Some Further Investigations of Demand and Supply Functions for Money

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STUDIES ON MONEY AND MONETARY POLICY

SOME FURTHER INVESTIGATIONS OF DEMAND AND SUPPLY FUNCTIONS FOR MONEY

KARL BRUNNER* AND ALLAN H. MELTZER†

I. ON SOME QUESTIONS LOST BETWEEN "MONETARY POLICY" AND "MONETARY THEORY"

FOR ALMOST TWO DECADES a virtually impenetrable curtain has separated two groups of monetary economists. One group has engaged in a learned and apparently interminable discussion concerned with the subtleties of Pigou effects, Lerner effects, and Keynesian effects, the existence or absence of money illusions, dichotomization of real and money prices, Hicksian weeks, and related esoterica. The other is ostensibly concerned with the "world’s work", a range of interests that encompasses the details of monetary policy operations. Included among the subjects of discussion in recent years have been the nature of open market operations, their restriction to Treasury bills, the nature and timing of other Federal Reserve actions, the comparative advantages of *ad hoc* institutional devices, the possibilities of "twisting" the yield curve, and similar issues.

It does not require an astute observer to notice that many of the issues that agitate monetary theorists have little or no bearing on the policy issues discussed. The analytical frames constructed and vigorously disputed with plausible and counter-plausible arguments yield neither propositions clarifying the policy issues nor verified propositions directly applicable to policy decisions. On the other hand, the discussions of monetary policy most often proceed without the benefit of validated theory or even an explicit frame. Such policy discussions supply a collection of statements unrelated to and independent of any systematically constructed empirical theory exposing the structure of the process under consideration.¹

The separation of theoretical endeavors and policy discussions seriously obstructs explication and appraisal of fundamental issues. In particular,

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the causal structure of monetary mechanisms remains unresolved. The basic facts, expressed by the close association of money stock and national income (or between money stock per unit of output and prices) cannot be reasonably disputed. The observations from many countries, differing time periods, and a variety of economic "climates" clearly exhibit the high correlation between the magnitudes indicated. While disputes concerning these facts are quite futile, the problem of interpreting the facts remains. Such an interpretation is completely determined by a theory that provides the explanation.

The theories offered by economists differ in important respects. Some maintain that the money supply has no relevant place in the formation of prices and the determination of national income. They attribute the association between money supply and national income to the demand for money and specify a demand function for money that contains national income as an important argument. The "pull" of money on national income is used to rationalize the observed association.

Two distinct notions are subsumed under the "pull" framework. Both deny the relevance of monetary processes in this context. One, the "pure pull" notion, suggests a linear ordering from non-monetary forces via the behavior of national income to the demand for money. The money stock is somehow called forth to meet the demand. The stock of money emerges as a residual entity without much importance for the operation of the economy. A second, more sophisticated alternative acknowledges the independent operation of monetary factors on both the supply and the demand side. But these factors only contribute to shape interest rates on financial assets with almost no spillover to prices and the pace of economic activity. The stock of money is either totally or largely irrelevant as a factor determining the level of income.

Other economists acknowledge the operation of a causal relation connecting money to prices and national income. The observed association between money and income is not imputed to the demand for money alone. The "push" of money on income and prices—exerted via the play of substitution processes linking money, financial assets, real capital, and current output—operates jointly with the "pull" of income on money via the demand for money. This conception yields suggestions for policy

2. A rather explicit example of the asserted dependence of M on demand factors only can be found in Goldsmith's Financial Intermediaries in the American Economy Since 1900, p. 21. "Under contemporary American conditions ... the volume of check (demand) deposits ... is determined primarily by the public's demand for cash." On the same page though, in footnote 7, Goldsmith asserts: "A banking system is able to keep in circulation more (or less) money than the public wants to hold at unchanging prices and interest rates." This statement admits a relatively independent (i.e. from demand factors) determination of the money stock.

A monetary system of most peculiar structure and exhibiting approximately a linear ordering of the causal chain relating income and money has been implicitly incorporated into the Klein-Goldberger model. See L. R. Klein and A. S. Goldberger, An Econometric Model of the United States, 1929-1952.
action that differ markedly from those that are consistent with the "pure pull" or the more sophisticated version of money irrelevance.

Systematic investigation of the supply and demand functions for money and their interaction appears to yield pertinent information bearing on the gross association observed between money stock and national income. A successful specification of the arguments of a money supply function, independent of the public's asset supply to banks or of national income, would lead us to reject the notion that the money stock is a residual entity dangling on the causal chain. Moreover, validated knowledge about the interaction of the demand and supply functions for money is capable of disentangling the notion of "money relevance" from the more sophisticated version of "money irrelevance".

In this paper, we continue our studies of monetary processes with a view toward providing a theory of money incorporating policy actions and the behavior of banks and the public. The following section develops a linear and non-linear theory of the supply of money. We then consider the demand function for money. Because our earlier published work suggested that the demand function for money is linear in the logarithms of the variables, while one of the supply functions is linear in the variables, we have provided a series of estimates for linear and non-linear functions. A discussion of our findings from one and two stage least squares regressions is provided in section IV. A final section of the paper discusses some estimates for the theories bearing on the interaction of banks and the public and the resultant determination of money stock and interest rates on the credit markets.

It should be noted, however, that while the propositions subsumed under the theories presented are deemed necessary for a rational evaluation of policy arrangements, they are not sufficient conditions for a good theory. The estimates that we provide contain information about the theories' compatibility with a given batch of observations. But such information, by itself, yields no sufficient basis to choose a particular theory, even if it is our own, over an alternative other than the null hypothesis.¹

II. Money Supply Theory

Despite abundant numerical examples, multiplier tables and balance-sheets distributed over textbooks, money supply theory has been essentially disregarded by economists. Recent years, however, indicate a growing awareness that our knowledge about this subject is quite inadequate. This awareness has already stimulated a variety of investigations. We may hope that the onsetting competitive supply of distinct hypotheses and pieces of analysis improves our comprehension of monetary processes.⁴

³. An appraisal of rival demand for money hypotheses, however, was developed in our paper, "Predicting Velocity: Implications for Theory and Policy," Journal of Finance, May 1963.

⁴. Several investigations bearing on money supply or monetary system behavior have
Our own tentative attempts go back to 1958, covering simultaneously the United States and France. The joint efforts applied over subsequent years enabled us to broaden our systematic inquiries. From these inquiries three distinct money supply theories emerged. One, actually more a group of theories than a single theory, centers on the banks’ adjustment to free reserves. Another theory centers on the banks’ response to surplus reserves. In combination with a careful specification of the processes generating surplus reserves, this theory yields a linear hypothesis of the money supply mechanism. A third theory is based on some homogeneity properties of demand behavior and centers on a mutual adjustment of actual and desired allocation ratios. The second and third theory will be concisely summarized in the next two sections.

already been published or presented at professional gatherings. Some of these studies may be mentioned in this footnote:

a. A. J. Meigs, Free Reserves and the Money Supply, Chicago 1962. This study does not present a money supply hypothesis. It contains, however, an excellent study of the banks’ demand for free reserves. This particular piece of the money supply mechanism is most competently investigated.

b. Orr and Mellon, “Stochastic Reserve Losses and Bank Credit Expansion,” American Economic Review 1961. This paper is marred by some mathematical flaws bearing on the conclusion about the system behavior. More seriously, it is not completely specified as a theory. This specification error bears on the interpretation of excess reserves typically remaining after banks partly absorbed new injections through asset expansions. Two interpretations are available. One yields a theory seriously contradicted by long sequences of observations. The other interpretation yields a theory implying results identical with the naïve non-stochastic theory discussed in the same paper. In either case no useful money supply hypothesis emerges.

c. A paper by Ronald Teigen presented at the meetings of the Econometric Society in December 1962. His paper included a money supply hypothesis. A logical appraisal of this hypothesis establishes that it yields statements about the response of the money stock to changes in the average requirement ratio and of the discount rate. But it supplies no information concerning open market operations or the effect of gold flows or changes in the distribution of Treasury balances between Federal Reserve account and tax and loan account. Moreover, a conversion of checking deposits into currency raises the money supply under the Teigen hypothesis.

d. Specification of money supply behavior as an integral part of a macro-model offers convincing evidence indicating the growing attention to money supply theory. Franco Modigliani included such a specification into his model II (mid-50’s) presented in “The Monetary Mechanism and Its Interaction with Real Phenomena,” The Review of Economics and Statistics, February 1963, p. 80. With the specifications laid down it is impossible to derive any statements concerning the response of the money stock to open market operations, to variations in requirement ratios and to the discount rate. As a matter of fact, the Modigliani hypothesis implies that the money supply is independent of variations in both requirement ratios and the discount rate. Furthermore, losses of bank reserves due to currency flows exert no effect on banks’ portfolio of earning assets and the money stock.


6. The first paper noted in footnote 5 indicates the frame of the non-linear theory based on allocation ratios. This framework has since been developed in detail in a large manuscript on “The Joint Determination of Money Stock and Interest Rate on Credit Markets.” The linear hypothesis has been first presented in the paper by Karl Brunner on “A Schema for the Supply Theory of Money,” International Economic Review, 1961. A further development of this formulation was presented in a large manuscript at the meetings of the Econometric
a. The Linear Hypothesis

This hypothesis contains the description of two mechanisms. One specifies the portfolio response of banks to surplus reserves; the other characterizes the processes generating or absorbing surplus reserves independent of the banks’ induced portfolio responses. The hypothesis grew out of the “Phillips’ tradition.” An analysis of this tradition indicated that it supplied useful ingredients for a theory, without actually providing one. At the very best it yielded a description of the multiplier mechanism, i.e. of the banks’ portfolio response to surplus reserves. But the specification of surplus reserves was usually missing. The resulting structures were thus unable to imply any statements on policy patterns that could be subjected to systematic appraisal.

Surplus reserves are the difference between actual and desired reserves. The latter are a function of interest rates (incl. the discount rate) and the volume of deposit liabilities. Banks respond to surplus reserves by adjusting portfolios until the surplus is eliminated. A positive surplus induces a bank to acquire earning assets. Such acquisitions typically generate a loss of reserves to other banks, an outflow of currency and an increase in required reserves. (The latter event occurs because some of the deposits created in the process of acquiring assets are likely to remain with the bank.) The magnitude of any particular bank’s response to surplus reserves, involving a re-adjustment of its asset portfolio, thus depends on the average loss of surplus reserves associated with portfolio adjustments. The larger this loss per dollar of portfolio adjustment, i.e. the larger the average loss coefficient, the smaller the response to any given volume of surplus reserves.

The portfolio adjustment of any bank modifies the reserve position of other banks; surplus reserves emerge at other locations in the system. The magnitude of these surplus reserves is affected by the fate of the deposits generated by an expanding bank. These deposits may be converted into currency at the receiving banks or re-allocated in different ways between checking and time accounts. These secondary responses must be differentiated from the primary responses summarized by the loss coefficient of the expanding bank. The secondary responses induce other banks to readjust their portfolios etc. until the repetitive redistribution of surplus reserves over the system and associated portfolio adjustments absorb the initially available surplus reserves.

We stipulate further that this absorption process is quite rapid relative to the time periods in terms of which reliable data are specified. This
stipulation enables us to derive a formula describing the total portfolio response of the system to a prevailing level of surplus reserves.

\[
dE = \frac{1}{\lambda - \mu} \ s
\]

where \(E\) designates the system's portfolio of earning assets, \(s\) the prevailing volume of surplus reserves; \(\lambda\) is the average loss-coefficient of the banks in the system, and \(\mu\) has the form \((1 - n)\) p. p denotes the average "spillover" of deposits in the system from expanding banks to other banks (per dollar of asset expansion); \(n\) is a linear combination of spillover-rates into currency and time deposits on the secondary level, occurring at the banks receiving deposits from expanding banks.

Formula (1) has been derived from two radically different higher level hypotheses. One made use of a number of simplifying assumptions. The other removed all the simplifying assumptions at the cost of a more complex mathematical analysis. In both the expression \((\lambda - \mu)^{-1}\) describes the built-in leverage of the system's response to prevailing surplus reserves. We may refer to it as the "monetary-multiplier."

The multiplier formula must be supplemented by a relation connecting surplus reserves with observable entities. This relation emerges from a systematic inquiry into the processes generating a surplus independent of the banks' portfolio adjustment. Surplus reserves generated by these processes trigger the multiplier mechanism and are absorbed in the ensuing process. This second relation may be specified as follows:

\[
s = a_\circ dB + dL - a_1c + as + ad - dv^6
\]

The symbol \(B\) denotes the monetary base, i.e. the amount of money issued by the government sector (incl. the Federal Reserve Banks); \(L\) designates the cumulated sum of changes in required reserves attributable to changes in requirement ratios and the distribution of existing demand deposits between classes of banks with different requirement ratios.

\(c, t,\) and \(v^6\) require a somewhat less terse description. The higher level theory underlying the linear money supply hypothesis formalized in equations (1) and (2) contains a demand function for currency by the public \(C^p\), a demand function for time deposits by the public, \(T\), and the banks' demand function for "available cash assets" (i.e., cash assets in excess of required reserves). The public's demand functions are assumed to depend on monetary wealth (i.e. money stock plus time deposits),

8. The monetary base satisfies the following equation:
\[B = A + F^2 + U + C - d - o - f - c\] where \(A = \) discounts and advances of Federal Reserve Banks; \(F^2 = \) Federal Reserve's portfolio of securities; \(U = \) gold stock; \(d = \) Treasury deposits at Federal Reserve Banks; \(C = \) Treasury currency outstanding; \(f = \) foreign deposits at Federal Reserve Banks; \(c = \) Treasury cash; \(o = \) Federal Reserve's other deposits plus other accounts. — The adjusted base is defined as \(B^- = B - A\).
non-monetary wealth and pertinent cost and yield entities. The banks' demand is dependent on a spectrum of relevant interest rates and deposit liabilities.

