheric system pressures.

Figure 22 shows the time lapse process of spray start in air, taken with the high speed camera at 1000 frames per second (fps) and exposure of 996 ns. The liquid that was never atomized is subsequently pushed out of the nozzle. In the last picture, a typical bell shape begins to form at the nozzle tip, showing the beginning of successful atomization. Figure 23 is taken with an ordinary 35 mm camera and shows the fully developed spray in air for comparison. All pictures use the same nozzle and pressure delivery system described in Chapter 3.
Figure 22 Spray Start Showing Increase Atomization and Unatomized Liquid time = 0 to 5 ms

Figure 23 Fully Developed Spray in Air long after Spray Start

Figure 24 Spray Start in Vacuum $t=0$ to 1 ms Showing Immediate Atomization
In contrast to startup in air, startup in vacuum shows no signs of any unatomized liquid as shown in Figure 24. The spray appears to start immediately and atomization begins in less than 1 ms, the frame rate of the camera. Although not shown, the end of the spray is exactly the same, with an apparent sharp cut-off. A possible explanation is that, without air in the nozzle chamber, the liquid entering the nozzle chamber from the four tangential ports can reach the working speed very fast. Since there is no shear from stagnant air to provide a retarding startup resistance, the ejected liquid is not retarded by the air during the flight.

3.4 SYSTEM PRESSURE RISE DURING SPRAY PULSE

The addition of water to the chamber causes the system pressure to rise. Ideally, the system pressure should remain constant. However, since this pressure change is a known quantity, it can be easily factored into the model for data reduction. For example, during an 8 ms spray pulse, the system pressure is observed to increase from 0.04 Torr to 0.1 Torr due to the introduction of water into the chamber for the situation where the pump is off and the system is “closed”. This is predicted by the ideal gas law relation that considers the water vapor as a gas at room temperature. The equilibrium saturation temperature corresponding to this system pressure increase is -47 C to -40 C during an 8 ms pulse. Figure 25 shows an *Igor* curve fit to scatter plot data produced by the data acquisition system’s interpretation of the analog signal coming from the pressure transducer. The limits of the A/D converter within the data acquisition system can be seen as discrete “lines” in the scatter plot.
Figure 25 System Pressure Increase due to Water Spray Mass Entering Chamber
It should be noted that Figure 25 has some error, because the response time of the pressure transducer is stated by the manufacturer as 20 ms. The diaphragm within the pressure transducer requires a finite time to physically move. This would produce a lag between measured pressure rise and real pressure rise at the spray nozzle. Since the spray pulse requires only 49 ms, but the pressure transducer takes about 1 second to finally reach steady state. The real value of the pressure transducer is therefore not its transient tracking ability; but rather, its ability to confirm that, after time, the ideal gas law relation predicts the real pressure rise and that the cause of the system pressure rise is well understood. It will be shown later in Chapter 5 in the modeling, this change in system pressure is very small and therefore not important in developing the model. The equilibrium thermodynamic equilibrium temperature corresponding to the system pressure is very far below the disc temperature. Therefore, the pressure rise during the pulse is negligible.

3.5 SPRAY ANGLE CHANGE IN VACUUM

From [31] it has been observed that at pressures higher than standard atmospheric, such as in Diesel combustion, the spray cone angle is smaller. The reason is that the high velocity within the spray cone leads to lower pressure there, as compared to the stagnant ambient pressure air. The ambient air is pulled into the cone, entraining the droplets, producing drag on the droplets as the ambient air moves toward the center of the spray, and leading to a smaller effective cone angle. For pressures in the range of 10 to 800 kPa absolute, DeCorso and Kemeny experimentally report the relationship of cone angle to system pressure:

\[
Spray \ Cone \ Angle \ \propto \frac{1}{P_{system}^{1.6}}
\]  

(1)
Since this was measured at systems pressures no lower than 10 kPa, the question is to discover whether this relationship in Equation (1) is valid for system pressures far below 10 kPa.

To answer the question of spray angle versus system pressure, the spray angles for various system pressure points below the triple point were measured graphically, as shown in Figure 27 through Figure 30. Over the range of system pressure increase discussed in the last section, the equilibrium temperature corresponding to the system pressure appears to have a large change as shown in Figure 26.

The spray cone angle was observed to increase in a linear relationship as the system pressure decreased. The plot in Figure 31 shows a linear relationship between system pressure and spray cone angle.

The following model might explain this behavior: the evaporated water exiting the nozzle at a nozzle pressure difference of 10 psid creates a positive pressure gradient from the inside of the spray cone near the core to the outside of the core, where low pressure vacuum exists. Evaporated water blows outward, and entrains and pushes the spray away from the spray core. In addition, the vacuum system pressure outside the spray cone offers relatively little air to entrain the downward-moving droplets and provide drag to lessen the cone angle, as compared to the case of high system pressure operation described by Corso and Kennedy. As the pressure decreases, the Knudsen number (the mean free path compared to the length scale) ranges from less than 1.0 to 26, and indicates non-continuum flow for the lowest pressure of 0.04 Torr. In non-continuum flow, kinetic theory describes the relationship
between evaporation flux and pressure. This rarified air at low system pressures near 0.04 Torr therefore offers little drag to the downward-moving droplets to decrease the cone angle. As a result, the cone angle is wider for low system pressures.

Thus, the simple model of water vapor in the rarified and transition regime blowing away from the spray core using kinetic theory may predict the trend in data. Equation (2) describes the experimentally determined cone angle relationship between cone angle and system pressure valid for system pressures below 5 Torr.

\[
\text{Spray Cone Angle} = 74^\circ - 3.5(P_{\text{system}}); \quad \text{for } P_{\text{system}} < 5 \text{ Torr} \tag{2}
\]
Figure 26 System pressure rise over a temperature range is large in the range of the present research.
Figure 27 System Pressure = 0.1 Torr

Figure 28 System Pressure = 1 Torr

Figure 29 System Pressure = 3 Torr
Figure 30  System Pressure = 5 Torr

Figure 31  Spray Angle versus System Pressure
3.6 PARASITIC HEAT LOSSES FROM THE DISC

A model for temperature change of the copper disc must consider all the possible heat transfer paths: convection from the surrounding air, conduction through the support surfaces to the copper disc, radiation from and to the walls of the vacuum chamber, infrared heat from the light used to illuminate the inside of the chamber, and water mass transfer/phase change away from the disc. All sources are negligible compared to the mass transfer of water sublimation.

The heat transfer via convection can be shown to be negligible if an estimate for the system pressure dependence is made using well-known natural convection correlations.

The conduction heat transfer from the disc to the Kevlar threads that support the disc is negligible for the reason that two crossed Kevlar threads suspend the disc on top of a plastic cylinder as shown in Figure 32. The diameter of the thread is very small, about 0.0035 inches (0.09 mm), and the thermal conductivity of Kevlar itself is approximately 0.04 W/mK.
Figure 32 Kevlar Threads Suspend Copper Disc to Prevent Parasitic Heat Leaks to the Disc
The radiation heat transfer from the disc to the walls is small because the temperature of the disc is close to the wall temperature and the emittance of gold is small. For all cases the chamber walls temperature are at the ambient air temperature of the room, about 22 °C. The lowest disc temperature is approximately -40 °C. If the gold coating emittance is 0.1, then the total heat transfer from the walls to the disc is approximately 0.01 Watts.

The light that illuminates the inside of the chamber for high speed video is very bright and does transfer heat to the copper disc. However, a test with just the light on for several minutes showed the change in temperature of the disc is on the order of 10 °C per minute. This is a negligible change in temperature compared to that caused by the water sublimation, which is on the order of 10 °C in 2 seconds.

Therefore, the impingement and evaporation or sublimation of the water is the only mode of heat transfer needed for the model.

SUMMARY

The experimental hardware developed for this research was a unique and new. The vacuum chamber and associated pressure and temperature diagnostic instrumentation were combined with diagnostic hardware such as the PDPA and tensiometer. As is typical in new experimental setups, debugging and learning caused the system to evolve. The next chapter discusses the experimental procedure and results.
4.0 DATA OBSERVATIONS

This chapter includes a discussion of the data observations made from the thermocouple temperature history and video data. Chapter 5 presents the physical model for the prediction of the cooling process, based upon the observations discussed in this chapter. Finally, Chapter 6 discusses the use of the final heat transfer predictive model and extends the modeling to other applications.

4.1 PULSE DURATION

Examination of the video frames determines the actual pulse duration. For example, the spray pulse duration was somewhere between 8 and 9 ms in actuality, even though the PLC was programmed for a 20 ms pulse duration. This lag is believed to be due to the electrical induction of the windings of the solenoid and possibly the inductive electrical properties of the electrical path between the PLC and solenoid. Similar lags exist for other pulse durations: a setpoint of 40 ms resulted in a spray pulse duration of 28 ms, and a setpoint of 60 ms resulted in a spray pulse duration of 49 ms. Figure 33 shows the method for determining the real pulse duration from high speed video, taking the 8 ms pulse as a typical example. Some judgement was used since the resolution of the camera framerate was 1 ms. In all cases, the real opening and closing occurred in between frames. Also, the spray start interval, from fully “off” to fully “on”, appears to be less than 1 ms.
Figure 33 Determining Pulse Duration to be between 8 ms and 9 ms (judged as 8 ms)
4.2 NOZZLE FLOW RATE

The amount of water that has left the nozzle is also known from measurements in air for the nozzle at the same nozzle pressure difference as in the vacuum. The nozzle pressure difference is the difference in pressure before the nozzle to the pressure after the nozzle. If the nozzle exhausts to air, the exhaust system pressure is 15 psia (pounds per square inch absolute). If the nozzle exhausts to vacuum, the exhaust system pressure is effectively 0 psia. This nozzle pressure difference was always 25 psid (pounds per square inch differential). So for measurements in air, the gauge pressure was 25 psig (25 psia over the exhaust pressure of 15 psia). For measurements in vacuum, the gauge reading was reduced to 10 psig, because the vacuum provided the extra 15 psia to bring the nozzle pressure difference to 25 psid. The assumption that the nozzle driving pressure results in the same flowrate in air and in vacuum is valid, since the flow resistance across the solenoid, nozzle and swirl chamber is much larger than any flow resistance due to sheer against the air at the nozzle exit. The real vacuum pressure was 0.04 Torr (~8E-04 psia), which can be assumed essentially zero for flowrate/nozzle pressure measurements. For the spray mass during one pulse, a simple manual measurement method determined the amount of spray mass per second as a function of nozzle pressure. Then, knowing the opening and closing times of the nozzle in vacuum via the high speed video, the amount of mass during a pulse is known. A 10 ml graduated cylinder was used for several repeat trials. The graduated cylinder was located beneath the spray to catch the water from a single pulse. The graduated cylinder was then carried to a nearby balance to determine the increased weight due to the water mass. This was done for three pulse durations for many trials and an average determined. Details of the trial measurements are in Appendix C.
Figure 34 A Linear Fit From Experimental Data Shows the Mass Flow Rate and Volume Held Up in the Nozzle (Sac Volume)
Figure 34 shows the curve fit to the “catch and carry” flow rate data for the nozzle measured in air. This linear fit suggests that some of the mass, 3.0 E-06 kg, is remaining in the nozzle for each pulse. This volume of this liquid is about 3 mm³ which corresponds to the approximate volume of the swirl chamber and the space anterior to the nozzle “button” filter. In the fuel injector engineering field, this volume is known as the “Sac” volume. During the vacuum operation, prior to spray start, the water in this volume has apparently sublimated and left the Sac volume dry. So the absence of this volume at spray start in vacuum does not affect the results. At spray stop however, this volume is ejected at the end of the spray, but is not ejected as unatomized liquid; it is ejected as a fairly well atomized liquid. During one atmosphere measurements, the opposite occurs: the liquid in the Sac volume is ejected at the beginning of the spray, but held up and not ejected at the end of the spray due to slower evaporation in air. The spray mass flow rate of the nozzle is equation (3).

\[
\text{Mass [kg]} = 0.00025 \times (\text{Pulse Duration}) + 2.969 \times 10^{-6} \tag{3}
\]

The continual mass flux of the spray at the disc surface (neglecting the amount held up in the Sac volume) is 0.5 kg/m² s. This value is within the range that has frequently been used in past spray cooling research. For example, Choi and Yao used nozzles which produced a range of mass flux of 0.11 to 1.8 kg/m² s [32]. Also, Hsieh and Yao used nozzles which produced a range of mass flux from 0.25 to .74 kg/m² s [33].

4.3 DROPLET COOLING IN FLIGHT

As the droplets fly from the nozzle to the disc located 1.8 cm from the nozzle, evaporation of the droplet occurs. The droplet loses mass due to evaporation of the 22 °C droplet to the vacuum environment of and equivalent -49 °C. Streak lines from the video camera view of the spray show the droplets traveling the approximately 1.8 cm distance from
nozzle to surface in less than two frames, or 0.002 seconds. A good estimate of the mean velocity is therefore 10 m/s. This estimate relies on the visual observation of those droplets that are large enough to refract light to the camera. The heat flux per the Arden Buck and Hertz-Knudsen equations for this driving temperature difference is $3.1 \times 10^5 \text{ W/m}^2$. A detailed thermal math model analysis considered the pure conduction of both ice and water. As the surface of the droplet evaporates, heat conduction from inside the droplet provides sensible heat to the surface. The droplet surface temperature is determined by balancing the latent heat taken away by evaporation with the sensible heat being supplied to the surface via conduction from the inside of the droplet. A reasonable assumption is no circulation in the droplet such that a temperature gradient exists from the center to edge. It is further assumed there is no circumferential temperature difference. Since only a small amount of water is evaporated during flight, the assumption is that the process can be modeled by a constant diameter solid sphere. The Hertz-Knudsen equation for heat flux in the free molecular regime and the Arden Buck vapor pressure over ice relation are used to apply a surface heat flux on the solid sphere. In order to make the math model simpler, this heat flux was assumed constant and not temperature dependent. This would produce the worst case temperature decrease during the ~1 ms flight from the nozzle to the disc. This heat flux is calculated in Mathematica and then imported into SINDA for finite difference analysis of conduction using 100 nodes as negative “heat load” on a surface. Although the droplet size distribution in vacuum is not known exactly, the SMD data taken in air may be able to be used as a reference. The SMD in-air data of 50 to 90 micron diameter indicates that perhaps analyzing a 50 micron diameter droplet would be representative of the spray. The temperature change during flight in smaller droplets will be greater than shown here, and for larger droplets, the temperature change during flight would be smaller than shown here. The
SINDA droplet thermal math model parameters were \( k = 0.6 \text{ W/m K} \), \( c_p = 4186 \text{ J/kg K} \), \( \rho = 1000 \text{ kg/m}^3 \), finite difference timestep = 1 E -08 seconds, finite difference output = 0.0001 seconds. The heat flux determined by the Arden Buck and Hertz-Knudsen analysis was -310,000 W/m\(^2\). The listing of the Mathematica code is shown in Figure 73 in Appendix D.

As shown in Figure 35, the 50 micron droplet does not reach freezing temperature during flight. From this analysis, we may conclude that it is probable the majority of the droplets probably do not freeze during flight. This is consistent with observations of the video, because none of the droplets are seen bouncing from the surface. If any large droplets had frozen, bouncing would have been seen as streak paths in the video frames. However, since the water utilization efficiency is not 1.0, there was probably some bouncing of very small droplets that were too small to be detected by the camera. For the ImageJ analysis, the smallest reported area is 2.52 E -10 m\(^2\). Multiplied by 3.156 to derive the true base area of the spherical cap droplet on the disc, the smallest reported droplet on the disc has a base area of 7.95 E -10 m\(^2\) and a radius of 1.6 E -05 m. Considering spherical cap geometry and the conservation of mass when an spherical droplet impinges on the disc and transforms into a spherical cap, the smallest spray droplet able to be seen by ImageJ, and probably therefore the camera, is 18 microns in diameter. It is possible that some of the droplets are too small to be detected by the camera but they could have frozen and therefore bounced away without leaving evidence of this from the video data. Figure 75 in the Appendix D shows the outermost and center droplet temperatures from an additional SINDA analysis for a 10 micron diameter droplet. It shows a 10 micron diameter droplet does indeed freeze in less than the one millisecond time-of-flight from the nozzle to the disc.
Depending on the size of the droplets, the evaluation shows that most of the larger droplets could be partially frozen. Video data (observing the first frames of the high speed camera) shows that most droplets landing on the disc appear to be partially frozen; appearing translucent rather than clear. Also, all of the observable droplets appear to stick to the disc, which means the large observable droplets are not fully frozen. Although no droplet bouncing was observed, some very small droplets may have been frozen and bounced and not been visible with the limitations of the camera. This could explain some of the reason for a lower water utilization efficiency discussed in a later section.

Figure 35 shows the numerical model temperature history for a single droplet cooled by water evaporation while in flight from the nozzle exit to the disc surface for a 50 micron droplet. For this 50 micron diameter droplet, the temperature does not decrease to the freezing temperature.
Figure 35 Droplet Cooling In-Flight: Center and Surface Temperature Predictions from Numerical Model for a 50 micron Diameter Drop
The temperature of the droplets upon landing on the disc is, however, not critically important. The total droplet mass, even for the 49 ms pulse duration, is estimated at 1.53 E-06 kg, while the copper disc mass is 0.0045 kg. This ratio of copper disc mass to water mass is about 300. It appears that the sensible heat of the water is far smaller than that of the copper disc.

4.4 WATER DEPOSITION AND WATER UTILIZATION EFFICIENCY

For a given pulse, some of the droplets in the spray may bounce off the disc rather than sticking. In addition, evaporation of the droplets appears to provide an outward blowing effect, causing some of the droplets to have a curved trajectory and miss the disc. The curved trajectory is caused by the evaporating vapor leaving the disc as well as the evaporating vapor leaving the droplets that are in-flight from the nozzle to the disc. Most of the evaporated vapor flow is from that water mass that is sublimating from the disc. A much lesser amount is due to the evaporated flow from the flying droplets. Thus, the angle of curvature for the droplet path is more severe near the disc. Figure 36 is a high speed video frame with 1 ms exposure to show the pathlines of the droplets. A larger curvature can be seen as for the portion of the pathline near the disc. Overall, a small portion of the droplets miss the disc.
Figure 36. Observation of Droplet Streaks bowing Outward, Leading Some Droplets to Miss the Disc.
These effects produce a cooling ability of the spray pulse that is somewhat less than would be assumed if the entire water mass from the pulse were available for phase change and disc cooling. This ratio of actual to ideal cooling ability of a spray is referred to as utilization efficiency \([34], \eta\).

