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The Levered Equity Risk Premium and Credit Spreads: A Unified Framework*

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Abstract

We embed a structural model of credit risk inside a dynamic continuous-time consumption-based asset pricing model, which allows us to price equity and corporate debt in a unified framework. Our key economic assumptions are that the first and second moments of earnings and consumption growth depend on the state of the economy which switches randomly, creating intertemporal risk, which agents prefer to resolve sooner rather than later, because they have Epstein-Zin-Weil preferences. Agents optimally choose dynamic capital structure and default times. For a dynamic cross-section of firms, our model endogenously generates a realistic average term structure and time series of actual default probabilities and credit spreads, together with a reasonable levered equity risk premium, which varies with macroeconomic conditions.

JEL Classification Numbers: E44, G12, G32, G33

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A growing body of empirical work indicates that common factors may affect the equity risk premium and credit spreads on corporate bonds. In particular, there is now substantial evidence that stock returns can be predicted by credit spreads and that movements in stock-return volatility can explain movements in credit spreads.¹ This demonstrates that there is “*overlap between the stochastic processes for bond and stock returns*” (Fama and French (1993, p. 26, our emphasis)).

The existence of common factors indicates that the two well-known puzzles, the equity risk-premium puzzle and the credit risk puzzle, are inherently linked.² In addition, there is evidence suggesting that the equity risk premium is related to fundamental macroeconomic risks. For example, Bansal and Yaron (2004) show that a substantial fraction of the equity risk premium could stem from exposure to long-run fluctuations in macroeconomic growth rates. Furthermore, dynamic corporate financing decisions, which are important determinants of credit spreads, are also likely to be driven by macroeconomic factors as shown, for example, in Korajczyk and Levy (2003). The aim of this paper is to provide a unified framework, which relates the equity risk premium, the term structure of credit spreads, and endogenous default probabilities to risks in macroeconomic fundamentals.

To be successful, our methodology must resolve the following tensions. First, corporate debt is a finite maturity claim, while equity is a perpetual claim. To resolve the credit spread puzzle, one must correctly price the term structure of risk. However, equity only depends on the risk term-structure’s far end. Second, while the equity risk premium and credit spreads reflect aggregate risk, they are impacted by individual firms’ leverage choices and hence the cross-section of firm level risks. Third, capital structure decisions are dynamic and history-dependent which affects the risk term-structure.³

To this end, we price equity and corporate bonds in a continuous-time consumption-based asset pricing model with a representative agent. Similar to Bansal and Yaron (2004), we assume the first and second moments of consumption and earnings growth are stochastic and the representative agent has Epstein-Zin-Weil preferences. Variation in the first and second moments of growth rates introduces intertemporal macroeconomic risk into our model, which affects *both* stock and corporate bond markets. The Epstein-Zin-Weil agent has a preference for how quickly intertemporal risks are resolved. We assume that she prefers the early resolution of intertemporal risk.

Equity and debt are claims to cash flow processes and issued by individual firms. Optimal leverage is chosen dynamically to maximize individual firm value, thus dividing firm earnings into coupon payments to bondholders and dividends to equityholders. The optimal default boundary is chosen by maximizing levered equity value. Thus, both leverage and default decisions are endogenous. Importantly, the prices of equity and debt are not only linked by a common state-price density, but they are also affected by the optimal leverage and default decisions. Essentially, we embed a structural model

¹See e.g. Chen, Roll, and Ross (1986), Keim and Stambaugh (1986), Campbell (1987), Fama and French (1989), Fama and French (1993), Whitelaw (1994), Jagannathan and Wang (1996), Cremers (2002), and Tauchen and Zhou (2006).

²The credit risk puzzle refers to the finding that structural models of credit risk generate credit spreads smaller than those observed in the data when calibrated to observed default frequencies. Recent evidence is presented in Eom, Helwege, and Huang (1999), Ericsson and Reneby (2003), and Huang and Huang (2003).

³E.g., Lemmon, Roberts, and Zender (2008) find empirically that financing decisions show strong path-dependence.

(e.g. Fischer, Heinkel, and Zechner (1989) and Leland (1994)) inside an equilibrium consumption-based asset pricing model. We call the resulting framework a *structural-equilibrium* model.⁴

Central to our approach is the importance of considering the dynamic behavior of the distribution of firms in an economy. Consequently, we require a model where leverage does not vanish in the long-run, and so, a dynamic capital structure is essential. To understand why distributional dynamics are important, consider first the credit spread of a single firm at the date the leverage decision is made (the refinancing date), as is standard in the literature (see, for example, Huang and Huang (2003) and Chen, Collin-Dufresne, and Goldstein (2008)). A typical study of this kind would set the firm's leverage exogenously equal to the average leverage of firms in a particular rating class, e.g. BBB, and choose the default boundary so the firm's default probability at a given maturity equals the corresponding average default probability for its rating.

This exercise, while widely practiced, misses several important empirical considerations. First, it ignores the fact that in reality equityholders choose the time of default and leverage optimally. Second, it considers credit spreads solely at refinancing points, i.e. when debt is issued, whereas the majority of empirically reported spreads are based on observations made at times when debt is not being issued. Most of the time most firms are *not* at their refinancing points. For example, as Fischer et al. (1989) show, even very small adjustment costs can lead to first-order deviations from the optimal target leverage ratio. Empirically, Welch (2004) and Leary and Roberts (2005) show that firms do indeed refinance infrequently. Third, it provides an estimate of the credit spread for a typical *individual* firm, with a default probability equal to the historically observed default probability for a *sample* of firms of which this individual firm is representative (where sample is defined e.g. by rating). However, empirical estimates of credit spreads are not based on an individual firm. Therefore, for consistent calibration, it is crucial to generate a model-implied sample of firms and match its average default probability with the average of the corresponding empirical sample.

Our approach enables us to account for all the above issues. First, financing decisions in our model are optimal. This makes resolving the credit spread puzzle more challenging, since we are concerned whether our model can generate realistic credit spreads *and* realistic *endogenous* default rates. In other words, we do not have the flexibility of setting default boundaries exogenously so that actual default probabilities match the data. Second, we compute an average credit spread and default probability for a cross-section of firms. Finally, the time evolution of the distribution of firms produces the 'true dynamics' of the model. By exploiting true dynamics, we can produce a realistic term structure of *both* credit spreads and default probabilities. In true dynamics, average five- and ten-year BBB credit spreads are, respectively, 75 and 115 basis points (bp), close to the term structure of the BBB credit spread's default risk component, and the corresponding cumulative default probabilities are in the range of 1–4.5% for five years and 3.5–13% for ten years, consistent with historical default data.

⁴In contrast with our structural-equilibrium model, in pure structural models, it is impossible to recover actual default probabilities since the mapping between the risk neutral and actual probability measure is not modeled. Consequently, such models alone cannot be used to explain the credit spread puzzle.

We also study the time-series properties of credit risk variables. The optimal earnings default boundary, default rates, and credit spreads implied by the model are profoundly countercyclical. Equityholders optimally prefer earlier default in bad times, since their default option is less valuable. As a result, corporate defaults cluster when the state of the economy worsens even if individual firms' earnings have not changed. Credit spreads reveal hysteresis and depend both on the current macroeconomic environment and the state of the economy when the firm refinanced. In addition to generating a realistic term structure of credit spreads, our model delivers a levered equity premium of above 3% and levered equity return volatility of just under 20%.

Our results are largely driven by intertemporal risk and the dynamics of default risk in the cross-section. To understand how intertemporal risk impacts asset pricing, recall that the representative agent uses risk-neutral probabilities for valuation. For example, the model-implied credit spread depends on the risk-neutral default probability rather than on the actual one. It is well-known that for a risk-averse agent, the risk-neutral probability of a bad event occurring exceeds its actual probability. What is crucial in the context of our model is that asset prices depend on the risk-neutral probability of the economy moving from the good state to the bad state. Even though bad states may not last long, if the agent prefers earlier resolution of uncertainty, the risk-neutral probability of entering the bad state increases the average duration of the bad state in the risk-neutral world. In other words, the agent prices assets as if bad states last longer than is actually the case, which raises risk premia.

To see why cross-sectional dynamics are important in our model, observe that the term structure of actual default probabilities for an *individual* firm is much steeper than in the data and the five-year default probability is particularly small. However, when we compute *average* default probabilities for a *cross-section* of BBB firms, there are two crucial changes. First, the resulting term structure is higher at five years and much closer to the data. Second, the term structure is less steep, and so the ten-year actual default probability is also much closer to the data. The first effect is caused by positive skewness in the long-run distribution of firm cash flows, which increases the number of firms close to default. The second effect is a consequence of the *time evolution* of the distribution of firm cash flows. Over time, firms' earnings on average increase which reduces positive skewness, diminishing the mass of firms close to the default boundary. Hence, the difference between ten- and five-year default probabilities is smaller in the cross-section than for an individual firm.

In the remainder of the introduction we discuss the relationship between our paper and the existing literature. Since our paper builds on both structural models from corporate finance and consumption-based models from asset pricing, the related literature is vast (see Table I for a brief overview). On one side, our paper inherits features of structural models of credit risk (Merton (1974), Fischer et al. (1989), Leland (1994), Goldstein, Ju, and Leland (2001), Hackbarth, Miao, and Morellec (2006), and Strebulaev (2007)). On the other side, our model builds on consumption-based asset pricing models (Bansal and Yaron (2004) and Calvet and Fisher (2008)).

We now discuss several papers with which our paper is particularly close. It is important to note that all these papers base their analysis on an individual firm, whereas ours is based on the dynamics of

a cross-section of firms. The contingent-claims structural model that our paper is most closely related to is Hackbarth et al. (2006).⁵ They also study the influence of macroeconomic factors on credit spreads. Importantly, they are the first to show that macroeconomic factors imply a countercyclical earnings default boundary. There are several key differences between our models. Firstly, Hackbarth et al. do not use a state-price density linked to consumption and therefore do not study the equity risk premium and its relation to credit spreads. Also, their model does not allow them to check the size of actual default probabilities. Finally, while we study the impact of macroeconomic factors on both cash flows and discount rates, Hackbarth et al. focus purely on the cash flow channel by assuming that firms' earnings levels jump down in recessions.

A second closely related paper is Chen et al. (2008). They study a pure consumption-based model and use two distinct mechanisms to resolve the equity risk premium and credit spread puzzles. The first mechanism is habit formation, which makes the marginal utility of wealth high enough in bad states so that the equity risk premium puzzle is resolved. This does not resolve the credit spread puzzle, because actual default probabilities and thus credit spreads are procyclical. To remedy this, Chen et al. use a second mechanism: they force the asset-value default boundary to be *exogenously countercyclical*. There are several key differences between our models. First, we only need one economic mechanism to generate a realistic credit spread and risk premium, as outlined above. Second, while the earnings default boundary in our model is countercyclical, the asset-value default boundary is *endogenously procyclical*.⁶ Third, the risk premium in our model is directly affected by default risk creating a *levered* risk premium and capital structure is endogenous. Finally, while Chen et al. have to exogenously calibrate the default probability, our model *endogenously* produces realistic default probabilities *and* credit spreads.

Also closely related is David (2008), the first paper to study the impact of unobservable regime shifts in fundamental growth rates on credit spreads. David restricts the state-price density to one that can be obtained from a representative agent with power utility, so that the agent is indifferent to the timing of intertemporal risk. He also does not endogenize corporate financing decisions.⁷

A central feature of our approach is the concept of the time evolution of the cross-sectional distribution of firms, which is essential for calibrating our dynamic model consistently. This concept is related to Strebulaev (2007), who was the first to demonstrate the importance of the cross-sectional distribution of firms for dynamic capital structure. Whereas Strebulaev (2007) studies how the cross-sectional

⁵While contemporaneous work by Chen (2008) seeks to resolve the low-leverage and credit spread puzzles, it considers only the results at the optimal refinancing point for an individual firm, which makes it impossible to address the impact of cross-sectional dynamics on default probabilities and credit spreads. Neglecting cross-sectional dynamics also leads Chen to the conclusion that he can resolve the low-leverage puzzle. As shown in Bhamra, Kuehn, and Strebulaev (2008), this conclusion is overturned when cross-sectional dynamics are accounted for. Chen also does not analyze the term structure of credit spreads (he focuses only on ten-year spreads) and does not study comovement between bond and stock markets and the equity premium puzzle.

⁶The model of Hackbarth et al. (2006) may also imply, in a different setting, that asset-value and earnings default boundaries can move in opposite directions.

⁷David (2008) also accounts for inflation risk and in addition to resolving the credit spread puzzle, he prices equity (his model matches the historical Sharpe ratio when risk aversion is 16). Tan and Yan (2006) use the same framework as David (2008), but with an observable mean-reverting growth rate for firm earnings and without explicitly accounting for inflation. See also David and Veronesi (2002).

distribution implied by the dynamic model affects leverage, we focus on how the time-evolution of the distribution affects the term structures of credit spreads and default probabilities.⁸

Our paper is not the first to consider default in a consumption-based model (see e.g. Alvarez and Jermann (2000), Kehoe and Levine (1993) and Lustig (2005)). These papers focus on default from the viewpoint of households, which have identical preferences, but are subject to idiosyncratic income shocks. Chan and Sundaresan (2005) consider the bankruptcy of individuals in a production framework, looking at its impact on the equity risk premium and the term structure of risk-free bonds.

The remainder of the paper is organized as follows. Section I describes the structural-equilibrium model with intertemporal macroeconomic risk and Epstein-Zin-Weil preferences. Section II explores the implications of the model for pricing corporate debt and levered equity and develops an intuitive decomposition for the Arrow-Debreu default claim when capital structure is static. In Section III we extend the model to dynamic capital structure. In Section IV, we study the model's implications. We conclude in Section V. Proofs and other additional material are contained in the Appendices.

I Model

In this section we introduce the structural-equilibrium model with intertemporal macroeconomic risk. The basic idea is simple: we embed a structural model inside a representative agent consumption-based asset pricing model. Debt and levered equity can therefore be valued using the state-price density of the representative agent. Two consequences of this modeling approach are worth noting. Credit spreads now depend on the agent's preferences and aggregate consumption, and the equity risk premium is now affected by default risk.

It is also important to mention what our structural-equilibrium model does not do. It does not account for the impact of default on consumption, because we model consumption as an exogenous process. Furthermore, our model ignores the impact of agency conflicts on the state-price density, because the state-price density in our model is the marginal utility of wealth of the representative agent. While incorporating these two important effects is beyond the scope of this paper, this type of approach with an exogenous state-price density is an important first step towards our understanding of how subtly the credit spread and equity risk premium puzzles are intertwined.

I.A Aggregate consumption and firm earnings

There are N firms in the economy. The output of firm n , Y_n , is divided between earnings, X_n , and wages and other human capital income, W_n , paid to workers. Aggregate consumption, C , is equal to aggregate output. Therefore, $C = \sum_{n=1}^N Y_n = \sum_{n=1}^N X_n + \sum_{n=1}^N W_n$. We model aggregate consumption and

⁸Berk, Green, and Naik (1999) and Bertola and Caballero (1994) exploit the cross-sectional distribution of firms study stock returns and aggregate investment dynamics, respectively.

individual firm earnings directly, and thus aggregate wages are just the difference between aggregate consumption and aggregate earnings.⁹

Aggregate consumption, C , is given by

$$\frac{dC_t}{C_t} = g_t dt + \sigma_{C,t} dB_{C,t}, \quad (1)$$

where g is expected consumption growth, σ_C is consumption growth volatility, and $B_{C,t}$ is a standard Brownian motion.

The earnings process for firm n is given by

$$\frac{dX_{n,t}}{X_{n,t}} = \theta_{n,t} dt + \sigma_{X,n}^{id} dB_{X,n,t}^{id} + \sigma_{X,n}^s dB_{X,t}^s, \quad (2)$$

where θ_n is the expected earnings growth rate of firm n , and $\sigma_{X,n}^{id}$ and $\sigma_{X,n}^s$ are, respectively, the idiosyncratic and systematic volatilities of the firm's earnings growth rate. Total risk, $\sigma_{X,n}$, is given by $\sigma_{X,n} = \sqrt{(\sigma_{X,n}^{id})^2 + (\sigma_{X,n}^s)^2}$. While firms account for total risk in making financial decisions, the idiosyncratic component generates cross-sectional heterogeneity in risks over time.¹⁰ The standard Brownian motion $B_{X,t}^s$ is the systematic shock to the firm's earnings growth, which is correlated with aggregate consumption growth:

$$dB_{X,t}^s dB_{C,t} = \rho_{XC} dt, \quad (3)$$

where ρ_{XC} is the correlation coefficient. The standard Brownian motion $B_{X,n,t}^{id}$ is the idiosyncratic shock to firm n 's earnings, which is correlated with neither $B_{X,t}^s$, $B_{C,t}$, nor with other firms' idiosyncratic shocks.

I.B Modeling intertemporal macroeconomic risk

To model intertemporal macroeconomic risk in a simple and realistic fashion, we assume that the first and second moments of macroeconomic growth rates are stochastic. Specifically, we assume that g_t , θ_t , $\sigma_{C,t}$, and $\sigma_{X,t}^s$ depend on the state of the economy, ν_t , which is either 1 or 2. Hence, the conditional expected growth rate of consumption, g_t , can take two values, g_1 and g_2 , where g_{ν_t} is the expected growth rate when the economy is in state ν_t , and, similarly for θ_t , $\sigma_{C,t}$, and $\sigma_{X,t}^s$.¹¹ We assume state 1 is the bad state and state 2 is the good state. Since the first (second) moments of fundamental growth rates are procyclical (countercyclical), $g_1 < g_2$, $\theta_1 < \theta_2$, $\sigma_{C,1} > \sigma_{C,2}$, and $\sigma_{X,1}^s > \sigma_{X,2}^s$.

