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The Meaning of Monetary Indicators

Karl Brunner
Ohio State University - Main Campus

Allan H. Meltzer
Carnegie Mellon University, am05@andrew.cmu.edu

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Karl Brunner and Allan H. Meltzer

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(Continued on inside back cover)
THE MEANING OF MONETARY INDICATORS*

Karl Brunner and Allan H. Meltzer

Among the topics that monetary economists discuss, few have been debated as much or as long as the meaning of a given change or rate of change in the stock of money or the interest rate. Some have stressed the importance of monetary or of interest rate changes as a guide to the future pace of economic activity. Yet it seems reasonable to conclude that the partisans have not succeeded in convincing others that one or the other of these variables is the most reliable measure of the effect of monetary policy. There is not even agreement that the choice is restricted to these two measures. Bank credit, free reserves, liquid assets, and other variables are mentioned frequently. Since it is not unusual to find that quite different—even opposite—conclusions are suggested by the various measures, a comparison of the information provided by some of the variables proposed as indicators is called for.

The problem may be restated as a series of questions. What information about monetary policy is conveyed by the position of, or change in, a particular variable? How do we choose from among the many available time series those that are to be watched more carefully than others? By what criteria do we decide that a particular variable is a good or better indicator of current or recent monetary policy?

Many discussions of monetary policy take a rather different starting point from the one taken here. A particular hypothesis relating monetary policy to prices and output is assumed—usually implicitly—to be well established. Policy implications are obtained from the hypothesis. Attention is then directed toward the problem of forecasting the future, selecting a policy goal, and/or measuring lags of various kinds.1 Without

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disputing the importance of these problems, an obvious point should be noted, that is, that the policy conclusions depend on the hypothesis selected. Equally important, the measurement of lags depends on two types of hypotheses: (1) relatively crude approximations to an underlying dynamic theory of economic activity or policy-making behavior and (2) hypotheses about the remaining structure of the economy.

In this paper, we acknowledge our ignorance—or relatively incomplete information—about the structure of the economy. Given this state of knowledge, particularly the absence of quantitative estimates of many of the parameters of a general-equilibrium macromodel, of the speeds of adjustment of many of the variables, and of the distribution of the effect of monetary policy through time, a number of questions arise. What information does the policy maker have available to decide on a future course of action, or to judge the present or recent position of the economy? How does he assess the results achieved by past policy or likely to be achieved in the near future? How does he decide whether his policy has resulted in relative restraint or relative ease?

One solution is to decide that additional knowledge is so difficult to obtain, so subject to error, or so hard to interpret that reliance must be placed on simple solutions, for example, monetary rules, very simple qualitative hypotheses, or perhaps on "color, tone, and feel." An alternative is to use the available quantitative and qualitative information, while recognizing the incomplete and uncertain character of the knowledge possessed.

A strategy for combining incomplete information about the structure of the economy with an assumption about the goal of policy is discussed here. The problem is referred to as the "indicator problem," since we are concerned with the relative merits of a number of variables often used to indicate the current direction of monetary policy or the future effect of recent policies. Our tentative results are not presented as a resolution of the problem, only as a means of opening for analysis a topic that is more frequently debated than analyzed. At the outset we acknowledge that many of the questions raised here can be answered more fully if (and only if) more useful knowledge about the structure of the economy is assumed or obtained. Put otherwise, the theorist may choose to ignore this problem by assuming the possession of reliable information.


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currently outside the scope of quantitative economics. The policy maker is not as fortunate.

THE GENERAL PROBLEM

By way of contrast with our present state of knowledge, consider a situation in which there is a fully identified, highly confirmed theory of macroeconomic processes. There is then quantitative information about the structural parameters, or useful knowledge, to use Marschak's apt phrase. Since the policy maker can infer, from the comprehensive model, the expected effect of policy action on any or all variables that are of interest to him, he can determine the amount that policy must contribute to achieve a particular set of social goals. If the structure is known and the goal or social utility function is specified, knowledge of the change in monetary policy permits the policy maker to evaluate the effects of his past actions. The indicator of monetary policy summarizes in an index the relative degree of monetary ease or restraint. Let \( I \) be the indicator of monetary policy. Then

\[
I = \frac{du}{dy_1} \frac{dy_1}{dx_1} + \frac{du}{dy_2} \frac{dy_2}{dx_1} dx_1
\]

where \( u \) represents the social utility function, \( y_1 \) and \( y_2 \) are endogenous variables of the system—for example, prices and real output—and \( dx_1 \) is the change in variables expressing monetary policy operations.

The derivatives of the endogenous \( y \) variables with respect to \( x_1 \) describe the responses of the goal variables to policy operations. These derivatives are, of course, total derivatives obtained by differentiating over the whole system of relations, and depend on the derivatives of the structural equations. Since the derivatives of each of the structural equations and of the utility function are assumed to be known, all that is required to measure changes in the degree of ease or restraint is a measure of the change in one or more of the monetary policy variables that has been altered.

Some of the elements in the more general problem have been introduced in the example just considered. First, there is a choice to be made about the nature of the utility function. Second, hypotheses must be

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The nature of the index depends, of course, on the goal or utility function. If the latter is unique only up to order-preserving transformations, \( I \) must be treated as an ordering. For most practical purposes this would be sufficient. Note that since \( y_1, y_2, \) and \( x \) are vectors, \( \frac{du}{dy_1}, \frac{dy_1}{dx} \) are vectors and the \( \frac{dy_1}{dx} \) are matrices.
selected. Economics has not advanced far enough as an empirical science to make the choice obvious. Moreover, very little is known about the values of the parameters of any set of hypotheses that might be used. The example suggests that any change in the structural relations is capable of altering the indicator. Hence, the index value of any particular policy depends on the hypotheses. Third, changes in institutional arrangements alter the channels and effects of monetary policy. With relatively complete knowledge, institutional rearrangements do not create serious problems for the construction of an indicator. However, we will note below that there is a trade-off between complex institutional arrangements and the amount of knowledge about the system that is required. Finally, the example suggests that there is only one relevant strategy, namely, to construct the appropriate indicator for the evaluation of policy. This simple strategy is no longer meaningful when we revert to the position of relatively incomplete knowledge. A choice of strategy must then be made. For example, it may be desirable to choose policies or indicators that minimize the extent to which knowledge is required about the structure of the system; or, the strategy may be one of choosing an indicator that minimizes the chance of serious misinterpretation of the result of policy actions.

The indicators of monetary policy usually mentioned by economists are not even approximately related to the indicator function introduced above. Most of them are endogenous variables. As such, their position or rate of change at any time is the result of the joint interaction of the whole system and reflects more than the effect of current monetary policy. Fiscal policies and noncontrolled exogenous variables also influence the endogenous indicators. Moreover, their current position or rate of change is the result of partial or incomplete adjustment to the long-run position implied by the expected response to changes in policy and other exogenous forces. Information is rarely available on the proportion of the adjustment which has already occurred.5

The danger of misinterpreting the current direction of monetary policy exists in principle when any endogenous variable is used as an indicator. Suppose interest rates on financial assets are accepted as the indicator. Relatively high rates are interpreted as a "tighter" policy, and rising rates are taken as an indication of "tightening." Further, assume that interest rates are accepted as the principal financial variable that transmits monetary impulses to the real variables and hence to the pace of economic activity. The bond yield is endogenous. Its behavior is determined by the interaction of the whole system under the impact of changes in the monetary and nonmonetary policy variables and in the noncon-

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5Even if the length of the lags is assumed to be well established, the time distribution of the adjustment may be known less precisely.
trolled variables. If there is sufficient information available to construct the indicator function $I$, there is no danger of misinterpretation. A rise in interest rates that results from nonmonetary forces, say an increase in the expected yield on real capital, can be separated from the effect on interest rates of expansive monetary policy. Without the information required for construction of the indicator function, the rise in interest rates will be interpreted as an indication that monetary policy has become tighter and more restrictive when the indicator, if it were available, would reveal the error in this interpretation.

