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The Nature of the Policy Problem

Karl Brunner and Allan H. Meltzer*

There is an enormous gap between monetary theory and the discussion or practice of monetary policy. Most economists agree on the general outline of a theory—at least the comparative statics framework—linking money to prices and output. As long as discussion is confined to general frameworks that have minimal content, one can easily agree on some vague principles by which monetary policy should be guided and on the measures to be used as a standard of performance for policy. Implications of the theory are translated into guidelines for action, such as: Maintain the market rate of interest above, below, or equal to the real rate on real capital required to induce investors to hold the inherited capital stock, or equal to the real rate plus the expected rate of change of prices; or maintain full employment and growth without inflation.

What relation, if any, is there between these statements of objectives (or of principle) and the current practice of raising or lowering some short term market interest rate, or changing the level of some money market variable such as free reserves, in order to obtain a desired level of short term market interest rates? Anna Schwartz (Chap. II) shows that many central banks confine their operations to the manipulation of specific money market variables although they state their goals in terms of growth, employment, price stability, and exchange rates. The framework that permits economists or central bankers to interpret changes in money market variables as a guide for policy, or a useful scale for measuring the thrust applied by policy, has not been clearly stated. It is not very useful for economists to be able to say, after the fact, that although short term market interest rates rose, they did not rise enough to prevent

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inflation. If monetary theory is to be useful as a framework for policy decisions, it must be capable of producing statements with more content. Moreover, there is considerable ambiguity about the meaning or interpretation assigned to changes in money market variables. Sometimes such variables are used as indicators of monetary policy; that is, as measures of the direction in which monetary policy has changed in the recent past and/or of the effect of monetary policy on employment, output, and prices at some unspecified future dates. In this role, money market variables serve as an indicator of current monetary policy in the same way that the full employment budget surplus serves as an indicator of current fiscal policy. At other times, money market variables are used as policy targets. In this usage, the central bank directs its agent to attain or maintain a particular value of some money market variable. Or the central bank uses some implicitly weighted average of money market variables as a target of monetary policy, as it does when it directs its agent to maintain unchanged the “tone and feel” of the money market. Some central banks use money market variables as both targets and indicators. Typical examples in U.S. practice are the use of a specific level of free reserves as a target of policy combined with the use of the Treasury bill rate as an indicator or the use of free reserves as both an indicator and a target of monetary policy.

The meaning given to the terms “indicator” and “target” is not always clear from the context of policy discussions. We have chosen to associate these terms with specific problems that arise in the interpretation and implementation of policy. The indicator problem of monetary policy is the problem of constructing a scale that is invariant up to a monotone transformation and that provides a logical foundation for statements comparing the thrust of monetary policy. The target problem is the problem of choosing an optimal strategy or strategies to guide monetary policy operations in the money markets under the conditions of uncertainty and lags in the receipt of information about the more remote goals of policy. We discuss each of these problems briefly here and more extensively in later sections.

The indicator of monetary policy provides a scale (or ordering of policy actions) that permits policy makers, economists, and others to compare the thrust of monetary policy on economic activity, that is, to characterize one policy as more expansive than another or to characterize policies as more (or less) expansive than before. Statements comparing current policies to other policies that might have been chosen or to policies chosen at other times require such a scale. Meaningful comparative statements cannot be made without reference to a scale, so the indicator problem cannot be avoided where comparative statements are made.

Policy makers are concerned not only with the interpretation of their actions, but also with the choice of desirable courses of action. Uncertainty obscures the mechanism connecting policy changes with the goals that the policy maker desires to achieve. The variability of the lags in the
effect of policy action and in the receipt of information makes policy
makers uncertain about the current position and direction of change of
variables, such as the price level, that they wish to adjust. From the
point of view of the Account Manager executing the vague directives
issued by the Board of Governors, the level of employment, prices, and
the pace of economic activity are far removed from his operations and
are of little use when he is deciding on his daily or hourly operations.
Even the stock of money is remote and slowly changing relative to the
time span within which the Account Manager operates. The size and
direction of current changes in money are not known at the daily or
hourly periods at which the manager makes decisions.

Central bankers respond to the uncertainty of knowledge about the
state of the economy and the lag in receipt of information by choosing
targets that reduce uncertainty and appear to increase the amount of
available information. Since information on variables such as the
Treasury bill rate or the level of free reserves (excess reserves minus
member bank borrowing) become available frequently, central bankers
tend to regard these variables as highly desirable, perhaps optimal,
target variables. Moreover, bill rates and other money market variables
respond quickly to policy operations, and thus give the policy maker
information about the extent to which his operations have achieved his
short term aims.

An analysis of the target and indicator problems is necessary to bridge
the gap between monetary theory and monetary policy. Central bankers
have bridged the gap by trial and error and, as might be expected, have
not analyzed the problem or examined the consequences of choosing
alternative targets and indicators. Most often, economists have avoided
the problem completely by presupposing complete, useful knowledge
about the structure of the economic process. With complete knowledge,
the choice of a scale, or indicator, becomes easy and the choice of an
optimal target becomes trivial.

The literature on optimal policy provides one current example of the
way in which economists avoid the problems of selecting targets and in-
dicators. In that literature, the policy maker is given complete knowledge
of the structural relations pertaining to the economy and a set of goal
variables that enter the utility function. The policy problem is to maxi-
mize social utility, given the structure and the goals. No indicator need
be chosen for this exercise; policies are compared to the optimal policy
as part of the process of finding the optimum, and the optimal policy
becomes the standard to which other policies are compared. And since
the policy maker is given complete knowledge of the structure and the
length of relevant lags, there is no uncertainty about the structure, and
no reason to choose an intermediate target.

Analyses of policy that are made without the optimal policy frame-
work have many of the same characteristics. They, too, avoid the target
and indicator problems by pretending that complete knowledge of the
structure is available. Unfortunately, we do not possess the full infor-
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tion presupposed in these analyses. The array of competing econometric models and rival hypotheses clearly reflects the uncertain state of knowledge, and the behavior of policy makers reveals that uncertainty is pervasive. Under the pressures resulting from continuous implementation of general policy directives, central bankers have found it advisable to select an indicator for gauging the thrust of policy and to choose a target to guide the continuous adjustment of policy operations.

In this chapter, we reconsider the indicator problem and analyze the target problem. The following section discusses some of the different types of statements that are used to characterize policy and their relation to the chapters in this volume. Next we show why interest rates or money market variables are unreliable indicators of monetary policy, using a "Keynesian-type" model in which interest rates have a most important role in transmitting the effect of monetary policy to output or spending. Then we present formal analyses of the indicator and target problems.

Policy Statements and Types of Policy Analysis

Although economists have avoided the indicator problem, they do not avoid using an indicator in discussions of monetary policy. Economists, policy makers, the financial press, Congressmen, and others make statements frequently about the character of policy operations. These statements range from such simple descriptive or normative statements as "monetary policy is easy," or "monetary policy should remain easy," to the more abstract characterizations found in discussions of optimal policy. Sometimes the statements are obtained from an explicitly stated frame of reference; at other times no frame of reference is provided. Sometimes there is general agreement about the direction of change in policy; at other times there are disputes about the direction of current policy.

