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A Credit Market Theory of the Money Supply and an Explanation of Two Puzzles in U.S. Monetary Policy

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Federal Reserve policy in the early postwar years has frequently been described as an "engine of inflation" (1). Continuation of the wartime policy of supporting the government bond market required the central bank to absorb securities sold by the private sector. Given the large stock of government securities left as a residue of wartime finance, the expectation that the support policy would lead to inflation was quite reasonable. The puzzling feature about the period is that substantial inflation did not occur. Instead, the growth rates of the money supply and of the monetary base collapsed shortly after the end of World War II. By 1949, the economy had moved to a position in which the policy problem was one of removing recession and deflation. Despite the continuation of the support policy, prices in 1949 were falling, not rising.

If the support policy prevented the Federal Reserve from achieving an effective counter-cyclical monetary policy, it seems reasonable to expect counter-cyclical policy to have emerged once the support policy ended. The famed "accord" with the Treasury in March 1951 terminated the

(*) Helpful discussion with Armen A. Alchian and Milton Friedman, and financial support from the National Science Foundation are acknowledged gratefully.

agreement to maintain a fixed «pattern of yields» on government securities. But it did not usher in a period of markedly counter-cyclical policy.
The expansion of the money supply in 1951 was larger than it had been in any of the preceding four years. In the decade following the accord the monetary base and the money supply generally moved in a pro-cyclical rather than a counter-cyclical direction. This pro-cyclical pattern is the most puzzling aspect of U.S. monetary policy in the post-accord years.

Attempts to explain the Federal Reserve's behavior frequently suggest that there is a delay in recognizing the appropriate time for policy changes or «recognition lag» (2). On the surface, this line of argument appears quite plausible. Unfortunately, it is not supported by an examination of the timing of Federal Reserve decisions. A comparison of post-accord policy decisions with cyclical turning points reveals an impressive ability to recognize turning points rapidly (3).

Thus observations invalidate the standard conclusion about the support policy, deny the policymaker's confident assertion of a systematic, counter-cyclical policy and reject the attempt to salvage the counter-cyclical flavor of policy through the medium of a recognition lag. But then the two major puzzles remain. How can we explain the collapse of the growth rates of the base and the money supply under the rigid support policy? In the absence of a recognition lag, how can we explain the pro-cyclical pattern in monetary policy and the divergences between professed and actual policy?

In this paper we expand a theory of the money supply presented elsewhere (4) to incorporate an analysis of bank earning assets. The extended theory describes the joint determination of money, bank credit — loans and investments of commercial banks — and some interest rates

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The General Framework

The Federal Reserve sets the reserve requirement ratios and the proportion of vault cash of member banks to be counted as required reserves, decides on its portfolio of government securities, announces the discount rate, regulates the discount window, and classifies member banks. The Treasury plays a minor role through the administration of its cash balances. Commercial banks respond to their environment by allocating total assets between cash assets and earning assets, distributing earning assets between loans and investments, and borrowing or repaying indebtedness. These decisions determine the relative size of four items on the asset side of the consolidated balance sheet of the banking sector: loans, investments, borrowed and unborrowed reserves.

The public confronts the banks on the loan market with a supply of loans and interacts with the banks on the securities market. Moreover, the public's allocation of money balances between currency and demand deposits and the partition of total deposits between demand and time account exert a significant effect on monetary and credit magnitudes. Like the banks' allocation decisions, the public's demand and supply behavior (with the exception of its currency demand) is shaped by the movement of interest rates on the bank credit market.

Policy decisions and some of the market responses just discussed can be summarized by five parameters and a policy variable. In this section, we present a compact statement of the underlying theory and, in the following section, summarize the broad outline of the money-bank credit process in a diagram. The simultaneous solution for money, bank credit and interest rates on the credit market is suppressed for this purpose. The restriction is removed in a later section where the effects on interest rates are more explicitly stated.

The adjusted monetary base

By consolidating the government sector accounts, the Federal Reserve's balance sheet and the monetary accounts of the Treasury, a statement of sources and uses of the monetary base is obtained. This statement
includes member bank borrowing, an endogenous variable determined jointly with interest rates, money and bank credit. The monetary base net of member bank borrowing — the adjusted base, $B^*$ — is taken as an exogenous variable. Equation (1) expresses the adjusted base in terms of its uses.

\[ B^* = R_u + C_p \]

where $R_u$ is total unborrowed reserves of all commercial banks and $C_p$ is currency held by the public. The adjusted base occupies a central position in the analysis that follows since it summarizes the operations of the monetary authority during periods of unchanged reserve requirement ratios and rediscount rates (5).

\textit{Two behavior parameters for commercial banks}

The reserve ratio $r$ describes the banks' division of total assets between earning assets and reserves. The latter consists of vault cash and deposits at Federal Reserve banks. The parameter is measured by the ratio of total reserves to the banking system's total deposits (including Treasury deposits). The reserve ratio is effectively explained by the average requirement ratios against demand and time deposits, the level and expected variability of interest rates, and the banks' anticipation with respect to the average value and the variability of currency flows. We omit a detailed statement of the hypothesis for the reserve ratio and simply write equation (2):

\[ r = r(r', r', i_o, p, i_L; \pi) \]

where $r'$ and $r'$ designate the average requirement ratios against demand and time deposits; $i_o$ denotes the average yield on government securities, $i_L$ an index of loan-rates, and $p$ is the discount rate. The parameter $\pi$,

(5) The source components of the base include policy, predetermined and stochastic variables. The gold stock and Treasury currency are predetermined relative to the current money supply process and are, therefore, taken as exogenous magnitudes. Treasury cash is treated as exogenous also, since it is under the control of the fiscal authorities. Reserve Bank credit net of borrowing includes a policy variable — the Federal Reserve's portfolio of securities — and a stochastic variable, float. The latter is related to the volume of bank debits and is affected somewhat by the current money supply process. Nevertheless, the base is taken as an exogenous variable since the uses of the base, equation (1) of the text, are the total monetary issue of the central bank and can be controlled completely.

For a more complete description of the sources and uses of the base, see our An Alternative Approach to the Monetary Mechanism, op. cit., pp. 9-12.
The borrowing ratio, $b$, expresses the banks' partition of total reserves between borrowed and unborrowed reserves. This behavior parameter is measured by the ratio of total bank borrowings from Federal Reserve banks to the banks' total deposits. It is explained in equation (3).

$$b = b(i_o, i_t, p; \pi_2)$$

The borrowing ratio responds positively to the market rates of interest and negatively to the discount rate. As before, the parameter $\pi_2$ reminds us of omitted factors that have played a sizeable role on occasion (e.g., the tax treatment of borrowing costs).