The fact that interest rates are taken as the central element transmitting monetary policy from the financial to the real sector does not establish that they are a better (or worse) indicator of current monetary policy than some other endogenous variable. But the central role assigned to interest rates would suggest that they are influenced greatly by feedbacks from the real to the financial sector, so that their position or direction of change at any time is a result of opposing influences. Similar statements apply to the money supply and other endogenous variables, although the relative size of policy and nonpolicy influences will differ with the endogenous variable selected as an indicator.

It is possible to rank many of the endogenous variables by their quality as indicators, if we are not completely ignorant about the goals of policy and the operation of the economic system. To do so, a class of hypotheses must be selected. In principle, this choice should be made by systematically comparing alternative classes. In practice, we have selected one with which we are familiar, on the grounds that it includes among the endogenous variables many of the indicators that are frequently suggested or used to appraise the direction of monetary policy and is sufficiently rich in detail to bring out clearly the problem of choosing an indicator.⁶

**AN OUTLINE OF THE HYPOTHESIS**

The indicator function introduced above expressed the connection between monetary policy ($x_t$) and the indicator $I$. In the model, changes in policy affect the level of output, prices, and other real variables by changing financial variables and relative prices. Absence of useful knowledge about many structural relations of the transmission process does not alter the need for a framework like the one underlying the indicator func-

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⁶The statement in the text should not suggest that no effort has been made to compare alternative theories. The velocity relation plays a key role in the analysis below. It has been compared to a variety of alternatives in our “Predicting Velocity: Implications for Theory and Policy,” *Journal of Finance*, May, 1963, pp. 319-54, and in “The Demand for Money: The Evidence from the Time Series,” *Journal of Political Economy*, June, 1963, pp. 219-46. See also the references cited in the next footnote.
tion. To appraise the effect of policy operations, we require both a theory and a utility function. Relations summarizing behavior on the financial and output markets must be presented before several of the variables proposed as indicators can be compared. The hypotheses used for this purpose are described briefly in this section. The appendix provides a condensed, formal statement.

The set of relations that determine the partial equilibrium solutions for the money supply, the banks' portfolio of earning assets, and interest rates have been presented in some earlier work. The equations of the monetary system underlying the equilibrium conditions summarize the allocation decisions of the banks and the public and their responses to the decisions of the monetary authority.

The public's behavior is expressed through three relations: (1) the allocation of money between currency and demand deposits, (2) the allocation of bank deposits between demand and time accounts, and (3) the supply function of earning assets to banks. Banks are assumed to allocate assets in response to changes in a number of cost, yield, stock and flow variables. Of particular importance, at present, is the dependence of the money supply and the banks' demand for earning assets on interest rates and real income. This dependence is a consequence of the effect of interest rates on the banks' demand for excess and borrowed reserves, of real income on the currency ratio, and of both interest rates and real income on the ratio of time to demand deposits. Interest rates, real income, and the expected yield on real capital also affect the equilibrium solution for interest rates and bank credit through the public's supply of earning assets to banks (see equation A7).

Six policy variables record the decisions of the monetary authorities. The open market portfolio is one of the sources of the adjusted base—an exogenous variable of the system. The adjusted base is equal to reserves plus currency minus member bank borrowing. The remaining policy variables affect the monetary system by changing the monetary and asset multipliers: equations (A8) and (A9) of the appendix. The rediscount rate and the ceiling rate on time deposits affect the multipliers through the desired free reserve and time deposit ratios, respectively. Changes in the reserve requirement ratios and in the proportion of vault cash included in required reserves change the weighted average of reserve requirement ratios, denoted by \( r \) in the appendix.


In the appendix this is expressed by the dependence of the free reserves and time deposit ratios on interest rates. (See equations (A8) and (A10).) Other variables on which \( f, k, \) and \( t \) are assumed to depend are not discussed, since they do not play an important role in the subsequent analysis. The references in the previous footnote provide more details about the determinants of these ratios.
The solutions for the money supply and interest rates from the financial sector are connected to the real sector by a quantity equation. A demand function for money, akin to the one emphasized in much of our recent work, is expressed as a velocity relation. The aggregate demand for output, $MV + G$, is the sum of government expenditure ($G$) plus private expenditure ($MV$). The quantity equation relates aggregate government and private demand with nominal supply. A price-setting function expresses the adjustment of supply prices to variations in current and past output and in capital stock.

Fiscal policy operations affect both financial and real variables. As noted above, the government's income-generating expenditure has been introduced as a component of total spending. In addition, equation (A5) relates the government's cash deficit to the volume of government expenditures and to the means by which the deficit is financed. A surplus or deficit in the cash budget raises or lowers the sum of interest- and noninterest-bearing government debt. Changes in both types of debt affect real output and the equilibrium position of the financial sector. If the deficit is financed by noninterest-bearing debt, the adjusted base is increased.

The relations just described do not constitute a specific hypothesis of the type required for construction of the indicator function introduced above, since little is known about the speeds of adjustment or other parameter values in the equations. Rather, the system remains quite general and is representative of a large class of hypotheses. This is particularly true of the relations describing the real sector.

Nevertheless, preliminary investigations suggest that sufficient information is available about the class of hypotheses to reach tentative conclusions about some of the indicators frequently used by economists. The reason is that the comparative merit of various indicators is not greatly affected by the particular relations used to describe the real system. The analysis presented in a later section furnishes a firmer foundation for this conclusion.

This is not the place to discuss the implications of the model in detail. However, a few of the implications bear directly on the indicator problem since they describe some of the effects of monetary policy on output and prices, the requirements of an effective monetary policy, and some determinants of the length of the lag between policy action and its effect.

First, the elasticity of real output with respect to the coordinates of the monetary policy vector, $x_v$, is taken as the measure of the effectiveness of monetary policy operations of given size. The necessary and sufficient conditions for an effective monetary policy depend on the interest elasticities of velocity, of the public's supply of assets to banks, and of the monetary and asset multipliers.

Second, the larger the interest elasticity of velocity and/or of the
monetary multiplier, the smaller the response of output to monetary policy (and the larger the response to fiscal policy). A large interest elasticity of the public's asset supply and of the banks' earning asset multiplier raises the size of the response to monetary policy.

Third, constant growth of monetary magnitudes affects the level of output and the rate of change of prices. Constant growth generates no fluctuations, although fluctuations may occur for other reasons—for example, through the delayed adjustment of supply prices. The variability of monetary magnitudes is more important than their level in explaining their contribution to large fluctuations in output. (Similar statements apply to fiscal policies.)

Fourth, a "Friedman lag" emerges whenever acceleration or deceleration of monetary magnitudes (that is, second differences in the stocks) becomes pronounced and persistent. The length of the lag is dependent on the length and magnitude of the acceleration or deceleration of the monetary variables.

Fifth, the delayed adjustment of output prices in response to past output and capital stock means that the short-term effects of monetary policy are on real output. Over the longer term, the price level absorbs the effects of policy.

THE TRUE OR IDEAL INDICATOR

The relations in the previous section do not provide all the information required for the derivation of the true indicator. A social utility function must be introduced and a policy goal must be selected. Since this paper is concerned primarily with the comparison of several suggested indicators and with the general problem, a rather simple utility function is used. Utility is treated as a monotonic, increasing function of real income which has the form $u(\log \frac{y}{c})$, where $y$ is real income and $c$ is the capacity level of output. The only goal of monetary policy is to increase real income.

Once the goal is selected, the theory outlined in the appendix can be combined with the goal in an index that permits monetary policy to be ordered and compared. If there is enough information to compute the index, vague terms such as "easing" and "tightening," often used to characterize policy, can be replaced by statements about the movement...
of the indicator. A rising index denotes an easier policy; reductions in the
index show that policy is tighter.