At the most primitive level are statements based on classifications used to describe the position of current policy. One such framework involving five classifications was developed for monetary policy by President Hayes of the Federal Reserve Bank of New York.¹ His scaling of policies ranges from "very easy" to "very tight" and uses levels of money market variables and open market rates to classify policies. For fiscal policy, three positions—expansive, neutral, and contractive—are now generally associated with the level of the full employment budget deficit or surplus. Neither of these scales has been justified analytically.

Many of the statements made about monetary policy are comparative statements. Such statements provide more information than classifica-

tions but require important constraints on the logical structure of the concepts used. Comparative statements must satisfy the logical structure of order relations.  

Many of the disputes about policy are, in fact, disputes about the scale used to classify or compare policies or changes in policy. One such dispute is about the direction of change in monetary policy during the period 1929–1933, although numerous other, similar disputes come to mind. Underlying these discussions are more or less implicit beliefs concerning the adequacy of various scales as indicators of future changes in economic activity or of the future effect of monetary policy on economic activity. For example, those who use levels of market interest rates to indicate changes in monetary policy describe policy as “much easier” in the summer of 1931 than in the fall of 1929. Those who describe the 1931 policy as “easy” or “easier” are likely to attribute the subsequent decline in output and income to a failure of expansive monetary policy to stem the contraction.

Three of the chapters in this volume begin to bridge the gap between monetary theory and discussion of monetary policy by examining the scale or scales used by policy makers to indicate the thrust of policy or the targets chosen to guide policy. Anna Schwartz (Chap. II), Leonall Andersen (Chap. III), and Thomas Atkinson (Chap. IV) discuss the role of money market variables as indicators and/or targets of monetary policy. As already noted, Anna Schwartz shows that money market variables are used as indicators and targets by central bankers in several countries. Andersen and Atkinson view the performance of money market variables as targets or indicators during specific periods. Andersen uses statistical techniques to construct a measure of money market “pressure” which combines the independent influences of a number of money market variables. He then shows that the official description of policy offered by the Manager of the Federal Open Market Account was consistent with the movements of the variable summarizing money market pressure. His findings suggest, therefore, that the Manager used

\[\text{\textsuperscript{3}}\text{Rudolf Carnap clarifies this situation as follows: “In defining a class concept, we can specify any conditions we please. Of course, if we include logically contradictory conditions . . . then we are defining a class that has, in any possible world, no members. Apart from this, we are free to define a class in any consistent way we wish, regardless of whether that class does or does not have members in our world. With respect to comparative concepts, the situation is quite different. Unlike class concepts, they imply a complicated structure of logical relations. If we introduce them, we are not free to reject or modify this structure. The four requirements stated by Hempel must be fulfilled. Thus, we see that there are two ways in which the comparative concepts of science are not entirely conventional; they must apply to facts of nature, and they must conform to a logical structure of relations.”} \textit{Philosophical Foundations of Physics} (New York, London: Basic Books, 1966), p. 48.\]

\[\text{\textsuperscript{4}}\text{This is precisely the interpretation given by many of the observers at that time, including many of the Governors who served on the Federal Reserve’s Open Market Policy Conference.}\]
money market variables as an indicator of monetary policy. At times—1960 is an example—a recession occurred because the officials failed to judge correctly the direction in which policy changed and to consider the future effects of the change. At other times, the consequences of using an inappropriate indicator were less serious.

Atkinson compares two targets of monetary policy—free reserves (excess reserves minus member bank borrowing) and “tone and feel of the money market” (measured by changes in Treasury bill rates). He concludes that, during the years he studied, one or the other of these measures was used as a target of monetary policy at virtually every meeting of the Open Market Committee. As his chapter suggests and as might be expected, the Manager of the Account argued frequently that the broader measure, “feel of the market,” is the more appropriate target. This measure was never defined, so its adoption by the committee shifted responsibility from the committee to the Manager and made it difficult for the committee to audit the Manager’s performance.

There is only a superficial relation between the central bankers’ use of money market variables and interest rates as targets or indicators and the economists’ use of interest rates as an indicator of monetary policy. Central bankers are accustomed by habit and past experience to equate the effects of monetary policy with the effects such policy has on the banking system and the money market. Economists, on the other hand, generally emphasize the effects of monetary policy that spread beyond the banking system and the money market to affect output, employment, and the price level. As George Horwich makes clear in Chapter VII, the implications of any relatively standard macrotheory for monetary policy can be expressed—as they most often are—in terms of maintaining the rate of interest that equates the amount saved to the amount invested at full employment. Horwich concludes, therefore, that the central bank should accept as its task the maintenance of a market rate of interest equal to the full-employment-equilibrium rate. In the language of our discussion, he wants the market interest rate to be used as a target of monetary policy. It is noteworthy that his analysis presupposes full information about the structure of the system.

The connection between the interest rate target or indicator used by central bankers and the interest rate desired by Horwich involves four subjects currently discussed in economics under the headings of (1) the term structure of interest rates, (2) the formation of price expectations and the timing of their effect on the difference between real and financial rates, (3) the effect of risk and attitudes toward risk on real and financial rates, and (4) the role of liquidity premiums (if any) and their relation to the three topics just mentioned. No doubt other topics could, and should, be introduced. Our point is partly the obvious one that there are differences between short and long term rates as well as differences between real and financial rates. These differences are well-known, even if a theory explaining the differences and changes in the differences is not well developed. The less obvious prob-
lem is that the use of changes in short term market rates as an indicator of monetary policy not only presupposes that such a theory has been developed, but also presumes that the theory is sufficiently powerful and well confirmed that it can be used to separate the effects of monetary policy from other effects on short and long term rates. This is but one example of the gap between monetary theory and concrete discussions of monetary policy mentioned earlier.

In Chapter VIII James Tobin discusses one other example of the gap between theory and practice. It is customary in macroeconomic theory to consolidate the banking system with other parts of the private sector so that the item called "money" in economic theory is the net monetary liabilities of the central bank, or in other words, the monetary base. If money (defined as demand deposits plus currency) is mentioned, it is assumed generally to be a constant multiple of the monetary base. In either case, by changing the monetary base the central bank increases or decreases the stock of money.

Although changes in the monetary base are the main determinant of longer term changes in the stock of money, short term changes in money are substantially influenced by factors other than the monetary base. Tobin emphasizes that the central bank cannot assume that each dollar of reserves adds a fixed and unchanging number of dollars to the stock of money within a given time period. The amount by which money changes per dollar change in the monetary base depends on such things as changes in the demand for reserves by banks, or changes in the demand for currency by the public. Since many of these changes depend on interest rates, prices, and real income, the response of the money stock to policy actions is not invariant under changing credit-market conditions. Moreover, changes in the money stock result from impulses that are independent of monetary policy. Tobin concludes that changes in money can be in a direction opposite to the direction in which monetary policy has changed, and consequently are not a reliable indicator of monetary policy.