**One parameter for the Treasury**

The Treasury holds deposits at both Federal Reserve and commercial banks. Treasury deposits at the Federal Reserve are included among the sources of the base. Tax and loan accounts at commercial banks are described for present purposes by a parameter, $d$, the ratio of Treasury deposits at commercial banks to the public's demand deposits. This parameter is assumed to be the result of tax and spending decisions by the Congress, cash management practices of the Treasury, etc., so that $d$ may be taken as exogenous.

**Two behavior parameters for the public.**

The public's decision to hold some proportion of their demand deposits as currency is expressed by $k$. The allocation of money balances between currency and demand deposits has at times played a decisive role in the monetary process. An adequate explanation of the short-term behavior of $k$ is badly needed. But, at the moment, there is no reliable hypothesis for this purpose. Since our concern here is with the relation of interest rates, money and bank credit and there is no evidence that $k$ is dependent on interest rates, $k$ is taken as a datum that influences — but is not influenced by — current monetary processes.

The ratio of time to demand deposits, $t$, is dependent on the current monetary process. The time deposit ratio appears to be highly sensitive to the rate paid on time deposits ($i_t$), to the yields on Treasury bills and
on longer-term securities. In addition, the ratio responds positively to the public’s real wealth, \( w \). Equation (4) states the relation.

\[
(4) \quad t = t(i, \ i_o, \ i_u, \ w)
\]

The derivatives with respect to \( i \) and \( w \) are positive; the others are negative. Competitive pressure between banks generates a delayed response of \( i \) to the two market determined rates. A more detailed analysis would also include yield and «quality» changes in saving and loan association shares.

The Stock of Bank Credit: A First Approach

A relatively simple solution for the stock of bank credit can be obtained if the effects of interest rates and the public’s behavior on the credit markets are ignored temporarily. The underlying system can be compressed into four equations represented in the four panels of Figure 1.

The banks’ balance sheet together with the definition of the reserve and borrowing ratio implies the following relation between the banks’ desired stock of earning assets \( E_b \) and the volume of unborrowed reserves \( R_u \):

\[
(5) \quad E_b = \frac{1 - (r - b)}{r - b} \cdot R_u
\]

The line in the first quadrant of Figure 1 corresponds to this relation. The slope of the line is expressed as a rational function of the parameters \( r \) and \( b \) occurring in equation (5). It follows that changes of requirement ratios, the discount rate and market rates of interest modify the slope and thus rotate the line around the origin.

Equation (5) associates « bank credit » with the volume of unborrowed reserves by means of the reserve and borrowing ratios, two parameters characterizing bank behavior. According to equation (1) the supply of unborrowed reserves depends on the adjusted base and the public’s absorption of currency. This is expressed in the fourth quadrant of Figure 1. The position of the line depends on the magnitude of the adjusted base. This magnitude is measured by the intercepts on the two axis. Every point of the line indicates a possible partition of the base between unborrowed reserves of banks and the public’s currency holdings. Every change in the adjusted base, whatever its origin, involves a parallel shift of the line in the fourth quadrant.

The supply of unborrowed reserves has been linked to the public’s currency holdings. The latter are associated, according to our discussion
in a previous section, to the banks' supply of demand deposits. The currency ratio $k$ establishes this association, which is appropriately expressed by the line in the third quadrant of Figure 1. The slope of the line measured with respect to the horizontal axis is equal to the currency ratio $k$. Any rotation of the line around the origin thus reveals a change in the public's desired currency ratio.

![Figure 1 - Determination of Money and Credit](image)

Our last building block links demand deposits with « bank credit ». Demand deposits ($D_p$) are generated through the acquisition of earning assets. While it has become customary to think of this relation as a fixed proportion, our analysis denies that the ratio of demand deposits to earning assets is constant. It implies instead that the slope of the line in the second quadrant is dependent on the parameters $d$, $t$, $r$, and $b$ introduced earlier and hence on interest rates, the rediscount rate and the requirement ratios. The line associates demand deposits and bank credit and may be expressed by the formula

$$D_p = \frac{1}{(1 + t + d) \left[1 - (r - b)\right]} \cdot E_b$$

The diagram may be used to trace the general direction of the response...
in \( E_b \) to variations in the underlying variables or parameters. The solid line connecting points in the four quadrants shows a (partial) equilibrium position of the monetary system and cuts off unique values for \( E_b, D_p, C_r \) and \( R_u \). Any increase in the adjusted base, e.g., an open market purchase, a gold inflow, etc. shifts the line in quadrant IV away from the origin. The new box, shown by the dotted line in Figure 1, contains the old box. Each of the monetary variables is increased.

The precise effects of policy operations depend on the values of the parameters. For example, economy in the use of currency, represented by a flatter line in quadrant III, produces larger responses in \( E_b \) and \( D_p \) for a given increase in the base. Increases in the time deposit ratio alter the slopes in the first and second quadrants. The larger the \( t \) ratio, the larger the response of bank credit relative to the response of demand deposits. Increases in the ceiling interest rate on time deposits, when a large proportion of banks have reached the prevailing ceiling, raise the time deposit ratio and rotate the line in quadrant II to the right. Reductions in the reserve requirement ratios have an effect similar to increases in the ceiling rate.

Other policy changes or institutional rearrangements can be investigated using the simple framework. Each such change can be shown to alter the ratio between the banks' demand for earning assets and the adjusted base. In a later section, we will return to consider this ratio in more detail. Before doing so, it is desirable to remove the assumption of constant interest rates and to introduce relations summarizing the behavior of the public on the credit market.

\[ A \ Description \ of \ the \ Credit \ Market \]

Policy actions affect the money supply and bank credit via two distinct channels. An open market purchase increases the base and lowers interest rates. The effect of a change in the base is suggested in Figure 1 above. Analysis of the second channel — the response of money and bank credit to changes in market interest rates — requires some discussion of the bank credit market. In this section, our argument is extended to cover the second channel. The response of money and bank credit to policy variables operating through the two channels is combined in the following section.