The movements of the “true” indicator are given by $I$, where

$$
\Delta = \epsilon(y, B^a) \frac{dQ}{Q}
$$

A multiplicative factor consisting of the marginal utility with respect to
log $y$ has been omitted. The first component, $\epsilon(y, B^a)$, is the elasticity
of real income (the goal) with respect to open market operations and
other changes in the adjusted base $B^a$. Since this component enters the
indicator function as a scalar, it can be neglected in the subsequent dis-
cussion. The second component, $\frac{dQ}{Q}$, provides all of the information
required to order monetary policy under the given hypothesis and a
simple goal. It contains three terms:

$$
\frac{dQ}{Q} = \frac{dB^a}{B^a} + \frac{dq}{q} \left[ \frac{h_1 - h_2a}{h_1} \right]
$$

(1)

The first two ratios contain all of the monetary policy variables—the
monetary base, the reserve requirement ratios, the rediscount rate, and
so forth. Policy variables other than $B^a$ operate through the monetary
multiplier and are expressed in $\frac{dq}{q}$ as weighted relative rates of change.

The third (bracketed) term in equation (1) is a combination of interest
elasticities on the credit market ($h_1$) and on the output market ($h_2$)
and a parameter ($a$) equal to the ratio of the money supply plus time
deposits to bank earning assets. These expressions are more fully defined
in section C of the appendix, where it is noted that $a$ is greater than
unity and that the bracketed expression on the right is positive, but less
than unity, under the hypothesis.

Reliable information is available about some, but not all, of the
components. The percentage rate of change of the base and of other
policy variables can be measured exactly. Some of the weights used to
combine the policy variables in $\frac{dq}{q}$ have been computed; others can
be placed within a narrow range. Much less is known about several of
the elasticities in $h_1$ and $h_2$. Without such information, the indicator cannot
be computed reliably.

The problem of inadequate information vanishes if policy action is

$$
\frac{dq}{q} = \epsilon(m, r^*) \frac{dr^*}{r^*} + \epsilon(m, r^*) \frac{dr^*}{r^*} + \ldots
$$

plus similar terms for the rediscount rate, the vault cash counted as required re-
serves, and the ceiling rate on time deposits. The expressions $\epsilon(m, x)$ are elasticities
of the monetary multiplier ($m$) with respect to a particular policy variable. For a
description of the variables see the symbol dictionary in the appendix. The definition
of $m$ is given as part of equation (A9).
restricted to open market operations. The base and the true indicator then coincide. Construction of the indicator is reduced to measurement of changes in an exogenous variable, and the policy position is completely and accurately described by the growth rate of the base.\textsuperscript{11}

It is apparent from the above that prevailing monetary arrangements increase the amount of information required by the policy maker. In short, there is a trade-off between knowledge and the complexity of policy arrangements. The unknown benefits of having a variety of policy instruments seem a high price to pay for the substantial increase in the amount of information required to measure the quantitative impact of policy. Until the requisite knowledge becomes available, it would be useful to restrict policy operations to changes in the adjusted base.

Much of the time, policy operations are dominated by open market operations, so that the true indicator and the percentage rate of change in the base coincide. Unfortunately, this happy coincidence occurs least frequently when the desire for correct information is most pressing. At or near the peak of an expansion, bill rates generally rise above the prevailing discount rate, and the discount rate is raised while the growth rate of the base is compressed. Our estimate of the response in $q$ to a 1 percent change in the discount rate is relatively small, but percentage changes in the discount rate are generally quite large and therefore capable of dominating the movement of the indicator at or near the upper turning point in real income.

A COMPARISON OF SOME PROPOSED ENDOGENOUS INDICATORS

Since construction of the true indicator requires information that is not available, it is useful to consider the merits of variables often accepted as indicators. Movements of interest rates, free reserves, the money supply, and other entities have been used to measure the effect of monetary policy. All of these variables have the advantage of being observable. But each is an endogenous variable in the hypothesis introduced earlier, so that current movements are in part the result of feedback from the financial or output markets. The fact that the endogenous indicators can be measured need not, therefore, be an advantage. Separation of the influence of monetary policy from other influences often requires more information than the construction of the true indicator.

\textsuperscript{11}The effect on the indicator of redistributions of deposits between classes of banks with different requirement ratios is ignored. Otherwise, the statement in the text must be changed to eliminate the minor effect on the indicator caused by the slight difference in the average reserve requirement ratio resulting from deposit redistribution. The conclusion that the relative change in the base is equivalent to the ideal indicator when $q$ is unchanged does not hold for every utility function. It holds in the present case because increased real income is chosen as the only goal of policy.
In this section, several endogenous indicators are discussed. Solutions for the rates of change are derived from the hypothesis, and the problems caused by the presence of structural parameters of unknown value are considered. Several of the indicators are expressed in terms of the true indicator, so that differences between them can be compared. The discussion of alternative strategies for choosing an appropriate, observable indicator is deferred, however, to a later section.

The Money Supply

A small but vociferous group of economists takes the rate of change of the money supply as a simple and straightforward indicator of monetary policy. Increases in the money supply are interpreted as expansive and decreases as contractive. Leaving aside, momentarily, differences of opinion about the suitable definition of the money supply, we will define the money supply as currency and demand deposits. The percentage rate of change of the money supply so defined is given by equation (2), where \( e(m, i) \) and \( e(m, y) \) are the interest and real income elasticities of the monetary multiplier, \( m \):

\[
\frac{dM}{M} = \frac{dB^a}{B^a} + \frac{dq}{q} + e(m, i) \frac{di}{i} + e(m, y) \frac{dy}{y} \tag{2}
\]

The money supply is by no means an ideal indicator. It misstates the magnitude of changes in the policy variables and records the effects of changes in other exogenous variables. The solution for \( \frac{dM}{M} \) incorporates the influence of monetary policy variables, feedback effects, and the influences of fiscal and noncontrolled exogenous variables. The latter are merged with the feedback effects and summarized by the variations in two endogenous variables, interest rates (\( i \)) and real income (\( y \)).

Changes in the adjusted base appear to have the same effect on the money supply as on the ideal indicator. But policies that expand the money supply lower interest rates on financial assets, so that a given percentage change in the base induces a smaller percentage change in the money supply. The size of the difference depends on the size of the interest elasticity of the monetary multiplier, \( e(m, i) \). Our estimates suggest that this elasticity is in the neighborhood of .1 or .2.\(^{12}\) If these

\(^{12}\) See "Some Further Investigations . . ." op. cit. Additional estimates have since been computed using both quarterly and annual data. While such estimates are tentative, \( e(m, i) \) is generally small and positive. It is, of course, possible that \( e(m, i) \frac{di}{i} = e(m, y) \frac{dy}{y} \), so that the error in using \( \frac{dM}{M} \) in place of the true indicator would be quite small. Our analysis gives no reason to believe that this fortunate result should be expected. Nor can the similar terms appearing below in the equations for free reserves and interest rates be expected to cancel.
estimates are correct, the effect of changes in interest rates on the money supply is attenuated. Nevertheless, the money supply slightly understates the effect of changes in \( B^a \) relative to the true indicator.

On the other hand, a money supply indicator overstates the direct effect of changes in the reserve requirement ratios, the rediscount rate, and other policy variables in \( q \). The reason is that \( \frac{dq}{q} \) appears in the solution for the ideal indicator, weighted by a ratio of elasticities with a value less than 1. Changes in interest rates (induced by the changes in \( q \) ) lower the amount by which \( \frac{dM}{M} \) overstates the effect on the ideal indicator of policies operating through \( q \). But it is unlikely that the induced change in interest rates offsets the error caused by the absence of the ratio \( \frac{h_1 - h_2 \alpha}{h_1 - h_2} \) in equation (2).

