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THE DEMAND FOR MONEY: THE EVIDENCE FROM THE TIME SERIES

ALLAN H. MELTZER
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The arguments or variables that enter the demand function for money, and the definition of the quantity of money appropriate for the demand function, have received substantial attention in both the recent and more distant past. For present purposes, it is useful to distinguish three separate disputes about these variables. First, there is the question of the constraint that is imposed on money balances—whether the appropriate constraint is a measure of wealth, income, or some combination of the two. A second dispute has centered on the importance of interest rates and price changes as arguments in the demand function. Third, the question of the definition of money balances has often been raised. Is a more stable demand function obtained if money is defined inclusive or exclusive of time and/or savings deposits, and perhaps other assets that have value fixed in money terms?

In his recent survey of monetary theory, Harry Johnson has suggested that the above issues—the definition of money to be used in the money demand function, the variables on which the demand for money depends, and the stability of the demand function—are the chief substantive issues outstanding in monetary theory. This paper deals with each of them and attempts to evaluate empirically the results obtained from some alternative money demand functions and some alternative definitions of money. These results are used to appraise some propositions that have been advanced in monetary theory.

With respect to one of these issues, the constraint on money balances the well-known work of Hicks, and the more recent studies of Friedman and Tobin suggest that monetary theorists are now agreed that the demand function for money is to be treated as a problem in balance sheet equilibrium or asset choice. However, there are important differences among those who view monetary theory as a part of capital theory. Two of these differences are noted here. One concerns the importance of money for macroeconomic theory; the other is the question of the definition of wealth.

1 This article is a part of the research on monetary theory and monetary policy conducted under the joint responsibility of Karl Brunner, of the University of California, Los Angeles, and myself. I wish to acknowledge many helpful discussions with Brunner. The suggestions made by Philip Cagan, Michael Hamburger, and Richard Nelson and the excellent assistance of George Haines and Peter Frost have contributed to the paper. I would like also to express appreciation for the research support made available by the Graduate School of Industrial Administration, Carnegie Institute of Technology.


One approach to the demand for assets, the “modern quantity theory,” views the demand for and supply of money as the most stable macro-relation and the one most significant for an analysis of economic behavior. For the quantity theorist, adjustments of the economy to policy variables or changes in desired asset composition operate principally, but not exclusively, by creating discrepancies between desired and actual money holdings. These in turn lead to adjustments of other assets in the portfolios of ultimate wealth holders. The non-quantity theorist views money as but one of a number of financial assets.

Neither the long-term bond rate nor the quantity of money is an adequate measure of the “strategic variable—the ultimate gauge of expansion or deflation, of monetary tightness or ease.” For Tobin, the strategic variable is the yield required to absorb the existing capital stock into the balance sheets of the public.

Thus within the general framework of a capital theoretic view of the demand for money, there is an important difference. One view emphasizes the importance of the supply-and-demand functions for money; the other group chooses the demand for and supply of real capital as the crucial equations of a general equilibrium model.

Moreover, there is disagreement about the procedures to be applied in measuring the constraint imposed by the balance sheets. In particular, if income is defined as the flow corresponding to a stock of human and non-human wealth, as Friedman has emphasized, there is substantial difficulty in separating wealth, interest, and income variables empirically. Despite the apparent general consensus that the demand for money should be treated as a part of the theory of asset choice, both the studies referred to and Johnson’s survey article suggest that precise specification of the variable that acts as a constraint on money balances remains an open question.

Differences in the specification of the variables in the money demand function have produced important differences in implications or results. For example, Tobin and Baumol separately considered the demand for transactions balances as a problem in capital theory and each obtained a demand function for cash balances which depends on costs and yields. Both Baumol and Tobin deduce from their models that there are economies of scale in holding transaction balances. An income or wealth elasticity less than unity would confirm this implication.

Friedman’s empirical findings suggest that money is a “luxury” and that the relevant elasticity is in the neighborhood of 1.8. Economists would be likely to accept Friedman’s empirical result in preference to the untested deductions of Baumol and Tobin were it not for some problems in the specification of the var-

4 Friedman, op. cit.
6 Tobin, “Money, Capital and Other Stores of Value,” op. cit., p. 35.
variables in his money demand function. Specifically, Friedman's use of per capita permanent income combines wealth, interest rates, population, and lagged income in a single variable and thus combines their separate effects. The inclusion of time deposits at commercial banks, in the definition of money that Friedman used, leaves open the question of whether his results reject the Baumol-Tobin implication when money is defined exclusively of time deposits.

Most of the work in monetary theory to which I have referred has been at the level of hypothesis building. Much less attention has been given in the literature to the appraisal of the proposed demand functions by empirical tests, although Friedman's work is a notable exception. As yet no attempt has been made to compare theoretical results by testing alternative equations.

The absence of detailed empirical studies has important bearing on several issues; among them one of the more important is the role of interest rates in a demand function for money that uses wealth as a constraint. Tobin gives rates of return on financial and non-financial assets an important role in his theory of asset choice. Friedman's essay on the quantity theory stresses a view of the quantity theory as a theory of the demand for money. He uses bond and equity yields as direct arguments in the demand function. But his empirical findings suggest the importance of per capita permanent income and exclude interest rates as direct arguments of the function or assign them a role of second order of importance. Bronfenbrenner and Mayer estimated the separate effects of wealth and interest rates along with income and lagged money balances. Their results show that interest rate, income, and lagged money balances are statistically significant by the usual tests, but the wealth variable is non-significant. Empirical studies that support the role of interest rates in the long-term demand function for money, notably Latané's, are based on the use of income, not wealth, as a constraint on money balances. Thus the role of interest rates in a demand function for money that takes wealth as a constraint, like the measurement procedure for wealth itself, remains an open empirical question.

Furthermore, the definition of money is itself an open question. Gurley and Shaw suggest that monetary theory should be concerned with a concept broader than the liabilities of commercial banks. Friedman's empirical work is based on a concept of money that includes the time deposit liabilities of commercial banks while Latané, Bronfenbrenner, and Mayer, "Liquidity Functions in the American Economy," *Econometrica*, 1960.

12 "The Demand for Money . . . .," *op. cit.* On p. 349 he refers to his "inability to find any close connection between changes in velocity from cycle to cycle and any of a number of interest rates." See also his discussion of the residuals from the velocity equation and his argument that a "highly sensitive" response to interest rates would imply that real cash balances would be highly variable both secularly and cyclically and his citation of the evidence on secular stability of cash balances.

13 In "Rejoinder to Professor Eisner," *Econometrica* (forthcoming), Bronfenbrenner and Mayer note that the definition of wealth that they used was inappropriate for a proper test of the wealth model.


brenner and Mayer and others have been chiefly concerned with money defined as the sum of demand deposits and currency.

An important issue in the dispute about the appropriate definition of money for monetary analysis is the stability of the demand function for money. The problem is one of defining money so that a stable demand function can be shown to have existed under differing institutional arrangements, changes in the social and political environment, and changes in economic conditions, or to explain the effects of such changes on the function. Very little effort has as yet been made to compare the performance of particular money demand functions for the same time periods to obtain insight into this question. Since the variables specified in alternative forms of the demand function for money are often highly correlated during short periods of time, a comparison of relative performance over long periods of time is one important means of resolving disputes about the definitions and measurement procedures for the variables that enter as arguments in the function.

This paper starts from the apparent consensus on the broad principle that the demand function for money is subject to a wealth constraint. Section I develops and tests a particular form of the function using both nominal and real balances and alternative definitions of money. Section II tests for differences in the results when some alternative definitions of wealth are used. Sections III and IV compare the model based on a wealth constraint to models that use measured or permanent income as a constraint. The velocity relation is discussed in Section V. A brief section on policy and a conclusion that suggests some implications of the results complete the paper.

I. THE DEMAND FOR MONEY

In this section the quantity theory of money is expressed as an aggregate demand function for money containing variables that reflect wealth and substitution effects on the amount of real cash balances that the private economy desires to hold at any point in time. The general approach follows the procedures developed by Friedman; money is an asset held for the services which it provides. These services and the low transactions cost associated with exchanges of money for other assets explain the existence of a positive demand for money.

Here I assume that such a function exists and that the arguments of the function are the variables $r^*, p, d^*$, the yield on physical assets, $d^*$, the yield on human wealth, and $W_n$, non-human wealth. Let $M$ represent the quantity of money demanded (nominal value). Then $M = f(r^*, p, d^*, W_n)$ is a general demand function for money that contains the principal variables consistent with recent theoretical and empirical work discussed in the introduction.

$W_n$ represents the constraint which wealth imposes on the demand for money. $r^*, p$ and $d^*$ measure the yields of assets other than money that compete for a place in the balance sheet of households and business. The derivative of $M$ with respect to each of these yields is, of course, negative. Assuming that no additional variables, for example, measures of dispersion, need to be introduced when this general function is aggregated over

15 Friedman, "The Quantity Theory . . .," op. cit.

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all money-holders, I will use the general equation as an aggregate demand function for money.

If the demand function is homogeneous of first degree in the money value of \( W_n \):\(^{17}\)

\[ M = f^*(r^*, \rho, \Delta^*, 1)W_n. \]

Let \( \Delta^* = Y_h/W_h \), the ratio of human income to human wealth. Multiply \( \Delta^* \) by \( Y_f/Y_g (= 1) \) where \( Y_g \) is the stream of income expected from a stock of human wealth.

The yield on human wealth is now viewed as the product of two components. The first term measures short-run deviations of actual from expected human income; it is an index of the transitory component of human income. Let this index be denoted by \( \beta \). In the short-run, wealth holders may adjust the composition of their portfolios in response to changes in \( \beta \), but \( \beta \) will equal one in the long-run. This paper is primarily concerned with the long-run demand function for money, and \( \beta \) is assumed to be constant. The second term in \( \Delta^* \), \( Y_f/Y_h \), is the ratio of expected human income to its capitalized value. This ratio is denoted by \( \rho \) and will also be taken as a constant. Finally, I will assume that \( r^* \) and \( \rho \) may be combined in \( r \).

The extent to which the rates \( r^* \) and \( \rho \) may be measured by a single number—the rate of interest, \( r \)—depends on the covariance of the rates. The question has often been prejudged in the absence of empirical evidence. However, over long periods of time, it would be surprising if asset adjustments did not lead to high covariance between rates like \( r^* \) and \( \rho \). Here I have assumed that at least one of the interest rates quoted in financial markets is an adequate measure of both \( r^* \) and \( \rho \). This question will be reopened in a later section after we have seen the evidence which can be brought to bear.\(^ {18} \)

With these assumptions introduced and the subscript \( n \) dropped from \( W \) the equation becomes \( M = g(r; \Delta, \beta)W \), which will be shortened to

\[ M = g(r)W. \quad (1) \]

Note that as a result of the assumption that the equation is homogeneous in \( W_n \), the non-human wealth variable appears as a multiplicative factor in the function with a coefficient of 1. Additional assumptions about the form of the \( g \) function and the coefficient of \( r \), the definition \( Y = rW \), and the assumed constancy of \( \beta \) permit equation (1) to be rewritten as the familiar quantity theory of money in Cambridge form.

In its present form, equation (1) reflects the wealth constraint and substitution effect on the demand for money. That is, the equation reflects the general view that the demand function for money is a part of the general theory of portfolio composition or asset choice to which reference was made in the introduction. However, to complete the construction of a testable hypothesis, operational definitions of the variables must be specified. To examine the stability of equation (1) for various definitions of money, I define the monetary assets of the public in three ways—(1) as the sum of currency plus

\(^{17}\) Some alternative assumption and the inclusion of some additional variables have been suggested. For example, Friedman assumes that the function is homogeneous in population. This question and the question of homogeneity in prices and financial assets are discussed below.

\(^{18}\) The question and some evidence on the short-run behavior of the function are considered also in a paper by the present author and Karl Brunner, "Predicting Velocity: Implications for Theory and Policy," *Journal of Finance*, May, 1963.
demand deposits, \( M_1 \), (2) \( M_1 + \text{time deposits at commercial banks, } M_2 \), and (3) \( M_1 + \text{savings deposits} = M_3 \). For the interest rate, I have used Durand's basic yield on twenty-year corporate bonds.

Wealth poses a more difficult problem. As noted earlier, we wish to choose the appropriate measure of wealth to act as a budget restraint in the demand function for money in a manner analogous to the role of income in household utility maximization.

Once we drop the arbitrary distinction between "transactions" and "asset" balances and regard money as an asset that, like the other stocks, is held for the services it provides, a variety of alternative definitions of wealth can be suggested. In particular, three questions are raised. First, should human wealth be treated along with non-human wealth as a constraint in the determination of asset equilibrium? This is the approach that Friedman, in particular, has used, but it differs from the approach used here because I have assumed that the ratio \( d \) can be treated as a constant. Second, should non-human wealth be consolidated or combined? That is, do we consider each economic entity (individual or firm) to be constrained by its gross wealth and the aggregate constrained by the unconsolidated sum? Such an approach involves substantial double counting but may nonetheless more closely approximate the behavior of economic units. Third, how do we treat the assets and liabilities of the government? A substantial literature has grown up around the Pigou or Patinkin effect that suggests that we should include the net liabilities of the government sector to the public but should exclude the assets held by government. But Goldsmith's measure of the consolidated wealth of the economy includes net government assets and excludes government debt and non-metallic money.

At heart the issue is empirical. The relevant definition is the one that includes those balance sheet items which when aggregated act as a constraint on the demand for money by households and business. In this section, \( W \) is defined as total wealth from Goldsmith's Table W-1 plus monetary and non-monetary government debt minus government assets. In the following section some alternative measures of wealth will be used. The three questions raised in the preceding paragraph with respect to the operational definition of the wealth constraint will then be reopened.

Let \( g(r) = \rho \). Equation (1), \( \left[ M = g(r)W \right] \), may then be rewritten in log form as \( \ln M = a + b \ln r + c \ln W \), where \( M, r, W \) are as defined and \( b \) and \( c \) are the interest rate and wealth elasticities, respectively. Again noting that \( W \)


20 The series on money \( M_1 \) was taken from the U.S. Bureau of the Census, Historical Statistics of the United States (Washington, 1960), Series X 267. Series X 266, denoted \( M_2 \) in the text, which includes time deposits at commercial banks, mutual savings banks, and the postal savings system, was also used. \( M_1 \) includes time deposits of commercial banks but not at other savings institutions. It was courteously supplied by Anna J. Schwartz. Interest rates are from Series X 346 as published in Historical Statistics. The data on wealth are from Goldsmith, op. cit., Table W-1, with adjustments to exclude government structures, inventories, public land, and the monetary gold and silver stock and to include the monetary and non-monetary (interest bearing and non-interest bearing) debt of state, local, and federal governments. The exclusions were based upon Goldsmith's estimates; I am indebted to Philip Cagan for supplying a series of unpublished estimates of interest bearing and non-interest bearing federal debt. I am also indebted to Goldsmith and Lipsey for providing balance sheets for the postwar years that will appear in their forthcoming National Bureau of Economic Research volume.
appears in (1) as a multiplicative factor with an implied coefficient $c = 1$, we test the hypothesis that $c = 1$. The regression estimate for the period 1900–1958 is shown in equation (2).

Note that $M$ and $W$ are measured in nominal terms and that the $t$ values and partial correlations are given below the variables.

A regression using deflated values of $M$ and $W$ for the same period is shown in (3).

Note that $\rho$ is the implicit price deflator for net national product while $P$ is the price level for assets obtained by computing the implicit price deflator from Goldsmith's nominal and real wealth estimates. To obtain (3) again let $g(r) = r^r$ in (1), divide both sides by $\rho$, and multiply the right side by

$$\frac{P^e}{p^e} \quad (= 1)$$

to give

$$\frac{M}{\rho} = a r^* W^e p^e \quad \frac{P^e}{p^e} \rho .$$

The wealth elasticity $c$, in (2), is approximately $= 1$ as the hypothesis implies. We may treat the ratio of asset to output prices $P^e/\rho = P/\rho$, as an index of interest rates. For the regression estimates in (3), $P/\rho$ has been combined with $r$. As might be expected, this results in a somewhat higher estimate for the coefficient of $\ln r$ in (3) than in (2).

Similar results are obtained if $M_2$ (including time deposits at commercial banks) is substituted for $M_1$. For the log of real balances, the regression result is shown in (3').

Three statistical details about these results merit comment. First, the two arguments of the demand function, interest rates and wealth, explain almost all of the observed variance in money balances whether money is defined inclusive or exclusive of time deposits at commercial banks. Second, the partial correlation coefficients suggest that both variables are of approximately equal importance in the long-run demand functions. Third, the interest rate and wealth elasticities differ substantially depending on the definition of money balances used.

These findings—and others using a third definition of nominal and real balances—bear on long-standing issues in monetary theory. They suggest that a stable demand function for money is consistent with more than a single definition of money balances. But they also suggest that the definition of money is important

\[
\begin{align*}
\ln M_1 &= -1.65 \quad -0.781 \ln r + 1.01 \ln W + u_t \bar{R} = .994 \quad (2) \\
 t \text{ values} & \quad 13.5 \quad 66.8 \\
 \text{Partial correlations} & \quad -.88 \quad +.99 \\
\end{align*}
\]

\[
\begin{align*}
\ln M_1 \rho &= -1.48 \quad -.949 \ln r + 1.11 \ln \frac{W}{p} + u_t \bar{R} = .992 \quad (3) \\
 t \text{ values} & \quad 21.8 \quad 42.0 \\
 \text{Partial correlations} & \quad -.93 \quad +.98 \\
\end{align*}
\]

\[
\begin{align*}
\ln M_2 \rho &= -1.98 \quad -.50 \ln r + 1.32 \ln \frac{W}{p} + u_t \bar{R} = .994 \quad (3') \\
 t \text{ values} & \quad 10.8 \quad 53.2 \\
 \text{Partial correlations} & \quad -.82 \quad +.99 \\
\end{align*}
\]
for a proper appraisal of issues in monetary theory. For if money is defined as \( M_1/p \) or \( M_3/p \), the effects of general and relative changes in interest rates are combined in the estimated interest elasticity.\(^{21}\) A general rise in market interest rates raises all quoted interest rates in proportion and sets off substitution effects between currency plus demand deposits, \( M_1 \), and other assets. When money is defined inclusive of time or savings deposits, part of these substitution effects are hidden by changes within the composition of money itself. Thus the interest elasticity of an equation like (3') makes it difficult to distinguish between two quite different changes: (1) the results of a general rise in interest rates and (2) the substitution between time deposits and other assets stemming from an increase (say) in the Federal Reserve Board's Regulation Q or higher posted rates at commercial banks, other interest rates remaining unchanged.