Bank credit is divided into its components, loans and investments. This division reflects the substantially higher marginal cost of acquiring information about loans, a cost that has important consequences, for the
The solid equilibrium \( v, E_b, D_p \), market pur-
from the reas of the presented \( v, D_p \), ratio alter-\n\( tory, \theta \), the demand when a the the quasi-increase.

adjustment of bank portfolios \(^1\). Let \( L_b \) and \( I_b \) denote the aggregate desired loan and investment (or securities) portfolios of commercial banks. We postulate that \( I_b \) and \( L_b \) are proportional to total demand, time and Treasury deposits \((D + T)\) with proportionality factors, \( a_t \) and \( a_2 \), dependent on interest rates and the ratio of actual to expected income, \( y/y_e \). Thus \(^2\) \[ I_b = a_t(D + T); \quad (7) \quad L_b = a_2(D + T) \]

where \[ a_t = a_t(i_c, i_L, y/y_e), \quad a_2 > 0 > a_3; \quad a_2 < 0 \]

and \[ a_2 = a_2(i_c, i_L, y/y_e), \quad a_2 < 0 < a_3; \quad a_2 > 0 \]

The restrictions imposed on the parameters \( a_t \) and \( a_2 \) are not completely independent. The balance sheet of commercial banks implies that the four parameters used to characterize bank behavior are constrained by a linear relation, \( r + a_t + a_2 = 1 + b \).

This relation and the derivatives of the reserve ratio \((r)\) and borrowing ratio \((b)\), introduced earlier, imply that the direct derivatives of the \( a_t \) are numerically larger than the respective cross derivatives, i.e., \( a_{12} > |a_{21}| \) and \( a_{22} > |a_{21}| \). An increase in \( i_c \) or in \( i_L \), other rates remaining unchanged, alters the portfolio composition of the banking system. However, the division of earning assets between loans and investments has been assumed to be independent of the discount rate. The balance sheet constraint therefore implies that the derivatives of the reserve ratio and the borrowing ratio with respect to the discount rate are numerically equal but of opposite sign. From these constraints it follows also that \( a_{12} = -a_{21} \), i.e., \[ \frac{\partial a_t}{\partial (y/y_e)} = -\frac{\partial a_2}{\partial (y/y_e)}. \]

Changes in the index of transitory income \((y/y_e)\) affect the allocation of earning assets between loans and investments but do not alter the distribution of total assets between cash and earning assets. The existence of

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\(^1\) For a more detailed discussion of the loan portfolio see Donald Hester, An Empirical Examination of the Commercial Bank Loan Offer Function, Yale Economic Essays, 1962.

\(^2\) \( a_{12} \) is the partial derivative of \( a_t \) with respect to \( i_c \); \( a_{21} \) is the partial of \( a_t \) with respect to \( i_L \).
earning assets with very low transaction and information costs is crucial to this result.

The public's behavior on the bank credit market is represented by equation (8), the public's stock demand for securities \( L_p \), and equation (9) the supply of loans to commercial banks \( L_p \). These variables are assumed to depend on prevailing interest rates, the index of transitory income and on expected nominal income, \( Y_e \), a measure of the current value of wealth (*)..

\[
(8) \quad I_p = s_1 (i_0, i_L, y/y_e, Y_e); \quad s_{11} > 0 > s_{12}; \quad s_{13} > 0 < s_{14}
\]

\[
(9) \quad L_p = s_1 (i_0, i_L, y/y_e, Y_e); \quad s_{21} > 0 > s_{22}; \quad s_{23} < 0 < s_{24}
\]

Interaction between the public and the banks on the credit market approximately determines the two interest rates, \( i_0 \) and \( i_L \), and distributes the outstanding stock of debt \( S \) between the two sectors. The credit market is thus described by the two equations

\[
I_b + I_p = S \quad \text{and} \quad L_b = L_p.
\]

After the appropriate substitution in equations (6) and (7), the credit market equations can be rewritten in terms of the exogenous and predetermined variables.

\[
a_1 (i_0, i_L, y/y_e, r^*, r^1, k, p) B^* + s_1 (i_0, i_L, y/y_e, Y_e) = S
\]

\[
a_2 (i_0, i_L, y/y_e, r^*, r^1, k, p) B^* = s_2 (i_0, i_L, y/y_e, Y_e)
\]

The parameters \( a_1 \) and \( a_2 \) depend on interest rates and the index of transitory income by construction, since they are proportional to \( a_1 \) and \( a_2 \) of equations (6) and (7) (**). They also depend on the reserve requirement ratios, the rediscount rate and the currency ratio and on the variables that enter as arguments of the reserve, borrowing and time deposit ratios.

The influence of monetary and fiscal policies on credit market interest rates can be obtained from the two equations. Monetary policy operates either through the parameters \( a_1 \) and \( a_2 \) or by changing the adjusted base.

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(*) Separation of the effects of changes in prices and real wealth is ignored. A more detailed analysis than is provided here would require some statement about the separate effects of these variables as well as the response of the public's desired indebtedness to anticipated and unanticipated price changes, changes in the real yield on real capital, etc. Introduction of these additional variables would increase the complexity of our analysis without altering the conclusions in any important way.

For similar reasons, no explicit attention is given to the separation of financial intermediaries from the rest of the public.

(**) Specifically, \( a_i = a_i \left[ \frac{1 + f + d}{(r - b)(1 + f + d) + k} \right] \)
The history of fiscal policy is reflected in the outstanding stock of securities held by the banks and public, $S$. The response of $i_G$ and $i_L$ to each of the policy variables, expressed as an elasticity, is shown in Table I of the appendix. The restrictions imposed on the various equations prove to be sufficient to determine a unique sign for each of the elasticities.

Among the implications obtained from the elasticities, three are of particular interest. First, variations in the volume of outstanding securities induce a larger response in the yield on securities than on loan rates. This is a consequence of the restriction which makes the cross derivatives smaller than the direct derivatives. A sale of debt to the banks or the public, e.g., the financing of a deficit, raises both securities and loan rates but increases the former relative to the latter. Second, the elasticities of the two interest rates with respect to the stock supply of debt are not constant. In particular, they appear to depend positively on the proportion of the debt held by the public. Thus deficits financed primarily by the sale of new issues to the public are likely to increase the responsiveness of $i_G$ and $i_L$ to debt operations. Third, open market operations exert a more pronounced effect on interest rates than other changes in the base or fiscal policy operations. An open market operation is characterized by the conditions $dS = -dB^s$. Hence, the response of interest rates to central bank purchases and sales is given by the expression

$$
\left[ \varepsilon (i_s, B^s) - \varepsilon (i_s, S) \frac{B^s}{S} \right] \frac{dB^s}{B^s}
$$

where $\varepsilon (i_s, B^s)$ and $\varepsilon (i_s, S)$ are the elasticities of a particular interest rate with respect to the adjusted base and the stock of outstanding debt. Since the bracketed expression exceeds each of its components in numerical value, the effect on interest rates of an open market purchase by the Federal Reserve exceeds the effect of any equivalent change in one of the other sources of the base or of an equivalent volume of debt retirement by the Treasury. The magnified effect of an open market operation is the result of the simultaneous change in the demand for and supply of interest-bearing debt.