A recent attempt by Gramley and Chase to defend or justify central bank practices is reprinted in Chapter X, along with some critical comments on the paper by Karl Brunner (Chap. XI) and criticisms of Brunner's chapter by Patric Hendershott (Chap. XII). Robert Weintraub surveys the entire discussion (Chap. XIII). Gramley and Chase take the position that the central bank should not attempt to control the stock of money or to use money as an indicator of monetary policy. The discussion brings out some aspects of the complex set of short run relationships between money, interest rates, and substitutes for money, and makes clear that disputes about the most reliable indicator of monetary policy reflect unresolved differences about the theory of money. These differences become apparent once money is treated as an endogenous variable. The discussion brings out some of the confusion about the nature of the indicator and target problems that the central banker faces.
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As we noted in an earlier paper, all endogenous nonpolicy variables are potentially misleading as indicators of monetary policy. If such variables are used as indicators, it is important to compare the errors that are likely to be made if one variable rather than another is used. Unless we are completely ignorant of the responses of monetary and real variables to changes in policy, the relative errors to be expected when using endogenous nonpolicy variables as indicators can be compared. Kamien and Schwartz (Chap. V) generalize our earlier formulation of the problem to the case in which the policy maker is permitted to choose linear combinations of variables rather than a single variable as indicator. Kamien and Schwartz use a minimax strategy to select a linear combination of variables as an approximate indicator. The linear combination is closer to the unobservable "ideal" indicator than is a single variable selected on the basis of our earlier analysis.

Recently, some official descriptions of policy have gone beyond the comparative statements discussed above to make quantified statements about the effects of current or proposed policies. For example, the Council of Economic Advisers forecasts GNP and discusses the effects of alternative fiscal policies on prices, output, and employment. Such statements require a numerical scale and a theory that is able to generate quantified predictions. Econometric models are frequently used to provide the numerical estimates of the effects of various policies or combinations of policies. More recently, simulation has been used as a means of comparing the longer term effects of alternative fiscal and monetary policies.

Ronald Teigen (Chap. IX) illustrates some of the ways in which one can use an econometric model to predict the future effects of monetary policy. Teigen's model, if correct, would permit the user to predict the effects of changes in a number of policy variables on the relatively large number of variables determined by the model. The output of econometric models is usually a set of predictions—a sequence of changes in economic variables expected at various times in the future. In principle a set of weights can be used to combine several variables into a single indicator so the effects of alternative policies can be compared. In practice this has not been done, perhaps because of the problem of finding the appropriate set of weights, or perhaps because of the lack of confidence in the ability of econometric models to forecast a number of the variables believed to be a part of the social utility function used to appraise the effects of policy.

One of the earliest attempts to provide a framework within which policy statements can be analyzed was developed by Tinbergen, who recognized that in a determinate system of relationships with a fixed

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number of set goals or ends to be achieved by changing policy variables, there is a determinate number of independent actions that can be taken. Tinbergen's analysis established necessary and sufficient conditions for the number of independent policy variables (called instruments) relative to the number of independently fixed goals (called targets). Also, he was able to express some logical relations between policy variables and fixed goals under various assumptions about the formal properties of the system and to explore the consequences of certain types of changes in the structure of the economy.

Tinbergen's analysis was extended in an important way by Theil, who introduced the notion of "optimal" policy. Using a quadratic utility function to represent the policy maker's view of the social consensus, Theil was able to generalize Tinbergen's analysis to the case in which the goal (target) was flexible rather than predetermined. Furthermore, Theil showed that additive, stochastic uncertainty does not prevent the policy maker from formulating an optimal policy.

Brainard (Chap. VI) provides a further generalization and extension of the Tinbergen analysis. Brainard introduces a major change in the maintained hypothesis about the structure of the system and examines the effect on Theil's concept of optimal policy. He dismisses the Tinbergen-Theil assumption of full nonstochastic information about the multiplicative part of the structure and assumes full stochastic information about the structure. The concept of optimal policy becomes substantially more complicated when these changes are introduced.

Brainard's chapter brings out the relation between the knowledge of the structure that the policy maker either has, or is assumed to have, available and the choice of optimal or even desirable policies. Once we assume full knowledge of the structure, we can proceed with the determination of desired policies. Little interest remains in measuring the extent to which policy has become more or less expansive once we know what current policy "ought to be." The choice of an appropriate scale and indicator becomes redundant. Moreover, there is no longer a problem of choosing an appropriate short term target to interpose between policy actions and policy goals.

Once again we find a gap between theory and practice, in this case between the requirements for the application of the theory of optimal policy and the information that is available to a central bank or government agency responsible for monetary policy decisions. In fact, the gap is wider than the large gap that is recognized in Brainard's analysis. Both the value of the parameters and the distribution of the parameters are uncertain, and in addition there is uncertainty about the form of the functions used to state hypotheses about the system and the variables admitted as arguments in these hypotheses. There is also un-
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certainty about the merits of competing theories of the monetary process.

The absence of perfect knowledge is not equivalent to total ignorance, and the existence of conflicting theories does not imply that a rational interpretation of policy actions is impossible. Most of the statements made by policy makers either classify policies according to some vague criterion (easy, tight, expansive) or compare policies or changes in policies. Comparative statements about policies require an ordinal scale, and if such a scale is chosen appropriately, the policy maker is able to decide rationally which policy is more or less expansive, given the goals he has selected and the information that is available. A reliable scale justifying comparative statements is a necessary condition for a rational determination of appropriate policy.

Preliminary Analysis of the Indicator Problem

Many observers consider market interest rates, or some variable highly correlated with market rates, to be most useful for describing the present state of policy or for comparing the present state to some previous or some desired state. Others consider one or another measure of the growth rate of money to be the most readily available and comparatively reliable indicator of monetary policy.

The reasons offered for using market interest rates as an indicator differ. Economists often describe monetary policy as affecting output, or prices, through a network of interest elasticities, and they have come to view interest rates as the link between financial and real markets. Central bankers use two partially related arguments. At times they view interest rates, particularly short term rates, as the direct result of their policies. At other times they suggest that the policy is best judged by interest rates because policy could have made rates higher or lower if the policy officials had desired to do so. It appears to follow that the important role of interest rates in the transmission of policy makes them useful as a scale for measuring the thrust of monetary policy.

Interpretation of monetary policy depends very much on the indicator that is chosen. Both market interest rates and the growth rate of money generally rise in periods of economic expansion and fall during contractions. If changes in market interest rates are the more reliable indicator, the procyclical movements of money and interest rates should be interpreted as a result of countercyclical monetary policy. On this interpretation, the fact that changes in money have a procyclical pattern is of no particular consequence. On the other hand, if changes in money are the more reliable indicator, the procyclical pattern in money is evidence of a procyclical policy.

In this section we discuss some of the issues that have arisen and some of the arguments that have been advanced in the lengthy dispute about the comparative merits of money and interest rates as indicators of
monetary policy. We show that the dispute can be traced to differing conjectures about the structure of the monetary process. In a later section we present a formal analysis of the problem.

There are two parts to the analysis in this section. First, we use a macromodel of the type found in most textbooks. The money stock is exogenous and is controlled completely by policy. In this framework the money stock is the policy variable and the indicator problem has a simple solution even under imperfect knowledge about the structure. Later, we consider systems that permit monetary variables such as money stock, bank credit, or free reserves to be determined along with income or interest rates. The second case yields a simple answer when knowledge is uncertain only if we restrict policy to operations on the monetary base because the use of multiple policy instruments increases the amount of information needed to assess the thrust of policy. And the choice between observable variables, money or interest rates, as approximate measures of the thrust of policy depends on specific properties of the economic structure.