The estimated elasticities also bear on two issues raised by Gurley and Shaw.\(^{22}\) First, Gurley and Shaw suggest that the definition of money should include more than the sum of currency plus demand deposits, \( M_1 \). This follows from their argument that the creation of liabilities by financial intermediaries is analogous to the creation of money by commercial banks. Substitution between the liabilities of financial intermediaries and \( M_1 \) is regarded as closer than substitution between \( M_1 \) and other assets. Second, the Gurley-Shaw analysis presupposes that the demand for \( M_1 \) has declined relative to other assets. The results of (3) and (3') suggest a different interpretation: that the growth of financial intermediaries was a wealth effect and not primarily a substitution effect. For the wealth elasticity of (3') is substantially above the wealth elasticity shown in (3). These elasticities indicate that, for a given percentage increase in real wealth, the community has chosen to increase its time deposits by a greater percentage than its demand deposits and currency. In short, the public has chosen to hold a larger proportion of its wealth in the form of income-yielding assets.

However, the wealth elasticity of (3) denies that real \( M_1 \) balances have declined relative to real wealth. If we extrapolate from this result to consider the relation between savings deposits at non-commercial banks and real wealth, we should expect to find that including the liabilities of financial intermediaries—for example, savings and loan associations and mutual savings banks—in the definition of money balances also raises the wealth elasticity of money thus defined above the elasticity shown in (3). The evidence from the time series estimates for the \( M_3 \) function supports this extrapolation. The relevant elasticity is 1.34. Thus the data are consistent with the view that the relative growth of financial intermediaries, exhibited by the Goldsmith data, reflects the allocation of an increased stock of wealth rather than substitution between the liabilities of competing financial institutions as in Gurley and Shaw.

Moreover, the stability of the long-run demand function for money defined as \( M_1 \) or \( M_1/p \) denies the necessity for incorporating the liabilities issued (or the rates of return paid) by financial intermediaries as a part of the definition of money or as arguments in the long-run demand function. And the greater interest elasticity of real money balances \( M_1/p \) suggests that a given change in interest rates over the business cycle...
changes $M_2$ and $M_3$ balances by a much smaller percentage than it changes $M_1$. We should expect from the interest elasticities that the principal liabilities of financial intermediaries other than commercial banks are much more stable than the demand liabilities of commercial banks when the Federal Reserve alters the determinants of the money supply. This in turn leads us to believe that changes in the liabilities of financial intermediaries may be a much smaller source of short-run instability than has been supposed. But direct empirical evidence on this point is required for a precise answer to this question.

Much of the theory of money and macroeconomics has been formulated on the explicit assumption that the money demand function is homogeneous of degree one in prices and financial assets. To my knowledge none of the empirical studies of the demand for money has explicitly considered this assumption. Either homogeneity of degree one in prices and financial assets has been assumed or the question has not been considered. The similarity of the parameters of equations (2) and (3) for nominal and real balances suggests that the assumption holds for the demand function developed here. A doubling of prices and the value of financial assets doubles the demand for nominal balances but leaves the demand for real balances unaffected.

Further, we should note that the results presented above are consistent with those which have been obtained in the 126 cross-section studies of the demand for money by business firms referred to earlier. Specifically, the existence of a positive wealth elasticity in the neighborhood of 1.0 and a negative interest elasticity are suggested by the cross-section estimates. While there may be some bias in the parameter estimates and some errors in the observations, the consistency of the two sets of results again implies the existence of a stable demand curve for money with properties similar to those I have suggested. Moreover, one difficulty that commonly affects the estimates from economic time series appears to be largely absent. For the fifty-nine years covered by our data, there is little significant correlation between interest rates and wealth. $r_{W,t} = -0.44$ and $r_{W^{1/2},t} = -0.47$.

The demand function for money developed here (eq. [1]) and the evidence that supports it contribute to an understanding of the issues in monetary theory that I set out to discuss. First, the long-run data strongly suggest that the demand function for money has been quite stable despite major institutional, social, and political changes. Second, the theory and the evidence support the view that the long-run demand function is consistent with the quantity theory of money and contains two principal arguments of almost equal explanatory power: interest rates and non-human wealth. Third, with respect to the definition of money, the results strongly indicate that the demand to hold currency plus demand deposits is at least as stable as other alternative demand functions. The evidence does not suggest any compelling reason for broadening the definition of money to include time deposits at commercial banks (Friedman), or liabilities of financial intermediaries (Gurley-Shaw). Fourth, the data support the assumption of homogeneity of first degree in prices and financial assets. Finally, the results suggest that the measurement of yields on a variety of alternative assets by a single financial rate provides a reasonable approximation for the long-run function.

I shall discuss the more general implications of these findings in the conclud-
ing section. Before doing so it is useful, first, to evaluate the measurement procedures used here by introducing alternative specifications of the variables in the function and, second, to compare the results obtained, and the theory that lies behind them, to some alternative theories of the demand for money.

II. SOME ALTERNATIVE FORMULATIONS

To obtain equations (2) and (3), several assumptions were made about the measurement of variables, wealth in particular, that enter into the demand function for money. Alternative specifications of the wealth variable permit a direct test of the extent to which the measurement of \( W_n \) used in Section I is critical for the results. Indirect evidence has also been obtained on the assumption that \( p \) and \( r^* \) may be approximated by \( r \). In this section we consider problems of measurement only. The following two sections assess some alternative hypotheses involving income rather than wealth as a constraint.

Previously wealth was defined as the consolidated net non-human wealth of the public. One alternative definition of wealth is the unconsolidated total assets of households, business, and government. If the money balances of households in the aggregate are constrained by their asset holdings and the money balances of business are constrained by the unconsolidated value of business assets, the aggregate demand for money may be related to the unconsolidated sum of assets held by the economy as a whole. Similarly, if the proper constraint for households and business is the net worth of the sector, aggregate net worth may be the constraint that enters the aggregate money demand function. Alternatively, the relevant constraint may be the consolidated wealth of households, business, government, and the rest of the world. The latter measure is the one that Goldsmith has computed on an annual basis.

Table 1 shows estimates based on alternative definitions of wealth and money balances. The estimated elasticities for wealth, however, are not used in Section I. The Goldsmith definition of wealth is approximately equal to the sum of tangible assets shown on the balance sheets in Tables W-9 through W-16. The difference arises because of the inclusion in wealth, but not on the balance sheets, of net foreign assets and the inclusion on the balance sheets, but not in wealth, of the Treasury's net holdings of silver certificates minus minor coins.) Goldsmith wealth exceeds tangible assets when net foreign assets are positive and vice versa. Goldsmith's total assets exceed wealth by the sum of intangible assets plus or minus the above differences. His measure of net worth is always larger than his measure of wealth. The difference is equal to (tangible assets minus wealth) plus (intangible assets minus liabilities). The latter difference is, of course, always positive and equals a currency plus \( \Delta \) deposits in other financial institutions plus receivables from business and households minus payables to financial intermediaries, other business, and households plus securities and equities minus bond and note liabilities plus other intangible assets minus other liabilities (\( \Delta = \) the difference between the corresponding asset and liability items). The principal difference between the two is the ownership of equity assets. If this item is excluded, the remaining difference is never more than 1.5 per cent of total intangible assets. Total assets \( (A) \) were taken from Goldsmith, op. cit., and were available for eight benchmark dates from 1900-1949. The series used in the estimates for 1900-1945 was obtained by taking the ratio of total assets, \( A \), to wealth, \( G \) (from Table W-1). Denote this ratio for any base year as \( b_0 \). Let \( b_n \) the next base year ratio, and let \( i = 0, 1, \ldots, n \) be an index of the years between base year dates inclusive of the end points. Then

\[
A_i = \left[ \frac{(n-i)}{n} b_o + \frac{i}{n} b_n \right] G_i.
\]

A similar procedure was used to construct the net worth series, denoted \( N \).

As noted, these measures of wealth, \( A \) and \( N \), involve double counting. Of particular importance is the inclusion of equities as a part of the assets of households while at the same time plant and equipment are included among the assets of business. I am indebted to Goldsmith and Lipsey for supplying preliminary estimates of their forthcoming data for the years 1945-58. These have been used to extend the estimates based on total assets through 1958 as shown in Table 1.
ties in equations (4), (5), and (6) for the period 1900-1949 vary from those presented earlier, and, in general, the more inclusive the definition of money, the lower the interest elasticity. This result is not surprising since the more inclusive the definition of money, the more likely that there will be both positive and negative responses to a relative rise in interest rates. The conclusions reached in the previous section are altered little by these results. \( M_1 \) again approaches unit elasticity with respect to interest rates and the wealth measures, \( A, A/P, \) and \( N. \)

Moreover, equations (4), (5), and (6) have some implications for the measurement of the interest rate. For the empirical work we have used the same measured interest rate in all cases, the yield on twenty-year corporate bonds. But the rate that theoretically enters (4) and (5) is a weighted average of the yield on all non-human assets—tangible and intangible, financial, and physical. In (6) \( r \) is

<table>
<thead>
<tr>
<th>TABLE 1</th>
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<tr>
<th>Equation</th>
<th>Definition of ( M )</th>
<th>Variable and Coefficient</th>
<th>( t ) value</th>
<th>Variable and Coefficient</th>
<th>( t ) value</th>
<th>Multiple Correlation</th>
</tr>
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<tbody>
<tr>
<td>Period: 1900-1949</td>
<td></td>
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<tr>
<td>(4)</td>
<td>( \ln M_t )</td>
<td>(-0.80 \ln r )</td>
<td>9.1</td>
<td>( 0.997 \ln A )</td>
<td>34.8</td>
<td>0.99</td>
</tr>
<tr>
<td>(5)</td>
<td>( \ln M_{t/p} )</td>
<td>(-0.90 \ln r )</td>
<td>10.7</td>
<td>( 1.15 \ln A/P )</td>
<td>20.5</td>
<td>0.97</td>
</tr>
<tr>
<td>(4')</td>
<td>( \ln M_t )</td>
<td>(-0.40 \ln r )</td>
<td>8.2</td>
<td>( 1.07 \ln A )</td>
<td>71.2</td>
<td>0.99</td>
</tr>
<tr>
<td>(5')</td>
<td>( \ln M_{t/p} )</td>
<td>(-0.45 \ln r )</td>
<td>8.5</td>
<td>( 1.37 \ln A/P )</td>
<td>38.2</td>
<td>0.99</td>
</tr>
<tr>
<td>(6)</td>
<td>( \ln M_t )</td>
<td>(-1.00 \ln r )</td>
<td>11.1</td>
<td>( 1.06 \ln N )</td>
<td>32.9</td>
<td>0.98</td>
</tr>
<tr>
<td>(6')</td>
<td>( \ln M_{t/p} )</td>
<td>(-0.65 \ln r )</td>
<td>11.5</td>
<td>( 1.17 \ln N )</td>
<td>56.0</td>
<td>0.99</td>
</tr>
<tr>
<td>(7)</td>
<td>( \ln M_1 )</td>
<td>(-0.50 \ln r )</td>
<td>5.6</td>
<td>(-0.02 \ln G )</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>Period: 1900-1958</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>( \ln M_t )</td>
<td>(-0.80 \ln r )</td>
<td>10.1</td>
<td>( 1.00 \ln A )</td>
<td>47.9</td>
<td>0.99</td>
</tr>
<tr>
<td>(5)</td>
<td>( \ln M_{t/p} )</td>
<td>(-1.04 \ln r )</td>
<td>11.6</td>
<td>( 1.07 \ln A/P )</td>
<td>22.2</td>
<td>0.98</td>
</tr>
<tr>
<td>(3)</td>
<td>( \ln M_{t/p} )</td>
<td>(-0.59 \ln r )</td>
<td>8.4</td>
<td>( 1.24 \ln A/P )</td>
<td>34.9</td>
<td>0.98</td>
</tr>
<tr>
<td>Period: 1900-1929</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>( \ln M_t )</td>
<td>(-0.15 \ln r )</td>
<td>1.1</td>
<td>( 0.90 \ln W )</td>
<td>27.1</td>
<td>0.99</td>
</tr>
<tr>
<td>(3)</td>
<td>( \ln M_{t/p} )</td>
<td>(-0.32 \ln r )</td>
<td>3.0</td>
<td>( 1.84 \ln W/P )</td>
<td>16.1</td>
<td>0.98</td>
</tr>
<tr>
<td>(3')</td>
<td>( \ln M_{t/p} )</td>
<td>(-0.42 \ln r )</td>
<td>3.9</td>
<td>( 1.41 \ln W/P )</td>
<td>27.2</td>
<td>0.99</td>
</tr>
</tbody>
</table>

* Definitions of symbols:
  \( M_t \) = nominal money balances
  \( M_{t/p} \) = real money balances
  \( r \) = interest rate
  \( A \) = gross (unconsolidated) value of total assets
  \( A/P \) = gross (unconsolidated) value of real assets
  \( N \) = aggregate (unconsolidated) net worth
  \( G \) = nominal (consolidated) wealth, Goldsmith, Table W-1, estimate
  \( W/P \) = real wealth as before
  \( W \) = nominal wealth

For further description see n. 11.
theoretically a weighted average over both assets and liabilities. The similarity of the interest elasticities at least suggests that over long periods of time these two average rates move in direct proportion to the yield on long-term corporate bonds or financial assets. The result of this indirect test is not inconsistent with the assumption that the interest rate used to measure the return on assets may be roughly proportional to other asset rates in the long run. Furthermore, there appears to be little difference between the elasticity of money balances with respect to those assets that have been included in net worth and those assets that have been included in total assets but not in net worth.

The largest group of assets excluded from net worth but included in total assets is, of course, financial assets—bonds, mortgages, insurance, pensions, and money itself. The computed elasticities may be taken as weighted averages of the elasticity of the demand for money with respect to the components of the various wealth measures. Thus the data suggest that, on the average, money is as much a substitute for real assets as for other financial assets. Moreover, the inclusion or exclusion of money itself from the measure of wealth does not alter the elasticities substantially.

We can pursue this point further by considering an additional measure of wealth—the net wealth estimates which Goldsmith obtained by consolidating the balance sheets for all sectors. For this definition of wealth, we get (eq. [7], Table 1) a result that is much less satisfactory than those obtained when total assets and net worth were used as the measure of wealth or when government assets were subtracted and government liabilities added as in equations (2) and (3) earlier.

There appears to be no effect of Goldsmith wealth, $G$, on the demand for money; this contrasts with the very strong effects exhibited when wealth is measured by net worth, total assets, or as in Section I above. Moreover the results for the three other measures of wealth are homogeneous of first degree in prices and financial assets, while the demand function does not have this property when wealth is measured as $G$. The principal cause of this difference appears to be that $G$ includes government assets (buildings, inventories, and public lands) but excludes government liabilities. Some government liabilities are included in the three other wealth measures used here as a part of the assets or net worth of the public. Including such liabilities as a part of the assets of the public appears to be important for the proper specification of the wealth variable.\[34\]

The lower part of Table 1 presents some estimates of the elasticities for both shorter and longer time periods. It appears that the parameters of the demand function are more stable when money is defined to include time deposits at commercial banks, $M_t$. This result is tentatively confirmed by the estimates that have been computed for a variety of time periods and a number of different demand functions. I will return to this point in a following section where some subperiod estimates are presented.

In summary, the alternative formulations that have been used here tend to support the conclusions reached at the end of Section I, most notably the impor-
DEMAND FOR MONEY: EVIDENCE FROM THE TIME SERIES

importance of variables representing substitution and wealth effects as arguments in the demand function for money and the stability of the function. For several definitions of money and non-human wealth, we have seen (1) that a stable demand function can be isolated using interest rates and real wealth as arguments and (2) that the demand for money is approximately unit elastic with respect to wealth when money is defined as the sum of currency plus demand deposits. In addition, the results of this section provide some evidence about the relative substitutability of particular assets for money and suggest that increases in aggregate wealth have roughly the same effect on the demand for money when non-human wealth is measured by any one of a number of aggregate values that include government debt as assets of the public.

III. COMPARISONS WITH SOME HYPOTHESES INCORPORATING INCOME

Traditionally theories of the demand function for money have used income as an argument in the function. The model used here like many of the models used in recent work in monetary theory has been developed in terms of a wealth constraint. The effects of human income and human wealth have been dropped from demand equation (1) above by assuming that \( d/3 \) remains constant or nearly so. This formulation has been shown to yield a stable demand function for money. Nevertheless, in this section we consider some alternative hypotheses where income is taken as the constraint or incorporated as a variable. The evidence seems to suggest that the money demand relation is more stable when equation (1) above is retained.

The use of income as a constraint in the demand function for money often has been associated with the notion that money is a commodity used principally to effect a given transactions volume. This is explicitly stated in the formulation of the Keynesian demand function \( M = kY + L(r) \) and in the so-called Cambridge equation \( M = kY \) (\( M \) and \( r \) are as defined earlier, \( Y \) is aggregate money income). In contrast to the view taken here, the Keynesian approach views a large part of money balances as a separable, fixed, linear function of income or transactions that is not dependent on interest rates. Much of the empirical evidence for this approach to the Keynesian demand function rests heavily on rather tenuous evidence from the interwar period and particularly on the prolonged fall in interest rates associated with the depression and war periods.

Even when it is not assumed that money balances can be divided into separable “transaction” and “speculative” balances, there are important differences between the income and wealth approaches. The use of current income as a constraint has generally been rationalized in terms of the desire to use money to carry out some transactions volume. The quantity theory of money in its Cambridge form posits a direct link between money and money income. As Friedman has noted, the link, \( k \), in the Cambridge equation, is written as if the average cash balance, \( M/Y \), can be treated as a technologically fixed constant. Of course quantity theorists did not regard it in that way, but they emphasized long-run institutional factors, habits of payment, etc., rather than costs

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Footnotes:

25 As is well known, Marshall introduced wealth into the money demand function in *Money, Credit and Commerce* (London: Macmillan & Co., 1923), chap. iv, when he described the function but dropped wealth in the rest of his discussion.

and yields associated with asset exchanges and portfolios as determinants of $k$.

The wealth constraint emphasizes the role of money as a productive asset and focuses attention on the equilibrium of the balance sheet, the allocation of assets or current income, that have been used as arguments in the demand function for money, correspond to the theoretical definition of income as the yield on wealth? Since the measure of wealth used here excludes human income, the empirical question is worth investigation.