Responses in $C_p$, $T$, and $v^d$ to changes in monetary wealth are essential elements of the multiplier mechanism, eq. (1). On the other hand, responses in $C_p$, $T$, and $v^d$ to other arguments (e.g. interest rates) generate or absorb surplus reserves as shown by eq. 2. The emergence (or disappearance) of surplus reserves triggers the multiplier process. The expressions $dc_0$, $dt_0$, and $dv^d_0$ thus signify the changes in (1) the public's currency holdings, (2) time deposit allocation, and (3) the banks' asset allocation to cash assets that occur independently of changes in the public's monetary wealth or the banks' deposit liabilities. These changes are viewed as emanating from changes in interest rates that induce banks to release (or absorb) surplus reserves, or induce the public to re-shuffle deposits between time and demand accounts, and similar occurrences.9

The last item to be considered, $e$, is an expression describing the structure of interbank deposits. It can be demonstrated that variations in this structure affect, in principle, the banks' surplus reserve position.

The second relation thus asserts that increases in the base, reductions in average reserve requirements, a currency inflow from the public to the banks (i.e. $dc_0 < 0$), a conversion of demand into time deposits ($dt_0 > 0$) and a compression of banks' desired reserves, generate surplus reserves. The opposite changes absorb surplus reserves. The resulting deficiency (i.e. relative to the banks' desired portfolio) is eliminated by the portfolio contraction ($dE < 0$) set off by a negative $s$.

The higher level theory also yields useful restrictions on the coefficients

9. The three demand functions discussed in the text may be described by the following formulae

$$C_p = \gamma(n^1, M); T = \tau(n^2, M); v^d = \nu(n^3, D + T)$$

The symbols $n^1$ are vectors containing non-money wealth and pertinent cost and yield elements associated with asset-holdings. $M$ denotes the inclusive money supply; i.e. monetary wealth. The derivatives of the three functions with respect to $M$ or $D + T$ occur as crucial ingredients of the multiplier mechanism. Appropriate aggregation of specific elements constituting the average loss coefficient $\lambda$ or the average coefficient $\mu$ can be shown to yield the derivatives indicated. The portfolio response of banks to surplus reserves is typically associated with the public's allocative behavior bearing on the redistribution of the newly created financial assets between currency, checking deposits and time deposits. These allocation patterns are suitably reflected by the derivatives mentioned above. The generation of surplus reserves triggering the multiplier mechanism, on the other hand, is associated with the operation of the vectors $n^1$. The differential $dc_0$, $dt_0$, and $dv^d_0$ are thus defined by the expressions

$$dc_0 = \tilde{\gamma}^1 dc^1; dt_0 = \tilde{\tau}^1 dt^2; dv^d_0 = \tilde{\nu}^1 dv^3$$

where $\gamma^1$, $\tau^1$ and $\nu^1$ are row-vectors of derivatives of $\gamma$, $\tau$ and $\nu$ with respect to the coordinates of $n^1$. The expression $dv^d_0$ denotes vectors of differentials applying to the coordinates in the vectors $n^1$. 
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in equation (2). All coefficients are positive. Furthermore, the following inequalities are implied

\[ a_1 \leq 1 - r^d; \quad a_2 \geq r^d - r^t \]

where \( r^d \) is the average requirement ratio imposed on demand deposits and \( r^t \) the ratio imposed on time deposits. Moreover, it can be demonstrated that \( d_e \) cannot reasonably be expected to exert a significant influence. The higher level theories imply that its omission does not create serious errors.10

Two functions may be derived from the above specification, one for an inclusive concept of the money stock \( M^2 \), (including time deposits) and one for an exclusive concept \( M^1 \) (excluding time deposits). The \( M^2 \) function is obtained more immediately by an adjustment of formula (1) to obtain (3)

\[ dM^2 = m^s + qdB. \] (3)

The symbol \( m^s \) designates the monetary multiplier—itself dependent11 on reserve requirements, currency "spillover" rates and other entities—and \( q \) specifies the proportion of base money injected that affects simultaneously the banks' reserve position and deposit liabilities. Such injections add an amount \( qdB \) directly to the money stock, independent of the response mechanism triggered by the associated emergence of surplus reserves.

The \( M^1 \) formula requires two adjustments. First, the appropriate monetary multiplier, \( m^1 \), is smaller than \( m^s \) because time deposits generated in the multiplier process are excluded from the money stock. Furthermore, the demand deposits converted into time deposits in response to entities other than monetary wealth must be subtracted from the "exclusive money" created by the multiplier process. We obtain thus

\[ dM^1 = m^1 \cdot s + qdB - dt. \] (4)

with \( m^1 \) dependent on reserve requirements etc. in a manner analogous to \( m^s \).

Under the specification of the higher level hypotheses, replacement of \( s \) in (3) and (4) with the aid of (2), suitable integration, and some assumptions about the magnitudes \( q \) and \( a_o \) yields the money supply functions (5) and (6).

\[ M^2 = m_o + m^s(B + L) - m^s a_1c_o + m^s a_2t_o - m^s v^2(i) \] (5)

\[ M^1 = n_o + m^1(B + L) - m^1 a_1c_o - [1 - m^1 a_2] \cdot t_o - m^1 v^2(i) \] (6)

The magnitude \( B + L \) will be referred to as the "extended base." Combination of both \( B \) and \( L \) in a single additive expression reflects the

10. See the argument on p. 96 of "A Schema..."

11. The spillovers into currency and time deposits are given by the derivative of the currency function and the derivative of the time deposit function with respect to monetary wealth, \( M^2 \). See footnote 9 above and for more details see "A Schema..." esp. appendix II.
empirical assumption, supported by numerous tests, that the response in both \( M^1 \) and \( M^2 \) to changes in \( B \) coincides with the response to changes in \( L \). The expression \( v^e(i) \) introduces the dependence of the money stock on interest rates \( i \) operating via the banks' desired cash asset position.

The derivation of the two money supply functions from the higher level theories, only sketchily indicated here, is admittedly tedious. Neither the derivation nor the higher level hypotheses have been introduced in the mistaken belief that the status of an empirical generalization is raised by subsuming it under a higher level hypothesis. The latter must either permit the marshalling of indirect evidence bearing on the appraisal of the empirical hypothesis or enable us to derive statements connecting, in our case, observable money supply behavior with prevailing institutional arrangements. Our stated aim—to develop a link between monetary theory and monetary policy—requires that prevailing institutional arrangements, shaping the relation between policy operations and the money supply, be incorporated within the frame.

The hypothesis constructed provides more than a formula for the money supply. A rich variety of institutional details and policy operations can be assessed with the framework developed. Among the details capable of evaluation are changes in the proportion of checks collected by debiting interbank deposits or changes in the reserve classification of smaller banks in outlying sections of reserve cities. These minor changes are a part of the prevailing monetary landscape and are among the determinants of the magnitude of the monetary multiplier. Analysis of more pertinent operations can also be obtained with the linear hypothesis. Some are indicated briefly.

Foremost, it should be noted that the linear hypothesis yields an answer to questions about the response to open-market operations and changes in requirement ratios. (Statements about discount policy require a transformation of equation (2), separating \( dB \) into \( dA \) and \( dB^* \) and replacing \( dA \) by a suitable function of interest rates.) Furthermore, we may assess the sensitivity of the monetary multiplier to changes in requirement ratios and to variations in the degree to which the banking structure is centralized. Or one may trace the consequences of a redefinition of "net demand deposits" and an abolition of differential requirement ratios. The derivations based on the higher level theories enable us, in particular, to evaluate the significance of currency "spillovers" in the multiplier mechanism. Multiplier values for \( m^2 \) substantially below the reciprocal of \( r^d \), the average requirement ratio, would offer strong evidence in support of the relevant operation of these "spillovers."

b. The Non-Linear Hypothesis

A different hypothesis is outlined in this section. It represents a specific view of the credit market on which banks operate. Money stock and an interest rate emerge from the interaction of the public's asset sup-
ply to banks and the banks' portfolio adjustment. The following relations constitute the basic theory.

Equation (7) describes the banks' desired rate of portfolio adjustment as a function of surplus reserves \( R - R^d \), i.e. the difference between actual reserves \( R \) and desired reserves \( R^d \).

\[
\dot{E^*} = h(R - R^d)
\]  

(7)

Desired reserves \( R^d \) are a function of demand deposits \( D \), time deposits \( T \), a vector of interest rates \( i \) and the discount rate \( \rho \).

\[
R^d = R^d(D, T, i, \rho)
\]  

(8)

The derivatives of \( R^d \) with respect to deposits and the discount rate are positive and with respect to the coordinates of \( i \) negative. Institutional constraints impose a simple partition of \( R^d \) into the following pattern

\[
R^d = R^r + R^e + v
\]  

(8a)

where \( R^r \) denotes required reserves, \( R^e \) designates excess reserves, and \( v \) indicates member banks' vault cash outside "legal reserves" and non-member banks' vault cash. The components can then be specified by (8b)

\[
R^r = r^d D + r^T T
\]

\[
R^e = R^e(i, \rho, D + T).
\]  

(8b)

The symbols \( r^d \) and \( r^T \) indicate average requirement ratios against demand deposits and time deposits. The magnitude \( \delta \) reflects the ratio of net demand deposits to demand deposits adjusted, \( D \). The latter entity is a closer approximation to the volume of checking deposits held by the public. It excludes in particular Treasury deposits, which are included in net demand deposits. While \( \delta \) is certainly not a constant, its variability has been of comparatively small order of magnitude in all but a few periods. We find it useful, therefore, to incorporate the ratio \( \delta \) as a constant. In periods exhibiting large variations in Treasury balances (provided such deposits are subject to reserve requirements) or large variations in the relative position of non-member banks, this restriction must be relaxed. \( \tau \) in eq. (8b) measures the ratio of member banks' time deposits to the system's total time deposits. It is also approximated by a constant. Further, we stipulate that the \( R^e \) function is homogeneous of degree one with respect to total deposits. This permits the following reformulation

\[
R^e(i, \rho, D + T) = e(i, \rho) (D + T)
\]  

(8c)

where \( e \) designates the banks' desired excess reserve ratio as a function of interest rates and the discount rate.

The banks' desired vault cash outside reserves, \( v \), can be exhibited as a function of specific cost and yield items closely related to a comparatively stable institutional pattern. Such a function can be usefully
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derived from a stochastic model bearing on a banks' currency flows. The function derived reveals at best a tenuous dependence of \( v \) on interest rates and explains \( v \) essentially in terms of specific institutional features shaping the banks' marginal costs of and revenues from currency inventories. This analysis also explains the effect of the Federal Reserve's new treatment of vault cash on \( v \).\(^{12}\)

Required reserves may also be rewritten in a form employed in our subsequent analysis

\[
R' = r(D + T) ; \quad r = \frac{\delta D + \gamma T + v}{D + T}
\]

Here \( r \) is a weighted average of reserve requirements and vault cash holdings that depends (1) on the distribution of total deposits between demand and time accounts and (2) the distribution of demand deposits between classes of banks.

The public's supply rate of assets to the banks is specified by relation (9):

\[
E^d = f(i, W, E)
\]

The \( f \)-function is assigned negative derivatives with respect to the \( i \)-coordinates and \( E \), a positive derivative with respect to wealth \( W \).

The public is, furthermore, assumed to adjust its currency holdings and time deposit accounts according to the following patterns

\[
\begin{align*}
\Delta C &= g_1(kD - C^p) \\
\Delta T &= g_2(tD - T)
\end{align*}
\]

Both \( k \) and \( t \) denote the public's desired currency ratio and desired time deposit ratio, relative to \( D \), and both depend on the public's wealth and pertinent cost and yield elements. In particular, the public's desired currency ratio depends negatively on wealth, positively on the banks' relative service charges on checking deposits and negatively on the public's marginal utility for services supplied by banks to lure customers. The desired time deposit ratio depends positively on wealth, positively on interest rates granted on time accounts and the value of incidental services associated with time deposits, negatively on market rates of interest.

Four equations complete the basic theory; one pertains to the banks' borrowing behavior, and the others emanate from the meaning of the terms specified. Equation (12) describes

\[
\dot{A} = a[b(D + T) - A]
\]

the banks' adjustment of their indebtedness to Federal Reserve Banks.

\(^{12}\) It should be noted that, after 1960, the element \( v \) must be adjusted to include only the non-member banks' vault cash.
The symbol $b$ indicates the banks' desired borrowing ratio, defined in a manner such that $b(D + T)$ yields the desired volume of outstanding indebtedness. The desired borrowing ratio is a function of interest rates, $i$, and the discount rate, $p$. The derivative with respect to $p$ is negative and with respect to the $i$-coordinates positive.

The definitional relations may be presented as a group ranging from (13) — (15)

$$B = A + B^*$$  \hspace{1cm} (13)

$$B = R + C^p$$ \hspace{1cm} (14)

$$R + E = D + T + A$$ \hspace{1cm} (15)

Equation (13) introduces the base as the sum composed of discounts and advances plus the adjusted (relatively exogeneous) base. The next equation describes allocation of the base between the banks' cash assets and the public's currency holdings. And equation (15) reproduces the consolidated statement of the banks. The banks' portfolio must be understood to be net of capital accounts and Treasury deposits.

One may certainly wish to explore the structure specified above directly. Some preliminary work, particularly pertaining to the desired $k$, $t$, $b$, and $e$ ratios has been performed. This work revealed the patterns assigned to these ratios in the summary discussion above. The present paper pursues the empirical exploration on a somewhat lower level and applies to the money supply function implied by the above structure. For this purpose we augment the specifications already made with the postulate that all adjustment processes are rapid relative to the time units associated with our data. This postulate entitles us to approximate all dynamic relations with static relations. The auxiliary postulate has testable implications that permit a critical assessment.

The augmented system is approximated by the set of relations

$$B = A + B^*$$ \hspace{1cm} (16)

$$B = R + C^p$$ \hspace{1cm} (17)

$$R = (r + e)(D + T)$$ \hspace{1cm} (18)

$$C^p = kD$$ \hspace{1cm} (19)

$$T = tD$$ \hspace{1cm} (20)

$$A = b(D + T)$$ \hspace{1cm} (21)

$$E = E(i, W)$$ \hspace{1cm} (22)

The reader should be reminded that $e$, $t$ and $b$ are functions of $i$ and/or $p$.