Most textbook models of the "Keynesian" type can be reduced to two equations, one describing the output market and the other the "money market."

\[ Y = \alpha(i, Y, A, e) \quad \alpha_1 < 0; \alpha_2, \alpha_3, \alpha_4 > 0 \]
\[ M = \lambda(i, Y, e) \quad \lambda_1, \lambda_2 < 0; \lambda_3 > 0 \]

Here, \( Y \) denotes income, \( i \) interest rate (or an index of rates), \( A \) is a measure of "autonomous" expenditures, \( e \) is the expected real yield on real capital, and \( M \) is the money stock. The derivatives of the aggregate expenditure function, \( \alpha \), and the demand for money function, \( \lambda \), are listed on the right. In a system of this kind, the thrust of monetary policy is expressed by the derivatives of the solution for \( Y \) with respect to \( M \). The two solution functions, which are definable under certain constraints, can be written

\[ Y = \eta(M, A, e) \quad \eta_1, \eta_2, \eta_3 > 0 \]
\[ i = j(M, A, e) \quad j_1 < 0; j_2, j_3 > 0 \]

The differential of \( Y \) in terms of \( M \) defined by the \( \eta \) function provides an obvious measure of the thrust exerted by monetary policy in this case. This differential is

\[ dY = \eta_1 dM \]

With perfect knowledge about the structure, the derivative \( \eta_1 \) is known so the differential determines a scale for measuring policy.

Neither the levels nor changes in interest rates exhibit any discernible regular relation to the size of the thrust exerted by monetary policy. Both level and changes are jointly determined by \( M, A, \) and \( e \), or their differentials. A low level or rapid decline in \( i \) can be associated with low or falling thrust measured by \( \eta_1 dM \). Similarly, a large or rising thrust
can be associated with a high or rising interest rate. In the case of complete knowledge, the effect of monetary policy on interest rates, \( j_1 dM \), can be separated from the effects on \( i \) of changes in \( A \) and \( e \). Without such knowledge it is easy to attribute the effects of \( A \) and \( e \) to monetary policy and this is bound to happen if the total differential, \( d_i \), is taken as the indicator. When rising interest rates, induced by rising \( e \) and \( A \), overwhelm the moderating effect of an increasing money stock, policy is characterized as tight, tighter, or less expansionary even if the thrust supplied by policy, \( \eta dM \), has increased. The choice of the total differential, \( d_i \), as an indicator of policy creates, in this case, a divergence between the consequences of policy with respect to economic activity and its interpretation.

Proponents of interest rates as an indicator modify the argument just considered in several ways. The modifications appear to recognize the inadequacy of either the level of rates or the total differential of interest rates as an indicator of policy. One modification suggests the partial differential, \( j_1 dM \), as an indicator. A second argues that the deviation of the total differential, \( d_i \), from an "appropriate change," \( n \), provides a useful scale for measuring policy thrust. The larger the increase in \( d_i \) relative to \( n \), the more deflationary policy has become and vice versa. Since policy could have prevented either the relative rise or fall in rates and did not do so, policy is said to have permitted the movement and is held responsible for it. The larger \( (d_i - n) \) the greater is the deflationary pressure for which policy is said to be responsible. In this way the scale \( (d_i - n) \) is linked with policy and provides an interpretation of monetary policy.

Both modifications can be examined with the aid of the framework previously introduced. Since the partial differential \( d_i \) of interest rates attributable to monetary policy is proportional to the partial differential of income, \( dY_{MP} \), attributable to policy, we obtain

\[
d_i \sim \frac{j_1}{\eta} dY_{MP}
\]

The partial differential \( d_i \) appears to provide just as good a scale as the differential \( dY_{MP} \). However, this appearance is misleading even in the present case of complete information. The use of interest rates as an indicator loses some of the information provided by \( \eta dM \). This differential provides the correct ratio scale, whereas \( \frac{j_1}{\eta} dY_{MP} \) provides a scale which is order-preserving relative to \( \eta dM \) but is not equivalent as a ratio scale. In the contexts of nonlinear systems the distortion implicit in the interest scale relative to the correct indicator is reinforced.

The second qualification adds the value of \( n \), the "appropriate change" in interest rates, to the prior knowledge we are presumed to have available. The deviation \( (d_i - n) \) used as a scale for monetary policy becomes

\[
d_i - n = j_1 dM - n
\]
Suppose that the stock of money remains unchanged so the effect of monetary policy on output $\eta_t dM$ is zero.\(^7\) Consider two cases. In one $A$ and $e$ increase and in the other $A$ and $e$ fall. The difference $d_i - n$ rises in the first case and falls in the second, so the use of $d_i - n$ as an indicator shows that monetary policy has become more restrictive in the first case and less restrictive in the second even though $M$ is completely controlled and remains unchanged. Clearly, the use of the "interest rate deviation" scale is misleading. Changes in interest rates resulting from nonmonetary factors are attributed to monetary policy.

Of course monetary policy could have been used to maintain a constant value of $d_i - n$ in both cases. For example, if $M$ increased sufficiently when $A$ and $e$ increased, $d_i - n$ could have remained constant. However, $dY_{int}$ increases in this case and monetary policy is more expansive.

The example provides one explanation of the procyclical changes in interest rates and money. If $d_i$ or $d_i - n$ is used as an indicator, the rise in interest rates induced by an increase in autonomous expenditure $A$ is interpreted as restrictive policy and central bankers increase $M$ to make policy less restrictive. Policy is judged to be tight or tighter despite the increase in $M$ which adds to the expansion and perhaps to inflation. In the downswing, the opposite occurs. Policy is described as easy or easier, because $d_i$ falls (absolutely or relative to $n$) despite the decline in $M$. As a result cyclical changes in interest rates and money are positively correlated.

Of course, the increase in interest rates resulting from a large $dA$ or $de$ could be prevented by a suitable $dM$, so it is true that policy could have prevented the rise in $i$. However, if this is done, $dY_{int}$ becomes relatively larger. In fact, there are a variety of policies, $dM$, sufficient to assure $d_i = n$. The set of such policies is defined by the expression

$$\{dM | j_1 dM = n - j_2 dA - j_3 de\}$$

The members of this set induce very different responses in economic activity so different consequences follow from policies each of which is equivalent on the "interest rate deviation" scale.

Some of those who propose or defend the use of interest rates as an indicator might accept much of our argument but regard it as irrelevant. Market interest rates (or variables closely correlated with market rates) are used frequently as a target of monetary policy. When interest rates are used as a target, changes in market rates are said to define a suitable indicator. Changes in money are described as the result of policy operations designed to achieve the interest rate target.

We can analyze this case with the framework we have developed. Let $i = \bar{i}$ be the interest rate target. Our previous equations become

$$\bar{i} = j(M_t A_t e)$$
$$Y = \alpha(i_t A_t e)$$

\(^7\)This does not mean that income is unchanged. Nothing in this discussion rules out changes in income resulting from the effects of changes in $A$ and $e$ on $\eta_t$.\n
The argument asserts that the proper scale on which to measure the thrust of monetary policy is

\[ dY = \alpha_i di \]

Since the stock of money must be adjusted to maintain \( i = \bar{i} \),

\[ dM = \frac{1}{j_1} di + \frac{j_2}{j_1} dA + \frac{j_3}{j_1} de \]

This equation shows the adjustments imposed on the authority by the choice of \( i \) as a target and by changes in the target value. The first component of the sum on the right side describes the portion of the total monetary impulse resulting from given changes in the target. The remainder of the sum measures the portion attributable to the adjustment of the authorities' behavior to a given target.