For it may be argued that if income is treated as the yield on wealth, the distinction between an income and wealth constraint is arbitrary. If $Y = rW$ is treated as a definition of income, the point is tautological, that is, there exists a definition of income for which the income and wealth constraints give precisely the same results. But the open question is: Do measures of permanent

| Table 2 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Equation       | Definition of $M$ | Variable and Coefficient | t value | Variable and Coefficient | t value | Variable and Coefficient | t value | Multiple Correlation |
| (8)            | $\ln M_1/p$      | $-0.69 \ln r$       | 7.4    | $0.03 \ln Y/p$          | 0.2    | $1.10 \ln W/P$            | 7.3    | .98                 |
| (9)            | $\ln M_1/p$      | $-1.15 \ln r$       | 11.8   | $-0.10 \ln Y/p$         | 0.8    | $1.35 \ln W/P$            | 8.7    | .99                 |
| (10)           | $\ln M_1/p$      | $-0.70 \ln r$       | 7.4    | $0.03 \ln Y/p$          | 0.2    | $1.10 \ln W/P$            | 7.3    | .98                 |

* Definitions of symbols:

$M_1/p =$ real money balances  
$r =$ interest rate  
$Y/p =$ net national product  
$W/P =$ real wealth  
$Z/p =$ $(A/P)r +$ real wage and salary payments

and the services that money provides. Effecting a volume of transactions is but one of these services. Empirical evidence is required to separate the income and wealth models of the demand for money and to evaluate the importance of the difference in constraints.

For it may be argued that if income is treated as the yield on wealth, the distinction between an income and wealth constraint is arbitrary. If $Y = rW$ is treated as a definition of income, the point is tautological, that is, there exists a definition of income for which the income and wealth constraints give precisely the same results. But the open question is: Do measures of permanent
money for three time periods in this century. It is clear from these data that the statistical significance of the interest elasticity for the entire period is largely derived from the inverse movements in real money balances and interest rates during recent decades, if real income, \(Y/p\), is used as a constraint. Estimates presented earlier, and others that appear in the following section, show that the interest rate remains significant when real non-human wealth is used as the constraint. Thus those who argue that the demand for money depends on real income must deny the importance of the interest rate during much of the time period we have considered. In short, the results are different and imply that the underlying models are different when \(F\) is defined as measured net national product.

The assumption that income should not appear along with wealth in equation (1) is examined in equations (9) and (10) of Table 2. Real income has no significant effect on the demand for real money balances when real non-human wealth appears in the equation. Interpretation of these findings is difficult owing to substantial multicollinearity due to the high correlation between real income and real wealth. However, we note that the interest rate and real wealth coefficients are generally quite similar to those estimated earlier while those for the real income coefficients are quite different. From this it appears that the addition of real income to the money demand equation adds little additional information.

There remains the problem of human wealth. Specifically, does the exclusion of human wealth from the right side of equations (2), (3), (4), (5), and (6) alter the estimated wealth elasticities? Statistical problems rather than questions of economic interpretation prevent a completely satisfactory answer to these questions. Nevertheless the evidence seems to suggest that the omission of human wealth (or the income received as wages and salaries) does not introduce a substantial bias. Compare the long-run income elasticity of equation (8) with the long-run wealth elasticity of equation (3) earlier. Note that they are of the same order of magnitude. This suggests that the elasticity of the demand for money with respect to human wealth or wage and salary income is approximately the same as the elasticity with respect to non-human wealth or non-property income. Similar evidence is given in equation (11), which attempts a crude estimate of the elasticity of the demand for money with respect to wage and salary income. While \(Z/p = W/p + r\) real wage and salary payments, that is, a measure of human plus non-human income) and \(r\) are highly correlated, it again appears that real money balances, \(M_t/p\), are approximately unit elastic with respect to \(Z/p\). A variety of other tests that I have attempted point to the same general conclusion.

The findings of this section suggest, first, that the demand function for money is more stable when the function is formulated in terms of a wealth constraint

\[ Z/p = W/p + r \]

The relatively high interest elasticity of eq. (11) (—1.77) has an interesting implication. Let \(Z/p\) be an approximation of permanent income from human and non-human wealth, that is, \(Z/p = W/p + r\). If \(Z/p = \ln W/p + r\) is substituted in (3) above, we obtain \(M_t/p = k - .95 \ln r + 1.11 (\ln Z/p - \ln r) = k - 2.06 \ln r + 1.11 \ln Z/p\) as the parameters of (11) which would be consistent with (3). In fact, the parameters of (11) are slightly lower owing most likely to errors of measurement.
rather than an income constraint. This conclusion is supported by numerous additional estimates, some of which are presented in the following section; it suggests that it is more fruitful to view the demand for money as influenced by cost and yield considerations rather than primarily as a means of effecting a volume of "transactions" and subject to an income constraint. Second, the results here support the assumptions made earlier that eliminated the direct effects of human income from the money demand relation. Third, the findings suggest that little bias results from the exclusion of human wealth from the measure of wealth used to test the hypothesis. While this third finding is subject to some qualification owing to problems of estimation, I have failed to uncover evidence which strongly suggests that a large bias has been introduced or which does not suggest that the bias is small.

Finally, one income hypothesis remains to be considered. This is the permanent income model suggested by Friedman. The following section compares the two formulations, attempts to reconcile them, and provides some additional evidence on the assumption of homogeneity of the money demand function.

IV. THE PERMANENT INCOME HYPOTHESIS

Among recent studies of the demand function for money, none is closer to the present study than Friedman's. But two of his conclusions differ substantially from those presented here. First, he is unable "to find any close connection between changes in velocity from cycle to cycle and any of a number of interest rates." However, from his secular results, he notes that the "yield on corpo-

rate bonds is correlated with the real stock of money and velocity in the expected direction... Bond yields, however, play nothing like so important and regularly consistent a role in accounting for changes in velocity as does real income." Second, he concludes that the demand for real money balances is highly elastic with respect to permanent real income. A 100 per cent increase in real permanent income leads individuals to increase their real money balances by 180 per cent on the average, that is, money is a "luxury."

In the two earlier sections, I have presented a variety of estimates which suggest the conclusions that interest rates do play a consistent and significant role in the long-run demand for money and that the wealth elasticity of the demand for real money balances, \( M_1/p \), does not support the view that money is a luxury. In this section I show the relation between Friedman's demand function for money and the one presented here and argue that the two demand functions are conceptually similar. Most of the empirical differences between the two can be accounted for by differences in specification of the variables rather than by inherently conflicting implications. Some evidence is then presented for a number of different time periods. It suggests that the money demand function presented here is more stable and somewhat more consistent with the evidence than the alternative proposed by Friedman.

Following Friedman, let \( Y_p = rW \) be the definition of permanent income as the income flow resulting from a stock of wealth. In equation (1), \( M = g(r)W \).

\[ \text{Ibid., p. 345.} \]

\[ \text{It should be noted that the precise meaning of } Y_p \text{ needs clarification. Friedman defines it as the yield on wealth, refers to it as expected income, and measures it as an exponentially weighted average of past incomes. Clearly, it could be all three in prin-} \]

\[ \text{Friedman, "The Demand for Money...", op. cit., p. 349.} \]
we define $W$ as human plus non-human wealth and $r$ as the yield applicable to total wealth, we may substitute $Y_p/r$ for $W$ to get

$$M = \frac{g(r)Y_p}{r} \quad (12)$$

If $g(r)/r$ is approximately constant, equation (12) becomes

$$M = kY_p \quad (12')$$

To compare Friedman's empirical evidence with my earlier results, we divide equations (1) and (12') by population, $N'$, and prices, $p$ (or permanent prices $P_p$), define money as $M_z$, take logs and get

$$\ln \frac{M_z}{N'p} = a - \beta \ln r + \gamma \ln \frac{W}{N'P} \quad (13)$$

and

$$\ln \frac{M_z}{N'P_p} = a + b \ln \frac{Y_p}{N'} \quad (13')$$

It should be clear from the preceding discussion that (13) and (13') are not mutually exclusive in the sense that evidence which confirms one must fail to confirm the other. Rather they are alternative forms of the same hypothesis since the definition of permanent income and the results of the preceding section suggest that $krW$ can be substituted for $ky_p$. They differ only in that the parameter estimates of (1) or (13) will explicitly reflect the separate effects of interest rates and wealth while those of (12') or (13') will not.

This point is of some relevance. Theoretical considerations used to obtain equation (1), Friedman's essay on the quantity theory, and a variety of empirical results stemming from short- and long-period estimates of the interest elasticity suggest that both velocity and the demand for money are significantly dependent on interest rates. Friedman's finding that permanent real income dominates is an empirical result. The evidence he presents in support of this conclusion is based on an analysis of the within cycle residuals from an equation like (13'). The evidence presented here comes from the explicit use of interest rates as an argument in the demand function for money. Friedman's conclusion that interest rates play a small role in the demand function for money seems to stem from the measurement procedure he employs. As we shall see, a similar argument holds for velocity.

Though the two hypotheses are theoretically consistent, the empirical evidence distinguishes between them. Table 3 compares the estimated elasticities of equations (13), (13'), and (3') for a number of different time periods. Note that the parameter estimates for equation (3') are a much more stable proportion of their 1900-1958 values than those for equation (13) and slightly more stable than those of equation (13').

I am indebted to Anna J. Schwartz for making available to me the as yet unpublished data that she and Milton Friedman will use in their forthcoming books and that served as the basis for Friedman's estimates.

A notable exception is the interest elasticity for 1940-49. We will return to this point in the discussion of velocity.
for the shorter periods) and generally does not affect the longer period interest elasticities or their significance. Finally, note that the proportion of the variance explained by equation (3') is usually greater than that explained by the permanent income hypothesis. The one striking exception to this last conclusion occurs when permanent per capita money balances are negatively related to permanent per capita income, the nine years 1950–58.

It would appear therefore that interest rates do enter significantly into the demand function for money. Indeed the results are so pervasive for widely different time periods and alternative definitions of money and wealth that the issue seems beyond dispute. Friedman's contrary findings are not supported by the evidence from one variant of the permanent income hypothesis itself. For equation (13") shows that when interest rates are used along with per capita permanent income, interest rates enter significantly into the money demand relation. For 1900–1958 the regression estimates are shown in equation (13") on page 237.

The interest elasticity is not altered greatly by the fact that Friedman deflates by population, though the permanent income elasticity is raised by this procedure. When (14) is not deflated by population, the interest elasticity = –.39 and the income elasticity = 1.23. These coefficients are quite similar to those obtained when the function is stated in terms of interest rates and real wealth.

<table>
<thead>
<tr>
<th>TABLE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>REGRESSION AND CORRELATION COEFFICIENTS FOR THREE EQUATIONS</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>(13) ln ( M_t/N'p )</th>
<th>(13') ln ( M_t/N'P_p )</th>
<th>(13&quot;) ln ( M_t/p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) and ( \gamma ) value</td>
<td>( \beta ) and ( \gamma ) value</td>
<td>( \beta ) and ( \gamma ) value</td>
<td></td>
</tr>
<tr>
<td>1900–1958</td>
<td>10.5</td>
<td>1.59</td>
<td>0.50</td>
</tr>
<tr>
<td>1900–1929</td>
<td>1.75</td>
<td>33.7</td>
<td>10.8</td>
</tr>
<tr>
<td>1930–1958</td>
<td>14.1</td>
<td>0.71</td>
<td>14.3</td>
</tr>
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By Decades

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) and ( \gamma ) value</td>
<td>0.42</td>
<td>0.61</td>
<td>1.6</td>
<td>0.73</td>
<td>2.06</td>
<td>0.03</td>
</tr>
<tr>
<td>( \gamma ) and ( \gamma ) value</td>
<td>1.72</td>
<td>1.42</td>
<td>2.5</td>
<td>0.95</td>
<td>1.37</td>
<td>0.71</td>
</tr>
<tr>
<td>( \beta ) and ( \gamma ) value</td>
<td>3.6</td>
<td>2.3</td>
<td>3.9</td>
<td>0.93</td>
<td>0.90</td>
<td>0.75</td>
</tr>
<tr>
<td>( \gamma ) and ( \gamma ) value</td>
<td>12.0</td>
<td>12.6</td>
<td>12.0</td>
<td>0.94</td>
<td>0.90</td>
<td>12.2</td>
</tr>
<tr>
<td>( \beta ) and ( \gamma ) value</td>
<td>2.3</td>
<td>3.9</td>
<td>2.0</td>
<td>0.94</td>
<td>0.90</td>
<td>12.0</td>
</tr>
<tr>
<td>( \gamma ) and ( \gamma ) value</td>
<td>1.38</td>
<td>2.4</td>
<td>0.64</td>
<td>0.90</td>
<td>0.64</td>
<td>0.65</td>
</tr>
</tbody>
</table>

* Definition of symbols:

\( M_t/N'p \) = real per capita money balances
\( M_t/N'P_p \) = real per capita permanent money balances
\( M_t/p \) = real aggregate money balances
\( r \) = interest rate
\( r/N' \) = real per capita permanent income
\( W/N'P \) = real per capita wealth
\( W/P \) = real aggregate wealth
A second major difference between the findings in this paper and those presented earlier by Friedman relates to the value of the wealth or permanent income elasticity. As noted above, Friedman concludes that money is a luxury. This conclusion is largely a result of the definition of money that he employs and the process of deflating by population which raises the wealth and permanent income elasticities when money is defined as $M_4$.

Equation (14) (see below) affirms that, when the parameters for permanent prices, population, and real per capita wealth (in permanent prices) are left free, regression estimates of the interest rate and wealth elasticity consistent with those presented earlier are obtained if money is measured as the sum of currency plus demand deposits.\(^{36}\)

Friedman has provided no compelling reason for selecting the population variable for consideration from the many others that might affect the demand for money. A plausible case can be made for a number of variables such as the age composition of the population, the number or distribution of commercial and savings banks, or the distribution of population between urban and rural areas. Including the latter variable might suggest that the money demand function is not homogeneous of first degree in population alone. A shift toward greater urban concentration accompanied by an increase in population might easily lead to an elasticity of the demand for money, $M_1$ or $M_2$, with respect to population, that is less than one. The costs and yields of holding money balances are not unaffected by the location of savings banks in shopping centers, the development of "bank by mail," and a host of other devices that have accompanied the increase in population and increasing urbanization.

In the absence of theory which specifies that the demand function for money is homogeneous in population, we are left to decide on the evidence and its possible rationale. Equation (14) is at least consistent with the view that increased urbanization or changes in age composition of the population (life-cycle) make the demand function for money less than unit elastic in population. While it is most likely that there is multicollinearity in the estimates of equation (14), as I noted above, all of the other variables—per capita non-human wealth, interest rates, and permanent prices—have coefficients that are consistent with the theory of Section I and the empirical estimates of equation (3) above. Moreover, a comparison of equations (3\') and (13) of Table 3 seems to suggest that the parameters of the wealth model are more stable and the proportion of the variance explained is always higher when homogeneity of the first degree in population is not assumed (equation [3\']) than when it is (equation [13]). I therefore reject the assumption that the demand function for

\[
\ln \frac{M_4}{N'P_p} = -2.8 - .37 \ln r + 1.41 \ln \frac{W}{N'} + \nu_4. \quad (13\''\)
\]

\[
\begin{array}{ll}
\text{t values} & 7.3 & 34.1 \\
\text{Partial correlations} & -.72 & .98
\end{array}
\]

\[
\ln M_1 = - .39 - .79 \ln r + 1.11 \ln \frac{W}{N'P_p} + .74 \ln N' + 1.08 \ln P_p. \quad (14)
\]

\[
\begin{array}{llll}
\text{t values} & 7.0 & 7.0 & 4.2 & 7.1
\end{array}
\]
money is homogeneous of the first degree in the stock of population. Of particular relevance to our discussion is the work of Latané. His two studies suggest a stable relationship between velocity and interest rates over long periods which is linear in the logarithms. His evidence is based on the period 1909–58 and is obtained when income velocity measured as the ratio of GNP to $M_1$ is related to the yield on long-term corporate bonds.

Attention is drawn to the latter point because specification of the measurement procedure for velocity is as critical for an understanding of this problem as the specification of the wealth constraint is for the demand function for money. Latané’s high correlations suggest that the log of velocity, measured as the ratio $\text{GNP}/M_1$, is directly related to the log of interest rates. However, we may formulate his model as a demand function for money subject to an income constraint $M = L(r, Y)$, as in Section III of this paper. Assuming homogeneity in prices and real income gives $M/Y = L^*(r)$, the velocity equation that Latané used. Since the real income elasticity of equation (8) (Table 2) is approximately unity, Latané’s implicit assumption about real income elasticity seems reasonable.

However the tests in Table 2 also show that the interest elasticity of equation (8) does not differ significantly from zero in

The literature of economics contains numerous suggestions about the velocity relation. These range from the assumption of constant velocity, that has been imputed to some authors, to the low-level liquidity trap of Keynes’ General

V. VELOCITY

See also n. 40 below and the paragraph to which it relates.
DEMAND FOR MONEY: EVIDENCE FROM THE TIME SERIES

...the period 1900-1929. Thus Latané's result is spurious since it combines the significant results of the later period with the non-significant results of the first two or three decades. 89

With respect to the definition of $M$, the results presented above strongly suggest that the effect of a rise in interest rates on the percentage change in the quantity of money demanded is much greater when time deposits are excluded from $M$ than when they are included in it. It would not be surprising therefore to find that measures of income velocity that include a large proportion of money substitutes in the denominator are less sensitive to interest changes than those that are based on currency and demand deposits alone. The problem is one of disentangling relative and general changes in asset prices just as in the demand for money itself.

The velocity function or its inverse, money per unit of income, which is used here, is derived from equation (1) by dividing by income measured as $rW$ to get

$$\frac{M_i}{rW} = \frac{1}{V_i} = v_i(r, W) \quad i = 1, 2, 3. \quad (15)$$

If interest rate and asset parameters of the money demand functions are denoted $b$ and $c$ respectively, simple algebra makes clear that the parameters of the velocity functions should equal $b - 1$ and $c - 1$. This is the case for the interest rate and the asset parameters shown in equations (15) and (15'). No changes were introduced on the right-hand side of the equation. These equations appear to be quite consistent with their money demand functions, a further check on earlier results and an indication of the absence of major bias from deflating. 40

For 1900-1958, the results are shown in (15) and (15') on page 240.