Equations (16) through (22) may be reduced to two equations by appropriate substitution and elimination using the definition of the money supply and the consolidated balance sheet of the banks, eq. (15). $B^*$ and $W$ are specified as relatively exogenous variables to obtain the solutions, eqs. (23) and (24) for the money supply and the bank oriented credit market.
\[ M^2 = m^2 B^a \]  \( (23) \)
\[ (m^2 - 1)B^a = E(i, W) \]  \( (24) \)

\( m^2 \) denotes the monetary multiplier appropriate for the non-linear hypothesis. This multiplier is defined by equation (25)

\[ m^2 = \frac{1 + k + t}{(r + e - b)(1 + t) + k} \]  \( (25) \)

The dependence of the behavior parameters \( e, b, \) and \( t \) on \( i \) or \( \rho \) renders the monetary multiplier dependent on interest rates by construction. Equation (24) may be interpreted to determine one coordinate of the \( i \)-vector, say \( i^1 \), the first one, in terms of the remaining interest rates, the discount rate, wealth \( W \), the adjusted base, the average requirement ratio and the desired currency ratio \( k \). This solution for \( i^1 \) derived from equation (24) may then be substituted at all occurrences of \( i^1 \) in the \( m^2 \) expression of equation (23). In this manner money supply emerges as a function of interest rates \( i^s(s \neq 1) \), discount rate, adjusted base, requirement ratio \( r \) and wealth \( W \). The two equations thus jointly determine the money supply and an interest rate.

It may also be noted that the difference \( e - b \) occurring in the denominator of (25) corresponds to the banks' free reserve ratio, \( f \). The hypotheses constructed thereby enable us to appraise the Federal Reserve's conception of money supply and bank credit centered on free reserves. Free reserves emerge as one of the elements connecting policy variables, summarized in \( B^a \), with the stock of money and the banks' portfolio. As noted, both \( e \) and \( b \), hence \( f \), depend on interest rates and the rediscount rate.

The response of the money stock to variations in the base, reserve requirements, discount rate, the public's wealth and the public's allocation of 'payment money' between checking deposits and currency can be derived by suitable differentiation of equations (23) and (25). The ensuing linear system in the unknown derivatives of \( M \) and \( i^1 \) with respect to the relatively exogeneous variables can be easily solved to yield the intended response patterns. These patterns have been collected in table 1.

An inspection of the table reveals the decisive significance of the comparative order of three elasticities, viz. the elasticity of the public's asset supply to banks with respect to the interest rate \( i^1 \), i.e. \( \varepsilon(E, i^1) \), and the corresponding interest elasticities of the monetary and asset multipliers,

13. To avoid a problem that seemingly bothered a discussant, we reiterate that \( m^2 \) (or \( m^1 \)) is dependent on interest rates and other factors specified earlier. Equations (23) and (24) cannot be read "\( M \) or \( E \) is proportional to \( B^a \)." The proportionality hypotheses are alternatives that can be derived by much simpler means than those we have used. Unfortunately the simpler hypotheses are not supported by the evidence we have examined. The \( m^2 \) are not constants. For an appraisal of the effect on \( m^2 \) of some of the factors shaping the money multiplier, see Table 2 below.
Demand and Supply Functions for Money

TABLE 1
ELASTICITIES OF M² WITH RESPECT TO RELATIVELY EXOGENOUS VARIABLES

Notation: The expression \( \varepsilon(x, y) \) denotes the elasticity of \( x \) with respect to \( y \). In particular, \( \varepsilon(m^2, i^1) \) designates the elasticity of the monetary multiplier \( m^2 \) with respect to the interest rate \( i^1 \), whereas \( \varepsilon(m^2 - 1, i^1) \) indicates the elasticity of the asset multiplier \( (m^2 - 1) \) with respect to \( i^1 \).

|\( a) \) | \( \varepsilon(M^2, B^*) = 1 - \frac{\varepsilon(m^2, i^1)}{\varepsilon(m^2 - 1, i^1) - \varepsilon(E, i^1)} \leq 1 \) |
|\( b) \) | \( \varepsilon(M^2, r^d) = \frac{\varepsilon(m^2, r^d) \cdot \varepsilon(E, i^1)}{\varepsilon(m^2 - 1, i^1) - \varepsilon(E, i^1)} < 0; \varepsilon(M^2, r^d) \geq \varepsilon(m^2, r^d) \) |
|\( c) \) | \( \varepsilon(M^2, r^t) = \frac{\varepsilon(m^2, r^t) \cdot \varepsilon(E, i^1)}{\varepsilon(m^2 - 1, i^1) - \varepsilon(E, i^1)} < 0; \varepsilon(M^2, r^t) \geq \varepsilon(m^2, r^t) \) |
|\( d) \) | \( \varepsilon(M^2, \rho) = \frac{\varepsilon(m^2, \rho) \cdot \varepsilon(E, i^1)}{\varepsilon(m^2 - 1, i^1) - \varepsilon(E, i^1)} < 0; \varepsilon(M^2, \rho) \geq \varepsilon(m^2, \rho) \) |
|\( e) \) | \( \varepsilon(M^2, k) = \frac{\varepsilon(m^2, k) \cdot \varepsilon(E, i^1)}{\varepsilon(m^2 - 1, i^1) - \varepsilon(E, i^1)} < 0; \varepsilon(M^2, k) \geq \varepsilon(m^2, k) \) |
|\( f) \) | \( \varepsilon(M^2, p^t) = \frac{\varepsilon(m^2, i^1) \cdot \varepsilon(E, i^1)}{\varepsilon(m^2 - 1, i^1) - \varepsilon(E, i^1)} \leq \left| \varepsilon(E, i^1) \right| \) |
|\( g) \) | \( \varepsilon(M^2, W) = \frac{\varepsilon(m^2, i^1) \cdot \varepsilon(E, W)}{\varepsilon(m^2 - 1, i^1) - \varepsilon(E, i^1)} \leq \varepsilon(E, W) \) |

Supplementary list of symbols not included in general list:

\( m^2 \) = monetary multiplier of non-linear hypothesis
\( i^1 \) = loan-rate of banks
\( k \) = public's currency ratio

\( \varepsilon(m^2, i^1) \) and \( \varepsilon(m^2 - 1, i^1) \). Because of the decisive role played by the latter multipliers their structure is exhibited in table 2 together with some other elasticities of the monetary multiplier that occur on the right side of the equations in table 1. It is shown under point d of table 2 that \( \varepsilon(m^2, i^1) \) results from the simultaneous operation of five interest elasticities, the (two) interest elasticities of the banks' borrowing and excess reserve ratios, \( b \) and \( e \), the elasticity of the public's time deposit ratio \( t \) with respect to rates offered on such accounts, \( i^1 \), modified by the dependence of \( i^1 \) on \( i^1 \), and lastly the direct (negative) elasticity of \( t \) with respect to \( i^1 \). The expressions constituting \( \varepsilon(m^2, i^1) \) can be grouped in a manner that yields comparatively stable coefficients. It is consequently exhibited as a linear combination of two linear combinations of interest elasticities pertaining to the behavior parameters \( b \), \( e \), and \( t \). The coefficients \( a_1 \) and \( a_2 \) are rational functions of the behavior parameters.
and can be shown to be relatively stable over time. The first "inside combination" applies to $\varepsilon(b, i^1)$ and $\varepsilon(e, i^1)$ with coefficients $b$ and $-e$. The second inside combination has $\varepsilon(t, i^1)$ and $\varepsilon(t, i^1)$ with coefficients $\varepsilon(i^1, i^1) \geq 0$ and unity respectively. The first inside combination is

\[ \alpha_1 = \frac{\delta D}{D + T}; \quad \alpha_2 = \frac{\tau T}{D + T} \]

**TABLE 2**

**SOME ELASTICITIES OF THE MONETARY AND ASSET MULTIPLIER**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $\varepsilon(m^2, r^d) = \frac{(1 + t) \cdot a_1}{(r + e - b) (1 + t) + k}$</td>
<td>$r^d \sim -2.2 r^d$</td>
</tr>
<tr>
<td>b) $\varepsilon(m^3, r^t) = \frac{(1 + t) \cdot a_2}{(r + e - b) (1 + t) + k}$</td>
<td>$r^t \sim -1.15 r^t$</td>
</tr>
<tr>
<td>c) $\varepsilon(m^3, k) = k \left[ \frac{1}{1 + k + t} \right] \sim -1.6 k$</td>
<td></td>
</tr>
<tr>
<td>d) $\varepsilon(m^3, i^1) = a_1 [\varepsilon(b, i^1) \cdot b - \varepsilon(e, i^1) \cdot e] + a_2 [\varepsilon(t, i^1) \cdot \varepsilon(i^1, i^1) + \varepsilon(t, i^1)]$</td>
<td></td>
</tr>
<tr>
<td>e) $\varepsilon(m^3 - 1, i^1) = b_1 [\varepsilon(b, i^1) \cdot b - \varepsilon(e, i^1) \cdot e] + b_2 [\varepsilon(t, i^1) \cdot \varepsilon(i^1, i^1) + \varepsilon(t, i^1)]$</td>
<td></td>
</tr>
<tr>
<td>f) $\varepsilon(m^3, \rho) = \frac{1 + t}{(r + e - b) (1 + t) + k} [\varepsilon(b, \rho) \cdot b - \varepsilon(e, \rho) \cdot e]$</td>
<td></td>
</tr>
</tbody>
</table>

Supplementary list of symbols not included in general list or special list attached to table 1:
- $r =$ banks' cash-asset ratio
- $e =$ banks' excess reserve ratio
- $b =$ banks' borrowing ratio
- $t =$ public's time deposit ratio
- $\delta =$ ratio of member banks' net demand deposits to the system's demand deposit adjusted
- $\tau =$ ratio of member banks' time deposits to system's time deposits
- $i^1 =$ interest rate on time-deposits
definitely positive. The second inside combination has not been sufficiently specified to determine the sign uniquely. It would be positive, if the public's desired time deposit ratio is sufficiently sensitive to \( i \). We stipulate tentatively that this is the case.

The interest elasticity of \( \varepsilon(m^2 - 1, i) \) is presented under point e in Table 2. It again has been arranged as the product of \( b_1 \) and \( b_2 \) with the same "inside combinations" occurring in the specification of \( \varepsilon(m^2, i) \). Inspection of the coefficients reveals that \( b_1 \) and \( b_2 \) are larger than \( a_1 \) and \( a_2 \) in \( \varepsilon(m^2, i) \). It follows, therefore, that \( \varepsilon(m^2, i) < \varepsilon(m^2 - 1, i) \). That is, the interest elasticity of the asset portfolio multiplier is larger than the interest elasticity of the inclusive money multiplier. Through these multipliers a given change in interest rates raises or lowers the response of the money supply and the asset portfolio to policy action and induces changes in the desired reserves of banks.

The reader is invited to shift his attention once more to Table 1. The elasticity of the money stock \( M^2 \) with respect to the adjusted base \( B^* \) is evidently less than unity by an amount determined by the relative order of \( \varepsilon(m^2, i^2) \) and \( \varepsilon(E, i) \). If the interest elasticity of the monetary (and asset) multiplier is small relative to the interest elasticity of the public's asset supply, then \( \varepsilon(M^2, B^*) \) tends to approximate unity quite closely. A comparatively small interest elasticity of the public expressed by \( \varepsilon(E, i) \) would seriously compress the response of the money stock to the variations in the base, and thus, in particular to open market operations.

The expressions under points b, c, d, and e in Table 1 show the same pattern. Monetary policy operating via requirement ratios, open market transactions and the discount rate will effectively modify the money stock, under this hypothesis, if and only if, the interest elasticity of the public's asset supply does not vanish. Under the opposite circumstances variations of the public's desired currency ratio and changes in reserve requirements or in the discount rate exert no influence on the money stock. Just as in the case of open market operations, \( \varepsilon(E, i) \) plays a crucial role in the effective transmission of monetary policy to the money stock.

Two situations have been mentioned by economists as productive of a breakdown of monetary policy. It has been asserted that under deflationary conditions, the public's asset supply becomes insensitive to the banks' loan-rate, \( i \). It has also been contended that at sufficiently low rates the interest elasticity of excess reserves, \( \varepsilon(e, i) \), converges to minus infinity (liquidity trap). Either condition is sufficient to make all elasticities of \( M^2 \) with respect to \( B^* \), \( r^* \), \( r^t \), \( \rho \), and \( k \) approach zero. The broad

---

The case of \( \varepsilon(E, i^t) \) approx. zero has been discussed. The liquidity trap, i.e. \( \varepsilon(e, i^t) \rightarrow -\infty \), means that \( \varepsilon(m^2, i^t) \) and \( \varepsilon(m^2 - 1, i^t) \) both \( \rightarrow \infty \) and the policy multipliers approach zero. See eqs. d and e of Table 2. This is but one of the propositions obtainable from our hypothesis that contradicts the proportionality hypothesis discussed in the previous footnote.
contours observable in the 30's yield little support for either contention asserting a breakdown of policy mechanisms. The money supply declined in early 1937 before the peak was reached, while W continued to rise. Furthermore, B was still expanding, and k was comparatively stable. The major deflationary force at work in the monetary system was the 100% increase in the requirement ratios. The effective response of the money supply to this policy action is not compatible with either the liquidity trap or the vanishing elasticity of the public's asset supply with respect to the loan-rate i.