As before, \( M \) is controlled completely even if it is an endogenous variable. In the systems considered, variations in \( M \) reveal simultaneously total monetary impulses and the behavior of the monetary authority. The proper scale for measuring the thrust of monetary action remains \( \eta dM \). The scale \( \alpha_i di \) does not measure the thrust of monetary policy exhibited by the authorities' behavior but only the thrust resulting from changes in the target level of interest rates. This measures only a component of the total impulse emitted by the monetary authorities' behavior.

The use of a framework in which money is completely controlled brings out some main points but perhaps disposes of the arguments in defense of interest rates as an indicator too easily. The analysis can be broadened to include the more relevant case in which money is an endogenous variable that depends on both policy and nonpolicy variables. To do so, let \( x \) be a column vector of monetary policy variables, and \( z \) a column vector of the remaining predetermined variables. Our system becomes

\[ Y = \eta(x, z) \]
\[ i = j(x, z) \]
\[ F = F(x, z) \]
\[ M = m(x, z) \]

where \( F \) is the level of free reserves and other variables are as defined previously. The partial differentials of interest rates and income expressing the effects of monetary policy are now

\[ dY_{IMP} = \eta dx \]
\[ di_{IMP} = j_i dx \]

where \( \eta, j_i \) (and the similar terms \( m_i \) and \( f_i \)) are row vectors of derivatives with respect to \( x \).

*The monetary policy variables include the monetary base, the various reserve requirement ratios, the various ceiling rates on time deposits, the rediscount rate, and so on.
Karl Brunner and Allan H. Meltzer

It is clear that in general neither \( d_1 \) nor \( d_2 \) are proportional to \( d_3 \), so, as before, the interval scale constructed from \( d_1 \) would be misleading if used to replace the relevant scale \( d_3 \). The scales based on \( d_1 \) and \( d_3 \) are not equivalent even in a linear system. The partial differential \( d_1 \) could at best be used as an ordinal scale under appropriate constraints on the system, but this again implies a loss of information relative to the state of perfect knowledge that permitted the construction of an interval scale based on \( \eta dx \).

Our discussion of systems in which there are a number of monetary policy variables and complete information can be extended to the case in which there are several, independent interest rates. It is easy to show that nothing is changed in the extended analysis and that the conclusions reached for a single rate or an index of rates apply to the case of multiple rates as well. A more useful extension of the analysis is made if we remove the prior condition that knowledge is complete.

None of the variables we have considered—money, interest rates, and free reserves—provides a reliable index of monetary policy. We cannot compute or observe \( d_1 \) once our knowledge is less than complete. The solution to the indicator problem requires a choice between variables none of which is a fully reliable measure of the true thrust \( d_3 \).

The variables that are used most often as indicators—\( d_1 \), \( d_2 \), and \( d_3 \)—have several properties in common. Each is observable; each depends partly on monetary policy variables, \( dx \), and partly on the remaining variables, \( dt \). This last fact is used frequently as an argument against the use of \( d_3 \) as an indicator and as a defense of \( d_1 \). The argument is fallacious. It does not follow that because monetary variables can differ from the true indicator, interest rates are more reliable. As we showed in our previous work on indicators, both monetary variables and credit market variables, such as market interest rates, are imperfect indicators. Once we recognize the uncertain nature of our knowledge, all endogenous variables are capable of overstating or understating the thrust of monetary policy. But all are not equally good or equally bad, and our problem is one of comparing the relative merits of the variables proposed as indicators. The general frame used above is sufficient to establish however, that a restriction of policy to a single instrument \( x \) yields a reliable ordering scale even in the presence of uncertainty. The differential of income with respect to \( x \) is \( dY = \eta dx \). It is clear that \( d_3 \) under such a constraint is proportional to \( dx \). It follows that \( dx \) provides the proper ordering scale.

Differences in assumptions about broad properties of the underlying structure generating the responses of interest rates, free reserves, and money stock to policy variables and predetermined variables probably influence the proxy selected. If the dependence of \( M^{on-z} \) is believed to be small relative to the dependence of \( F \) and \( i \) on \( z \), \( M \) is preferred as a proxy to both \( F \) and \( i \). Those preferring \( F \) or \( i \) over \( M \) tend to assume

*See note 4.
that $M$ is dominated by $z$, whereas the movements of $F$ or $i$ are believed to be dominated by $x$. These considerations appear plausible. But it must be noted that even substantial evidence in support of the contention that $M$ is least dependent on $z$ is not a sufficient argument to establish a conclusive case on behalf of $M$. Such evidence provides only a prima-facie case until adequate logical foundation is supplied. Some foundations for the proper choice of an indicator in the absence of complete knowledge are developed in the next section. While our analysis provides a conceptual solution to the target and indicator problems, our solution is not unique.

### The Indicator Problem

The problem of selecting an indicator of monetary policy is equivalent to the problem of finding a scale that allows us to make reliable statements comparing the thrust of various policy combinations. In the preceding sections and in an earlier work, we discussed the relative merits of particular variables that are used frequently as indicators of monetary policy and illustrated how our general framework could be applied. In this section, we present a formal analysis of the problem of choosing an indicator and show the relation between the amount and kind of information available and the properties of the indicator that is selected. In the following section, we extend our analysis to discuss the problem of choosing a target.

Two different procedures are discussed. The first postulates that our information can be expressed as probability statements about hypotheses and classes of hypotheses. A scale can be constructed that is reliable given the available information, and that can be adjusted as new information is received. This procedure permits us to assign numbers to specific combinations of policy variables, and thus provides a scale or indicator of policy. In this case the optimal indicator is not one of the variables (or a combination of the variables) in the system.

The second procedure postulates that our information is not adequately expressed by probability statements about hypotheses. In this case, the optimal scale must be chosen from a set of observable variables or combinations of observables. This procedure selects a best approximation to the unknown and noncomputable true indicator. The meaning of "best approximation" is determined by the procedure, as discussed below.