⁹In assuming so, we follow such papers as Kandel and Stambaugh (1991), Cecchetti, Lam, and Mark (1993), Campbell and Cochrane (1999), and Bansal and Yaron (2004).

¹⁰For examples of this approach in the asset pricing literature see, e.g. Gomes et al. (2003) and Zhang (2005).

¹¹To ensure idiosyncratic earnings volatility, $\sigma_{X,t}^{id}$, is truly idiosyncratic, we assume it is constant and thus independent of the state of the economy. We also assume that the correlation coefficient, ρ_{XC} , is constant.

The state changes according to a 2-state Markov chain, defined by λ_{ν_t} , $\nu_t \in \{1, 2\}$, which is the probability per unit time of the economy leaving state ν_t .¹² The Markov chain gives rise to uncertainty about the future moments of consumption growth. This intertemporal consumption risk impacts the state-price density only if the representative agent cares about the intertemporal distribution of risk. To ensure this, we assume the representative agent has the continuous-time analog of Epstein-Zin-Weil preferences. Consequently, the representative agent's state-price density at time- t , π_t , is given by

$$\pi_t = \left(\beta e^{-\beta t} \right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t} e^{\int_0^t p_{C,s}^{-1} ds} \right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, \quad (4)$$

where β is the rate of time preference, γ is the coefficient of relative risk aversion (RRA), and ψ is the elasticity of intertemporal substitution under certainty (EIS).¹³ The Epstein-Zin-Weil agent cares whether news about consumption growth and hence future consumption is good or bad. Her state-price density then, unlike that of the power-utility agent, depends on the value of the claim to aggregate consumption per unit consumption, i.e. the price-consumption ratio, p_C . We assume $\gamma > 1/\psi$, which implies the agent prefers intertemporal risk to be resolved sooner rather than later. Consequently, bad news about consumption growth increases the state-price density, as can be seen from (4).¹⁴

The intertemporal distribution of risk is affected not only by whether news about consumption growth is good or bad, but also by the *speed* at which this news arrives. The rate of news arrival is governed by the rate at which the distribution for the Markov chain converges to its long-run distribution. The convergence rate is given by $p = \lambda_1 + \lambda_2$. The long-run distribution is given by $(f_1, f_2) = (\lambda_2/p, \lambda_1/p)$, where f_i is the long-run probability of being in state i . A smaller p means news arrives more slowly and thus there is more intertemporal risk. When quantifying the level of intertemporal risk, it is more intuitive to think in terms of units of time rather than the reciprocal of time. Therefore, we measure the amount of intertemporal risk in the economy via the *half-life* of the Markov chain, $t_{1/2}$, given by $t_{1/2} = \ln 2/p$. The larger the half-life, the greater the level of intertemporal risk. Increasing intertemporal risk slows the rate of news arrival. Since $\gamma > 1/\psi$, a slower news arrival rate decreases the price-consumption ratio, which increases the state-price density.

Importantly, the switching probabilities per unit time, λ_{ν_t} , $\nu_t \in \{1, 2\}$, are not directly relevant for valuing securities. Instead, we must account for risk by using the *risk-neutral* switching probabilities

¹²The extension to more than two states does not provide any further economic intuition and is straightforward. In particular, the 3-state version of our model and its quantitative implications are very close to that of the 2-state model. Details are available upon request.

¹³The continuous-time version of the recursive preferences introduced by Epstein and Zin (1989) and Weil (1990) is known as stochastic differential utility (SDU), and is derived in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness. Kraft and Seifried (2008) show the version of SDU we use is well defined under a mixed Brownian-Poisson filtration. The derivation of (4) is in the Supplement, which is available upon request.

¹⁴We do not yet require the assumption that $\psi > 1$, since the state-price density always increases in the bad state, independent of the EIS. One can see this from Equation (4), by noting that when $\psi > 1$ ($\psi < 1$) and p_C is procyclical (countercyclical), the state-price density increases in the bad state, since p_C is raised to a negative (positive) power. Intuitively, the cyclicalities of the price-consumption ratio is governed by the trade-off between cash flow and discount rate effects and hence by the agent's preferences over time, i.e. the EIS, which is distinct from the agent's preference for whether uncertainty about the intertemporal distribution of risk is resolved sooner or later.

per unit time, which we denote by $\hat{\lambda}_{\nu_t}$. The following proposition relates the risk-neutral to the actual switching probabilities (per unit time) via the state-price density.

Proposition 1 *The risk-neutral switching probabilities per unit time are related to the actual switching probabilities per unit time by the risk-distortion factor, ω ,*

$$\hat{\lambda}_1 = \lambda_1 \omega^{-1} \quad \text{and} \quad \hat{\lambda}_2 = \lambda_2 \omega, \quad (5)$$

where ω measures the size of the jump in the state-price density when the economy shifts from state 2 to state 1, i.e.¹⁵

$$\omega = \frac{\pi_t}{\pi_{t-}} \bigg|_{\nu_{t-}=2, \nu_t=1}. \quad (6)$$

The size of the risk-distortion factor depends on the representative agent's preferences for resolving intertemporal risk: $\omega > 1$ ($\omega < 1$) if $\gamma > 1/\psi$ ($\gamma < 1/\psi$), and $\omega = 1$ if $\gamma = 1/\psi$.

The risk-neutral switching probabilities are obtained from the actual probabilities by ‘distorting’ them via the *risk distortion factor*, ω , which is defined in (6) as the change in the state-price density when the state of the economy changes from good to bad. The price of risk associated with a change in the state of the economy from good to bad is given by $\omega - 1$. Therefore, the risk-neutral probability per unit time of switching from the good state to the bad state is higher than the actual probability, i.e. $\hat{\lambda}_2 > \lambda_2$. Conversely, the risk-neutral probability per unit time of switching from the bad state to the good state is lower than the actual probability, i.e. $\hat{\lambda}_1 < \lambda_1$. Since risk-neutral probabilities are used for pricing instead of actual ones, it follows that securities are priced *as if the bad state lasts longer and the good state finishes earlier* when $\gamma > 1/\psi$.

Based on the risk-neutral switching probabilities, we can compute the risk-neutral rate of news arrival, $\hat{p} = \hat{\lambda}_1 + \hat{\lambda}_2$, and the long-run risk-neutral distribution $(\hat{f}_1, \hat{f}_2) = (\hat{\lambda}_2/\hat{p}, \hat{\lambda}_1/\hat{p})$. Hence, we can compute the risk-neutral half-life, $\hat{t}_{1/2} = \ln 2/\hat{p}$, which quantifies the price impact of intertemporal risk by adjusting the actual half-life for risk. In Sections II and III we provide exact closed-form expressions for asset prices in terms of the risk-neutral rate of news arrival and long-run distribution.

While we could use any stochastic process to model the first and second moments of fundamental growth rates (as long as the underlying probability distribution is non-stationary, but converges to a stationary distribution), employing a Markov chain has two advantages. First, it allows us to ensure that bad states of the economy are of shorter mean duration than good states. The mean duration of state i is $1/\lambda_i$ and we can assume that $1/\lambda_1 < 1/\lambda_2$. This is important since, empirically, recessions are shorter than booms. This is in contrast to an Ornstein-Uhlenbeck process (the continuous time analog of the AR(1) process used in Bansal and Yaron (2004)) or a Cox-Ingersoll Ross process which have

¹⁵To distinguish between the state of the economy before and after the jump, denote the time just before the jump occurs by $t-$, and the time at which the jump occurs by t . We give precise definitions of the left and right limits, ν_{t-} and ν_t , respectively, in Equations (A1) and (A2) in Appendix A.

symmetric transition probabilities, which forces the duration of good and bad states to be equal.¹⁶ Second, we can derive asset prices exactly in closed-form rather than using the log-linear approximation of Campbell and Shiller (1988).

II Asset valuation with static capital structure

While our main goal is to explore the behavior of the economy in a dynamic financing equilibrium, to provide clearer intuition, in this section we derive the prices of all assets in the economy assuming that capital structure is static. As we shall see the valuation principles are identical in both cases.

To introduce benchmark corporate securities, equity and debt, we follow standard EBIT-based capital structure models (see e.g. Goldstein et al. (2001), Hackbarth, Hennessy, and Leland (2007), and Strebulaev (2007)) where the earnings of a firm, X , are split between a coupon, c , promised to debtholders in perpetuity and a dividend, $X - c$, paid to equityholders. If corporate income is taxed at the rate η , the after-tax distribution to equityholders is $(1 - \eta)(X - c)$.¹⁷ Equityholders have the right to default, which they exercise if earnings drop below a certain level, which is the default boundary. As we discuss in detail in Section II.D, the default boundary is endogenously state-dependent. For now we assume that default occurs in state ν_t when X reaches the default boundary, X_{D,ν_t} , for the first time. Upon default, bondholders recover a proportion of the firm's assets in lieu of coupons, i.e. a fraction α_{ν_t} of the after-tax present value of the firm's earnings.

Since the value of any security, such as debt and equity, can be written in terms of the prices of a set of Arrow-Debreu default claims, we now derive the values of these fundamental securities.

II.A Arrow-Debreu default claims

The Arrow-Debreu default claim, denoted by $q_{D,\nu_t\nu_D,T-t}$, is the time- t value of a unit of consumption paid upon default where the subscripts ν_t and ν_D denote the current state and the state at the moment of default, respectively, provided default occurs before time T . In other words, if the time- t state is ν_t and earnings hit the boundary X_{D,ν_D} from above for the first time in state ν_D at some time before T , one unit of consumption is paid that instant. Since each Arrow-Debreu default claim is effectively a digital put, their values can be derived by solving a system of ordinary differential equations

$$E_t^{\mathbb{Q}}[dq_{D,\nu_t\nu_D,T-t} - r_{\nu_t}q_{D,\nu_t\nu_D,T-t}dt] = 0, \nu_t, \nu_D \in \{1, 2\}. \quad (7)$$

Intuitively, the above conditions hold because of the no-arbitrage restriction.¹⁸

The next proposition shows that the price of the Arrow-Debreu default claim can be decomposed into several intuitive components.

¹⁶Other papers which use Markov chains to model stochastic mean growth rates and volatilities, such as Kandel and Stambaugh (1991) and Calvet and Fisher (2007), do not exploit this feature of the Markov chain approach.

¹⁷We assume this particular tax code for simplicity. Introducing other taxes (such as partial loss offsets in financial distress and personal income taxes) change neither the structure of the model nor economic intuition.

¹⁸For the case when $T \rightarrow \infty$, which we denote as $q_{D,\nu_t\nu_D}$, closed-form solutions are given in the Supplement, which is available upon request

Proposition 2 *The price of the Arrow-Debreu default claim, $q_{D,\nu_t\nu_D,T-t}$, can be written as*

$$q_{D,\nu_t\nu_D,T-t} = p_{D,\nu_t\nu_D,T-t} \mathcal{T}_{\nu_t\nu_D,T-t} \mathcal{R}_{\nu_t\nu_D,T-t}, \quad (8)$$

where $p_{D,\nu_t\nu_D,T-t}$ is the actual probability of default occurring in state ν_D before time T : $p_{D,\nu_t\nu_D,T-t} = \Pr(t < \tau_D \leq T | \nu_t, \nu_{\tau_D})$, where τ_D is the default time; $\mathcal{T}_{\nu_t\nu_D,T-t}$ is a time-adjustment factor given by the weighted-average discount factor:

$$\mathcal{T}_{\nu_t\nu_D,T-t} = \int_t^T E_t^{\mathbb{Q}} \left[e^{-\int_t^s r_u du} \right] h_{\nu_t\nu_D}(s) ds, \quad (9)$$

where $E_t^{\mathbb{Q}} \left[e^{-\int_t^s r_u du} \right]$ is the time- t discount factor for risk-free cash flows arriving at time $s > t$ and the weight $h_{\nu_t\nu_D}(s)$ is the risk-neutral density for defaulting in state ν_D , conditional on default occurring in state ν_D before time T .

$\mathcal{R}_{\nu_t\nu_D,T-t}$ is a risk-adjustment factor given by

$$\mathcal{R}_{\nu_t\nu_D,T-t} = \frac{\hat{p}_{D,\nu_t\nu_D,T-t}}{p_{D,\nu_t\nu_D,T-t}}, \quad (10)$$

where $\hat{p}_{D,\nu_t\nu_D,T-t}$ is the risk-neutral probability of defaulting in state ν_D before time T .

Existing literature on credit spreads (e.g. Chen et al. (2008)) notes that to resolve the credit spread puzzle, risk-neutral default probabilities must be high, while actual default probabilities must be low. As the above decomposition shows, *Arrow-Debreu default claims* must be relatively high while actual default probabilities are low. That can be achieved via a high risk-adjustment factor, leading to a high risk-neutral default probability, *and* a high time-adjustment factor.

The time-adjustment factor is a weighted-average discount factor. To see this suppose that default can only occur at T_1 and $T_2 > T_1$. The time-adjustment factor is then the risk-neutral expectation of the weighted average of the standard time-discount factors, $E_t^{\mathbb{Q}}[e^{-\int_t^{T_1} r_u}]$ and $E_t^{\mathbb{Q}}[e^{-\int_t^{T_2} r_u}]$, where the weights are the risk-neutral probabilities of default occurring at dates T_1 and T_2 , respectively (both probabilities are conditional on default occurring to ensure they sum to one). The intuition is that when the agent thinks default under the risk-neutral measure is more likely at the earlier date T_1 , more weight is put on the larger discount factor, $E_t^{\mathbb{Q}}[e^{-\int_t^{T_1} r_u}]$, and the time-adjustment gets larger. When default can occur at any time, the two weights are replaced by a continuum of weights, given by the conditional risk-neutral default density, $h_{\nu_t\nu_D}$, as shown in (9).

The risk adjustment is a risk premium for default. In contrast with existing literature, we can explore the dependence of the risk adjustment on preferences and the statistical properties of consumption. For a power-utility agent ($\gamma = 1/\psi$), $\mathcal{R}_{\nu_t\nu_D}$ is equal to one, i.e. when the agent is indifferent to the timing of intertemporal risk, the risk associated with not knowing the timing of default is not priced. When earlier resolution of uncertainty is preferred (as we assume throughout), the agent cares about the state of the economy at the time of future default. Since the risk price associated with

a random shift into the bad state is positive ($\omega > 1$), the agent demands extra compensation for defaults occurring in bad states. This leads to a larger gap between risk-neutral and actual default probabilities. Conversely, default in the good state is less costly. Therefore, the risk-adjustment factor for defaulting in the bad (good) state is higher (lower) than unity, $\mathcal{R}_{\nu_1} > 1$ and $\mathcal{R}_{\nu_2} < 1$.

Recent empirical studies estimate the values of the risk-adjustment, unconditional on the economy's state at default time (see e.g. Berndt et al. (2005) and Almeida and Philippon (2007)). Our model predicts that the risk-adjustment factor should exhibit procyclical time-variation for any time to maturity, $\mathcal{R}_{1\nu_D} < \mathcal{R}_{2\nu_D}$. Intuitively, as the economy worsens, the chance of the future state of the economy being better than the current state increases, thereby reducing the risk-adjustment factor.

II.B Abandonment value

The firm's state-conditional liquidation, or abandonment value, denoted by A_{ν_t} , is the after-tax value of the unlevered firm's future earnings, when the current state is ν_t :

$$A_{\nu_t} = (1 - \eta)X_t E_t \left[\int_t^\infty \frac{\pi_s X_s}{\pi_t X_t} ds \middle| \nu_t \right], \text{ for } \nu_t \in \{1, 2\}. \quad (11)$$

The liquidation value in (11) is a function of the current earnings level and is time-independent, $A_{\nu_t} = A_{\nu_t}(X_t)$. The next proposition derives the value of A_{ν_t} in terms of fundamentals of the economy.

Proposition 3 *The liquidation value in state $\nu_t \in \{1, 2\}$ is given by*

$$A_{\nu_t}(X_t) = \frac{(1 - \eta)X_t}{r_{A, \nu_t}}, \quad (12)$$

where

$$r_{A, \nu_t} = \bar{\mu}_{\nu_t} - \theta_{\nu_t} + \frac{(\bar{\mu}_j - \theta_j) - (\bar{\mu}_{\nu_t} - \theta_{\nu_t})}{\hat{p} + \bar{\mu}_j - \theta_j} \hat{p} \hat{f}_j, \quad j \neq \nu_t, \quad (13)$$

and

$$\bar{\mu}_{\nu_t} = r_{\nu_t} + \gamma \rho_{XC} \sigma_{X, \nu_t}^s \sigma_{C, \nu_t} \quad (14)$$

is the discount rate in the standard Gordon growth model.

To understand the intuition behind the discount rate (13), note that if the economy stays in state i forever, the discount rate reduces to the standard expression $r_{A, i} = \bar{\mu}_i - \theta_i$. If the economy is, say, in state 1 (recession), then the discount rate (13) is obtained by adjusting $\bar{\mu}_1 - \theta_1$ downwards by $\frac{(\bar{\mu}_2 - \theta_2) - (\bar{\mu}_1 - \theta_1)}{\hat{p} + \bar{\mu}_2 - \theta_2} \hat{p} \hat{f}_2 < 0$, to account for time spent in state 2 (boom) at future times. The magnitude of the adjustment increases with θ_2 and $\hat{\lambda}_1 = \hat{p} \hat{f}_2$. As news arrival under the risk-neutral measure speeds up (i.e., \hat{p} increases), the adjustment approaches $[(\bar{\mu}_2 - \theta_2) - (\bar{\mu}_1 - \theta_1)] \hat{f}_2$ and the discount rate approaches $(\bar{\mu}_1 - \theta_1) \hat{f}_1 + (\bar{\mu}_2 - \theta_2) \hat{f}_2$, which is the long-run risk-neutral mean of the difference between the discount rate and the expected earnings growth rate.

