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Abstract. The PRODIGY project is primarily concerned with the integration of planning and learning. Members of the PRODIGY research group have developed many learning algorithms for improving planning efficiency and plan quality, and for automatically acquiring knowledge about the properties of planning domains. The details of the PRODIGY planning algorithm, however, have not been described in the literature.

We present a formal description of the planning algorithm used in the current version of the PRODIGY system. The algorithm is based on an interesting combination of backward-chaining planning with simulation of plan execution. The backward-chainer selects goal-relevant operators and then the planner simulates the application of these operators to the current state of the world. The system can use different backward-chaining algorithms, two of which are presented in the paper.

1 Introduction

PRODIGY is an integrated planning and learning system, which includes a planning algorithm and procedures for learning control knowledge to improve planning efficiency and plan quality [Veloso et al., 1995]. PRODIGY is able to learn control rules [Minton, 1988, Veloso and Borrajo, 1994], generate abstraction hierarchies [Knoblock, 1994], use analogical reasoning to recognize and exploit similarities between planning problems [Veloso, 1994], and conduct experiments to acquire missing knowledge about the properties of a planning domain [Gil, 1991a, Wang, 1994].

A solution to a planning problem is represented in the PRODIGY system as a sequence of operators, which can be applied to the initial state of the world to achieve some state that satisfies the goal statement. PRODIGY’s core, the planning algorithm, is designed to search for such sequences of operators.

The PRODIGY system includes a variety of mechanisms that enhance the operator-based planner. The system is able to guide the planner’s search using control rules, hierarchical organization of operators, and a library of planning cases. The knowledge used in guiding the

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search may be encoded by the user or learned automatically. The development of methods for learning the guiding knowledge automatically is the main purpose of the PRODIGY project.

The PRODIGY planning algorithm has been improved over years. The initial algorithm, PRODIGY2.0 [Minton et al., 1989], was succeeded by NOLIMIT [Veloso, 1989] and then by the current PRODIGY4.0 [Carbonell et al., 1992, Veloso et al., 1995]. All versions of PRODIGY were developed by members of the PRODIGY research group at Carnegie Mellon University. We list the authors of the system in the acknowledgements section.

PRODIGY4.0 is a nonlinear planning system, which uses means-ends analysis to reason about multiple goals and multiple alternative plans achieving the goals. The dynamic goal selection enables the planner to interleave plans and exploit common subgoals.

The planning system is based on a combination of backward-chaining with simulation of plan execution, similar to forward-chaining planning. The system consists of two parts: a simulator of the plan execution and a backward-chaining algorithm. The system may use different backward-chaining algorithms, two of which are described in the paper.

The backward-chaining algorithm is responsible for goal-directed reasoning, whereas the execution-simulator enhances the goal-directed search with elements of forward-chaining. The execution-simulator searches among states of the world that can be achieved from the initial state by applying different operators. Unlike usual forward-chainers, the simulator uses only goal-relevant operators, selected by the backward-chaining algorithm.

The goal of our paper is to give a formal description of the PRODIGY planner. We describe the backward-chaining and execution-simulating planning algorithms used in PRODIGY and show how this algorithms are combined into a single system.

2 Domain and Problem Definitions

We define a planning domain by a set of object classes and a library of operators that act on objects of these classes. PRODIGY’s language for describing operators is based on the STRIPS domain language [Fikes and Nilsson, 1971], extended to express disjunctive preconditions, universal and existential quantification, functions, and conditional effects [Carbonell et al., 1992].

An operator is defined by its preconditions and effects. The description of preconditions and effects of an operator can specify classes of variables and restrict the values of variables by logical formulas. The preconditions of an operator are represented as a logical expression containing conjunctions, disjunctions, negations, and universal and existential quantifiers. The effects of an operator are a list of predicates to be added to or deleted from the current state of the domain.

In Figure 1, we show an example of a planning domain, a simplified version of the drilling operations in the PRODIGY Process-Planning Domain [Gil, 1991b, Gil and Pérez, 1994]. The drill press in the simplified domain uses two types of drill bits, a spot drill and a twist drill. A spot drill makes a small spot on the surface of a part. This spot guides the movement of a twist drill, which makes a deeper hole.