The framework for our discussion consists of a system of differential equations

$$\dot{y} = h(x, y, z)$$

See note 4.
where \( y \) is a vector of endogenous variables, \( x \) is a vector of monetary policy variables, \( z \) is a vector of other policy variables and predetermined variables, and \( \dot{y} \) is a vector of time derivatives.\(^\dagger\)

Suppose that monetary policy changes at time \( t_0 \) by a vector \( \mu \). We assume that for any \( t \geq t_0 \), policy differs by a constant vector \( \mu \) from whatever policy would have been. The system responds to \( \mu \) according to the hypothesis \( h \) and moves in a path described by the vector equation:

\[
\dot{y} + \beta = h(y + \beta, x + \mu, z)
\]

The vector \( \beta(t) \) for \( t \geq t_0 \) shows the net effect of \( \mu \) on the system’s behavior. Without loss of generality, we let the first coordinate of \( y \) represent economic activity. It follows that \( \beta(t) \) expresses the effect of \( \mu \) on economic activity \( t \) time units after the policy action expressed by \( \mu \). The dependence of \( \beta \) on \( \mu \), on the time elapsed, and on the hypothesis \( h \) is shown explicitly. It should be noted that the linearity assumption discussed in a footnote assures the independence of \( \beta(t) \) from \( x(t) \) and \( z(t) \), so the \( \beta \) function yields an unambiguous scale for policy changes \( \mu \), at any \( t \geq t_0 \), for any hypothesis \( h \). Movements along the time axis beyond \( t_0 \), however, may change the sense of the scale. Since we are not interested in temporary effects or judgments based on temporary effects of policy, we want to choose a scale that reveals the dominant effects of a policy change. Under suitable stability constraints an appropriate scale is given by the limit of \( \beta(t) \). This limit is

\[
\beta_s = \beta_s(\mu; h)
\]

and is a function only of \( \mu \) for any hypothesis. If we know \( h \), this function provides the desired indicator and yields substantially more than the logical basis for comparative statements that we seek. This indicator provides a ratio scale and permits numerical comparisons of distinct policy combinations, if we know \( h \). But we do not know which of the competing hypotheses is correct. Our uncertainty is expressed by the statement that \( h \) belongs to a class \( H \). This class summarizes both our partial knowledge and our partial ignorance. Restriction of \( h \) to \( H \) reveals that we feel justified in disregarding hypotheses outside \( H \). This restriction, to the extent that it is based on relevant evidence, summarizes our shared knowledge.

The distinctive assumption of our first procedure is that the support for competing beliefs can be expressed by means of probability statements. A conditional probability measure \( p(h|H) \) is defined for the class \( H \). All relevant information can now be summarized by the ordered pair \((p, H)\) of a probability measure and its domain and the indicator of

\(^\dagger\)The formula is sufficiently general to include in \( \dot{y} \) time derivatives of higher order of selected coordinates of \( y \). In spite of the somewhat formal nature of the discussion we do not proceed in fullest generality. We constrain our discussion to linear systems. This is less restrictive than it appears. The coefficient matrices are not restricted to constancy and the results can be easily generalized to piece-wise linear systems.
monetary policy action \( \mu \) can now be defined relative to the available information. We define

\[
\beta_\mu(\mu|H) = \int \beta_\mu(\mu; h) \rho(h|H) dh
\]

The indicator is computed by averaging the ratio scales. The averaging does not remove this property. The scale \( \beta_\mu(\mu|H) \) is also a ratio scale.

The equation defining the indicator expresses the dependence of the indicator on the information that is available. This is the best indicator we can find in a given state of knowledge \((\rho, H)\). This state changes, however, as new data become available and modify the density \( \rho \) just as new information modifies the probabilities in Bayesian analysis. Thus, as new information becomes available, the indicator changes.

If the density \( \rho \) is relatively small for all \( h \in H \) because of great uncertainty about the structure, further examination of the indicator problem seems advisable. We partition the class \( H \) into a finite number of subsets \( H_i \) such that the function \( \beta_\mu(\mu; h; H) \) maintains a constant form for all \( h \in H_i \). For example, if \( H \) is a collection of econometric models, each \( H_i \) is a specific model and each \( h \in H_i \) is an admissible structure of this model. For each subset \( H_i \) a scale can be constructed using the expression

\[
\beta_{Hi} = \beta_\mu(\mu; H_i) = \int_{H_i} \beta_\mu(\mu, h, H_i) \rho(h|H_i) dh
\]

The space of admissible policy actions is also subdivided into a finite number of rectangular cells. Each cell is represented by its central point \( \mu_s \), and there is now a finite class of admissible policy vectors \( \mu_s \) describing a finite number of distinct policy states. The variation of \( \mu \) over this class generates \( n \) series of values \( \beta_{Hi} \). The covariation of these series can be computed and appropriate tests can be performed to decide whether or not the \( n \) series are drawn from the same population. If we accept the hypothesis that they are drawn from the same population, we may select an arbitrary scale or modify the result by suitable probability mixtures of the various scales. However, if the covariation is unsatisfactory, we should inquire whether the inadequate covariation results from a few of the series \( \beta_{Hi} \). One will probably find that major differences in policy interpretations can be reduced to a few broad subsets of the set \( H \).

If the covariation is inadequate, we should also consider the extent to which the covariation improves substantially in portions of the admissible policy action space. For example, the covariation may be close to unity if ceiling rates are held constant or if policy actions are restricted.
restricted to changes in the monetary base. Even if we are unable to construct a reliable indicator for the entire policy space, we can select some particular scale for ranking restricted policy actions. Our preliminary investigations suggest that there is an inverse relation between the number of independent policy actions and the reliable interpretation of policy. This aspect will be considered further.

The use of probability statements is not a necessary condition for the construction of an indicator. An alternative, formal decision procedure, can be developed. Our partial ignorance is expressed again by a class of hypotheses, $H$. As before, we do not have sufficient information that will permit us to compute the value $\beta_i(\mu; h)$ associated with any $\mu$ since to do so requires knowledge of $h$. Moreover, $\beta_i(\mu, h)$ is also a nonobservable magnitude.

In the first procedure we averaged over the class of indicators or scales generated by $H$ using probabilities as weights. The second procedure replaces the probabilities with observables (or linear combinations of observables); the observable magnitude that is the best approximation to the true but nonobservable and noncomputable indicator is chosen. This observable indicator provides a scale that assures the least misinterpretation of policy actions.

A number of observable magnitudes—such as interest rates, money, and free reserves—are used as indicators and, as noted earlier, these variables frequently support opposing interpretations of monetary policy. One indicator may suggest that policy is more restrictive, while another indicator suggests that policy is less restrictive. The problem is to develop a procedure for choosing the most nearly correct interpretation. Let $O_j$ denote an observable magnitude or a linear combination of such magnitudes belonging to a set $S$ of observables. Every $O_j$ is either a coordinate of the vector $(y + \beta)$, a coordinate of the vector of time derivatives $(\dot{y} + \dot{\beta})$, or a linear combination of the coordinates of the policy vector $\mu$. Under appropriate but comparatively weak constraints on the vector function $h$, each of the $O_j$ can be expressed as a functional of the time functions $z(\tau)$ and $x(\tau)$. We obtain solutions for the endogenous variables used as indicators in terms of current and past values of policy ($\mu$) and nonpolicy ($z$) variables.

The association of $O_j$ with given time functions of $x$ and $z$ and with given monetary policy operations, $\mu$ in $t_0$, depends on $h$, but unlike $\beta_i$, we can observe $O_j$ directly and independently of our knowledge of $h$. However, since many observables compete for selection as indicator, we...
require a procedure or a strategy for selecting one of the $O_i$ (or some linear combination of $O_i$).

Before discussing the formal choice of a strategy for choosing an indicator, we wish to point out two properties of the variables that are used as indicators. First, the observables selected are believed by those who choose them to be closely related to policy actions during some period, $t = t_0 + dt$, or at a point of time, $t = t_0$, and are believed to be affected relatively little by $x(t)$ and $z(t)$. The policy variables that are elements of the vector $\mu$, or a weighted combination of these variables, have this property. Proponents of variables such as the money supply or interest rates suggest that one or another of these variables has this property also, as noted earlier. However, dependence on $x(t)$ and $z(t)$ is not a sufficient condition for rejecting a variable as indicator. The reason is that variations in $x(t)$ during any interval $(t_0, t)$ may have little effect on $O_i(t)$. However, the dependence of many elements in $S$ on $x(t)$ and $z(t)$ does justify the construction of two distinct types of indicators: one that is invariant over the space of policy histories, and a second type that depends on the history of past policy. The first type can be used to make comparative statements that are independent of time, whereas the second type can be used only to rank alternative policy combinations at a particular point (or interval) of time. Both types of scales will be constructed in our formal analyses.