II.C Credit spreads and the levered equity risk premium

In this section, we provide closed-form expressions for perpetual corporate debt and levered equity prices. We then use these expressions to derive credit spreads and the levered equity risk premium.¹⁹ Note that these expressions are for values at the individual firm level. However, empirically, credit spreads and the equity risk premium are based on equally-weighted and value-weighted averages, respectively. Therefore, we compute the model-implied equally-weighted average credit spread and value-weighted equity risk premium when we explore the implications of our model in Section IV.

The generic value of debt at time t , conditional on the state being ν_t , denoted by B_{ν_t} , is given by

$$B_{\nu_t} = E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} cds \middle| \nu_t \right] + E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \alpha_{\tau_D} A_{\tau_D} \middle| \nu_t \right], \nu_t \in \{1, 2\}. \quad (15)$$

The first term in (15) is the present value of a perpetual coupon stream until default occurs at a random stopping time τ_D . The second term is the present value at time t of the asset recovery value the debtholders successfully claim upon default, where $\alpha_{\nu_t} \in \{\alpha_1, \alpha_2\}$ is the date- t recovery rate. We show (see Proof of Proposition 4, Appendix B) that (15) reduces to

$$B_{\nu_t} = \frac{c}{r_{P,\nu_t}} \left(1 - \sum_{\nu_D=1}^2 l_{D,\nu_t\nu_D} q_{D,\nu_t\nu_D} \right), \quad (16)$$

where

$$l_{D,\nu_t\nu_D} = \frac{\frac{c}{r_{P,\nu_D}} - \alpha_{\nu_D} A_{\nu_D} (X_{D,\nu_D})}{\frac{c}{r_{P,\nu_t}}} \quad (17)$$

is the loss ratio at default, when the current state is ν_t and default occurs in state ν_D . The first factor in (16) is the price of the equivalent riskless consol bond, $c/r_{P,\nu_t}$, and the second factor is a downward adjustment for default risk, where $l_{\nu_t\nu_D,t} q_{D,\nu_t\nu_D}$ is the present value of the loss ratio. The discount rate for a riskless perpetuity when the current state is ν_t is given by

$$r_{P,\nu_t} = r_{\nu_t} + \frac{r_j - r_{\nu_t}}{\hat{p} + r_j} \hat{p} f_j, j \neq \nu_t. \quad (18)$$

Note that r_{P,ν_t} is not equal to the risk-free rate in state ν_t , r_{ν_t} , because the risk-free rate is expected to change in the future whenever the state of the economy switches.²⁰

The next proposition gives the corporate bond spread in terms of the discount rate for a risk-free perpetuity, loss ratios, and Arrow-Debreu default claims. Note that we define the credit spread as the yield on corporate debt less the yield on an equivalent risk-free security of the *same maturity*, thus ensuring that the credit spread is not falsely inflated by a term spread.

¹⁹The corresponding closed-form expressions for finite maturity corporate debt are derived in the Supplement, which is available upon request.

²⁰Note that (18) can be obtained from (13) by replacing the Gordon growth model discount rate, $\bar{\mu}_{\nu_t}$, with the risk-free rate, r_{ν_t} , and setting the expected growth rate of earnings, θ_{ν_t} , equal to zero.

Proposition 4 *The credit spread in state ν_t , s_{ν_t} , is given by*

$$s_{\nu_t} = \frac{c}{B_{\nu_t}} - r_{P,\nu_t} = r_{P,\nu_t} \frac{\sum_{\nu_D=1}^2 l_{D,\nu_t\nu_D} q_{D,\nu_t\nu_D}}{1 - \sum_{j=1}^2 l_{D,\nu_t\nu_D} q_{D,\nu_t\nu_D}}, \nu_t \in \{1, 2\}. \quad (19)$$

Current levered equity value is given by the expected present value of future cash flows less coupon payments up until bankruptcy, conditional on the current state:

$$S_{\nu_t} = (1 - \eta) E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} (X_s - c) ds \middle| \nu_t \right], \nu_t \in \{1, 2\}.$$

We can show (see Proof of Proposition 5, Appendix B) that the above equation simplifies to give

$$S_{\nu_t} = A_{\nu_t}(X_t) - (1 - \eta) \frac{c}{r_{P,\nu_t}} + \sum_{\nu_D=1}^2 q_{D,\nu_t\nu_D} \left[(1 - \eta) \frac{c}{r_{P,\nu_D}} - A_{\nu_D}(X_{D,\nu_D}) \right], \nu_t \in \{1, 2\}. \quad (20)$$

The first two terms in the above equation are the present after-tax value of future cash flows less coupon payments, if the firm were never to default. The last term accounts for the fact that upon default shareholders no longer have to pay coupons to bondholders and at the same time they lose the rights to any future cash flows from owning the firm's assets.

In the next proposition we derive the levered equity risk premium of an individual firm.

Proposition 5 *The conditional levered equity risk premium in state ν_t is*

$$\mu_{R,\nu_t} - r_{\nu_t} = \gamma \rho_{XC} \sigma_{R,\nu_t}^{B,s} \sigma_{C,\nu_t} + \Pi_{\nu_t}, \nu_t \in \{1, 2\}, \quad (21)$$

where Π_{ν_t} is the jump risk-premium in state ν_t , given by

$$\Pi_{\nu_t} = \begin{cases} (1 - \omega^{-1}) \sigma_{R,1}^P \lambda_1, & \nu_t = 1 \\ (1 - \omega) \sigma_{R,2}^P \lambda_2, & \nu_t = 2 \end{cases}, \quad (22)$$

$$\sigma_{R,\nu_t}^P = \frac{S_j}{S_{\nu_t}} - 1, \nu_t \in \{1, 2\}, j \neq \nu_t \quad (23)$$

is the volatility of stock returns caused by Poisson shocks, and $\sigma_{R,\nu_t}^{B,s}$ is the systematic volatility of stock returns caused by Brownian shocks, given by

$$\sigma_{R,\nu_t}^{B,s} = \frac{\partial \ln S_{\nu_t}}{\partial \ln X_t} \sigma_{X,\nu_t}^s, \nu_t \in \{1, 2\}. \quad (24)$$

Interestingly, there are two opposite effects of leverage on the equity risk premium. Firstly, the levered equity risk premium is larger than the unlevered risk premium, simply because the act of paying coupons makes the equityholders' residual claim riskier. Secondly, corporate debt brings in default risk, which increases the value of the option to default and therefore decreases the risk premium.

II.D Optimal default boundary and optimal static capital structure

Equityholders maximize the value of their default option by choosing when to default and also optimal capital structure. Intuitively, the state-contingent endogenous default boundary X_{D,ν_t} , $\nu_t \in \{1, 2\}$, depends on the current state of the economy. The default boundaries satisfy the following two standard smooth-pasting conditions:

$$\left. \frac{\partial S_{\nu_t}(X)}{\partial X} \right|_{X=X_{D,\nu_t}} = 0, \nu_t \in \{1, 2\}. \quad (25)$$

In Appendix B we prove that the optimal default boundary is weakly countercyclical, i.e. $X_{D,1} \geq X_{D,2}$.

Equityholders choose the optimal coupon to maximize firm value at date 0 by balancing marginal tax benefits from debt against marginal expected distress costs. There are two important features to note which also apply to the dynamic case. First, as is standard in the capital structure literature (e.g., see Leland (1994)), by maximizing firm value equityholders internalize debtholders' value at date 0. However, in choosing default times they ignore the considerations of debtholders. This feature creates the usual conflict of interest between equity and debtholders. Second, the optimal coupon depends on the state of the economy at date 0. To make this clear, we denote the date-0 coupon by $c_{\nu_0,0}$, where ν_0 is the date-0 state of the economy. We assume a proportion ν_{ν_0} of the bond's value is lost due to issuance costs when the initial state is ν_0 .²¹ Therefore equityholders choose the coupon to maximize date-0 firm value net of issuance costs, $F_{\nu_0,0} = B_{\nu_0,0}(1 - \nu_{\nu_0}) + S_{\nu_0,0}$, i.e.

$$c_{\nu_0,0} = \argmax F_{\nu_0,0}(c).$$

Optimal default boundaries depend on the coupon. This implies hysteresis in the sense that the default boundaries not only depend on the current state of the economy, but also on its initial state. The next section expands and enriches this intuition to the case when firms make dynamic financing decisions.

III Asset valuation and dynamic capital structure

In this section, we extend our model to incorporate dynamic capital structure by allowing equityholders to restructure firm's financial obligations over time. This extension is necessary for two reasons. First, empirical evidence suggests that firms follow a target leverage ratio even though they restructure infrequently. Second, to correctly compare the implications of our model with the data for credit spreads and the risk premium, we must compute the credit spread and equity risk premium as cross-sectional averages over individual firm values. The cross-sectional distribution of firms used to compute these averages should be the one implied by our model.²² Since this exercise is impossible with static capital (in this case, leverage attenuates in the long-run), we introduce dynamic capital structure.

²¹We introduce issuance costs to make the static capital structure results more comparable with the dynamic capital structure results.

²²A growing literature highlights the importance of doing this, as opposed to simply averaging over equilibrium credit spreads and risk premia, where every firm has the same earnings level (see e.g., Berk et al. (1999), Carlson et al. (2004), and Strebulaev (2007)).

The pricing of unlevered assets, such as the firm's abandonment value, is the same under static and dynamic capital structure, and so in the following sections we explain how dynamic capital structure is modeled and how debt and equity are priced.

III.A Refinancing

The key difference between dynamic and static capital structure lies in the possibility to restructure a firm's debt obligations. In the static model, debt is issued only at time 0. In the dynamic model, firms may restructure at the time of their choice. They prefer to refinance infrequently, since each refinancing is costly (Fischer et al. (1989)). Intuitively, at each refinancing, equityholders choose a new coupon to maximize their value. We now explain how we implement this in an economy where the state changes stochastically and what happens exactly at each refinancing date.

There are two corporate events in the model: default and refinancing. Since leverage is altered at refinancing dates it is convenient to divide time into periods. A *period* is the time interval between two consecutive refinancing dates. It is convenient to denote the beginnings of such periods by date 0. Within each period, default occurs when a firm's cash flow level reaches a lower boundary, $X_{D,\nu_0\nu_D}$, where ν_0 is the state at the most recent refinancing date and ν_D is the state of the economy at default. Our notation emphasizes that the default decisions depend on both the initial state and the current state. Since our economy has two states, there are four default boundaries for each coupon value:

$$\{X_{D,11}, X_{D,12}, X_{D,21}, X_{D,22}\}.$$

Restructuring occurs when earnings reach an upper boundary, $X_{U,\nu_0\nu_U}$, which again depends on ν_0 and the state of the economy at restructuring, ν_U . Hence, there are four possible restructuring boundaries for each coupon value:

$$\{X_{U,11}, X_{U,12}, X_{U,21}, X_{U,22}\}.$$

III.B Homogeneity property

In the dynamic capital structure model specification that we consider below, a homogeneity property holds, which is an extension of the homogeneity property in the one-state model (see e.g. Fischer et al. (1989) and Goldstein et al. (2001)) to the case of regime switching. Let $\xi_{\nu_U}(c_{\nu_0})$ and $\varsigma_{\nu_D}(c_{\nu_0})$ to be scaling factors that depend on the state at restructuring and default, respectively, and the coupon chosen at the beginning of the refinancing period. These scaling factors are defined as:

$$\xi_{\nu_U}(c_{\nu_0}) = \frac{X_{U,\nu_0\nu_U}(c_{\nu_0})}{X_0} \text{ and } \varsigma_{\nu_D}(c_{\nu_0}) = \frac{X_{D,\nu_0\nu_D}(c_{\nu_0})}{X_0}. \quad (26)$$

The homogeneity property holds when: (i) ξ and ς above are time-invariant and level-invariant quantities; and (ii) the following conditions are satisfied:

$$\xi_i(c_i) = \xi_i(c_j) \frac{c_i}{c_j}, \quad i, j = 1, 2; \quad (27)$$

$$\varsigma_i(c_i) = \varsigma_i(c_j) \frac{c_i}{c_j}, \quad i, j = 1, 2. \quad (28)$$

Condition (i) above is the homogeneity property of Fischer et al. (1989). Condition (ii) allows us to map optimal decisions in one state into another state.

Using the homogeneity property, we can relate optimal coupons and boundaries between two consecutive periods. In particular,

$$c'_i(c_i) = \xi_i(c_i)c_i \text{ and } c'_i(c_j) = \xi_i(c_j)c_i, \quad (29)$$

and

$$\frac{X_{U,i\nu_U}(c_{\nu_i})}{c_i X_0} = \frac{X'_{U,j\nu_U}(c'_{\nu_j})}{c_j X'_0}, \quad (30)$$

where $'$ denotes a variable for a new period.

III.C Debt and equity valuation

Denote by $B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)$ the value of debt where ν_0 is the state at the previous refinancing, c_{ν_0} is the current coupon, and ν_t is the current state. We can write the value of debt in terms of fundamental Arrow-Debreu securities as

$$\begin{aligned} B_{\nu_t}(X_t, c_{\nu_0}, \nu_0) = & \frac{c_{\nu_0}}{r_{P,\nu_t}} + \sum_{\nu_D=1}^2 q_{D,\nu_t\nu_D}(X_t, \nu_0) \left(\alpha_{\nu_D} A_{\nu_D}(X_{D,\nu_0\nu_D}) - \frac{c_{\nu_0}}{r_{P,\nu_D}} \right) \\ & + \sum_{\nu_U=1}^2 q_{U,\nu_t\nu_U}(X_t, \nu_0) \left(R_{\nu_0\nu_U} - \frac{c_{\nu_0}}{r_{P,\nu_U}} \right). \end{aligned} \quad (31)$$

The Arrow-Debreu restructuring claim $q_{U,\nu_t\nu_U}(X_t, \nu_0)$ pays out a unit of consumption at restructuring if it occurs in state ν_U , the current state is ν_t , and the firm has not yet defaulted. Importantly, since the claim's value depends on the optimal default and restructuring thresholds, its value depends on the state of the economy at the most recent refinancing, ν_0 , as the notation $q_{U,\nu_t\nu_U}(X_t, \nu_0)$ signifies. The key difference between the default claims, q_D , in the static and dynamic cases is that in the dynamic case q_D pays off provided that restructuring has not already occurred in the current period.²³

In (31), $c_{\nu_0}/r_{P,\nu_t}$ is the value of a risk-free consol bond paying coupon c_{ν_0} and r_{P,ν_t} is the relevant risk-free perpetual interest rate in state ν_t . The second term in the first line is the recovery value of the firm received by bondholders if default takes place less the value of the coupon payments lost due to default, multiplied by an Arrow-Debreu default claim, q_D . The second line is the payment made to bondholders at refinancing, denoted by R , less the value of the coupons lost, multiplied by an Arrow-Debreu default restructuring claim, q_U .

²³We derive exact closed-form expressions for perpetual Arrow-Debreu claims under dynamic capital structure in the Supplement, which is available upon request.

Observe that (31) holds for a general refinancing payment, $R_{\nu_0\nu_U}$. The exact form of the refinancing payment depends on the bond indenture provisions such as callability and seniority. For example, if debt is callable at its book value, then $R_{\nu_0\nu_U}$ is the original par value of debt. If debt is non-callable, $R_{\nu_0\nu_U}$ is the continuation value of debt. For simplicity, we assume that debt is non-callable and issued *pari passu*, i.e. all outstanding debt issues have equal seniority. Dilution is on a per-coupon basis, so that if the coupon at the previous refinancing is c_{ν_0} , and at restructuring the new coupon is $c_{\nu_U}(c_{\nu_0})$, then the continuation value of the original debt issued at the previous refinancing date is²⁴

$$R_{\nu_0,\nu_U} = \frac{c_{\nu_0}}{c_{\nu_U}(c_{\nu_0})} B_{\nu_U}(X_{U,\nu_0,\nu_t}, c_{\nu_U}(c_{\nu_0}), \nu_U). \quad (32)$$

Based on the above structure of the refinancing payment, we can derive bond prices at refinancing dates and hence at all dates, as shown in the following proposition.