For this research, the initial disc temperature is known. Also, the final disc temperature after all the water has evaporated is also known. Since there is no other source of cooling in the short duration of the cooling process, then this entire temperature change will be mainly due to the amount of water that had sublimated on the disc. Thus, the actual amount of water that landed on the disc can be deduced.

The amount of mass during a spray pulse is known, and the amount of mass that actually reached the disc can be deduced from the total disc temperature change. The water utilization efficiency can be calculated.

\[
\eta = \frac{\text{Spray Mass Deduced from Disc Temperature Change}}{\text{Spray Mass Measured During a Pulse}}
\]

A curve fit of the data in Figure 37 shows a power relationship between water utilization efficiency and pulse duration, \(\eta = 0.32 + 175 (\Delta t_p)^{0.273}\). For the three pulse durations presented, \((0.008, 0.028, 0.049)\) seconds, the water utilization efficiency is \((0.48, 0.62, 0.73)\).
Figure 37 Water Utilization Efficiency Increases with Pulse Duration
The reason for the increase in efficiency with pulse duration is that the longer pulse durations have more wetted area. The wetted area intercepts incoming droplets, causing a sticking effect, and therefore less bounce of fine drops. More dry patches of disc exist in the shorter pulse cases. A dry surface is more susceptible to causing incoming fine droplets to bounce.

4.5 NON-UNIFORM DROPLET SPRAY PATTERN ON THE DISC

The spray pattern of the hollow cone nozzle produces a non-uniform distribution of droplets on the disc. This pattern consists of a relatively dry area in the center of the disc. Figure 38 shows the scaling of relatively dry area to totally wetted area.
The picture is deceptive because of the 17° look-angle perspective. It exaggerates the relatively dry area such that it appears larger than it really is. For example, the percentage of relatively dry area is only 11% of the total area for Figure 39 which was produced by spray pulse duration of 28 ms. After scaling from the pictures for various spray durations and nozzle pressures, the relatively wetted area to total area ratio is about 84% +/- 5%. After consideration that the “line” between wetted area and dry area is rather subjective, the 84% value was used in all cases.
4.6 STATIC CONTACT ANGLE MEASUREMENT

The contact angle between the water and gold-coated copper disc was measured with a tensiometer that produced the pictures in Figure 42 and Figure 43. Water was collected from the nozzle after it passed through the system. The value obtained was 47° for a disc cleaned with alcohol, and 70° for as-delivered disc. The 47° value is appropriate for this research because the disc was cleaned with an alcohol wipe prior to closing the tank. The test points were taken over a span of four days during which the chamber remained closed at a vacuum pressure that did not exceed 0.1 Torr. Since the copper disc surface was wiped with alcohol to clean it before taking data, possibly an organic monolayer was established. Since the surface preparation is the same for each test the contact angle will be consistent for each test. According to [35], even a small amount of organic contaminant, will greatly affect contact angle. The assumption is that the organic monolayer capable of affecting the contact angle remained on the disc and therefore provided the same contact angle. Ensuring a repeatable contact angle is important to correlating a model across different data sets.

The contact angle measurement was made with water that was circulated through the spray nozzle system. The water was sprayed into a small (approximately 10 ml) clean glass vial with plastic screw top. The Attension Theta Lite TL100 Optical Tensiometer measurement was made at standard atmospheric pressure and room temperature.

The contact angle between a liquid and a surface depends upon the conditions at the so-called “triple line”, sometimes called the line of contact, or the contact line. The term “contact line” seems the most common term and will be used here. The contact line is the line formed from the circle located at the circumference at the base of a drop. At this line, the water, the gold, and the air are in contact with each other. In the case of the present research,
the air is replaced with vacuum. In the notation of deGennes [36], Young’s relation balances the surface tension forces at the contact line: \( \gamma \cos(\theta_E) = \gamma_{SO} - \gamma_{SL} \). As shown in Figure 40, \( \gamma \) is the surface tension between the liquid and air, \( \gamma_{SO} \) is the surface tension between the solid and air, \( \gamma_{SL} \) is the surface tension between the solid and liquid, and \( \theta_E \) is the contact angle.

![Figure 40 Diagram for Relating Surface Tension Forces and Contact Angle at the Contact Line [36]](image)

As these three forces balance each other, the contact angle is determined. The value of a gas-solid surface tension is difficult to find in the literature. Gas has a very low density compared to liquid water. Based on this, it is possible that the contact angle measurement in vacuum would be very similar to the contact angle measurement in vacuum. No literature results on vacuum water contact angle were found. However, a measurement of the surface tension of mercury in both vacuum and air showed close agreement [37]. The reason that mercury was available is probably because of the low vapor pressure and lack of a phase change when testing in an atmospheric pressure environment versus a vacuum environment. After 1 hour, the surface tension in air was 43.7 and in vacuum, 43.4 mg/mm. Based on this data, it is possible the contact angle of water in vacuum, the droplet being deposited on the surface in
the manner described in the present research, is nearly the same as was measured with the
tensiometer.

A computer simulation the surface tension of water in a vacuum environment showed
the value to be 70.2 +/- 1.7 dynes/cm versus 72.75 dynes/cm for an experimental
measurement in air [38]. This gives additional support to the possibility that the contact
angle of water would change very little as compared to the contact angle measured in air.

The change in water surface tension with temperature is well documented. How this
translates into a contact angle change is not well established. Since the measurement of 47°
for the present research was made at room temperature, there is a desire to know the possible
change in contact angle if the water is cold, near the freezing point, as is the case in the
present research. Vargaftik [39] reported the results of the variation of water surface tension
as 72.75 mN/m at 20 °C and 75.64 mN/m at 0.01 °C. This change of approximately 4% in
the water surface tension might translate into an increase in contact angle of also 4 %, or 49 °,
if the reasoning of the Young’s relation is followed. Since this value is small and within the
error values that are described later, the uncertainty in contact angle at a lower temperature
should not change the results or the conclusions of the present research. A different reference
makes different conclusions about the direction of the change in contact angle with
temperature, but confirms the change is relatively small [40]. The trend in data presented in
Figure 41, which was taken from Osborne [40], shows that the contact angle might actually
decrease slightly in the range of room temperature to near freezing. Osborne then refers to
an analytical prediction that the contact angle trend should be opposite of this data, but offers
no conclusive explanation in his paper.
Since the literature shows some indication that the true contact angle changes little when considering a vacuum environment versus an air environment, and when considering a temperature near freezing instead of one at room temperature, the factors used in the uncertainty analysis (for the parameters that depend on contact angle) should envelope these effects.
Figure 41 Contact angle vs Temperature measurements of a 1.5 L droplet of water on gold. The shark-kink approximation predicts a monotonically decreasing contact angle as temperature is increased, while this figure shows contact angle growing as temperature is increased [40].
Figure 42 Clean As-delivered Gold Coated Copper Disc shows a Static Contact Angle of 70 °

Figure 43 Alcohol Wiped Gold Coated Copper Disc shows a Static Angle of 47 °.
4.7 OBSERVATIONS OF DROPLETS ON THE DISC

Visual observation of the droplets on the disc provides an estimate of the true base area of each deposited droplet. The 17° camera look-angle allows only a projection of the true droplet base area as illustrated in Figure 44. The approximate relationship between the true base area to the projected area seen by the camera viewing at a 17° look-angle is:

\[
\text{True Base Area} = \frac{\text{Projected Area Seen by Camera}}{\sin[17^\circ]} 
\]

Since the contact angle of the droplet is about 47°, the droplets are not perfect hemispheres. Their shape is a spherical cap, as illustrated in Figure 45. This volume of a spherical cap versus a simple hemisphere is resolved through consideration of spherical cap geometry which is discussed later in this chapter.
Figure 44 Look Angle from Camera Enables Only a Projection of Droplet Base Area

Figure 45 Geometry of a Spherical Cap Showing 47 ° Contact Angle
The Phantom V9 high speed camera images are 8 bit black-and-white pixels leading to a grayscale from 0 to 255. The illumination is from a light source that is 150° from the point of view in order to cause refraction within the droplets and allow greater contrast between the droplet and gold coated copper disc. The light shines towards each droplet also at a 17° elevation angle and is refracted through the droplets on the disc towards the camera. Figure 46 shows the view of the disc from the camera to illustrate the relative angles of light source and camera viewing angle.
Figure 46. Relationship between Light Source and Disc as Seen from the Camera’s Point of View
Imaging software, “ImageJ”, estimates the base area of each water droplet on the disc during the first frame after the spray stops by counting the illuminated pixels recorded by the camera. ImageJ has a number of thresholding methods available to the user to convert a grayscale image to a binary image [41],[28],[27]. ImageJ then establishes outlines of the boundaries of white and dark. In this research, the copper disc looks dark because it is a diffuse surface and reflects little light specifically in the direction of the camera. However, because of internal refraction, the ice droplet refracts light preferentially in the direction of the camera. The result is the droplets are bright while the copper is dark. The ImageJ thresholding algorithm uses sophisticated weighting of light and dark pixels to arrive at an outline of each droplet. An example of the first frame for the 8 ms pulse duration is shown in Figure 47. Plotting each droplet’s area value shows a double exponential form for the droplet size distribution, as shown in Figure 48.
Figure 47  ImageJ Outline Plot of “Bowtie” Section

Figure 48  ImageJ Shows Droplet Area Distribution has a Double Exponential Form
ImageJ provides a list of each of the droplet areas. With the spherical geometry relations, the droplet volumes corresponding to these areas can be calculated. The total mass of ice droplets on the disc is therefore estimated by multiplying by the density of ice, 920 kg/m$^3$.

Many researchers have provided meaningful particle size or droplet size distribution relations in the literature. In reference [42], three common continuous probability distribution functions for number-of-droplets versus the drop diameter are listed: the log-normal distribution, the upper limit distribution, and the root-normal distribution. All involve an exponential term. Although not a probability distribution, a plot of the droplet size area versus each droplet resolved by ImageJ also shows an exponential distribution in Figure 48.

In section 4.4, the discussion on water utilization efficiency used the disc temperature before-and-after change as a true measure of the amount of water deposited on the disc. This ImageJ technique provides an estimate of total droplet mass that agrees very well in the 8 ms and 28 ms cases as shown in Table 1. For the other 49 ms case, the agreement is good, but not as good as the 8 ms and 28 ms case. The excellent agreement for the 8 ms and 28 ms cases may be due to the fact that the droplets are relatively separated from each other and very few odd-shaped puddles exist. ImageJ can better resolve the white and dark pixels when some distance exists between them. For the longer pulse duration of 49 ms, the droplets are relatively crowded together. ImageJ’s ability to resolve the image into separate droplet outlines is not as good in that case.
Table 1 Droplet Mass Derived from Spherical Cap Geometry (Theory) versus Mass Derived from Experimental Disc Temperature Change Data (Measured)

<table>
<thead>
<tr>
<th>Pulse Duration</th>
<th>Mass Deduced from Disc Temperature Change</th>
<th>Mass Deduced from ImageJ Software</th>
<th>Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 ms</td>
<td>2.85 E-06</td>
<td>2.81 E-06</td>
<td>-2 %</td>
</tr>
<tr>
<td>28 ms</td>
<td>8.30 E-06</td>
<td>8.09 E-06</td>
<td>-3 %</td>
</tr>
<tr>
<td>49 ms</td>
<td>1.53 E-05</td>
<td>1.92 E-05</td>
<td>+25%</td>
</tr>
</tbody>
</table>
4.8 COOLING OF THE DISC TO DETERMINE THE TIME CONSTANT

When the spray starts, the water begins to deposit on the disc. The heat transfer from the disc is not high enough to evaporate the water at the same rate as it arrives. Hence, the mass of the water on the disc begins to increase. As water evaporates, the droplets cool, but also mix with warm arriving droplets. This is a difficult portion of the spray from which to deduce any type of evaporation rate or heat transfer mechanism, because 1) the exact temperature of the incoming droplets is not known, 2) the high speed video frames show a rapidly changing pattern of droplet formation, coalescence, and breakup, due to the impacting droplets’ momentum and 3) the spray itself obscures the view of the disc surface.

However, a clearer picture develops after the spray stops. At that point, no incoming droplets exist. The mass of the water on the disc is a fixed quantity that steadily sublimes away. From the point just after the spray stops, a smooth disc temperature history can be recorded that is solely based on the evaporation of the water that existed at the point of spray stop. Since the cooling of the disc is caused by the evaporation of the water, the evaporative heat transfer can be quantified. Then, a heat transfer mechanism can be proposed and verified.

There are two thermocouples on the backside of the disc, one in the center and one near the edge. The center and outer edge thermocouples are area-averaged to represent a lumped parameter temperature of the entire disc.

The primary source of non-uniformity of the disc temperature is caused by the non-uniformity of the deposited droplets. This deposition pattern is complex. To simplify the complexity, a linear temperature profile across the disc is assumed when averaging the two thermocouples. There is a secondary source of non-uniformity caused by the radial heat flow.
from the dry center of the disc outward to the wetted portion of the disc. This radial heat flow alone would cause a radial temperature profile that is not linear, but logarithmic. There is a third source of local temperature non-uniformity caused by the alternating dry and wetted portions of the disc all throughout the deposition pattern. This third source causes a radial temperature profile that varies with deposited droplet location at a small scale. At the disc location just below a 50 micron diameter drop, for example, that small portion of the disc covered by the drop will decrease faster than the dry disc area that immediately surrounds the 50 micron drop.

The proper choice of determining whether an average temperature is acceptable might depend on the application. If the surface to be cooled is the surface of a heat exchanger, such as the shuttle, where the spatial temperature variation is not important to reducing the bulk heat exchanger coolant temperature, the assumed temperature profile is not critical. In this case, assuming a linear radial temperature profile may be adequate for the purposes of averaging the two thermocouple temperatures. The present research reports an average cooling ability for the spray as an average of the total circular impacted area. However, if the surface to be cooled is an integrated circuit or some surface with a distributed and highly non-uniform heat source pattern, then the concept of using an average temperature, as presented in this research, may not be appropriate. The use of copper in the present research is a high conductivity metal when compared to other possible surfaces such as gallium arsenide, silicon, or even other metals such a nickel. The use of copper here in the present research does not reveal the greater temperature non-uniformity that might arise in an application where a lower conductivity surface is to be cooled.

Assume a linear temperature distribution:
\[ T = T_{\text{center}} + \frac{r}{R} (T_{\text{edge}} - T_{\text{center}}) \]  

(3)

Define an average disc temperature based on the total area

\[ T_{\text{Average}} A_{\text{total}} = \int_{0}^{R} T(r) 2\pi r \, dr \]  

(4)

Substitute equation (3) into equation (4)

\[ T_{\text{Average}} = \frac{\int_{0}^{R} T_{\text{center}} 2\pi r \, dr + \int_{0}^{R} (T_{\text{edge}} - T_{\text{center}}) \frac{r}{R} 2\pi r \, dr}{A_{\text{total}}} \]  

(5)

Perform the integration across the disc, from 0 to R

\[ T_{\text{Average}} = \frac{T_{\text{center}} \frac{2\pi R^2}{2} + (T_{\text{edge}} - T_{\text{center}}) \frac{2\pi R^3}{3} \frac{R}{3}}{\pi R^2} \]  

(6)

Simplify

\[ T_{\text{Average}} = T_{\text{center}} + (T_{\text{edge}} - T_{\text{center}}) \frac{2}{3} \]  

(7)

Finally,

\[ T_{\text{Average}} = T_{\text{center}} + \frac{2}{3} (T_{\text{edge}} - T_{\text{center}}) \]  

(8)

Therefore, the temperature gradients in the lateral direction are accounted for by area-averaging an estimate of the temperature gradient.
The thickness of the copper disc is 1 mm. For conduction in this thru-thickness direction, the lumped parameter consideration of the disc could be valid, according to the generally accepted criterion based on the Biot number [43].

\[
Biot = \frac{h L}{k} < 0.1
\]  \hspace{1cm} (9)

The heat transfer coefficients were not known at the time of experiment formulation, and had to be estimated from a reasonable assumption of evaporative heat transfer. The highest heat flux might be conservatively estimated at that for water boiling heat transfer at atmospheric pressure [43]. Although very high and unlikely, this yields the highest Biot number and the worst case value.

Assuming a heat transfer coefficient definition of:

\[
h = \frac{q''}{(T_{disc} - T_{sink})}
\]  \hspace{1cm} (10)

and substituting 20 °C for \(T_{disc}\), -47 °C for \(T_{sink}\), and 1 E+04 W/m² for \(q''\), then

\[
h = 150 \text{ W/m}^2 \text{ K}
\]

\[
Bi = 0.0005
\]

The lowest heat flux value might be 1 W/cm², based on data from Grissom[16]. For these values listed above, the Biot number ranges from 0.0005 to 0.04. Since this range of Biot numbers are below the criterion of 0.1, the temperature difference between the top surface of the disc where the water impinges and the thermocouple measurement location on the bottom of the disc is negligible.
The data acquisition system produces 6400 temperature data points per second which appears as a scatter plot. Since the thermocouples wires are effectively antennas, they receive electrical signals generated from a variety of noise sources in the frequency of radio waves, MHz, and the electrical main of 60 Hz. The random error in the data causes the data to fluctuate within a 2 C range. The mean of all the high frequency noise sources, when averaged over the time scale of the disc cooldown of ~ 250 milliseconds, is zero. Therefore, a least squares curve fit will give the true temperature of the thermocouple with time. A least squares curve fit using Igor data analysis software provides a temperature versus time plot of the disc average temperature [44]. Figure 49 shows all three temperature histories plotted together with the lower y-axis value equal to the equilibrium ice vapor pressure corresponding to -49° C. It also indicates the temperature changes of the disc are a small fraction of the overall temperature differences between the disc to the ice vapor pressure. Figure 50 through Figure 52 show the temperature history for pulse durations of 8 ms, 28 ms, and 49 ms for nozzle pressure of 25 psid. A time constant of an exponential decay is defined here in the traditional way: \( \tau = 1/e = .368 \) of the original value; the time it takes for the temperature to decay to .368 of the total temperature change that will occur.
Figure 49. Temperature History Data for Pulse Durations of 8 ms, 28 ms, and 49 ms

Figure 50 Temperature Scatterplot Data with a Least Squares Curve Fit for the 8 ms Pulse Duration
Figure 51 Temperature Scatterplot Data with a Least Squares Curve Fit for the 28 ms Pulse Duration

Figure 52 Temperature Scatterplot Data with a Least Squares Curve Fit for the 49 ms Pulse Duration
Figure 53 shows the time constants of the three cases, where the bars indicate the 95% dispersion band, or extent of temperature data point scatter from the mean. This is not error in the sense of uncertainty analysis, but is a characterization of the extent of date point scatter from the true temperature determined by the least squares curve fit.