The second property of variables generally proposed or used as indicators is that information becomes available with a minimal lag. Economic activity, $y_i(t_0 + \theta)$, $\theta$ time units after the policy action $\mu$ may reflect policy very well because $x(t)$ and $z(t)$ during the interval $(t_0, t_0 + \theta)$ have a minimal effect on $y_i(t_0 + \theta)$. However, $y_i$ depends on the previous histories of $x$ and $z$, so it could only be used as an indicator of the second type. To construct a scale that permits more than a description of past records and that can be used in rational policy making, we require a magnitude that becomes available (or computable) at the time of the policy action under consideration.

The formal analysis of the indicator problem developed in our previous paper4 made the optimal scale a function of the difference $|\beta_1 - O|$ between the true measure of the impact of $\mu$ and the particular approximation $O$. The optimal indicator was found by first maximizing the norm $|\beta_1 - O|$ of the above difference with respect to the hypotheses $h \in H$, and past histories $x(t)$ and $z(t)$ and then minimizing the resulting expressions. This procedure gave us the minimax of the norm and thus protected us against the worst possible misinterpretations of policy; but without introducing additional constraints, we could not be certain that the scale selected was a reliable approximation to the true or desired scale. The indicator computed in this way provides no measure of its own reliability.

The correlation function used here is an improvement on our previ-

\*See note 4.
ous argument. This function defines a rank correlation between the unknown and unobservable measure of policy thrust $\beta_i$ and the proposed approximations $O_j(t)$. To define this function we require specification of a space with elements $[x(\tau), z(\tau), \mu]$ where $x(\tau)$ and $z(\tau)$ are functionals. An upper bound $T$ is imposed on time and the vectors $[x \cdots (\tau), z(\tau), \mu]$ form the encompassing space $S_T$ underlying our discussion. A

similar space $S_T$ is obtained for any $t$ satisfying the condition $t_0 \leq t \leq T$. Each space $S_t$ is partitioned into a lattice of cells. The formulation of the functions $\beta_i$ and $O_j(t)$ associates a value to each cell in the spaces $S_t$ for any given $h$. The time index is restricted for our purposes to the interval $(t_0, T)$, where $t_0$ denotes the location of the policy action, $\mu$. Variations between cells in $S_t$ ($t > t_0$) which are not in $S_{t_0}$ do not affect $\beta_i$. This follows from the definition of the function governing the values associated with $\beta_i$. Particular policy histories up to $t_0$ are expressed by definite pairs of functionals denoted by $[x_0(\tau), z_0(\tau)]$. For every pair of histories a subspace of $S_t$ can be constructed for each $t$ in the interval $(t_0, T)$. This subspace consists of all vectors $[x(\tau), z(\tau), \mu]$ with a fixed history $[x_0(\tau), z_0(\tau)]$ up to $t_0$. These subspaces are denoted with $S_t(x_0, z_0)$.

The construction of the spaces $S_t$ (or $S_t(x_0, z_0)$) together with the appropriate lattice of cells defines a set of discrete domains for the $\beta_i$ and $O_j$ functions. Each domain generates two series via the $\beta_i$ function and any selected $O_j$ function. For any given domain $S_t$ or $S_t(x_0, z_0)$ and any given selection of $O_j$, the two series yield a unique rank correlation expressed by the following function.

$$R_t = C[S_t; j, h]$$

or

$$R_t(x_0, z_0) = C[S_t(x_0, z_0); j, h]$$

For any given space the rank correlation depends on the variable $O_j$ selected, on the hypothesis $h$, and on the spaces $S_t$. Of greatest interest are the spaces $S_{t_0}$ and $S_{t_0}(x_0, z_0)$. But other spaces are not totally irrelevant and may provide useful check points for revising the interpretation of policy.

The formal determination of the optimal choice is described by the expression below. There are two distinct operations to obtain the maximin.

$$\max_j \min_h C[S_t, j, h]$$

First the rank correlation is minimized over the set of hypotheses $h$. The minimization finds the lowest rank correlation between $O_j$ and the true but unknown indicator resulting from our uncertain knowledge about the properties of $h$. The $O_j$ may have a higher correlation with $\beta_i$ than the correlation we have computed, but we do not know this. We do know that there is at least some minimal correlation between $j$ and the
true indicator. Second, from among all the \( j \)'s under consideration, we select as indicator the one that maximizes the lowest rank correlations resulting from partial information. This \( O_j \) is the indicator of the thrust of monetary policy.

One last implication of our analysis concerns the role of uncertainty. If there is incomplete knowledge about the structure, our analysis shows that it becomes more difficult to judge the direction in which policy changed. The reason for this is that the rank correlations are reduced as knowledge becomes less certain, so the maximin is reduced and we are forced to use a less reliable indicator. On the other hand, the maximin of the rank correlation rises if policy action is restricted to a single magnitude. We gain this reliability by sacrificing a larger array of instruments. In particular, we conjecture that in the context of many econometric models, the restriction of policy to manipulations of the base raises the maximin of the rank correlation to unity.

The Choice of a Target

We have already defined the target problem as the problem of choosing an optimal strategy to guide policy operations. Under conditions of uncertainty about the structure and lags in the receipt of information about prices, output, employment, or other variables that are among the more remote goals of policy, policy makers try to relate policy actions to current conditions. For example, Treasury bill rates are used as a target at times. During recessions, the policy target may be to keep the Treasury bill rate at some relatively low level if member bank borrowing does not increase. More generally, the target problem is to find the optimal relation between changes in policy and conditions in the environment.

This analysis of the target problem uses the framework developed in the previous section. The behavior of the economy is described in the most general terms by the differential vector equation

\[
y = h(y, x, z)
\]

The notation emphasizes again that the form of the equation or some of its broad properties depend on the hypothesis \( h \).

For our discussion of the target problem, the description of the process must be supplemented by a definition of policy patterns; a scale must also be defined to measure the desirability of various targets that might be selected as a means of achieving the goals of policy. The

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\[15\] The procedure for choosing an optimal indicator can be applied if the policy variables are endogenous variables determined by the system. Our analysis neither requires nor implies that policy variables are exogenous.
A desirability scale is constructed with the aid of a performance integral:

\[ P = \int_{t_0}^{\infty} w(\dot{y} - c)dt \]

where \( c \) is a given preferred vector of constant time rates of changes. The vector \( \dot{y} \) in the integrand is a function of time. The function \( w \) expresses the relative weights assigned by policy makers to deviations of various components of \( \dot{y} \) from the preferred constant rates of change. The weight function, \( w \), is subject to three constraints: (1) \( P \) is not negative; (2) \( P = 0 \) if and only if \( \dot{y} = c \); and (3) the derivatives of \( w \) with respect to any component of \( (\dot{y} - c) \) assume the same sign as the sign of the component deviation. These properties are satisfied by a quadratic form in \( (\dot{y} - c) \) without cross products. \( P \) ranges, therefore, over the set of nonnegative numbers. The larger \( P \), the larger and far more frequent the deviations of \( \dot{y} \) from \( c \). Since the rate of change in economic activity or of its major components is almost always chosen as a component of the integrand, the analysis implies that less volatile growth is preferred to more volatile growth. The performance integral formulated above will be modified slightly below. This modification yields an expression that can be interpreted as a time average of a variance generated by the information process confronting the policy maker.