Proposition 6 *Suppose that the refinancing payment R is given by (32). Then the homogeneity property (see (26) – (28)) holds, the date- t debt value is given by*

$$B_{\nu_t}(X_t, c_{\nu_0}, \nu_0) = \frac{c_{\nu_0}}{r_{P,\nu_t}} \left(1 - \sum_{\nu_D=1}^2 q_{D,\nu_t\nu_D}(X_t, X_{D,\nu_0\nu_D}) l_{D,\nu_t\nu_D} - \sum_{\nu_D=1}^2 q_{U,\nu_t\nu_D}(X_t, X_{U,\nu_0\nu_D}) l_{U,\nu_t\nu_D} \right), \quad (33)$$

and the credit spread, $s_{\nu_t}(X_t, c_{\nu_0}, \nu_0)$, is given by

$$s_{\nu_t}(X_t, c_{\nu_0}, \nu_0) = r_{P,\nu_t} \frac{\sum_{\nu_D=1}^2 q_{D,\nu_t\nu_D}(X_t, X_{D,\nu_0\nu_D}) l_{D,\nu_t\nu_D} + \sum_{\nu_U=1}^2 q_{U,\nu_t\nu_U}(X_t, X_{U,\nu_0\nu_U}) l_{U,\nu_t\nu_U}}{1 - \sum_{\nu_D=1}^2 q_{D,\nu_t\nu_D}(X_t, X_{D,\nu_0\nu_D}) l_{D,\nu_t\nu_D} - \sum_{\nu_U=1}^2 q_{U,\nu_t\nu_U}(X_t, X_{U,\nu_0\nu_U}) l_{U,\nu_t\nu_U}}, \quad (34)$$

where loss ratios conditional on default and restructuring are given, respectively, by (17) and

$$l_{U,\nu_t\nu_U} = \frac{\frac{c_{\nu_0}}{r_{P,\nu_U}} - B_{\nu_U}(X_{U,\nu_0\nu_U}, c_{\nu_U}, \nu_U)}{\frac{c_{\nu_0}}{r_{P,\nu_t}}} \quad (35)$$

and $B_{\nu_U}(X_{U,\nu_0\nu_U}, c_{\nu_U}, \nu_U)$ is the bond value at restructuring, given by (B36) in Appendix B.

Comparing (34) with (19), we can see that for the same coupons and default boundaries, credit spreads are larger under dynamic capital structure. Intuitively, refinancing is costly for bondholders since their ownership stake is diluted, and so they demand a higher premium ex-ante.

To value equity, we must distinguish between equity value just after refinancing, S_0 , and equity value just prior to refinancing, E_0 . The value of equity just after refinancing is

$$S_{\nu_t}(X_t, c_{\nu_0}, \nu_0) = \text{Div}_{\nu_t}(X_t, c_{\nu_0}, \nu_0) + \sum_{\nu_U=1}^2 q_{U,\nu_t\nu_U}(X_t, X_{\nu_0\nu_U}) E_{\nu_0\nu_U}, \quad (36)$$

²⁴Other definitions of R can be incorporated in the model but with a loss of the homogeneity property.

where Div_{ν_t} is the present value of dividends paid to equityholders during the current refinancing period when the current state is ν_t , and can be written as

$$\begin{aligned} Div_{\nu_t}(X_t, c_{\nu_0}, \nu_0) &= A_{\nu_t}(X_t) - (1 - \eta) \frac{c_{\nu_0}}{r_{P, \nu_t}} + \sum_{\nu_D=1}^2 q_{D, \nu_t \nu_D}(X_t, X_{D, \nu_0 \nu_D}) \left[(1 - \eta) \frac{c_{\nu_0}}{r_{P, \nu_D}} - A_{\nu_D}(X_{D, \nu_0 \nu_D}) \right] \\ &\quad + \sum_{\nu_U=1}^2 q_{U, \nu_t \nu_U}(X_t, X_{U, \nu_0 \nu_U}) \left[(1 - \eta) \frac{c_{\nu_0}}{r_{P, \nu_U}} - A_{\nu_U}(X_{U, \nu_0 \nu_U}) \right]. \end{aligned} \quad (37)$$

The second term in (37) shows that, if default occurs, equityholders no longer pay coupons but lose the right to future dividends. The third term shows a similar adjustment for the effect of restructuring.

The second term in (36) is present because, if restructuring occurs, equityholders derive value from cash flow payments made after restructuring. In the case of the *pari passu* covenant assumed above for the valuation of debt, equity value just prior to refinancing can be written as

$$\begin{aligned} E_{\nu_0 \nu_U}(X_{U, \nu_0 \nu_U}) &= [(1 - \iota_{\nu_U}) B_{\nu_U}(X_{U, \nu_0 \nu_U}, c_{\nu_U}(c_{\nu_0}), \nu_U) - R_{\nu_0 \nu_U}(X_{U, \nu_0 \nu_U}, c_{\nu_0}, c_{\nu_U}(c_{\nu_0}), \nu_U)] \\ &\quad + S_{\nu_U}(X_{U, \nu_0 \nu_U}, c_{\nu_U}(c_{\nu_0}), \nu_U), \end{aligned} \quad (38)$$

where a proportion ι_{ν_U} of the newly issued bond's value is lost due to restructuring costs, when restructuring takes place in state ν_U .

Given the above expression for equity value just prior to refinancing, we can derive equity values at refinancing dates and hence at all dates, as shown in the following proposition.

Proposition 7 *Suppose that the bond refinancing payment R is given by (32), so that the homogeneity property (see (26) – (28)) holds and the value of equity just before refinancing is given by (38). Then equity values are given by*

$$\begin{aligned} S_{\nu_t}(X_t, c_{\nu_0}, \nu_0) &= Div_{\nu_t}(X_t, c_{\nu_0}, \nu_0) \\ &\quad + \sum_{\nu_U=1}^2 q_{U, \nu_t \nu_U} \left[B_{\nu_U}(X_0, c_{\nu_U}, \nu_U) \left((1 - \iota_{\nu_U}) \xi_{\nu_0, \nu_U} - \frac{c_{\nu_0}}{c_{\nu_U}} \right) + \xi_{\nu_0, \nu_U} S_{\nu_U}(X_0, c_{\nu_U}, \nu_U) \right], \end{aligned} \quad (39)$$

where $S_{\nu_U}(X_0, c_{\nu_U}, \nu_U)$ is the equity value at restructuring, given by (B39) in Appendix B.

Note that expressions for the equity risk premium and return volatility are the same as in the static case (see Proposition 5). The only difference is in the functional form of the elasticity, $\partial \ln S_{\nu_t} / \partial \ln X$.

III.D Optimal default boundary and optimal capital structure

Relative to the static case, equityholders must now also decide on optimal restructuring boundaries as well as optimal coupons and default boundaries. The homogeneity property implies that

$$X_{U, 2i}(c_2) = X_{U, 1i}(c_1) \frac{c_2}{c_1}, \quad X_{D, 2i}(c_2) = X_{D, 1i}(c_1) \frac{c_2}{c_1}, \quad i = 1, 2, \quad (40)$$

so equityholders choose only six variables: $X_{D,11}$, $X_{D,12}$, c_1 , c_2 , $X_{U,11}$, and $X_{U,12}$.

Given the coupons and restructuring boundaries, the optimal default boundaries $X_{D,11}$, $X_{D,12}$ are determined by the following smooth pasting conditions:

$$\left. \frac{\partial S_1(X_t, c_1, 1)}{\partial X_t} \right|_{X_t=X_{D,11}} = 0, \quad \left. \frac{\partial S_2(X_t, c_1, 1)}{\partial X_t} \right|_{X_t=X_{D,12}} = 0. \quad (41)$$

Given the coupon chosen in state 2, c_2 , the optimal coupon c_1 and the restructuring boundaries $X_{U,11}$ and $X_{U,12}$ are then chosen to maximize levered firm value at restructuring, conditional on restructuring taking place in state 1, i.e.

$$(c_1, X_{U,11}, X_{U,12}) = \operatorname{argmax} F_1(c_1, X_{U,11}, X_{U,12}),$$

where $F_1 = B_1(X_0, c_1, 1)(1 - \iota_1) + S_1(X_0, c_1, 1)$. Finally, coupon c_2 is determined by maximizing levered firm value at restructuring, conditional on restructuring taking place in state 2, i.e.

$$c_2 = \operatorname{argmax} F_2(c_2),$$

where $F_2 = B_2(X_0, c_2, 2)(1 - \iota_2) + S_2(X_0, c_2, 2)$.

IV Model implications

In this section we study the quantitative implications of the model. We start by obtaining conditional estimates of parameter values. In Sections IV.B and IV.C, we see whether our model can resolve the credit spread and equity premium puzzles. We then look at cross-market comovement (Section IV.D).

IV.A Parameter Estimation

To estimate parameter values we use aggregate US data at quarterly frequency for the period from 1947Q1 to 2005Q4. Consumption is real non-durables plus service consumption expenditures from the Bureau of Economic Analysis. Earnings data are from S&P and provided on Robert J. Shiller's website. We delete monthly interpolated values and obtain a time-series at quarterly frequency. The personal consumption expenditure chain-type price index is used to deflate the earnings time-series. Unconditional parameter estimates are summarized in Panel A of Table II. With intertemporal macroeconomic risk, conditional estimates are given in Panel B of Table II. We now discuss the estimation exercise in more detail.

We obtain estimates of λ_1 , λ_2 , g_1 , g_2 , θ_1 , θ_2 , $\sigma_{C,1}$, $\sigma_{C,2}$, $\sigma_{X,1}^s$, $\sigma_{X,2}^s$, and ρ_{XC} by maximum likelihood.²⁵ For simplicity, we assume expected earnings growth and volatility parameters are identical across firms. Our estimates are similar to those obtained by other authors who jointly estimate consumption and dividends with a state-dependent drift and volatility, as opposed to consumption and

²⁵The approach is based on Hamilton (1989) and details specific to our implementation are summarized in the Supplement, which is available upon request.

earnings (see Bonomo and Garcia (1996)). We calibrate idiosyncratic earnings volatility so that the total asset volatility is approximately 25%, the average asset volatility of firms with outstanding rated corporate debt (see Schaefer and Strebulaev (2008)). This yields an idiosyncratic earnings volatility of 22.8%. Andrade and Kaplan (1998) report default costs of about 10–25% of asset value. Moreover, Thorburn (2000), Altman et al. (2002), and Acharya et al. (2007) find that bankruptcy costs, $1 - \alpha_{\nu_t}$, are countercyclical, i.e., $\alpha_1 < \alpha_2$, so we assume $\alpha_1 = 0.7$ and $\alpha_2 = 0.9$.²⁶ Unconditional average debt issuance costs are in the order of 1.3% (Altinkilic and Hansen (2000)). We assume that the issuance cost, ι_{ν_t} , is 3% (1%) in a recession (boom). The corporate tax rate, η , is set at 15%.

The annualized rate of time preference, β , is 0.01. We assume that relative risk aversion, γ , is equal to ten and the EIS, ψ , is equal to 1.5, as in Bansal and Yaron (2004).²⁷ Our assumption that ψ is greater than one ensures that the price-consumption ratio is procyclical.²⁸

IV.B The credit spread puzzle

In this section, we present the credit risk implications of our model. In particular, we investigate the credit risk puzzle, a well-known result that pure structural models of credit risk find it difficult to generate credit spreads of the same magnitude as observed in the data while keeping actual default probabilities (and hence leverage ratios) realistically low. Our methodology consists of two crucial points which reconcile model implications and the way the data is analyzed. First, we compute model-implied credit spreads based on the *average* credit spread over the *distribution* of firms. Second, we pay particular attention to the impact of the *time evolution* of the distribution on the *term structure* of actual default probabilities and credit spreads. Our main empirical message is that, once our method is applied, the model is consistent with empirically observed credit spreads and actual default probabilities at several maturities.

Table III reports our main results on credit risk. We focus on five- and ten-year spreads rather than perpetual spreads, since only finite maturity spreads are observable in the data. In this table we report unconditional rather than state dependent results. For example, the reported five-year credit spread is a weighted average of the state-dependent credit spreads, $s_{\nu_t,5}$, where the weights are given by the long-run distribution of the Markov chain, i.e., $f_1 s_{1,5} + f_2 s_{2,5}$.

We restrict our attention, along with a number of other researchers, to BBB-rated debt since it has been argued that the pricing of very high-grade investment firms (such as AAA or AA firms) is

²⁶While empirical evidence unequivocally suggests bankruptcy costs are countercyclical, estimates are numerically imprecise. Therefore, assumptions on numerical values are necessarily to some extent ad hoc. Since empirical data is often reported on recovery rates, we measure the recovery rate implied by our model, using the definition in Acharya et al. (2007) (the ratio of the payoff to corporate bond in default to the original principal value), and find that for the optimally levered firm it is 49% in an expansion and 35% in a contraction. This corresponds to 53% and 41% in Acharya et al. (2007) and 35 – 50% and 20 – 30% in Cantor et al. (2008).

²⁷We also considered the following values of relative risk aversion and the EIS: $\gamma = 10$ & $\psi = 0.5$, $\gamma = 7.5$ & $\psi = 1.5$, $\gamma = 7.5$ & $\psi = 0.5$, and found that all these parameter choices lowered the equity premium, consistent with Bansal and Yaron (2004). Further details are in the Supplement, available on request.

²⁸Some studies (see, e.g. Hansen and Singleton (1982), Attanasio and Weber (1989), Vissing-Jorgensen (2002), Guvenen (2006)) estimate that the EIS is more than one. Others (see, e.g. Hall (1988) and Campbell (1999)) find the opposite. The latter studies assume consumption growth volatility is fixed and Bansal and Yaron (2004) show that doing so biases estimates of the EIS downward. See also footnote 14.

dominated by factors other than credit risk and structural models perform much better for low-grade bonds (such as bonds with junk rating of B and below). Firms with the lowest investment-grade rating, i.e. BBB, represent the natural testing ground for our model.²⁹

We consider the implications of our model along three dimensions. The first two constitute a traditional way of looking at the credit risk puzzle analyzing an individual firm and the third one offers a new perspective by studying the distribution of firms. In the first exercise, we take leverage and default boundaries as exogenous. We set coupon levels so unconditional leverage at the moment of refinancing equals 40%. We choose this value, because it is the average leverage ratio of firms with a BBB credit rating (see Schaefer and Strebulaev (2008)). Then, for each maturity, the default boundaries are set such that actual default probabilities match closely their empirical historical counterparts.³⁰ In the second exercise, default boundaries are chosen optimally according to the model, while leverage is still exogenous. For both of these exercises we consider static and dynamic capital structure, and compute credit spreads and other variables related to corporate bonds based on values at the refinancing dates. Thirdly, in Panel C, we report credit spreads and other variables based on cross-sectional averages over the distribution of individual firm values obtained by simulating the model over time, thereby extracting the ‘true dynamics’ of the model. We argue that the third exercise represents the correct procedure of testing this and all other dynamic credit risk models. We discuss each exercise in turn.

IV.B.1 Exogenous default boundary and leverage

For the case when default boundaries and leverage are set exogenously, our results are based on the credit spread for a bond with maturity T , computed for an *individual* firm, i.e.

$$s_T(\bar{L}, \bar{p}_{D,T}, X_0), \quad (42)$$

where s_T is the unconditional T -year spread for an individual firm, with unconditional leverage ratio set equal to average leverage for BBB firms, $\bar{L} = 40\%$; unconditional actual T -year default probability set equal to 2% and 5% for $T = 5$ and $T = 10$ years, respectively, closely matching the corresponding historical default probabilities for BBB firms, $\bar{p}_{D,T}$; and the cash flow level set equal to X_0 to ensure the firm is at the refinancing point.³¹

Panel A of Table III shows that both static and dynamic models produce similar results of approximately 45 bp for a five-year spread and 75 bp for a ten-year spread for BBB firms. A common way of measuring the credit risk component of the BBB–Treasury spread is via the BBB–AAA spread, since AAA bonds are almost default free and their spreads most likely result from factors not included in

²⁹See De Jong and Driessen (2005) for evidence of a tax component in spreads over treasuries and Ericsson and Renault (2005), Longstaff, Mithal, and Neis (2005), and Chen, Lesmond, and Wei (2007) for evidence of a liquidity component.

³⁰This exercise is the same as in Huang and Huang (2003) who report that a variety of structural models can match only a small fraction of the credit spread, especially for low-risk high-grade credit ratings, when subjected to this test. Chen et al. (2008) also carry out a similar exercise, where the state-price density used for pricing comes from a representative agent with habit formation as in Campbell and Cochrane (1999).

³¹To match the historical default probability and average leverage, we keep the ratio of default thresholds across states, $X_{D,1}/X_{D,2}$, the same as in the optimal case, select the default boundary in state 2 exogenously, use optimal restructuring thresholds, and choose the coupon exogenously.

our model, such as personal taxes and liquidity. To gauge the relation to empirical data, Table IV provides a set of empirical numbers taken from a variety of sources. Panel A reports 5, 10, and 15 year actual default probabilities over the time periods 1920–2007 and 1970–2007 as reported by Moody’s. The estimates for historical credit spreads shown in Panel B exhibit non-trivial variation due to fluctuations of credit spreads, alterations in definitions of ratings, and differences in option-like features of corporate bonds. For example, Duffee (1998) estimates the BBB–AAA spread for debt between 2 and 15 years of maturity to be about 70–75 bp. However, Huang and Huang (2003) report higher values with a noticeable upward sloping structure of the BBB–AAA spread, 103 and 131 bp for four and ten years, respectively. Another way is to estimate the fraction of the BBB credit spread responsible for default risk directly. Based on CDS data, Longstaff et al. (2005) estimate that the default component of BBB credit spreads is 71%, while Duffee (1998) reports that the BBB credit spread is about 145 bp. According to these results, the BBB credit spread due to default risk is about 100 bp. Overall, the fraction of the credit spread due to credit risk for these maturities is higher than the model suggests, even though the model captures a larger fraction of the spread than pure structural models.