The non-linear hypothesis has thus been shown to generate statements covering the response patterns to all policy actions. Furthermore, the structure of the hypothesis permits the derivation of other statements relating these response patterns with specific institutional details of the monetary system. In particular, with appropriate semantic rules specified, the hypothesis enables us to assess the impact of variations in the interbank deposit structure, modifications in the definition of net demand deposits and the abolition of reserve requirements. Moreover, it can be applied to an evaluation of Tobin's proposal to have Federal Reserve Banks pay an interest rate equal to the discount rate ρ on excess reserves. Implementation would most likely raise the elasticity ε(M2, ρ) and lower slightly ε(m2, i) and thus raise ε(M2, B) by a probably insignificant amount. The consequences for M of variations in the relative position of member and non-member banks or required membership in the Federal Reserve System, can be traced with the aid of the hypothesis. Similarly, the substantial contribution to the growth rate in the money supply induced by the admission of member banks' vault cash to reserves (December 1959, August 1960, November 1960) can be established and properly subsumed under the hypothesis. We will return to the latter point in our discussion of the empirical results.

III. THE DEMAND FOR MONEY

Until the General Theory stimulated interest in the demand for money, economists' discussions centered around the assumption of more or less constant velocity or a long-run time trend. On the other side of the Atlantic, money demand theory focused on the Marshallian k. Its emphasis on "volitional elements" was on occasion adduced in praise of the relative advantage of the Marshallian k. But this focus remained unspecified and at best a program for investigation. The General Theory introduced a specific class of hypotheses satisfying the following properties:

\[ M = \lambda(i, Y); \quad \lambda_1 < 0 < \lambda_2 \]

i.e. the public's desired money balances are negatively associated with interest rates and positively associated with current national income. The Keynesian class immediately suggested an explication of the
Marshallian k. For this purpose $\lambda$ can be postulated to be homogeneous of degree one with respect to $Y$. We obtain consequently

$$\lambda(i, Y) = \lambda^*(i) Y$$

and $k$ emerges as the function $\lambda^*(i)$.

The analytical tightening of money demand theory introduced by the *General Theory* paved the way for a number of empirical investigations. The central contribution of these investigations pertained to the role of interest rate in the public's demand behavior. The evidence gradually accumulated was consistent with the role attributed to interest rates in the Keynesian class of hypotheses. But it should also be acknowledged that none of these investigations went beyond an inference from a given sample, drawn to appraise the Keynesian type hypothesis against a null or chance hypothesis. To our knowledge, the evidence accumulated was not the result of a critical examination of rival *systematic* hypotheses. Still, the evidence supported the relevant operation of interest rates in the demand behavior for money.

Friedman's implicit denial of the role of interest rates, inherent in his permanent income hypothesis, is therefore rather surprising. He essentially disregarded the accumulated evidence and advanced a hypothesis that competes with the Keynesian class of hypotheses. Prima facie evidence is adduced, but he has made no attempt, thus far, to assess the status of the permanent income hypothesis relative to Keynesian type hypotheses. Friedman's contribution most certainly "stirred up the natives." Several important issues, while not entirely new in money demand analysis, have been most forcefully thrown up for our attention and clamor for critical examination. The issues may be listed as follows:

i. the role of interest rates
ii. the comparative relevance of wealth and current income
iii. the comparative significance of human and non-human wealth

A more fundamental issue bears on the formulation of a higher level hypothesis explaining the characteristic occurrence of money in asset portfolios. Only the vaguest suggestions are provided in the *General Theory*. Tobin developed the suggestions bearing on the "asset demand for money" and explicated the Keynesian notions with two distinct higher level hypotheses. While Keynes separated demand into transaction

15. This task was performed to some extent in our paper, "Predicting Velocity . . . " op. cit.
16. "The Demand for Money: Some Theoretical and Empirical Results," *The Journal of Political Economy* 1959. There is no doubt that Friedman never asserts in his verbal argument that "interest rates do not matter." But we must differentiate carefully between a hypothesis and the surrounding verbal accompaniments. The associated discussion may be interpreted to reveal the processes which led Friedman to formulate his hypothesis. And the crux of the matter is that the irrelevance of interest rates is implied by the stated hypothesis.
demand and asset demand, he offered no analysis and suggested only that transaction balances were determined by the volume of transactions within an environment of predetermined payment schedules. Baumol and Tobin attempted to reformulate and develop this idea. In both cases a function is derived relating transaction balances with the volume of transactions, interest rates and some cost elements associated with conversion of financial assets. Both higher level theories involved a set of "pre-ordained" payment schedules. The payment patterns are treated as entities independent of any volitional elements; they appear in the nature of imposed characteristics of nature simply to be suffered by the economic agents. The only choice available is the time pattern of possible conversions between money and other financial assets, but the time pattern of payments between money and "non-financial commodities", or money and liabilities seems to be historical fate.

The relevance of the Baumol-Tobin analysis cannot be assessed without auxiliary specifications. No semantic rule characterizing "transaction-balances" in terms of observable entities is indicated, nor is there an explicit merger of a subhypothesis bearing on transactions balances with a subhypothesis covering asset balances. Of course, it is possible to merge the subhypotheses constructed by Tobin. The resulting demand hypothesis would exhibit an elasticity less than unity with respect to the "volume of transactions," a positive elasticity with respect to wealth, a negative elasticity with respect to interest rates and a positive elasticity with respect to some measure of uncertainty concerning interest rates. But such a demand hypothesis could be obtained without the partition of money-balances into transactions and asset balances. In order to render this partition substantively significant, a semantic characterization of transactions balances appears mandatory.

To indicate the direction of our own work on a higher level theory, we propose to outline an idea that clarifies a concept of transaction costs and reduces these costs to heterogeneous information patterns about a specific group of phenomena. Information about characteristics of

18. William Baumol, "The Transactions Demand For Cash: An Inventory Theoretical Approach." Quarterly Journal of Economics 1952. James Tobin, "The Interest Elasticity of the Transactions Demand For Money." Review of Economics and Statistics 1954. It is frequently asserted that Baumol derived a "square root formula" describing the transactions demand for money. Actually he specified two distinct sets of conditions. Under the first set balances are refilled only by conversion of financial assets into money. No money is received from the sale of non-financial commodities. Under the second set of specifications laid down, money is received by selling non-financial commodities and conversion from financial assets. The square root formula emerges only under the first set of specifications. Under the second set he derives a rather complex combination of formulae, one of which is a square root formula. Furthermore, the first set of conditions is inherently inconsistent: everybody spends money but nobody seems to receive any.

19. The idea, very roughly sketched in the text, emerged from interminable discussions with A. A. Alchian. It should be noted that we do not absolve Alchian from responsibility for our errors. Further development of this idea will be included in our book on the supply and demand for money.
assets is a commodity that has a non-zero cost. All members of the social system do not acquire the same set of information about each type of asset. For example, security and commodity specialists are observed. The distribution of information among members of society gives rise to the joint occurrence of a positive demand for money and differential transaction costs for distinct asset types.

Suppose there exist N agents and A asset items. Exchanges may occur between (1) asset items, (2) asset items and more or less limited rights to their yields, (3) or between rights to yields. We indicate by \( e(i, j; r, s) \) an exchange between the i\(^{th}\) and the j\(^{th}\) agent, whereby the i\(^{th}\) agent supplies the r\(^{th}\) asset and acquires the s\(^{th}\) asset (j supplies s and acquires r). The expression \( e(i, j; r, s) \) is used as a function of the discrete values associated with the four arguments. It should be understood to measure the exchange ratio between agents i and j involving assets r and s and measured as the price of r in terms of s. A non-uniform distribution of information about the characteristic properties of assets supplies the basic element in our idea. The degree of knowledge about assets is postulated to vary substantially between different agents composing the social group. This postulate implies, in particular, a specialization of information between agents concerning assets and a division of labor concerning acquisition and transmission of information. A knowledge about asset characteristics is derived from a variety of information sources; not the least among these sources is the potential asset supplier.

The unequal distribution of relevant knowledge and information among agents affects the exchange ratios formed in the transactions between the members of the social group. A specialist in asset r, for example, will experience (on the average) a higher exchange ratio of r relative to any asset s than a non-specialist in asset r. Furthermore the degree of information specialization and the resulting variability in the degree of knowledge is sharply differentiated between assets. Some assets exhibit a persistently high variability in the distribution of knowledge, and others tend to have more uniformly distributed information patterns. These differences tend to be reflected in the pattern of exchange ratios defined above. The variability in degrees of knowledge generates, on the average, a corresponding variability in exchange ratios occurring between pairs of agents. It appears to us that “liquidity” and “moneyness” may be usefully associated with this variability in exchange ratios. The greater “liquidity” and closer “moneyness” could then be represented by a smaller (relative) variability. Money could then be defined as the asset item with a minimal (relative) variability of pertinent exchange ratios.

20. An excellent survey published by Gilbert, “The Demand for Money,” Journal of Political Economy 1934, indicates that the existence of a positive demand has traditionally been reduced to “uncertainty” and “time.” His second category seems to cover essentially the same phenomena as our transaction-costs.

J. Marshak derived a positive demand from differential transaction-costs between assets in a paper on “The Rationale for the Demand for Money” Metro economica 1949.
This programmatic summary of our underlying analysis may be slightly elaborated with the aid of the notation already introduced. For every pair of transactors \((i, j)\) and every triad of assets \((r, t, s)\) we define the exchange ratio \(e(i, j; t, r/s)\) implicit in the two exchange ratios \(e(i, j; r, s)\) and \(e(i, j; t, s)\), i.e.

\[
e(i, j; r, t/s) = \frac{e(i, j; r, s)}{e(i, j; t, s)}
\]

This implicit exchange ratio between \(t\) and \(r\) (measured as the price of \(t\) in terms of \(r\)) is simply the arithmetical consequence of dividing the price of \(r\) in terms of \(s\) (occurring between the supplier of \(r\) (\(i\)) and the supplier of \(s\) (\(j\))) by the price of \(t\) in terms of \(s\) (occurring again between the supplier of \(t\) (\(i\)) and the supplier of \(s\) (\(j\))). The variability of this implicit exchange ratio \(e(i, j; t, r/s)\) relative to the associated actual ratio \(e(i, j; t, r)\) emerges as the central element of the market situation determined by the information patterns. We introduce therefore, the following variance measure

\[
\sigma^2(r, t/s) = \frac{1}{N(N-1)} \sum_{i \neq j} [e(i, j; r, t/s) - e(i, j; r, t)]^2
\]

Unevenly distributed knowledge about asset item \(s\) tends to generate a comparatively large variance \(\sigma^2(r, t/s)\). This measure will of course depend in general on the specific assets \(t\) and \(r\). "Liquidity" and "money-ness" of \(s\) are thus properties which involve a comparison of \(\sigma^2\) over the whole spectrum of possible pairs of asset combinations. In particular, we define an asset \(s\) to be money in case the variance \(\sigma^2(r, t/s)\), for any pair \(t\) and \(r\), does not exceed the corresponding variance for any \(s^1\). More specifically:

the asset \(s\) is money \(\text{if and only if} \sigma^2(r, t/s) \leq \sigma^2(r, t/s^1)\)

Substantial variation between assets as to the distribution of pertinent information thus induces a pattern of exchange ratios which will favor the asset with the most uniform knowledge distribution and the least information specialization. This particular asset will occur with a dominant frequency in exchange transactions. If the distributions of information vary enough, it will become advantageous for most agents to intercalate this asset into most of their transactions. This result may be further formalized in a manner leading us to the notion of transaction costs.

Consider an initial asset position of some agent \(i\) denoted by \(p(i; 0)\). He wishes to transform this position into another asset position at prevailing relative prices—which are held constant for our purpose. This constancy permits us to express all assets in terms of some asset arbitrarily chosen as numéraire. With every asset position \(p\) there is consequently
associated its wealth value $W(p)$, a scalar expressed in terms of the numéraire. The initial asset position $p(i, 0)$ may be transformed into terminal positions by a variety of transaction chains, denoted by $c$. The wealth of the terminal position may then be expressed as a function of the initial position $p(i, 0)$ and the transaction chain $c$, i.e. $W[p(i, 0), c]$. The cost associated with the transaction chain $c$ can be defined as the difference

$$W[p(i, 0)] - W[p(i, 0), \times c] \geq 0.$$  

Under the conditions described above, pertaining to the variations in information specialization, a transaction chain $c^1$ will emerge that minimizes the transaction costs over the set of all possible transaction chains simultaneously for all agents in the social group. This condition may be rewritten more explicitly with the aid of the formula:

$$c^1 \text{ is an optimal transaction chain } \overset{\text{def.}}{=} (i)(c)[c^1 \neq c \supset W[p(i, 0), c^1] \geq W[p(i, 0), c]]$$

With this definition of an optimal transaction chain, we assert that the divergent degrees of knowledge discussed above imply the existence of an optimal transaction chain and, furthermore, that this chain is centered on the asset that had previously been defined in terms of the relative variance of exchange ratios.

Our sketchy outline of a more complete analysis (to be delivered in a later context) thus yields the following basic proposition. Divergent distributions of knowledge about the quality characteristics of assets generate a pattern of implicit and actual exchange ratios that induce cost-saving and wealth-maximizing agents to use some specific assets as a focal point of all transaction chains. Holding such assets thus becomes advantageous. Some positive amount enables the transformation of asset positions at smaller cost and consequently larger terminal wealth position. If the variability of information patterns is extended to cover the possible exchange operations emerging in the future, the advantages of holding some positive amount of the asset with minimal relative variance (of exchange ratios) and to use it as the focal point of optimal transaction chains becomes even more pronounced.

Money is thus characterized in terms of relative transaction costs, and the existence of a positive demand is derived from divergent patterns of information specialization pertaining to assets. At this point we begin to move on somewhat more familiar ground. Having outlined the existence of a positive demand, i.e. an allocation of a portion of wealth to money holdings, general demand theory will usefully guide our further consideration. This theory contends that transformation conditions offered on the market and the range of opportunities available to an agent determine the desired amount of any particular asset. This formulation is naturally at best a direction mark, but yields, per se, no hypotheses. The transformation conditions and the opportunity range
must be specified. The latter may be inferred from a description of an individual agent's position. At any moment he inherits from the past an asset position and is confronted with evolving market conditions indicating the relative prices of various assets. The agent is then visualized as adjusting his asset position to the changing market conditions. Such adjustment proceeds within the constraints imposed on the agent by his initial asset position. We propose, therefore, a measure of this asset position as an indicator of the range of opportunities. This measure eliminates "human wealth" in a non-slave economy and reflects only "non-human wealth", denoted by \( W_n \). We do wish to suggest that our argument "establishes" the irrelevance of "human wealth" in demand behavior for money. But our hypothesis does imply the irrelevance of "human wealth" \( W_h \). This issue cannot be settled by plausible and counter-plausible statements. The issue bears on two alternative empirical hypotheses, each quite "plausible" a priori, and must be settled by detailed further investigation.