The Determination of Bank Credit and Money

The solutions for interest rates on the credit market implicitly define an equilibrium stock of money and bank credit. This may be recognized formally by inserting the interest rates determined by the credit market equations into the relations defining $L_b$ and $L_o$. The sum of the two stocks,
$E_b$, is the volume of bank credit net of Treasury deposits at commercial banks. By a simple rearrangement of the relations shown in Figure 1 earlier, the money supply, currency and demand deposits, is obtained also. This is expressed in equation (10).

(10) \[ M = \beta E_b \]

The ratio $\beta$ will be referred to as the issuing quotient, the amount of money issued by the monetary system per dollar of earning assets. The discussion of Figure 1 has already indicated that the issuing quotient is not a constant. It depends on the ratios $r$, $b$, $k$, and $t$. But, unlike the analysis in Figure 1 where interest rates were held constant, $\beta$ depends on interest rates and the rediscount rate as well. Specifically, $\beta$ responds positively to changes in the reserve requirement ratios, the currency ratio and the rediscount rate, whereas it reacts negatively to changes in market interest rates. The precise way in which the ratios affect $\beta$ is shown in footnote 11 below.

The analysis underlying Figure 1 can now be extended to combine the effects of policy variables and interest rates on the stocks of money and bank credit. Part a of Figure 2 describes the (proximate) determination of the loan portfolio and the loan rate. The position of the two schedules depends, of course, on the securities yield as well as on the other variables of the $L_a$ and $L_p$ functions. The securities market is described in part b. The stock supply, taken as a given magnitude in the present analysis, must be absorbed into the portfolios of the banks and the public. Their combined demands, $I_a$ and $I_h$, depend on the loan rate, policy variables, and other arguments previously introduced. The equilibrium stock of bank credit resulting from the interaction of the banks and the public on the two markets is shown on the horizontal axis of panel c. Corresponding to each equilibrium stock of bank credit and a particular value of the issuing quotient, there is an equilibrium money supply. The slope of the line in Figure 2c is the issuing quotient. The money supply can be read on the vertical axis.

Solid lines on the diagram are used to indicate an initial position. We will consider in turn three changes which disturb this equilibrium: an open market purchase, a reduction in reserve requirement ratios, and an increase in the public’s demand for loans. Dotted lines have been used to indicate the position attained after the effects of the open market purchase on the credit market have been achieved.

Open market purchases raise the base and consequently expand the banks’ demand for loans and securities, shifting $L_a$ and $L_p$ to the right.
and reducing interest rates on the loan and securities markets. In panel b, the decline in interest rates is magnified by the reduction in the stock supply of securities by the amount of the open market purchase. Securities are redistributed between the banks and the public in response to the changes in loan and securities rates, and the public's supply of loans to banks increases slightly. The interplay between the two markets continues until the rates $i_{L2}$ and $i_{S2}$ are established. At these rates, the outstanding volume of loans is $L_2$ and the banks hold an amount of securities equal to $I_{S2}$. The sum of these items is the increased stock of bank credit, $E_{b2}$, in panel c.

*a: The Loan Market*  
*b: The Securities Market*  
*c: The Issuing Quotient*

**FIG. 2.** The Effect of an Open Market Purchase

However, since interest rates fall, the issuing quotient increases and the line in panel c rotates to the left. The total response of the money supply is thus made up of two components. One expresses changes which are directly proportional to the change in bank credit; the other results from the operation of interest rates on the issuing quotient. In the particular case considered, the change is larger for money than for bank credit. More generally, the analysis suggests that the elasticity of the money supply with respect to open market operations exceeds the corresponding elasticity for bank credit ($e^m$). This is a consequence of the smaller interest elasticity for the money supply than for bank credit.

Let us now consider the effect of an increase in the public's supply

(10) Relative rates of change in money and bank credit differ substantially at different phases of the cycle. The hypothesis explains the mechanism generating these differences.
of loans to commercial banks, \( L_p \). Interest rates rise on both markets. Since the reduction in the banks' securities portfolio is smaller than the increase in loans, total bank credit expands just as it did in response to an open market purchase. This time, however, the effect on the money supply (panel c) is not amplified by an increase in the issuing quotient. Instead, the issuing quotient falls, since interest rates have increased, and the line in panel c rotates to the right. The effect on the money supply of an expanded stock of bank credit remains positive but is less than proportional to the change in bank credit.

The opposite result is obtained if the initial change is assumed to be an increase in the public's demand for securities, \( I_p \). Interest rates fall in response to the increased demand for the given stock of debt. The issuing quotient rotates to the left, and the money supply rises by more than the proportion indicated by the initial value of \( \beta \).

Our argument may be summarized as follows: Credit market changes that induce changes in interest rates in the opposite direction affect the money supply more than bank credit. Open market operations, other changes in the base, or changes in the demand for securities have this effect. Variations in loan demand or in the stock of securities induce movements of interest rates in the same direction. Consequently, these variations have a larger effect on bank credit than on money.

What is the effect of the public's currency behavior and of other policy instruments on money and credit? Analysis suggests that there are partially compensating influences on the issuing quotient. A reduction in the reserve requirement ratios or in the rediscount rate induces an increase in the banks' portfolio and lowers interest rates in panels a and b. In panel c, the effect is the same as in the case of an open market purchase. Money and bank credit expand along the existing issuing quotient, and the issuing quotient rotates to the left, increasing the rate of monetary expansion. But now there is an additional effect. The issuing quotient depends, by construction, on the reserve requirement ratios, the discount rate, etc. Reductions in the reserve requirement ratios and in the discount rate cause the line to rotate to the right, offsetting, at least in part, the movement induced by the decline in interest rates.