Finally, \( \frac{dM}{M} \) is affected by the exogenous, noncontrolled, and fiscal policy variables which affect \( i \) and \( y \) but which do not appear in the solution for the ideal indicator. Since \( e(m,i) \) is quite small, variations in interest rates induced by changes in interest-bearing government debt, capital stock, and so forth do not greatly reduce the usefulness of \( \frac{dM}{M} \) as an indicator. However, the combined effects of the many changes summarized by \( \frac{dy}{y} \) have not been carefully estimated. Although the evidence which has been collected strongly suggests that monetary policy operations and changes in the currency ratio are the dominant influences on the money supply, the problem of separating the effects of monetary policy from other influences remains if the money supply is used as an indicator.

Other Monetary Variables

The rates of change of commercial bank credit (\( E \)) and the money supply plus time deposits (\( M + T \)) are also used as indicators of monetary policy. The solutions for these variables contain terms very similar to the solution for the money supply. Relative to the true indicator, they, too, overstate the effect of policy variables combined in \( q \), and understate the effects of policies operating through the adjusted base. Moreover, \( M + T \) and \( E \) are more responsive than the money supply to relative changes in interest rates and in the variables operating through the relative change in real income. The reason is that the elasticities of the multipliers of \( M + T \) and \( E \) with respect to interest rates and real income are larger than the similar elasticities for the money supply.
Our analysis suggests that because the elasticities are largest for $E$ and smallest for $M$, the difference, relative to the ideal indicator, is also largest for $E$ and smallest for $M$. This is particularly true for open market operations and other changes in the base. In addition, nonmonetary policy changes are more fully reflected in $E$ and $M + T$ than in $M$. The money supply is likely to be a better indicator of current or recent monetary policy than the other monetary variables. However, it bears repeating that the money supply is not equivalent to the ideal indicator.

**Interest Rates**

Market rates of interest are perhaps the most popular indicators of monetary policy among academic economists. One reason may be that the transmission of monetary policy is generally described as operating through a number of interest rates before reaching the components of aggregate expenditure. The theory underlying the discussion of indicators is not incompatible with general statements about the importance of interest rates in the transmission mechanism. But it does not follow from such statements that interest rates are (or are not) closely related to the ideal indicator. The quality of interest rates as an indicator is given by the comparison of the information they convey with the information conveyed by the ideal indicator.

Equation (3) states the solution for the relative change in $i$ in terms of the ideal indicator, where $z$ is a vector of fiscal and noncontrolled variables including the capital stock, the stock of outstanding government debt, and other variables treated as exogenous under the hypothesis; $g_1$ and $g_2$ are combinations of elasticities with respect to real income on the output and credit markets, defined in section C of the appendix; and $\epsilon(i, B^a)$ and $\epsilon(y, B^a)$ are the elasticities of interest rates and output with respect to the extended base:

$$\frac{di}{i} = \epsilon(i, B^a) \left[ \frac{dQ}{Q} + \frac{dq}{q} \frac{(\alpha - 1)}{(g_1 - g_2) \epsilon(y, B^a)} \right] + \epsilon(i, z) \frac{dz}{z} \quad (3)$$

The bracketed expression contains all of the monetary policy variables. Since $\epsilon(i, B^a)$ is negative, and the ratio modifying the effect of $\frac{dq}{q}$ is positive, expansive policies (rising $B^a$ or $q$) lower interest rates on financial assets. Lower interest rates are interpreted, therefore, as an indication of expansive policies.

\[^{18}\text{To obtain the solution for} \frac{di}{i} \text{in terms of} Q, \text{we make use of the solution for} \epsilon(y, B^a) \text{derived from the hypothesis:}\]

$$\epsilon(y, B^a) = \frac{h_x - h_xa}{g_1h_1 - g_2h_2}$$
Two important consequences follow from this interpretation of interest rate movements. First, the bracketed expression containing the monetary policy variables exceeds the true indicator whenever policy operates through \( q \). Thus the effects of changes in the reserve requirement ratios, the rediscount rate, vault cash policy, or the ceiling rate on time deposits are subject to misinterpretation. This difficulty can be removed by restricting monetary policy to open market operations. The relative change in interest rates is then proportional to the relative change in the indicator. But there would then be no reason to use interest rates or any other endogenous variable as an indicator. Monetary policy would be measured correctly by the relative change in the adjusted base.

The second consequence is the result of a more fundamental problem. The choice of \( i \) as an indicator attributes the effects of changes in the fiscal and noncontrolled variables, represented by \( dz \), to monetary policy. Changes in the outstanding stocks of interest-bearing government debt or of real capital are important influences on \( \frac{di}{i} \). A large government deficit financed by new debt issues imparts substantial momentum to rising interest rates. Even if an increasing portion of the new issues is absorbed through expansion of the monetary base so that \( Q \) rises, \( \frac{di}{i} \) may increase. Reliance on interest rates would lead to the conclusion that monetary policy is restrictive, despite the rise in \( \frac{dQ}{Q} \) indicative of an acceleration of expansive policy action.

With the exception of the thirties, interest rates generally move procyclically. The influence of \( \frac{dQ}{Q} \) is usually overwhelmed by the effects of variables combined in \( \frac{dz}{z} \). Interpretations of monetary policy based on interest rate movements neglect the powerful influence on interest rates of new issues of interest-bearing debt or of changes in the capital stock. Policies are judged to be restrictive whenever the effect of \( \frac{dQ}{Q} \) is dominated by the effect of \( \frac{dz}{z} \), despite the possible large value of \( \frac{dQ}{Q} \) and the true effect of monetary policy on output and economic activity. Similarly, monetary policy is judged to be easy early in the downswing because of the delayed effects of a slower growth rate of \( Q \). The slower growth rate of \( Q \) gradually reduces the growth rate of income and prices and thus lowers \( \frac{dz}{z} \). The effect of falling \( \frac{dz}{z} \) more than offsets the fall in \( \frac{dQ}{Q} \), generating falling interest rates or falling \( \frac{di}{i} \). The interpretation of policy as easy when the true indicator decelerates convinces the monetary
authority that a policy of expansion has been initiated, when their actual policies are in the opposite direction.\textsuperscript{14}

**Free Reserves**

Few indicators of monetary policy have been used more consistently than the level of free reserves. Nearly every week, the financial press describes Federal Reserve policy in terms of the movement of free reserves, and evidence of the Federal Reserve's attachment to free reserves has been presented elsewhere.\textsuperscript{15} Increases in free reserves are interpreted as expansive or indicative of easier policy; reductions are presumed to reflect less ease or greater restraint.

The solution for the volume of free reserves, $F^*$, implied by our theory,

$$F^* = \phi B^*$$

makes free reserves depend on all of the monetary policy variables. Open market operations affect free reserves through the base, $B^*$, and other monetary policies affect $F^*$ through $\phi$, where $\phi$ is a rational function of the free reserve ratio and of the time deposit, currency, and weighted average reserve requirement ratios.\textsuperscript{16} In short, $\phi$ depends on the monetary policy variables summarized in $q$, on the components of the monetary and asset multipliers, and thus indirectly on output and interest rates as well. The joint determination of these central endogenous variables in terms of the policy and noncontrolled exogenous variables makes $\phi$ dependent on all of the exogenous variables—fiscal, monetary, and noncontrolled. Equation (4) is a compact statement of the result for nonzero $F^*$.\textsuperscript{17}

$$\frac{dF^*}{F^*} = \frac{dB^*}{B^*} + \frac{dq}{q} + \epsilon(f, \rho) \frac{dp}{p} + \epsilon(\phi, i) \frac{di}{i} + \epsilon(\phi, y) \frac{dy}{y}$$

\textsuperscript{14}It may appear that the problem just discussed arises if $\frac{dM}{M}$ is used as an indicator. Since $\frac{dM}{M}$ depends on $\frac{di}{i}$ and, in addition, on the components of $\frac{dx}{x}$, the relative change in the money supply can lead to a misinterpretation of monetary policy. However, the data suggest that the behavior of the money supply is generally dominated by the currency ratio and the monetary policy variables, as noted earlier, so that the problem is much less serious for $M$ than for $t$.