The domain includes two classes of objects, PART and DRILL-BIT. The DRILL-BIT class consists of two subclasses, SPOT-DRILL and TWIST-DRILL. We use these classes to limit the scopes of variables in the operator description.

A planning problem is defined by a list of objects, an initial state I, and a goal statement G. The initial state is represented as a set of literals. The goal statement is a logical formula, which may contain conjunctions, disjunctions, negations, and universal and existential quantifiers. We show an example of a planning problem in Figure 2. In this example, we have five objects:
two metal parts, called part-1 and part-2, the spot drill drill-1, and two twist drills, drill-2 and drill-3.

A solution to a planning problem is a sequence of operators that can be applied to the initial state to achieve the goal. A sequence of operators is called a total-order plan. A plan is valid if the preconditions of every operator are satisfied before the execution of the operator. A valid plan that achieves the goal is called correct.

For the problem in Figure 2, the plan “put-part(part-1), put-drill-bit(drill-1)” is valid, since it can be executed from the initial state; however, this plan does not achieve the goal and hence it is not correct. We may solve the problem by the following correct plan: “put-part(part-1), put-drill-bit(drill-1), drill-spot(part-1, drill-1), remove-drill-bit(drill-1), put-drill-bit(drill-2), drill-hole(part-1, drill-2).”

A partial-order plan is a partially ordered set of operators. A linearization of a partial-order plan is a total order of the operators consistent with the plan’s partial order. A partial-order plan is correct if all its linearizations are correct. For example, the first two operators
in our solution sequence need not be ordered with respect to each other, thus giving rise to
the partial-order plan in Figure 3.

\[
\begin{align*}
\text{put-part (part-1)} & \quad \rightarrow \quad \text{put-drill (drill-1)} \quad \rightarrow \quad \text{drill-spot (part-1, drill-1)} \quad \rightarrow \quad \cdots \cdots \quad \rightarrow \\
\end{align*}
\]

Figure 3: The beginning of the partial-order solution to the problem in Figure 2.

3 Representation of Plans

Before presenting the planning algorithms used in PRODIGY, we describe the representation
of incomplete plans generated by these algorithms during their search for a solution plan.

Given a problem, most planning systems start with the empty plan and modify it until
a solution plan is found. A plan may be modified by inserting a new operator or imposing
an ordering constraint. The plans considered during the search for a solution are called
incomplete plans. We view incomplete plans as nodes in the search space of the planning
algorithm. Modifying a current plan corresponds to expanding a node. The branching factor
of search is determined by the number of possible modifications to the current plan.

Different planning systems use different ways of representing incomplete plans. A plan
may be a totally ordered sequence of operators (as in Strips [Fikes and Nilsson, 1971]) or a
partially ordered set of operators (as in Tweak [Chapman, 1987], NONLIN [Tate, 1977], and
SNLP [McAllester and Rosenblitt, 1991]); the operators of the plan may be instantiated (for
example, in NOLIMIT [Veloso, 1989]) or contain variables with codesignations (for example,
in Tweak); the relations between operators and the goals they establish may be marked by
causal links (for example, in NONLIN and SNLP).

In PRODIGY, an incomplete plan consists of two parts, the head-plan and tail-plan (see
Figure 4). The tail-plan is built by a backward-chaining algorithm, which starts from the
goal statement \( G \) and adds operators, one by one, to achieve goal literals and preconditions
of tail-plan operators that are not satisfied in the current state.

The head-plan is a valid total-order plan, that is, a sequence of operators that can be
applied to the initial state \( I \). All variables in the operators of the head-plan are instantiated,
that is, replaced with specific values. The head-plan is generated by the execution-simulating
algorithm described in Section 4. If the current incomplete plan is successfully modified to
a correct solution to the problem, the current head-plan will become the beginning of this
solution.

The state \( C \) achieved by applying the head-plan to the initial state is called the current
state. Note that, since the head-plan is a total-order plan that does not contain variables,
the current state is uniquely defined. The backward-chaining algorithm responsible for the
tail-plan views \( C \) as its initial state. If the tail-plan cannot be validly executed from the current
state \( C \), then there is a “gap” between the head and tail. The purpose of planning is to bridge
this gap.
We show an example of an incomplete plan in Figure 5. PRODIGY can construct this plan while solving the problem of making a spot in part-1. We can bridge the gap in this plan by a single operator, put-drill-bit(drill-1).