The three data points presented in Figure 50 though Figure 52 were taken over the span of four days as the very first operation of the day. The test equipment was allowed to sit overnight. In that way, the entire rig, nozzle, flow delivery, and water reservoir were all at the same temperature. Normally, when a pulse occurs, there is some cooling of the nozzle, because a small amount of ice is observed on the nozzle after shut off. Since the viscosity of water is very temperature dependent near its freezing point, the extra caution of using data from the first operation of the day was carried out to minimize that possible source of error.

Figure 54 shows additional data points as well as the three baseline data points with their uncertainty bands. For repeated data points that were not the first operation of the day, the data points were generally repeated later after several minutes or hours of waiting for the test equipment to warm to room temperature. This waiting time produced reasonable results as shown in Figure 54. However, the three baseline points chosen for further analysis were considered the most reliable points because they were taken as the first spray operation of the day, and because at least one of the other points was very close to each of these three baseline points. These extra data points support repeatability of the data and most are within the calculated uncertainty bands. The entire uncertainty analysis is discussed later in Chapter 5.
Figure 53 Time Constant Derived from Experimental Temperature Data with 95% Prediction Bands based on Dispersion of Data Points
Figure 54 Time constant versus pulse duration shows additional data points which support the uncertainty analysis and repeatability of data.
4.9 VIDEO DATA

Figure 55 and Figure 56 show eight frames of the sublimation process, for the 8 ms pulse duration case, in 20 ms increments. These lend visual support to the temperature data as determined by the thermocouples. The first frame shown is that one just after the spray pulse has ended. It appears that the speed of disappearance; that is, sublimation, takes place rapidly at first, and the decreases as time goes forward. This is visual experimental evidence that supports the temperature thermocouple data, whose form is an exponential temperature decay.
Figure 55 Time lapse pictures for the 8 ms case show more rapid change of deposited mass in the early time frames.
Figure 56 Time lapse pictures for the 8 ms case show a slower change of deposited mass in the later time frames.
4.10 SPHERICAL CAP GEOMETRY TO DETERMINE DROP VOLUME AND VOLUME/AREA PARAMETER

For an ensemble of droplets on a disc, the disc transfers heat to each droplet, leading to various rates of sublimation for each droplet, depending on size. For modeling purposes, there exists a droplet whose diameter represents a mean droplet diameter for all the droplets. When this mean droplet sublimes, it has the same effect on the disc as the ensemble. This mean droplet is analogous to a Sauter Mean Diameter (SMD), which is the averaged volume to surface area ratio that frequently characterizes fuel evaporation of sprays in combustion applications. It is noticed, however, since droplets on a surface are only partial droplets. These partial droplets are characterized by a contact angle. The shape is mathematically known as a spherical cap.

The static contact angle of a sessile drop is defined as the angle the liquid apparently makes with the solid substrate. A contact angle less than 90° leads to a spherical cap. The contact angle is a characteristic of a fluid and the surface upon which the fluid is resident. To a much lesser extent, the contact angle is also determined by the third fluid, which is typically air. For this research, the third fluid is extremely low pressure air; that is, vacuum. Since most contact measurements are made in air, and since the air typically plays a very small role, if any, in determining contact angle, the data in the literature for gold and water in an air environment is reasonably assumed to apply to vacuum environment.

Spherical cap geometry relationships are available in the literature [45, 46]. They can relate parameters such a volume, surface area, base area, height, and contact angle. From Weisstein [46], Figure 57 shows the parameters of a spherical cap. Note the spherical cap internal angle, $\theta_M$, is defined differently than the usual contact angle $\theta_C$, used in fluid
mechanics, and must be subtracted from $90^\circ$ to arrive at the common definition of contact angle used in fluid mechanics. The relation is

$$\theta_m = \frac{\pi}{180}(90 - \theta_c)$$

For a sessile droplet on a disc, the base of the spherical cap, described by $\pi a^2$, contacts the disc.

In Figure 57, the parameter “R” is the radius of a sphere that contains the spherical cap. The sublimation takes place on the surface of the droplet and therefore “R” decreases with time. This would cause the frozen droplet to appear to shrink and sink into the disc. In our model, we assume the spherical cap configuration stays the same for all drops, and is maintained through the history of the drop sublimation. In other words, if “R” decreases, then all the other parameters would decrease proportionally and the spherical cap would always be similar in shape.

The spherical cap geometry assumption is a key part of the simplification that allows a rather complicated process to perhaps be modeled in a simple way. As will be shown later in the modeling section, an estimate of the heat transfer is adequate but could be improved by better knowledge of the exact shape of the deposited droplets. It appears the smaller droplets are more dominated by surface tension and are perhaps close to a spherical cap geometry. The small droplets during a short pulse are mostly singular: follow-on drops have not landed on top of them. For the longer pulses that create medium sized drops and small puddles, some drops are larger because they are aggregates of multiple landings of the small drops. And for the more extreme puddles for the longest pulse durations, some drops appear to not be spherical caps at all, but odd shapes that would be better estimated with some other
geometry, if only a simple way to define the complicated shape were available. In the present research, a spherical cap geometry applied to all droplets produces an acceptable model and understanding of the mechanism. However, the results point to a need for a perhaps better video resolution for improved understanding of the mechanism, especially for longer pulses. Accordingly, a sub-model for the puddles can be developed to broaden the applicability of the present spherical cap model.
Figure 57  Definition of geometry parameters for a spherical cap used in Mathematica [46]

Figure 58 Contact angle definition of contact angle typically used by the thermal-fluids community
4.11 PDPA DATA TO CORROBORATE EXPERIMENTAL MODEL

The Dantec PDPA provided an estimate of SMD, the Sauter Mean Diameter, for the spray in air. This is a standard instrument used to measure spray characteristics. It is a “turnkey” system that requires no specialized knowledge of optics. The system is associated with software that works together with the laser heads to evaluate the Sauter Mean Diameter (SMD), and other parameters. The PDPA is not applied in the vacuum chamber due to substantial difficulties in alignment through the window caused by various reflections.

The SMD is an important parameter for evaporation and combustion of sprays. It is conceptually a diameter represents the ratio of the volumes to surface areas of all the drops in heat-mass transfer. The classic definition of SMD [47]:

\[
SMD = \frac{\sum N_i D_i^3}{\sum N_i D_i^2}
\]  

(11)

Where Ni is number of drops in the size range i

D is the diameter of size range i

There are other mean diameters. For example, the D_{10} is the straight average of all the droplet diameters. Figure 59 shows a comparison between SMD and D_{10} for the same spray. As shown in Figure 59, the reported values range from about 60 microns to 90 microns at various positions across the spray. It has always been a challenge to predict a mean diameter in a particular problem in sprays [48].
Since there is experimental evidence that the spray is different in air than in vacuum (cone angle change, pattern differences, startup differences), it is reasonable to assume that the SMD might also be different. If the additional shear and turbulence provided by air causes better atomization, the SMD in air should be smaller than in vacuum. Therefore, this SMD data is only a good reference for this study. It is important to point out that the Volume/Area ratio of the deposited drops is developed not using the PDPA, but using the high speed camera and an imaging analysis software code called ImageJ.
Figure 59 PDPA Laser Particle Sizing Results for the Spray in Air.
4.12 MULTIPLE PULSES

Successive 50 ms pulses cause an increase in chamber pressure and a corresponding decrease in disc temperature, with ice buildup on the disc. At some point the equilibrium pressure and temperature of ice is reached whereby no additional evaporative cooling takes place. A plot of water vapor pressure over ice is presented in Figure 60.

In this process, two cases are possible: 1) the vacuum pump is off and the chamber is closed, and 2) the vacuum pump remains on.

Results for case 1 are shown in Figure 61. After 6 successive pulses of 50 ms, ice/vapor equilibrium is reached and additional disc cooling stops. Pulsing is continued and the incoming water is immediately frozen and added to the ice already resident on the disc. The disc temperature and chamber pressure slowly rise in equilibrium. The equilibrium temperature and pressure after 6 pulses is measured at about -14 C and 1.41 Torr. This compares well with the most recent accepted relations between vapor pressure over ice to temperature, the Arden Buck equation of 1996 [49] shown in Figure 60, which predicts 1.36 Torr.

\[
Vapor\;Pressure\;over\;Ice = \frac{760.101307.78}{101307.78} 611.11 e^{\frac{(23036-T)}{27982+T}} \tag{12}
\]
Figure 60. Vapor Pressure over Ice as a Function of Temperature
Figure 61 Disc Temperature and Chamber Pressure When Ice is Resident on the Disc

(Case 1 Pump Off)
Results for case 2 are shown in Figure 62. The equilibrium temperature and pressure after multiple pulses is measured as -23°C and 0.62 Torr. This value compares well with the 0.58 Torr value predicted by Equation 4. At this point of equilibrium, the vacuum pump is removing water at the rate it is being introduced to the chamber by the multiple 50 ms pulses. An effective steady state/steady flow process is achieved. After the pulsing is stopped at about 85 seconds, the vacuum pump continues to remove mass from the chamber. The pressure and temperature decrease according Equation 4, with some difference due to the disc thermal mass causing a small lag in the transient temperature curve.
Figure 62 Disc Temperature and Chamber Pressure When Ice is Resident on the Disc

(Case 2 Pump On)
SUMMARY

In this chapter, the dynamics and heat transfer of drops are examined through the whole process. The true pulse duration and the nozzle flow rate are determined. The difference between spray in air and in vacuum is discussed. The effect of drop cooling during flight is discussed. The spray cone angle variation, the water deposition pattern and deposition efficiency are studied. Also, the observation of contact angle effect on the shape of landed drops and confirmation of the deposition amount, by visual analysis, are reported.

From the cooling rate of the disc, the cooling time constants of different pulse durations are obtained. The changing rate of evaporation is confirmed by observations. Observation and a basic model of the spherical cap shape of landed drops are provided. In the next chapter of modeling, this information of a spherical cap is used to understand the heat transfer mechanism and to predict the experimental results. Finally, the cooling results of multiple pulses are reported.
This chapter discusses the analytical models. A physical model is developed based upon the observations described in Chapter 4. Since higher heat transfer rates are desired in cooling applications, the interest lies mainly with understanding the physics of discrete depositions of smaller drops in short pulses during the cooldown phase after the spray has stopped.

A basic experimental model was developed based on a transient energy balance of the drop-disc system during evaporation. A basic solution is derived for the time constant \( \tau \) of the cooling process. The time constant \( \tau \) depends upon two major parameters: the accommodation coefficient \( \alpha \) of drop evaporation in vacuum and the volume to surface ratio of the deposited drops on surface, \( w \).

The sublimation rate evaluation section intends to find the accommodation coefficient of drop evaporation in vacuum, which is to be used in the predictive model. In this section, the volume to surface ratio of the deposited drops, \( w \), of 8ms pulse, is measured using ImageJ. Since the time constant, \( \tau \), is available from the temperature history of experimental data, the accommodation coefficient \( \alpha \) is determined from trial-and-error to fit the data of \( \tau \) for 8 ms. This accommodation coefficient is used as input to the predictive model, which is one of the final products of this research. The accommodation coefficient of 0.036 is obtained, which is a reasonable value based on literature of many investigations of accommodation coefficient of water [50]. Finally, using this value of accommodation coefficient, 0.036, together with the experimental results of time constant \( \tau \) of 28 and 49 ms pulse cases, the corresponding volume to surface ratios of deposited drops, \( w \), are back calculated. These calculated \( w \)'s are
compared with the experimental observed w’s from ImageJ and give a reasonable agreement. This implies the value of accommodation coefficient chosen in this problem is acceptable.

The *spacing parameter model* section generates a spacing parameter “n” of the drop pattern on the surface. With the accommodation coefficient $\alpha$ determined, the drop pattern and the consideration of spherical cap droplet shape lead to the derivation of the volume to surface ratio, w, of the deposited drops. Based upon the experimental data of time constant $\tau$ and the determined accommodation coefficient $\alpha$, and the corresponding volume to surface ratio “w”, the corresponding spacing parameter “n” is determined. The spacing parameters of 1.9, 1.45, and 1.25 for the 8, 28, and 49 ms pulse durations, respectively, were found to correlate the experimental data of $\tau$ well. Accordingly the model is fully established.

The *predictive heat transfer model* section shows the streamlined procedure of using the established predictive model. In this model, only the incoming water mass flux is required. With the evaluation of the spacing parameter, n, and the determined accommodation coefficient $\alpha$, this predictive heat transfer model predicts not only the experimental temperature data in the present research, but also could be applied generally to similar sprays in relevant applications. A flowchart of the process of establishing the models and the use of the models are given in Figure 63.
Figure 63 Summary of Modeling
Regarding the range of applicability of this model, a consideration of pulse duration and the physical appearance of the water deposition is warranted. Based upon experimental observation, for spray pulses of 8 ms or shorter, the droplets on the disc are fairly discrete and separate. For spray pulse durations greater than 8 ms, the droplets on the disc are mostly discrete and separate, but some appear to coalesce and form a few small puddles. The apparent trend for longer pulse durations, as exhibited by examining the data and noting the change in appearance from the 8 ms pulse duration through to the 49 ms pulse duration, is that a complex coalescence of the droplets occurs and more random puddles form. It appears that the longer pulse durations create larger and more frequent odd-shaped puddles, and this phenomenon causes a more difficult characterization. A spray pattern that creates a large number of odd-shaped puddles is a different physical situation than one with only discrete small droplets. The model proposed in this research applies better to shorter pulse durations, because the model assumes all discrete spherical cap geometry droplets. The model appears to be also acceptable for those that contain a limited number of small puddles. The model may not apply very well to pulse durations longer than 49 ms because of the large number of puddles.
5.1 BASIC EXPERIMENTAL MODEL

As discussed in chapter 4, the pressure rise in the tank during a short pulse is very small. The mass of water injected during a pulse does not substantially affect the system pressure. Therefore, there exists practically an “infinite sink” of vacuum which can absorb the water. The equilibrium temperature corresponding to this system pressure is called the “sink” temperature, $T_{sink}$. Presently, the consideration is evaporating and sublimating in a true vacuum, non-continuum regime environment.

The small mass of water deposited on the disc in the short spray pulse durations is insufficient to reduce the disc temperature all the way to the sink temperature. Since the final disc temperature is still far from the $T_{sink}$, the driving temperature difference, $(T_{disc} - T_{sink})$ is relatively constant. In Chapter 4, Figure 49 shows a scale of temperature for both the sink temperature and the temperature change in one pulse duration to support this claim that the driving temperature difference is relatively constant. Further support is provided by the relation of vapor pressure as a function of temperature for the kinetic theory.

\[
\text{Driving Pressure Difference} = \frac{760}{101307.78} \times 611.15 \left( e^{\frac{23.036}{333.7} T_{disc}} - e^{\frac{23.036}{333.7} T_{sink}} \right) \quad (13)
\]

For values of 22 C as the disc temperature and -47 C as the sink temperature, the first term is 24 Torr, and the second term is .04 Torr. Equation (13) shows that the difference between the vapor pressure at 22 C and – 47 C is so large, that the second term in the equation can be removed, with less than 1% error.

The basic model is an approximate analysis, which is constructed with the following assumptions:
1. The value of “T” represents the disc-average temperature.

2. The evaporation and droplet mass loss occurs at the surface of the droplet.

3. The conduction resistance through the frozen water is small such that the cooling is controlled by the sublimation at the droplet surface.

4. The copper disc can be considered a lumped mass at uniform temperature in the cooling process since the radially averaged temperature is used.

5. The sensible heat of the water is negligible compared to that of the copper disc.

The latent heat of the water mass loss is equal to the temperature change of the disc and the temperature change of the water:

\[ m \ C_p \ w \ \frac{\partial T}{\partial t} + M \ C_p \ d \ \frac{\partial T}{\partial t} = h_{fg} \ \frac{\partial m}{\partial t} \]  

(14)

Since the mass of the water is very much smaller when compared to the disc mass, the temperature change of the water, the first term on the left hand side of Equation (14), can be neglected. Then, the change in mass of water with time scales linearly with the change in disc temperature with time.

\[ M \ C_p \ d \ \frac{\partial r}{\partial t} = h_{fg} \ \frac{\partial m}{\partial t} \]  

(15)

The lumped parameter governing equation is of disc cooling:

\[ M \ C_p \ d \ \frac{\partial T_{disc}}{\partial t} = -h \ \ A_{droplet\ surface}(t) \ (T_{disc} - T_{sink}) \]  

(16)

The left hand side of the equation (16) can be written in terms of water mass loss. Substituting Equation (15) into Equation (16):
\[ h_{fg} \frac{\partial m}{\partial t} = -h \ A_{\text{droplet surface}}(t) (T_{\text{disc}} - T_{\text{sink}}) \]  

(17)

Consider that a particular droplet mass is related to its volume as:

\[ m = \rho \ A_{\text{droplet surface}}(t) \frac{\text{Volume}(t)}{A_{\text{droplet surface}}(t)} \]  

(18)

Call the quantity \[
\frac{\text{Volume}}{A_{\text{droplet surface}}}
\] with the letter \( w \).

\[ m = \rho \ A_{\text{droplet surface}}(t) \ w(t) \]  

(19)

In this way, “\( w \)” is a mean droplet length parameter, but could also be thought of as a volume to area ratio of deposited drops.

For general heat and mass transfer processes, the major parameter involved in the drop evolution is the volume (for size change) and the surface (for heat mass transfer to occur). The \( w \) may be similar to the Sauter Mean Diameter (SMD) used generally in sprays and characterizes the overall volume to surface ratio of a spray droplet ensemble that is composed of many droplets of different diameters. Also, the “\( w \)” could be considered the length parameter; that is, it could represent an equivalent mean height of a droplet ensemble.