\textsuperscript{16}A coefficient of the third term has been assumed to be unity and hence omitted. The coefficient is $1 - \epsilon(m,f)$, where $\epsilon(m,f)$ is the elasticity of the monetary multiplier with respect to the free reserve ratio. The omitted term is almost always between .99 and 1.01.
The first three terms contain the monetary policy variables. The direct effect of changes in $B^*$ are recorded correctly by the free reserves indicator. But changes in the base modify interest rates on financial assets so that the full effect of open market operations and other changes in $B^*$ on $\frac{dF^*}{F^*}$ includes the response represented by $\epsilon(\phi,i) \frac{di}{i}$. While the effect of $i$ on the money supply is small, its effect on free reserves is quite large. Moreover, the sign of $\epsilon(\phi,i)$ is opposite to the sign of $F^*$. When free reserves are negative, the free reserves indicator exaggerates the magnitude of expansive policy actions and understates the size of contractive policies. Positive free reserves have precisely the opposite effect. They cause the free reserves indicator to overstate the size of reductions in the base and underestimate the size of increases.

Additional problems arise when monetary policy operates through $q$. Since $\frac{dq}{q}$ is not multiplied by the ratio of interest elasticities that appears in the true indicator, the direct effects of variations in the reserve requirement ratios, the rediscount rate, and other policies are misstated. Moreover, the rediscount rate, $\rho$, appears as a separate term in addition to its effect on $\frac{dF^*}{F^*}$ through $dq$. This term is another source of differences between the free reserves indicator and the true indicator when the discount rate is changed. Furthermore, changes in any of the terms in $q$ induce changes in interest rates and hence lead to either overestimates or underestimates of the magnitude of policy operations, depending on the sign of $F^*$. Those who rely on free reserves as an indicator are likely to be misled by their interpretation of the size of policy actions.

Toward the end of a period of economic expansion, free reserves are generally negative and monetary policy is frequently moving in a deflationary direction. The use of free reserves as an indicator underrates the size of the deflationary impulse. A drastic application of policies designed to "prevent inflation" might seem called for because of the attenuated response in free reserves. During a downswing, free reserves are generally positive. Expansions in the base or in the variables denoted by $q$ are understated by the free reserves indicator. If the $F^*$ indicator is used by policy makers as a guide to desired policy, it is likely to produce abrupt changes in the direction and magnitude of the size of policy action.

Variations in interest rates and in free reserves induced by monetary policies cannot be separated from the influence of fiscal and noncontrolled forces without reliable information about the structure of the process. Major changes in free reserves are often a response to changes in real variables. For example, a decline in the expected yield on real capital induces an increase in free reserves. This typically happens to-
ward the end of an expansion phase and is one of the forces contributing to the termination of the expansion. Influenced by the expansion in free reserves, the monetary authority becomes convinced that policy is "aggressively easy." This error in interpretation is superimposed on the underestimation of deflationary policies associated with the sign of $F^*$, discussed above. Attributing the movements in free reserves to monetary policy misconstrues the effect of actual policy operations.

THE CHOICE OF STRATEGY AND INDICATOR

None of the endogenous variables we have considered indicates the exact effect of monetary policy on real income. Each is potentially or actually misleading. But the imperfections are not the same and vary substantially with the choice of endogenous indicators. In the absence of knowledge about the structure, some choice must be made among imperfect alternatives.

One solution is to confine policy operations to changes in the base, so that relative changes in the base become the ideal indicator. Another is to choose a strategy or criterion under which the performance of the various endogenous indicators can be compared. A third alternative is to search for measurable indicators that bracket the ideal indicator between an overestimate and an underestimate. In this section, some alternative solutions are discussed.

The choice of a strategy depends on the utility function of the policy maker. He may wish to minimize the expected loss of utility or minimize some function of the mean and variance of the deviations between measured and ideal indicators. Most choices of strategy require substantially greater additions to knowledge than is required to compute the ideal indicator.

The strategy we have selected for illustrative purposes is a minimax strategy. Given the hypothesis and the simple goal, we minimize the worst possible outcome (misinterpretation) attributable to incomplete knowledge about the effect of a specific set of policy changes occurring through changes in $B^a$ and $q$. In the particular case, this means that the maximum deviations between the various endogenous indicators and the ideal indicator are computed, using our limited knowledge of the structural parameters. The endogenous variable that has the smallest maximum deviation is then chosen as the optimal indicator.

Theories of search provide another means of investigating the problem of choosing an optimal indicator. Optimal search procedures can generally be stated as a minimax problem. See D. J. Wilde, Optimum Seeking Methods (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1964). We are indebted to M. Kamien for this reference.

Appendix D presents the formal outline of the procedure. The appendix was added in the last draft of the paper.
The material for the solution has been provided. The ideal indicator, \( \frac{dQ}{Q} \), is given in equation (1) above. The difference between the value of \( \frac{dQ}{Q} \) and the solution for each of the endogenous variables can be computed easily. When the computations are performed, the money supply (currency and demand deposits) emerges as the optimal indicator.

Investigation of five frequently used financial variables thus establishes that, relative to the goal function, hypothesis, and strategy, the money stock is the best approximation to the true indicator. It is not ideal and has been quite misleading on occasion in the postwar period. Nevertheless, it is the least misleading and least dangerous single guide to the position of monetary policy.

A solution of quite a different kind is to combine two indicators, one more responsive and one less responsive than the ideal indicator. Fortunately, a prolonged search for measures with these properties is not required if the particular hypothesis and goal are maintained. Let one approximate indicator be

\[
\frac{dQ^*}{Q^*} = \frac{dB^*}{B^*} + \frac{dq}{q}
\]

It is clear from inspection of the ideal indicator that \( \frac{dQ^*}{Q^*} \) is identical to the ideal indicator whenever \( q \) is unchanged. Otherwise, this approximation misstates the influence of \( q \) on the ideal indicator. When \( \frac{dq}{q} \) is positive, \( \frac{dQ^*}{Q^*} \) exceeds \( \frac{dQ}{Q} \) by an amount \( \left[ h_2(\alpha-1) \right] \frac{dq}{q} \); reductions in \( q \) are understated by the same amount. We will refer to the bracketed expression as the "error of overstatement," since it exaggerates the influence of \( q \) relative to the ideal indicator.

For the "error of understatement," a measure is required that always is less than \( \frac{dQ}{Q} \) when \( \frac{dq}{q} \) is positive and always is greater than the ideal indicator when \( \frac{dq}{q} \) is negative. Relative changes in the base have this property. The error of understate is the bracketed term in

\[
\frac{dQ}{Q} - \frac{dB^*}{B^*} = \left[ \frac{h_1 - h_2\alpha}{h_1 - h_2} \right] \frac{dq}{q}
\]

Since the hypothesis implies that the error of understatement is numerically larger than the error of overstatement, the true indicator is nearer to \( \frac{dQ^*}{Q^*} \) than to \( \frac{dB^*}{B^*} \) whenever \( q \) changes. The sum of the two
errors is always unity. Unless the change in \( q \) is extremely large, a reasonable approximation to the ideal indicator is given by

\[
\frac{1}{2} \left( \frac{dQ^*}{Q^*} + \frac{dB^a}{B^a} \right) = \frac{dB^a}{B^a} + \frac{dq}{2q}
\]

The difference between the ideal indicator and this approximation has the same sign as \( \frac{dq}{q} \) and hence is known. The approximate indicator is greater than the ideal when \( \frac{dq}{q} \) is negative and less than the ideal indicator for positive \( \frac{dq}{q} \).

Thus the approximate indicator has three distinct advantages. First, it is easily computed, since \( \frac{dB^a}{B^a} \) can be measured exactly and most of the components of \( \frac{dq}{q} \) can be approximated quite closely. Second, the direction of error is known—an advantage that is not generally obtained with the use of endogenous indicators. Third, for small changes in \( \frac{dq}{q} \), the error is quite small.