Initial State  
part-holder-empty  
tool-holder-empty  

Current State  
put-part  
(part-1)  
(holding-part part-1)  
tool-holder-empty  
gap  

drill-spot  
(part-1, drill-1)  

Goal Statement  
(has-spot part-1)  

Figure 5: Example of an incomplete plan.

4 Simulating Plan Execution

We next describe an execution-simulating algorithm, the purpose of which is to enhance the backward-chaining planner with elements of forward chaining. The execution-simulator calls the backward-chainer to select operators relevant to the goal and simulates the application of these operators.

Given an initial state \( I \) and a goal statement \( G \), PRODIGY starts with the empty plan and modifies it, step by step, until a correct solution plan is found. The empty plan is the root node in PRODIGY’s search space. The head and tail of this plan are, naturally, empty, and the current state is the same as the initial state, \( C = I \).

At each step, PRODIGY can modify the current incomplete plan in one of two ways (see Figure 6). First, it can add an operator to the tail-plan (operator \( t \) in Figure 6). Modifications of the tail are handled by the backward-chaining planning algorithm. The backward-chainer views the current state \( C \) as the initial state. Almost any backward-chaining planner may be used as PRODIGY’s backward-chainer. In Section 5, we describe two backward-chainers designed specifically for PRODIGY.

Second, PRODIGY can move some operator \( op \) from the tail to the head (operator \( x \) in Figure 6). The preconditions of \( op \) must be satisfied in the current state \( C \). The operator \( op \) becomes the last operator of the head, and the current state is updated to account for the effects of \( op \). PRODIGY usually has to select between several operators that can be applied to the current state.

Intuitively, we may imagine that the head-plan is being carried out in the real world and PRODIGY has already changed the world from its initial state \( I \) to the current state \( C \). If the tail-plan contains an operator whose preconditions are satisfied in \( C \), PRODIGY can apply it, thus changing the world to a new current state. Because of this analogy with the real-world
changes, moving an operator from the tail to the end of the head is called the application of the operator. Note that the term “application” refers to simulating an operator application. Even if the application of the current head-plan is disastrous, the world does not suffer: PRODIGY can backtrack and consider an alternative execution sequence.

Note that, if we apply some operator \( op \) (that is, move \( op \) to the head-plan), then \( op \) will precede all other operators of the tail. Therefore, we can apply an operator only if it is not ordered after any other operator of the tail. For example, in the top plan of Figure 6, we can apply operator \( x \) or \( y \), but not \( z \). If the tail-plan is a total-order plan, we can apply only its first operator.

Moving an operator from the tail to the head is the only way of updating the head-plan. PRODIGY never inserts a new operator directly into the head. Thus, only goal-relevant operators are used in PRODIGY’s forward chaining.

PRODIGY recognizes a plan as a solution to the problem when the head-plan achieves the goal; that is, the goal statement \( G \) is satisfied in \( C \). PRODIGY may terminate after finding a solution or it may search for another plan.

We summarize the execution-simulating algorithm in Table 1. The places where PRODIGY chooses among several alternative modifications to the current plan are marked as decision points. Exploring different alternatives in decision points gives rise to different branches in the search space.

**Prodigy4.0**

1. If the goal statement \( G \) is satisfied in the current state \( C \), then return Head-Plan.
2. Either
   (A) Backward-Chainer adds an operator to Tail-Plan, or
   (B) Operator-Application moves an operator from Tail-Plan to Head-Plan.
   *Decision point: Decide whether to apply an operator or to add an operator to the tail.*
3. Recursively call Prodigy4.0 on the resulting plan.

**Operator-Application**

1. Pick an applicable operator \( op \) in Tail-Plan, that is, such an operator that
   (A) there is no operator in Tail-Plan ordered before \( op \),
   (B) and the preconditions of \( op \) are satisfied in the current state \( C \).
   *Decision point: Choose one of the applicable operators.*
2. Move \( op \) to the end of Head-Plan and update the current state \( C \).