Generally, the SMD of a spray (not deposited drops) might be available in three ways: 1) the nozzle manufacturer of a particular nozzle may provide the SMD data, 2) there exist some empirical formulas for estimating SMD, and 3) a Phase Doppler Particle Analyzer (PDPA) or other optical device may be used to measure the spray droplet size and velocity and may calculate a SMD. In this research, neither the manufacturer’s data nor the empirical correlations would apply exactly, because those are based on data in system pressures much higher than vacuum. The PDPA data and analysis is in air, and is presented in this study is
only as a reference. Nozzle spray physics changes when a nozzle discharges into a vacuum as compared to when it discharges into standard atmospheric pressure. Therefore, we shall address the w separately in later sections.

Rearranging equation \((19)\) to solve for the droplet surface area:

\[
A_{\text{droplet surface}}(t) = \frac{m}{\rho w(t)}
\]

Then equation \((17)\) becomes:

\[
h_f g \frac{dT}{dt} = -\frac{h m}{\rho w(t)} (T - T_{\text{sink}})
\]

This is a first order differential equation in \(m(t)\).

An important concept in equation \((21)\) is that this first order differential equation assumes that the initial disc temperature and initial “w” value are constant over the duration of the cooldown. This allows equation \((21)\) to be mathematically consistent with a simple ordinary differential equation in \(m(t)\). Recalling the plot presented in Figure 49, the temperature difference between the disc temperature and the “sink” temperature, \((T - T_{\text{sink}})\), is very large compared to the disc temperature change. Also, the \(w(t)\) represents the mean volume/area value for all droplets. As the spray droplets sublime, smaller droplets disappear and drop out of the sum. This leads to a larger \(w(t)\) as time progresses. However, the larger droplets shrink in size and their volume/area ratio decreases. This leads to a smaller \(w(t)\) as time progresses. It is possible therefore, that \(w(t)\) is constant with time. But this is not derived from mathematical analysis nor is it available from rigorous video analysis of the droplet size. In the present research, the assumption that the ratio \((T - T_{\text{sink}}) / w(t)\) is constant,
allows equation (21) to be an ordinary differential equation in \( m(t) \). The \( w(t) \) is replaced with \( w_0 \) to show that the value is calculated with the video data of the first frame and the temperature data at the early moment.

The qualitative argument of \( w(t) \) remaining constant seems reasonable, since an exponential solution is adequate to fit the temperature history scatterplot data. Nevertheless, better knowledge of how “\( w \)” varies with time would be a strong addition to the present first attempt model.

Consider the solution to be an equation with the form of an exponential solution in \( m(t) \).

\[
m(t) = m(t = 0) e^{-t/\tau}
\]

Where

\[
\tau = \frac{h_{fg} \rho w_0}{h (T - T_{sink})}
\]

The values of \( h_{fg} \) and \( \rho \) vary negligibly over the small temperature change of the experiments.

The other variables are assumed constant as part of the basic modeling assumptions:

\[
\frac{w_0}{h(T - T_{sink})}
\]

5.2 SUBLIMATION RATE EVALUATION

Until now, the heat transfer has been kept general and expressed as a convection term, \( h (T - T_{sink}) \). This will now be specified for this application of sublimation in a vacuum. The classic kinetic theory is appropriate here. Although not considered high vacuum, the
working system pressure used in this research is sufficiently low enough to consider the non-continuum flow assumption applies. Both water vapor molecules and air molecules do exist, but are not present in large enough numbers to make conventional convection heat transfer correlations valid.

To determine whether the assumption of free molecular flow is appropriate, the Knudsen number is calculated in
Appendix E. The Knudsen number is the ratio of the mean free path of a molecule to a reference length parameter. In this case, the reference length is the droplet diameter; that is, the diameter of the base of the spherical cap. An estimate of the mean free path of water vapor for the conditions of system pressure at 0.04 Torr and water vapor temperature of 0 °C, is 1900 microns. Since the largest droplet diameter derived for the 0.008 s pulse duration determined by ImageJ is 385 microns, the Knudsen number is 1900/385, or approximately 5. Thus, the assumption of free molecular flow is valid.

Therefore, the model uses a classical kinetic theory relationship between heat transfer and temperature. The equation that describes the heat flux leaving an evaporating surface has many forms and names but has recently been referred to as the Hertz-Knudsen equation [50-52]. This equation requires an evaluation of the equilibrium vapor pressure over ice or water. For this, the equilibrium vapor pressure of ice or water as a function of temperature is available from the literature [49], [53], [54]. Since most of the total droplet evaporation lifetime is that of sublimation of ice rather than evaporation of not-yet-frozen water, the Arden-Buck correlation of vapor pressure over ice was used [49].

\[
P_{\text{vapor}} = 611.15 e^{\left(\frac{23.036 - \frac{3.088}{T+237.3}}{279.82 + T}\right) T}
\]

where \(T\) is in Celsius and \(P_{\text{vapor}}\) is in Pascals.

Although there are many empirical correlations of water vapor versus temperature in the literature for various temperature ranges, the Arden-Buck formula was used because it was one of the most cited and established relationships. The heat transfer coefficient approach is replaced with the classic kinetic theory. The kinetic theory approach considers the motion of individual gas molecules to model the heat transfer process [55].
The heat flux leaving an evaporating surface is taken as Equation 4.141 of [55].

\[
q^* = \frac{2\alpha}{2 - \alpha} h_{fg} \left( \frac{\text{Molecularweight}}{2\pi R} \right)^{\frac{5}{2}} \left( \frac{P_{\text{source}}}{(T_{\text{source}})^{\frac{5}{2}}} - \frac{P_{\text{sink}}}{(T_{\text{sink}})^{\frac{5}{2}}} \right)
\]  

(26)

The “source” and “sink” subscripts are added here for this research to designate the sublimating water droplets as the “source” and the surrounding vacuum system pressure as the “sink”. Further, for the low system pressures in this research, the “sink” term is negligible when compare to the source term, and is omitted in the analysis.

When equation (25) is substituted into (26), the following equation is used to predict \(\tau\) for use in the analytical model (recall equation (23)):

\[
\tau = \frac{\rho w_0}{2^\alpha \left( \frac{\text{Molecularweight}}{2\pi R} \right)^{\frac{5}{2}} \left( 611.15e^{\frac{23036 - 273.15}{(T + 273.15)^2}} \right)}{(T + 273.15)^{\frac{5}{2}}}
\]

(27)

The accommodation coefficient, \(\alpha\), is a parameter for which only empirical estimates can be provided. The accommodation coefficient of 0.036 has been published in the literature. This is reasonable for the experimental setup according to literature sources [56], [57], [58], [59], [60]. From the work of Marek [50], the summary of accommodation coefficients provides a sense of uncertainty in this parameter, but also a reasonable bound for this research. A range of between 0.01 and 0.1 appears reasonable. Figure 64 shows a reproduction of Marek’s accommodation coefficient literature survey. In Eames, 1995, a literature review lists values of the accommodation coefficient, called the “true evaporation coefficient” in his paper, as 0.03, 0.052, .042, 0.047, and 0.05 from sources that reported evaporation to either low pressure or an evacuated chamber. In Bonaci [52], the
measurement was for ice and used a copper disc similar to this research. In general, the wide variety of reported results seems due to very different measurement methods and possibly different instrument accuracy. Some of the literature quotes results, apparently using a temperature measurement and evaporation setup similar to this research, that support the accommodation coefficient used in this research. However, a precise value for the accommodation coefficient is not available, and appears to be a function of how the measurement was taken.

Figure 64 The Accommodation Coefficient Literature Survey of Marek [50]
In order to derive a suitable accommodation coefficient, $\alpha$, the Equation (28) was used.

$$\tau = \frac{w_0 \rho_{ice}}{2 \alpha \left(\frac{\text{Molecularweight}}{2\pi R}\right)} \left(\frac{611.15e^{\frac{(23.036-T_{\text{droplet}})}{333.7}} \frac{T_{\text{droplet}}}{279.82+T_{\text{droplet}}}}{(T_{\text{droplet}} + 273.15)^{\frac{1}{2}}}ight)$$

(28)

The video data and ImageJ analysis of the 8 ms case provides the value for “w” as an input to the above equation. The accommodation coefficient, $\alpha$, is adjusted until the $\tau$ predicted by the above formula matches the $\tau$ derived from the experimental temperature history for the 8 ms case. The $T_{\text{droplet}}$, which is the starting temperature, is known from the experimental temperature history. The ice density, molecular weight of water, and “R” are all known.

The experimental video data with ImageJ provides an estimate of the base area of each droplet for the first frame. For a spherical cap internal angle $\theta_M$, spherical cap geometry relates the base area to the volume of the droplet:

$$\text{Droplet Volume} = \frac{A_{\text{base}}^{3/2}(2 - 3\sin[\theta_M] + \sin[\theta_M]^3)}{3\sqrt{\pi}(-1 + \frac{2}{1 - \sin[\theta_M]})^{3/2}(1 - \sin[\theta_M])^3}$$

Where

$$\theta_M = \frac{\pi}{180}(90 - \theta_c)$$

Substituting $\theta_c = 47^\circ$,

$$V_{\text{drop}} = 0.1303883 A_{\text{base}}^{3/2}$$
The droplet surface area, $A_{\text{drop}}$, is the area where sublimation occurs and is larger than the base area due to the shape of the spherical cap:

$$A_{\text{surface}} = \frac{2}{1 \pm \sin[\theta_M]} A_{\text{base}}$$

with the *Mathematica* spherical cap internal angle to the fluids-thermal contact angle relation,

$$\theta_M = \frac{\pi}{180}(90 - \theta_c)$$

and the thermal-fluids contact angle of 47°:

$$\theta_c = 47 \, ^\circ$$

then

$$A_{\text{surface}} = 1.19 \, A_{\text{base}}$$

Then, the volume/area parameter, “w”, determined for the first frame is the ratio of the sum of the drops’ volumes divided by the sums of the drops’ surface areas. Call this the “$w_{\text{video}}$”, because it is determined from the video data.

$$w_{\text{video}} = \frac{\sum_{j=1}^{N} V_{\text{drop},j}}{\sum_{j=1}^{N} A_{\text{surface},j}}$$

The $w_{\text{video}}$ parameter is calculated from ImageJ for each of the three pulse durations. Details of the Excel worksheet are in the Appendix F.
Using the 8 ms ImageJ data of w and the experimental time constant of cooling, the accommodation coefficient $\alpha$ of this present study is found to be $\alpha = 0.036$. Then, with $\alpha$ and the known time constant of cooling for each case of 28 ms and 49 ms, the corresponding “w” can be evaluated, and is listed in Table 2 in the first row.
Table 2 Comparison of the Volume/Area Ratio or $w_0$ Parameter

<table>
<thead>
<tr>
<th></th>
<th>$w_0$, [m] for 8 ms pulse</th>
<th>$w_0$, [m] for 28 ms pulse</th>
<th>$w_0$, [m] for 49 ms pulse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determined from</td>
<td>.000028</td>
<td>.000060</td>
<td>.000092</td>
</tr>
<tr>
<td>Video Data</td>
<td>Determined from</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Determined from</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Experimental Temperature</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Data and $\tau$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha = 0.036$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.000028</td>
<td>.000048</td>
<td>.000060</td>
</tr>
</tbody>
</table>

The comparison of the $w$ of ImageJ and $w$ calculated from $\alpha = 0.036$ for the longer pulses (28 and 49 ms) in the second row of the table are reasonable but not accurate. The refined model in the next section is able to predict them better. Nevertheless, the difference between ImageJ values of $w$ and derived values for the 28 and 49 ms cases may have an explanation: the video data estimates the droplet surface area assuming a smooth surface. However, if the surface is not smooth, but generally follows a spherical cap geometry, as shown in Figure 65, the area of that surface will be greater than the spherical cap geometry predicts. In that case the $w$ determined from the experimental temperature data should be smaller. Further calculations which support an irregular surface explanation are in Appendix F.
Figure 65  Droplet shaper for ideal (left) and possible realistic larger surface area (right) will lead to underestimated effective surface area if a perfect spherical cap geometry is assumed when the real surface area has ridges, peaks, and valleys.
5.3 SPACING PARAMETER MODEL

With the accommodation coefficient determined, the prediction of the volume to surface ratio of deposited drops shall be addressed to complete the modeling of predicting the disc cooling. With the known pulse duration and the known mass flux, there will be a corresponding liquid film of thickness “δ”. Assuming the film will evolve into spherical cap shaped drops with an average size and presence with a specific pattern of drop arrangement, the average size of the drops can be determined. Accordingly, the volume to surface ratio “\( w_0 \)” can be obtained. In this section a model is established to give the drop pattern on the surface.

To calculate a mean droplet diameter that is appropriate to characterize an impacting spray, first assume a uniform film of thickness \( δ \) over the wetted area of the surface. This film thickness is calculated by multiplying the mass flow rate of the nozzle times the pulse duration, then multiplying by the deposition water utilization efficiency, and then dividing by wetted area.

\[
\delta = \frac{V_p}{A_{\text{disc}} P_{\text{wett}}} \quad (29)
\]

where for the present study:

\[
V_p = \left( 0.00025 \Delta t_p + 0.000003 \right) \eta / \rho_{\text{ice}}
\]

\[
A_{\text{disc}} = 0.000506707 \, m^2
\]

\[
P_{\text{wett}} = 0.84
\]

\[
\rho_{\text{ice}} = 920 \, kg/m^2
\]
\[ \eta = 0.32 + 175 (\Delta t_p)^{0.273} \]

Then assume that the uniform film changes into a regular hexagon packing pattern of spherical caps, as shown in Figure 66, with a specified contact angle and radius “a”. The hexagon packing pattern was chosen because it is the least ordered, most random, and therefore most likely. Also, the hexagon packing pattern is the most compact of all the packing patterns, triangle, square, etc. If the droplets’ edges touch, the spacing is zero. Establish a spacing parameter, n, that gives the distance between the centers of drops divided by the diameter of the drops in a hexagon packing pattern. As shown in Figure 66, each droplet is then surrounded by a hexagon of half-chord length, n a. For example, if the droplets all touch edges, n = 1.

If there is a small space between the droplets, the n will be larger than 1. As mentioned earlier with respect to the analytical model developed in this research, this spacing parameter model applies best for shorter pulse durations with a small percentage of the droplets’ mass as puddles. This is a good approach, because the experiment and model show the highest heat transfer rates with smaller droplets. Since the greatest interest is in producing a cooling system with the highest heat transfer rates, long pulse durations but they are of lesser interest in reality anyway.
Figure 66 Spacing Parameter "n"
To determine the n spacing parameter, we shall recall the equation (28), which relates the cooling time constant $\tau$, accommodation coefficient $\alpha$, and the volume to surface ratio $w$. Experimentally, we know the $\tau$ and previously determined $\alpha$. Therefore, the $w_0$ can be evaluated. The $w$ is the volume to surface ratio, which is controlled by the known contact angle and the radius of the base area “a”. Therefore “a” can be found. The “a” also determines the volume of the drop. However, consider that the volume conservation of a spherical cap comes from a hexagonal prism of deposited water layer. For one of the hexagonal cells shown in Figure 66, the hexagonal base area multiplied by the film thickness, $\delta$, equals the volume of the drop in the hexagon. Since the film thickness, $\delta$, is known from the equations developed earlier in this section, the hexagonal base area is determined. However the base area is dependent upon the “a” and “n”. With a known “a”, the “n” can be found. All these equations can be put together in a reverse order as shown in the following sequence to solve for the “n” parameter:

The volume of a spherical cap shaped drop as a function of contact angle and base area radius “a” is shown in equation (30):

$$V_{drop} = \frac{a^3 \pi (2 - 3\sin[\theta_M] + \sin[\theta_M]^3)}{3(-1 + \frac{2}{1 - \sin[\theta_M]}^{3/2}(1 - \sin[\theta_M])^3)} \tag{30}$$

where

$$\theta_M = \frac{\pi}{180}(90 - \theta_C) \tag{31}$$
The volume of water over a hexagon base area is shown in equation (32):

\[ V_{\text{hexagon}} = \delta \ 3.462 \ (n\ a)^2 \] (32)

The volume of water over a hexagon is conserve and transformed into a spherical cap shaped drop:

\[ V_{\text{drop}} = V_{\text{hexagon}} \] (33)

Substituting equations (30) and (32) into equation (33):

\[ \frac{a^3 \pi (2 - 3 \sin[\theta_M] + \sin[\theta_M]^3)}{3(-1 + \frac{2}{1 - \sin[\theta_M]^{3/2}})} = \delta \ 3.462 \ (n\ a)^2 \] (34)

Solving equation (34) explicitly for “n”:

\[ n = \frac{0.54 \left[ a \left(2.09 - \pi \sin[\theta_M] + \sin[\theta_M]^3\right)\right]}{\sqrt{\frac{1}{1 - \sin[\theta_M]^{3/2}} - 1} \ (\sin[\theta_M] - 1)^2 \ (\sin[\theta_M] + 1)} \] (35)

and “w_0” relates to a spherical cap radius “a”:

\[ w_0 = \frac{a \left(2 - 3 \sin[\theta_M] + \sin[\theta_M]^3\right)}{3(-1 + \frac{2}{1 - \sin[\theta_M]^{3/2}}) (1 - \sin[\theta_M])^3} \] (36)

The \( \tau \), \( T_{\text{droplet}} \), and \( \alpha \) in the following equation are all known:
\[
\tau = \frac{w_0 \rho_{ice}}{2 \alpha \left( \frac{M_w}{2\pi R} \right)^2 \left( \frac{(23.036 - \frac{T_{droplet}}{333.7}) T_{droplet}}{279.82 + T_{droplet}} \right) \left( \frac{611.15e}{T_{droplet} + 273.15} \right)^{1/2}}}

(37)

In equation (37), \(\tau\) is know from experimental temperature versus time data.

In summary, since the cooling time constant is known experimentally, the spacing parameter \(n\) can be found. The Mathematica notebook with calculations and details of finding the \(n\) values are included in the Appendix G.

It appears that these \(n\) versus film thickness values may apply to any spray, because they are only dependent upon the amount of deposition, but not the nozzle type or the model. This enables general applicability of the model.