IV.B.2 Endogenous default boundary and exogenous leverage

The exercise above, while widely practiced (e.g., see Huang and Huang (2003) and Chen et al. (2008)), misses several important empirical considerations. First, it ignores the fact that in reality equityholders and managers choose the time of default optimally. Second, it considers credit spreads solely at refinancing points, i.e. when debt is issued, whereas the majority of empirically reported spreads are based on observations made at times when debt is not being issued. Third, it considers the credit spread of a typical *individual* firm, but compares it with the historical default probability observed for a *sample* of firms. For example, a contemporaneous paper by Chen (2008) studies the 10-year spread of an individual BBB firm at refinancing, thus ignoring the second and third points above. However, our results imply that controlling for these effects would make a dramatic quantitative difference to Chen’s conclusions. We address the first consideration here and the other two in the next section.

In our model, firms default optimally. As a result, our task of resolving the credit spread puzzle is more challenging than in e.g. Chen et al. (2008), since we are concerned whether our model can generate realistic prices of corporate claims *and* realistic *endogenous* default rates. In other words, in our exercise it is not sufficient to set default boundaries exogenously so that actual default probabilities match the data. Panel B of Table III reports spreads and other bond market related variables for an individual firm with optimal default boundaries. Importantly, we continue to keep leverage fixed at 40% so we can compare default rates for the two cases. The default rate when equityholders make default decisions is about eight times smaller than the empirically observed default rate for a five-year maturity and about twice as small for a ten-year maturity (see Panel A of Table IV for empirical default rates). Thus, exogenously specifying default boundaries to match default probabilities substantially distorts the value of the option to default. Not surprisingly, for such low default rates, credit spreads are also low, especially for shorter maturity debt (as can be seen by comparing Panel B of Table III

with Panel B of Table IV). Moreover, since the default option is more valuable under dynamic capital structure (because equityholders have the opportunity to refinance), default boundaries and therefore credit spreads are even lower than in the static case.

IV.B.3 Endogenous leverage and default: ‘True dynamics’

The results above may suggest that our model is unlikely to be capable of pricing corporate debt and predicting default rates simultaneously. This conclusion, however, neglects the following crucial observation. All results in Panels A and B are obtained under the assumption that the firm is at the refinancing point. But, most of the time, most firms are *not* at their refinancing points. Due to refinancing costs, they deviate from their optimal target leverage ratios and refinance only when restructuring thresholds are hit. As Fischer et al. (1989) show, even small adjustment costs can lead to first-order deviations from the optimal target leverage ratio. Empirically, Welch (2004) and Leary and Roberts (2005) show that firms do indeed refinance infrequently. Another piece of evidence is that while all BBB firms are frequently lumped together and their average characteristics are used to compute the credit spread for a typical individual firm, BBB firms differ widely in their leverage ratios, among other characteristics, both from each other and over time. In order to obtain the empirical implications of our dynamic framework, we must consider ‘true dynamics’ by simulating a cross-section of BBB firms that resembles its empirical counterpart.

An important difference between this and previous exercises becomes apparent when we consider the treatment of default probabilities in the existing literature. The usual task performed in credit risk studies is to get an estimate of the credit spread for a typical *individual* firm that would be consistent with the historically observed default probability for a *sample* of firms of which the individual firm above is representative (where sample is defined e.g. by rating), i.e. the reported spread is calculated using (42). For example, the five-year default probability of all BBB-rated firms is historically about 2% which would be then used to compute the credit spread of the typical individual BBB firm. The necessity of such an exercise is dictated by the impossibility of observing default probabilities for individual firms. Since Moody’s and S&P report default probabilities across ratings, credit risk studies mainly consider rating-based samples.

As we argue, the traditional exercise is to some extent futile. The point is that a sample of, say, BBB-rated firms, contains a wide array of firms with different characteristics and, as Figure 1 shows, widely different leverage ratios. That this sample of firms may experience a 2% default rate on average is possible. What is unlikely, however, is that the typical firm in this sample would experience the same default rate since default likelihood is very non-linear in firm characteristics. Therefore, it is crucial to generate a *sample* of firms implied by the model and match its average default probability with the corresponding empirical average. For this purpose, we start by simulating the model-implied dynamic economy, populated by 3,000 firms. We closely follow the simulation procedure given in Strebulaev (2007) and details are given in Appendix C. This gives us the starting point for all our investigations based on ‘true dynamics’.

To obtain the ‘true dynamic’ implications of the model for the BBB credit spreads, we proceed as follows. First, for every month over the period from 1997 to 2004 we take the empirical sample of BBB firms and construct the cross-sectional distribution of their quasi-market leverage ratios (the ratio of book debt to the sum of book debt and the market value of equity).³² Figure 1 shows an average cross-sectional distribution of quasi-market leverage for BBB firms over the entire period.

Second, we match this average distribution with its model counterpart. For each observation in the empirical sample we find the observation in the simulated sample that is closest in distance in terms of the quasi-market leverage ratio. While we assume firms are identical at refinancing points, they are different in dynamics since the evolution of asset values is driven not only by systematic but also by idiosyncratic shocks and therefore most firms refinance at different times. As a result, the distribution of leverage ratios in the model is non-trivial on any given date.

The above matching procedure allows us to construct a cross-sectional distribution of leverage ratios which is almost identical to the one empirically observed.³³ Three further points are worth noting. First, we do not assume that our model-implied economy contains only BBB firms (as indeed it should not). Rather, we choose only those firms within the economy which can be identified as BBB firms on a particular date. Second, it is critical to match empirical and simulated samples only at one date (which we call “the initial date”). Our exercise is equivalent to comparing returns (in our case, default probabilities and credit spreads) of two investment portfolios. Once these portfolios are matched based on firm characteristics, there is no need to match the composition of these portfolios at any other time. Finally, any simulated sample is a function of a particular realization of past systematic shocks and states of the economy. We therefore repeat the procedure a number of times to explore the distributional properties of our results.

We compute both the average default rate and the average credit spread for the resulting simulated sample. To compute the average default rate, we simulate future cash flow paths for each firm. Importantly, the five-year default rate in this exercise depends on the *time evolution* of the distribution of the initial sample over five years (and similarly for default rates of any maturity), since the average five-year default rate is given by

$$\frac{1}{M} \sum_{m=1}^M \left[\frac{1}{N} \sum_{n=1}^N p_{D, \nu_m, 5}(x_{n,m}) \right], \quad (43)$$

³²The data is from the sample of BBB firms for which pricing data is available in Merrill Lynch corporate bond data (see Schaefer and Strebulaev (2008)). The sample excludes all bonds issued by utility and financial firms. Since many firms issue a number of bonds, one unit of observation is a corporate bond rather than a firm.

³³One could argue that instead of the simulation, one can pick distances to default for a collection of firms, which ensures that their leverage ratios match our empirical sample. However, then we would have to exogenously specify the conditions under which the previous refinancing occurred, e.g. the state of the economy. The simulation ensures that these characteristics are determined endogenously. Another reason why simulation is necessary is that since capital structure decisions depend on macroeconomic history and firms vary in the timing of their financing decisions, we cannot simply assume the number of firms N is large and compute the state-conditional cross-sectional densities of distances-to-default by solving a system of forward Kolmogorov equations.

where $p_{D,\nu_m,5}(x_{n,m})$ is the five-year actual default probability for firm n , ν_m is the current state of economy m , and $x_{n,m}$ is the current distance-to-default of firm n in economy m , $x_{n,m} = \ln(X_n/X_{n,D,\nu_m})$. Crucially, ‘current’ refers to the date of matching empirical and model-implied samples.

We obtain a distribution of values for distances-to-default for N firms by simulating our initial distribution forward for five years. We choose N to be equal to 891, the average, over time, of the number of BBB bonds in the empirical sample. We carry out the simulation M (equal to 1000) times. To compute the average credit spread, we average over credit spreads for each firm in the sample. Panel C of Table III reports the results of this exercise. The panel shows that, while at the refinancing points the model-implied credit spreads and default rates are unrealistic, in ‘true dynamics’ the same model generates an average credit spread and expected default rates which are consistent with their empirical counterparts at both five and ten years. Crucially, an average credit spread and expected default rate are now produced consistently with each other for the identical sample of firms.

While model-implied average default rates are marginally larger than the historical default rates reported in Panel A of Table IV, it is important to realize that the ‘true dynamic’ ex-ante credit spreads are consistent with a wide distribution of *realized* ex-post default rates in the future. Since shocks to firms’ growth rates include a systemic component, observed default rates vary depending on a particular realization of good or bad times. To investigate default rate variation, we generate data for a large number of economies (which differ only with respect to their future shock realizations and are identical at the time the BBB and simulated samples are matched). Table III reports the 25% and 75% percentiles for model-implied default rates showing that historically observed default rates easily fall within this range. In particular, the median economy experiences a default rate very similar to observed historical default rates over the period 1920–2007.

As our results show, for an *individual* firm, actual default probabilities are too low and their term structure is too steep relative to the data. For the *distribution* of firms, average actual default probabilities are higher and their term structure is flatter, thus matching the data more closely. The reason for the higher average actual default probabilities based on the distribution of firms is that some firms will be near default and actual default probabilities are convex in the distance-to-default. The intuition for the flattening of the term structure is quite different and is based on the *time evolution* of the distribution of firms. As can be seen from Figure 2, starting from a given distribution, after five and ten years the distribution of firms’ cash flows (and hence distances-to-default) looks different. The right tail is fatter at ten years than five years. Consequently, the slope of the term structure of average default probabilities for a distribution of firms is flatter than the term structure of default probabilities for an individual firm.³⁴

³⁴We also study the implications of our model for A, BB, and B rated corporate debt. The simulated dynamic cross-section implied by our model is able to match both the term structure of credit spreads and historical default probabilities for A and BB debt. However, for B debt our model underpredicts both credit spreads and default probabilities, most likely because original junk issues frequently default for liquidity reasons and thus the default boundary for these firms may be somewhat higher than is optimal. Details are in the Supplement, which is available on request.

IV.B.4 Risk adjustment

To gain a better understanding of credit spreads, we can use our decomposition of Arrow-Debreu default claims, which depend on the time and risk-adjustment. Unfortunately, empirical estimates of the time-adjustment do not exist. However the literature does provide estimates of the risk-adjustment, which is the ratio of risk-neutral to actual default probabilities. However, since actual default probabilities are less than one, relatively small changes in their values lead to large variations in the risk-adjustment. For example, Almeida and Philippon (2007) report a risk adjustment of 4.00 for BBB, while Berndt et al. (2005) report an average risk-adjustment of 2.76. Our model produces a risk-adjustment of 1.83 and 2.31 at five and ten years, respectively, for BBB debt (Panel C of Table III), close to the latter estimates.

IV.B.5 Time variation of credit spreads and default rates

In this section we analyze the time series properties of credit risk variables. Importantly, the time series behavior of credit spreads reveals hysteresis, i.e. credit spreads depend not only on the current state of the economy but also on the state of the economy at the time of the previous refinancing. To explore this path-dependence, Table V shows the relative values of credit risk variables with respect to the states of the economy at the present and last refinancing dates. For comparison purposes we set the leverage ratio at refinancing equal to 40% and report all equilibrium values relative to the values when both the refinancing and the current states are equal to 1 (contraction). To concentrate on the impact of the state, we also fix the level of earnings at its refinancing value, so that all comparative statics is due to the variation in the state of the economy alone. The first result that Table V shows is that the term structure of credit spreads in a boom is more upward-sloping than in a recession (Column 4). For short maturities, corporate debt is safer in good times due to a lower default likelihood. For longer maturities, however, there is a higher chance that the firm refinances, increasing credit risk.

The second result concerns the impact of the current state of the economy. For example, comparing columns 1 and 2 (refinancing in the bad state), we see that credit spreads are countercyclical with respect to the current state. This countercyclicity is driven by the time-variation in the Arrow-Debreu default claims. To understand why these claims are countercyclical, recall the decomposition of Section II.A (see Equation (8)). First, the actual default probability is higher in bad states. It is driven by earnings growth volatility (higher in the bad state) and the countercyclicity of the optimal earnings default boundary (higher in the bad state since the option to retain equityholders' rights is less valuable). Second, time-variation in the moments of consumption growth makes the time adjustment countercyclical. The countercyclicity of the time adjustment and the actual default probabilities robustly dominate the procyclicality of the risk adjustment (discussed in Section II.A), making the Arrow-Debreu claims countercyclical. Importantly, our results on the cyclicity of credit spreads are different from Chen et al. (2008), where actual default probabilities and hence credit spreads are procyclical, unless the asset-value default boundary is exogenously countercyclical.

Interestingly, in our framework while the earnings default boundary is countercyclical, the precautionary motive leads the discount rate, $r_{A,i}$ (see Equation (13)), also to be countercyclical, implying that the asset-value default boundary can be endogenously procyclical. Indeed, for our calibrations, the asset-value default boundary is procyclical. The model of Hackbarth et al. (2006) may also imply, in a different setting, that asset-value and earnings default boundaries can move in opposite directions. Chen et al. (2008), however, demonstrate that habit formation preferences with i.i.d. consumption growth can generate a realistic spread only with a countercyclical asset-value default boundary. Future empirical research could exploit this difference to determine which of the intertemporal risk or habit-formation classes of models are more suitable for the joint valuation of debt and equity.

The third set of results describes the impact of the refinancing state. For e.g., comparing Columns 1 and 3 (the current state is bad), we see that credit spreads are higher when the firm refinances in the good state, *ceteris paribus*. This path-dependence is due to the different relative valuation of marginal debt and equity claims in two states. In good times, the same amount of debt increases the probability of default by less than in bad times and more valuable future growth options make equity more valuable. Therefore, more debt should be issued in the good state for leverage ratios to be equal across the refinancing states. Higher debt levels increase the likelihood of failure when the state changes leading to higher credit spreads. The hysteresis effect is particular strong at short maturities, but remains quantitatively non-trivial over the long horizon as well (for example, credit spreads in perpetuity are still higher by 10% as the comparison between Columns 1 and 3 demonstrates).

We now study the time-variation of five-, ten- and, fifteen-year credit spreads and realized actual default rates implied by the ‘true dynamics’ of our model. To generate the data, we start with the matched BBB sample (see Section IV.B.3 for details) and then simulate the behavior of the matched cross-section in the model. The left graph of Figure 3 shows the behavior of generated credit spreads at different maturities for a typical economy. The main result is that credit spreads are significantly countercyclical. For this economy the correlation between ten-year spreads and the state of the economy is -0.43. When we simulate 1,000 economies (starting with the same BBB sample), the correlation’s range is between -0.22 and -0.60. Empirically, the correlation between Moody’s BBB–AAA spread and the NBER recessions (with 1 denoting the NBER recession; and 2 otherwise) over the 1920–2007 period is -0.36.³⁵ As the economy stays in the good state longer, credit spreads tend to decline as distances-to-default increase with continuously positive shocks. Conversely, as the economy stays longer in contractions, credit spreads tend to rise as distances-to-default decline with, on average, negative shocks. The latter result is mostly due to a negative earnings growth rate in contractions. The right graph of Figure 3 shows the realization of default rates for the same generated data. The figure shows that default rates are strongly countercyclical, with the correlation between default rate and the state of the economy is -0.45 (and the range across 1,000 economies is between -0.15 and -0.73).

³⁵Our results on the time-variation of credit spreads and default rates are not directly comparable to the empirical ones, since, while we start with the BBB sample, the subsequent behavior of the dynamic cross-section follows the optimal financial policy prescribed by the model and firms stay in the sample unless they default. Moody’s constructed BBB sample is modified every time firms leave and enter the BBB rating category.

Empirically, the correlation between Moody’s BBB issuer-weighted default rate and NBER recessions over the 1920–2007 period is -0.18.³⁶ The graph also shows that default rates spike at the start of bad times, causing default to cluster (see e.g. Duffie, Saita, and Wang (2007) and Das et al. (2008) for empirical evidence). This result is driven by an increase in the default boundary for all firms, forcing a number of firms immediately into default even though their earnings have not changed, consistent with the results of Hackbarth et al. (2006) on optimal default boundaries in a risk-neutral setting.

IV.C The levered equity premium

In this section, we consider the implications of our model for the levered equity risk premium. Previous work, which studies the impact of leverage on risk premia (Kandel and Stambaugh (1991), Abel (1999), Gennotte and Marsh (1993), Bansal, Dittmar, and Lundblad (2005)), takes default and capital structure decisions as exogenous and does not study how the cross-sectional distribution of firms affects the aggregate risk premium. We endogenize default and capital structure decisions and compute the value-weighted average levered equity risk premium. This allows us to make two contributions to studies of the levered equity premium. First, we can study the impact of the default and refinancing options, when leverage is endogenous. Second, we can study the impact of aggregation.

Panel A of Table VI shows how leverage, with the default and refinancing options present, affects the levered equity risk premium at refinancing, whereas Panel B shows the effects of aggregation. We start by focusing on the results in Panel A, where we compute the equity premium under the following assumptions: (i) no leverage; (ii) leverage without default and refinancing options (obtained by setting both the Arrow-Debreu default and restructuring claims set equal to zero); (iii) leverage with the default option but without the refinancing option (obtained by setting just the Arrow-Debreu refinancing claims set equal to zero); (iv) the benchmark case of leverage with both default and refinancing options; (v) the Modigliani & Miller (MM) case with taxes and leverage equal to the leverage in the benchmark case, where the cost of equity capital is given by

$$\mu_R = \mu_R^U + (1 - \eta) \frac{B}{S} (\mu_R^U - \mu_R^B), \quad (44)$$

where μ_R is the cost of levered equity, μ_R^U is the cost of unlevered equity, η is the rate at which corporate earnings are taxed, B is the corporate bond price, S is the levered equity price, and μ_R^B is the cost of debt.