Note that $V_1$ depends only on interest rates while $V_2$ depends on both interest rates and wealth as the hypothesis (and the arithmetic) implies. The results for $V_1$ are quite consistent with those obtained by Latané, except that the interest elasticity is much higher (in absolute value) as a consequence of the measurement procedure used here.

Evidence of the time trend in velocity is given by the asset elasticities. This question has often been discussed by economists, and Selden in particular has noted that much of the difference in findings has been due to differences in definition. 41 However, Selden chooses to in-

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89 Problems of specification are also involved in the choice of the ratio of debits to demand deposits as a measure of velocity. There is little a priori reason to suggest that this ratio should behave in a manner similar to that of income velocity. Assume, for example, that changes in expectations of future yields cause readjustment of asset portfolios such that the quantity of money demanded remains the same. Specifically, at the existing asset prices, assume that there is an excess demand for land and an excess supply of equities. Portfolio adjustments will change the relative prices until asset equilibrium is restored. Income velocity, the ratio of income to money stock, need not change, since neither the flow of income nor the stock of money necessarily changes in the process of adjustment. But debits to demand deposits have risen and this measure of velocity is higher. Other explanations of the differences in these ratios have been suggested elsewhere and need not be elaborated here (see Selden, op. cit., pp. 215-17).

40 Note two points: first, the definition of $V_i$ differs from the usual definition since $rW$ is a measure of the "permanent" income from nonhuman assets; second, eq. (15) and (15') provide a further check on the question of homogeneity of first degree in population. If the absence of population, $N'$, as an argument in the development of eq. (1) introduces bias, the empirical estimates of (15) and (15') would not be consistent with the estimates for (3) and (3'). Since $M_i/rW = M_iN'/rWN'$, the consistency of the two sets of estimates is another test of whether the population variable should be included in a proper specification of the demand function for money. Of course we cannot be certain about the results of this test since other (unnamed) variables may be operating to make the money demand and velocity estimates consistent. Nevertheless, the consistency of the $M_i$ and $V_i$ parameter estimates does increase confidence in the demand function for money developed here.

include time deposits in his measure of velocity and concludes that there was a marked decline in V from 1839-1939 or from 1909-46, and little net change from 1929-51. A similar result, obtained by Warburton earlier, is also due to the inclusion of time deposits in the definition of money.42

The results here clearly indicate that for 1900-1958 the secular trend in velocity holds only for \( V_2 \). \( V_1 \) is dependent solely on the interest rate—a trend-free variable. But \( V_2 \) depends on real assets, which increased approximately 275 per cent during the first half of the century. Hence during that period \( V_2 \) fell by approximately 40 per cent according to the estimate of equation (15'). This fall, however, is largely a result of measurement procedures representing as it does the increase in time deposits relative to other components of the money stock.

Moreover, the denial of a time trend in \( V_1 \) does not depend on the choice of \( \bar{r}W \) as the denominator in equation (15). Similar results with respect to secular trend are obtained for the \( V_1 \) if velocity is defined as the ratio of net national product to money stock.

Those who have argued that there has been a secular decline in velocity have often pointed to the latter part of the nineteenth century, particularly the last two decades, in support of their position. Indeed the fall in measured velocity during that period is most striking. My results do not directly bear on this point. However, a "prediction" of the results for earlier years has been obtained by extrapolating beyond the sample period.

From 1839-99, Selten's series V-39 fell by 70 per cent.44 Macaulay's Railroad Bond Yields fell approximately 50 per cent during the same period. Since Selten's V-39 includes time deposits, we multiply the percentage fall in bond yields by the estimated elasticity from equation (15') (1.34). The predicted fall in velocity for the forty-year period is therefore 67 per cent—that is, within 3 per cent of the actual fall in velocity.

Friedman's measure shows that velocity (again including time deposits) peaked about 1880. In 1879, his graph shows V as approximately 4.75. By 1899 it had fallen to 2.5, a drop of roughly 48 per cent in twenty years.44 Macaulay's Bond Yields declined by 34 per cent during the same period. Again multiplying by the interest elasticity from equation (15'), the predicted fall in velocity is 46 per cent, or 2 per cent less than the actual fall.

Thus it appears that the demand function for money has been stable for almost a century. But these results are too good, for as noted, there has been a secular fall in \( V_2 \) in this century resulting from the inclusion of time deposits in this measure. Applying the asset elasticity would indicate a further fall in velocity and an overestimate of the decline in the nine-

\[ \ln \frac{1}{V_1} = 2.21 - 1.78 \ln r + .02 \ln \frac{W}{P} + w_1 \bar{R} = .98 \]  
\( t \text{ values} \quad (30.4) \quad (61) \)

\[ \ln \frac{1}{V_2} = 1.37 - 1.34 \ln r + .23 \ln \frac{W}{P} + w_2 \bar{R} = .96 \]  
\( t \text{ values} \quad (19.5) \quad (6.4) \)
teenth century. Nevertheless, the "predictions" do suggest that the much discussed fall in velocity at the end of the last century can be interpreted as a response to the decline in interest rates when the elasticity is estimated for $M/rW$ rather than $M/Y$. If we interpret the secular trend in $V_2$ as a reflection of the growth of time deposits relative to other components of the money stock, the "prediction" suggests that the growth of financial intermediaries and thus the non-zero asset elasticity of equation (15') are phenomena associated with the twentieth rather than the late nineteenth century.

The successful prediction of the fall in velocity at the end of the nineteenth century as an interest rate phenomenon is of course critically dependent on the value of parameters. These in turn are the result of a particular specification for the velocity variable—that is, the use of $r \cdot W$ as the numerator in place of the more commonly used measure of income. A good prediction is not obtained when measured net national product (or for Latané, measured gross national product) is used as the numerator of velocity. Therefore this result lends strong support to the wealth hypothesis developed here and to the use of permanent velocity, $M/rW$, in the explanation of economic behavior over time.\textsuperscript{46}

Ritter's notion that there is an upper bound to velocity seems to be denied by the data. For interest rates that have ranged from 2.35 to 5.31 per cent there is little evidence of a velocity ceiling. If such a ceiling exists, evidence for it should take the form of large positive values for $w_1$ in equation (15) during periods of high interest rates; that is, the measured value of $1/V_1$ should exceed the predicted value by a large amount. In fact, the measured value did exceed the predicted value when interest rates were above 5 per cent, but the size of the residual is not substantial. This suggests that the velocity relation has not reached an upper bound for any of the interest rates observed in this century.

The results of this section, like those for the money demand function in earlier sections, support the conclusions reached at the end of Section I. They strongly suggest that the demand function for money constructed here, based on a theory of asset preference, is stable. The theory and the empirical tests have required only two arguments—interest rates and real non-human wealth—to approximate the recorded behavior of real money balances for almost sixty years and for a number of shorter sub-periods. Of the two arguments, one (real wealth) appears to be unnecessary for the velocity relation when money is defined as currency plus demand deposits. Both arguments appear to be relevant when time deposits are included in the definition of money. This suggests that the much discussed issue of a trend in velocity is analogous to the problem of whether or not money is a luxury. Both depend on the use of a definition of money which is broader than $M_1$.

The velocity estimates provide additional evidence on the stability of the demand function for they suggest that the prolonged fall in velocity at the end of the nineteenth century can be explained in terms of the hypothesis developed here. We now turn briefly to discuss some additional evidence of the stability of the money demand function and some policy considerations before indicating some implications of the results for monetary theory.

\textsuperscript{46} Scatter diagrams plotting the relation between $1/V_1$ and $r$ confirm the stability of the velocity function during the first fifty-nine years of this century.
VI. POLICY CONSIDERATIONS

Policy actions are designed primarily for their short-run effects and are a response principally to short-run changes. The results here are based on longer periods of time and refer more to the secular than to the cyclical performance of the economy. Two methods of relating short and long-run functions are (1) comparison of the estimates obtained for sub-periods, for example, cycles or decades, with those for the entire period, and (2) comparison of related crossection studies. A considerable body of evidence of both types has now been accumulated.

TABLE 4
ELASTICITY OF $V_1$ WITH RESPECT TO INTEREST RATES

<table>
<thead>
<tr>
<th>Interest Elasticity</th>
<th>1900–1958</th>
<th>1.78</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1900–1909</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td>1910–19</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>1920–29</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>1930–39</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>1940–49</td>
<td>2.14</td>
</tr>
<tr>
<td></td>
<td>1950–58</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Some of the major findings are reported here before I briefly consider some policy implications.

The most impressive finding from the decade estimates is the relative stability of the money demand or velocity function. The major exceptions (see Table 4) include observations drawn from the forties, particularly from the postwar, pre-Accord period. Interest rate ceilings were effectively maintained on financial assets during this period, but it is likely that such ceilings were largely irrelevant with respect to reducing the rising yields on non-financial assets.

The earlier discussion of the measure of assets which is relevant for money holding decisions suggested that over time the yields on most aggregate measures of wealth move in roughly parallel fashion. Suppose velocity (or money demand) adjusts to the relative yield on real wealth (or the spread between yields on financial and real assets). Then for the period up to World War II, the choice between broadly based measures of asset yields is largely a matter of convenience. But once price control is instituted for financial assets, this is no longer true. In short, my study of the interest elasticity for various cycles suggests that the interest rate to which velocity adjusted during the forties was considerably higher than the quoted corporate bond yield, the measure that works well for earlier and later periods.

Of course there is an alternative interpretation. The result might suggest that the velocity function, hence the demand for money, was unstable in the war and immediate postwar years. I reject this explanation. In a regime of price controls, it would be surprising to find the price ratios equal to the marginal rates of substitution between commodities. The effect of price controls is to distort observed economic relationships. Similar results undoubtedly hold for bond price controls or pegged interest rates.

The decade of the thirties would require little comment were it not for the many attempts to find evidence in support of the low-level liquidity trap. The data deny that the interest elasticity of the demand for money or velocity became exceptionally large during that decade. Indeed, the interest elasticity of $V_1$ (1.66) was slightly below the average for the fifty-nine years (1.78). Substantially similar conclusions are reached if we consider the data for individual cycles. Moreover, interest rates on long-term corporate bonds continued to adjust downward during the decade from the high of 4.70 reached in 1932. Annual rates fell to 2.90 in 1937, rose very
slightly in the recovery of 1938, and fell again to 2.50 in 1941. Like the evidence on velocity that I have examined, the almost continual fall in interest rates denies the existence of the low-level liquidity trap.

None of the foregoing implies that a given change in the interest rate is equally effective with respect to absolute money holdings in deep and mild depression. The money demand function is a curve of relatively constant elasticity—not constant marginal propensity to hold money. Taking derivatives of (say) equation (3) shows that the marginal propensity to hold money with respect to interest rates is negative and dependent on the ratio $M/r$ as well as the parameters of the equation. At low interest rates the slope of the function becomes quite large in absolute value. Relatively large increases in the money supply would therefore be required to restore equilibrium once a deep depression has occurred.

The cross-section studies give results that are quite consistent with those presented here. In particular, they suggest strongly that the demand for money is approximately unit elastic with respect to wealth. However, they imply that some additional variables must be considered in discussions of the reaction of the economy to cyclical and policy changes.

For one important implication of the cross-section studies is that changes in the distribution of output between industries result in quite different totals for the aggregate demand for cash balances. Similar effects appear to operate if the composition of output changes such that large firms within an industry increase their sales relative to small firms or vice versa. In short, during cycles the demand for money appears to depend on the composition of output.

It follows then that both the flow of aggregate income (or its rate of change) and the money supply (or its rate of change) may be the same at corresponding points in any two cycles while the excess demand for money is different in one than the other. The variables that serve as guides to policy, for example, excess or free reserves and the rate of change of prices, need not be the same in the two cases. Under these circumstances policymakers can easily be misled into making excessive, insufficient, or ill-timed changes.

If additional studies confirm the above results, the policy-maker must be equipped with more, or at least different, information than he now collects. In particular, the existence of a stable demand function for money suggests that the quantity of money demanded would be a most useful addition to the list of variables that the policy-maker studies. Certainly the quantity demanded would be helpful in deciding whether money is "tight" or "loose" in the aggregate. But if distribution effects are important, as the cross-section studies seem to suggest, knowledge of the aggregate quantity of money demanded and supplied will be useful but insufficient for policy purposes.

VII. CONCLUSION

Three issues in contemporary monetary theory—(1) the variables or arguments on which the demand for money depends, (2) the stability of the demand function, and (3) the definition of money to be used in monetary analysis—are noted in the introduction. Other issues, such as the assumption of homogeneity of the demand function with respect to prices and financial assets or the conclu-
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sion that money is a "luxury," were introduced at other points in the paper. It seems useful to summarize the empirical findings with respect to these questions before considering some more general implications of the results.

First, with respect to the arguments of the function, we have found that, during the first six decades of this century, non-human wealth and interest rates have been of almost equal importance in explaining the demand for real cash balances. Interest rates have played the predominant role in determining the level of velocity.

Second, a comparison of estimates for various measures of wealth and various definitions of money strongly suggest that the demand function developed here is stable for several alternative definitions of money. These results seem to hold for both nominal and real cash balances and for a number of short and long periods. When the wealth model was compared to other hypotheses, based on measured or permanent income, the results supported the present hypothesis. The parameters of the wealth model appear to be more stable over time than those obtained from the estimates of the income and permanent income equations.

Third, with respect to the definition of money, the evidence from the time series does not support the view that monetary theory must be concerned with a concept more inclusive than currency plus demand deposits to obtain a stable demand relation. The demand functions for money defined inclusive of time or time plus savings deposits are no more stable in the long run than the demand function for money defined exclusive of these financial assets. Moreover, the time series evidence suggests that the use of more inclusive definitions of money mixes the effects of general and relative changes in interest rates on desired money holdings. The evidence further suggests that the observed growth of financial intermediaries relative to commercial banks reflects the effect of increased wealth on the desired allocation of wealth rather than a substitution effect as Gurley and Shaw have suggested.

Taken together the evidence bears on some additional issues. It supports the interpretation that the demand function developed here is homogeneous of first degree in prices and financial assets, a result that has been assumed in much aggregate theory. It denies that money is a "luxury." Examination of the evidence shows that the "luxury" proposition holds only when money is defined to include time or savings deposits. And the data deny the existence of a low-level liquidity trap, a proposition that has received attention in discussions of the ineffectiveness of monetary policy.

A more general issue in monetary and macroeconomic theory is involved in the disputes about the variables that enter the demand function for money and the stability of the function. Once monetary theory is formulated in terms of the demand to hold an asset subject to cost and yield considerations, rather than as a direct relation among money, income, and prices, attention is shifted to the determinants of the desired asset composition of households and business.

The importance of the stability of asset demand functions to this approach is obvious. The definition of money balances is also important. If stability of the demand function for money requires that a number of financial assets be included in the definition of money, the problem of finding stable relations is shifted to the supply function for money. The exogenous stock of base money, the part di-
money demanded is always equal to the quantity supplied, the economy is in a low-level liquidity trap. The addition of the yield on real capital and the value of expected human income to the model of the demand for money developed here, the substitution of wealth for income as the principal constraint on money holdings, and/or the specification of a supply function for money in place of an exogenously determined quantity of money do not then alter the conclusion that changing the exogenous determinants of the money supply will not change the level of income.

The model and the evidence presented here together with recent developments in money supply theory (see n. 47) suggest that changes in the policy variables that are exogenous determinants of the supply of money give rise to desired money holdings that differ from actual money holdings. The relevant elasticities suggest that exogenous changes in base money initiated at the discretion of the central bank lead to changes in the actual supply of money. Denial of the liquidity trap suggests that desired money balances neither rise nor fall as much as actual money balances in the initial response to increases or decreases in the policy variables. Thus the exogenous change in base money leads to a reallocation of assets in the portfolios of households and business by altering relative yields on financial assets or other determinants of the demand for money. Attempts by the public to readjust its portfolios, to find new balance sheet equilibriums, give rise to further changes in asset composition, to changes in the flow variables income, consumption, and investment, and to attendant changes in total wealth.

One alternative view of economic behavior, the income-expenditure approach,
concentrates directly on the determinants of the flow variables and ignores portfolio adjustments or considers only the adjustment between money and bonds. A third macroeconomic model looks at the economy in terms of the allocation of assets in portfolios but, as noted in the introduction, considers the demand-and-supply equations for physical capital as the relations of central theoretic and practical interest. In this model, money is but one of a number of financial assets.

The problem for the economist is to decide between these three broad approaches and the many variants of each approach. Central to such a decision is the ability to find more stable functional relations in one approach than in the others, to be able to predict more accurately with one than with the others. The evidence from the time series suggests that the monetary approach presented here is promising.

The term "income-expenditure approach" refers to a class of models rather than to a specific set of hypotheses. In its simplest form, the income-expenditure approach is perhaps best represented by the familiar textbook "models of income determination." Cf. G. L. Bach, Economics, or P. A. Samuelson, Economics: An Introductory Analysis, for examples. Numerous more sophisticated versions of the general approach exist. Perhaps the most familiar version is the Klein-Goldberger model. Cf. L. R. Klein and A. S. Goldberger, An Econometric Model of the United States, 1929-1952 (Amsterdam: North Holland Publishing Co., 1955).


(Continued on inside back cover)
MONEY AND BANKING

PREDICTING VELOCITY: IMPLICATIONS FOR THEORY AND POLICY*

KARL BRUNNER† AND ALLAN H. MELTZER‡

I. INTRODUCTION

It has become commonplace for economists to say, “The quantity theory of money, \( M = kY \), holds only, if at all, in the long run. The ratio of money per unit of income, or its reciprocal, income velocity, is subject to substantial short-run fluctuation. The model is at best a useful long-run approximation.” As yet there is no accepted theoretical explanation of the short-run behavior of velocity, which is to say that there is no theory of the demand function for money that is sufficiently in accord with the observed short-run behavior of velocity to be useful for prediction and that explains relatively constant long-run velocity.

Recent theoretical work has suggested a promising reformulation of monetary and macroeconomic theory in terms of the demand and supply functions for assets. Behavior functions for the economy have been formulated in terms of a general wealth-adjustment process which connects monetary processes with the pace of economic activity. There

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* This paper and the following discussions by Albert Ando, Martin J. Bailey, and Alvin Marty were presented at a joint meeting of the American Finance Association and the Econometric Society in Pittsburgh, Pa., on December 29, 1962. The program was under the chairmanship of Eli Shapiro.