The transformation conditions confronting an agent seem to be usefully summarized by a vector of relative asset yields, extended to include relative yields on human wealth. The vector is partitioned for our purposes into three subvectors applying to financial assets, physical assets and human wealth. Each subvector is then represented by a single measure; \( r^* \) denotes an interest rate on financial assets, \( p \) designates a relative yield on physical assets and \( d \) indicates a relative yield on human wealth. Our aggregative hypothesis can thus be formulated with the aid of the following expression:

\[
M = f(r^*, p, d, W_n)
\]

\[
f_1 < 0 > f_2 ; \quad f_3 < 0 < f_4.
\]

The function sign with a subscript attached denotes a derivative with respect to the argument in the position indicated by the subscript. This hypothesis is supplemented by auxiliary assumptions to yield the form previously subjected to detailed empirical investigation. One set of assumptions bears on the interrelation between the financial rate \( r^* \) and the physical rate \( p \). Economic theory indicates that we should expect a substantial covariance in relative yields on financial and physical assets. We replace, therefore, the two rates \( r^* \) and \( p \) by a single measure of interest rates provided by the bond yield \( r \).

Another set of assumptions deals with the rate \( d \) on human wealth. This rate may be written as the ratio of income from human resources, \( Y_h \), to human wealth \( W_h \), i.e.

\[
d = \frac{Y_h}{W_h}
\]

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Y_h^* is introduced to denote "expected or permanent income from human resources." The rate d can then be reformulated as the product of a "permanent" component of the discount rate d and a transitory component:

\[ d = \frac{Y_h}{Y_h^*} \cdot \frac{Y_h^*}{W_h} \]

The "permanent" component \( Y_h^* \cdot W_h^{-1} \) is furthermore either assumed to be relatively constant or to be sufficiently covariliated with \( r^* \) and \( p \) over the longer pull to be absorbed into the measure \( r \).

These auxiliary specifications lead us ultimately to

\[ M = g\left(r, \frac{Y_h}{Y_h^*}, W_a\right) \]

\[ g_1 < 0 \text{ and } g_2 > 0. \]

The signs of the derivatives follow immediately from the signs of the initial demand hypothesis. It should be noted particularly that the sign of the transitory component \( Y_h^* \cdot W_h^{-1} \) is negative. An increase in this transitory component thus lowers the desired money balance. The function \( g \) may also be subjected to additional constraints, particularly homogeneity properties. We assume, therefore, explicitly that \( g \) is homogeneous of some degree with respect to each argument separately.\(^{22}\)

We terminate our discursive argumentation covering money demand theory with a short indication of the relation between payment schedules and demand behavior. The notions based on the partition into transaction and asset demand typically convey an impression that timing patterns of payments for liabilities and the disposition of non-financial assets are part of man's environment. We contend that payment schedules have a position similar to market prices. They confront the individual agents as part of the environment, but his optimizing adjustment to this environment modifies both prices and payment schedules. These schedules are not imposed on the group once and for all but emerge from the interacting, searching and adjusting behavior of the agents composing the group. We contend more specifically that the determination of desired money balances and the determination of optimal payment schedules are closely related problems. An average money balance per time unit is uniquely associated with a class of payment schedules. Explanation of variations in average balances is thus equivalent to explanation of changes in classes of payment schedules.

\(^{22}\) It should be noted that the analysis of several problems (changes in degree of anticipation of inflation, variations on institutions bearing on interest offered on deposits etc.) indicates the useful separation of a specific item \( r^* \) from the subvector of financial rates to express the relative yield on money. This yield may be positive or negative, and exhibits a positive derivative. One may probably disregard this yield in situations of unanticipated inflation and deflation or "plain inflation," and in situations showing small variations in rates offered on checking deposits. Even large variations in the latter may be disregarded if one accepts that such variations do not affect the desired total but only the allocation between currency and checking deposits.
Of course, the adherents of the transactions idea quite clearly indicate that schedules pertaining to conversions between money and financial assets result from optimizing behavior. But their argument seems to deny such adjustments for the remaining payment schedules. There may very likely be a difference between various components of the total schedule, similar to the divergent costs applying to the adjustment of different types of capital that give rise to the traditional differentiation between short-run and long-run. We simply suggest that the costs of readjusting various parts of the total payment schedule differ substantially. With minor variations in the cost of holding money, only those parts of the payment schedule will be adjusted that permit modifications at correspondingly small marginal costs. When the costs of holding money rise sufficiently high, the range of adjustment of payment schedules extends over the complete range. Even the most immutable portion of these schedules can be adjusted, at a finite cost. The experiences from hyper-inflation yield some impressive evidence supporting this contention.

IV. ONE-STAGE AND TWO-STAGE LEAST SQUARES ESTIMATES ASSOCIATED WITH THE LINEAR AND NON-LINEAR HYPOTHESES

Some of our accumulated results are discussed in this section. The empirical patterns collected in tables 3 to 8 do not provide a critical test of the hypotheses. Such tests would have to go beyond the classical t-tests of statistical significance (even when reformulated to adjust for the loss in degree of freedom attributable to auto-correlation of residuals present in some cases). These tests only appraise the hypothesis under consideration relative to the perennial chance alternative. No information is thereby provided about the status of a hypothesis relative to a systematic (i.e. non-chance) alternative. We contend that such tests are ultimately much more important than the choice between “naive” one-stage least squares and “sophisticated” two-stage least squares procedures.

a. The Linear Money Supply and Linearized Money Demand

Tables 3, 4, and 5 contain information about the linear money supply hypothesis. If we had restricted ourselves to single equation estimation
TABLE 3a

ESTIMATES DERIVED FROM OSLS AND TSLS FOR LINEAR APPROXIMATION OF
SEVERAL SPECIFICATIONS OF MONEY DEMAND, EXCLUSIVE CONCEPT OF
MONEY STOCK. ANNUAL DATA 1930-1959

<table>
<thead>
<tr>
<th>Specification</th>
<th>Method</th>
<th>$r$</th>
<th>$W/P$</th>
<th>$Y/Y_p$</th>
<th>$P_r$</th>
<th>$e_1$</th>
<th>$R^2$</th>
<th>$D-W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. linear</td>
<td>TSLS</td>
<td>-14.609</td>
<td>.249</td>
<td>.164</td>
<td>.969</td>
<td>.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>regression</td>
<td>OSLS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>elasticity</td>
<td>TSLS</td>
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<td>1.813</td>
<td>.266</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>at mean</td>
<td>OSLS</td>
<td>-.526</td>
<td>1.711</td>
<td>.354</td>
<td></td>
<td></td>
<td></td>
<td>.73</td>
</tr>
<tr>
<td>2. linear</td>
<td>TSLS</td>
<td>-18.994</td>
<td>.201</td>
<td>-54.722</td>
<td>.347</td>
<td>.992</td>
<td>1.76</td>
<td></td>
</tr>
<tr>
<td>regression</td>
<td>OSLS</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td>-.772</td>
<td>.644</td>
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<td>1.420</td>
<td>-.597</td>
<td>.661</td>
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<td></td>
<td>1.57</td>
</tr>
<tr>
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<td>.992</td>
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</tr>
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<td>1.426</td>
<td>-.597</td>
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<td>1.57</td>
</tr>
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<td>.992</td>
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<tr>
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<td>OSLS</td>
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<td>1.420</td>
<td>-.597</td>
<td>.661</td>
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<td>1.57</td>
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TABLE 3b

ESTIMATES OF LINEAR MONEY SUPPLY DERIVED FROM OSLS AND TSLS.
EXCLUSIVE CONCEPT OF MONEY STOCK. ANNUAL DATA 1930-1959

<table>
<thead>
<tr>
<th>Method</th>
<th>$B+L$</th>
<th>$c_0$</th>
<th>$t_0$</th>
<th>$\rho$</th>
<th>$r$</th>
<th>$R^2$</th>
<th>$D-W$</th>
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<td>- .694</td>
<td>- .010</td>
<td>3.341</td>
<td>.996</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>(29.63)</td>
<td>(- 1.38)</td>
<td>(- .03)</td>
<td>(2.14)</td>
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<td></td>
<td></td>
</tr>
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<td>- .408</td>
<td>6.068</td>
<td>1.20</td>
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<td></td>
</tr>
<tr>
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<td>(3.03)</td>
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</tr>
<tr>
<td>OLS</td>
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<td>- .694</td>
<td>- .010</td>
<td>3.341</td>
<td>.996</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>(29.63)</td>
<td>(- 1.38)</td>
<td>(- .03)</td>
<td>(2.14)</td>
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<td></td>
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<tr>
<td>TSLS</td>
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<td>-1.052</td>
<td>- .411</td>
<td>6.068</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(25.74)</td>
<td>(- 1.09)</td>
<td></td>
<td>(3.04)</td>
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</table>

OSLS

<table>
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<th>$t_0$</th>
<th>$\rho$</th>
<th>$r$</th>
<th>$R^2$</th>
<th>$D-W$</th>
</tr>
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<td>OLS</td>
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<td>-1.059</td>
<td>- .413</td>
<td>6.103</td>
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<tr>
<td>(26.09)</td>
<td>(- 1.92)</td>
<td>(- 1.11)</td>
<td>(3.15)</td>
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<td>(20.63)</td>
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<td>(- .02)</td>
<td>(1.60)</td>
<td>(1.48)</td>
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<tr>
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<td>- .716</td>
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<tr>
<td>(19.52)</td>
<td>(- 1.88)</td>
<td>(- 1.36)</td>
<td>(- .97)</td>
<td>(1.95)</td>
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</tr>
</tbody>
</table>
we could easily have juxtaposed the linear money supply with a logarithmic money demand. But we wished to compare the statistical results obtained under OSLS with the results from a joint estimation procedure. This forced us to replace the logarithmic demand hypothesis with a linearized approximation. One-stage and two-stage least squares estimates of this linearized approximation are collected in tables 3a and 4a. Estimates pertaining to the linear money supply are presented in tables 3b and 4b. Tables 3a and 3b present estimates using the exclusive concept (\(M^1\) = currency outside banks plus demand deposits adjusted) and tables 4a and 4b are the corresponding estimates for the inclusive concept (\(M^2 = M^1 + \text{time deposits at commercial banks}\)).

The reader is invited to inspect the results for the demand function. Three different specifications are presented. All four include interest rate \(r\) (bond yield), an adjusted Goldsmith measure of the public's tangible and non-human wealth deflated according to an appropriate price index supplied by Goldsmith, and a deflator of national income \(P_y\). Rows 2, 3, and 4 contain in addition the transitory component \(Y/Y^*\), i.e. the ratio of current net income to (Friedman's) permanent income. This ratio was introduced as a crude but available index of the intended magnitude \(Y^*/Y^{*}\). We surmise the existence of a close correlation between the intended magnitude and its proxy. The last specification considered (rows 3 and 4 in tables 3a and 4a) insert one more variable, viz. a moving

### TABLE 4a
**ESTIMATES OF LINEARIZED APPROXIMATION TO MONEY DEMAND DERIVED FROM OSLS AND TSLS. INCLUSIVE CONCEPT OF MONEY STOCK. ANNUAL DATA 1930-1959.**
*(t statistic in parentheses)*

<table>
<thead>
<tr>
<th></th>
<th>(r)</th>
<th>(W/P_a)</th>
<th>(Y/Y^*)</th>
<th>(P_y)</th>
<th>(\sigma_1)</th>
<th>(R^2)</th>
<th>(D-W)</th>
</tr>
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<tr>
<td>1. linear regression</td>
<td>—13.206</td>
<td>.379</td>
<td>.100</td>
<td>.996</td>
<td>1.01</td>
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<tr>
<td></td>
<td>(—10.39)</td>
<td>(10.69)</td>
<td>(.79)</td>
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<tr>
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<td>OSLS</td>
<td>—.346</td>
<td>1.925</td>
<td>.170</td>
<td>.96</td>
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</tr>
<tr>
<td></td>
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<td>.635</td>
<td>.111</td>
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<tr>
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<td>—44.810</td>
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<td>.997</td>
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<tr>
<td></td>
<td>(—11.56)</td>
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<td>(—3.99)</td>
<td>(2.40)</td>
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<td>.350</td>
<td>1.66</td>
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<tr>
<td></td>
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<td>—.457</td>
<td>.339</td>
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<tr>
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<td>—59.445</td>
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<td>.997</td>
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<td>(—4.23)</td>
<td>(2.96)</td>
<td>(2.01)</td>
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<td>—.448</td>
<td>1.694</td>
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<td>.410</td>
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<tr>
<td></td>
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<td>(—3.99)</td>
<td>(2.95)</td>
<td>(1.62)</td>
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<tr>
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<td>1.694</td>
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<td>1.668</td>
<td>—.518</td>
<td>.443</td>
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</table>
Demand and Supply Functions for Money

Three-year standard deviation of the bond yield as a measure of uncertainty concerning credit market conditions. Under the Tobin theory of asset demand for money, uncertainty about interest-rates operates to raise the desired allocation to money balances. Unfortunately, no analysis developing the significance of such uncertainty ever pursued the argument to the semantic characterization of this uncertainty. A tentative choice was made in the selection of the standard deviation indicated. Rows 3 and 4 in tables 3a and 4a exhibit the same specification of demand. Their OLS estimates are, therefore, necessarily identical. Their TSLS estimates differ slightly because of combination with a different specification of the linear money supply hypothesis to be discussed shortly.