**LIMITATIONS**

Computation of the approximate indicator would not reveal the magnitude of the change in real income induced by monetary policy. At best, the indicator would correctly scale the size of the impulse that policy is directing toward the final goal. Measurement of the magnitude of the change in real income induced by monetary policy requires information about the structure, and particularly about the elasticity of output with respect to the base, \( \epsilon(y, B^a) \), as noted earlier.

Successive computations of the indicator would not furnish information useful for the computation of the correct lags or of \( \epsilon(y, B^a) \). The length of the lags depends, in part, on the acceleration and deceleration of monetary policy, while the computation of the "true" value of \( \epsilon(y, B^a) \) depends on knowledge of the structure. More importantly, changes in income depend on the fiscal and noncontrolled exogenous variables. The indicator, therefore, does not provide a forecast of future output, but only a scale on which the relative magnitudes of the monetary impulse can be measured.

Moreover, the scale is not unique. It depends on the particular class of hypotheses and goal selected. Some of the more complicated goal
functions require substantially more information about structural properties than is required in the case discussed here. Although we have not investigated the problem thoroughly, this is likely to be true of alternative hypotheses as well. The analysis underlying our effort has the advantage of reducing, perhaps to a minimum, the required amount of knowledge about structural details.

Random elements have been ignored. The structural equations have been treated as if they held exactly. Errors in these equations will appear in the partially reduced forms used to compare the endogenous indicators and in the equation for the ideal indicator. However, it should be noted that the problem of errors in the equation is likely to be at least as serious for the endogenous indicators that are in common use as for the approximate indicator we have developed.21

CONCLUSION

One means of obtaining information about the direction and effect of monetary policy has been illustrated in this paper. While our solution is neither exact nor ideal, it is likely to be less misleading than many of the variables used to describe the content of monetary policy.

Policy makers must continuously interpret the effects of their past decisions. Their future actions depend on these interpretations. If the indicators they select are misleading, their policy decisions will be inappropriate or misinterpreted and will introduce fluctuations in output, prices, and other goal variables. One need only look at 1962-63 to recognize that when the Federal Reserve described its action as a shift to "slightly less ease," our suggested indicator began growing at the fastest rate in the postaccord period.

Numerous other years reveal the same pattern. When the Federal Reserve describes its action as a move toward restraint, the indicator accelerates. Description of policies as "increased ease" is frequently followed by deceleration of the indicator summarizing the effect of monetary policy on real output. Clearly, the problem is worthy of more attention than it has received from government or academic economists.

APPENDIX A

Alphabetical List of Symbols

- $a$ the banks' earning asset multiplier
- $B^*$ the adjusted monetary base: the monetary base minus member bank borrowing

21The introduction of errors in the equations would not alter the comparative merits of the indicators considered here. Under the hypothesis and goal we have discussed, the ordering of indicators would remain identical.
The Meaning of Monetary Indicators

APPENDIX A—Continued

E commercial banks' earning assets net of Treasury deposits and the banks' net worth

\( E_b \) banks' demand for earning assets

\( E_p \) public's supply of earning assets to banks

\( f \) the ratio of free reserves to total deposits

\( G \) government expenditures (national income accounting definition)

\( I \) the ideal indicator

\( i \) an index of interest rates on financial assets

\( i_e \) expected rate of interest

\( i_t \) interest rate paid on commercial bank time deposits

\( k \) the ratio of currency to demand deposits

\( K \) the stock of real private capital

\( m \) the monetary multiplier

\( M \) the money supply: currency and demand deposits of the public

\( n \) the real yield on real capital

\( p \) the income deflator

\( P_a \) the deflator for wealth

\( q \) the component of the indicator incorporating monetary policy variables not included in \( B^* \)

\( Q \) the "true" indicator

\( r \) a weighted average of reserve requirement ratios including the vault cash ratio

\( S \) the stock of interest bearing government debt

\( t \) the ratio of time to demand deposits

\( T \) the stock of time deposits

\( V \) circuit velocity of private expenditures

\( W \) the nominal stock of wealth held by the public

\( Y \) nominal income or output

\( y \) real income or output

\( y_e \) expected real income

\( a \) the ratio of money plus time deposits to bank earning assets

\( \beta \) the ratio of money supply to bank earning assets

\( \gamma \) money expenditures of the government sector

\( \epsilon \) an elasticity

\( \theta \) money receipts of the government sector

\( \rho \) the rediscountrate

APPENDIX B

A Compact Statement of the Underlying Model

A. The Macromodel

\( (A 1) \quad Y = MV + G \) A quantity equation.

\( M = \text{currency and demand deposits} \) 

\( (A 2) \quad V = V (i_e, i, y/y_e, W/P_a, P_a/p, n) \) The velocity relation.

\( (A 3) \quad M_s = \beta(i)E_b \) The money supply generated by the banks' desired portfolio position.

where \( \beta = \frac{1 + k}{(1 + t)(1 - r - f)} \)
APPENDIX B—Continued

(A 4) \[ p = p(y, K, w) \]
\[ p_1 > 0 > p_2 \]

The price level. The parameter \( w \) formalizes the revision of supply prices on the basis of changing market conditions. \( w \) depends on past \( y \) and \( K \).

(A 5) \[ \Delta S + \Delta F = \gamma(G) - \Theta \]

The change in interest-bearing and noninterest-bearing debt equals the government's cash flow deficit.

B. The Condensed Monetary System

(A 6) \[ E_b = E_p = E \]

Equilibrium on the bank credit market.

(A 7) \[ E_p = s(i, y/y_p, W/P_a, p, n) \]
\[ s_1 < 0 < s_2; s_3 > 0 < s_4; s_5 > 0 \]

The supply of financial assets to banks.

(A 8) \[ E_b = a(i, \rho, i^t) B^a \]

where \( a = \frac{(1 + t)(1 - r - f)}{(1 + t)(r + f) + k} \)

The banks' demand for earning assets. The earning asset multiplier is dependent on interest and rediscount rates through the components of \( a \).

(A 9) \[ M_s = M(i, \rho, i^t) B^a \]

where \( m = \frac{1 + k}{(1 + t)(r + f) + k} \)

The money supply function.

The monetary multiplier depends on the same parameters as the earning asset multiplier, but with different derivatives.

(A 10) \[ \delta = \delta(i, \rho, \pi) \]

The ratio of free reserves to total deposits.

(A 11) \[ M + T = \alpha(i) E_b \]

The ratio of money supply plus time deposits to bank earning assets. The ratio is greater than one by definition.
Other behavior relations of the monetary system and discussion of the adjusted base may be found in several of our papers. See especially "Some Further Investigations of Demand and Supply Functions for Money," *Journal of Finance*, May, 1964, and "Liquidity Traps for Money, Bank Credit and Interest Rates," forthcoming.

C. The Reduced System

A condensed statement of the reduced forms is given by substituting equations (A7) and (A8) into equation (A6) and substituting (A4), (A9), and (A2) into equation (A1) to obtain

\[ a(B^a) = s(B^a) \]

\[ y_p(B^a) = [m(B^a)] V(B^a) + G \]

The monetary policy variables enter either directly as \( B^a \) or through the monetary and asset multipliers.

Differentiating the system and stating the results as elasticities yields the following expressions, where \( \epsilon(j, k) \) is the elasticity of \( j \) with respect to \( k \). For example, \( \epsilon(y, B^a) = \frac{h_1 - h_2 \alpha}{g_1 h_1 - g_2 h_2} \), where

\[ h_1 = \epsilon(a, i) - \epsilon(s, i) \epsilon(i, i) - \epsilon(s, W/P_a) \epsilon(W/P_a, i) \]

\[ h_2 = \epsilon(m, i) + \epsilon(V, i) \epsilon(i, i) + \epsilon(V, i) + \epsilon(V, W/P_a) \epsilon(W/P_a, i) \]

The class of hypotheses implies that \( h_1 \) and \( h_2 \) are positive and that \( h_1 > h_2 \alpha \), where \( \alpha \) is the ratio of the money supply plus time deposits to bank credit. The ratio in the text, \( \frac{h_1 - h_2 \alpha}{h_1 - h_2} \), is a proper fraction.