The diagrams do not yield information about the net effect of policies that change both interest rates and other components of the issuing quotient. However, solutions for the money supply and bank credit in terms of the policy variables and the parameters can be obtained from the preceding analysis. The volume of bank credit, \( E \), and the stock of
money, $M$, are expressed in the equations below as the product of a multiplier and the adjusted base. Parentheses are used to indicate the dependence of the multipliers on the interest rates, the rediscount rate and the parameters introduced earlier (11).

$$ M = m(\ )B^* \ ; \ E = a(\ )B^* . $$

In some previous work, we have demonstrated that the interest elasticity of the monetary multiplier exceeds the interest elasticity of the bank credit multiplier and that both elasticities are positive (12). By combining the interest elasticities of the two multipliers with the response of the multipliers to changes in the reserve requirement ratios (or other parameters), some information about the direction of change in the issuing quotient can be obtained. The elasticities of the money supply and bank credit with respect to policy variables have been collected in Table II of the appendix. Items 3 and 4 of the table show the responses to the reserve requirement ratios and other parameters. Each response is expressed as a combination of the simple response in the money or bank credit multiplier and the modification introduced by the operation of the interest mechanism.

The simple or direct responses of the multipliers are negative. While these elasticities are attenuated by the interest mechanism, the direction of change in money and credit is not reversed. Both money and bank credit are affected negatively by changes in the requirement ratios, the rediscount rate and the currency ratio.

To obtain the direction of change in the issuing quotient, $g$, the elasticities have been rearranged in Table II. The effect on $g$ is equal to the difference between the responses in $M$ and in $E$. This difference is shown on line 5 of the table. The hypothesis implies, as the discussion of Figure 2 suggested, that changes in the base have a larger effect on money than on bank credit. This suggests that other policy operations also have larger effects on money than on bank credit and that the issuing quotient increases

\[ m = \frac{1+k}{(r+b)\ (1+t+d)+k} \] and \[ a = \frac{(1+t+d)\ (1-(\gamma-b))}{(r+b)\ (1+t+d)+k} . \]

Hence, the issuing quotient, $g$, is

\[ g = \frac{1+k}{(1-t+d)\ [1-(\gamma-b)]} . \]

(11) In these equations,

(12) Money, Bank Credit, Interest Rates and the Liquidity Trap (forthcoming).
when the reserve requirement ratios or the discount rate are reduced. However, the second component on line 5 is the difference between the direct elasticities of the credit and money multipliers. Both are negative, but the asset elasticity is numerically larger than the monetary elasticity. Thus the second term on line 5 affects the issuing quotient in the direction opposite to the first. On balance, the issuing quotient moves very little. It follows that bank credit is comparatively more sensitive to requirement and discount policy than to variations in the base.

Additional inspection of Table II shows three distinct patterns, one for the base, one for the parameters operating directly on the multipliers and one for the variables that work neither through the base nor the multipliers. In the absence of the interest mechanism, the elasticities of money and credit with respect to the base would be unity. The interest mechanism attenuates the size of these elasticities and makes the elasticity larger for the money supply than for bank credit. In our discussion of the effect of changes in reserve requirement ratios, we noted that the interest mechanism also reduced the elasticity of money and credit with respect to policies operating through the multipliers. These elasticities are approximately equal for money and credit and are substantially smaller than the elasticities with respect to the base.

The third pattern in Table II is exemplified by the elasticities with respect to the outstanding stock of securities, but similar solutions are obtained for variables like expected income, \( Y_r \), which do not operate through the base or the multiplier. Bank credit is more responsive to these variables than the money supply, but both elasticities are substantially smaller than the elasticities with respect to the base. The reason is that each is multiplied by the interest elasticity of the appropriate multiplier. Preliminary investigations suggest that these interest elasticities are substantially below one-half. It follows that variations in the public's demand for loans or the government's fiscal policy exert a comparatively small effect on the volume of bank credit and money.

Additional implications of credit market behavior and their influence on the money supply can be obtained from the hypothesis. Analysis of the effects of interest rates on the time deposit ratio, for example, furnishes a useful clarification of some developments in the United States during recent years. More detailed consideration of the response of money and bank credit to changes in the currency ratio reveals the precise effects of increases in the demand for currency that are important for understanding U. S. monetary history. Discussion of these problems has been
deferred, however, to permit more detailed examination of two puzzling features of recent monetary history.

**Money Stock and Interest Rates under the Support Program: Explanation of the First Puzzle.**

Detailed concern about stable interest rates was not an alien commitment imposed on the Federal Reserve by the Treasury. Monetary policy had been greatly influenced by such concern at least since 1937. The constraint imposed in 1942, however, formalized the obligation and removed discretionary authority from the Federal Reserve for almost ten years. The accord of March 1951 ended the Federal Reserve’s obligation to maintain particular yields on Treasury securities.

There are two quite distinct phases during the support period. Both the money supply and the adjusted base grew at a rapid rate during the War, while in the postwar period both exhibited sharply declining growth rates which became negative for a time in 1948-49. Both accelerated in the second half of 1950 following the outbreak of the Korean War. Thus the support policy appears to have been an engine of inflation during both wars but not during the postwar readjustment.

Many of the consequences of the support policy can be investigated within the framework developed in previous sections. Some modification is required, however, since the support policy imposes a constraint on the operation of the credit market. This constraint takes the form of an inequality. If the market-determined rate on government securities is higher than the ceiling rate set by the authorities, the ceiling rate is effective. The yield on government securities remains at the ceiling and is, therefore, a predetermined variable. The adjusted base becomes an endogenous variable determined by the support policy and the operation of the monetary system \(^{(13)}\). If the market-determined yield of government securities is less than the ceiling rate, the patterns discussed in the preceding sections

\(^{(13)}\) The formal properties of the system do not require that the base becomes endogenous. Any of the exogenous variables — \(S, r^*, r^, \) etc. — would satisfy the formal requirement. The choice of the adjusted base results from supplementary considerations. Had \(S\) been chosen, fiscal policy would become dependent. The budget would have to be continuously readjusted. During part of the period, “tap” securities permitted the private sector to call forth issues of government debt. But the public was given no means of reducing the deficit. Under the circumstances, the base seems a more likely choice than \(S\). Similar considerations apply to the reserve requirement ratios. The choice of these variables or the rediscount rate would require their continuous adjustment or violation of the constraint.
apply. The base remains exogenous and the credit market elasticities shown in Tables I and II are applicable.