Each row of tables 3a and 4a is subdivided into two subrows. The first subrow exhibits the estimated (TSLS) coefficients of the linear approximation with the computed t-values indicated in parentheses below the coefficient values. A rapid glance will convince the reader that all coefficients of the interest rate are statistically significant at the usual levels, even if we adjust for the loss in degrees of freedom due to the auto-correlation of residuals. Similarly, deflated wealth and the index of transitory income are always significant at the standard levels.

TABLE 4b
(t statistic in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>B + L</th>
<th>B* + L</th>
<th>e_0</th>
<th>t_e</th>
<th>p</th>
<th>r</th>
<th>R^2</th>
<th>D - W</th>
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<tr>
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<td>.4764</td>
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<td>(2.70)</td>
<td>(2.74)</td>
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<td>(3.71)</td>
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<tr>
<td>3.</td>
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<td>(28.307)</td>
<td>(-1.99)</td>
<td>(1.16)</td>
<td>(3.71)</td>
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<tr>
<td>4.</td>
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<td>(-.93)</td>
<td>(2.57)</td>
<td>(1.30)</td>
<td>(.73)</td>
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<td>3.454</td>
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<td>-6.285</td>
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<td></td>
<td></td>
<td>(20.55)</td>
<td>(-2.02)</td>
<td>(.14)</td>
<td>(-1.22)</td>
<td>(2.28)</td>
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</table>
income deflator (Py) on the other hand occurs significantly (at the standard levels) only when the transitory component is included. The uncertainty measure, on the other hand, does not seem to be significant. In all other regressions appraised in our work, we were never able to infer a significant operation of "uncertainty" on desired money balances. But this may be entirely due to the specific measure chosen and may only reflect the semantic characterization incorporated in our analysis. This issue seems worthy of further investigation, particularly in the context of "shorter-run data." The reader may also verify that all signs are compatible with the hypotheses under consideration. This is particularly important for the transitory component \(Y/Y_p\), for which the demand hypothesis implies a negative sign.

The second subrow of each row presents elasticities evaluated at the sample means of the magnitudes specified in the regression. Each cell in these subrows contains two elasticities; the upper one is inferred from TSLS estimates; the lower has been obtained from the OSLS estimates. With the exception of row 1 of table 4a, i.e. the \(M^2\) function with arguments \(r, W/P_a\) and \(Py\), the OSLS and TSLS-estimates practically coincide. A look at the last column of each table reveals however that the joint estimation procedure lowers the degree of auto-correlation of the residuals. The Durbin-Watson statistic is consistently higher in the TSLS case than in the OSLS case. We also note that inclusion of the transitory component \(Y/Y_p\) contributes to remove a substantial portion of the auto-correlation, revealed by the Durbin-Watson statistic, and makes the presence of auto-correlation indeterminate or non-significant.

A comparison with some of our previously published results is intriguing but poses some questions to be analyzed. The absolute value of the interest elasticity in the \(M^1\) function exceeds the value of the corresponding coefficient in the \(M^2\) function. This has been shown in some of our previous publications to be an implication of the hypothesis. Our previous analysis also indicated that the wealth elasticity of the \(M^1\) function is less than the wealth elasticity of the \(M^2\) function. This still holds for the new results presented (with the exception of row 1), but the wealth elasticity of the \(M^2\) function substantially exceeds unity and thus deviates from our previous results. But our previous results were either inferred from a long sequence of observations or averaged from short samples moving over a long sequence. In either case the war years were comparatively submerged or omitted. In the present context the years from 1941 to 1946, dominated by control of interest rates and only partly extended, via price-controls, to control over the yield on physical capital, loom with substantial weight in the sample. This modification of the sample could explain the larger wealth elasticity. The reader should further notice a peculiar relation between wealth elasticity and price elasticity; their sum is without exception very close to 2. Under the hypotheses the sum of the two elasticities would be 2 for the \(M^1\) function and somewhat larger for the \(M^2\) function. It appears that a
given total effect is improperly distributed between the two components, attributable very likely to a considerable measure of multicollinearity.

Tables 3b and 4b collect some results bearing on the linear money supply hypothesis. In order to obtain these results, the marginal propensities c and t of the public's currency and time deposit holdings relative to monetary wealth were estimated by a variety of procedures. These estimates centered on \( \frac{1}{2} \) for the marginal propensity to hold currency with respect to monetary wealth and \( \frac{1}{2} \) for the corresponding entity associated with time deposits. The magnitudes \( c_t \) and \( t_t \) were then computed by subtracting \( cM^2 \) and \( tM^2 \) from \( C^o \), currency outside banks and \( T^o \), time deposits. We notice that the multiplier value estimated under the TSLS procedure is persistently higher than the value obtained with OLS. A similar pattern holds for the interest coefficients. The \( t \)-values of the multiplier estimates are lowered by TSLS but their magnitude still reveals the fundamental significance of the extended base. On the other hand, the \( t \)-values of the interest coefficients are raised by the TSLS procedure, and so is the Durbin-Watson statistic with one exception. The latter would suggest in almost all cases the presence of auto-correlated residuals.

The reader may also note that all the constraints imposed by the hypothesis on orders of magnitudes of the coefficients associated with \( B + L \), \( c_o \) and \( c_t \) are satisfied with the exception of the last row of table 4b. The \( t_t \) coefficient violates the inequality implied by the hypothesis, viz. \( m^2 \cdot a_2 \geq r^d - r^t \). Other comparative orders implied by the hypothesis are reflected in the empirical patterns. The \( m^1 \) multiplier is consistently below the \( m^2 \) multiplier. Similarly the \( c_o \) coefficients associated with the \( M^1 \) functions are numerically smaller than the corresponding coefficients in the \( M^2 \) functions. Furthermore, the \( t \) coefficients are always negative in the \( M^1 \) hypothesis (table 3b) and always positive for the \( M^2 \) function (table 4b) as the linear money supply hypothesis specifies.

The inferred multiplier values yield important information concerning the significant operation of currency patterns in the monetary mechanisms. Both pronouncements by Federal Reserve authorities and the discussions of economists deny, more or less explicitly, the relevant occurrence of currency patterns in the money supply process. If such patterns were inoperative, only variations in the banks' desired cash asset position could explain a response to changes in the base by less than the reciprocal of suitably averaged reserve requirement ratios. The linear hypothesis has been formulated in a manner to eliminate the banks' response to evolving credit market conditions from the multiplier mechanism. This separation permits, therefore, an appraisal of the relative importance of currency patterns in this mechanism. With the

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24. The response of one of the discussants suggests to us to emphasize once more: The values of the marginal properties c and t were estimated independently of the money supply function and then used to compute \( c_t \) and \( t_t \).
banks reserve adjustment to changing interest rates eliminated, the multiplier values for \( M^1 \) would have to be around 6 or 7 (and for \( M^2 \) above those values) if currency patterns do not operate in the multiplier mechanism. We remind the reader that \( m^1 \) moves in the neighborhood of 2.7 and 2.8, while \( m^2 \) moves around 3.3 to 3.5. These results strongly suggest the relevant operation of currency patterns.

The last row in tables 3b and 4b applies to a reformulation of the linear hypothesis previously discussed in part II.a. The total base, \( B \), was partitioned into discounts and advances and the remaining (endogeneous) base, \( B^a \). The discounts and advances were made dependent on interest rates and discount rate \( p \). The remaining part of the base, \( B^a \), is called the adjusted base. The hypothesis implies that regressions with the adjusted base yield numerically larger interest coefficients than regressions with the base. This pattern is confirmed by the TSLS estimates in the table. An inspection of the last rows in these tables also reveals the marked difference in the OSLS and TSLS estimates for all coefficients with the exception of the monetary multiplier (i.e. the \( B^a + L \) coefficient). In particular, we note that the sign of the \( p \) coefficient obtained under OSLS is not compatible with the hypothesis, whereas both interest coefficients are compatible with the hypothesis under the TSLS procedure.

The reader will note, that in general, the two estimation procedures do not yield the same results for the money supply function. The \( c_0 \) and \( t_0 \) interest coefficients satisfy more clearly the constraints of the linear hypothesis when estimated by TSLS than by OSLS. Only the monetary multiplier is little affected by the change in estimation procedure.

Our previous statistical investigations indicated that it was usually difficult to catch the operation of \( c_0 \) and \( t_0 \) in the money supply process from annual data covering long periods. Our inquiries suggested to us that the factors shaping the behavior of the (partial) differentials \( dC_0 \) and \( dL \) of the demand functions for currency and time deposits are dominated by cyclic patterns and become easily submerged in the very long-run. The \( c_0 \) and \( t_0 \) coefficients rarely emerged with t-values indicating clear statistical significance in the long-period estimates.

To consider this problem somewhat further, quarterly data were used for a variety of periods. Tables 5 summarize some typical results obtained for both concepts of money stock, the base \( B \) and the adjusted base \( B^a \). All regressions presented are based on first differences between the values in corresponding quarters of adjacent years. Even so, the smallest \( R^2 \) is .862. The signs and constraints imposed by the hypothesis are satisfied by the regressions involving the adjusted base. The bondrate \( i^1 \) occurs non-significantly and at times has a negative sign. The bond rate has never played an important role in our estimates. The Treasury bill rate and discount rate dominate the scene completely in all the statistical results accumulated thus far.

Table 5b covers a sample period which goes four years beyond the terminal year used in table 5a. The regressions presented in 5b thus
Demand and Supply Functions for Money

TABLE 5a
(t statistic in parentheses)

<table>
<thead>
<tr>
<th>Model</th>
<th>(\Delta M_1)</th>
<th>(\Delta M_2)</th>
<th>(\Delta M_3)</th>
<th>(\Delta M_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(0.418 + 2.556 \Delta(B + L) - 2.204 \Delta c_0 + 0.492 \Delta t_0 - 0.693 \Delta t^1 + 0.186 \Delta t^2 + 0.137 \Delta p)</td>
<td>(0.53 + 2.285 \Delta(B + L) - 1.588 \Delta c_0 - 0.511 \Delta t_0 - 2.809 \Delta t^1 - 0.351 \Delta t^2 - 2.570 \Delta p)</td>
<td>(0.464 + 2.580 \Delta(B + L) - 2.671 \Delta c_0 + 0.564 \Delta t_0 + 0.759 \Delta t^1 - 0.206 \Delta t^2 + 0.147 \Delta p)</td>
<td>(0.502 + 2.538 \Delta(B + L) - 1.764 \Delta c_0 + 0.543 \Delta t_0 + 3.121 \Delta t^1 + 0.391 \Delta t^2 - 2.855 \Delta p)</td>
</tr>
<tr>
<td></td>
<td>(17.29)</td>
<td>(8.55)</td>
<td>(17.29)</td>
<td>(8.55)</td>
</tr>
<tr>
<td></td>
<td>(—8.42)</td>
<td>(—5.00)</td>
<td>(—8.42)</td>
<td>(4.78)</td>
</tr>
<tr>
<td></td>
<td>(2.50)</td>
<td>(6.68)</td>
<td>(2.49)</td>
<td>(6.68)</td>
</tr>
<tr>
<td></td>
<td>(.27)</td>
<td>(.28)</td>
<td>(.27)</td>
<td>(.28)</td>
</tr>
<tr>
<td></td>
<td>(.21)</td>
<td>(.21)</td>
<td>(.21)</td>
<td>(.21)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>.956</td>
<td>.862</td>
<td>.963</td>
<td>.885</td>
</tr>
</tbody>
</table>

Supplementary list of symbols:
i* = treasury bill rate
i1 = long-term bond rate

TABLE 5b
(t statistic in parentheses)

<table>
<thead>
<tr>
<th>Model</th>
<th>(\Delta M_1)</th>
<th>(\Delta M_2)</th>
<th>(\Delta M_3)</th>
<th>(\Delta M_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(-0.212 + 2.445 \Delta(B + L + \pi) - 2.400 \Delta c_0 - 0.593 \Delta t_0 + 0.679 \Delta t^1 + 0.306 \Delta t^2 - 0.386 \Delta p)</td>
<td>(-0.236 + 2.717 \Delta(B + L + \pi) - 2.711 \Delta c_0 + 0.452 \Delta t_0 + 0.755 \Delta t^1 + 0.340 \Delta t^2 - 0.429 \Delta p)</td>
<td>(-0.236 + 2.717 \Delta(B + L + \pi) - 2.711 \Delta c_0 + 0.452 \Delta t_0 + 0.755 \Delta t^1 + 0.340 \Delta t^2 - 0.429 \Delta p)</td>
<td>(-0.236 + 2.717 \Delta(B + L + \pi) - 2.711 \Delta c_0 + 0.452 \Delta t_0 + 0.755 \Delta t^1 + 0.340 \Delta t^2 - 0.429 \Delta p)</td>
</tr>
<tr>
<td></td>
<td>(18.59)</td>
<td>(18.60)</td>
<td>(18.60)</td>
<td>(18.60)</td>
</tr>
<tr>
<td></td>
<td>(—13.36)</td>
<td>(—13.36)</td>
<td>(—13.36)</td>
<td>(—13.36)</td>
</tr>
<tr>
<td></td>
<td>(3.05)</td>
<td>(3.05)</td>
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</tr>
<tr>
<td></td>
<td>(.50)</td>
<td>(.57)</td>
<td>(.57)</td>
<td>(.57)</td>
</tr>
<tr>
<td></td>
<td>(.84)</td>
<td>(.84)</td>
<td>(.84)</td>
<td>(.84)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>.957</td>
<td>.882</td>
<td>.985</td>
<td>.958</td>
</tr>
</tbody>
</table>

Supplementary list:
\(\pi\) = cumulated sum of vault cash released for absorption in legal reserves
subsume a major institutional change in our monetary system, viz. the stepwise absorption of member banks' vault cash into reserves (December 1959, August and November 1960). This institutional modification can be incorporated easily into the linear hypothesis. The specifications made under the hypothesis imply that the cumulated sum of the stepwise release must be incorporated additively with the extended base to form an expression $B + L + \pi$, where $\pi$ designates the indicated cumulated sum. A comparison of 5a and 5b thus yields additional material bearing on the quality of the hypothesis. Inconsistent results would have seriously damaged the hypothesis, whereas the strongly consistent pattern exhibited by the two tables only shows successful passage of another hurdle. We also wish to draw attention to the intercepts in the regressions. According to the hypothesis taking first differences eliminates any constant term. The statistical result support this implication also. In particular, the intercept contributes very little to the average first difference of the dependent variable.