Table 1: Execution-simulating algorithm.

Note that, by construction, the head-plan is always valid. PRODIGY terminates when the goal statement \( G \) is satisfied in the current state \( C \). Therefore, upon termination, the head-plan is always a valid plan that achieves \( G \), that is, it is a correct solution to the planning problem. We conclude that PRODIGY is a sound planner, and its soundness does not depend on the backward-chaining algorithm.

The use of an unsound backward-chaining algorithm often improves the performance, because ensuring correctness in backward-chaining planning may be expensive, especially for partial-order plans. The comparative analysis of PRODIGY and SNLP [Veloso and Blythe, 1994] has demonstrated that an unsound backward-chainer makes PRODIGY much more efficient on a number of problems.

## 5 Backward-Chaining Algorithms

We now turn our attention to the backward-chaining techniques for constructing PRODIGY’s tail-plan. We present two of the PRODIGY’s backward-chainers, one of which operates with
total-order tail-plans, whereas the other is a partial-order planner.

**Total-order backward-chainer**

We describe an algorithm that represents the tail-plan as a total-order sequence of operators (see Figure 7). New operators are added to the beginning of the sequence. When the backward-chainer is called to modify the tail, it chooses an unachieved goal literal or operator precondition and adds a new operator to achieve this literal. A literal $l$ is considered unachieved if

1. $l$ does not hold in the current state $C$.
2. $l$ is not established by any preceding operator of the tail-plan.

For example, given the plan in Figure 5, the backward-chainer may add the operator `put-drill-bit(drill-1)` to achieve the precondition `(holding-tool drill1)` of the `drill-spot` operator, thus generating the tail-plan `“put-drill-bit(drill-1), drill-spot(part-1,drill-1).”`

The set of unachieved goal literals and unachieved operator preconditions in the tail-plan is the current goal of planning. We denote this set by $G'$. For example, the current goal $G'$ of the plan in Figure 5 includes the unachieved precondition `(holding-tool drill1)` of the `drill-spot` operator.

Initially, when the tail-plan is empty, the current goal comprises the goal literals that are not satisfied in the initial state: $G' = G - I$. When a new operator is added to the beginning of the tail-plan, the backward-chainer removes from $G'$ the literals achieved by this operator and adds to $G'$ the preconditions of the operator that are not satisfied in the current state $C$.

When the execution-simulator moves an operator from the beginning of the tail-plan to the end of the head-plan, the current goal $G'$ must also be updated. To perform this update, the planner has to know the goal literals achieved by the moved operator. For this reason, the planner establishes links from every operator of the tail-plan to the literals achieved by the operator.

When inserting an operator into the tail-plan, the backward-chaining algorithm instantiates all variables in the operator description with specific values. Since PRODIGY’s domain language allows complex constraints on the preconditions of operators, finding the set of possible instantiations may be a difficult problem. PRODIGY uses a constraint-based matching algorithm that considers all applicable instantiations of an operator [Wang, 1992].

For example, suppose that the planning goal is to drill a hole in `part-1` and the backward-chaining algorithm adds the operator `drill-hole(<part>,<drill-bit>)` to achieve the goal literal `has-hole(part-1)` (see Figure 8). First, PRODIGY instantiates the variable `<part>` in the description of the `drill-hole` operator with the value `part-1` from the goal literal. Then, the planner has to instantiate the other free variable, `<drill-bit>`. Since our domain contains two twist drills, `drill-2` and `drill-3`, this variable has two possible instantiations, as shown in Figure 8.

We present a summary of the total-order backward-chaining algorithm in Table 2.

**Partial-order backward-chainer**

We now describe an algorithm that operates with tree-structured tail-plans (see Figure 9). The root of the tree is the goal statement $G$, the other nodes are operators, and the edges are the
**Total-Order-Backward-Chainer**

1. Pick a literal $l$ from the current goal $G'$.
   
   *Decision point: Choose one of the literals of $G'$.

2. Pick an operator $op$ that achieves $l$.
   
   *Decision point: Choose one of such operators.