Even if the interest rates had been substantially below the ceiling rates at the start of the war, they would not have remained there. The large budget deficit increased the upward pressure on interest rates generated by expanding income and a higher currency ratio. Increases in the reserve requirement ratios, under these circumstances, would not abate inflationary pressure, since such increases would accelerate the approach of \( i_c \) to the ceiling rate. However, increases in the base would operate to keep interest rates below the ceiling.

The initial choice of the ceiling rate, the large budget deficit, and the increased demand for currency made the constraint operative early in the war. The system was transformed to one in which the ceiling rate was effective and the responses were altered. Some of the elasticities that are of interest have been collected in Table III of the appendix.

Much of the War was financed by government deficits. The table shows that the elasticity of the base with respect to the deficit (line 1) is positive. The larger the deficit, the greater is the increase in the base. Moreover, this elasticity contains the ratio of total outstanding securities to the banks' holdings of securities, \( S/S_b \). Sales of debt to the public rather than to the banks, therefore, raised this elasticity. Whatever anti-inflationary effect was achieved by such sales through reduction in the public's propensity to spend was offset, at least in part, by the increased elasticity of the base.

In the absence of a support policy, increases in the desired currency ratio, the discount rate and reserve ratios lower the money supply and have no effect on the base. The support policy substantially changed the monetary effect of changes in these parameters. The increased wartime demand for currency raised both the base and the money supply, but raised the former more than the latter. Discount policy was adjusted to the support program. The discount rate was maintained at a relatively low level and preferential borrowing rates were introduced. These served to reduce the growth rate of the base. Their elimination early in the postwar period and the subsequent increase in the discount rates worked in the opposite direction. Under the support policy, increases in the discount rate have a slightly expansive effect on the base and the money supply.

Required reserve ratios were changed several times during the period. In 1942, the average ratio was lowered slightly. Had the support policy not been in effect, the reduction would have lowered interest rates on the
loan market. Under the support policy, they had no direct effect on loan rates; instead, they had the effect of reducing the growth rate of the base. Postwar increases in the reserve requirement ratios, of course, had the opposite effect. It is not surprising, therefore, to find that the contractive effect of each of the increases in reserve requirement ratios in 1948 was rather rapidly offset by increases in the base (14).

The principal explanations of the inflationary experience during the early years of the support policy are given by the behavior of the currency ratio and the budget deficit. Both increased substantially during the war. Minor reductions in the average requirement ratio or introduction of a preferential lending rate on securities had only a slight effect in the opposite direction, an effect that was far from sufficient to reverse the expansion in the money supply.

The same variables — Treasury deficit and currency ratio — explain the puzzling features of postwar monetary policy. Shortly after the war, the deficit was replaced by a substantial surplus, and the currency ratio fell sharply. Each of these induced a decline in the growth rate of the base and the money supply. The budget surplus and the decline in the currency ratio pushed market rates to the ceiling rate and thus permitted the Federal Reserve to decelerate the monetary base. Given the Federal Reserve's traditional concern for "orderly money market conditions," deceleration of the base was a response to the absence of pressure from market rates on the ceiling rate.

The return to an inflationary monetary policy after the start of the Korean War can be explained in much the same way. The budget surplus was replaced by a deficit, and the downward adjustment of the currency ratio was completed. The slack in the system disappeared, so that the base was pulled up by the renewed market pressures. The Federal Reserve continued to respond to these pressures until fears of inflation encouraged them to demand and eventually to exercise renewed authority.

A Brief Digression on Recent Policy.

Recent monetary history has produced a mirror image of the constrained system just discussed. Treasury and Federal Reserve concern for the balance of payments position and the outflow of gold encouraged them to seek policies that would prevent interest rates from falling. Despite sub-

(14) For a more detailed discussion of the timing and magnitude of the offsetting movements in the base, see An Alternative Approach to the Monetary Mechanism, op. cit., p. 66.
stantial unemployment in 1961, expansionary monetary policy was regarded as inappropriate. Domestic expansion was to be achieved by fiscal policy. According to Federal Reserve descriptions, their policy would seek to improve the balance of payments positions by slightly increasing monetary restraint.

We accept the description of fiscal policy. The characterization of monetary policy, however, is quite misleading. From 1962-65, the monetary base grew at the fastest rate since World War II. Moreover, this growth rate was achieved by the expansion of the Federal Reserve's portfolio of securities. Never before has the Federal Reserve engineered a growth rate of similar size and duration in peacetime. Yet, the yield on government securities has not declined.

The key to an understanding of recent monetary experience is provided by the constraint on interest rates. The ceiling on interest rates imposed during the War has been replaced by a floor. The constraint, therefore, works in the opposite direction. If the market-determined rate is above the floor, the constraint is inoperative, and the elasticities of interest rates are those shown in Table I. Whenever the market rate becomes less than the minimum rate, the base must be reduced, or some other policy must be introduced, to raise interest rates. If the base becomes endogenous, the constraint becomes effective and the elasticities in Table III are again applicable.

Without the substantial tax reduction and the resulting increase in the deficit, sustained expansion of the base would have been inconsistent with the floor on interest rates. Fiscal policy, therefore, permitted an expansionary monetary policy of record proportion. By increasing the upward pressure on interest rates, fiscal policy permitted the base to expand while the floor on interest rates was maintained.

The currency ratio and the public's demand for loans contributed to the admissible rate of expansion in the base. The currency ratio has increased persistently since 1961, and banks have expanded their loan portfolios substantially during the period. Both movements operated to raise interest rates and thereby permitted an acceleration of the growth rate of the base.

The Divergence of Actual from Stated Policy: Explanation of the Second Puzzle.

A second puzzling feature of U. S. monetary policy is the frequent divergence between the policies pursued by the Federal Reserve and their
interpretation of these actions. The period since 1962 provides an excellent example. Policies have been described as «less easy» despite acceleration of bank credit, of the Federal Reserve’s securities portfolio and the monetary base.

An explanation of the divergence between actual and stated policy requires some understanding of the various roles assigned to free reserves in the Federal Reserve’s view of the monetary mechanism and the role that is implicit in the theory developed here (13). The Federal Reserve has assigned an important, causal position to free reserves: — excess reserves minus member bank borrowing — since the 1920’s. High levels of free reserves are said to induce low interest rates, expansion of bank credit, and conversely. In this role, free reserves are viewed as the center of the mechanism connecting monetary policy to the stocks of money and bank credit. On occasion, free reserves are used also as the target of monetary policy. Open market operations are conducted so that free reserves are kept at a prescribed level or within a specified range. In their third role, free reserves are taken as an indicator of monetary policy and of the position of the bank credit market. Movements of free reserves are interpreted as changes in the extent of «money market pressure». A reduction in free reserves is interpreted as a sign of increased pressure on the banks and a slowing of the rate of credit expansion. Higher levels of free reserves are viewed as an indication of an «easier» policy, one designed to expand the pace of economic activity; lower free reserve levels are treated as a sign of «tighter» policy.