We find that the levered equity risk premium is always higher than the unlevered equity risk premium (1.91%), a result in line with MM. In particular, the risk premium in the absence of default and refinancing options is 3.19%. However, we also find that the default option lowers leverage and hence *reduces* the levered equity risk premium to 2.69%. The intuition is that costly default makes shareholders issue less debt and thus lowers the probability of default. Similarly, introducing the

³⁶If we extend the definition of a recession to include the year immediately after its end, this correlation becomes -0.24.

refinancing option lowers leverage, since equityholders now have a real option of adding more debt later. Therefore, this option reduces the levered risk premium, giving a benchmark value of 2.66%.

Perhaps more interestingly, Panel A also shows that if we compute the levered equity risk premium using (44) and the leverage ratio from the benchmark case (Column (iv)), we obtain the levered equity risk premium of 2.33%. Thus, once leverage is controlled for, the impact of the default and refinancing options on the levered equity risk premium is 0.33%. This finding, however, may be construed to undermine the true importance of these options. It is crucial to recognize that the possibility to default and refinance leads to the *optimal* leverage ratio, which is derived endogenously within our theoretical framework. It can then be used in Equation (44) to estimate the levered equity risk premium. Therefore, the default and refinancing options affect the risk premium mainly via the optimal leverage channel, and this effect is quantitatively non-trivial.

In Panel B we compute the risk premium and equity return volatility under three scenarios: at date 0 under static capital structure, at the refinancing date under dynamic capital structure, and in the cross-section under dynamic capital structure, i.e. in ‘true dynamics’. There are two dimensions to our results: the impact of leverage and the impact of a dynamic cross-section of firms, i.e. aggregation. Leverage has two effects on the risk premium. First, the dividend payment to equityholders becomes riskier, which increases the risk premium. Second, the presence of a default option (driven by limited liability) shifts value from debtholders to equityholders and thus decreases the premium. Panel B shows that the first effect dominates and leverage increases the risk premium from 1.91% to 3.08% in the static capital structure case.

In dynamic capital structure, the levered equity premium at the refinancing point is lowered to 2.66%, since firms issue less debt (they can lever up later) and initial leverage is lower. Importantly, dynamic capital structure also allows us to compute the risk premium based on the model-generated cross-section of firms. To this end, we simulate model-implied cash flow paths. Appendix C provides details on the starting point of this simulation. We then simulate future cash flow paths for each firm and measure the risk premium as the following weighted average across economies, time, and firms:

$$\frac{1}{M} \sum_{m=1}^M \left[\frac{1}{T} \sum_{t=1}^T \left(\sum_{n=1}^N w_{n,t}^m r p_{n,t}^m \right) \right],$$

where M is the number of economies, T the length of each simulation, N the number of firms, $r p_{j,t}^m$ is the risk premium at date- t for firm n in economy m , $r p_{j,t}^m = \mu_{R,\nu_t,j}^m - r_{\nu_t}$, $w_{n,t}^m$ is the weight associated with this risk-premium observation, $w_{n,t}^m = S_{n,t}^m / \sum_{n=1}^N S_{n,t}^m$, with $S_{n,t}^m$ being the value of levered equity at date t for firm n in economy m . In the benchmark case, M is equal to 1,000 economies, N is 3,000 firms, and T is 100 years. Panel B shows that the levered equity risk premium under ‘true dynamics’ is 3.22%. Thus, accounting for aggregation raises the levered equity premium. The intuition is that in the dynamic cross-section, most firms are not at their refinancing points, and for these firms leverage in general is not equal to leverage at the most recent refinancing point. In fact, average leverage is higher, as has been shown by Strebulaev (2007). The intuition is quite general: unsuccessful firms refinance

later than successful firms and, as a result, have higher leverage ratios, especially so because firms which opt for higher leverage at refinancing also choose a lower refinancing boundary. Consequently, the risk premium under ‘true dynamics’ is higher than at the refinancing point.

To see the impact of market leverage on our results, in Figure 4 we show what happens to the levered equity risk premium under ‘true dynamics’ if we condition the average leverage in the economy to equal a certain benchmark. The risk premium is an increasing and convex function of the leverage ratio. In particular, for an exogenously set economy-wide leverage ratio of 66% in dynamics, our model produces a much larger levered equity risk premium of above 5%.

The model-implied aggregate levered equity return volatility in ‘true dynamics’ is 19.80%, close to the empirical estimate of 19.42% for the US data over the period of 1929–1988 (Bansal and Yaron (2004)). This is somewhat lower than for an individual firm, since the idiosyncratic components of individual equity return volatilities cancel out in the aggregate as a result of the well known diversification effect. It is also lower than the aggregate volatility at the refinancing date since in ‘true dynamics’ profitable firms are more likely to both have larger market capitalization and be low-levered, and therefore lower levered equity volatility of these firms has a higher weight in the estimation.

The Sharpe ratio for an individual firm is 7.59% under static capital structure at date zero and 7.52% under dynamic capital structure at the refinancing date. Under ‘true dynamics’, we compute the ratio of the cross-sectional average levered equity risk premium to the cross-sectional average equity return volatility and obtain a model-implied Sharpe ratio of 16.26%.³⁷ The Sharpe ratio under ‘true dynamics’ is higher than for an individual firm because of the diversification effect.

IV.D Comovement between equity and debt markets

In this section, we study the relation between stock and corporate bond market variables. Our focus is on the link between (i) stock-return volatility and credit spreads, (ii) stock-return volatility and credit spread volatility, and (iii) stock-return volatility and default rates. To measure the above relationships, as implied by our model, we simulate it over time and, as above, we consider the case of dynamic capital structure. The simulation procedure is identical to the one used in Section IV.C. Table VII summarizes our results by reporting cross-panel averages for regression coefficients. Importantly, within our model, both the dependent variables (credit spreads and default rates) and independent regressors (jump and total stock return volatility) are endogenous, so the regressions explore the association between the model variables rather than any causality relation.

³⁷The speed of mean reversion of drifts (and volatilities) in our model is given by $p = 0.7646$, and changes little when we assume there are 3 states or more. Bansal and Yaron (2004) obtain an estimate for the rate of mean reversion for expected consumption growth of 0.2547, when they calibrate an AR(1) process to consumption data. If we set $p = 0.2547$ in our model, intertemporal risk is higher and the levered risk premium (‘true dynamics’) is also higher at 6.65%, close to the value of 6.84% reported by Bansal and Yaron. Also, the Sharpe ratio for the aggregate stock market under true dynamics becomes 31.66%. Our empirical evidence combined with that in Bansal and Yaron (2004) suggests that the drifts and volatilities of macroeconomic fundamentals contain different components, which revert to their means at different speeds (see, e.g. Calvet and Fisher (2002) for evidence of such behavior in asset returns). Our simple Markov chain approach assumes that only one speed of mean reversion is possible. One can overcome this restriction by modeling the time variation of drifts and volatilities via a multifrequency regime-switching process (see Calvet and Fisher (2001)).

To estimate empirical counterparts, we use US data over the period from 1926 to 2007. Credit spreads are measured as the difference between yields on Moody’s monthly Baa (equivalent to S&P’s BBB rating class) and Aaa (AAA) indices (hereafter, Moody’s credit spread) and stock returns are measured using the CRSP monthly value-weighted index. In Panel A, we extract the monthly jump-component of stock returns, following the approach of Andersen et al. (2007), using daily returns. In Panels B and C, we estimate the monthly credit spread and stock return volatility process with a GARCH(1,1) model. In Panel D, we use Moody’s actual (annual) default probabilities.

Investors have long been aware of the positive relationship between volatility indicators from equity markets and credit spreads, and have exploited this to forecast spreads.³⁸ Several academic studies also use measures of total equity return volatility to explain changes in credit spreads, e.g., Campbell and Taksler (2003), Collin-Dufresne, Goldstein, and Martin (2001), and Hull, Nelken, and White (2004). Recent empirical work by Cremers, Driessen, and Maenhout (2008), Zhang et al. (2008), and Tauchen and Zhou (2006) finds that an important determinant of credit spreads is the jump-component of stock return volatility. Similar to Tauchen and Zhou (2006), we find empirically that a 1% increase of the jump component of stock market return volatility is associated with an increase in the Moody’s annual credit spread of 12.5 basis points. We replicate Tauchen and Zhou’s exercise using simulated data from our model. Panel A of Table VII shows that our model generates a mean regression coefficient of 0.105 with a 25%-quantile (across economies) of 0.090 and 75%-quantile of 0.123. Thus, in our model, a 1% increase in the jump component of stock return volatility is associated with an increase in the average credit spread of 10.5 basis points. Crucially, our evidence suggests that both in the data and the model large movements in credit spreads are associated with relatively small changes in the jump component of stock return volatility.³⁹

To study the comovement between credit spreads and total stock market return volatility, we regress the monthly Moody’s credit spread on monthly total stock return volatility. As Panel B shows, in the model, a 1% increase in total stock return volatility is associated with a 6.5 basis point increase in credit spreads compared to 6.8 basis points observed in the data. The comparison between Panels A and B suggests that both in the data and in the model credit spreads are more sensitive to large and relatively infrequent macroeconomic shocks (which we model as Markov regime-shifts) than to small and very frequent shocks (which we model via a Brownian motion).

Next, we turn to study the comovement between credit spread volatility and total equity return volatility. In the data, as panel C shows, a 1% increase in equity return volatility is associated with an increase in credit spread volatility by about 6.2 basis points. The corresponding model-implied value is 3.1 basis points. In the data credit spread volatility is thus more strongly associated with changes

³⁸For example, Reuters (Apr 18, 2008) reports that according to John Lonski, Chief Economist for Moody’s Investors Service, ‘A fall in the Chicago Board Options Exchange Volatility Index, or VIX, should also support credit spreads and it also implies a firming of the market value of the collateral that supports corporate debt’.

³⁹Cremers et al. (2008) study credit spreads in a pure structural model, where firm value is subject to exogenous jumps. In our model, firm value is subject to endogenous jumps, which occur because jumps in the state-price density occur at the same time as shifts in the first and second moments of cash flow growth rates, as discussed in Section I.B.

in stock return volatility. This suggests that our model does not account for a significant source of comovement between credit spreads and equity return volatility, such as liquidity or monetary policy.

Panels A, B, and C of Table VII report the relation between equity and corporate bond markets using variables derived from prices. In contrast, in Panel D, which reports the results of regressing total stock return volatility on actual default probability, the independent variable is not price based. The regression coefficient is 0.147 in the model, compared to 0.138 in the data. In other words, a 1% increase in total stock return volatility is associated with an increase in actual default probability of about 14 basis points both in the model and in the data. Overall, the results in Table VII support our assumption that corporate debt and equity are subject to the same set of risk factors.

V Conclusion

We develop a theoretical framework that jointly prices corporate debt and equity in order to deliver a unified understanding of how macroeconomic risks drive the equity risk premium and credit spreads. To this end, we embed a structural model of credit risk with optimal dynamic financing decisions inside a representative agent consumption-based model. Intertemporal macroeconomic risk is present, because the first and second moments of consumption and earnings growth processes switch randomly. Furthermore, we ensure the representative agent dislikes these regime shifts, by assuming she has Epstein-Zin-Weil preferences and prefers uncertainty to be resolved sooner rather than later.

Since leverage and default decisions are made optimally, we obtain an endogenous term structure of actual default probabilities. We estimate aggregate consumption and earnings cash flows and then simulate our dynamic model over time, with many firms. Firms are heterogeneous because their earnings growth rates are subject to idiosyncratic shocks. We then match the distribution of leverage ratios implied by our model with the empirical distribution of BBB firms. For these BBB firms, we compute the average default probabilities and credit spreads at five and ten years. The resulting term structure of average default probabilities is close to that observed in the data for the BBB sample. Furthermore, the model-implied term structure of BBB credit spreads is close to the default risk component of the observed BBB credit spread. Thus, our model offers a potential resolution of the credit spread puzzle. We also compute the value-weighted stock market index from all firms in our simulated economy and obtain a reasonable levered equity premium and levered stock return volatility.

This paper is only a first step towards the development of a fully-fledged consistent framework for pricing corporate equity and debt and the unification of existing asset pricing and corporate finance paradigms. Interesting possibilities for further research include studying the effects of default on consumption and introducing heterogeneous agents to distinguish between equity and debtholders.

A Appendix: The state-price density

First, we introduce some notation related to jumps in the state of the economy. Suppose that during the small time-interval $[t - \Delta t, t)$ the economy is in state i and that at time t the state changes, so that during the next small time interval $[t, t + \Delta t)$ the economy is in state $j \neq i$. We then define the left-limit of ν at time t as

$$\nu_{t-} = \lim_{\Delta t \rightarrow 0} \nu_{t-\Delta t}, \quad (\text{A1})$$

and the right-limit as

$$\nu_t = \lim_{\Delta t \rightarrow 0} \nu_{t+\Delta t}. \quad (\text{A2})$$

Therefore $\nu_{t-} = i$, whereas $\nu_t = j$, so the left- and right limits are not equal. If some function E depends on the current state of the economy i.e. $E_t = E(\nu_t)$, then E is a jump process which is right continuous with left limits, i.e. RCLL. If a jump from state i to $j \neq i$ occurs at date t , then we abuse notation slightly and denote the left limit of E at time t by E_i , where i is the index for the state. i.e. $E_{t-} = \lim_{s \uparrow t} E_s = E_i$. Similarly $E_t = \lim_{s \downarrow t} E_s = E_j$. We shall use the same notation for all processes that jump, because of their dependence on the state of the economy.

Using simple algebra we can write the normalized Kreps-Porteus aggregator in the following compact form:

$$f(c, v) = \beta(1 - \gamma)v u(c/h^{-1}(v)), \quad (\text{A3})$$

where

$$\begin{aligned} u(x) &= \frac{x^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}}, \psi > 0, \\ h(x) &= \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma \geq 0, \gamma \neq 1. \\ \ln x, & \gamma = 1. \end{cases} \end{aligned}$$

The representative agent's value function is given by

$$J_t = E_t \int_t^\infty f(C_t, J_t) dt. \quad (\text{A4})$$

Proposition A1 *The state-price density of a representative agent with the continuous-time version of Epstein-Zin-Weil preferences is given by*

$$\pi_t = \begin{cases} (\beta e^{-\beta t})^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t} e^{\int_0^t p_{C,s}^{-1} ds} \right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, & \psi \neq 1 \\ \beta e^{-\beta \int_0^t [1+(\gamma-1) \ln(V_s^{-1})] ds} C_t^{-\gamma} V_t^{-(\gamma-1)}, & \psi = 1 \end{cases} \quad (\text{A5})$$

When $\psi \neq 1$, the price-consumption ratio in state i , $p_{C,i}$, satisfies the nonlinear equation system:

$$p_{C,i}^{-1} = \bar{r}_i + \gamma \sigma_{C,i}^2 - g_i - \left(1 - \frac{1}{\psi}\right) \lambda_i \left(\frac{(p_{C,j}/p_{C,i})^{\frac{1-\gamma}{1-\frac{1}{\psi}}}}{1-\gamma} - 1 \right), i \in \{1, 2\}, j \neq i. \quad (\text{A6})$$

where

$$\bar{r}_i = \beta + \frac{1}{\psi} g_i - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi}\right) \sigma_{C,i}^2, i \in \{1, 2\}. \quad (\text{A7})$$

When $\psi = 1$, define V_i via

$$J = \ln(CV). \quad (\text{A8})$$

Then V_i satisfies the nonlinear equation system:

$$\beta \ln V_i = g_i - \frac{\gamma}{2} \sigma_{C,i}^2 + \lambda_i \frac{(V_j/V_i)^{1-\gamma} - 1}{1-\gamma} i \in \{1, 2\}, j \neq i. \quad (\text{A9})$$

The proof of Proposition A1 is given in the Supplement, which is available upon request.

B Appendix: Proofs

Proof of Proposition 1. We define $N_{i,t}$ as the Poisson process which jumps upward by one whenever the state of the economy switches from i to $j \neq i$. The compensated version of this process is the Poisson martingale, $N_{i,t}^P = N_{i,t} - \lambda_i t$. We start by proving that the state-price density satisfies the stochastic differential equation

$$\left. \frac{d\pi_t}{\pi_{t-}} \right|_{\nu_{t-}=i} = -r_i dt + \left. \frac{dM_t}{M_{t-}} \right|_{\nu_{t-}=i}, \quad (\text{B1})$$

where M is a martingale under \mathbb{P} such that

$$\left. \frac{dM_t}{M_{t-}} \right|_{\nu_{t-}=i} = -\Theta_i^B dB_t + \Theta_i^P dN_{i,t}^P, \quad (\text{B2})$$

r_i is the risk-free rate in state i given by

$$r_i = \begin{cases} \bar{r}_1 + \lambda_1 \left[\frac{\gamma - \frac{1}{\psi}}{\gamma - 1} \left(\omega^{-\frac{\gamma-1}{\psi}} - 1 \right) - (\omega^{-1} - 1) \right] & , i = 1; \\ \bar{r}_2 + \lambda_2 \left[\frac{\gamma - \frac{1}{\psi}}{\gamma - 1} \left(\omega^{\frac{\gamma-1}{\psi}} - 1 \right) - (\omega - 1) \right] & , i = 2, \end{cases} \quad (\text{B3})$$

and $\omega_2 = \omega_1^{-1} = \omega$, where ω is the solution of

$$G(\omega) = 0, \quad (\text{B4})$$

and

$$G(x) = \begin{cases} x^{-\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}} - \frac{\bar{r}_2 + \gamma\sigma_{C,2}^2 - g_2 + \lambda_2 \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{\frac{\gamma-1}{\psi}} - 1 \right)}{\bar{r}_1 + \gamma\sigma_{C,1}^2 - g_1 + \lambda_1 \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{-\frac{\gamma-1}{\psi}} - 1 \right)}, & \psi \neq 1; \\ \ln x^{\frac{1}{\gamma-1}} - \frac{g_2 - \frac{1}{2}\gamma\sigma_{C,2}^2 + \lambda_2(x-1)}{g_1 - \frac{1}{2}\gamma\sigma_{C,1}^2 + \lambda_1(x^{-1}-1)}, & \psi = 1. \end{cases} \quad (\text{B5})$$

Θ_i^B is the market price of risk due to Brownian shocks in state i , given by

$$\Theta_i^B = \gamma\sigma_{C,i}, \quad (\text{B6})$$

and Θ_i^P is the market price of risk due to Poisson shocks when the economy switches out of state i :

$$\Theta_i^P = \omega_i - 1. \quad (\text{B7})$$

We begin the proof by noting that if we define

$$\omega_i = \left. \frac{\pi_t}{\pi_{t-}} \right|_{\nu_{t-}=i, \nu_t=j}, j \neq i, \quad (\text{B8})$$

then (A5) implies that

$$\omega_i = \begin{cases} \left(\frac{p_{C,j}}{p_{C,i}} \right)^{-\frac{\gamma-1}{1-\frac{1}{\psi}}}, & \psi \neq 1; \\ \left(\frac{V_j}{V_i} \right)^{-(\gamma-1)}, & \psi = 1. \end{cases} \quad (\text{B9})$$

The above equation implies that $\omega_2 = \omega_1^{-1}$, so we can set $\omega_2 = \omega_1^{-1} = \omega$.