The spacing parameter \(n\), predicted from the present model, can be compared with the experimental observation of drop pattern. The outline plots of the deposited drops produced by ImageJ are shown in Figure 67 through Figure 69. Within each plot caption is the spacing parameter, \(n\), obtained from the present spacing parameter model. A qualitative visual examination of these plots and comparison to the respective derived spacing parameter appears to show an acceptable confirmation of the apparent average spacing between the droplets.
Figure 67 ImageJ Droplet Outlines 8 ms, n= 1.9

Figure 68 ImageJ Droplet Outline 28 ms, n = 1.45

Figure 69 ImageJ Droplet Outline 49 ms, n = 1.25
A curve can be drawn to fit the three discrete points of spacing parameter “n” versus film thickness “δ” that were derived from the experiments as shown in equation (38). Three different pulse durations produce three discrete values each for: spacing parameter “n”, droplet radius “a”, time constant “τ”, and equivalent film thickness “δ”.

\[
  n = \frac{0.458 \sqrt{0.000077 + 5 \delta}}{\sqrt{\delta}}
\]  

Figure 70 Plot of the spacing parameter "n" versus the equivalent film thickness “δ”, where δ is derived from the nozzle flow rate, the pulse duration, the deposition efficiency, and the wetted area.
5.4 COOLING PROCESS – HEAT TRANSFER PREDICTIVE MODEL

The complete procedure of evaluating the spray cooling in vacuum described in this study can be summarized here with the step by step evaluations. For any spray pulse, there exists an equivalent deposition film thickness based on pulse duration. Use the following steps to calculate the cooling process of the deposited drops. In Chapter 4, the parameter “n” was found for three different pulse durations: \( n = 1.9, 1.45, \) and \( 1.25 \) for pulse durations of \( 8 \) ms, \( 28 \) ms, and \( 49 \) ms, respectively. The longer pulse durations produce a more crowded droplet pattern and therefore smaller spacing between drops, as represented by a smaller “n” spacing parameter.

1. With the film thickness known by calculations considering nozzle flowrate and pulse duration, and with the “n” value known from the curve of equation (38), use the following relation to determine the “a” value, the drop base radius, for the general case of the contact angle, \( \theta_c \):

\[
n = \frac{0.54 \sqrt{a (2.09 - \pi \sin[\theta_M] + \sin[\theta_M]^3)}}{\sqrt{1 - \sin[\theta_M]^2}} \frac{1}{\sqrt{\sin[\theta_M]^2 - 1} (\sin[\theta_M]^2 - 1)^2 (\sin[\theta_M] + 1)}
\]

(39)

where the Mathematica spherical cap internal angle, \( \theta_M \), is related to the fluids-thermal contact angle, \( \theta_C \), by the following equation:

\[
\theta_M = \frac{\pi}{180} (90 - \theta_C)
\]

(40)

2. Estimate the “\( w_0 \)”, the spherical cap volume to surface area ratio, by using the “a” value found in step 1.
\[ w_0 = \frac{0.397 \alpha (2 - 3\sin[\frac{\pi}{180}(90 - \theta_C)] + \sin[\frac{\pi}{180}(90 - \theta_C)]^3)}{(-1 + \frac{2}{1 - \sin[\frac{\pi}{180}(90 - \theta_C)]})^{3/2} (1 - \sin[\frac{\pi}{180}(90 - \theta_C)])^3} \]  

(41)

3. Estimate the τ value, the time constant, using the following equation (42) with \( \alpha = 0.036 \) as the accommodation coefficient, the “\( w_0 \)” value found in step 2, and the initial surface average temperature as \( T_{\text{droplet}} \).

\[ \tau = \frac{w_0 \rho_{\text{ice}}}{2\alpha \left( \frac{M_w}{2\pi R} \right)^5 \left( \frac{611.15 e^{\left( \frac{23.036}{333.7} T_{\text{droplet}} - \frac{279.82}{T_{\text{droplet}}} \right)}}{(T_{\text{droplet}} + 273.15)^2} \right)} \]  

(42)

4. Use the total water deposition, surface characteristics, and latent heat to find the total temperature change of the surface upon cooling.

\[ T_i - T_f = \frac{h_{fg} m_p}{M c_p} \]  

(43)

5. Apply the time constant found from equation (42) and total temperature change to get the prediction of temperature versus time.

\[ T = (T_i - T_f) e^{-t/\tau} + T_f \]  

(44)

The surface temperature, \( T \), can be scaled to the difference in initial and final temperatures with the following equation:
\[ T^* = \frac{T - T_f}{T_i - T_f} = \frac{T - T_f}{\frac{h_{fg} m_p}{M c_p}} \]  

(45)

Scaled by the expression in equation (45), equation (44) becomes:

\[ T^* = e^{-t/\tau} \]  

(46)

The heat transfer is proportional to the differential of equation (46) with respect to time:

\[ \frac{dT^*}{dt} = -\frac{1}{\tau} e^{-t/\tau} \]  

(47)

The surface-averaged heat flux is the derivative of (44) multiplied by \( M c_p / A_{disc} \):

\[ q'' = \frac{M c_p}{A_{disc}} \frac{dT}{dt} = -\frac{M c_p}{A_{disc}} (T_i - T_f) \frac{1}{\tau} e^{-t/\tau} \]  

(48)

A non-dimensional heat flux is possible with the following scaling:

\[ q''^* = \frac{\tau A_{disc}}{h_{fg} m_p} q'' \]  

(49)

Substituting (48) and (43) into (49):

\[ q''^* = -e^{-t/\tau} \]  

(50)
5.5 UNCERTAINTY ANALYSIS

The uncertainty analysis follows standard rules for error propagation as outlined in Taylor [61]. The modeling centers on the variable $\tau$, the time constant. Determining the error in $\tau$ essentially determines the important error in the temperature history prediction.

5.5.1 THE UNCERTAINTY OF ACCOMODATION COEFFICIENT

The experimental model estimates the accommodation coefficient, “$\alpha$”, and has an associated error band with that value. Second, the spacing parameter model estimates the spacing parameter, “$n$”, and also has an error band. Finally, the heat transfer predictive model estimates the $\tau$, the time constant of an exponential decay curve, and also has an associated error.

The expression for $\tau$ is:

$$\tau = \frac{\rho \, w_0}{2\alpha \left( \frac{\text{Molecularweight}}{2\pi R} \right)^5 \left( \frac{611.15e^{\left( \frac{23.036 - \frac{T}{333.7}}{279.82 + T} \right)}}{(T + 273.15)^{\frac{5}{2}}} \right)}$$

Following the “general formula for error propagation” outlined in Taylor [61], the equation for $\tau$ is differentiated with respect to all the variables that have uncertainty. Then, each of those expressions is multiplied by the uncertainty, or estimated change in the value due to uncertainty. They are then all added in quadrature. The details of the error calculations are included in the Appendix H.

For the evaluation of accommodation coefficient, the values of $\tau$, $w$, $\rho$, and $T$ are known. The goal is to find “$\alpha$”, the accommodation coefficient. A brief rationale for the estimated uncertainty of the input parameters used in determining the uncertainty in “$\alpha$” is listed here.
δT=1.0 °C

This is the uncertainty in the temperature. One standard deviation of the temperature error is approximately +/- 1 °C, since the scatter plot data appear to show a dispersion of about +/- 2 °C maximum. Per error analysis convention, the error then is perhaps one standard deviation, which would be about 1° C.

δρ =50 kg/m³

This is the uncertainty of the density of ice. The density of ice is fairly well known, but some of the ice could be water that has not yet frozen. The density of ice is reported as 920 kg/m³ [43]. Some of the droplet could be at some point partially water, and have a density of 1000 kg/m³. The calculation of 67% (one standard deviation) between 920 kg/m³ is 1000 kg/m3 is approximately 50 kg/m³.

δτ=.002 seconds

This is the uncertainty in τ which is calculated from the experimental temperature history in Chapter 4 and is a result of the scatterplot of temperature points with time. Igor software’s least squares curve fitting algorithm reported the largest uncertainty for the curve fit as 0.002 seconds.

δw= 0.2 w

This parameter is the volume/area ratio. This uncertainty is taken as 20 % of the w value. This unknown arises primarily from the uncertainty in how ImageJ resolves the surface area of the droplet. The chosen thresholding algorithm shows the mass resolved by ImageJ as 32% higher than the mass calculated by considering the disc temperature change. Also, the droplet could have more sublimating area than assumed with a simple spherical cap because of waves or other surface irregularities. These would effectively increase the sublimating area but not
significantly increase the droplet volume. Thus, the estimate of the droplet volume may be “correct” while the area is underestimated. This would cause uncertainty in the “w” parameter even though the uncertainty in the volume may be very small. One standard deviation of 0.32 is approximately 0.2.
5.5.2 UNCERTAINTY IN THE SPACING PARAMETER MODEL

From the spacing parameter model, the values of spray pulse duration and water utilization efficiency are known. With the uncertainties in these parameters, the uncertainty in “n” can be estimated.

\[ \delta \text{ spray pulse duration} = 0.0005 \text{ s} \]

This was illustrated in Figure 33 when the estimate of pulse duration was presented. The frame rate is 1 ms. The pulse could start anywhere in between frames; thus, +/- 0.0005 seconds is the estimated uncertainty.

\[ \delta \text{ water utilization efficiency} = 0.05 \]

The water utilization efficiency varied from about 46% to 75% as shown in Figure 37. The power law fit is within +/- 5%.

5.5.3 UNCERTAINTY IN THE PREDICTIVE HEAT TRANSFER MODEL

For the final predictive heat transfer model that uses the “n” fit curve and other previously listed known parameters, the final uncertainty in \( \tau \) can be estimated. The final uncertainty in predicting the \( \tau \) is estimated here and would be used with the final predictive heat transfer model. Figure 71 shows how the predictive model developed in this present research predicts the current test data, including error uncertainty bands developed in this section.
Figure 71 Uncertainty Analysis in the Predictive Model for the Time Constant
Figure 72 Time constant versus pulse duration shows additional data points which support uncertainty analysis and repeatability of data.
5.5.4 UNCERTAINTY IN DATA REDUCTION FOR THE TEMPERATURE HISTORY

The major cooling of the disc is near the edge, and the central portion of the disc is not cooled very much. During the cooling process, heat comes from the central portion toward the disc edge to provide extra heat. As a result, a temperature gradient occurs. For a cylindrical geometry, the temperature profile can be considered as the standard logarithm profile of the steady heat transfer across the wall of a hollow cylinder. For the present case, the radius of the inner wall of the hollow cylinder will be at the midpoint of the radius of the dry portion at the center of disc, because this is the representative location or the heat origin within the dry center portion.

The assumed radial temperature profile used in the present research data reduction is linear. There is no direct evidence that this simple temperature profile exactly fits the true temperature profile. However, since the temperature difference between the two thermocouples is small, assuming a different temperature profile should result in a small change in the resulting calculation of the time constant \( \tau \). In order to test the sensitivity of the results to the assumed temperature profile, we can assume a logarithmic temperature profile, recalculate the average disc temperature, and then fit the new scatterplot data with a new exponential solution and deduce the new time constant \( \tau \). We can then compare the original \( \tau \) value of 0.152 seconds for 8 ms pulse to the new exponential solution \( \tau \) value calculated by assuming a logarithmic temperature profile. We know that the one dimensional, steady state conduction for radial heat flow in a hollow cylinder has a natural logarithmic temperature distribution [1]:
\[ T(r) = T_{Edge} + \frac{\ln \frac{r}{R_1}}{\ln \frac{R}{R}} (T_{Center} - T_{Edge}) \]  \hspace{1cm} (52)

where \( r_1 \) is the radial midpoint of the dry portion of the disc. Since 84\% of the disc is wet, and the disc radius, \( R \), is 0.0127 m, this midpoint, \( r_1 \) = 0.00254 m.

Define an average disc temperature based on the total area:

\[ T_{Average}A_{total} = \int_0^R T(r) 2 \pi r \, dr \]  \hspace{1cm} (53)

Substitute equation (3) into equation (4)

\[ T_{Average} = \frac{\int_0^R (T_{Edge} + \frac{\ln \frac{r}{R_1}}{\ln \frac{R}{R}} (T_{Center} - T_{Edge})) 2 \pi r \, dr}{A_{total}} \]  \hspace{1cm} (54)

Perform the integration across the disc, from 0 to \( R \), with \( r_1 \)=0.00254, and \( A_{total} = \pi R^2 \):

\[ T_{Average} = \frac{R^2 (1.57 T_{Center} + 27.36 T_{Edge}) + R^2 (\pi T_{Edge}) \ln[R]}{\pi R^2 (9.21 + \ln[R])} \]  \hspace{1cm} (55)

Simplify with \( R=0.0127 \):

\[ T_{Average} = T_{Center} + .69 (T_{Edge} - T_{Center}) \]  \hspace{1cm} (56)

This compares to the disc average temperature that assumes a linear temperature profile:

\[ T_{Average} = T_{Center} + \frac{2}{3} (T_{Edge} - T_{Center}) \]  \hspace{1cm} (57)
The data reduction for a scatterplot that uses a logarithmic profile to average the two thermocouples shows a $\tau = 0.150$. The original $\tau = 0.152$ for the linear temperature profile assumption. The difference is 0.002 seconds and is negligibly small.

Since the both the linear and logarithmic temperature profile assumptions lead to about the same final model prediction, a better estimate of the error in data reduction of the experimental temperature data is desired. An additional temperature profile might be used to represent the average disc temperature, in order to assess the uncertainty in the final model prediction. Consider that the center thermocouple is always dry and therefore always higher in temperature. Also, the edge thermocouple always has some water deposition and therefore is always lower in temperature than the center thermocouple. However, the portion of the disc with the lowest temperature is in between the edge and center thermocouples, because this is the portion of the disc with the most water deposition. Consider that the edge thermocouple, therefore, might be a good estimate of the average disc temperature, since it is in between the center temperature and the lowest disc temperature. In reality, we do not have any temperature measurement in the portion of the disc that is most covered with water; that portion of the disc in between the edge and center thermocouples. However, the edge temperature location might be a good estimate of the disc average temperature. The data reduction for a scatterplot that uses only the edge thermocouple shows a $\tau = .119$. Compare this to the original $\tau = 0.152$ for the linear temperature profile assumption. The difference is 0.033 seconds and is a small difference.

To determine a resulting uncertainty bar in the final predictive model, we can use the following method:
1. Only the edge thermocouple temperature is used to generate a temperature history scatterplot.

2. A least squares curve fit is used to get a new experimental data reduction time constant $\tau$.

3. Use the difference between the baseline $\tau$ and this new $\tau$ as the uncertainty in the experimental data reduction time constant, the “$\delta\tau$”.

With this method, the uncertainty analysis considers the uncertainty in the time constant as 0.033. The “delta $\tau$” value is $\delta\tau=0.033$ seconds, because the baseline $\tau$ and this new $\tau$ arising from this sensitivity analysis differ by about 0.033 seconds. A lower value of $\delta\tau$ is available from the *Igor* software which provides the least squares curve fit. If just the $\delta\tau$ from the *Igor* error involved in calculating a least squares curved fit, $\delta\tau=0.002$, were included as the input to the uncertainty analysis, the total error bar value would be 0.07 seconds for the uncertainty in final model prediction of $\tau$. But by increasing the $\delta\tau$ value to 0.033, the final model prediction of $\tau$ is 0.085. In fact, this $\delta\tau$ is only one part of the overall uncertainty for the final model prediction error bar, and the presence of many other errors makes this particular effect small. The 8 ms case is provided as an example here. The other pulse duration error bars are established with the same method. Considering this edge thermocouple method of estimating the uncertainty in $\delta\tau$, the final error bars shown in *Figure 72* are 0.085, 0.095, 0.12, 0.0125 and 0.13 rather than previous value of 0.07, 0.085, 0.10, 0.11, and 0.12 for pulse durations of 0.08, 0.20, 0.28, 0.040, and 0.49 seconds.
6.0 DESIGN AND APPLICATION

The chapter is to demonstrate how the research results might be used in an application of spray cooling a surface in vacuum. The surface to be cooled might be a component in a spacecraft, a laser diode, or other electronics with time-dependent heat generation, or a liquid heat exchanger. To demonstrate the use of the research results, a constant or variable heat generation source within the copper disc is simulated using a finite different numerical model. There are many commercially available thermal solver software packages available, such as ANSYS, FEMAP, and Thermal Desktop SINDA. For this demonstration, Thermal Desktop SINDA was chosen.

6.1 APPLICATION

Assuming repeated pulses will produce the same results as the repeated single pulse, the temperature of a surface that has a steady state heat generation can be held relatively constant over time, if both the pulse duration and timing between pulses are chosen that each pulse becomes independent. The following demonstration takes the current copper disc and applies a heat generation source within the copper disc. This heat generation source might be, for example, the waste heat from a CPU. With no cooling, heat dissipation will cause the CPU temperature to rise. With intermittent spray cooling, the CPU temperature will remain within a small temperature band. The sublimating water will absorb the heat. The vacuum system pressure could possibly maintain the CPU or other electronics near the freezing temperature of water. Since the latent heat of water is relatively high when compared to other working fluids, cooling with water allows for an efficient use of mass and pumping power needed to move the working fluid. Whether this type of cooling is optimum for a
particular application depends on the details and requirements of this application. The few potential demonstrations in this chapter are not meant to be a complete list of possible applications.

As demonstrated in this research, the model can estimate the time constant, $\tau$, from knowledge of the spray mass flux, together with an experimentally determined spacing parameter “n” that varies from 1.25 to 1.9. A surface with a constant low heat flux might be able to use a longer pulse, because a high time-averaged heat transfer coefficient is not necessary. A surface with a high heat flux will need many successive short pulses, because a high time-averaged heat transfer coefficient is needed. In both cases, the disc temperature will decrease at first, and then recover to its original temperature, as the water disappears. This cycle will repeat with each pulse. The maximum and minimum surface temperature will occur within a specific band. Some surfaces to be cooled might require a very small temperature band, but not necessarily a high time-averaged heat transfer coefficient. An example might be a laser diode cooling application, where the wavelength varies as the temperature varies. In a laser diode application, the goal is to limit the wavelength variation. Another application might be electronics cooling where a large cyclic temperature band might limit the life of the hardware due to structural fatigue. In those cases, a short pulse duration combined with a short waiting period between pulses would be appropriate. In fact, the type of heat dissipation might be a combination of all these mentioned, and depend on the operating mode of whatever is being cooled. For example, the CPU temperature is a function of the intensity of CPU calculations. Intermittent spray cooling, combined with an intelligent way to modulate the pulse duration and waiting period, might be an ideal cooling solution for such applications.
In addition, limitations of the hardware might bound the problem if, for example, the nozzle solenoid cannot generate pulses shorter than 8 ms (as is the case with the nozzle solenoid used in this research). The nominal setting for open/close solenoid in this research was 20 ms. However, due to delay in the electrical signal and inductance of the magnetic solenoid, the real observed open/close spray pulse duration was 8 ms when the Programmable Logic Controller was set to a nominal duration of 20 ms. For a nominal setting of 10 ms, there was no observed spray at all.

