1985

Logic as a Tool for Discovery: What Sorts of Knowledge Can We Acquire Through Logic?

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What Sorts of Knowledge Can We Acquire Through Logic?

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> *". . . people often say that logic can tell us no more than we already know: but the trouble is that, until logic tells us, we may not know just what are the things which in this odd sense we are said to know already."* [P. T. Geach, "On Teaching Logic," *Philosophy*, 54 (1979), 5.]

Professor Geach is speaking, of course, about *deductive* logic, which, by popular if misleading textbook accounts, allows us to draw only those conclusions that are already implicated or 'contained' in the premises of our reasonings. By this account, deductive inference 'adds' no 'new' information to our present store.

But, as Geach points out, until we analyze our present store of information and deduce various of its implications, we do not know what information we have 'already' at hand.

Whether 'given' by sociological survey or the human sensorium, whether described in natural language or technical terminology -- *data* do not constitute *information* until processed and interpreted by some inferential machinery. *Access* to information is always mediated by *inference*. Determining exactly what information is at hand often requires some logical artifice and analysis. This is true on the most elementary level of information exchange.

For example: suppose you are a student in my class and one day I come into class and announce the following:

> *(1) It's not the case that either you will not fail the final exam or you will not pass the course.*

This could be crucial information, but what have I said? It is unfortunately formulated in a very convoluted and pedantic fashion. When I have made this announcement to my classes, they understandably have asked me what I meant. Suppose I respond:

> *I mean: (1a) It's not the case that if you fail the final you won't pass the course.*

Some students begin to get the drift: They may fail the final exam but still pass the course. But who would ever guess offhand that both of my statements above are logically (and demonstrably) equivalent to the following bizarre prediction?

> *(1b) You will fail the final but pass the course.*

There is more to statement (1) than meets most students' eyes (or ears). It may be that the information conveyed by (1b) is somehow already 'contained' in (1); (1b) is in fact logically implied by both (1) and (1a). But until a bit of logical analysis confirms that (1) or (1a) implies (1b), most of us are not likely to realize (let alone accept) this fact as something we *know*.

Imagine the folowing case: I approach you on a dark, deserted street, with my hand in my coat pocket. I give you an important piece of information; I say:

(2) I'll shoot you unless you hand over your money.

Whether you take the situation seriously depends, of course, on your making certain presuppositions; namely, on whether you believe me armed, serious and capable of shooting you. Suppose we stipulate these presuppositions. What information do you have about the conditions for saving your life? What do you *know* about the conditions for saving your life? Have I told you, can you infer, for example, that

(2a) I will NOT shoot you if you hand over your money ?

This depends.

The interpretation of (2) depends, in particular, on the *logical* interpretation of the tricky if ordinary term *unless*. Logic cannot tell you what I, the speaker, have in mind; but logic *can* tell you -- definitively and exhaustively -- what the basic logical alternatives are for construing my statement. They are three:

(2a) I will NOT shoot you IF you hand over your money.

(2b) I will NOT shoot you ONLY IF you hand over your money.

(2c) I will NOT shoot you IF AND ONLY IF you hand over your money.

In the case of (2), unlike (1), there may be *less* to the statement than meets the eye: If (2b) is what I mean, then your handing over your money may not be enough to save your life.

Logic can tell you that (2) is ambiguous, and recommends that, if you dare, you ask me which of the above alternatives (2a) - (2c) I have in mind. Without this information, logic tells you, you don't know what it will take to save your life -- you actually do not *know* what information you may already have been given.

What logic can tell us about what we do or do not have at hand in the way of information will not always be a life-or-death matter; but, often enough, what information we have is no better than the logic we have available to unpack or interpret it.

These simple examples should suffice to show that even on the level of 'everyday' discourse, the meaning and information conveyed by simple-seeming statements is often neither clear nor explicit. The question of what information is conveyed by language cannot be separated from the question of what the language means; and issues of meaning often hang on the interpretation of key *logical expressions* (like *either...or, not, if, only if, unless*) whose proper and precise force requires some analysis.