A potential objection has been raised about the meaning of some statistical results. It was asserted that the computation of $c_o$ and $t_o$ introduced a serious measure of implicit correlation attributable to the implicit occurrence of $M^2$ (or $M^1$) on the right side of the regressions. The implicit occurrence is correctly stated, but we question the assertion that therefore implicit correlation occurs and makes impossible the separation of systematic from spurious (i.e. in our case induced purely by measurement procedures) ingredients in the regression. This objection is too impressionistic. The structure of the magnitudes occurring in the regressions must be more carefully considered. The manner of introducing $c_o = C^o - .2M^2$ and $t_o = T - .1M^2$ into the regressions yields some indication about the pattern of implicit correlation generated. In both $M^1$ regressions ($i = 1, 2$) $c_o$ and $t_o$ introduce negative implicit correlation. But the hypothesis asserts a divergent pattern of systematic associations. In particular, the hypothesis implies opposite signs for the $t_o$ coefficient. Actually, the hypothesis implies a much more constraining statement, viz. that the sum of the numerical value of the $t_o$ coefficients in the $M^1$ and $M^2$ regression be slightly above unity. Confirmation of these patterns in tables 3, 4 and 5 yields some measure of support for our contention that the regression results reveal the systematic association traced by the hypothesis and are not just the spurious creatures of measurement procedures.

But more can be done to appraise the nature of the $c_o$ and $t_o$ coefficients. Only a few results of these additional appraisals are collected in table 6. Analysis begins with the replacement of $c_o$ and $t_o$ according to the formulae

$$c_o = C^o - .2(C^o + D + T); \quad t_o = T - .1(C^o + D + T).$$
Demand and Supply Functions for Money

**TABLE 6**
**INDIRECT ESTIMATES OF THE LINEAR MONEY SUPPLY.**
**ANNUAL FIRST DIFFERENCES OF QUARTERLY DATA.**

The equation estimated by OSLS has the form

\[ \Delta D^p = b_0 + b_1 \Delta(B + L + \pi) + b_2 \Delta C^p + b_3 \Delta T + b_4 \Delta I^p + b_5 \Delta P \]

The coefficients of the linear hypothesis can be exhibited as functions of the \( b_i \) coefficients and the marginal propensities to hold currency and time deposits with respect to \( M^2 \). These marginal propensities have been estimated independently to approximate \( 1/5 \) and \( 1/10 \). The values \( 1/5 \) and \( 1/10 \) were used in conjunction with the estimated \( b_i \) coefficients to compute a set of coefficients for the linear money supply hypothesis for the inclusive concept of the money stock. For two postwar periods, the coefficients are:

1. \( \frac{I}{1949} - \frac{IV}{1958} \)  
   - \( B + L \) coefficient: 2.763  
   - \( c_0 \): -1.860  
   - \( t_0 \): 0.644

2. \( \frac{I}{1949} - \frac{IV}{1962} \)  
   - \( B + L + \pi \) coefficient: 2.634  
   - \( c_0 \): -1.861  
   - \( t_0 \): 0.619

The reader is once more reminded that the values of the marginal propensities (i.e. \( \frac{1}{5} \) and \( \frac{1}{10} \)) have been estimated independently in a variety of ways to remove all possible influences of implicit correlation affecting the estimates of these marginal propensities. The magnitudes \( c_0 \) and \( t_0 \) are then replaced with the suitable expression, and the resulting equation is solved for demand deposits adjusted. Given the marginal propensities of currency demand and time deposit demand, the coefficients of the resulting D functions are rational functions of the coefficients in the original \( M^2 \) functions. Inspection of this D function will assure the reader that no trace of implicit correlation remains. The coefficients should therefore only reveal chance operations or systematic associations. To reduce multi-collinearity, only first differences between values of corresponding quarters in adjacent years were used. The monetary multiplier for \( M^2 \) and the \( c_0 \) and \( t_0 \) coefficients of the \( M^2 \) function were computed from the estimated coefficients of the D function by means of the connecting link provided by the rational functions indicated above. The results are assembled in table 6. First we note the close clustering of the multiplier estimates with the values presented in table 5. Inspection of the \( c_0 \) coefficients in tables 5 and 6 also suggests the likely operation of implicit correlation in the case exhibited in table 5. The \( c_0 \) coefficients are substantially smaller (numerically) than the indirect estimates inferred under removal of implicit correlation. Implicit correlation appears to have raised the numerical value of the \( c_0 \) coefficients in table 5 beyond the level of systematic association expressed by the hypothesis. It should be added that the indirect estimates of the interest coefficients are essentially the same as the estimates collected in table 5.
b. The Non-Linear Hypothesis

The non-linear money supply hypothesis yields the following expressions

\[ M_1 = \frac{1 + k}{(r + e - b)(1 + t) + k} \cdot B^a; \]
\[ M_2 = \frac{1 + k + t}{(r + e - b)(1 + t) + k} \cdot B^a \]

Logarithmic differentiation of either expression leads to the formula

\[ \frac{dM}{M} = \left[ \frac{dB^a}{B^a} + \varepsilon_1 \cdot \frac{dr^4}{r^4} + \varepsilon_2 \cdot \frac{dr^1}{r^1} + \varepsilon_3 \cdot \frac{dk}{k} \right] \]
\[ + \varepsilon_4 \cdot \frac{de}{e} + \varepsilon_5 \cdot \frac{db}{b} + \varepsilon_6 \cdot \frac{dt}{t} \]

This formula describes the relative change of the money stock in terms of relative changes in the adjusted base, requirement ratios \( r^4 \) and \( r^1 \), the public's currency ratio \( k \) and time deposit ratio \( t \), and the banks' excess reserve and borrowing ratios \( e \) and \( b \). The coefficients \( \varepsilon_1 \), constituting the linear combination of relative changes on the right side of the expression, are the elasticities of the appropriate multiplier \( m^1 \) with respect to the parameter associated with \( e^1 \). These elasticities are rational functions of the behavior parameter \( k, t, r, e, \) and \( b \). Our investigations indicate that these elasticities exhibit a radically smaller coefficient of variation than the associated parameter (or its relative change).

We note a useful partition of the differentiated formula. The first three terms mirror all the major policy actions of the Federal Reserve authorities. The fourth term reflects the public's currency patterns. The first four terms are thus policy imposed on the system or currency behavior essentially independent of credit market conditions. We do not wish to convey that these currency patterns are independent of other feedback relations from the economic process. We strongly suspect a cyclical component in \( k \) related to the cyclical patterns of income, retail sales and wages. But we do not specify this link at present and only separate influences operating via the credit markets. These influences work via the last three terms of the differentiated expression. The relative changes of \( e, b, \) and \( t \) are thus visualized to depend on similar changes in interest rates, including the discount rate.

Integration of the differentiated and partitioned formula thus yields the following relations

\[ \log M^1 = K^5 + \varepsilon_1 (\log i) \]
\[ \log M^2 = K^6 + \varepsilon_2 (\log i) \]

\( K^5 \) and \( K^6 \) are defined as linear combinations of \( \log B^a, \log r^4, \log r^1, \) and \( \log k \) with the appropriate elasticities. The expressions \( \varepsilon_1(\log i) \) are
functions of several interest rates (in logs) including the log of the discount rate.

Similar formulae may be derived involving the base in lieu of the adjusted base. The logarithmic differentials of $M^1$ and $M^2$ would be exhibited as linear combinations of relative changes in the base $B$ and the behavior parameters. However, the borrowing ratio $b$ has been eliminated and the elasticities $\varepsilon^n$ will differ somewhat from the previous case. Integration again yields two relations

$$\log M^1 = K^1 + \varepsilon^{11} (\log i)$$
$$\log M^2 = K^2 + \varepsilon^{22} (\log i)$$

of a type already introduced. $K^1$ and $K^2$ differ from $K^5$ and $K^6$ only by the replacement of $\log B$ by $\log B^*$. A major implication of the non-linear hypothesis, revealed by the omission of the borrowing ratio, $b$, in the differentials involving the base, $B$, are the comparatively low interest elasticities associated with the $K^1$ and $K^2$ variables.

The money supply functions involving $K^1$, $K^2$, $K^5$, and $K^6$ were combined with a logarithmic demand function. The statistical results from OLS and TSLS estimates are presented in tables 7a and 7b. Four rows are presented in each table, and the TSLS estimates refer to rows in matching position. Only one demand hypothesis is presented for both $M^1$ and $M^2$, containing as arguments the bond yield $r$, deflated wealth, $W/P_a$, and the income deflator, $P_y$. The $M^1$ demand function was combined with the $K^1$ function and the $K^5$ function.

A comparison of the first two rows in table 7a shows no relevant difference in the elasticities of the demand function. Similarly the elasticities of the $M^2$ demand function are not sensitive to combination with a $K^2$ or $K^6$ supply function. As before, the OLS and TSLS estimates of the demand elasticities practically coincide. Serial correlation of the residuals is not significant in the $M^1$ functions despite the absence of $Y/Y_P$. Consistent with our previous results the interest elasticity of $M^2$ demand is numerically smaller than the interest elasticity of $M^1$ demand. But the larger wealth elasticity of $M^1$ demand is not compatible with our previous results and raises some problems for further inquiry. Once again, the

25. The regressions of the demand function apply to a further elaboration of the demand hypothesis discussed in Meltzer's JPE (1963) article. If $M = f(r, Y/Y_p, W_n)$ is homogeneous of degree $s$ with respect to $W_n$, we may write $f(r, Y/Y_p, W_n/P_p)P^*_s = f(r, Y/Y_p, W_n)$ where the degree $s$ depends on the definition of money balances. Dividing both sides by $P^*_s$, we obtain, real balances

$$M = f(r, Y/Y_p, W_n/P_p)P^*_s$$

The ratio $P_s/P^*_s$ may be related to the rate of interest and absorbed with the latter magnitude as in the earlier formulation. Multiplication with $P_s$ then yields

$$M = f^*(r, Y/Y_p, W_n/P_p)P_s$$
price elasticity appears to be too small and the wealth elasticity too
large, but the meaning of this statement should be cautiously watched.
It presupposes the relevance of the underlying hypothesis.

The overpowering importance of the forces operating in the money
supply process which are summarized by the K magnitudes is clearly
brought out in table 7b. The coefficients of these K factors barely differ
significantly from unity. Even with proper adjustment in the t-values for
the loss in degrees of freedom attributable to auto-correlated residuals,
the elasticities with respect to the K's remain statistically very significant.
We also notice that OLS and TLS yield almost identical estimates for
the K elasticities. On the other hand TLS definitely improves the
quality of the estimated interest coefficients and also lowers somewhat
the auto-correlation of residuals. This auto-correlation could very likely
be caused by the approximation involved in our procedure which used
a differential to approximate finite differences. Non-linear terms in-
volving relative changes, in k t, b etc. were thus omitted. The observed
auto-correlation may possibly reflect this omission. Additional inquiries
seem appropriate on this point.

The pattern of interest elasticities obtained confirms the hypothesis.
The $M^2$ supply function always shows a larger interest elasticity than the
Demand and Supply Functions for Money

TABLE 7b
OSLS and TSLS Estimates of Non-Linear Money Supply.
Annual Data 1929-1959.
(t statistic in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>K1 = 1, 2</th>
<th>K1 = 5, 6</th>
<th>r</th>
<th>q</th>
<th>R²</th>
<th>D — W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>OLS</td>
<td>.987 K1</td>
<td>—</td>
<td>.013</td>
<td>.996</td>
<td>.667</td>
</tr>
<tr>
<td></td>
<td>(71.61)</td>
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<td>(—.27)</td>
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<tr>
<td></td>
<td>TSLS</td>
<td>.991</td>
<td>.019</td>
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<td>.662</td>
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<tr>
<td></td>
<td>(70.34)</td>
<td></td>
<td></td>
<td>(.37)</td>
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</tr>
<tr>
<td>2.</td>
<td>OLS</td>
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<td>.164</td>
<td>—</td>
<td>.118</td>
<td>.994</td>
</tr>
<tr>
<td></td>
<td>(58.27)</td>
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<tr>
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<td>TSLS</td>
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<td>.312</td>
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<tr>
<td></td>
<td>(43.83)</td>
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<td></td>
<td></td>
<td>(—3.05)</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>OLS</td>
<td>.945 K2</td>
<td>.244</td>
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<td>(81.54)</td>
<td>(5.99)</td>
<td></td>
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<td></td>
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<td>.949</td>
<td>.276</td>
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<td>.765</td>
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<td>4.</td>
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<td>.069</td>
<td>.995</td>
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<td>(68.23)</td>
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<td>.721</td>
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<td>.456</td>
<td>.998</td>
</tr>
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<td></td>
<td>(50.68)</td>
<td>(3.52)</td>
<td></td>
<td></td>
<td>(—1.87)</td>
<td></td>
</tr>
</tbody>
</table>

M₁ function, and the regressions involving the adjusted base (i.e. K₅ and K₆) yield substantially larger interest elasticities than the regressions involving the base (i.e. K₁ and K₂) in accord with the underlying hypothesis.

Two additional sets of estimates, not reported in the tables, were obtained for the demand and supply functions. One set included the transitory income ratio, Y/Yₚ, in the demand functions juxtaposed with the non-linear supply functions. The coefficients for r, W/P₀, and Pₚ were generally unchanged, and the Durbin-Watson statistic was generally higher. The coefficient of the index of transitory income always carried the negative sign expected from the hypothesis but was never significant. Since the long-run expected value of the ratio is one, the expected value of the logarithm is zero. Thus the absence of a significant coefficient for the 31 year span is not a refutation of this feature of the underlying hypothesis.