Our analysis denies that there is a close association between levels of free reserves and the direction of monetary policy. It suggests that there is no unique association between movements of free reserves and changes in the rate of monetary or credit expansion. The role of free reserves is implicit in the analysis that has been presented. The desired reserve ratio, \( r \), and the desired borrowing ratio, \( b \), were introduced in an earlier section. The reserve ratio includes the ratio of excess reserves to total deposits, \( c \), as one component. The difference between the desired

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(13) While there is not a uniform «Federal Reserve view» accepted by all members of the Federal Open Market Committee, there is a strong tendency to discuss or analyze the monetary situation in terms of free reserves. For more detail on this and related points, see our The Federal Reserve’s Attachment to the Free Reserve Concept, op. cit. See also A. J. Metz’s Free Reserves and the Money Supply (Chicago: University of Chicago Press, 1962) and W. DeWald, Free Reserves, Total Reserves and Monetary Control, «Journal of Political Economy», April 1963.
excess reserve and borrowing ratios, \( e-b \), is the desired free reserve ratio. The dependence of \( e \) and \( b \) on interest rates and the discount rate determines a corresponding dependence for the free reserve ratio. The interest elasticity of the free reserve ratio is negative and appears to be substantially greater than unity.

A solution for the level of free reserves, denoted \( F \), may be obtained from the equations of the system. The equation below expresses this solution as the product of the adjusted base, \( B^* \), and a proportionality factor, \( \Theta \). Parentheses are used to indicate that \( \Theta \) is not a constant but depends on the ratios \( r, b, t, \) and \( k \) and hence on interest rates, the discount rate, the reserve requirement ratios, etc.

\[
F = \Theta (\ldots)B^*
\]

The desired level of free reserves has been drawn as a function of the yield on securities, \( i_G \), in Figure 3. The curve has a negative slope. Its position depends on the base and the parameters included in \( \Theta \). Given the values of the base and the parameters, there exists a value of \( i_G \) for which \( \Theta = 0 \) and hence \( F = 0 \). At higher rates \( \Theta \) and \( F \) become increasingly negative and at lower rates increasingly positive. Changes in the base, in the loan rate, in the ceiling rate on time deposits, and in the other parameters change the position of the free reserve relation in a manner determined by the above equation.

The diagram clarifies the effect of monetary policy and other events on the volume of free reserves. The solid line is the initial position of the free reserves function. An open market operation expands the base and the stock of bank credit. (See Figure 2 for the effect on loans and investments and the credit markets). The free reserve curve rotates around its intersection point on the \( i_G \) axis as a result of the increase in the base. The slope becomes flatter as shown by curve 2 of the diagram. The reduction in loan rates resulting from the expansion of bank credit, however, shifts the curve up and to the right. The new curve, labelled 3 in the diagram, is flatter and has a higher intersection point on the \( i_G \) axis. The volume of free reserves in the banking system is higher after the open market operation. Similar results are obtained when other sources of the base expand. Open market sales, or other reductions in the base, reduce the volume of free reserves.

Reduction in the reserve requirement ratios or the discount rate raise the free reserve level. Both policy actions lower interest rates since they induce increases in \( L_8 \) and \( L_6 \). Furthermore, these policies alter the value of \( \Theta \) and raise the position of the curve in Figure 3. The upward
shift of the curve is thus the result of two forces: the direct effect on $\Theta$ of the lower requirement ratios or lower rediscount rate, and of the lower interest rate on the loan market.

Fiscal policies exert an effect on the volume of free reserves opposite to the effect of monetary policies. Sales of government securities to finance a deficit raise interest rates on the securities market. The higher rates on securities spill over to the loan market. Free reserves therefore fall for two reasons. First, increases in $i_0$ involve a movement along the curve in Figure 3 in the direction of higher values of $i_0$. Second, higher loan rates shift the curve to the left and thus lower the intersection point. On the other hand, a Treasury surplus used to retire outstanding debt raises the free reserve level.

![Figure 3](image)

Fig. 3 - The Response of Free Reserves to Monetary Policy

Thus changes in the level of free reserves are not necessarily an indication of changes in monetary policy. They may result from fiscal policies. Moreover, monetary and fiscal policies are not the only influences on free reserves. All of the variables in the public's loan and securities demand functions affect the level of free reserves. These variables enter as determinants of interest rates on the credit market. The relatively high interest elasticity of free reserves suggests that their effect on free reserves is not negligible.

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(16) This statement applies to the money supply and bank credit as well. But there are substantial differences in the effect on money, bank credit and free reserves. These differences depend primarily on the interest elasticities of $M$, $E$, and $F$. The effect of changes in the loans or securities demand on money and bank credit are damped by an interest elasticity substantially below unity. The interest elasticity of free reserves appears to be above unity.
The frequent divergence between actual and alleged policy can be traced to two sources: (i) the Federal Reserve's inappropriate choice of free reserves as an indicator, and (ii) the sensitivity of free reserves to the operation of non-policy variables. In particular, free reserves respond substantially to changes in the factors affecting the public's loan-demand or stock demand for securities. Suppose that the public's supply of loans to banks, \( L_p \), shifts in response to forces arising from the income process. If, for some reason, \( L_p \) declines relative to \( L_b \), interest rates decline and free reserves increase. The Federal Reserve interprets this as an increase in « ease ». In our framework, the increased free reserves are viewed as a part of the financial adjustment. If the base and policy parameters have not changed, monetary policy has not changed.

Or, assume that the growth rate of the base falls substantially as it did in 1952-3 and in 1958-9. The retardation in \( L_b \) and \( I_b \) is gradually distributed through the economy. Interest rates rise and free reserves fall markedly. The Federal Reserve correctly infers that policy is more restrictive. But after a lag of several quarters, economic activity reaches a peak and starts to decline. The public's supply of loans falls relative to the banks' demand and may even be reduced. Interest rates fall and the level of free reserves rises. Interpretations of policy based on the level of free reserves suggest that policy has become easier. The Federal Reserve may respond by further reducing the growth rate of the base through open market sales. Such interpretations are incorrect and delay appropriate action, since the Federal Reserve is convinced by the free reserve indicator that appropriate action has been taken.