For an ensemble of droplets on a disc at the start of sublimation, if a mean value of volume/area, which is the \( w \) parameter, can be obtained, then the present model can estimate the time constant. Further, since the amount of water on the disc is known, the eventual total disc temperature change can be determined. Then, by differentiating the temperature history exponential curve, the heat flux history can be determined. A time-averaged heat flux is obtained by integrating the heat flux over the total cooldown time; that is, the time it takes for all the frozen water to disappear. For a practical solution to total cooldown time, an end time value corresponding to 95% of the total temperature change was used. At that point, the next spray pulse could occur. The assumption is that if 5% of the water is remaining on the disc, it could easily be swept away by the next pulse. In this way, there would be no build-up of water.

6.2 TEMPERATURE EXCURSION VARIATION

The analytical procedure outlined in the previous chapter can be used to produce temperature histories of the copper disc that extend beyond the experimental results. In addition, since the copper disc is represented by an “M”, total mass, and a “\( c_p \)”, specific heat, then any other values for these parameters could be used to represent a different material.
Figure 73 shows a plot of variation in pulse duration generated by the stated computation procedure using SINDA.

The pulse duration is set to 1 ms with the assumption that the results of the present research apply to a pulse duration outside the present research experimental results. The SINDA results show that in that case, the time for disappearance of 95% of the frozen water is .219 seconds, making the total cycle time 0.22 seconds. The pulse repeats every 0.22 seconds. It is assumed that a small amount of residual frozen droplets, 5%, could be easily swept away by the next pulse.

For the longer pulse duration of 8 ms, the wait time between successive pulses is 0.492 seconds. The cycle time is 0.5 seconds. The temperature excursion during the cycle is greater than in the 1 ms discussed in the previous paragraph.

For a long 49 ms pulse with a wait time of 1.58 seconds, the total cycle time is 49 ms + 1.58 seconds = 1.63 seconds. The large temperature excursion is evident in Figure 73.
Figure 73 Application of Spray Cooling Shows Trade-Off between Higher Heat Flux and Smaller Temperature Swing

6.3 HEAT FLUX RESULTS COMPARISON

Typical heat transfer rates for this type of spray cooling appear to be low compared to heat transfer rates reported in the literature for spray cooling at higher system pressures. Vacuum spray cooling appears to yield heat flux values in the range of 1 to 5 W/cm². At higher system pressures, other researchers have reported much higher heat flux values. Mudawar reported heat fluxes of 10 to 100 W/cm² while investigating spray cooling near the critical heat flux for FC-72 [62], Rini reported results for boiling spray cooling near 40 W/cm² for FC-72 [63], Yao and Choi reported heat flux values near 100 W/cm² for spray cooling in the film boiling regime [64], and Panao reported 8 to 24 W/cm² for HFE-7100 for intermittent spray cooling [65]. This is likely due to the difference that at atmospheric or higher pressure environment, the heat transfer comes from evaporation or boiling of liquids.
which has a high vapor pressure. In vacuum, the heat transfer comes from sublimation of ice solid where the vapor pressure is very low. As a result, the rate of spray heat transfer in vacuum is lower.
7.0 FUTURE WORK

The use of water for cooling was important for several reasons. Water is an undeniably safe working fluid for human health. It can evaporate or sublimate over a range from room temperature to water freezing, which is useful for many things: electronics cooling, desalination, and human environment temperature regulation. However, spray cooling in a vacuum may allow other fluids such as methane, nitrogen, or the common refrigerants (e.g. R-134a) to cool at temperature much below what has been demonstrated here for water. Experiments of spray cooling in a vacuum with these other fluids would possibly validate this model and lead to cooling over a wider range of temperatures.

The microgravity operation of spray cooling has been demonstrated by the Flash Evaporator System (FES) aboard the Space Shuttle Orbiter for years. However, the nominal operation has always been slightly above the triple point of water. When the temperature accidentally dropped below the freezing point, a failure mode could have occurred whereby the system was allowed time to recover, while waiting for the ice to slowly sublimate. In this research, it has been shown that operation below the triple point is a valid operational mode and is understood. It remains to demonstrate the experimental work here again, but in a microgravity environment. Since the frozen droplets “stick” to the substrate being cooled, the operation in this temperature and pressure range should be gravity independent. There appears no reason why this would not work the same in a microgravity environment as in a 1-g environment. The experimental hardware developed here could be slightly modified to allow operation in a 2 second drop tower. The disc temperature history created by the spry pulse is finished within about 2 seconds, which fits within the microgravity duration allowed
by a 2 second drop tower. Microgravity testing would demonstrate that this vacuum spray cooling approach is gravity independent.

In this research a specific spray nozzle was used to maintain uniformity over all the experiments. If a different type of droplet generator, perhaps a piezoelectric mono-disperse droplet generator, could be made to work in a vacuum, very small drops might be evenly distributed over the surface. In this case, an even more effective water cooling system might be possible.

The camera resolution and ability to define a true three dimensional picture of the deposited droplet pattern was limited. If two cameras could focus on the disc, and if specialized software could then create a more accurate three dimensional image of the deposited drops, it may be possible to expand the video data for further analysis.

The PDPA is a capable instrument that can characterize a spray in terms of droplet velocity, number and size. It is possible to measure a spray in vacuum, but this would require the laser beams passing through two windows, as discussed in Chapter 3. Since this expertise needed to do this is more in the area of optical diagnostics, collaboration between those with expertise in evaporative heat transfer in a vacuum and PDPA optical diagnostics might produce a team to investigate this type of measurement. Although not needed to perform this research described in this thesis, the knowledge of how a spray varies from vacuum to standard atmospheric pressure would be of interest to the spray community.
8.0 CONCLUSION

For sprays or any droplet deposition system that can generate discrete small droplets on a surface, this research provides an accurate predictive model to estimate the heat transfer. For a system that includes partial flooding of the water on the surface, this research provides a reasonable estimate of heat transfer for this condition as well. For a system that generates total flooding or a film, this model is not applicable. Further, the concept of an average heat flux should be carefully considered. For applications involving many highly non-uniform or many localized discrete heat sources, the present model may not be applicable. In any event, the research shows the more desirable operating point is small discrete droplets, when considering the most efficient use of a working fluid, for a water spray in a vacuum cooling a substrate at near room temperature.

The major accomplishments from this research are:

1. This research provides the first attempt at a qualitative and quantitative understanding of spray cooling at low system pressures below the triple point of water, where water will freeze. To the best of the author’s knowledge, there have been no published data on spray cooling for system pressures well below the triple point of water. Many researchers have reported on spray cooling at system pressures ranging from just above the triple point of water to very high system pressures. This investigation of a water spray in vacuum is the first ever such study. This is the first time to acquire data of spray cooling at these conditions. This research proves it is possible to successfully perform spray cooling at vacuum conditions.
2. This is the first mechanistic model of the sublimation process and subsequent comparison to experimental data. With this modeling approach, an estimate of heat transfer under these conditions is now possible, whereas prior to this research, there was no engineering method. This work has resulted in a predictive model which can estimate heat transfer for a pulsed spray to remove heat from a steady state heat source. Input parameters needed by this model are available without requiring prior testing or characterization of the spray.

3. This experimental data shows that a spray behaves very differently in a vacuum. First, the cone angle change with pressure does not follow the same relationship as the cone angle change at high system pressures. Second, droplet freezing does not occur immediately upon exiting the nozzle, but requires a finite amount of time. Video data from these experiments and a compatible model both show that drops in the spray appear to behave like a liquid in that they form spherical cap-shaped drops after landing on the surface. Third, startup and shutdown behavior seen by in-air high speed video, the “tail” and “leading edge” effect, is absent in a vacuum. Consequently, this effect will give an advantage for a pulsed spray in vacuum, whereas it will limit the performance of intermittent spray cooling in air.

4. The pulse duration is a very important parameter. Short pulses allow spherical cap-shaped frozen drops to deposit discretely on the surface, while long pulses can cause odd-shaped puddles, in addition to the small spherical cap-shaped drops. The short pulses are more desirable, because the odd-shaped puddles that occur with longer pulses lead to a slower cooldown process as seen in the present research as a longer time constant. A system with a short pulse duration capability is a priority.
REFERENCES


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Http://Demonstrations.Wolfram.Com/Filteringawhitenoisesequence/


[34] Groeblacher, H., 1996, "Method of and Apparatus for the Cooling of Extruded Plastic Profiles or Sections", Patent# 5484557,


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Excel Macros to convert DAQ output text file into an excel spreadsheet with temperature versus time plots.

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    Range("A2").Select
    ActiveCell.FormulaR1C1 = ":=R[-1]C+1/6400"
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    Selection.Copy
    Range("A3").Select
    Range(Selection, Selection.End(xlDown)).Select
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    Application.CutCopyMode = False
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    Selection.NumberFormat = "0.00000000"
    Range("H1").Select
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    Range(Selection, Selection.End(xlUp)).Select
    ActiveSheet.Paste
    Application.CutCopyMode = False
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    Selection.End(xlUp).Select
    Columns("G:G").Select
    ActiveCell.Offset(0, -6).Range("A1:F1").Select
    Range(Selection, Selection.End(xlDown)).Select
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    ActiveCell.Offset(0, 7).Range("A1").Select
    Selection.End(xlUp).Select
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    Range("H1").Select
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    Range("H2").Select
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Range("H3").Select
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ActiveSheet.Paste
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ActiveCell.Offset(0, 7).Range("A1").Select
Columns("H:J").Select
Selection.Copy
Range("O1").Select
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:=False, Transpose:=False
Application.CutCopyMode = False
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ActiveWorkbook.Worksheets("Sheet1").Sort.SortFields.Add Key:=Range( _
"Q1:Q73666"), SortOn:=xlSortOnValues, Order:=xlAscending, DataOption:= _
xlSortNormal
With ActiveWorkbook.Worksheets("Sheet1").Sort
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 .Header = xlGuess
 .MatchCase = False
 .Orientation = xlTopToBottom
 .SortMethod = xlPinYin
 .Apply
End With
End With
Range("O1:Q10").Select
ActiveWorkbook.Worksheets("Sheet1").Sort.SortFields.Clear
ActiveWorkbook.Worksheets("Sheet1").Sort.SortFields.Add Key:=Range("O1"), _
SortOn:=xlSortOnValues, Order:=xlAscending, DataOption:=xlSortNormal
With ActiveWorkbook.Worksheets("Sheet1").Sort
 .SetRange Range("O1:Q10")
 .Header = xlGuess
 .MatchCase = False
 .Orientation = xlTopToBottom
 .SortMethod = xlPinYin
 .Apply
End With
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ActiveCell.FormulaR1C1 = "=MATCH(RC[-4],C[-11])"
Dim Copyrange As String
myvalue = Range("S1").Value
Startrow = 1
Lastrow = myvalue
Let Copyrange = "H" & Startrow & ":" & "J" & Lastrow
Range(Copyrange).Select
Selection.Copy
Range("T1").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
Columns("O:S").Select
Selection.ClearContents
Columns("T:V").Select
Selection.Cut
Columns("O:O").Select
ActiveSheet.Paste
Range("R1").Select
Range("S1").Select
    ActiveCell.FormulaR1C1 = "=AVERAGE(RC[-16]:R[19]C[-16])"
Range("T1").Select
ActiveCell.FormulaR1C1 = "=MIN(C[-3])"
Range("U1").Select
ActiveCell.FormulaR1C1 = "=RC[-2]-RC[-1]"
Range("U2").Select

Range("S1").Select
Selection.Copy
Range("S3").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
Range("T1").Select
Application.CutCopyMode = False
Selection.Copy
Range("S4").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
Range("U1").Select
Application.CutCopyMode = False
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Range("S5").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
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Range("S2").Select
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ActiveChart.Location Where:=xlLocationAsNewSheet
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ActiveChart.SeriesCollection(2).Select
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   .MarkerSize = 7
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Selection.MarkerStyle = -4118
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Selection.MarkerSize = 5
Selection.MarkerSize = 4
Selection.MarkerSize = 3
Selection.MarkerSize = 2
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Selection.MarkerSize = 6
Selection.MarkerSize = 5
Selection.MarkerSize = 4
Selection.MarkerSize = 3
Selection.MarkerSize = 2
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Range("H1").Select
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ActiveChart.Location Where:=xlLocationAsNewSheet
ActiveChart.ChartArea.Select
ActiveChart.SeriesCollection(2).Select
With Selection
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   .MarkerSize = 7
End With
Selection.MarkerStyle = -4118
Selection.MarkerSize = 6
Selection.MarkerSize = 5
Selection.MarkerSize = 4
Selection.MarkerSize = 3
Selection.MarkerSize = 2
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   .MarkerSize = 7
End With
Selection.MarkerStyle = -4118
Selection.MarkerSize = 6
Selection.MarkerSize = 5
Selection.MarkerSize = 4
Selection.MarkerSize = 3
Selection.MarkerSize = 2
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ActiveChart.SeriesCollection(1).Select
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    .MarkerStyle = 2
    .MarkerSize = 7
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Selection.MarkerStyle = -4118
Selection.MarkerSize = 6
Selection.MarkerSize = 5
Selection.MarkerSize = 4
Selection.MarkerSize = 3
Selection.MarkerSize = 2
Sheets("Sheet1").Select
Range("V1").Select
    Range("O:O,P:P").Select
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ActiveChart.Location Where:=xlLocationAsNewSheet
ActiveChart.ChartArea.Select
ActiveChart.SeriesCollection(1).Select
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    .MarkerSize = 7
End With
Selection.MarkerStyle = -4118
Selection.MarkerSize = 6
Selection.MarkerSize = 5
Selection.MarkerSize = 4
Selection.MarkerSize = 3
Selection.MarkerSize = 2
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Range("V1").Select
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Range(Selection, Selection.End(xlDown)).Select
Selection.Copy
Selection.End(xlDown).Select
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Range("R1").Select
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Selection.Copy
Range("R2").Select
Range(Selection, Selection.End(xlDown)).Select
ActiveSheet.Paste
Application.CutCopyMode = False
Range("O:O,R:R").Select
Range("R1").Activate
ActiveSheet.Shapes.AddChart.Select
ActiveChart.ChartType = xlXYScatter
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ActiveChart.SeriesCollection(1).Select
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  .MarkerSize = 7
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Selection.MarkerSize = 6
Selection.MarkerSize = 5
Selection.MarkerSize = 4
Selection.MarkerSize = 3
Selection.MarkerSize = 2
Sheets("Sheet1").Select
Range("U4").Select
End Sub
Appendix B

Details Of Converting An Imagej File To The Outline Plots

The ImageJ macros were written for each pulse duration.

**For the 8 ms pulse duration:**
makeLine(24, 804, 1624, 826);
run("Set Scale...", "distance=1600.15 known=0.0254 pixel=1 unit=m");
run("Invert");
makePolygon(68,774,1614,874,1618,806,64,840);
setBackgroundColor(255, 255, 255);
run("Clear Outside");
run("Measure");
setAutoThreshold("Moments");
setOption("BlackBackground", false);
run("Make Binary");
run("Watershed");
run("Analyze Particles...", "size=0-Infinity circularity=0.00-1.00 show=[Bare Outlines] display summarize add");

**For the 28 ms pulse duration**
makeLine(24, 804, 1624, 826);
run("Set Scale...", "distance=1600.15 known=0.0254 pixel=1 unit=m");
run("Invert");
makePolygon(68,774,1614,874,1618,806,64,840);
setBackgroundColor(255, 255, 255);
run("Clear Outside");
run("Measure");
setAutoThreshold("Shanbhag");
setOption("BlackBackground", false);
run("Make Binary");
run("Watershed");
run("Analyze Particles...", "size=0-Infinity circularity=0.00-1.00 show=[Bare Outlines] display summarize add");

**For the 49 ms pulse duration**
makeLine(24, 804, 1624, 826);
run("Set Scale...", "distance=1600.15 known=0.0254 pixel=1 unit=m");
makeLine(1510, 540, 1510, 540);
//setTool("polygon");
run("Invert");
makePolygon(44,774,1606,888,1602,786,50,880);
setBackgroundColor(255, 255, 255);
run("Clear Outside");
run("Measure");
setAutoThreshold("Shanbhag");
setOption("BlackBackground", false);
run("Make Binary");
run("Watershed");
run("Analyze Particles...", "size=0-Infinity circularity=0.00-1.00 show=[Bare Outlines]
display summarize add");

Each of the selected bowtie areas were adjusted slightly such that each included a similar number of droplets:

8 ms – 354 droplets using the Moments thresholding algorithm
28 ms – 362 droplets using the Shanbhag thresholding algorithm
49 ms – 353 droplets using Shanbhag thresholding algorithm

These algorithms were chosen because they produced the best agreement in the prediction of water mass on the disc, among all of the available thresholding techniques provided by ImageJ. The generality of ImageJ means that each user must decide which thresholding algorithm will work the best for the particular application. In the present research a comparison was made for all the thresholding techniques and then the best was chosen.
Appendix C

Details Of The Measurement Of The Mass Flow Rate

Details of the measurement of the mass flow rate are discussed here and the data presented in Table 3. A 10 ml graduated cylinder was placed around the nozzle such that all water from the nozzle deposited into the cylinder. The equation for the shorter sprays shows the SAC volume is about 0.003 grams of water. Although not exactly known due to the proprietary issues, the nozzle orifice and volume that might hold up the water appears to be 0.2 mm³, which is in agreement with 0.003 grams of water. The pressure gauge reading was 25 psig, which is 10 psid.