At any moment of time, policy makers estimate the value denoted \( y^* \) and \( \dot{y}^* \) of the vectors \( y \) and \( \dot{y} \). The difference between the estimated and actual values is an error term \( \varepsilon \).

\[ y^* = y + \varepsilon_1 \quad \text{and} \quad \dot{y}^* = \dot{y} + \varepsilon_2 \]

The behavior of the error terms is governed by the properties of the production function governing the acquisition of information. This production function and the associated cost function imply the existence of measurement errors and of a lag in acquiring information. Differences in the information cost functions explain why some components of the vector \( \varepsilon = (\varepsilon_1, \varepsilon_2) \) remain comparatively small whereas other components are very large at times. It should be noted particularly that \( \dot{\varepsilon}_1 \) is in general not equal to \( \varepsilon_2 \). Partly independent and partly overlapping procedures may be used to estimate \( y \) and \( \dot{y} \) directly. The error vector is thus governed by a joint density \( p(\varepsilon|I) \) conditional on the vector \( I \) characterizing the pattern of investments made to produce and prepare information. It is postulated that the variances and the means converge toward zero as \( I \) increases.

The last building block is a policy reaction function which defines a strategy for the policy maker.

\[ \dot{x} = \lambda(y^*, \dot{y}^*, x) \]

Each strategy determines a continuous adjustment of policy variables in the light of the policy maker's appraisal of current conditions. The
The target problem consists of the optimal choice of a member \( \lambda \) from the class of strategies \( I \).

Appropriate substitution for \( y^* \) and \( \dot{y}^* \) yields

\[
\dot{x} = \lambda(y + \epsilon_1, \dot{y} + \epsilon_2, x) = \lambda(y, \dot{y}, \epsilon, x)
\]

The vector \( y \) in the last expression may be replaced by means of \( \dot{y} = h(y, x, z) \) and we obtain

\[
\dot{x} = \lambda(y, x, z, \epsilon; h)
\]

where the notation emphasizes again the dependence on \( h \). The last equation is now combined with the description of the economic process. The vector \( v = (y, x) \) is introduced to summarize the expression

\[
\dot{v} = \alpha(v, z, \epsilon)
\]

and \( \alpha \) is used to denote the vector function defined by \( \alpha = (h, \lambda) \).

The weighting function \( w(\dot{y} - c) \) can be extended into a function \( \overline{w}(v - a) \) such that \( w(\dot{y} - c) = \overline{w}(\dot{v} - a) \) for every \( t \). The integrand of the performance integral can thus be rewritten as \( \overline{w}(\alpha - a) \). The form of this integrand implies that its expectation with respect to \( \rho(\epsilon|I) \) is a variance defined by the expression

\[
E_s[v; z; h, \lambda; I] = \int_{-\infty}^{\infty} \overline{w}(\alpha - c)\rho(\epsilon|I)d\epsilon
\]

This variance is a function of \( h \), and particularly of the policy function \( \lambda \). It is also a function of investment in information. If we integrate the \( E \) function over a finite horizon \( (0, T) \), where \( O \) denotes the present time, we obtain an expression proportional to the average variance over the horizon,

\[
J = \int_0^T E_s[v(t); z(t); h, \lambda; I]dt
\]

This integral is optimized subject to three sets of constraints to obtain the optimal \( \lambda \).

i) \( \dot{v} = \alpha(v, z, \epsilon) \)

ii) \( f_i(y, \dot{y}, x, \dot{x}) \leq 0 \quad i = 1 \ldots N_1 \)

iii) \( \int_0^T f_j(y, \dot{y}, x, \dot{x})dt \geq C \quad j = 1 \ldots N_2 \)

The first constraint describes the restrictions on our choices resulting from the nature of the economic mechanism. Once we have chosen \( z(t) \) and \( \lambda \), the path expressed by \( v(t) \) is determined for any \( h \). The second and third constraints reflect political-institutional restriction of various types. For example, central bankers try to keep output in the housing sector (or other sectors) above a minimum, or they try to maintain par-
ticular patterns of asset allocations to financial institutions. These con-
straints are expressed by the inequality, \( ii \). Examples of the third type 
of constraint are found in discussions of the balance of payments and 
the fiscal budget deficits. The cumulated international deficit (defined 
as a negative value) must exceed a critical value \( C \) or the full employment 
budget must show a surplus.

The target is chosen by a minimax procedure

\[
\min_{\lambda} \max_{\mathcal{H}} \max_{h} J
\]

involving the three sets of constraints. First we obtain the worst possible 
performance attributable to our partial information (or ignorance) ex-
pressed by \( h \) and also expressed by the admissible range for the prede-
termined functional \( z(t) \). For every strategy or policy behavior, \( \lambda \), one 
can associate the worst possible performance resulting from partial in-
formation. Then we minimize these worst possible performances with 
respect to \( \lambda \) and choose the policy function \( \lambda \) which yields the smallest 
of all the worst possible outcomes. The arguments with nonvanishing 
derivatives occurring in this optimal \( \lambda \) function are the optimal targets 
to be used in the continuous adjustment of policy variables.

The formal solution to the target problem is applicable to other prob-
lems that economists have discussed. For example, the literature con-
tains a number of papers on the use of "rules," and a number of different 
rules have been proposed. In our framework these rules are claims or 
statements that a particular strategy (or rule) permits us to reach the 
lowest value of \( J \). Our analysis implies that the choice between rules or 
discretionary action is a false issue. The arguments on behalf of discre-
tionary policies describe the advantages to be gained from the con-
tinuous adjustment of policy in response to changing conditions. The 
very nature of this argument involves the operation of a \( \lambda \) function, so 
the choice between rules and discretion reduces to the optimal choice 
of a policy function. If our knowledge is very limited, the optimal 
strategy may be to choose a target that eliminates all variability of policy 
such as the "constant growth rate of money" rule. On the other hand, 
we may possess sufficient information to improve on the performance 
of the economy by changing policy infrequently or by relatively small 
amounts. Formal analysis does not tell us the optimal amount of vari-
ability. It does suggest that differences in performance resulting from 
more variable policies depend on the frequency with which we change 
other policy variables and on the amount of reliable information we 
have about the structure.

Conclusion

In the final sections of this chapter, we attempted to clarify the con-
cepts of indicator and target of monetary policy that were discussed at
the UCLA conference. Our analysis shows the formal relation of these concepts to the frameworks used to analyze policy operations or other operations affecting the economy. These frameworks permit us to compare the properties of endogenous variables used by many economists and policy makers either as indicators or as targets of policy. Moreover, the analysis provides a general framework within which it becomes meaningful to discuss issues such as "rules versus authority" or, perhaps more relevantly, the desirable range of variability in monetary policy.