\( g \) and \( g_2 \) summarize the elasticities on the output market:

\[ g_1 = 1 - \epsilon(V, W/P_a) [1 - \epsilon(y_p, y)] - \epsilon(V, W/P_a) \epsilon(W/P_a, y) - \epsilon(V, n) \epsilon(n, y) + \epsilon(p, y) - \epsilon(m, t) \epsilon(t, W/P_a) \epsilon(W/P_a, y) \]

\[ g_2 = \epsilon(s, W/P_a) [1 - \epsilon(y_p, y)] - \epsilon(s, W/P_a) \epsilon(W/P_a, y) - \epsilon(s, n) \epsilon(n, y) - \epsilon(s, p) \epsilon(p, y) + \epsilon(a, t) \epsilon(t, W/P_a) \epsilon(W/P_a, y) \]
Under the class of hypotheses, \( g_1 > g_2 \), hence
\[
\frac{g_1 - g_2}{g_1} > 1
\]

**D: An Outline of the Formal Procedure**

Let \( H \) denote a class of hypotheses, \( h \) any member of the class and \( u \) the goal or social utility function. The general form of the index function is expressed by the relation
\[
I = f(dx; h; H, u)
\]

This formulation reveals three important properties of the index function: (1) the general format of the function depends on the class of hypotheses \( H \) and the goal function \( u \); (2) given \( H \) and \( u \), the computation of \( I \) associated with the policy vector \( dx \) requires knowledge of \( h \); (3) the value of \( f \) associated with any \( dx \)—and the derivative of \( f \) with respect to \( dx \)—depend on the particular \( h \) and the class \( H \).

If we knew which \( h \) in the class \( H \) is the true hypothesis, we could construct the appropriate index function. Until we have that information, an alternative is to choose an observable indicator that approximates the true indicator as closely as possible. Let \( A \) denote a set of approximations, and let \( S \) be a member of the set \( A \). In the text, \( A \) contains the money supply, the money supply plus time deposits, bank earning assets, interest rates and free reserves; the class \( H \) is given in appendix A; and the utility function has been specified.

The formula for selecting an optimal approximation is:
\[
\min_S \max_{dx} \max_h \left| f(dx; h; H, u) - S(dx; h) \right|
\]

The function \( S(dx; h) \) expresses the dependence of each of the observable indicators on the policy vector, \( dx \), and on the particular hypothesis, \( h \).

Our procedure involves the following steps:

1. Compute the absolute value of the difference between the true indicator function and the approximation, \(|f - S|\).
2. Maximize this difference over the class of hypotheses, \( H \). We thus obtain a function expressing the maximum deviation of \( S \) from \( I \), for given \( dx \), attributable to our uncertainty about the structure of the world.
3. Maximize the function obtained in step 2 for the feasible (closed) set of policy actions. For every value of \( S \), we obtain a function of \( S \) which expresses the maximum deviation of \( S \) from \( I \) due to our uncertain knowledge and the range of feasible policy actions.
4. Minimize the function obtained in step 3 with respect to \( S \) and obtain
the $S$ which has the smallest maximum deviation. This is the optimal approximation.

The analysis can be extended to cover alternative classes of hypotheses $H_1$ and $H_2$ and alternate utility functions. A comparison of the ranking of the elements $S$ achieved under $H_1$ and $H_2$ or $u_1$ and $u_2$ may yield a compromise candidate usefully guiding our interpretations even if we are more uncertain than in the case described here.

E: An Illustration of the Macrosystem's Characteristics

The money and bank credit subsystem and the output subsystem express, in a formal way, the responses in financial and nonfinancial markets to changes in relative prices, output, and policy operations. In this section, we discuss some properties of the two subsystems, using a diagram to illustrate the effect of a change in the adjusted base and to clarify the meaning of the four parameters denoted $h_1$ and $g_1$. The discussion will make clear why the use of a quantity equation as an equilibrium condition has no bearing on the comparison of endogenous indicators of monetary policy. The reason, stated briefly, is that the transmission of policy changes depends on the interest rate and relative price mechanisms and not on the equilibrium conditions.

There are two subsystems, financial and nonfinancial. The nonfinancial sector contains a description of aggregate demand and supply on the output markets. Interest rates, output, wealth, government expenditure, and asset and output prices affect aggregate demand directly or through their effects on velocity and the supply of money; aggregate supply is expressed by means of price-setting and price adjustment equations for the prices of output and nonfinancial assets. The financial subsystem describes the allocation of financial assets by the banks and the public. This interaction on the credit market generates the money stock and hence feeds into the aggregate demand function, $MV + G$.

Each subsystem can be reduced to a single equation, shown in section C, so that the two reduced equations express the interaction between the output and the credit markets and the proximate determination of interest rates and output. Since the reduced system contains the policy variables, it can be used to derive the response of output and interest rates to monetary and fiscal policy changes or to other exogenous changes. By differentiating over the reduced system and expressing the results as elasticities, four combinations of elasticities—$h_1$, $h_2$, $g_1$ and $g_2$—are obtained. These combinations are formally stated in section C of this appendix. The $h_1$ are combinations of elasticities with respect to interest rates; the $g_1$

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This section was added just prior to publication at the request of the editor. It was not available at the conference or to the discussant.
are combinations with respect to output. Changes in monetary or fiscal policies change output and interest rates by amounts that depend on the $h_i$ and $g_r$.

As an illustration, consider the effect of a change in the adjusted base, $B^a$. The elasticity of output, $y$, with respect to the base is denoted $e(y, B^a)$ where

$$e(y, B^a) = \frac{h_1 - h_2 \alpha}{g_1 h_1 - g_2 h_2}$$

It is apparent from the formula that the interest elasticities combined in $h_1$ and $h_2$ have a dominant role in determining the size of the response of output. The combinations of output elasticities, $g_1$ and $g_2$, play a similar role in determining the response of interest rates. Moreover, the class of hypotheses implies that the combinations $h_1$, $h_2$, and $g_1$ are positive while the combination $g_2$ is negative. It follows that a sufficient (but not a necessary) condition for an open market operation to affect output in the expected direction is that the combined response to changes in interest rates on the credit market ($h_1$) is large relative to the response of aggregate demand to changes in interest rates ($h_2$).

A diagram further clarifies the meaning of each combination of elasticities. In Figure 1, the slopes of the curves depend upon the elasticities, and all variables are measured positively starting at the common origin. We will, first, explain the relation of the curves in each quadrant to the equations in the appendix above, using an increase in the base to illustrate the adjustment but concentrating on the determinants of the slopes and positions of each curve. We will then discuss the adjustment from one equilibrium to another. A discussion of a reduction in the reserve requirement ratios or in the rediscount rate or an increase in the percentage of vault cash counted as part of required reserves would be similar, apart from minor details. Illustration of the effects of an increase in the government's interest-bearing debt or of a larger government deficit will not be attempted here.

In the upper left quadrant, we have drawn the public's supply of earning assets to banks ($E_p$) and the banks' demand for earning assets ($E_b$). The quantities demanded and supplied depend on interest rates ($i$), and the positions of each curve depend on the other arguments of the $E_p$ and $E_b$ functions given in the formal statement of the system above. An increase in $B^a$ moves the $E_b$ curve to the left from the solid line $E_{bl}$ to the broken line $E_{b2}$, lowering interest rates. The percentage change in interest rates depends on $h_1$, since the reciprocal of $h_1$ is the relative change in interest rates—holding output constant—resulting from a shift of the $E_b$ curve by 1 percent.
The reduction in interest rates is shown also by the credit market (CM) curve in the upper right hand quadrant. The slope of this curve is $-\frac{1}{h_1}$, so it shows the responsiveness of the bank credit market to interest rate changes, holding output constant but acknowledging the interaction within the financial system. The larger $h_1$, the flatter is the CM curve and the smaller is the response of interest rates to changes in the base. But a large $h_1$ and "flat" CM curve imply a large value for $\epsilon(y, B^a)$ in the formula above and hence a large increase in output in response to a given change in the base.