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Pulse Duration [seconds]</th>
<th>Grams</th>
<th>0.003</th>
<th>0.03</th>
<th>0.049</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
<td>10</td>
<td>2.5329</td>
<td>2.5329</td>
<td>0.0658</td>
<td>0.01</td>
<td>0.0155</td>
<td>0.01</td>
</tr>
<tr>
<td>Trial 2</td>
<td>10</td>
<td>2.4881</td>
<td>2.4881</td>
<td>0.0653</td>
<td>0.0099</td>
<td>0.0143</td>
<td>0.008</td>
</tr>
<tr>
<td>Trial 3</td>
<td>10</td>
<td>2.5574</td>
<td>2.5574</td>
<td>0.0641</td>
<td>0.01</td>
<td>0.0154</td>
<td>0.028</td>
</tr>
<tr>
<td>Trial 4</td>
<td>10</td>
<td>2.5664</td>
<td>2.5664</td>
<td>0.0647</td>
<td>0.01</td>
<td>0.0149</td>
<td>0.009</td>
</tr>
<tr>
<td>Trial 5</td>
<td>10</td>
<td>2.4986</td>
<td>2.4986</td>
<td>0.0658</td>
<td>0.01</td>
<td>0.0155</td>
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<tr>
<td>Trial 6</td>
<td>10</td>
<td>2.5083</td>
<td>2.5083</td>
<td>0.0641</td>
<td>0.01</td>
<td>0.0154</td>
<td>0.028</td>
</tr>
<tr>
<td>Trial 7</td>
<td>10</td>
<td>2.5247</td>
<td>2.5247</td>
<td>0.0658</td>
<td>0.01</td>
<td>0.0155</td>
<td>0.01</td>
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<tr>
<td>Trial 8</td>
<td>10</td>
<td>2.5134</td>
<td>2.5134</td>
<td>0.0641</td>
<td>0.01</td>
<td>0.0154</td>
<td>0.028</td>
</tr>
<tr>
<td>Trial 9</td>
<td>0.068</td>
<td>0.068</td>
<td>0.068</td>
<td>2.5134</td>
<td>4.97E-08</td>
<td>9.966E-06</td>
<td>1.523E-05</td>
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<tr>
<td>Trial 10</td>
<td>0.068</td>
<td>0.068</td>
<td>0.068</td>
<td>2.5134</td>
<td>4.97E-08</td>
<td>9.966E-06</td>
<td>1.523E-05</td>
</tr>
<tr>
<td>Trial 11</td>
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<td>0.068</td>
<td>0.068</td>
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<td>4.97E-08</td>
<td>9.966E-06</td>
<td>1.523E-05</td>
</tr>
<tr>
<td>Trial 12</td>
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<td>0.068</td>
<td>0.068</td>
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<td>4.97E-08</td>
<td>9.966E-06</td>
<td>1.523E-05</td>
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<tr>
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Table 3 Raw Data for Nozzle Flow Rate Determination
Figure 74 Trial-averaged nozzle flowrate data plotted as: blue diamonds are short pulses: 0.008, 0.028, 0.049 seconds and brown squares are the 10 second spray pulse (only 0.01 second duration shown here for compactness).
Appendix D

Details Of The Thermal Desktop SINDA Thermal Math Model

This describes the SINDA model used to predict the droplet cooldown in flight. A reasonable assumption is no circulation in the droplet such that a temperature gradient exists from the center to edge. It is further assumed there is no circumferential temperature difference. Since only a small amount of water is evaporated during flight, the assumption is that the process can be modeled by a constant diameter solid sphere. The Hertz-Knudsen equation for heat flux in the free molecular regime and the Arden Buck vapor pressure over ice relation are used to apply a negative heat flux on the solid sphere. In order to make the math model simpler, this heat flux was assumed constant and not temperature dependent. This would produce the worst case temperature decrease during the ~ 1 ms flight from the nozzle to the disc. This heat flux is calculated in Mathematica and then imported into SINDA as negative “heat load”. Although the droplet size distribution in vacuum is not known exactly, the SMD data taken in air may be able to be used as a reference. The SMD in-air data of 50 to 90 micron diameter indicates that perhaps analyzing a 50 micron diameter droplet would be representative of the spray. The temperature change during flight in smaller droplets will be greater than shown here, and for larger droplets, the temperature change during flight would be smaller than shown here. The droplet thermal math model parameters were \( k = 0.6 \text{ W/m K} \), \( c_p = 4186 \text{ J/kg K} \), \( \rho = 1000 \text{ kg/m}^3 \). The heat flux determined by the analysis in Mathematica was \(-310,933 \text{ W/m}^2\).

As shown in Figure 35 which was presented earlier on page 63, the 50 micron droplet does not reach freezing temperature during flight. From this analysis, we may conclude that it is probable the majority of the droplets probably do not freeze during flight. This is
consistent with observations of the video, because none of the droplets are seen bouncing from the surface. If any large droplets had frozen, bouncing would have been seen as streak paths in the video frames. However, since the water utilization efficiency is not 1.0, there was probably some bouncing of very small droplets that were too small to be detected by the camera. For the ImageJ analysis, the smallest reported area is 2.52 E -10 m^2. Multiplied by 3.156 to derive the true base area of the spherical cap droplet on the disc, the smallest reported droplet on the disc has a base area of 7.95 E -10 m^2 and a radius of 1.6 E -05 m. Considering spherical cap geometry and the conservation of mass when an spherical droplet impinges on the disc and transforms into a spherical cap, the smallest spray droplet able to be seen by ImageJ, and probably therefore the camera, is 18 microns in diameter. This discussion is for completeness and reference only, since the actual detection capabilities of the camera would require some complicated optics and opto-electronics understanding. The objective of the discussion is to argue that some of the droplets are too small to be detected by the camera. These could have frozen solid and therefore bounced away without leaving evidence of this from the video data. Figure 75 shows the outermost and center droplet temperatures from an additional SINDA analysis for a 10 micron diameter droplet. It shows a 10 micron diameter droplet does indeed freeze in less than the one millisecond time-of-flight from the nozzle to the disc. The following is a listing of Mathematica code to generate heat flux for droplet cooling.
Droplet Temperature Change during flight

Universal Gas Constant in Joules/ mol K

\( R = 8.3144621 \);  
Molecular weight of water in kg/mol

\( \text{Molecular weight} = 0.01801528 \);  
accomodation coefficient

\( \alpha = 0.036 \);

\( \text{hfg} = 2411.323 \);  
The following is the vapor pressure over ice

\[ P_{\text{sat}}[\text{mmHg}] = 611.15 e^{\left(\frac{\text{In}}{\text{mmHg}}\right)^\alpha} \]

\( T_{\text{droplet}} = 22 \);  

\( \text{Wattspersquaremeter} = \text{hfg} \frac{2 \alpha}{2 - \alpha} \left( \frac{\text{Molecular weight}}{2 \pi R} \right)^{0.5} \left( \frac{P_{\text{sat}}[\text{Tdroplet}]}{(T_{\text{droplet}} + 273.15)^{\frac{1}{2}}} \right) - \left( \frac{P_{\text{sat}}[-47]}{(47 + 273.15)^{\frac{1}{2}}} \right) \)

\( \omega \equiv 310.933 \).
Figure 75  Thermal math model droplet temperature during flight for outermost location and center location shows that for a 10 micron diameter droplet, complete freezing could possibly occur.
Appendix E

Knudsen Number and Free Molecular Flow Discussion

Throughout this present research, free molecular flow is assumed to be the heat transfer mode that occurs during sublimation of the water and ice. To verify this is valid, the Knudsen number is calculated based on the system pressure of 0.04 Torr.

\[
\text{Knudsen number} = \frac{\text{mean free path}}{\text{characteristic length}}
\]

The key parameter, the appropriate length scale, is somewhat arbitrary, or at least, application-based. For the hardware setup here, the length scale could be the tank diameter. This would be inappropriate, because the tank wall is relatively far away from the small disc is where the sublimation occurs. The length scale could be the 0.0254 disc, but even this is large compared to the droplet diameter, which is on the order of 0.0001 m. Since evaporation occurs on the surface of the droplet, the appropriate length scale is assumed to be 100 microns, which is much larger than the majority of the droplets’ base area diameter. From ImageJ, the very largest puddle seen in the longest pulse duration of 49 ms is 1.60 E -06 m². This translates to a base diameter of .0026 m. The very largest droplet base area diameter of the shortest pulse durations of 8 ms is 1.02 E -07 m², which translates to a base diameter of .00064 m. The criterion for continuum flow versus free molecular flow is generally cited as:

- Knudsen number > 10 Free molecular flow
- 0.01 < Knudsen number < 10 Mix of continuum and free molecular flow
- 0.01 < Knudsen number Continuum flow
To take a worst case, if the very largest puddle seen in the longest pulse duration of 49 ms is used as a characteristic length, the Knudsen number is 0.73. If the largest droplet base area in the 8 ms pulse duration is used, the Knudsen number is 3. However, for the case of the vast majority of the droplet sizes; for 100 microns, the Knudsen number is 20. This means that the Hertz-Knudsen equation applies and that free molecular flow is appropriate and the assumptions in the present research of the Hertz-Knudsen equation governing the heat transfer is valid.

Figure 76 Plot shows that most drops are within free molecular flow regime and all are very far from continuum flow criteria.
**Mathematica** Free Molecular Flow Calculations to Verify Kinetic Theory Approach to Sublimation Heat Transfer:

Boltzmann constant, \( k \)

Diameter of the gas particle in meters, \( d \)

Pressure in Pascal, \( p \)

Temperature in Kelvin, \( T \)

\[
\text{Torr} = 0.04; \\
p = \frac{101300 \text{Torr}}{760}; \\
\text{Out}[2]= 5.33158
\]

Diameter of a water molecule is 0.29 E-09

\[
d = 0.29 \times 10^{-9}; \\
k = 1.3806503 \times 10^{-23}; \\
T = 273.; \\
\text{MeanFreePath} = \frac{k T}{\sqrt{2} \pi d^2 p}; \\
\text{Out}[3]= 0.00189204
\]
Appendix F

Details of the Method to Calculate the Volume of Water and the “W” Parameter from The Video Data

Here are the details of the method to calculate the volume of water and the “w” parameter, the sum of volume/sum of the areas ratio, on the disc prior to the cooldown. Also included at the end of this appendix is an illustration of a drop showing surface irregularities that may explain the apparent increased surface area for the longer pulse duration of 49 ms.

ImageJ uses the macros presented in Appendix B to generate a list of all the droplets’ areas for each video frame. Of importance is the first steady frame just after the spray stops. At that point, the droplets are motionless because no spray is impinging. Also no additional spray will impinge on the disc, so that that volume observed at this point is the volume whose latent heat will cause the disc temperature change. Thus, the disc temperature change, known from the thermocouple measurement is known well and can be used reliably to calibrate or compare to the ImageJ prediction of the water volume at this point. In Figure 77, an example of the ImageJ output is presented. Imported into Excel, the ImageJ output is then transformed with macros and formulas to get an estimate of volume and area for the first frame. The volume of each droplet is calculated assuming a 47° contact angle.

\[ V_p = 0.1303883 \ A_{\text{base}}^{3/2} \] (59)

The volume of all droplets are summed and then multiplied by the ratio of the entire disc area to the bowtie area. Further, the ratio of the sum of the volumes to the sum of the areas is calculated. This is the “w” parameter.
Figure 77  The example of the ImageJ output from the video data shows the listing of frame number (946) and area measurement of each droplet [m²].
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Figure 79: ImageJ provides a listing of all droplet areas [m²] for the first frame for the 8 ms pulse.

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<td>1.06E-08</td>
<td>290</td>
<td>1.54E-08</td>
<td>324</td>
<td>2.27E-08</td>
<td></td>
</tr>
<tr>
<td>223</td>
<td>9.32E-09</td>
<td>257</td>
<td>5.04E-10</td>
<td>291</td>
<td>2.02E-09</td>
<td>325</td>
<td>4.03E-09</td>
<td></td>
</tr>
<tr>
<td>224</td>
<td>4.28E-09</td>
<td>258</td>
<td>2.52E-09</td>
<td>292</td>
<td>2.52E-10</td>
<td>326</td>
<td>1.11E-08</td>
<td></td>
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<tr>
<td>225</td>
<td>1.08E-08</td>
<td>259</td>
<td>2.02E-08</td>
<td>293</td>
<td>9.83E-09</td>
<td>327</td>
<td>9.07E-09</td>
<td></td>
</tr>
<tr>
<td>226</td>
<td>2.02E-09</td>
<td>260</td>
<td>3.28E-09</td>
<td>294</td>
<td>1.04E-08</td>
<td>328</td>
<td>1.76E-09</td>
<td></td>
</tr>
<tr>
<td>227</td>
<td>2.52E-10</td>
<td>261</td>
<td>2.02E-09</td>
<td>295</td>
<td>3.35E-08</td>
<td>329</td>
<td>2.52E-10</td>
<td></td>
</tr>
<tr>
<td>228</td>
<td>5.04E-10</td>
<td>262</td>
<td>2.02E-09</td>
<td>296</td>
<td>2.77E-09</td>
<td>330</td>
<td>6.55E-09</td>
<td></td>
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<tr>
<td>229</td>
<td>2.77E-09</td>
<td>263</td>
<td>1.51E-09</td>
<td>297</td>
<td>2.52E-10</td>
<td>331</td>
<td>1.18E-08</td>
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<tr>
<td>230</td>
<td>1.51E-09</td>
<td>264</td>
<td>2.52E-10</td>
<td>298</td>
<td>5.04E-10</td>
<td>332</td>
<td>2.52E-10</td>
<td></td>
</tr>
<tr>
<td>231</td>
<td>2.52E-10</td>
<td>265</td>
<td>4.03E-09</td>
<td>299</td>
<td>1.76E-09</td>
<td>333</td>
<td>3.28E-09</td>
<td></td>
</tr>
<tr>
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<td>1.01E-09</td>
<td>266</td>
<td>4.75E-09</td>
<td>300</td>
<td>1.01E-09</td>
<td>334</td>
<td>1.44E-08</td>
<td></td>
</tr>
<tr>
<td>233</td>
<td>5.04E-10</td>
<td>267</td>
<td>3.04E-10</td>
<td>301</td>
<td>2.75E-08</td>
<td>335</td>
<td>2.97E-08</td>
<td></td>
</tr>
<tr>
<td>234</td>
<td>3.28E-09</td>
<td>268</td>
<td>3.04E-10</td>
<td>302</td>
<td>1.51E-09</td>
<td>336</td>
<td>2.52E-10</td>
<td></td>
</tr>
<tr>
<td>235</td>
<td>2.52E-10</td>
<td>269</td>
<td>2.52E-10</td>
<td>303</td>
<td>7.56E-10</td>
<td>337</td>
<td>1.26E-09</td>
<td></td>
</tr>
<tr>
<td>236</td>
<td>2.52E-10</td>
<td>270</td>
<td>2.52E-10</td>
<td>304</td>
<td>5.04E-10</td>
<td>338</td>
<td>5.04E-10</td>
<td></td>
</tr>
<tr>
<td>237</td>
<td>5.04E-10</td>
<td>271</td>
<td>2.90E-08</td>
<td>305</td>
<td>3.02E-09</td>
<td>339</td>
<td>2.52E-10</td>
<td></td>
</tr>
</tbody>
</table>
The following listings are the Excel macros that take ImageJ data and create a spreadsheet with volume and volume/area ratio, or w spacing parameter.

Sub SumVolumes()

' Macro1 Macro
Dim i As Integer
i = 1
Do Until i > 30000
    Dim MyResults
    Dim myrange
    Selection.End(xlDown).Select
    Selection.End(xlDown).Select
    Set myrange = Range(ActiveCell.Offset(-1, 0), ActiveCell.End(xlUp))
    MyResults = WorksheetFunction.Sum(myrange)
    ActiveCell.Value = MyResults
Loop
End Sub

Sub CreateVolumesOneAtATime()

' Macro17 Macro
Dim i As Integer
i = 1
Do Until i > 30000
    ActiveCell.Offset(1, 0).Range("A1").Select
    ActiveCell.FormulaR1C1 = "+=0.130388*(3.156*RC[-2])^1.5"
Loop
End Sub

Sub SumAColumnforAreasorVolumes()
Macro14 Macro
Dim i As Integer
i = 1
Do Until i > 30000
    Dim MyResults
    Dim myrange
    Selection.End(xlDown).Select
    Selection.End(xlDown).Select
    Set myrange = Range(ActiveCell, ActiveCell.End(xlUp))
    MyResults = WorksheetFunction.Sum(myrange)
    ActiveCell.Offset(1, 0).Value = MyResults
    Selection.End(xlDown).Select
Loop
End Sub
Sub CreateVolumes()
' Macro2 Macro
Dim i As Integer
i = 1
Do Until i > 30000
    ActiveCell.FormulaR1C1 = "=RC[-2]*.13038*(3.156*D3)^1.5"
    ActiveCell.Select
    Selection.Copy
    ActiveCell.Offset(1, 0).Range("A1").Select
    Range(Selection, Selection.End(xlDown)).Select
    ActiveSheet.Paste
    Selection.End(xlDown).Select
    ActiveCell.Offset(2, 0).Range("A1").Select
Loop
End Sub
Sub SumtheAreas()
' Macro1 Macro
Dim i As Integer
i = 1
Do Until i > 30000
    Selection.End(xlDown).Select
    ActiveCell.Offset(1, 0).Range("A1").Select
    ActiveCell.FormulaR1C1 = "=SUM(R[130]C:R[-1]C)"
    ActiveCell.Offset(2, 0).Range("A1").Select
Loop
End Sub
Sub makelabel()
    makelabel Macro
    ActiveCell.Offset(0, 2).Columns("A:A").EntireColumn.Select
    Selection.Insert Shift:=xlToRight
    ActiveCell.Offset(0, -1).Columns("A:A").EntireColumn.Select
    Selection.TextToColumns Destination:=ActiveCell, DataType:=xlDelimited, _
    TextQualifier:=xlDoubleQuote, ConsecutiveDelimiter:=False, Tab:=False, _
    Semicolon:=False, Comma:=False, Space:=False, Other:=True, OtherChar _
    :=".", FieldInfo:=Array(Array(1, 1), Array(2, 1)), TrailingMinusNumbers:=True
    ActiveCell.Offset(0, -1).Range("A1").Select
End Sub
Sub InsertSheetRowsAboveZero()
' Macro4 Macro
Dim i As Integer
i = 1
Do Until i > 30000
    Range(Selection, Selection.End(xlDown)).Select
    Selection.Find(What:="0", After:=ActiveCell, LookIn:=xlValues, LookAt:= _
    xlPart, SearchOrder:=xlByRows, SearchDirection:=xlNext, MatchCase:=False _
    , SearchFormat:=False).Activate
    ActiveCell.Rows("1:1").EntireRow.Select
Sub CreateVolumeSumAndSMD()
" Macro2 Macro
Dim i As Integer
i = 1
Do Until i > 30000
    Selection.End(xlDown).Select
    Selection.End(xlDown).Select
    ActiveCell.Offset(1, 1).Range("A1").Select
    Selection.Copy
    ActiveCell.Offset(0, 1).Range("A1").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
        :=False, Transpose:=False
    ActiveCell.Offset(0, 1).Range("A1").Select
    Application.CutCopyMode = False
    ActiveCell.FormulaR1C1 = "/RC[-2]/RC[-4]"
    ActiveCell.Offset(0, 1).Range("A1").Select
    Application.CutCopyMode = False
    ActiveCell.FormulaR1C1 = "=(RC[-3]/(RC[-5])^0.5)^0.5"
    ActiveCell.Offset(0, -4).Range("A1").Select
Loop
End Sub
Sub CreateVolumeSum()
' ' Macro2 Macro
Dim i As Integer
i = 1
Do Until i > 30000
    Selection.End(xlDown).Select
    Selection.End(xlDown).Select
    ActiveCell.Offset(1, 1).Range("A1").Select
    Selection.Copy
    ActiveCell.Offset(0, 1).Range("A1").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
        :=False, Transpose:=False
    ActiveCell.Offset(0, 1).Range("A1").Select
    Application.CutCopyMode = False
    ActiveCell.FormulaR1C1 = "="
    ActiveCell.Offset(0, 1).Range("A1").Select
    Application.CutCopyMode = False
    ActiveCell.FormulaR1C1 = "=(RC[-5]*RC[-5]/(RC[-3])/7.4)"
187
IRREGULAR SURFACE DISCUSSION