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appears to be substantial agreement on this general approach, but there is much less agreement on substantive issues.²

Indeed, the general conception of a wealth-adjustment process has yielded two (approximative) macroeconomic theories. In one view "money matters." Emphasis is placed on the demand and supply functions for assets, particularly the demand and supply functions for money. Friedman's recent work, emphasizing the importance of money as a determinant of money income, is one example of this approach. A second approach also maintains the importance of viewing the economy in terms of the demand and supply functions for assets but takes the demand and supply functions for real assets as the relations of greatest interest for policy and prediction. Tobin, for example, has argued that the "strategic variable" is neither the quantity of money nor the rate of interest. For him, the supply price of real capital is, or should be, the object of direct attention by economists.

Both stock-dominated theories compete with a more familiar and widely accepted flow conception that views income as emerging essentially from the interaction of flow magnitudes. Monetary processes are usually assigned a marginal position or even totally disregarded in flow models.

These differences in approach challenge economists to provide evidence capable of distinguishing between alternative theories. They impose four tasks on the economist who views the demand and supply functions for money as the relations of greatest stability and significance in the determination of income: (1) to show that macrotheories formulated as demand and supply functions for assets provide relations that are more stable than theories based on flows; (2) to establish that the demand and supply functions for money are more useful tools for predicting income than other stock or flow relations; (3) to indicate the arguments, or variables, and the form of the particular demand and supply functions for money that can be used for policy and prediction; and (4) to resolve the controversies about the appropriate definition of money.³

In recent papers, we have attempted to formulate and test demand and supply functions for money and to compare our results to existing alternative theories.⁴ A substantial volume of evidence has now been


3. Johnson, op. cit., pp. 344-45, has a similar discussion.

accumulated for both the supply and the demand functions. The evidence relates to different time periods, countries, and monetary and political arrangements. It strongly suggests that the demand and supply functions which we have developed and tested are more stable than any demand and supply function for real capital or any investment function—the important relations stressed in alternative approaches to macro-theory—which we have found in the literature.

Of particular relevance for this paper is the work on the demand function for money or the velocity relation. One of us has presented elsewhere a long-run demand function for money. This function will be referred to as the "wealth model." The following conclusions are suggested by the evidence which supports the wealth model: (1) A relatively stable demand function for money can be isolated. (2) The principal arguments of the function are interest rates and non-human wealth. (3) The stability of the wealth model in the long run is independent of the definition of money balances if money is defined inclusive or exclusive of time and/or savings deposits. (4) The more inclusive the definition of money, the more substitution effects between, say, demand deposits and savings deposits are concealed within the monetary variable. (5) The parameter estimates for a 59-year period and for several shorter periods suggest that the parameters of the wealth model are more stable than the parameters of a number of alternative money-demand relations which use measured or permanent income as variables.

These results seem to support the view taken above that the formulation of macroeconomic theory which emphasizes the demand and supply functions for money provides more stable relations than alternative approaches that stress the demand and supply for real capital or are based on flows. But monetary and fiscal policies are primarily a response to short-run changes and an attempt to influence the short-run behavior of the system. As yet, we have not provided a comparable body of evidence on the short-run stability of the demand function for money based on the wealth-adjustment process.

Moreover, our interest in the demand and supply functions for money is related to our conceptual framework, which views the money func-
tions as of central importance in the determination of money income. In our view, policy or instrument variables directly controlled by central banks or governments operate through the behavior relations of the banks and the public to alter the actual or existing quantity of money. The demand function for money tells us the total money balance which the public desires to hold. If, as a result, say, of a change in policy, the actual money balance differs from the desired money balance, the public attempts to substitute money for other assets, or conversely. If the actual money supply exceeds desired money holdings, the public attempts to increase non-money assets and to decrease holdings of money in their (aggregate) portfolio. These changes operate through the relative price mechanism to induce changes in the flow variables, in the level and composition of output, in the demand for and supply of labor or capital, and in other variables. Given the supply function for money, the public has only negligible power to alter the nominal quantity of money directly. But their efforts to exchange money for other assets affect relative asset prices and alter the value of real money balances as well as their distribution.

Central to the view that "money matters" is the hypothesis that differences between the demand for and the supply of money, i.e., between desired and actual money holdings, clearly indicate the direction of change in money income. The hypothesis imposes the requirement that the arguments of the demand and supply functions for money permit us accurately to predict income in the short run from a knowledge of these two functions, perhaps with the auxiliary aid of some other relations. Numerous attempts have been made to show that a particular theory of velocity or the demand for money accounts for observed behavior reasonably well. But few attempts have been made to compare particular theories under similar conditions, for precisely similar time periods, or to compare their ability to predict money income.

In this paper the problem of predicting velocity in the short run is reopened. The predictive performance of the wealth model is compared with the performance of a number of alternative theories of the demand for money to obtain both relative and absolute measures of the power of the various demand-for-money or velocity relations in the short run. We examine the extent to which the stability of the long-run wealth model is shown by the short-run results and reopen the question of the appropriate definition of money. First, the procedures used in the paper are explained in Section II. In Section III, several variants of the demand function for money proposed by Keynes and developed by Latané, Tobin, and others are used to predict velocity. These predictions are compared with those based on the "permanent income" model.
and a "naïve" model. Section IV presents the evidence on the predictive power and the stability of the short-run wealth model—the model of the demand for money that we propose. A discussion of some implications for monetary theory and policy and a conclusion complete the paper.

II. DESCRIPTION OF PROCEDURES

Empirical tests of economic theories often consist of a few regression estimates which purport to exhibit the properties of a model, its ability to explain some proportion of the joint variation of the "dependent variable" and the arguments of the function, or the statistical significance of the arguments in the function developed. However, discrimination between hypotheses is not only shown by such tests. Continued exposure of hypotheses to new data generated in a variety of economic climates adds to our understanding of the range of the hypotheses and their comparative explanatory powers.

The particular procedure chosen here is the use of a number of demand functions to predict velocity (or in some cases "idle" balances) for 39 separate years. The predictions with which we are concerned are the information that the various models would have given about velocity in the year following the sample period used to estimate regression coefficients. These predictions are not forecasts, since in most cases the data on which the predictions are based would not have been available until after the year had passed. Our interest throughout is the comparison of the explanatory or predictive power of the various models to increase our understanding of monetary theory. Nevertheless, a brief digression on forecasting will be found in a later section.

Two criteria were used to judge the predictive performance of the alternative equations. The first of these—the mean absolute per cent error—was obtained by subtracting actual values from predicted values in a given year and dividing the absolute error by the actual value for that year. The absolute per cent errors were summed up for 39 years and for four shorter periods, and the averages were computed for the relevant number of years. If the money supply is assumed to be given, the error in predicting velocity is equal to the error in predicting income. The root-mean square, or standard deviation (hereafter "RMS"), our second criterion, was computed from the absolute per cent errors for the 39-year period.

6. Infrequently the empirical evidence for the theory in question is compared with a "naïve" model. Direct comparisons of alternative economic theories are rare indeed. Comparison of a demand-for-money hypothesis with a "naïve" model instead of with an alternative hypothesis implicitly assumes that there is little or no knowledge of the factors influencing the demand for money worthy of consideration. Such assumptions overlook the many demand functions previously developed and the substantive knowledge about their properties that has been obtained.
Predictions were made for each of the years in the 1910-40 and 1951-58 periods. Data for 1941-50 have been excluded from both the computations and the predictions. During these ten years, bond prices were pegged by the Federal Reserve. In many of the demand-for-money functions with which we will be concerned, interest rates are taken as a measure of the yield received if wealth or income is held in non-monetary form. Of course, bonds are but one of the alternative means of holding wealth. But it has been shown in a long-run study elsewhere that a reasonably stable demand-for-money equation can be obtained by using the yield on bonds as a measure of the weighted average of the yields on a variety of financial and real assets. When bond prices are controlled and rates of return on real assets are free to fluctuate, this assumption is patently false. Precisely these conditions prevailed from 1946 through March, 1951. Bond prices were controlled, but commodity prices were free to fluctuate. During the earlier part of the period, 1942-46, both bond and commodity prices were controlled by fiat, but the controls on the former were undoubtedly more effective than those applied to the latter.

Predictions were obtained by regressing each money-demand function or velocity equation for a moving 10-year period. The parameters thus obtained were used with the value of the independent variable in the "eleventh" year to predict the dependent variable for that year. The absolute per cent error and the RMS were then calculated as described above.

The use of a 10-year period is, of course, quite arbitrary, as is any choice of period. Most often, the period is chosen because of the availability of reasonably consistent data. In this case, that would have meant using regression coefficients based on 10 years of information for the first prediction, and \( t + 9 \) years for a prediction in the \( t \)th year \((t = 1, \ldots, 39)\). Three major reasons motivated the use of a moving 10-year period. First, economic time series often are subject to wide fluctuations during short periods of time. The velocity series are no exception, as the data for 1918, 1921, or 1929 attest. The use of \( t + 9 \) observations in the regression equations means that extreme values influence all

7. Meltzer, op. cit. We used data for the entire period in a number of cases. The relative performance of the various hypotheses appears to be altered little by the omission of the years 1941-50.

8. Thus the prediction for 1930 is based on the parameter estimates for 1920-29 inclusive and the actual value of the independent variables in 1930; the prediction for 1953 was obtained by using parameter estimates from a regression for the 10 years 1953-40 plus 1951-52 and the values of the independent variables in 1953. Note that, while the "predictions" are not "forecasts," they are made for values which have not been included in the sample used to estimate the parameters. A description of the sources used for the variables is in the appendix.
Predicting Velocity

predictions that follow the occurrence of what may be a unique event. Second, we are interested here in short-run predictions, based on short-run elasticities or marginal propensities to hold money. Using only 10 observations in the regression equations provides a better approximation to short-run behavior than the use of \( t + 9 \) observations, particularly when \( t \) is large. Finally, it is of interest to compare changes in the predictive performance of various money-demand equations. As noted below, some models perform better in the early years; others more accurately predict the data for more recent years. Maintaining the same number of regression observations facilitates the comparison of predictive performance over time.

III. Predictions from the Keynesian, Friedman, and Naive Models

Efforts to predict the future values of economic variables or to describe their time path often begin by extrapolating from ratios or their time trend. Accountants and businessmen have used such ratios with some success. But economists have learned that predictions based on such methods can be unfortunately wide of the mark. In the absence of a theoretical framework which implies the short-run stability of some ratio, researchers have sought to isolate stable functions with the expectation that stable functional relations would serve as a better guide to policy and prediction.

Developments in monetary theory illustrate this process. Predictions of income or the demand for money based on the assumed short-run constancy of income velocity are subject to relatively large errors. Numerous efforts have been made to isolate a time trend in velocity as a means of improving predictions. Following Keynes, many economists abandoned the notion that the velocity ratio is sufficiently stable in the short run to predict the level of income and substituted an analysis based on the components of aggregate expenditure. To predict or describe the demand for money, these writers assumed "motives for holding money." One portion of the quantity of money was assumed to be described by the constant ratio of "transaction balances" to income, i.e., constant transactions velocity; the remainder became functionally dependent on interest rates. Post-Keynesians provided some empirical estimates that suggested that the functional relation was stable.\(^9\)


Other writers abandoned both the concept of constant velocity and the analysis of time trend. They sought to isolate a stable velocity function dependent on interest rates and/or other variables, for example, income. More recently two studies of the demand for money have emphasized the importance of wealth as a constraint on total money balances. Both imply the stability of a velocity function, but the definitions of velocity used in these models differ from those commonly employed. From one model it follows that the ratio of permanent income to permanent money stock is a stable function of real per capita permanent income. The other—the wealth model—suggests that the ratio of the yield on wealth divided by money stock is dependent on interest rates. Both theories imply a relation between the particular measure of velocity used in the study and velocity as it is commonly measured, and additional variables may be introduced to obtain predictions of the more common measure of velocity—the ratio of measured income to money stock.

Thus theories of money demand have moved from the assumption of constant short-run velocity ratio to the implied stability of a function that predicts or describes only a part of measured velocity. In this section, we compare the predictive performance of several of these theories. A naive model is introduced and used as a measuring rod to discriminate among the hypotheses. In the following section, the wealth model is tested in the same way.

The money demand hypothesis of Keynes's General Theory is discussed first. The general Keynesian function is equation (1), where $M$ is money, $r$ is a rate of interest, $Y$ is money income, and $L_r$ and $L_y$ are the derivatives of the $L$ function with respect to the two arguments.

\[ M = L(r, Y), \quad L_r < 0 < L_y. \]  

(1)

Since Keynes did not specify a unique set of measurement procedures for the variables or properties of the $L$ functions, a number of hypotheses, all of which are consistent with equation (1), have been developed.

One well-known form of the Keynesian money-demand function merges the assumed constancy of velocity from the Cambridge equation with the Keynesian interest-rate hypothesis. The resulting merger divides money into two components by assuming that the demand


function for money is separable into "transactions" \( (M^T) \) and "speculative" \( (L) \) balances. The demand function is then written \( M_1 = M^T + L = kY + L(r) \). Tobin (1947) provided some empirical support for this hypothesis. He assumed that the average cash balance \( k \) could be computed from time-series data by choosing the minimum ratio of \( M_1/Y \), selected 1929 from his data as the year in which velocity reached a maximum, and assumed that there were no "speculative" balances in that year.\(^{13}\)

Tobin's specification of the Keynesian hypothesis is generally written as

\[
L = M_1 - \min \left( \frac{M}{Y} \right) Y = L(r) \tag{2}
\]

Empirical tests of equation (2) have used data for the post–World War I period only. While there has been some discussion of the year in which velocity reached its maximum value, 1929 has generally been taken as the base year or the year in which there were no "speculative" balances. For the longer time series used here, maximum velocity occurs in 1918 and an additional set of predictions has been made for the 1918 base.

Two assumptions about the form of the function have been made. Tobin used a linear relation; others have assumed that equation (2) is linear in the logarithms. The use of logarithms introduces a difficulty for empirical testing, since there are extreme values of the logarithm in years of relatively high or maximum-observed velocity. Extreme values of log \( L \) bias tests of the predictive power of the model, since relatively small absolute errors appear as extremely large percentage errors. Following Eisner, we have attempted to overcome this difficulty by adding a constant term (1.0) to the Tobin form of the Keynesian model.\(^{14}\) We thus obtain two alternative Keynesian-type hypotheses:

\[
\ln (L + 1) = b_1 \ln r + a_1 \tag{K1}
\]

and

\[
L + 1 = b_2 r + a_2 \tag{K2}
\]

13. Eisner \textit{(op. cit.)} and Tobin (1947) \textit{(op. cit.)}, among others, discuss these procedures and some alternatives in greater detail. At times, some other variables, e.g., wealth, are included in the \( L \) function along with interest rates. Note that here and in other relations discussed below, variables without time subscripts are assumed to be dated as of time \( t \).

14. Eisner, \textit{op. cit.} Note that the addition of 1.0 to the non-log equation (K2) is a pure scale effect. In the log form, it distorts the relation, since it is not a pure scale effect. But, as noted, omitting the constant also prevents an unbiased test of equation (K1). The use of the constant term reduces the average absolute per cent error.
The results for these two equations are shown in Table 1, where they may be compared with other demand-for-money hypotheses considered in this section. The predictions from the two hypotheses are extremely poor for the period as a whole and for each of the subperiods, although the use of non-logs and the 1918 base (K2) improves the predictive performance somewhat. Arbitrarily eliminating the most extreme errors further improves the predictive performance, as noted in the footnote to Table 1. But the model is virtually useless as a guide to predicting "speculative" balances, a result which casts doubt on the usefulness of the 1947 Tobin form of the Keynesian hypothesis.

One reason for the comparatively large percentage errors from the (K1) and (K2) equations is the method of computation. As we noted in the previous section, the percentage error for any year is computed by subtracting the actual from the predicted value and dividing by the actual value. For the velocity equations considered below, the computed error is \( \frac{V - V_a}{V_a} \) or \( \frac{V_e}{V_a} - 1 \), where \( V \) and \( V_a \) are predicted and actual velocity, respectively. Assume that the income, \( F \), in the numerator of predicted and actual velocity is the same, then

\[
\frac{V - V_a}{V_a} = \frac{M}{M_a} - 1, \quad (3)
\]

where \( M \) is the demand for money. In equations (K1) and (K2) the percentage error is \( \frac{L_a}{L_a} - 1 \). If we accept one assumption of the 1947 Tobin form of the Keynesian hypothesis, we can take \( M_T \), "transactions balances," as given with certainty. Then equation (K2) becomes (K2'):

\[
\frac{L_a + M_T - (L_a + M_T)}{L_a} = \frac{L_a - L_a}{M_a} = \left( \frac{L_a - L_a}{L_a} \right) \frac{L_a}{M_a}. \quad (K2')
\]

Since \( L_a/M_a < 1 \), this term reduces the error from the (K1) or (K2) equation.

The percentage errors for equation (K2') were computed by dividing the absolute errors from equation (K2) by the actual quantity of money rather than the amount of "speculative balances." The results are also shown in Table 1. The mean absolute per cent error shown there is still slightly higher than the error which would be obtained if the 1947 Tobin hypothesis had been treated like the velocity equations. For comparative purposes, the absolute per cent error for the 39-year period is 7.6 per cent. However, this value must be used with caution. We have

15. From equation (3) of the text, the percentage errors computed from the velocity equations equal \( \frac{M_a}{M} - 1 \) if we assume that numerators of predicted and actual velocity are the same. From equation (K2') of the text, we know that \( \frac{L_a - L_a}{M_a} = \frac{M_T - M_T}{M_a} \) if we again assume that \( M_T \) is known exactly. (The \( e \) and \( a \) subscripts of course refer to
shown above that the mean absolute per cent error from equation (K2) equals \( \frac{L_a}{M_a} \) times the error from equation (K2). For the 39-year period, the ratio of these two errors, i.e., the average value of \( \frac{L}{M} \) is 2/9. Thus the error is computed on the assumption that 7/9 of the demand for money is known with certainty, on the average.