**FIGURE 1**

The effect of the increase in $B^a$ on aggregate demand is shown in the upper right quadrant also. Since the money supply ($M$) can be expressed as the product $mB^a$, the increase in the base ($dB^a$) increases aggregate demand ($MV_1 + G$) by $VmdB^a$, as shown by the position of the broken curve in the diagram. The position of the aggregate demand curve also depends on the volume of government expenditures and, to a minor extent, on the level of output. The slope of the aggregate demand curve shows the response of aggregate excess demand to changes in interest
rates, holding output constant but incorporating the interaction over the nonfinancial subsystem, that is, the combination summarized by $h_2$.

More precisely, the slope of $MV + G$ is $\frac{1}{h_2}$.

The broken line drawn between the upper quadrants connects the new, partial equilibrium positions. The percentage increase in aggregate demand (given by the movement from the solid to the broken aggregate demand curve along the curve labeled $CM_1$) is attenuated by the reduction in interest rates. However, the hypothesis implies that the increase in the base generates aggregate excess demand that spills over to the output markets. The broken line, therefore, continues into the lower right quadrant.

The curve in the lower right panel shows aggregate supply (at current prices) as a function of output and thus expresses the combined response of output and prices. The slope of the curve, relative to the output or $y$ axis, is given by the dominant component of $[1 + \epsilon(p,y)]$ in $g_x$. At very low levels of output, the curve has a slope equal (or very close) to unity. Since $\epsilon(p,y)$ increases with $y$, the curve turns away from the 45° line at higher levels of output. Given the effect of an increase in the base on aggregate demand, the effect on output is larger the steeper the relevant portion of the curve and the smaller is $g_v$. The size of the response in prices also depends on the slope of the curve. The flatter the curve, the greater is the inflationary impact of an increase in the base.

However, the position of the curve depends on current and prior changes in the capital stock, on the level of output in the recent past, and upon information about the adjustment of supply prices. Increases in the stock of real capital, for example, rotate the curve toward the 45° line and thus enlarge the response of output and diminish the change in prices resulting from a given increase in aggregate demand.

Increases in output induce additional changes in aggregate demand and in the public's supply of assets to banks and thus change the positions of the aggregate demand and of the $E_p$ curve. The new position of the former is shown by the dotted line, $MV_2 + G_1$, in the upper right quadrant. The increase from the broken to the dotted line reflects the response of $V$ to changes in output and real wealth, that is, to components of $g_y$. The shift from $E_{p1}$ to $E_{p2}$ in the upper left quadrant shows the effect of increased income and wealth on the public's supply of assets to banks (including an increased volume of borrowing). The elasticities that determine the size of this response in $E_p$ are the principal components of $g_v$.

The other components of $g_v$ shift the curve $MV + G$ in the quadrant above. There is a feedback from this quadrant to the quadrant above that will be considered below.
These changes in aggregate demand and in $E_p$ induce further changes in interest rates. The increase in $E_p$ raises interest rates and shifts the CM curve to the right; the increase in aggregate demand lowers interest rates. We will discuss these interactions and their further effects more fully below.

The effect of a change in output on $E_p$ is also shown in the lower left quadrant. The slope of the curve in that quadrant shows the effect of output on $E_p$ and thus a principal component of $g_2$, as just noted. However, the elasticity of $E_p$ with respect to output appears in $g_2$ with a negative sign, so the slope in the lower right quadrant is a main component of $-g_2$.

We may summarize the discussion of the four combinations of elasticities by noting that each of the slopes is represented in the diagram. The CM curve depends on the interest elasticities of the financial sector and has slope $\frac{1}{h_1}$. The slope of the aggregate demand curve is $\frac{1}{h_2}$. The principal component of $g_1$ determines the slope of the curve in the lower right quadrant, and the other components of $g_1$ shift the aggregate demand curve. The percentage change of the horizontal distance between the $E_p$ and $E_b$ curves at each level of interest rates is a measure of the components of $g_2$, one component of which is shown also by the slope of the curve in the lower left quadrant.

The response of output to monetary (or fiscal) policy depends on the four combinations. Policy actions have a larger effect on output if the curves in the two upper quadrants are flat. Moreover, a flat $E_p$ curve means that there is a large feedback from the upper left to the lower left quadrant and a correspondingly larger effect on output. A flat curve in the lower left quadrant suggests that relatively small increases in output (or prices) generate relatively large increases in the supply of assets to the banking system and a relatively large induced shift in the CM curve. Similarly a relatively large feedback from output and prices to aggregate demand raises the response of aggregate demand and the further response of output and prices. Thus the size of the "multiplier" effect resulting from a change in monetary or fiscal policy depends on the slopes of the output and credit market curves.

It is a by-product of the class of hypotheses that the size of the response of output to monetary policy is not positively related to the size of the response of interest rates. On the contrary, if the combinations of elasticities magnify the response of interest rates to monetary or fiscal policy, they attenuate the response of output. Although interest rates
have an important role in transmitting changes in policy between the two sectors, the size of the response in the nonfinancial sector decreases with the magnitude of the change in interest rates.

We can now trace the effect of an increase in the base on interest rates, output, and bank credit using the diagram shown in Figure 1. Initially the system is in an equilibrium shown by the solid line connecting the four quadrants and passing through the intersection of $E_{p1}$ with $E_{n1}$ and of $MV_1 + G$ with $CM_1$. Suppose this equilibrium is disturbed by a gold inflow, at constant exchange rates, that raises the base. A new, partial equilibrium is reached in the bank credit market by an increase in the banks’ demand for credit, shown by the movement from $E_{p1}$ to $E_{p2}$ along the existing $E_{p1}$ curve. The increased demand for bank credit reduces interest rates and increases the money supply. Both changes carry over to the upper right quadrant. The reduction in interest rates is shown by the movement down the $CM_1$ curve; the increase in the money supply by the shift to the right of the aggregate demand curve. The broken line connecting the two upper quadrants connects the new partial equilibrium points and continues into the lower right quadrant to show the increase in output (and prices) resulting from the increase in the base. The initial increase in the base also increases the horizontal distance between the $E_p$ and $E_b$ curves at each level of interest rates. This shifts the $E_y$ curve in the lower left quadrant to the left; the new position is shown by the $E_{y2}$ curve.

The increase in output induces an increase in the public’s supply of earning assets to banks. The broken line connecting the lower right and lower left quadrants intersects the $E_{y2}$ curve at $B$. The distance from $E_{p1}$ to $E_{y2}$ at point $A$ shows the increase in the quantity of bank credit supplied by the public as a result of the fall in interest rates. The horizontal distance between $A$ and $B$ is, therefore, the amount by which the supply of bank credit must increase to maintain the existing levels of output, prices, and interest rates.

However, an increase in the public’s supply of earning assets to banks (shown by the shift from $E_{p1}$ to $E_{p2}$) moves the $CM$ curve to the right raising interest rates, as shown by the intersection of $E_{y2}$ and $E_{p2}$ or by the position of the $CM_2$ curve. The feedback from the output market to aggregate demand (resulting from the initial increase in aggregate demand) shifts the aggregate demand curve to the position shown by the dotted $MV_2 + G$ curve and partially offsets the effect on interest rates of an increased supply of earning assets to banks.

Interaction between the financial and nonfinancial sector continues until a new equilibrium is established. The solid line connecting the intersection of $E_{y2}$ with $E_{p2}$ and $CM_2$ with $MV_2 + G$ is used to represent
the new equilibrium positions. As the curves are drawn, the new equilibrium "box" lies outside the old "box." Output and interest rates are higher than before the gold inflow.

^In the lower left quadrant, this line intersects the $E_{q1}$ curve (not shown) that lies slightly to the left of $E_{q2}$. 


(Continued on back cover)
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