The second set of additional estimates provide a test of an alternative hypothesis. The measured value of real net national product is introduced as an argument in the demand function in place of the transitory index. The results of our OLSs and TSLS are quite consistent with earlier results reported elsewhere, covering the same and longer time periods.
The coefficient of income is never significant in the $M^1$ or $M^2$ functions. Moreover, in the TSLS estimates, the coefficient of real income is of vanishing order, .08 in the $M^1$ function and — .05 in the $M^2$. Coefficients for $W/Pa$, $r$, and $Py$ remain approximately in the ranges obtained earlier, and their “$t$” values are effectively unchanged. The asymptotic estimates of the “$t$” values in the TSLS for $Y/Py$ do not exceed .56 in absolute value.

Thus the data once again appear to deny the relevance of real income as an argument in the specified demand function for money. With this denial, we find no evidence from the application of more sophisticated statistical procedures to reverse the tentative conclusion that we reached in an earlier paper—that it is the cost of making transactions, and not the volume of transactions, that has important bearing on the demand for money. Our results are therefore consistent with our hypothesis and our denial of a foreordained payments schedule or the view that money, inclusive or exclusive of time deposits, is a substitute only for bonds and not for real assets.

Finally, we conjecture about the meaning of the coefficient of real income. In the context of the joint estimates of money supply and demand functions, the estimated value may be an approximation of the “pull” of income on money. If this is correct, it is noteworthy that the TSLS coefficients of the arguments in the supply function are unaffected by the presence of real income in the reduced form. But such speculation once again presupposes a validated theory, a claim that we do not wish to make.

V. EXTENSION TO THE CREDIT MARKET

Equation (22) of the non-linear hypothesis formalized the stock adjustment on the credit market and enabled us to obtain the reduced form for the credit market within the frame developed. We may therefore use the specified relations to investigate the joint interaction of the demand for and supply of money with the public's supply of assets to banks. This section briefly considers some of the issues raised and some of the available evidence.

We have previously indicated that the elasticity of the money supply with respect to $B$ or $B^*$ is smaller when the hypothesis is augmented to cover the credit market. This result is obtained from our appraisal of the derivates of the money supply with respect to the policy variables and the modifications introduced by the elasticity of the public's supply of earning assets with respect to interest rates. The latter elasticity reduces the magnitude of the multiplier of the policy variables $B$ or $B^*$ by an amount approximated by the following expression

$$q = \frac{\varepsilon(m, i)}{\varepsilon(m, i) - \varepsilon(E, i)}$$

where $m$ and $i$ are the money multiplier and an interest rate, $E$ is the
public's stock supply of assets to banks and $\varepsilon$ is an elasticity as before.
Similar considerations hold for the elasticity of the money supply with respect to the rediscount rate. This elasticity is reduced by a factor approximately equal to $1 - q$.

The magnitude $q$ is the ratio of the banks' interest elasticity to the sum of the banks' and the public's interest elasticity. It summarizes the operation of the interest mechanism on the credit market. Provided that $q$ is not equal to unity, the hypothesis implies that the incorporation of the credit market and the interest mechanism attenuates the effect of policy variables operating on the money supply without eliminating their relative orders of magnitude. All the relevant elasticities are modified by the factor $(1 - q)$.

Thus the effectiveness of monetary policy on the money stock depends on the relative size of the banks' and the public's sensitivity to interest rates measured by the appropriate elasticities. Whatever the size of the elasticities, a comparatively sensitive interest elasticity of the public lowers $q$, while a relatively large sensitivity of the money multiplier raises $q$. The larger the interest elasticity of the money multiplier, the more $q$ approaches unity with consequent reduction toward zero of the elasticities of the money supply with respect to the policy variables. Conversely, an extremely sensitive response by the public to interest rate variations lowers the value of $q$ and maintains the elasticities of the money supply with respect to the policy variables in their former neighborhood.

From equation (24) we note that the joint interaction of money stock with the credit market leads to the incorporation of nominal wealth in the reduced form. We have also specified an additional interest rate from the vector, the rate on short-term government securities, as a part of the specific hypothesis estimated. The two rates of interest appear in the demand equation and in the joint credit market-money supply equation along with the other variables used in previous estimates. The results are shown in tables 8a and 8b. We comment on the TSLS estimates only.

The interest elasticity of the demand functions is affected little by the introduction of the credit market and the incorporation of nominal wealth in the reduced form for the money supply. The long term rate emerges with larger elasticity and higher “$t$” value for the equations juxtaposed against the $K^4$ and $K^6$ versions of the modified supply function. Multicollinearity seems to account for the distribution of the interest elasticity over the long- and short-rate when the $K^1$ and $K^2$ versions of the supply function are estimated. The D-W statistic rises in all the estimated demand functions, and the coefficient of transitory income maintains the expected sign, though it is generally of doubtful significance in terms of the usual standard. The coefficient of $W/Pa$ remains significant in all cases, as does the coefficient of $Py$. But the values of one or the other of these two coefficients continue to indicate a problem worthy of further investigation.
### TABLE 8a

OSLS and TSLS Estimates of Non-linear Money Demand 1929–1959
when Credit Market is Incorporated
(t statistic in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>( r_1 )</th>
<th>( r_s )</th>
<th>( W/\text{Pa} )</th>
<th>( P_y )</th>
<th>( Y/Y_p )</th>
<th>( R^2 )</th>
<th>( D - W )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M^1</strong> OSLS</td>
<td>- .806</td>
<td>- .088</td>
<td>1.292</td>
<td>1.122</td>
<td>- .117</td>
<td>.994</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td>(- 6.68)</td>
<td>(- 2.27)</td>
<td>(5.22)</td>
<td>(5.00)</td>
<td>(- .68)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSLS</td>
<td>- .478</td>
<td>- .236</td>
<td>1.037</td>
<td>1.550</td>
<td>- .358</td>
<td>1.85</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(- 1.72)</td>
<td>(- 2.32)</td>
<td>(2.97)</td>
<td>(4.06)</td>
<td>(- 1.472)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>M^1</strong> TSLS</td>
<td>same as above</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>M^2</strong> OSLS</td>
<td>- .646</td>
<td>- .021</td>
<td>1.245</td>
<td>.975</td>
<td>- .189</td>
<td>.994</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>(- 5.79)</td>
<td>(- .594)</td>
<td>(5.43)</td>
<td>(4.70)</td>
<td>(- 1.17)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSLS</td>
<td>- .352</td>
<td>- .161</td>
<td>.999</td>
<td>1.385</td>
<td>- .441</td>
<td>1.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(- 1.34)</td>
<td>(- 1.67)</td>
<td>(3.03)</td>
<td>(3.86)</td>
<td>(- 1.91)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>M^2</strong> TSLS</td>
<td>same as above</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>M^3</strong> OSLS</td>
<td>- .854</td>
<td>.040</td>
<td>1.339</td>
<td>.814</td>
<td>- .198</td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(- 5.88)</td>
<td>(.87)</td>
<td>(5.40)</td>
<td>(3.50)</td>
<td>(- 1.15)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 8b

OSLS and TSLS Estimates of Non-linear Money Supply 1929–1959
when Credit Market is Incorporated
(t statistic in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>( K^1 = 1, 2 )</th>
<th>( K^1 = 5, 6 )</th>
<th>( r_1 )</th>
<th>( r_s )</th>
<th>( \phi )</th>
<th>( W )</th>
<th>( R^2 )</th>
<th>( D - W )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong></td>
<td><strong>M^1</strong> OSLS</td>
<td>.854 ( K^1 )</td>
<td>.058</td>
<td>- .066</td>
<td>.183</td>
<td>.996</td>
<td>.71</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.54)</td>
<td>(.49)</td>
<td>(- 2.21)</td>
<td>(2.14)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSLS</td>
<td>.922 ( K^1 )</td>
<td>.098</td>
<td>- .052</td>
<td>.107</td>
<td>.71</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.1)</td>
<td>(.58)</td>
<td>(- .99)</td>
<td>(1.04)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2.</strong></td>
<td><strong>M^2</strong> OSLS</td>
<td>.647 ( K^2 )</td>
<td>.141</td>
<td>- .034</td>
<td>- .195</td>
<td>.396</td>
<td>.977</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>(7.56)</td>
<td>(1.27)</td>
<td>(- .90)</td>
<td>(- 2.91)</td>
<td>(4.84)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSLS</td>
<td>.677 ( K^2 )</td>
<td>.283</td>
<td>- .044</td>
<td>- .229</td>
<td>.377</td>
<td>1.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.09)</td>
<td>(2.00)</td>
<td>(- .93)</td>
<td>(- 3.02)</td>
<td>(4.36)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>3.</strong></td>
<td><strong>M^3</strong> OSLS</td>
<td>.823 ( K^2 )</td>
<td>.172</td>
<td>- .009</td>
<td>.131</td>
<td>.996</td>
<td>.61</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.90)</td>
<td>(1.63)</td>
<td>(- .34)</td>
<td>(1.67)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSLS</td>
<td>.882 ( K^2 )</td>
<td>.194</td>
<td>.006</td>
<td>.063</td>
<td>.71</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.3)</td>
<td>(1.29)</td>
<td>(.13)</td>
<td>(.67)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>4.</strong></td>
<td><strong>M^2</strong> OSLS</td>
<td>.630 ( K^2 )</td>
<td>.256</td>
<td>.190</td>
<td>- .178</td>
<td>.328</td>
<td>.997</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>(8.25)</td>
<td>(2.69)</td>
<td>(.58)</td>
<td>(- 3.07)</td>
<td>(4.56)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSLS</td>
<td>.630 ( K^2 )</td>
<td>.354</td>
<td>.324</td>
<td>- .236</td>
<td>.077</td>
<td>1.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.34)</td>
<td>(2.86)</td>
<td>(.77)</td>
<td>(- 3.58)</td>
<td>(4.26)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The modified or extended supply equations are of particular interest. The data appear to discriminate between the equations incorporating $B^*$ (containing $K^5$ and $K^6$) and those based on $B(K^1$ and $K^2)$. First, the serial correlation of the residuals is indeterminate for the $K^6$ and $K^8$ version of the extended hypothesis. Second, the coefficient of nominal wealth emerges with substantial significance in both the $M^1$ and $M^2$ versions. Third, long-term interest rates have a positive and significant effect in the supply functions with relative orders of magnitude corresponding to the hypothesis. Fourth, the rediscount rate has the appropriate sign and is significant in both equations. Only the short-term interest rate remains of tenuous significance.

All of the elasticities with respect to policy variables are reduced in the extended supply equations as implied by the hypothesis. Moreover, the coefficients of $K^5$ and $K^6$ are reduced by more than those of $K^1$ and $K^2$, further support for our proposition that the operation of interest rates on the $B^*$ multiplier is substantially larger than on the $B$ multiplier. But the values of the elasticities with respect to $K^1$, though reduced, do not vanish. This suggests that $q$ is substantially less than unity. These results, applicable to the period of the thirties, from which the estimates were in part obtained, seem to indicate that monetary policy was not without power to alter the position of the public and the banks and to affect the supply of assets offered by the public to the banks on the bank oriented credit market.

These brief remarks and the limited evidence cannot be expected to provide a thorough analysis of the interaction of the demand and supply for money with the credit market. But the results are perhaps sufficient to suggest that monetary theory may be capable of furnishing a framework for monetary policy.

**List of Principal Symbols**

- $M^1 =$ Currency plus demand deposits adjusted
- $M^2 =$ $M^1$ plus time deposits at commercial banks
- $C_p =$ Currency outside banks
- $T =$ Time deposits of commercial banks
- $D =$ Demand deposits adjusted
- $B =$ Monetary base
- $B^* = B - A$
- $A =$ Member bank borrowing
- $L =$ Cumulated sum of changes in required reserves attributable to changes in reserve requirements
- $d_c = $ Change in $C_p$ independent of money wealth
- $d_t = $ Change in $T$ independent of money wealth
- $dv =$ Change in banks' desired cash assets independent of their deposit liabilities
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\[ R \] Base money held by banks  
\[ \Pi^B \] Bank's desired portfolio of base money  
\[ E^B \] Banks' desired portfolio of earning assets  
\[ E^P \] Public's desired portfolio of earning assets  
\[ \rho \] Rediscount rate  
\[ R^* \] Required reserves  
\[ R^e \] Excess reserves  
\[ \bar{r} \] Average reserve requirement against demand deposits  
\[ r^t \] Reserve requirement against time deposits  
\[ W \] Public's non-human wealth at current prices  
\[ Y \] Net national product at current prices  
\[ P_7 \] Deflator of non-human wealth  
\[ Y_p \] Friedman's permanent income at permanent prices  
\[ \sigma \] 3-year moving standard deviation of bond yields  

(in Tables 3 and 4)

\[ r \] Bond yield  

(in table 7b)

\[ K^1 = \log B^1 + \epsilon(m^1, r^d) \log r^d + \epsilon(m^1, r^t) \log r^t + \epsilon(m^1, k) \log k \]

(in table 7b)

\[ K^2 = \log B^2 + \epsilon(m^2, r^d) \log r^d + \epsilon(m^2, r^t) \log r^t + \epsilon(m^2, k) \log k \]

(in table 7b)

\[ K^6 \] and \[ K^8 \] derived from \[ K^1 \] by replacing \[ B \] with \[ B^A \]. \[ K^8 \] derived from \[ K^2 \] by replacing \[ B \] with \[ B^A \]

(in table 8a and 8b)

\[ r_l \] Long-term rate (bond yield)  

(in table 8a and 8b)

\[ r_s \] Short-term rate

**Sources and Nature of Data**

1. All data occurring in estimates of demand functions were described in two previous papers. See footnote 21 for references.

2. All data occurring in money supply functions and not in demand functions were obtained from the *Federal Reserve Bulletin and Banking and Monetary Statistics*, Washington D.C. The nature of the data required for the linear hypothesis was described in "Schema ...". The data
applied to the non-linear hypothesis are straightforward computations proceeding according to the explicit specifications of the variables. The computations apply to the data published in the Federal Reserve publications indicated. Detailed descriptions of data and sources will be made available on request to research workers.

**Units**

Interest rates are in per cent. All other variables are in billions of dollars.