Moreover, the ratio of \( \frac{L}{M} \) for the last 9 years is in the neighborhood of 1/3, while the ratio of \( \frac{L}{M} \) for the decade of the 1930's is approximately 0.37. This suggests that there was little difference in the proportion of the money supply assigned to "transactions" and "speculative" balances in the two periods. But one period, the 1930's, is characterized by falling interest rates, while the last 9 years were principally years of rising interest rates. The fact that we observe little difference in the proportion of money balances assigned to "speculative" and "transaction" components in radically different economic climates casts doubt on one of the central features of Keynesian money-demand theory—the low-level liquidity trap.\(^{16}\)

The data here strongly suggest that the 1947 Tobin form of the Keynesian money-demand hypothesis has relatively poor predictive power. This conclusion holds even if 7/9 of the demand for money is assumed to be known with certainty. These results cast doubt on the analysis of the demand for money based on separate motives for holding money.\(^{17}\)

We turn now to the Keynesian money-demand functions tested by predicted and actual values.) Since \( \frac{(M_a - M_s)}{M_s} = \frac{M_a}{M_s} - 1 \), the absolute per cent error for comparative purposes is, on the assumptions made, \( -\left( \frac{M_a}{M_s} - 1 \right) (\frac{M_a}{M_s} - 1) = -0.082/1.082 = -0.076 \), or 7.6 per cent, since we are considering absolute percentage errors.

16. Equations (K1) and (K2) also cast doubt on the substantive content of the liquidity-trap proposition. True (K1) and (K2) perform their best during the 1930's. But the errors are large and variable. For equation (K1), they range from \(-67\) per cent in 1932 to \(-4\) per cent in 1933 and \(+26\) per cent in 1937. Equation (K2) shows similar wide variations; indeed, extreme variability seems to be characteristic of the predictions made from these two equations during the 39 years. Long-term interest rates fell during the 1930's, and, as noted, the errors are negative in the early years, positive in the later years of the decade. "Speculative" balances were therefore greater than the prediction of the model in the early years of the decade, while in the later years they were less than the predicted amount. This is particularly true for equation (K1). Thus the actual amount of "speculative" balances rose much less rapidly than predicted "speculative" balances. At a minimum, this suggests that the hypothesis overstates the magnitude of the marginal propensity to hold cash balances at low rates of interest. For additional evidence against the liquidity-trap hypothesis cf. Meltzer, "Yet Another Look at the Low Level Liquidity Trap," *Econometrica* (forthcoming).

17. Of course, the error in predicting money demand or velocity from the Keynesian model should not be confused with the error in predicting income from that model. As is well known, the Keynesian model relies on the analysis of expenditures for predicting income.
Latané. He avoided the Tobin-type assumption that the general hypothesis, equation (1), could be tested by means of the equation \( L = L(r) \). Instead, he presented three additional forms of the Keynesian money-demand functions shown as equations (4), (5), and (6).

\[
M = a - b r + c Y, \quad \text{(4)}
\]
\[
M = f(r) Y, \quad \text{(5)}
\]
\[
\frac{1}{r} = g \left( \frac{M}{Y} \right) + h. \quad \text{(6)}
\]

Equation (4) follows from equation (1) above on the assumption that the function is linear. Equation (5) follows from the general Keynesian money-demand function on the assumption that equation (1) is homogeneous of first degree in \( Y \), and equation (6) differs from equation (5) only according to statistical assumptions about the correlation of residuals and independent variables.

Latané devoted most of his attention to equation (5) expressed as a velocity equation. We have modified his definition of income from GNP to NNP to obtain data for the years 1900-1909 and to make the predictions from his model comparable to the other velocity relations. Our predictions from his model are obtained from equation (K3). Following Latané, we assume that (K3) is linear in the logarithms:

\[
\ln V_1 = a + b \ln r. \quad \text{(K3)}
\]

Although Latané did not explicitly do so, his assumption that the Keynesian money-demand equation is homogeneous of first degree in money income can be tested explicitly. From the demand-for-money equation \( M = a r^{c} Y \), the corresponding velocity equation is \( V = (1/a) r^{c} Y^{(1-c)} \). When the income elasticity of the demand function for money, \( c \), equals 1, i.e., when the demand function is homogeneous of first degree in \( Y \), the velocity equation becomes (K3). Some earlier results suggest that the income elasticity is unity in the long run. This supports the implicit assumption that Latané made. However, we tested both (K3) and (K4) and obtained the results shown in Table 1:

\[
\ln V_1 = a + b \ln r + c \ln y. \quad \text{(K4)}
\]

18. Latané, op. cit.

19. The long-term results are heavily dependent on the co-movement of bond rates and velocity after 1929. The results for earlier years show a relation that is much less stable and, in particular, one in which the parameters change in about 1929. Prior to that time, income is the major and often the only significant explanatory variable. After 1929, bond rates played an important independent role. No explanation of this change in behavior has been provided. Cf. Meltzer, "... The Evidence from the Time Series," op. cit. We consider the relation of equation (K3) to the wealth model in a following section.
La**tane rejected the linear hypothesis, equation (4), on the grounds that it implies

$$\frac{M}{V} = a - b Y + c,$$  

(7)

or, as a velocity equation in real terms,

$$V = \frac{Y}{a - b r + c y}. \quad (7')$$

Latané interpreted (7) or (7') as showing that a rising income level and a constant rate of interest lowers average cash balance, $M/Y$, or raises velocity in the long run. Since he argued that a positive long-run association of income and velocity has not been observed, he conducted no further test of the linear hypothesis. In fact, equation (7') does not necessarily imply a positive relation between income and velocity. Since the partial derivative of $V$ with respect to $y$ equals $(a - b r)1/M^2$, the signs and magnitudes of the parameters $a$ and $b$ are required to evaluate the sign of the derivative. We, therefore, test the linear Keynesian equation (K5), using the demand for real balances to avoid the complicated velocity relation shown in equation (7'):

$$\frac{M}{\bar{p}} = a - br + cy. \quad (K5)$$

Equation (K5) predicts with smaller average per cent absolute error and RMS than any of the other Keynesian-type hypotheses. However, the data support Latané's reason for rejecting the linear hypothesis. The partial derivative of $V$ with respect to $y$, $V_y = (a - b r)1/M^2$, is positive in 35 of the 39 years. In 37 of the 39 years, $a$ is positive and ranges in value from 0.05 in 1910 to 48.5 in 1937. Moreover, the interest-rate coefficient, $-b$, is positive in every year 1910–13 and 1915–27 inclusive. Thereafter, $b$ is always negative. This suggests the omission of a significant factor from the demand function or a structural change in the function in 1928 that is difficult to explain. Still, unsatisfactory results and false implications are not sufficient by themselves to dispose of a hypothesis. Alternative hypotheses may perform even worse, and, as we have seen, the (K5) equation predicts the demand for money more accurately than any of the other equations we have tested that are based on the general Keynesian hypothesis, equation (1).

The results for equations (K3) and (K4) support Latané's implicit assumption that the log form of the Keynesian model is homogeneous of first degree in income, since (K3) generally predicts with smaller error than (K4). Moreover, they support the choice of $M_1$ as the
definition of money balances appropriate for the demand function. When the denominator of velocity includes time deposits at commercial banks, the mean and RMS absolute per cent error for that version of equation (K3) become 9.6 and 9.3, respectively. The results for (K4) are even poorer when the more inclusive definition of money is used. This suggests the tentative conclusion that $M_1$ is the more appropriate definition of money for predicting velocity, but we shall continue to use the two definitions to obtain additional evidence.

Equation (K4) is the dividing line between those demand functions for money that predict income velocity worse and those that predict at least as well as the "naive" models to be considered presently. The former group consists of models in which measured income enters as the constraint on money balances; they are alternative forms of the general Keynesian hypothesis. The latter group contains those that are constrained by wealth or permanent income plus equation (K5), which contains false implications, as we have seen. The meaning of these results seems clear: Measured income alone is not the proper constraint in the demand function for money. We shall reopen this discussion in Section V below.

Friedman has suggested one demand-for-money hypothesis that contains permanent or "theoretical" income rather than measured income. The particular form of his equation is

$$\frac{M_2}{NP} = b\left(\frac{Y_p}{NP}\right)^a,$$

where total real money balances inclusive of time deposits at commercial banks ($M_2$) and the money value of permanent income ($Y_p$) are deflated by population ($N$) and permanent prices ($P_p$). From this he obtains an equation for permanent velocity ($V_{sp}$) associated with total money balances analogous to the one shown here as equation (F1):

$$V_{sp} = \frac{1}{b} \left(\frac{Y_p}{N}\right)^{-a}. \quad (F1)$$

This velocity equation was used to test the predictive power of the demand-for-money model which Friedman has proposed. The data in Table 1 show that equation (F1) has a smaller mean and root-mean square absolute per cent error than any of the money demand functions (K1)–(K4) considered above. But we do not know whether this improvement results from the definition of money balances, which in this case includes time deposits at commercial banks, from a more appropriate specification of the demand function for money, or from both.

Moreover, permanent money balances and permanent velocity do not jointly predict measured income, the concept with which economists and policy makers are most often concerned. To obtain predictions of
Predicting Velocity

measured income and measured money balances from Friedman's model, we derive equation (F2):  

\[ V_1 = \frac{Y}{M_1} = \frac{y}{y_p} \frac{P}{P_p} \frac{1}{b} \left( \frac{y_p}{N} \right)^{1-a}. \]  

(F2)

When equation (F2) is used to predict measured velocity, it performs with lower average absolute per cent error and lower RMS than any of the equations that we have discussed heretofore. And it predicts measured velocity more accurately than (F1) predicts permanent velocity, perhaps reflecting only that permanent velocity is a long-run concept and is not well defined by the short-run procedures used here.

The results for the permanent-income model suggest that the short-run response of measured velocity to variables like transitory income or transitory prices is more predictable than the short-run response to measured income itself. When time deposits at commercial banks are excluded from the definition of money and velocity, the results are poorer on the average and somewhat more variable (eq. [F3]):

\[ V_1 = \frac{Y}{M_1} = \frac{y}{y_p} \frac{P}{P_p} \frac{1}{b} \left( \frac{y_p}{N} \right)^{1-a}. \]  

(F3)

Thus the evidence seems to indicate that the \( M_3 \) function is a more stable function than the \( M_1 \), in marked contrast to the earlier result when the two definitions of money were used in the (K3) and (K4) equations. Further evidence is therefore needed to discriminate between the two definitions of money and to examine the extent to which the results for Friedman’s model reflect the difference in the constraint imposed on money balances rather than the difference in the definition of money itself.

The data do suggest that the Friedman-type equation (F2) is a more useful approximation than any of the five Keynesian models that we have examined. Nevertheless, the error in predicting velocity from (F2) is sufficiently large that it would fail to distinguish between predictions of prosperity or recession in a given year. The Friedman-type equations are helpful to policy-makers, however, since both (F2) and (F3) have smaller average errors than the “naïve” equations which we use as a measuring rod:

\[ \frac{Y}{M_1} = V_{1, t} = N \left( V_{1, t-1} \right) \]  

(N1)

or

\[ \frac{Y}{M_2} = V_{2, t} = N \left( V_{2, t-1} \right). \]  

(N2)

20. Friedman, op. cit., p. 336. Definitions of symbols for this equation: \( Y \) = money net national product, \( M_3 \) = total money balances inclusive of time deposits at commercial banks, \( y \) = real net national product, \( P \) = national product deflator, \( P_p \) = permanent prices, \( y_p/N \) = per capita permanent income, \( a \) and \( b \) are parameters.
Both (N1) and (N2) were estimated in log form following the procedures described earlier. The results require little additional comment. We note that (N2) is more variable in our test than (N1), a fact which suggests that the (F2) equation predicts a measure of velocity that is subject to greater short-run variation. But more accurate predictions of velocity are within the scope of monetary theory, as the following section indicates.

### TABLE 1
**PREDICTIVE PERFORMANCE OF SOME ALTERNATIVE MONEY-DEMAND FUNCTIONS**
1910–40 and 1951–58

<table>
<thead>
<tr>
<th>MONEY-DEMAND FUNCTION</th>
<th>TYPE</th>
<th>MEAN ABSOLUTE PER CENT ERRORS</th>
<th>ROOT MEAN SQUARE ABSOLUTE PER CENT ERROR, 39 YEARS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>39 Years</td>
<td>First 10 Years</td>
</tr>
<tr>
<td>(K1)</td>
<td>Base 1929 logs</td>
<td>94.5*</td>
<td>225.2</td>
</tr>
<tr>
<td>(K2)</td>
<td>Base 1918 non-logs</td>
<td>36.4†</td>
<td>48.9</td>
</tr>
<tr>
<td>(K3)</td>
<td>Non-logs</td>
<td>8.2‡</td>
<td>5.3</td>
</tr>
<tr>
<td>(K4)</td>
<td>Logs</td>
<td>8.0</td>
<td>8.0</td>
</tr>
<tr>
<td>(K5)</td>
<td>Logs</td>
<td>9.8</td>
<td>7.9</td>
</tr>
<tr>
<td>(F1)</td>
<td>Logs</td>
<td>5.7§</td>
<td>4.5</td>
</tr>
<tr>
<td>(F2)</td>
<td>Logs</td>
<td>7.0</td>
<td>5.1</td>
</tr>
<tr>
<td>(F3)</td>
<td>Logs</td>
<td>5.2</td>
<td>4.3</td>
</tr>
<tr>
<td>(N1)</td>
<td>Logs</td>
<td>6.2</td>
<td>4.6</td>
</tr>
<tr>
<td>(N2)</td>
<td>Logs</td>
<td>7.0</td>
<td>8.5†</td>
</tr>
</tbody>
</table>

* Omitting 8 years with errors > 70 per cent reduces average to 35.5 per cent.
† Omitting 3 highest values, 1917, 1918, and 1922, this falls to 16.4 per cent.
‡ 7.6 per cent for comparative purposes. See text and n. 15.
§ 5.4 per cent for comparative purposes.
† Based on 38 years only; 1910 prediction omitted due to lagged value.
# Based on 9 years; 1910 omitted due to lagged value.

**Note:** (K1) are Keynesian-type equations; (F1) are Friedman-type equations; (N1) are "naïve" models.

### IV. PREDICTIONS FROM THE WEALTH MODEL

The results of the previous section raise three important questions for monetary theory. First, why do the Keynesian demand-for-money equations fail to predict income velocity as well as the "naïve" models? Second, why does the interest rate alone (K3) fail to explain the short-run behavior of velocity, as Latané has maintained? Third, why does the permanent income theory predict better than both the naïve and the Keynesian models? These questions are of importance for theory and policy. This and the following section attempt to explore them in
detail after we present the findings based on our model of short-run velocity.

Consider the wealth model of the demand function for money,\(^2\) where
\[
M = f(r^*, \rho, d, W_n) \quad f_{r^*}, f_d < 0 > f_{W_n}
\] (8)
is the general money demand hypothesis, and the \(f_j\) specifies the signs
of the derivatives for each argument of the function. Assume that (8)
is homogeneous of first degree in \(W_n\), to get
\[
M = f^* (r^*, \rho, d, 1)W_n.
\] (9)

In the long run, expected and actual income are equal. In the short run,
differences between expected and actual income introduce variations in
the ratio \(d\) which raise or lower the demand for money. We expect that,
apart from changes in expectations, \(d\) changes slowly. Moreover, we do
not know an adequate or even a reasonably approximate measure of \(d\)
or \(W_n\). We therefore multiply \(d = \frac{Y_h}{W_h}\) by \(Y_h/\bar{Y} = 1\) and rearrange
terms in
\[
M = g^* \left( r^*, \rho, D \left( \frac{Y_h}{\bar{Y}} \frac{Y^*_h}{\bar{W}_h} \right) \right) W_n.
\] (10)

Further, we assume that the covariance of \(r^*\) and \(\rho\) is sufficiently high
that they can be combined in a single measure, denoted \(r^*\). Finally,
we make explicit in (11) our inability to measure the ratio \(Y^*_h/W_h\)
by assuming that it may be taken as a given:
\[
M = g^* \left( r, \frac{Y_h}{\bar{Y}} \frac{Y^*_h}{\bar{W}_h} \right) W_n.
\] (11)
The derivatives of \(g^*\) with respect to \(r\) and \(Y_h/\bar{Y}\) are negative; with
respect to \(W_n\), the derivative is positive. These signs follow from the
specification of the signs of the derivatives in equation (8).

In the long run, the ratio of expected to actual human income equals

21. Meltzer, op. cit., contains a discussion of the long-run model and several tests of
the proposition that the elasticity of \(M\) with respect to \(W_n\) is unity. As we noted earlier,
this paper attempts to provide additional evidence about the relative performance of a
variety of demand-for-money equations. Therefore we do not test the wealth elasticity
directly here. The definitions of the symbols to be used here are \(M = \) money; \(Y_h\) the money
value of human income; \(Y^*_h\) the expected value of \(Y_h\); \(W_h\) = the value of human wealth;
\(W_n = \) the value of non-human wealth defined as the value of wealth from Goldsmith’s
\textit{A Study of Saving}, Vol. III, Table W-1, minus government-owned assets plus government
liabilities to households and business; \(r^* = \) interest rates on financial assets; \(\rho = \) interest
rates on real capital; and \(d = \) the discount rate applicable to human wealth.

22. Ibid., contains a detailed discussion.
The long-run wealth model of the demand for money in ratio form therefore becomes

\[ \frac{M_1}{W_n} = h(r) \quad \text{or} \quad \ln \frac{M_1}{W_n} = a - b \ln r \tag{W1} \]

when money is defined as the sum of demand deposits and currency. The long-run velocity relation based on equation (11) is

\[ \frac{rW_n}{M_1} = H(r) \quad \text{or} \quad \ln \frac{rW_n}{M_1} = -a + (1 + b) \ln r, \tag{W2} \]

where the income from non-human wealth is used as the numerator of the velocity relation in the long run.

But, again, we are concerned primarily with the short-run model in this paper. We therefore test our short-run demand function for money in ratio form. In the short run, \( Y_h/Y_p \) need not equal unity; a measure of this ratio is required for the demand function. \( Y/Y_p \) is introduced to measure \( Y_h/Y_p \), since there are no adequate measures of human income which date back to 1900. Equation (W3) is assumed to be linear in the logarithms:

\[ \frac{M_1}{W_n} = h^* \left( r, \frac{Y}{Y_p} \right). \tag{W3} \]

To predict measured velocity or measured income rather than the short- or long-run demand for money, equation (W3) must be modified. Measured velocity is the ratio \( Y/M_1 \). But

\[ \frac{Y}{M_1} = \frac{Y}{W_n} \cdot \frac{W_n}{M_1} = U \left( r, \frac{Y}{Y_p} \right) \frac{Y}{W_n} = U \left( r, \frac{Y}{Y_p}, \frac{P_s}{P_s} \right) \frac{Y}{W_n}. \tag{W4} \]

The wealth model views measured velocity as composed of two principal elements. The first isolates a long-run functional relation expressed in equation (W2); this may be called "permanent velocity." It reflects the long-run desire to hold part of wealth in the form of money. Superimposed on permanent velocity is the response to essentially short-run events which affect the short-run demand for money through \( Y_h/Y_p \).