We now show how to determine ω . Using (B9) we rewrite (A6) and (A9) as

$$p_{C,i} = \frac{1}{\bar{r}_i + \gamma \sigma_{C,i}^2 - g_i + \lambda_i \frac{1-\frac{1}{\psi}}{\gamma-1} \left(\omega_i^{\frac{\gamma-1}{\gamma-1/\psi}} - 1 \right)}, \quad i \in \{1, 2\}, \quad (\text{B10})$$

and

$$\beta \ln V_i = g_i - \frac{1}{2} \gamma \sigma_{C,i}^2 + \lambda_i \frac{\omega_i - 1}{1 - \gamma}, \quad i \in \{1, 2\}, \quad (\text{B11})$$

respectively. Therefore, from (B9) and the above two equations it follows that ω is the solution of Equation (B4).

We now derive expressions for the risk-free rate and risk prices. Ito's Lemma implies that the state-price density evolves according to

$$\begin{aligned} \frac{d\pi_t}{\pi_{t-}} &= \frac{1}{\pi_{t-}} \frac{\partial \pi_{t-}}{\partial t} dt + \frac{1}{\pi_{t-}} C_t \frac{\partial \pi_{t-}}{\partial C_t} \frac{dC_t}{C_t} + \frac{1}{2} \frac{1}{\pi_{t-}} C_t^2 \frac{\partial^2 \pi_{t-}}{\partial C_t^2} \left(\frac{dC_t}{C_t} \right)^2 \\ &\quad + \lambda_{\nu_{t-}} \frac{\Delta \pi_t}{\pi_{t-}} dt + \frac{\Delta \pi_t}{\pi_{t-}} dN_{\nu_{t-}, t}^P, \end{aligned} \quad (\text{B12})$$

where $\Delta \pi_t = \pi_t - \pi_{t-}$. The definition (B8) implies

$$\left. \frac{\Delta \pi_t}{\pi_{t-}} \right|_{\nu_{t-}=i, \nu_t=j} = \omega_i - 1, \quad j \neq i.$$

Together with some standard algebra that allows us to rewrite (B12) as

$$\left. \frac{d\pi_t}{\pi_{t-}} \right|_{\nu_t=\nu_i} = - \left(\kappa_i + \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 + \lambda_i (1 - \omega_i) \right) dt - \gamma \sigma_{C,i} dB_{C,t} + (\omega_i - 1) dN_{i,t}^P.$$

Comparing the above equation with (B1), which is standard in an economy with jumps, gives (B6) and (B7), in addition to

$$r_i = \kappa_i + \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 + \lambda_i (1 - \omega_i),$$

where

$$\kappa_i = \begin{cases} \beta \left[1 + \left(\gamma - \frac{1}{\psi} \right) \frac{(\beta p_{C,i})^{-1} - 1}{1 - \frac{1}{\psi}} \right], & \psi \neq 1; \\ \beta \left[1 + (\gamma - 1) \ln(V_i^{-1}) \right], & \psi = 1. \end{cases} \quad (\text{B13})$$

We use Equations (B10) and (B11) to eliminate $p_{C,i}$ and V_i from (B13) to obtain

$$\kappa_i = \begin{cases} \bar{r}_i - \left(\gamma - \frac{1}{\psi} \right) \lambda_i \left(\frac{\omega_i^{\frac{\gamma-1}{\gamma-1/\psi}} - 1}{1 - \gamma} \right) - \left[\gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right], & \psi \neq 1; \\ \bar{r}_i + \lambda_i (\omega_i - 1) - \left[\gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right], & \psi = 1, \end{cases} \quad (\text{B14})$$

so

$$r_i = \begin{cases} \bar{r}_i - \left(\gamma - \frac{1}{\psi} \right) \lambda_i \left(\frac{\omega_i^{\frac{\gamma-1}{\gamma-1/\psi}} - 1}{1 - \gamma} \right) + \lambda_i (1 - \omega_i), & \psi \neq 1; \\ \bar{r}_i, & \psi = 1. \end{cases} \quad (\text{B15})$$

Taking the limit of the upper expression in the above equation gives the lower expression, so (B3) follows. The total market price of consumption risk in state i accounts for both Brownian and Poisson shocks, and is thus given by

$$\Theta_i = \sqrt{(\Theta_i^B)^2 + \lambda_i (\Theta_i^P)^2}, \quad i \in \{1, 2\}. \quad (\text{B16})$$

Because the Poisson and Brownian shocks in (B2) are independent and their respective prices of risk are bounded, M is a martingale under the actual measure \mathbb{P} . Thus, M defines the Radon-Nikodym derivative $\frac{d\mathbb{Q}}{d\mathbb{P}}$ via $M_t = E_t \left[\frac{d\mathbb{Q}}{d\mathbb{P}} \right]$. It is a standard result (see Elliott (1982)) that the risk-neutral switching probabilities per unit time are given by

$$\hat{\lambda}_i = \lambda_i E_t \left[\frac{M_t}{M_{t-}} \middle| \nu_{t-} = i, \nu_t = j \right], j \neq i.$$

The jump component in $d\pi$ comes purely from dM . Thus, using (B8), we can simplify the above expression to obtain $\hat{\lambda}_i = \lambda_i \omega_i$, which implies (5).

We deduce the properties of the risk distortion factor, ω , from the properties of the function G defined in (B5). We restrict the domain of G to $x > 0$. First we consider the case where $\psi \neq 1$. We assume that the price-consumption ratios, $p_{C,i}$, $i \in \{1, 2\}$, are strictly positive. Therefore, G is continuous. We observe that if G is monotonic, then by continuity, $G(1)$ and $G'(1)$ are the same (opposite) in sign iff $\omega < 1$ ($\omega > 1$). Clearly, in both cases, ω is unique. To establish monotonicity note that

$$\begin{aligned} G'(x) = & -\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} \left[\frac{1}{\left(\bar{r}_1 + \gamma \sigma_{C,1}^2 - g_1 + \lambda_1 \frac{1 - \frac{1}{\psi}}{\gamma - 1} \left(x^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right) \right)^2} \left(\bar{r}_1 + \gamma \sigma_{C,1}^2 - g_1 + \lambda_1 \frac{1 - \frac{1}{\psi}}{\gamma - 1} \left(x^{-\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right) \right) \lambda_2 x^{\frac{1/\psi-1}{\gamma-1/\psi}} \right. \\ & \left. + \left(\bar{r}_2 + \gamma \sigma_{C,2}^2 - g_2 + \lambda_2 \frac{1 - \frac{1}{\psi}}{\gamma - 1} \left(\omega^{\frac{\gamma-1}{\gamma-\frac{1}{\psi}}} - 1 \right) \right) \lambda_1 x^{-\frac{1/\psi-1}{\gamma-1/\psi}-2} \right] + x^{-\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} - 1} \end{aligned}$$

The above equation implies that for $x > 0$, if $p_{C,1}$ and $p_{C,2}$ are strictly positive, then $G'(x)$ does not change sign. Therefore, G must be monotonic. Now we use the following expressions:

$$G(1) = 1 - \frac{\bar{r}_2 + \gamma \sigma_{C,2}^2 - g_2}{\bar{r}_1 + \gamma \sigma_{C,1}^2 - g_1},$$

and

$$G'(1) = -\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} \left[1 + \frac{(\bar{r}_1 + \gamma \sigma_{C,1}^2 - g_1) \lambda_2 + (\bar{r}_2 + \gamma \sigma_{C,2}^2 - g_2) \lambda_1}{(\bar{r}_1 + \gamma \sigma_{C,1}^2 - g_1)^2} \right],$$

to relate the signs of $G(1)$ and $G'(1)$ to the properties of the agent's preferences. Note that $G'(1) < 0$, ($G'(1) > 0$) iff $\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} > 0$,

$\left(\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} < 0 \right)$. We assume that $\bar{r}_i + \gamma \sigma_{C,i}^2 - g_i > 0$ for $i \in \{1, 2\}$, which is equivalent to assuming that if the economy were always in

state i , then the price-consumption ratio would be positive. Simple algebra tells us that $\bar{r}_i + \gamma \sigma_{C,i}^2 - g_i = \beta + \left(\frac{1}{\psi} - 1 \right) \left(g_i - \frac{1}{2} \gamma \sigma_{C,i}^2 \right)$.

We know that $g_1 - \frac{1}{2} \gamma \sigma_{C,1}^2 < g_2 - \frac{1}{2} \gamma \sigma_{C,2}^2$. Therefore $G(1) < 0$, ($G(1) > 0$) iff $\psi > 1$, ($\psi < 1$). Consequently, $G(1)$ and $G'(1)$ are the same (opposite) in sign iff $\gamma < 1/\psi$ ($\gamma > 1/\psi$). It then follows that $\omega > 1$ ($\omega < 1$) iff $\gamma > 1/\psi$ ($\gamma < 1/\psi$), assuming that $\psi \neq 1$.

Similarly, when $\psi = 1$, if we assume that $V_i > 0$ for $i \in \{1, 2\}$, then we can prove that: $\omega > 1$ if $\gamma > 1$ ($\gamma < 1$) and $g_i - \frac{1}{2} \gamma \sigma_{C,i}^2$, $i \in \{1, 2\}$ are of the same (opposite) sign. Now, if $\gamma < 1$, then $\bar{r}_1 + \gamma \sigma_{C,1}^2 - g_1 > 0$ implies $g_1 - \frac{1}{2} \gamma \sigma_{C,1}^2 > 0$, which means $g_i - \frac{1}{2} \gamma \sigma_{C,i}^2$, $i \in \{1, 2\}$ cannot be of opposite sign. Therefore, $\omega > 1$ iff $\gamma > 1$.

So, for $\psi > 0$, $\omega > 1$ ($\omega < 1$) iff $\gamma > 1/\psi$ ($\gamma < 1/\psi$). It follows that $\omega = 1$ iff $\gamma = 1/\psi$. ■

Proof of Proposition 2. By definition

$$q_{D,ij,T-t} = E_t \left[\frac{\pi_{\tau_D}}{\pi_t} 1_{\{\tau_D \leq T \text{ \& } \nu_D = j\}} \middle| \nu_t = i \right], \quad (\text{B17})$$

Under the risk-neutral measure, \mathbb{Q} , the above equation can be written as

$$q_{D,ij,T-t} = E_t^{\mathbb{Q}} \left[e^{-\int_t^T r_u du} 1_{\{\tau_D \leq T \text{ \& } \nu_D = j\}} | \nu_t = i \right].$$

Therefore,

$$q_{D,ij,T-t} = \int_t^T E_t^{\mathbb{Q}} [e^{-\int_t^s r_u du}] \hat{f}_{\tau_D,ij}(s) ds.$$

where $\hat{f}_{\tau_D,ij}(s)$ is risk-neutral probability density function for default occurring in state j , conditional on the current state being i . The risk-neutral probability of default occurring in state j before time T , conditional on the current state being i is

$$\hat{p}_{D,ij,T-t} = \int_t^T \hat{f}_{\tau_D,ij}(s) ds.$$

The time-adjustment factor, $\mathcal{T}_{ij,T-t}$, is defined by $\mathcal{T}_{ij,T-t} = \frac{q_{D,ij,T-t}}{\hat{p}_{D,ij,T-t}}$. Therefore,

$$\mathcal{T}_{ij,T-t} = \int_t^T E_t^{\mathbb{Q}} [e^{-\int_t^s r_u du}] \frac{\hat{f}_{\tau_D,ij}(s)}{\int_t^T \hat{f}_{\tau_D,ij}(s) ds} ds.$$

From Bayes' Law,

$$h_{ij}(s) = \frac{\hat{f}_{\tau_D,ij}(s)}{\int_t^T \hat{f}_{\tau_D,ij}(s) ds}$$

is the risk-neutral probability density of default occurring in state j , conditional on the current state being i and default occurring in state j before time T . Therefore,

$$\mathcal{T}_{ij,T-t} = \int_t^T E_t^{\mathbb{Q}} [e^{-\int_t^s r_u du}] h_{ij}(s) ds,$$

which implies (9). Equation (10) holds by definition. ■

Proof of Proposition 3. Suppose the economy is currently in state i . Then, the risk-neutral probability of the economy switching into a different state during a small time interval Δt is $\hat{\lambda}_i \Delta t$ and the risk-neutral probability of not switching is $1 - \hat{\lambda}_i \Delta t$. We can therefore write the unlevered firm value in state i as

$$A_i = (1 - \eta) X \Delta t + e^{-(\bar{\mu}_i - \theta_i) \Delta t} \left[(1 - \hat{\lambda}_i \Delta t) A_i + \hat{\lambda}_i \Delta t A_j \right], \quad i, j \in \{1, 2\}, j \neq i. \quad (\text{B18})$$

The first term in (B18) is the after-tax cash flow received in the next instant and the second term is the discounted continuation value. The continuation value is the average of A_i and A_j , weighted by the risk-neutral probabilities of being in states i and $j \neq i$ a small instant Δt from now. For example, with risk-neutral probability $\hat{\lambda}_i \Delta t$ the economy will be in state $j \neq i$ and the abandonment value will be A_j . The continuation value is discounted back at a rate reflecting the discount rate $\bar{\mu}_i$ and the expected earnings growth rate over that instant which is θ_i .

We take the limits of (B18) as $\Delta t \rightarrow 0$, to obtain

$$0 = (1 - \eta) X - (\bar{\mu}_i - \theta_i) A_i + \hat{\lambda}_i (A_j - A_i), \quad i \in \{1, 2\}, j \neq i.$$

To obtain the solution of the above linear equation system, we define

$$p_i = \frac{1}{(1 - \eta)} \frac{A_i}{X},$$

the before-tax price-earnings ratio in state i . Therefore

$$\left(\text{diag}(\bar{\mu}_1 - \theta_1, \bar{\mu}_2 - \theta_2) - \hat{\Lambda} \right) \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (\text{B19})$$

where $\text{diag}(\bar{\mu}_1 - \theta_1, \bar{\mu}_2 - \theta_2)$ is a 2×2 diagonal matrix, with the quantities $\bar{\mu}_1 - \theta_1$ and $\bar{\mu}_2 - \theta_2$ along the diagonal and

$$\hat{\Lambda} = \begin{pmatrix} -\hat{\lambda}_1 & \hat{\lambda}_1 \\ \hat{\lambda}_2 & -\hat{\lambda}_2 \end{pmatrix}$$

is the generator matrix of the Markov chain under the risk-neutral measure. Solving (B19) gives (13), if $\det(\text{diag}(\bar{\mu}_1 - \theta_1, \bar{\mu}_2 - \theta_2) - \hat{\Lambda}) \neq 0$. We now define $P_i^X = p_i X$, the before-tax value of the claim to the earnings stream X in state i . Hence, from the basic asset pricing equation

$$E_t \left[\frac{dP^X + X dt}{P^X} - r dt \middle| \nu_{t-} = i \right] = -E_t \left[\frac{d\pi}{\pi} \frac{dP^X}{P^X} \middle| \nu_{t-} = i \right],$$

we obtain the unlevered risk premium:

$$E_t \left[\frac{dP^X + X dt}{P^X} - r dt \middle| \nu_{t-} = i \right] = \gamma \rho_{XC} \sigma_{X,i}^s \sigma_{C,i} dt - (\hat{\lambda}_i - \lambda_i) \left(\frac{p_j}{p_i} - 1 \right) dt, i \in \{1, 2\}, j \neq i.$$

Applying Ito's Lemma,

$$dP^X = p_i dX_t + \lambda_i (p_j - p_i) dt + (p_j - p_i) dN_{i,t}^P, i \in \{1, 2\}, j \neq i.$$

Thus, the unlevered volatility of returns on equity in state i is given by

$$\sigma_{R,i} = \sqrt{\sigma_{X,i}^2 + \lambda_i \left(\frac{p_j}{p_i} - 1 \right)^2}, j \neq i,$$

where $\sigma_{X,i} = \sqrt{(\sigma_{X,i}^s)^2 + (\sigma_{X,i}^{id})^2}$. ■

Proof of Proposition 4. In this proof it is not necessary to distinguish between the state of the economy at dates $t-$ and t . First we show that (16) holds. The central part of our proof consists of proving that

$$E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] = \frac{1}{r_{P,i}} - \sum_{j=1}^2 \frac{q_{D,ij}}{r_{P,j}}, \quad (\text{B20})$$

where $r_{P,i}$, the discount rate for a risk-free perpetuity, when the economy is in state i , is given by

$$r_{P,i} = \left(E_t \left[\int_t^\infty \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] \right)^{-1}, \quad (\text{B21})$$

and

$$E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \alpha_{\tau_D} A_{\tau_D}(X_{\tau_D}) \middle| \nu_t = i \right] = \sum_{j=1}^2 \alpha_j A_j(X_{D,j}) q_{D,ij}. \quad (\text{B22})$$

Using the above result, (16) follows immediately from (15).