Whether or not we understand a statement is often a function of whether we understand its logical *implications*: whether we understand what, given certain assumptions, logically follows from the statement. For example: assuming the following

(3) You hand over your money

does it 'follow logically' from (2) that

(4) I will not shoot you ?

To decide, we need employ logic to the analysis of two matters:

1. The logical force of *unless*: whether (2) is to be interpreted as (2a), (2b), or (2c).

2. Whether (3) together with (2a), (2b), or (2c) logically entails (4).

Re/ matter 1: Logic will tell us the alteratives, but not which interpretation I, the speaker, has in mind. Re/ matter 2: Logic will tell us that (3) together with (2a) or (2c) logically implies (4); but that (4) does *not* follow from (3) and (2b).

From this last example we see that *reasoning* is at least tacitly involved in the very interpretation of meaning. We can reconstruct the reasoning involved in the three interpretations of the implications of (2) as follows: We'll let letters stand in for the component sentences in order to more conveniently and graphically depict the *logical form* of the following pieces of reasoning.

Let: S = I will shoot you. H = You hand over your money.

(2a) Not S if H	(2b) Not S only if H	(2c) Not S if and only if H
(3) H	(3) H	(3) H
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(4) Not S	(4) Not S	(4) Not S

The reasoning from (2a) or (2c) to (4) is deductively *valid*, by virtue of its *logical form*. For example, the *form* of reasoning from (2a), the *structure* resulting from the deployment of key logical expressions like *not, if, . . .*, is such that any reasoning of this form with true premises will have a true conclusion. This requires a bit of artifice to prove, but it is the case. By contrast, the reasoning from (2b) to (4) is *invalid*: the *logical form* of the reasoning is such that reasoning of this form could lead from true premises to a *false* conclusion.

Invalidity in the logical *form* of a piece of reasoning can be demonstrated by example: we find an analog of the reasoning in question that has the same relevant form and that has obviously true premises but an obviously false conclusion. The following reasoning has the same basic *form* as that from (2b) to (4), and leads from *true* premises to a *false* conclusion:

Let: P = You are a porpoise. M = You are mammalian.

(2b′) P only if M

(3′) M

(4′) P

Why do we say that the reasoning from (2b′) to (4′) has *the same* basic form as that from (2b) to (4)? This discovery requires a bit of formal artifice; but it is demonstrable. The point here is that the deductive *validity* of reasoning as well as the meaning or implications of statements is a function of *logical form.* Logical form is conveniently represented by means of symbolic or schematic abstraction, as in the schemas above. These abstractions provide *x-rays*, as it were, of the essential logical structures underlying our reasoning in natural language. Without a grasp of underlying logical form or structure, we can precisely assess *neither* the logical meaning and implications of statements *nor* the validity of our reasonings.

Thus, three crucial categories of discovery made possible by a bit of formal apparatus and artifice are:

1. Exactly what statements that purport to convey information *mean.*

2. Exactly what the information we have already at hand, given whatever our background assumptions, *implies.*

3. Whether 'new' information (some inference), purportedly derived or implied from information already or expressly at hand (premises), in point of logical fact *follows*; whether our inferences are in fact *valid* and reliable vehicles of 'new' information.

Another crucial issue for assessing information at hand or new information is consistency of fit with what we already know, believe or accept. Another category of discovery that often requires a bit of formal artifice is the discovery of inconsistency:

> *If a man's assumptions do not include a pair of flagrantly inconsistent ones, he may still be inconsistent in his position, but in a way that it takes logical artifice to bring out.* [Geach, *opus cit.*, 5.]

What, now, in the way of logical artifice is required to facilitate these categories of discovery? What manner of deductive apparatus is required to process and access information, even on the level of everyday discourse? What logical tools are wanted to test and maintain the deductive inferential machinery that delivers reliable information from the logically untutored deliverances of ordinary language?