3. Add $op$ in the beginning of Tail-Plan and establish a link from $op$ to $l$.

4. Instantiate the free variables of $op$.
   
   *Decision point: Choose an instantiation for the variables of the operator.

5. If the instantiated $op$ achieves other literals of $G'$, establish links to these literals as well.

6. Modify $G'$: remove literals achieved by $op$ and add the preconditions of $op$ that are not satisfied in the current state $C$.

| Table 2: Total-order backward-chaining algorithm. |

ordering constraints. The planner adds operators to the tail-plan to achieve goal literals or preconditions of other tail-plan operators.

A goal literal or operator precondition $l$ in a tree-structured tail-plan is considered unachieved if

1. $l$ does not hold in the current state $C$
2. and $l$ is not linked with any operator of the tail-plan.

When the backward-chaining algorithm is called to modify the tail, it chooses some literal $l$ from the set $G'$ of unachieved literals, adds a new operator $op$ that achieves $l$, and establishes a link from $op$ to $l$ and the corresponding ordering constraint.

We present a summary of the backward-chaining algorithm in Table 3.

### 6 Conclusion

The PRODIGY planning system interleaves backward-chaining planning with simulation of plan execution. The backward-chainer is responsible for the goal-directed planning: it selects operators relevant to the goal of the planning problem. The simulator models the application of these goal-relevant operators to the initial state and computes the resulting current state.

When searching for a solution, PRODIGY operates with incomplete plans that consist of two parts: (1) the valid total-order head-plan, built by the execution-simulator and (2) the goal-directed tail-plan, constructed by the backward-chainer.
Partial-Order-Backward-Chainer
1. Pick an unachieved goal or precondition literal \( l \), from \( G' \).
   *Decision point: Choose an unachieved literal.*
2. Pick an operator \( op \) that achieves \( l \).
   *Decision point: Choose an operator that achieves this literal.*
3. Add \( op \) to the plan; establish a link from \( op \) to \( l \) and the corresponding ordering constraint.
4. Instantiate the free variables of \( op \).
   *Decision point: Choose an instantiation for the variables of the operator.*
5. Modify \( G' \): remove \( l \) and add preconditions of \( op \) that are not satisfied in \( C' \).

Table 3: Backward-chaining algorithm that generates tree-structured plans.

The efficiency of PRODIGY depends on its search space and on the order of expanding nodes of the search space. The search space is determined by the backward-chaining algorithm used by PRODIGY, whereas the order of expanding nodes depends on the branching decisions made in decision points. On each step, the planner decides which branch of the search space to explore first. PRODIGY’s branching decisions are summarized in Figure 10.

A formal analysis of advantages and drawbacks of PRODIGY as compared to other planners is still an open research problem. However, multiple experiments have demonstrated PRODIGY’s ability to solve efficiently a wide range of complex problems (see, for example, [Veloso, 1994] and [Gil, 1991a]).

We have recently described and compared different search strategies used in PRODIGY [Stone et al., 1994]. We have also compared the PRODIGY planning algorithm with the SNLP partial-order planner [Veloso and Blythe, 1994] and identified situations in which PRODIGY is considerably more efficient than the planners that do not use execution simulation.

The results of comparing PRODIGY with other planners suggest that no single planning algorithm is universally superior for all planning domains. The effectiveness of a planning algorithm depends on a particular planning problem; different planning systems perform efficiently on different problems.

We conclude with a summary of important features of the PRODIGY planning and learning system.

**Rich domain language.** PRODIGY’s language for operator representation includes disjunctive preconditions, universal and existential quantification of variables, and conditional and functional effects. The execution-simulator, which maintains the description of the current state of the world, allows us to use this powerful operator representation without
compromising the efficiency of planning.

**Combining execution simulation and backward chaining.** The execution-simulator enables PRODIGY to use information about the updated world state in planning and learning. The application of operators often considerably reduces the size of the tail-plan, thus improving the efficiency of the backward-chaining algorithm.

**Learning opportunities.** PRODIGY uses several learning algorithms, which improve the efficiency of the planner. The learning algorithms use information about the current state to identify the reasons for local and global planning successes and failures. Partial-order constraints of the tail-plan enable learners to determine which aspects of the operator ordering in the head-plan are relevant to the solution.

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**References**