Like the Friedman model, the wealth hypothesis uses a definition of long-run velocity that differs from the conventional ratio of income to money stock. A comparison of the predictive performance of eq. (W1) or (W2) in Table 2 with the long-run Friedman equation (F1) in Table 1 shows that the long-run wealth model predicts more accurately than the permanent-income model and has substantially smaller RMS. Indeed (W1) and (W2) perform substantially better than any of the money-demand functions that we have considered heretofore. In particular, (W2) predicts better than (K3), the Latané form of the Keynesian hypothesis. Both (K3) and (W2) depend solely on interest rates, and, of course, the same measure of interest rates was used in both equations. We interpret this result as strong evidence for our hypothesis that only a part of measured velocity, the ratio \( rW_n/M_1 \), is a stable function of interest rates alone.
Predicting Velocity

(or \( Y/Y_s \)) in equation (W3) and affect measured velocity through \( Y_p^e/Y_s \) (or \( Y_s^e/Y \)) and \( Y/W_n \) in equation (W4).

Thus the wealth model itself specifies that additional variables are required for an analysis of the short-run behavior of velocity. These variables have economic meaning. The first \( Y_p^e/Y_s \) is the ratio of expected to actual human income, i.e., is an index of transitory human income inverted. Measuring this index by the ratio of actual to expected net national product, \( Y/Y_p \), makes the index above 1 when transitory income \( Y_T \) is positive and conversely. We have chosen in equation (W4) to distribute the effect of transitory income in two inverted indexes—an index of real transitory income and an index of transitory prices. Since \( d \) is negatively related to \( M \) in our hypothesis (eq. [8]), \( M \) is negatively related to \( Y_s/Y_p^e \) and \( Y/Y_p \). An increase in the index of transitory income will lower the demand for money and increase velocity in the short run if our hypothesis about the sign of this derivative is confirmed.

The second new variable which enters the short-run velocity equation (W4) is the ratio of \( Y/W_n \), the ratio of net national product to net non-human wealth of households and business. This ratio is a crude measure of the rate at which the stock of wealth generates income—i.e., the capitalization rate applicable to income. We shall denote this ratio as \( \rho \) and use it as a measure, albeit a crude one, of the rate of return on physical assets; \( \rho \) (and \( \tau \)) have a negative effect on the demand for money, a positive partial derivative in equation (W4).

The results for measured velocity, equation (W4), support the wealth model. The predictions obtained from this equation are substantially closer to the actual values of measured velocity and are much less variable than any of those in Table 1. This is particularly true of the predictions for the most recent nine years—1940 plus 1951–58. Recall that our procedure bases the parameter estimates used in the predictions for the 1950's on the data for the 1930's, at least in part. This alone suggests that the demand function for money based on the wealth model has remained remarkably stable.

When \( M_t \), money exclusive of time deposits, was replaced by \( M_2 \) (inclusive of time deposits) in equations (W1)–(W4), the errors were larger on the average and much more variable. The earlier long-run study indicates that \( M_2 \) (unlike \( M_1 \)) is a luxury; the elasticity of real non-human wealth, \( W_n/\rho \), is greater than 1 in the long-run \( M_2/\rho \) regression and positive in the \( rW_n/M_2 \) regression. \( W_n/\rho \) is therefore included as an argument in the short-run demand function for \( M_2/W_n \), the analogue to equation (W3). The mean absolute error rose to 7.2, and the RMS became 12.2. These are substantially greater than the error and RMS from the Friedman-type equation (F2), which uses \( M_2 \) as a definition of money.
A number of questions are posed by this result and the comparison of the predictive performance of the wealth model and the Friedman model for the two definitions of money. We note that the arguments of (W4) and (F2) differ only in that (W4) contains \( r \) and a measure of \( \rho \), while (F2) uses \( \gamma p/N \). Further, we note that the Friedman-type equation (F3) does better than the corresponding (W4) equation when both are used to predict \( V_2 \), but (W4) predicts better than (F2) when velocity is defined as \( V_1 \).

Thus the differences in predictive performance between the best of the Friedman-type equations (F2) and the wealth equation (W4) raise three questions about the rival hypotheses. First, do interest rates play a direct and important role in predicting short-run velocity or demand for money as the wealth hypothesis implies and the Friedman model denies? Second, is \( M_1 \) or \( M_2 \) a more predictable variable for the demand function for money? Third, do important differences in predictive performance occur if \( \gamma p/N \) replaces \( Y/W \) in the \( M_1 \) function or \( Y/W \) and \( W_n/p \) in the \( M_2 \) function? Evidence on the answers to these three questions can be obtained by adding an interest-rate variable to equations (F2) and (F3). The modified equations are denoted (WF2) and (WF3) in Table 2, since they can be regarded as a variant of equation (W4) of the wealth model with \( Y/W \) and \( W_n/p \) replaced by \( \gamma p/N \) when money is measured as \( M_2 \) and \( Y/W \) replaced by \( \gamma p/N \) when money is defined as \( M_1 \).

24. Specifically, \( M_1 \) is used with (WF2); \( M_1 \) is used in (WF3). The equations are:

\[
V_t = v_t (y / \gamma p, p_t / p_{pt}, r, \gamma p/N).
\]
The evidence is slightly mixed but, on the whole, supports the wealth model and suggests the following conclusions:

1. Including interest rates in (WF2) and (WF3) substantially reduces both the mean and the RMS absolute per cent error below those obtained in the Friedman-type equations (F2) and (F3). Clearly, the inclusion of interest rates improves the predictive performance of the Friedman model for either definition of money. This suggests that interest rates play an important, independent role in the prediction of short-run measured velocity and income, a conclusion which Friedman has all but completely denied. Since the wealth hypothesis specifies the inclusion of interest rates in the demand function for money, the evidence supports the wealth model with respect to this variable.

2. We find that demand functions for money defined as \( M_1 \) are subject to smaller errors of prediction than those in which money is defined as \( M_s \). A comparison of (WF2) and (WF3) shows that this conclusion holds when the permanent-income model is modified to include an interest-rate variable, just as it does for other demand-for-money equations we have considered. The only contrary evidence comes from a comparison of (F2) and (F3). Including interest rates in (F2) and (F3) reverses the previous result with respect to the definition of \( M \).

The interpretation seems clear. To predict \( M_s \) balances more accurately than in (WF2), the positive influence on the amount of time-deposit balances of increase in rates paid on time deposits must be separated from the negative influence exerted by an increase in alternative yields. That is, general and relative changes in interest rates must be separated. The absence of an adequate series on time-deposit rates for the whole period prevents further analysis here. But the results do show that a more stable short-run demand function has been developed for money defined as \( M_1 \).

3. When money is defined as \( M_1 \) and \( Y/W \) is replaced by \( y_p/N \) in equation (W4) to get (WF3), the results are mixed but, in general, support the wealth model. It is true that in some periods (WF3) predicts better than (W4) by our test and that, on the average, the absolute per cent error is slightly smaller when per capita permanent income replaces \( Y/W \) as a variable. But the RMS is considerably larger for (WF3) than for (W4). Moreover, we have found no other equation which predicts as well as the short-run money demand function in ratio form (W3). Equation (W3), following our hypothesis, contains neither \( Y/W \) nor \( y_p/N \). We interpret the comparison of (W3), (W4), and (WF3) as confirmation that \( Y/W \) is a relatively poor measure of \( \rho \), since it lowers the predictive power of (W4) relative to (W3), on the

average. This suggests that errors of measurement and the effects of variables like \( \frac{Y_1/W_4}{W_4} \) which have been excluded may be assigned to \( Y/W_4 \) in equation (W4) and to \( y_p/N \) in equation (WF3).

The mean elasticity of \( V_1 \) with respect to \( y_p/N \) in (WF3) is positive, on the average, and in 29 of the 39 periods (see appendix). This is directly contrary to the Friedman hypothesis and supports the interpretation that we have just given. For the positive elasticity of \( V \) with respect to \( y_p/N \) suggests that a rise in \( y_p/N \) lowers the demand for money, other things equal, a result which is difficult to accept if \( y_p/N \) is a measure of wealth or permanent income. We shall consider the role of \( y_p/N \) in the demand function for money in Section V and will attempt a further explanation of the differences in the wealth and permanent-income model there.

One additional task remains in this section. We want to show the relation between predicted velocity and predicted income. Following the assumption made earlier, the money-supply function is known. We can therefore multiply the predicted value for velocity given by equation (W4) by the actual money stock in the year to obtain predictions of NNP. The percentage error in predicting income is, of course, the same as the percentage error in predicting velocity if \( M \) is assumed to be known. But the income predictions are of interest for the role that they assign to velocity in business cycles and for the close relation between business cycles and monetary changes which they reflect. The income predictions therefore contribute to our understanding of the extent to which the demand and supply functions for money can be used to predict income accurately—a basic contention of those who believe that "money matters."

Chart 1 shows the predicted and actual values of NNP for 39 years. Although the predictions are somewhat more volatile than actual income, note that the turning points match in all but a few of the years in which income changed direction. Comparison of the changes in the predicted value and the changes in actual income shows that changes in both were in the same direction in all but three years (1910, 1915, 1926. Note that including interest rates in the (F2) equation also reduces the mean absolute error obtained. As we have stated in the text, there is a confusion between general and relative changes in interest rates when money is defined as \( M_s \). Moreover, the computed elasticity of \( V_3 \) with respect to \( y_p/N \) remains non-negative, contrary to the Friedman hypothesis (see appendix). We therefore attempt no further explanation of the relative roles of \( y_p/N \), \( Y/W_n \), and \( W_d/p \) in the short-run \( M_s \) equations. However, we will note in Sec. V that it is quite consistent with our present view of the demand function for money that the factors summarized by \( y_p/N \) have a different effect on the \( M_s \) function than on the \( M_t \) function.

27. For a discussion of money supply theory see Brunner, op. cit.
and 1924). Thus the changes in the predicted series correspond to the changes in the actual series at most of the turning points for which the analysis has been made. Equally important, the predicted series does not indicate turning points which did not occur. This close correspondence between actual and predicted values at turning points increases our confidence in the underlying demand function for money and suggests the importance of money, asset yields, and the transitory variables in the explanation of cyclical behavior.

CHART 1
ACTUAL AND "PREDICTED" NET NATIONAL PRODUCT
1910-40, 1951-58

The predictions in Chart 1 are synchronous, like those we have discussed throughout this paper. To forecast, i.e., to predict future values of the variables, lagged values must be introduced into equation (W4). Specifically, the parameters of the moving 10-year regressions were re-estimated with values of yr, p, r, and r lagged one year. The results of these predictions of income are shown in Table 3, where they are compared with the actual values of income and the values predicted from the synchronous or non-lagged equation for the years 1952-58.

The predictions from the lagged model correspond to income reasonably well. While the lagged equation does not predict the negative change in income in 1954, the prediction for that year would have
suggested the presence of a recession. And there is no reason to believe that a lag of one year is appropriate.

It is of interest to compare briefly the results from the lagged and the untagged model for the implications that they contain. Using the model without lags implies that the desired stock of money is always equal to the actual stock. A one-year lag suggests that the money supply and the desired demand for money adjust slowly. Since the introduction of a one-year lag lowers the predictive power of the model,

<table>
<thead>
<tr>
<th>YEAR</th>
<th>ACTUAL Yt</th>
<th>PREDICTED Yt—EQUATION WITHOUT LAGS</th>
<th>PREDICTED Yt—LAGS IN THE EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amount $ Billions</td>
<td>Annual Change</td>
<td>Amount $ Billions</td>
</tr>
<tr>
<td>1953</td>
<td>323.0</td>
<td>16.0</td>
<td>336.0</td>
</tr>
<tr>
<td>1954</td>
<td>338.9</td>
<td>15.9</td>
<td>344.3</td>
</tr>
<tr>
<td>1955</td>
<td>334.3</td>
<td>-4.6</td>
<td>333.3</td>
</tr>
<tr>
<td>1956</td>
<td>365.5</td>
<td>31.2</td>
<td>351.1</td>
</tr>
<tr>
<td>1957</td>
<td>384.8</td>
<td>19.3</td>
<td>365.4</td>
</tr>
<tr>
<td>1958</td>
<td>405.9</td>
<td>0.6</td>
<td>403.9</td>
</tr>
</tbody>
</table>

the results suggest that shorter lags would improve our ability to predict economic events. To do so, however, quarterly estimates of $V/W_n$, and the transitory variables are required. These are not yet available. But the power of the model to predict is sufficient to indicate that compiling such estimates would be a useful undertaking.28

28. We have been urged to compare the predictive performance of our model with the forecasting performance reported in D. B. Suits, "Forecasting with an Econometric Model," American Economic Review, March, 1962. We think such comparison is unnecessary. Suits' model is designed solely for forecasting. Many of the equations depend heavily on the lagged value of the variable which is being forecast and a constant term. The constant term frequently dominates the forecast, contrary to Suits' hypothesis. We are convinced that Suits' procedure may be a useful forecasting device. But its relation to economic theory is at best tangential. Our approach is one of basing predictions on economic theory, for we are convinced that useful predictions can be obtained by that route. Meanwhile it is helpful for policy makers to have reasonably accurate forecasts from models of the type which Suits has used. As is well known, such models must be used with caution. Moreover,
Predicting Velocity

We have now compared the predictive performance of almost 20 short- and long-run demand functions for money. We could continue to compute error and RMS values for additional velocity equations, but the results are sufficiently clear for the Keynesian, Friedman, and wealth models to permit a summary of our findings for some of the issues which we are investigating.

In the introduction, four problems in monetary theory and macroeconomics were presented. The tests which we have conducted provide some answers to these questions and, of course, pose new questions, like those at the start of this section. In particular, the evidence which we have accumulated seems to show the following: (1) The demand function for money is more stable when it is subject to a wealth constraint than when it is constrained by measured or permanent income. (2) The most stable demand function that we have found, the wealth model, has as arguments yields, and non-human wealth. This demand function implies that short-run velocity depends on asset yields and two "transitory" variables—the ratio of expected to actual human income and the ratio of expected to actual prices. (3) Money matters. Measured income has been shown to respond closely to the demand and supply for money. We know of no other model of income determination that predicts income as accurately as the money supply and the wealth model of the demand for money for as long a period and under as many changes in social, political, and economic institutions. (4) Predictive performance of the wealth model, as well as the Keynesian and the modified Friedman model, is improved when money is defined as the sum of currency plus demand deposits. Hence $M_1$ seems to be the appropriate definition of money for monetary theory. Broader definitions predict velocity or income less accurately, presumably because they mix general and relative changes in interest rates.

Finally, the results of our theoretical and empirical analysis have indicated some responses to the questions at the start of this section. The following section will attempt to evaluate the evidence on the several money-demand functions and to explain why the wealth model is more stable and more useful as a tool for predicting velocity.

we have just noted that shorter lags and more frequent observations are required to test the forecasting power of our model adequately. Nevertheless, for those who wish to see the comparison, it is provided. On the average, for the seven years 1952-58 our lagged model made an average absolute error of 2.24 per cent in forecasting velocity or net national product. The Suits model predicts GNP. For the seven years 1953-59, his model had an average absolute error of 1.77 per cent. The largest error made by Suits is greater than any of the errors from the lagged velocity model. Both models do well in 1953, 1954, and 1958, and both do poorly in 1955. The Suits model clearly forecasts better than the lagged wealth model in 1956 and 1957.
V. IMPLICATIONS FOR MONETARY THEORY

Historically it has been assumed that money is held principally to effect transactions. The usual formulation of the transactions approach subjects the demand function for money to an income constraint. The results above cast doubt on the use of an income constraint, since they show that the explicit or implicit use of the income constraint introduces substantial error into the prediction of the demand for money or velocity. The question arises: If $W$ is the capitalized value of $Y$, how can a model based on $W$ describe the short- and long-run behavior of $M$ or $V$ better than those based on $Y$?

One answer is that there can be no difference other than error of measurement if income is measured as the yield on wealth or if wealth is the capitalized value of income. This answer suggests that income is measured inappropriately in the national accounts, since we have seen that both the permanent income and the wealth models have smaller errors in prediction than the Keynesian demand functions. But the measure of wealth which we have used is non-human wealth, $W_n$. Clearly, this measure differs in magnitude from total wealth, the capitalized value of income or permanent income.

A second answer is that $W_n$ and total income move together, that the ratio of non-human to human income ($Y_n/Y_h$) is constant, or that changes in the ratio $Y_n/Y_h$ are offset by changes in the capitalization rates applicable to the two streams of income. If the ratio $Y_n/Y_h$ is constant, it would matter little whether we chose total income or $W_n$ as the constraint in the demand-for-money equation. In fact it would be difficult to separate the two hypotheses. But clearly in the short-run there is sufficient variability in the ratio of wage to non-wage income that we would expect money-demand functions based on $Y$ or $Y_p$ to perform better than those based on $W_n$ if, in fact, human wealth is an important part of the constraint on money balances. Put otherwise, we would expect that the error caused by the omission of human wealth, $W_n$—the procedure we have followed—would distort rather than improve the short-run relation. But we observe the opposite.

In our hypothesis, no direct long-run role is given to the income variable. Equations (W1) and (W2) are dependent solely on interest rates. Our short-run demand function for money (W3) includes the income variable in ratio form only. The data in the appendix assign only a small positive elasticity to this variable in the velocity equation (W4). Hence the transitory income ratio has a negative effect on the short-run demand for money, as our hypothesis implies. In the modified Friedman model, equations (WF2) and (WF3), the elasticity of per capita perma-
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nent income is positive in the velocity equation, hence negative in the demand functions for money, and contrary to the Friedman hypothesis, as we noted earlier. None of these results suggest an important role for human income or human wealth in the demand function for money.

Let us therefore investigate the arguments advanced by those who use $Y$ or $y$ as a constraint. The most commonly advanced rationale for the importance of $Y$ or $Y_1$ in the demand function for money is the “transactions” argument, as we have noted. This is a major point in the “motives” analysis, in the view that a primary motive for holding money is to maintain a technologically fixed transactions balance as a constant proportion of income. Our results cast doubt on the usefulness of studying the demand function for money in terms of separate motives.