The spherical cap geometry assumption appears to be an adequate model even for the longer pulse duration of 49 ms. For the longer pulse durations, water drops land on top of already-deposited drops. This possibly creates an irregular surface. The spherical cap model appears to give good estimates of the mass on the disc, so this irregular surface is not a problem when using the ImageJ data to estimate mass. However, considering the $w$ parameter (the volume to area ratio) for the longer pulse durations, the $w$ parameter using the spherical cap model and ImageJ picture data is larger than what is calculated using the temperature data: 0.00092 m versus 0.00060 m. A possible explanation is that the real surface has small ripples and waves, such that the surface area is greater than would be predicted by assuming a smooth surface. A greater surface area, while maintaining the same volume, would then decrease the $w$ parameter; that is, the volume/area ratio by making the effective sublimating surface area larger. If this is true, the 0.000090 value would be closer to the 0.00060 value, if some accounting of the surface irregularities could be made. The volume would be the same in both cases, provided the mean surface shape for the rippled surface is that of the smooth surface. If the surface area of the largest puddle increases 50% via ripples, bumps, waves, or other irregularities, then the value of $w$ in the 49 ms case calculated from the temperature data, 0.000060 m, matches the value of $w$ calculated by the ImageJ video picture data. Mathematica can draw a picture showing the relative size of the ripples that would be required to increase the surface area. It is possible these ripples are too small to be clearly seen in the resolution of the video pictures. In Figure 80, Mathematica
illustrates the surface profile when a smooth surface is replaced with a sine wave surface with amplitude 3.5% of a unit radius and frequency of 50 waves for the entire sphere. The effective surface area increases 50% from that of a smooth surface. Although this proposed explanation of the w value difference is reasonable and seems to be supported by a Mathematica calculation, the resolution of the video is not adequate to actually measure the ripples in the picture.

Figure 80 A Mathematica illustration of the surface profile assuming the smooth spherical cap surface is replaced with a sine wave surface.

Figure 81 The largest puddle (frozen) in the 49 ms case appears to have some surface irregularities that may explain the under-prediction of the w parameter when assuming a smooth spherical cap geometry.
Figure 82 The full view of the 49 ms pulse. Figure 81 is a close up view of the large puddle on the right side of this picture.

Figure 81 shows the real ImageJ picture of the largest drop (frozen puddle) for the 49 ms case. Even though the resolution is poor for discerning any surface texture, it does appear that some surface texture exists. This picture therefore lends good qualitative support to the proposed explanation of differing w value predictions for the 49 ms case.

As mentioned earlier, the final predictive model includes a spacing parameter curve fit that takes this effect into account, because it uses the temperature data rather than the video data to benchmark the “n” versus equivalent film thickness curve. So this discussion is important for the qualitative understanding of the sublimation rate, but not important with respect to the final heat transfer predictive model and quantitative analysis.
The following is a listing of the Mathematica code that generated Figure 80. The bottom line value of “1.50346” is the ratio of the irregular surface area to the smooth surface.

\[
magnitude = 0.035;
\]

\[
\text{Plot}\left[1 - \frac{\text{magnitude}}{\pi/2} + \text{Abs}[\text{magnitude} \sin[50 \theta]], \{\theta, 0, \pi\}\right]
\]

\[
\text{SphericalPlot3D}\left[1 - \frac{\text{magnitude}}{\pi/2} + \text{Abs}[\text{magnitude} \sin[50 \theta]], \{\theta, 0, \pi/5\}, \{\phi, 0, 2 \pi\}, \text{BaseStyle} \to \{\text{FontSize} \to 18\}\right]
\]

\[
r = 1 - \frac{\text{magnitude}}{\pi/2} + \text{magnitude} \sin[50 \theta];
\]

\[
\text{term} = \partial_\theta r;
\]

\[
\text{NIntegrate}\left[2 \pi r \cos[\theta] \sqrt{r^2 + (\text{term})^2}, \{\theta, 0, \frac{\pi}{2}\}\right]
\]

9.44654

\%  

\[
\frac{2 \pi}{1.50346}
\]
Appendix G

Experimental “w” Calculated From Temperature Data

The Mathematica notebook listing that calculates the experimental w calculated from temperature data is presented here. The accommodation coefficient has been iteratively chosen as 0.036 with this Mathematica notebook so that “w008”, the w for the 0.008 ms pulse duration, matches the video data ImageJ-determined w. The value of w008 is 0.00028 m as can be seen in the Mathematica notebook listing.

<table>
<thead>
<tr>
<th></th>
<th>w, [m] for 8 ms pulse</th>
<th>w, [m] for 28 ms pulse</th>
<th>w, [m] for 49 ms pulse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determined from Video Data</td>
<td>.000028</td>
<td>.000060</td>
<td>.000092</td>
</tr>
<tr>
<td>Determined from Experimental Temperature Data</td>
<td>.000028</td>
<td>.000048</td>
<td>.000060</td>
</tr>
</tbody>
</table>
The spacing parameter is determined from the following *Mathematica* notebook. The spacing parameter “n” is chosen iteratively as 1.9, 1.45, and 1.25 as shown below. Examine the 0.008 ms calculations as an example. First, the equivalent film thickness, δ, is calculated from the pulse duration. Then, the radius of the base area of the droplet is calculated as a008. The a008 value is then multiplied by 0.273 which converts the base area radius to the w parameter based on not the base area, but on the droplet surface area where the water mass sublimates. This w parameter, the same as used elsewhere is calculated here at 0.000029. This is close enough to the 0.000028 value calculated in the above spreadsheet that the iterative procedure was ended. At the bottom of the Mathematica spreadsheet on page 197, one can see that this tuned spacing parameter model predicts the time constant for the temperature cooldown curve very well. With these “n” values based on pulse duration, the model may be used to predict the cooldown curve for any general spray.
Universal Gas Constant in Joules/mole K

\[ R = 8.3144621; \]

Mass of copper disc

M = 0.0045;

Molecular weight of water in kg/mol

Molecular weight = 0.01801528;

hfg = 2411.329.;

Joules per kg per C

cp = 383.;

The following is the vapor pressure over ice.

\[ P_{sat}[T] = 611.15 e^{\left(\frac{-33.01+\frac{T}{273.15}}{273.15}\right)} \]

\( a = 0.036; \)

\( T_{sink} = -47.; \)

Experimental Thermocouple Data = (.152, .34, .43);

Experimental Vovera = (.000028, .000060, .000092);

Tdroplet008 = 26.;

\[ \text{Solve } \left( \begin{array}{c}
0.152 &=& \\
\begin{array}{c}
hfg \frac{2.8}{2.0} (\text{Molecular weight}) \frac{2.0}{2.0} \\
(\text{Tdroplet008-273.15})
\end{array} \\
\begin{array}{c}
611.15 e^{\left(\frac{-33.01+\frac{T}{273.15}}{273.15}\right)} \\
(\text{Tdroplet008-273.15})
\end{array}
\end{array} \right) \right] \]

\([0.0000079431]\)

\[ \text{Tdroplet = 22.;} \]

\[ \text{Solve } \left( \begin{array}{c}
0.34 &=& \\
\begin{array}{c}
hfg \frac{2.8}{2.0} (\text{Molecular weight}) \frac{2.0}{2.0} \\
(\text{Tdroplet-273.15})
\end{array} \\
\begin{array}{c}
611.15 e^{\left(\frac{-33.01+\frac{T}{273.15}}{273.15}\right)} \\
(\text{Tdroplet-273.15})
\end{array}
\end{array} \right) \right] \]

\([0.0000476544]\)

\[ \text{Solve } \left( \begin{array}{c}
0.43 &=& \\
\begin{array}{c}
hfg \frac{2.8}{2.0} (\text{Molecular weight}) \frac{2.0}{2.0} \\
(\text{Tdroplet-273.15})
\end{array} \\
\begin{array}{c}
611.15 e^{\left(\frac{-33.01+\frac{T}{273.15}}{273.15}\right)} \\
(\text{Tdroplet-273.15})
\end{array}
\end{array} \right) \right] \]

\([0.0000602688]\)
SprayPulseDuration = {0.008, 0.028, 0.049}
{0.008, 0.028, 0.049}

WaterUtilizationEfficiency = {0.48, .66, .76}
{0.48, 0.66, 0.76}

SprayPulseVolume = {0.000025 SprayPulseDuration * 0.000003 \over WaterUtilizationEfficiency / 920.}
{2.6087 \times 10^{-7}, 7.1781 \times 10^{-7}, 1.25978 \times 10^{-6}}

Multiply by 84% to reflect that only 84% of the disc is occupied by water droplets

\[ \delta = \frac{SprayPulseVolume}{0.000006787 \times .84} \]
{6.12897 \times 10^{-8}, 0.0000168547, 0.0000295978}

a008 is the base radius for the pulse duration of 8 ms

a008 = a028 = .5 a049 = .

sola008 = \text{solve} \left[ \frac{130388 \pi^{1.5} a008^3}{3.462 (1.9 a008)^3} \right] \rightarrow \theta[[1]], a008 \right] \rightarrow [1]
{a008 = 0.0001055502}

The radius, a008, is the radius of the droplets for the 8 ms pulse duration

a008 = a008 / sola008
0.0001055502
sola028 - Solve \[ \frac{130388 \pi^{1.5}}{3.462 (1.45 \text{ a028})^2} \rightarrow 6 \{[2], \text{ a028}\}[[1]] \]
(a028 = 0.000168974)

sola049 - Solve \[ \frac{130388 \pi^{1.5}}{3.462 (1.25 \text{ a049})^2} \rightarrow 6 \{[3], \text{ a049}\}[[1]] \]
(a049 = 0.000220516)

Universal Gas Constant in Joules/mol K.

\[ R = 8.3144621 \]
8.31446

Mass of copper disc

\[ M = 0.0045 \]
0.0045

Molecular weight of water in kg/mol

\[ \text{Molecularweight} = 0.01801528 \]
0.0180153

\[ hfg = 2411.323 \]
2.41132 \times 10^8

Joules per kg per C

\[ cp = 383. \]
383.

\[ T = -\]

The following is the vapor pressure over ice, whereas the previous is over water.

\[ P_{\text{sat}}[T] = 611.15 e^{\left(\frac{23.436 - 353.7}{298.15 - 273.15}\right)} \]

\[ T = -\]

This is the Arden Buck equation over ice, not water, good to -80 C. Journal of Applied Metrology 1981. The source of 1991 Buck Research Manuals also lists this for ice. The 1991 reference gave an update for water, though, and that is listed above.

These are the beginning temperatures of the droplets

\[ T_{\text{droplet}} = \{26., 22., 22.\} \]
\{26., 22., 22.\}
"a" is accommodation coefficient.

\[ a = 0.036 \]

\[ w = 0.075 \{ a008, a_{028}, a_{049} \} \]

\[ \{0.0000260129, 0.0000464679, 0.0000666425\} \]

\[ T_{\text{link}} = -49. \]

\[-49. \]

\[ \frac{W \cdot h_{\text{fg}}}{h_{\text{fg}}} \text{ Predicted} = \frac{\text{wt} \cdot \text{h}_{\text{fg}}}{24} \left( \frac{\text{Weight}_{\text{test}}}{232} \right)^3 \left( \frac{P_{\text{sat}} \left( \text{test} \right)}{P_{\text{sat}} \left( \text{test} \right)} \right) \]

\[ \{0.159768, 0.393959, 0.492453\} \]

Text Values from Experimental Temperature Thermocouple Data Least Squares Curve Fit: 0.152, 0.34, 0.43
Appendix H

Calculation Of The Uncertainty Analysis

Uncertainty analysis *Mathematica* notebook is presented here. The major variables are:

Errora - the error in the accommodation coefficient

Errorn – the error in the spacing parameter “n”

ErrorTauPredicted – final error considering all of the error analysis

The listing presented here is for the 8 ms pulse duration. The other calculation *Mathematica* notebooks are similar, with the variables for 28 and 49 ms substituted.
\[ R = 8.3144621 \]
\[ \text{Molecularweight} = 0.01801528 \]
\[ \text{Error} = 0.0100293 \]

\[ \alpha = \cdot \]
\[ T = \cdot \]
\[ \rho = \cdot \]
\[ w = \cdot \]
\[ \tau = \cdot \]
\[ \delta T = \cdot \]
\[ \delta \rho = \cdot \]
\[ \delta w = \cdot \]
\[ \delta \tau = \cdot \]

The basic experimental model

\[ \text{sol} = \text{Solve}[\tau = \frac{\rho w}{2 \omega \left( \frac{\text{Molecularweight}}{2 \pi R} \right)^{0.5} \left( \frac{611.15 \omega}{(T.273.15)^{0.5}} \right)}, \alpha][[1]]; \]
\[ \alpha = \alpha / . \text{sol}; \]
\[ \text{Error} = \sqrt{\left( (\partial_\tau \alpha \delta T)^2 + (\partial_\rho \alpha \delta \rho)^2 + (\partial_w \alpha \delta w)^2 + (\partial_\tau \alpha \delta \tau)^2 \right)}; \]

\[ T = 22.; \]
\[ \rho = 920.; \]
\[ w = 0.000028; \]
\[ \delta T = 1.; \]
\[ \delta \rho = 50.; \]
\[ \tau = 0.152; \]
\[ \delta \tau = 0.002; \]
\[ \delta w = .2 w; \]
\[ \text{Errorvalue} = \text{Error} \]
\[ \text{Out}[209]= 0.0100293 \]
The spacing parameter model

\[ \text{SprayPulseDuration} = \text{WaterUtilizationEfficiency} = \]

\[ \text{Pwett} = . \]

\[ \delta \text{SprayPulseDuration} = . \]

\[ \delta \text{WaterUtilizationEfficiency} = . \]

\[ \delta \text{Pwett} = . \]

\[ n = . \]

\[ a008 = 0.000144; \]

\[ \text{sola008} = \text{Solve} \left[ \frac{0.130388 \pi^{1.5} a008^3}{3.462 (n \ a008)^2} = \right. \]

\[ \left. ((0.00025 \ \text{SprayPulseDuration} + 0.000003 \text{WaterUtilizationEfficiency} / 920.) / (0.000506707 \ Pwett), \ n) \right]^{[1]}; \]

\[ n = n / . \text{sola008}; \]

\[ \partial \text{WaterUtilizationEfficiency} n; \]

\[ \partial \text{SprayPulseDuration} n; \]

\[ \partial \text{Pwett} n; \]

\[ \text{Errorn} = \]

\[ \sqrt{((\partial \text{SprayPulseDuration} n \ \partial \text{SprayPulseDuration})^2 + ((\partial \text{WaterUtilizationEfficiency} n \ \partial \text{WaterUtilizationEfficiency})^2 + ((\partial \text{Pwett} n \ \partial \text{Pwett})^2)} ; \]
Now, heat transfer model is the predictive model with all the uncertainties.

\[
\begin{align*}
\text{SprayPulseDuration} & = . \quad \text{WaterUtilizationEfficiency} = . \\
\delta\text{WaterUtilizationEfficiency} & = . \\
\delta\text{SprayPulseDuration} & = . \\
T\text{droplet} & = . \\
\delta T\text{droplet} & = . \\
n & = . \\
a008 & = . \\
s0a008 & = . \\
\text{TauPredicted} & = . \\
\alpha & = . \\
\end{align*}
\]

\[
\text{s0a008} = \text{Solve} \left[ \frac{.130388 n^{1.5} a008^3}{3.462 \ (n\ a008)^2} = (0.00025 \ \text{SprayPulseDuration} + 0.000003) \ \text{WaterUtilizationEfficiency} / 920. / (0.000506707 \ Fwett), \ a008 \right] \left[1\right];
\]
\[ a_008 = a_008 \div \text{sola008} \]
\[ 12.177 n^2 \left( 3 \times 10^{-6} + 0.00025 \text{SprayPulseDuration} \right) \]
WaterUtilizationEfficiency

\[ \frac{.194 a_008 920}{2n \left( \frac{\text{Molecularweight}}{2n} \right)^{.5} \left( \frac{511.15 \div \left( \text{Droplet} \cdot 237.15 \right)^{.2}}{\text{Droplet} \cdot 237.15^{.2}} \right)} \]

\[ \frac{\text{Error} \text{TauPredicted}}{\text{Error} \text{SprayPulseDuration}} \]
\[ \text{Error} \text{WaterUtilizationEfficiency} \]

\[ \text{Error} \text{Droplet} \]

\[ \text{Error} \text{WaterUtilizationEfficiency} = 0.05; \]
\[ \text{WaterUtilizationEfficiency} = .46; \]
\[ \text{SprayPulseDuration} = 0.005; \]
\[ \text{SprayPulseDuration} = .008; \]
\[ \text{Pwett} = 0.84; \]
\[ \text{Pwett} = 0.05; \]
\[ n = 2.2; \]
\[ \text{Error} = \text{Error} \text{value} \]
\[ 0.143332 \]
\[ \text{Error} = \text{Error} \text{value} \]
\[ 0.0100293 \]
\[ \alpha = 0.036; \]
\[ \text{Droplet} = 2.; \]
\[ \text{Droplet} = 22.; \]
\[ \text{Error} \text{TauPredicted} \]
\[ 0.0671899 \]