The lag between turning points in economic activity and changes in Federal Reserve policy has recently been studied by Kareken and Solow (17). They attribute the delay at turning points to a « recognition lag » and suggest that the Federal Reserve is slow to act at turning points. But if the Federal Reserve interprets increases in free reserves as an indication of easier policy, movements in bank credit or maximum earning assets are not a useful measure of the timing of policy changes. The misinterpretation of the rising (or falling) volume of free reserves induces a fall (rise) in the growth rate of the base and hence in money and bank credit. This occurs even if there is speedy recognition of turning points and cannot be attributed to a long delay in recognition of the need for action.

(17) Lags in Monetary Policy, op. cit.
APPENDIX

TABLE I

Elasticities of interest rates with respect to policy variables
Notation: \( \varepsilon (x, y) \) denotes the elasticity of \( x \) with respect to \( y \)

\[
\varepsilon (i_c, B^p) = \frac{-\varepsilon (a_1, i_c) + \varepsilon (a_2, i_c) + \varepsilon (s_1, i_c) + \varepsilon (s_2, i_c)}{\Delta} < 0
\]

\[
\varepsilon (i_l, B^p) = \frac{-\varepsilon (a_1, i_c) + \varepsilon (a_2, i_c) - \varepsilon (s_1, i_c) - \varepsilon (s_2, i_c)}{\Delta} < 0
\]

\[
\varepsilon (i_c, x) = \varepsilon (a_1, x) \varepsilon (i_c, B^p) > 0 \quad \text{for } x = r', r', p, k
\]

\[
\varepsilon (i_l, x) = \varepsilon (a_2, x) \varepsilon (i_l, B^p) > 0
\]

\[
\varepsilon (i_c, S) = \frac{\varepsilon (a_2, i_c) - \varepsilon (s_1, i_c)}{\Delta} \cdot \frac{S}{S - S_p} > 0
\]

\[
\varepsilon (i_l, S) = \frac{-\varepsilon (a_2, i_c) - \varepsilon (s_2, i_c)}{\Delta} \cdot \frac{S}{S - S_p} > 0
\]

\[
\Delta = \left[ \varepsilon (a_1, i_c) + \varepsilon (s_1, i_c) \frac{S_p}{S - S_p} \right] \left[ \varepsilon (a_2, i_l) - \varepsilon (s_2, i_l) \frac{S_p}{S - S_p} \right]
\]

\[
- \left[ \varepsilon (a_2, i_l) - \varepsilon (s_2, i_l) \right] \left[ \varepsilon (a_2, i_c) - \varepsilon (s_2, i_c) \frac{S_p}{S - S_p} \right]
\]

SYMBOLS:

- \( S_p \) = Stock of securities held by the public
- \( a_1 \) = Banks' demand function for securities
- \( a_2 \) = Banks' demand function for loans
- \( s_1 \) = Public's demand function for securities
- \( s_2 \) = Public's supply function of loans
- \( i_c \) = Interest rate on securities
- \( i_l \) = Interest rate on loans

TABLE II

Elasticities of the Money Supply and Bank Credit

1. \( \varepsilon (M, B^p) = 1 + \varepsilon (m, i_c) \varepsilon (i_c, B^p) + \varepsilon (m, i_l) \varepsilon (i_l, B^p) < 0 \)

2. \( \varepsilon (E, B^p) = 1 + \varepsilon (a, i_c) \varepsilon (i_c, B^p) + \varepsilon (a, i_l) \varepsilon (i_l, B^p) < \varepsilon (M, B^p) \)

3. \( \varepsilon (M, x) = \varepsilon (m, x) + \varepsilon (m, i_c) \varepsilon (i_c, x) + \varepsilon (m, i_l) \varepsilon (i_l, x) > \varepsilon (m, x) < 0 \)

4. \( \varepsilon (E, x) = \varepsilon (a, x) + \varepsilon (a, i_c) \varepsilon (i_c, x) + \varepsilon (a, i_l) \varepsilon (i_l, x) > \varepsilon (a, x) < 0 \)

Note: \( \varepsilon (a, x) < \varepsilon (m, x) < 0 \)
5. \( \varepsilon (M, x) - \varepsilon (E, x) = \varepsilon (m, x) [\varepsilon (m, B^a) - \varepsilon (E, B^a)] - [\varepsilon (a, x) - \varepsilon (m, x)] \)

6. \( \varepsilon (M, S) = \varepsilon (m, i_0) \varepsilon (i_0, S) + \varepsilon (m, i_x) \varepsilon (i_x, x) > 0 \)

7. \( \varepsilon (E, S) = \varepsilon (a, i_0) \varepsilon (i_0, S) + \varepsilon (a, i_x) \varepsilon (i_x, S) > \varepsilon (M, S) \)

Note: \( m = \) monetary multiplier,
\( a = \) asset-multiplier.
Both were defined in footnote 11.

**TABLE III**

Elasticities of the Base and the Loan Rate Under an Effective Support Policy

1. \( \varepsilon (B^a, S) = \frac{\varepsilon (a_2, i_2) - \varepsilon (s_2, i_k)}{[\varepsilon (a_2, i_2) - \varepsilon (s_2, i_k)] - [\varepsilon (a_1, i_2) + \varepsilon (s_1, i_k)]} \frac{S}{S - S_p} > 0 \)

2. \( \varepsilon (B^a, x) = -\varepsilon (a_1, x) > 0 \) since \( \varepsilon (a_1, x) = \varepsilon (a_2, x) \)

for \( x = r^f, r^i, \rho, k \)

3. \( \varepsilon (i_L, S) = \frac{1}{\Delta} \frac{S}{S - S_p} > 0 \)

4. \( \varepsilon (i_L, x) = \frac{\varepsilon (a_1, x) - \varepsilon (a_2, x)}{\Delta} = 0 \)

5. \( \varepsilon (i_L, c) = \frac{\varepsilon (a_1, i_0) + \varepsilon (s_1, i_0) \frac{S_p}{S - S_p} - \varepsilon (a_2, i_0) + \varepsilon (s_2, i_0)}{\Delta} > 0 \)

Symbols are given at the bottom of **TABLE I**.
\( c = \) ceiling rate.