But if equation (2), the fundamental equation for those who view transactions balances as technologically fixed, is applied to current money balances, using either the 1918 or 1929 minimum ratio of $M/Y$, approximately 30 per cent of current net national product is held in transaction balances. At the end of 1961, NNP was approximately $500 billion. Desired “transaction” balances would be $150 billion from equation (2). Total demand deposits and currency holdings of the public at the end of 1961 were about $140 billion. Desired “transactions” balances were almost $10 billion more than the money stock $M_1$. The fact that “desired transaction” balances exceed “actual transaction” balances is inconsistent with the basic assumption that “transaction” balances are technologically fixed.

Let us attempt an alternative approach to the analysis of the demand for money in terms of the technologically fixed “transactions” motive. At the end of 1961, disposable income was $375 billion. Households held about $60 billion of the $140 billion in total currency and demand deposits. The ratio of household balances to household receipts (disposable income) was 1/6, suggesting that households have about 2 months’ income in the form of money, on the average. If these balances are regarded as largely held for “transactions,” they suggest a substantial technological lag between the receipt and the expenditure of money.

Given the payments schedules for most workers, the lag of expenditure behind the receipt of money cannot be more than $\frac{1}{4}$ week, on the average. Allowing for semimonthly, monthly, and quarterly income receipts probably does not raise the weighted average cash balance from this source much beyond the income from 1 week, i.e., 2 or 3 per

29. It may appear that this result is inconsistent with our earlier statement that, in the predictions for the last nine years, the ratio of $L/M$ was 1/3. There is no inconsistency. Since 1958, the money supply has grown more slowly than NNP.
cent of disposable income. In any case the technologically fixed lag is less than the 2-month average cash balance. 30

The wealth model, equation (8), on the other hand, views the demand for money in terms of costs and yields. Suppose that the public adjusts each of the assets in its balance sheet position in response to relative prices—costs and yields—to achieve a desired balance-sheet position. Current output or the volume of income transactions then emerges as part of a general wealth-adjustment process. 31

It is by no means the same to state (1) that both transactions volume and money balances depend on cost and yield factors and to state (2) that money balances directly depend on transactions volume. In the latter statement there is usually assumed to be a fixed short-run relation between $M$ and $Y$; only long-run changes in tastes and technology alter the proportion of income held as money. The former statement suggests that there is short-run variation in the ratio of $M$ to $Y$ precisely because $M$ and $Y$ both change in response to cost and yield factors or relative prices. In statement 1, transactions volume and the demand for money are viewed as jointly determined by cost and yield factors together with wealth position. These entities imply a desired payments schedule suitably reflected by a desired money balance. In long-run equilibrium, adjustments have been made so that desired and actual payments schedules are equal, and desired money balances equal actual money balances. Cost and yield factors have operated to obtain the long-run adjustment; hence it would not be surprising to find that, in the long run, the relation of $M$ to $Y$ would appear to be constant.

It should be clear that our denial of the role of income rather than wealth as the constraint on money balances and of the “transactions” motive for holding money is not a denial of the relation between income and money balances. 32 Rather it is a reformulation of the way in which actual human income is related to the demand function for money. One effect of $Y_h$ or $y_h$ comes through $d$; another effect of human income on the demand for money comes through $Y^* _h$ (or $y^*_h$). If an increase in


32. A regression of ln $M/p = a + b \ln Y/p + c \ln W/p + d \ln r$ for the years 1900-1958 is reported in Meltzer, *op. cit.* The estimate of $b$ is 0.13; the estimate of $c$ is 0.9. (This footnote was added to respond to a comment of one of the discussants.)
actual human income leads to an expectation of greater increases, $V_\tau$ and $W_\lambda$ rise relative to $Y_h$, and the quantity of money demanded falls.

Turning from the general money-demand hypothesis, equations (8)-(11), to the specific short-run velocity equation (W4), we note that income variables directly affect the equation in two ways. First, there is the ratio of $y_\tau/y_\pi$, which we have measured by the (inverted) ratio $y/y_p$ and which we have called transitory income, $y_T$. The elasticity of $V_\tau$ with respect to $y_T$ is $+0.14$ on the average, as the data in the appendix show; $y_h/y_\pi$ therefore has a positive effect on velocity and a negative effect on the demand for money, as our hypothesis implies. A similar conclusion holds if the separate effects of transitory prices, $p_T$, and $y_T$ are combined in a single measure $Y_T$.

The quantity $Y_h$ affects the short-run velocity equation in a second way. It appears in the ratio $Y/W_n$ or $\rho$. If $Y_h$ should be included in the numerator of $\rho$, the larger $Y_h$ given $Y$ and $W_n$, the larger the return from holding non-financial assets, the smaller the quantity of money held. The data show that the mean elasticity of $V_\tau$ with respect to $\rho$, given $r$, is positive. Hence they confirm the expected negative effect of $\rho$ on the demand function for money.

The effect of human income on the demand for money is a combination of positive and negative forces. These include the negative effect of a rise in transitory income, given the other variables; the negative effect of a rise in $\rho$, given $r$ and $W_n$; the positive effect of human income on non-human wealth and, through non-human wealth, to the demand for money, plus the effect of $y_h$ on $y_T$ and, through $y_T$, to the desired money balance.

The small effect of $y_h$ on money balances holds for the definition of money as the sum of currency plus demand deposits. It need not hold for more inclusive definitions of money, say $M_3$, or concepts of velocity like $V_5$. It is consistent with our view of the relation of human income to money demand that the effect of income on $M_3$ may reflect a small influence on demand deposits and currency holdings but a relatively large influence on time-deposit holdings. Time deposits are a common means for holding current or past savings. As income increases, savings increase. It is not unlikely that changes in time-deposit accounts are positively related to $y_h$. Such a positive relation between $y_h$ and the change in some asset item would be obtained if the elasticity of saving (change in total wealth) with respect to $y_h$ and the elasticity of $W_n$ with respect to $W(W_\lambda + W_n)$ are both positive. Indeed, our discussion presupposes that $W_n$ yields a stream of income which is used to acquire additional assets and thus becomes a part of $W_n$. The only alternatives, as noted above, are that all the income from $W_\lambda$ is consumed or used.
to acquire additional $W_h$, and both of these alternatives are most unlikely.

Friedman's permanent-income model of the demand for money uses a definition of money which includes time deposits at commercial banks, $T$. We now see that the use of this definition of money not only mixes the effects of general and relative changes in interest rates but includes a component $T$. Changes in $T$ are more responsive to $y_h$. This helps to explain why equation (F2), in which velocity is defined as $V_x$, predicts more accurately than equation (F3), which predicts $V_1$ of the permanent-income model. It also explains why (WF3), a $V_1$ equation, predicts with more accuracy than (WF2), the $V_2$ equation, when the Friedman model is modified to include interest rates in the equation.

Moreover, we have increased our understanding of the reasons why the Friedman-type equations generally predict better than the Keynesian equations but not as well as the wealth model. Friedman's variable $y_p/N$ includes one component, $W_n$, which we have found to be dominant; it also includes a component $y_h$ or $W_h$ which has a much less important role in the demand function for money when money is defined as $M_1$.

We have now provided answers for two of the questions which we raised at the start of Section IV. First, we have shown some routes by which $y_h$ is related to $M$ and why the effect of $y_h$ on the demand for money is much smaller than the effect of $W_n/p$. An analysis which assumes a strong positive relation between $Y$ and $M$—like the "motive" analysis—is based on the incorrect assumption that an increased desire to "transact" will be reflected in a higher demand for money rather than in higher expenditure. Models like the Keynesian hypotheses, in addition to other deficiencies, inappropriately specify income rather than wealth as the constraint on money balances. Second, we have tried to show that $y_p/N$ is an inappropriate measure of the wealth concept which enters the $M_1$ function. Our third question, why the interest elasticity in the Latané version of the Keynesian hypothesis (K3) is inappropriately specified, remains to be discussed. But, of course, the answer is implicit in the earlier discussion of the wealth model, which argued that other variables must be included for a correct specification of the short-run money-demand or velocity function.

Nevertheless, there are some points worthy of brief exploration. First, the short-run interest elasticity has itself been the subject of dispute. As the appendix shows, the short-run (or perhaps intermediate-run) interest elasticities for $V_1$ in the wealth model are between 0.40 and 0.54, depending on the variables which are included in the equation.\[33\] Note that the elasticity of $rW/M_1$ permanent velocity with respect to $r$ is approximately larger by 1.0 than the other short-run elasticities and closer to the elasticity of
This compares with the average elasticity computed from the Latané equation, 0.34. This is further evidence of the specification bias in the Keynesian model.

Second, as noted earlier, the Latané equation can be viewed as a special case of the Keynesian demand function, which uses income as a constraint on money balances. It suffers from the defects of that approach, which have been discussed above. In particular, the omission of yields on alternative assets, e.g., \( \rho \), and the strong emphasis on current income rather than wealth are responsible for the relatively low interest elasticity in comparison with the wealth model.

The foregoing discussions are perhaps sufficient to indicate the relation of the wealth model to contemporary monetary theory. We turn now to a summary of the major findings and a brief discussion of their more general implications for macro-economic theory and policy.

VI. CONCLUSION

We opened this paper by noting that economists often view the quantity theory as, at most, a theory of the long-run response of money income to the money stock. Our view of the quantity theory as a short-run theory has not been adequately tested heretofore. The major purpose of this paper has been to compare a number of different demand-for-money hypotheses, using identical tests, identical time periods, and the same measurement procedures where possible. The results speak clearly when the tests are performed. Compared with more than a dozen alternative equations which we have considered, the short-run quantity theory, equations (W3) and (W4) of the text, perform significantly better than any of the alternative hypotheses.

The alternative hypotheses represent the bulk of the substantive empirical research on money during the past three or more decades. They differ substantially in several respects: the roles assigned to interest rates, income, human and non-human wealth are quite different in the Keynesian, Friedman, and wealth models. The definition of money used in the Keynesian and wealth-demand functions for money is the sum of currency plus demand deposits. Friedman includes time deposits at commercial banks as a part of money balances.

The results here reinforce and extend the evidence obtained from a comparison of regressions, using long-run time series for the three

\[ \frac{rW}{M_1} \] obtained in our long-run study (1.8). This elasticity includes the effect of \( \rho \). The fact that this elasticity is larger by 1.0 bears on the issue of implicit correlation, which results from the use of \( r \) on both sides of the equation. For a more detailed discussion see Sec. 5 of the time-series study. Note also that \( V_1 \) is much less interest-elastic than \( \frac{rW}{M_1} \) but that the effects of the positive elasticities of \( V_1 \) with respect to \( y_t \) and \( P_t \) suggest that \( V_1 \) responds to economic events more strongly than has been sometimes suggested.
models. First, the data suggest that interest rates enter significantly in the velocity equations and in the demand-for-money equations from which the velocity equations are derived. Including interest rates as an additional variable in Friedman's permanent income model improves the prediction of measured velocity from the model. But the use of the interest rate alone, the Latané form of the Keynesian model, does not provide the most accurate prediction of short-run velocity obtained here. Indeed, the predictions from Latané's equation are less accurate than those of a "naïve" model. Moreover, a comparison of the wealth model and the Latané version of the Keynesian model shows that the exclusion of other relevant variables from the Latané equation biases the interest rate downward and reduces predictive performance.

Second, the tests sharply discriminate between the effects of income and wealth on the demand for money. Although our theory and empirical results assign a role to income as an argument in the wealth model, income appears to play a much smaller role than wealth as a determinant of desired money balances. The evidence from a number of Keynesian-type equations that take income as a constraint and ignore the effect of wealth suggests that, in general, such equations will not predict velocity or desired money balances as well as a "naïve" model.

Third, a comparison of the roles of human wealth and non-human wealth (or non-human wealth and total wealth) seems to suggest that non-human wealth is a more important determinant of the demand for money. But, in the absence of adequate measurement procedures for non-human wealth, a direct test has not been made, and the question must remain open.

Fourth, the appropriate definition of money for monetary analysis has been debated at length. Our results seem to suggest clearly that currency plus demand deposits is the more appropriate definition. More inclusive definitions of money appear to mix the effects of general and relative changes in interest rates and to obscure a part of the wealth-adjustment process.

A number of conclusions with respect to monetary policy follow directly from the theory. Space permits us to note only two such conclusions briefly.

First, the liquidity-trap proposition is denied by the evidence. The elasticity of the demand function for money with respect to its arguments has not been of sufficiently large order to render demand and supply identical. Moreover, the data for the 1930's were used to predict velocity in the 1950's. We observe that the predictive errors are relatively small and that the computed elasticities remain within their customary range. This is added evidence against the trap hypothesis.

Second, we conclude that a comparatively stable demand function
for money has been obtained from the wealth model. Stability of the demand function is a necessary condition for money relevance. Our results seem to suggest that money affects the pace of economic activity, that monetary policy operates on interest rates or relative prices and through wealth and substitution processes on the level of income.

APPENDIX I

DATA SOURCES

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source</th>
</tr>
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<tbody>
<tr>
<td>$M_1$</td>
<td>Series X 267 from <em>Historical Statistics of the U.S.</em> 1960</td>
</tr>
<tr>
<td>$r$</td>
<td>1900–1945 from Goldsmith, <em>A Study of Saving</em> (the series values at original cost were used); 1946–58 from Department of Commerce estimates</td>
</tr>
<tr>
<td>$Y_p$</td>
<td>Obtained by multiplying $y_p$ by $P_p$. Both underlying series are unpublished estimates of the National Bureau of Economic Research and were generously supplied by Milton Friedman and Anna J. Schwartz</td>
</tr>
<tr>
<td>$y_p/N$</td>
<td>Same as the source for $y_p, P_p$</td>
</tr>
<tr>
<td>$W_n$</td>
<td>Unpublished estimates computed by Meltzer. Based on Goldsmith, <em>A Study of Saving</em>, Table W-1; revised to eliminate asset accounts and to include liability accounts of the government sector</td>
</tr>
<tr>
<td>$P_p$</td>
<td>See $y_p$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Sources are same as those for $Y$</td>
</tr>
<tr>
<td>$V_1, V_2$</td>
<td>Ratio of $Y/M_1$ and $Y/M_2$, respectively</td>
</tr>
<tr>
<td>$V_n, \rho$</td>
<td>Ratio of $Y_p/M_2$</td>
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APPENDIX II

DISTRIBUTION OF COMPUTED ELASTICITIES

A. SUMMARY

<table>
<thead>
<tr>
<th>ELASTICITY OF</th>
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<th>GIVEN</th>
<th>MEANS</th>
<th>MEDIAN</th>
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<tr>
<td>1. $V_1$</td>
<td>$Y/W_n$</td>
<td>$r$</td>
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<td>0.93</td>
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<td>2. $V_1$</td>
<td>$Y/W_n$</td>
<td>$r, V/Y_p$</td>
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<td>0.64</td>
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<td>3. $V_1$</td>
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<td>0.36</td>
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<td>5. $rW/M_1$</td>
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<td>$r$</td>
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<td>6. $V_1$</td>
<td>$r$</td>
<td>$Y/Y_p$</td>
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<td>0.31</td>
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<tr>
<td>7. $V_1$</td>
<td>$Y/Y_p$</td>
<td>$Y/W_n, r$</td>
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<td>0.52</td>
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<td>8. $V_1$</td>
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<td>$P_p, P_n, r, V/W_n$</td>
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<td>9. $V_1$</td>
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<td>12. $V_1$</td>
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## B. DISTRIBUTIONS*

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<tr>
<td>1.40</td>
<td>1.00</td>
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<td>0.90</td>
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<td>1.20</td>
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<tr>
<td>1.21</td>
<td>1.50</td>
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<td>1.80</td>
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<td>-0.90</td>
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<tr>
<td>1.50</td>
<td>-1.20</td>
</tr>
</tbody>
</table>

*The following notation is employed: $V_i, Y/W_m; r, Y/Y_p$ should be read "the elasticity of $V_i$ with respect to $Y/W_m$ given $r$ and $Y/Y_p."
### B. DISTRIBUTIONS—Continued

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<th>Elasticity</th>
<th>Frequency</th>
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<td>At Least</td>
<td>No Greater Than</td>
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<table>
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<tr>
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<tr>
<td>$V_t, y/y_p; \phi/P_p, r, V/W_n$</td>
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<thead>
<tr>
<th>11. $V_t, \phi_p/N; r, y/y_p, \phi/P_p$</th>
<th>12. $V_t, \phi_p/N; y/y_p, \phi/P_p$</th>
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<th>13. $V_t, \phi_p/N$</th>
<th>13. $V_t, \phi_p/N$</th>
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<td>$V_t, \phi_p/N$</td>
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353
APPENDIX III

DISTRIBUTION OF SERIAL CORRELATION COEFFICIENTS (DURBIN-WATSON) OF RESIDUALS FROM EQUATION (W4)

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<td>1.25 4</td>
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<td>1.26</td>
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<tr>
<td>2.66</td>
<td>3.00 8</td>
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<tr>
<td>3.01</td>
<td>3.35 4</td>
</tr>
<tr>
<td>3.36</td>
<td>3.70 4</td>
</tr>
</tbody>
</table>

Mean 2.40  
Median 2.58

The Durbin-Watson test statistic is not given for sample sizes smaller than 15. We used the 1 per cent level for $n = 15$ as a test for the presence of serial correlation in our samples of size 10.

<table>
<thead>
<tr>
<th>For Values</th>
<th>Autocorrelation Is</th>
<th>Observed Number of Such Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 0.49</td>
<td>Significant and positive</td>
<td>0</td>
</tr>
<tr>
<td>Above 3.51</td>
<td>Significant and negative</td>
<td>1</td>
</tr>
<tr>
<td>Between 1.70 and 2.30</td>
<td>Not significant</td>
<td>6</td>
</tr>
<tr>
<td>Between 0.49 and 1.70</td>
<td>Indeterminate, possibly positive</td>
<td>10</td>
</tr>
<tr>
<td>Between 2.30 and 3.51</td>
<td>Indeterminate, possibly negative</td>
<td>22</td>
</tr>
</tbody>
</table>

The results of the test suggest that serial correlation of the residuals from equation (W4) is either not significant or indeterminate (cf. J. Durbin and G. S. Watson, "Testing for Serial Correlation in Least Squares Regression," Biometrika, Volume XXXVIII [1951]).

(Continued on back cover)
The present series begins with articles written by the faculty of the Graduate School of Industrial Administration and published during the 1957-58 academic year. Single copies may be secured free of charge from: Reprint Editor, G.S.I.A., Carnegie Institute of Technology, Pittsburgh, Pa. 15213. Additional copies are 50 cents each, unless otherwise noted.

(Continued)