To prove (B20), we note that

$$E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] = E_t \left[\int_t^\infty \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] - E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \int_{\tau_D}^\infty \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_t = i \right],$$

and conditioning on the event $\{\nu_{\tau_D} = j\}$, we obtain

$$E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \int_{\tau_D}^\infty \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_t = i \right] = \sum_{j=1}^2 E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \int_{\tau_D}^\infty \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_t = i \right].$$

Since consumption is Markovian, so is the state-price density, which implies that

$$\begin{aligned} & E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_t = i \right] \\ &= E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i \right] E_t \left[\int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_{\tau_D} = j \right]. \end{aligned}$$

Therefore

$$\begin{aligned} E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] &= E_t \left[\int_t^{\infty} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] \\ &\quad - \sum_{j=1}^2 E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i \right] E_t \left[\int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_{\tau_D} = j \right]. \end{aligned} \quad (\text{B23})$$

Conditional on being in state i , the value of a claim which pays one risk-free unit of consumption in perpetuity is $E_t \left[\int_t^{\infty} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right]$, so the discount rate for this perpetuity, $r_{P,i}$, is given by (B21). Consequently, (B23) implies

$$E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] = \frac{1}{r_{P,i}} - \sum_{j=1}^2 \frac{E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i \right]}{r_{P,j}}. \quad (\text{B24})$$

To obtain (B20) from the above expression, we note that (B17) implies

$$q_{D,ij,t} = E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i \right]. \quad (\text{B25})$$

We do not have to evaluate $r_{P,i}$ from scratch based on (B21), because we can infer its value from (13), by setting $\nu_t = i$, and then $\theta_i = \sigma_{X,i} = \rho_{XC,i} = 0$, to obtain (18). To prove (B22), we condition on the event $\{\nu_{\tau_D} = j\}$ to obtain

$$E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \alpha_{\tau_D} A_{\tau_D}(X_{\tau_D}) \middle| \nu_t = i \right] = \sum_{j=1}^2 \alpha_j A_j(X_j) E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i \right].$$

Using (B25) to simplify the above expression we obtain (B22). The credit spread in state i is

$$s_i = y_i - r_{P,i} = \frac{c}{B_i} - r_{P,i}. \quad (\text{B26})$$

Substituting (16) into the above equation and simplifying gives (19). ■

Proposition B1 *The volatility of the credit spread in state i , $\sigma_{s,i}$, is given by*

$$\sigma_{s,i} = \sqrt{\left((s_i + r_{P,i}) \frac{\partial \ln B_i}{\partial \ln X} \right)^2 \left[(\sigma_{X,i}^{id})^2 + (\sigma_{X,i}^s)^2 \right] + \lambda_i (s_j - s_i)^2, \quad i \in \{1, 2\}, j \neq i,} \quad (\text{B27})$$

where, in state i , the elasticity of the bond price with respect to earnings under static capital structure is given by

$$\frac{\partial \ln B_i}{\partial \ln X} = -\frac{s_i + r_{P,i}}{r_{P,i}} \sum_{j=1}^2 \left(\frac{\partial \ln q_{D,ij}}{\partial \ln X} \right) q_{D,ij} l_{D,ij}, \quad i, j \in \{1, 2\}. \quad (\text{B28})$$

Proof of Proposition B1. Equation (B27) follows from applying Ito's Lemma to (B26) and identifying the diffusion term. From (16), we can show that, in state i , the elasticity of the bond price with respect to earnings is given by (B28). ■

Proof of Proposition 5. The derivation of (20) follows the same approach as the derivation of (16). Applying Ito's Lemma to (20), we obtain

$$dR_t|_{\nu_{t-}=i} = \frac{dS_t + (1-\eta)(X_t - c)dt}{S_{t-}} \Big|_{\nu_{t-}=i} = \mu_{R,i}dt + \sigma_{R,i}^{B,id} dB_{X,t}^{id} + \sigma_{R,i}^{B,s} dB_{X,t}^s + \sigma_{R,i}^P dN_{i,t}^P,$$

where

$$\begin{aligned} \mu_{R,i} &= \frac{A_i(X) + \sum_{j=1}^2 \left[X q'_{D,ij} \theta_i + \frac{1}{2} X^2 q''_{D,ij} \sigma_{X,i}^2 \right] \left[(1-\eta) \frac{c}{r_{P,j}} - A_j(X_{D,j}) \right]}{S_i} \\ &\quad + \left(\frac{S_k}{S_i} - 1 \right) \lambda_i + \frac{A_i(X)}{S_i}, \quad k \neq i, \end{aligned}$$

$\sigma_{R,i}^P$ and $\sigma_{R,i}^{B,s}$ are given in (23) and (24), respectively. Idiosyncratic stock return volatility, $\sigma_{R,i}^{B,id}$, caused by Brownian shocks is given by

$$\sigma_{R,i}^{B,id} = \frac{\partial \ln S_{i,t-}}{\partial \ln X_t} \sigma_{X,i}^{id}, \quad i \in \{1, 2\}, \quad (\text{B29})$$

where the elasticity of levered equity with respect to earnings is

$$\frac{\partial \ln S_{i,t-}}{\partial \ln X_t} = \frac{\frac{A_i(X_t)}{X_t} + \sum_{j=1}^2 q'_{D,ij} \left[(1-\eta) \frac{c}{r_{P,j}} - A_j(X_{D,j}) \right]}{S_{i,t-}/X_t}, \quad i \in \{1, 2\}. \quad (\text{B30})$$

Therefore,

$$-E_t \left[dR \frac{d\pi}{\pi} \Big| \nu_{t-}=i \right] = \left\{ \gamma \rho_{XC,i} \sigma_{R,i}^{B,s} \sigma_C - \sigma_{R,i}^P (\omega_i - 1) \lambda_i \right\} dt,$$

and because the levered equity risk premium is given by

$$E_t [dR - rdt | \nu_{t-}=i] = -E_t \left[dR \frac{d\pi}{\pi} \Big| \nu_{t-}=i \right],$$

we obtain (21). ■

Proposition B2 *Conditional levered stock return volatility in state i is given by*

$$\sigma_{R,i} = \sqrt{\left(\sigma_{R,i}^{B,id} \right)^2 + \left(\sigma_{R,i}^{B,s} \right)^2 + \lambda_i \left(\sigma_{R,i}^P \right)^2}, \quad i \in \{1, 2\}. \quad (\text{B31})$$

Proof of Proposition B2. Overall levered stock return volatility in state i is given by combining the variances from Brownian and Poisson shocks to obtain (B31). ■

Proof of Weak Countercyclicality of the Default Boundary. It is clear that $S_1(X) \leq S_2(X)$ and that both $S_1(X)$ and $S_2(X)$ are monotonically increasing in X . Thus, for $i \in \{1, 2\}$, if there exists $X_{D,i}$ such that $S_i(X_{D,i}) = 0$, then $X_{D,i}$ is unique. From $S_1(X) \leq S_2(X)$ it follows that $S_2(X_{D,1}) \geq S_1(X_{D,1}) = 0$. Since $S_2(X)$ is monotonically increasing in X , it follows that $X_{D,2} \leq X_{D,1}$. ■

Proof of Proposition 6. For ease of notation, we define $\xi_{\nu_0, \nu_U} = \xi_{\nu_U}(c_{\nu_0})$. At restructuring existing debt is diluted on a per coupon basis, so that (32) holds. Note that

$$c_{\nu_U}(c_{\nu_0}) = c_{\nu_U} \frac{X_{U, \nu_0, \nu_U}}{X_0} = c_{\nu_U} \xi_{\nu_0, \nu_U}.$$

Therefore,

$$B_{\nu_U}(X_{U,\nu_0,\nu_U}, c_{\nu_U}(c_{\nu_0}), \nu_U) = B_{\nu_U}(\xi_{\nu_0,\nu_U} X_0, \xi_{\nu_0,\nu_U} c_{\nu_U}, \nu_U),$$

which can be simplified using the homogeneity property to give

$$B_{\nu_U}(X_{U,\nu_0,\nu_U}, c_{\nu_U}(c_{\nu_0}), \nu_U) = \xi_{\nu_0,\nu_U} B_{\nu_U}(X_0, c_{\nu_U}, \nu_U). \quad (\text{B32})$$

It then follows from (32) that

$$R_{\nu_0,\nu_U}(X_{U,\nu_0,\nu_U}, c_{\nu_U}(c_{\nu_0})) = \frac{c_{\nu_0}}{c_{\nu_U}} B_{\nu_U}(X_0, c_{\nu_U}, \nu_U). \quad (\text{B33})$$

Thus (31) implies that

$$\begin{aligned} B_{\nu_t}(X_t, c_{\nu_0}, \nu_0) &= \frac{c_{\nu_0}}{r_{P,\nu_t}} + \sum_{\nu_D=1}^2 q_{D,\nu_t\nu_D}(X_t, \nu_0) \left(\alpha_{\nu_D} A_{\nu_D}(X_{D,\nu_0\nu_D}) - \frac{c_{\nu_0}}{r_{P,\nu_D}} \right) \\ &\quad + c_{\nu_0} \sum_{\nu_D=1}^2 q_{U,\nu_t\nu_D}(X_t, \nu_0) \left(\frac{B_{\nu_D}(X_0, c_{\nu_D}, \nu_D)}{c_{\nu_D}} - \frac{1}{r_{P,\nu_D}} \right). \end{aligned} \quad (\text{B34})$$

Evaluating the above expression at, $X_t = X_0$, gives debt values at refinancing:

$$\begin{aligned} B_{\nu_0}(X_0, c_{\nu_0}, \nu_0) &= \frac{c_{\nu_0}}{r_{P,\nu_0}} + \sum_{\nu_D=1}^2 q_{D,\nu_0\nu_D}(X_0, \nu_0) \left(\alpha_{\nu_D} A_{\nu_D}(X_{D,\nu_0\nu_D}) - \frac{c_{\nu_0}}{r_{P,\nu_D}} \right) \\ &\quad + c_{\nu_0} \sum_{\nu_D=1}^2 q_{U,\nu_0\nu_D}(X_0, \nu_0) \left(\frac{B_{\nu_D}(X_0, c_{\nu_D}, \nu_D)}{c_{\nu_D}} - \frac{1}{r_{P,\nu_D}} \right). \end{aligned} \quad (\text{B35})$$

Since $\nu_0 \in \{1, 2\}$, we obtain the following matrix equation

$$\begin{pmatrix} 1 - q_{U,11}(X_0, 1) & -\frac{c_1}{c_2} q_{U,12}(X_0, 1) \\ -\frac{c_2}{c_1} q_{U,21}(X_0, 2) & 1 - q_{U,22}(X_0, 2) \end{pmatrix} \begin{pmatrix} B_1(X_0, c_1, 1) \\ B_2(X_0, c_2, 2) \end{pmatrix} = \begin{pmatrix} b_1(X_0, c_1) \\ b_2(X_0, c_2) \end{pmatrix},$$

where $b_i(X_0, c_i)$, $i \in \{1, 2\}$ are defined by

$$b_i(X_0, c_i) = \frac{c_i}{r_{P,i}} \left(1 - \sum_{\nu_D=1}^2 q_{D,i\nu_D}(X_0, i) l_{D,i\nu_D} - \sum_{\nu_U=1}^2 q_{U,\nu_U}(X_0, i) \frac{r_{P,i}}{r_{P,\nu_U}} \right), \quad i \in \{1, 2\},$$

and $l_{D,i\nu_D}$ is defined in (17). Solving the above matrix equation gives us

$$B_i(X_0, c_i, i) = \Delta_B^{-1} \left[(1 - q_{U,jj}(X_0, X_{D,jj})) b_i(X_0, c_i) + \frac{c_i}{c_j} q_{U,ij}(X_0, X_{D,ij}) b_j(X_0, c_j) \right], \quad j \neq i, \quad (\text{B36})$$

where $\Delta_B = (1 - q_{U,11}(X_0, 1))(1 - q_{U,22}(X_0, 2)) - q_{U,21}(X_0, 2)q_{U,12}(X_0, 1)$. Thus, we can use (B34) to compute $B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)$, where the current state is not necessarily the same as the state at restructuring. Note that the expression for $B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)$ in (B34) can be rewritten as (33). Therefore, the credit spread at date t , $s_{\nu_t}(X_t, c_{\nu_0}, \nu_0) = \frac{c_{\nu_0}}{B_{\nu_t}(X_t, c_{\nu_0}, \nu_0)} - r_{P,\nu_t}$, is given by (34). ■

Proof of Proposition 7. For ease of notation, we define $\xi_{\nu_0,\nu_U} = \xi_{\nu_U}(c_{\nu_0})$. If refinancing occurs in state j , the value of equity just before refinancing is given by (38). Since the homogeneity property implies that (B32), (B33), and

$$S_j(X_{U,\nu_0j}, c_j(c_{\nu_0}), j) = S_j(\xi_{\nu_0,j} X_0, \xi_{\nu_0,j} c_j, j) = \xi_{\nu_0,j} S_j(X_0, c_j, j),$$

it follows that

$$E_{\nu_0,j}(X_{U,\nu_0j}) = B_j(X_0, c_j, j) \left((1 - \nu_j) \xi_{\nu_0,j} - \frac{c_{\nu_0}}{c_j} \right) + \xi_{\nu_0,j} S_j(X_0, c_j, j).$$

Hence,

$$S_{\nu_t}(X_t, c_{\nu_0}, \nu_0) = Div_{\nu_t}(X_t, c_{\nu_0}, \nu_0) + \sum_{j=1}^2 q_{U, \nu_t j}(X_t, \nu_0) \left[B_j(X_0, c_j, j) \left((1 - \iota_j) \xi_{\nu_0, j} - \frac{c_{\nu_0}}{c_j} \right) + \xi_{\nu_0, j} S_j(X_0, c_j, j) \right]. \quad (\text{B37})$$

Evaluating the above expression at, $X_t = X_0$, gives equity values at refinancing:

$$S_{\nu_0}(X_0, c_{\nu_0}, \nu_0) = Div_{\nu_0}(X_0, c_{\nu_0}, \nu_0) + \sum_{j=1}^2 q_{U, \nu_0 j}(X_0, \nu_0) \left[B_j(X_0, c_j, j) \left((1 - \iota_j) \xi_{\nu_0, j} - \frac{c_{\nu_0}}{c_j} \right) + \xi_{\nu_0, j} S_j(X_0, c_j, j) \right].$$

Since $\nu_0 \in \{1, 2\}$, we obtain the following matrix equation

$$\begin{pmatrix} 1 - q_{U, 11}(X_0, 1) \xi_{11} & -q_{U, 12}(X_0, 1) \xi_{12} \\ -q_{U, 21}(X_0, 2) \xi_{21} & 1 - q_{U, 22}(X_0, 2) \xi_{22} \end{pmatrix} \begin{pmatrix} S_1(X_0, c_1, 1) \\ S_2(X_0, c_2, 2) \end{pmatrix} = \begin{pmatrix} s_1(X_0, c_1) \\ s_2(X_0, c_2) \end{pmatrix},$$

where $s_i(X_0, c_i)$, $i \in \{1, 2\}$ are defined by

$$s_i(X_0, c_i) = Div_i(X_0, c_i, i) + \sum_{\nu_U=1}^2 q_{U, i \nu_U} B_{\nu_U}(X_0, c_{\nu_U}, \nu_U) \left((1 - \iota_{\nu_U}) \xi_{1 \nu_U} - \frac{c_i}{c_{\nu_U}} \right). \quad (\text{B38})$$

Solving the above matrix equation gives

$$S_i(X_0, c_i, i) = \Delta_S^{-1} [(1 - q_{U, jj}(X_0, j)) \xi_{jj} s_i(X_0, c_i) + q_{U, ij}(X_0, i) \xi_{ij} s_j(X_0, c_j)], \quad j \neq i, \quad (\text{B39})$$

where $\Delta_S = (1 - q_{U, 11}(X_0, 1) \xi_{11})(1 - q_{U, 22}(X_0, 2) \xi_{22}) - q_{U, 12}(X_0, 1) q_{U, 21}(X_0, 2) \xi_{21} \xi_{12}$. Using the above solution, we can exploit (B37) to rewrite $S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)$ as (39). The conditional levered equity risk premium in state i is the same as in Proposition 5.

Therefore, to compute the equity risk premium, we must compute the elasticity $\frac{\partial \ln S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}{\partial \ln X_t} \sigma_{X, i}^s$. To make it clear how to do this, we define $S_{\nu_0, \nu_t} = S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)$. So we must compute four equity risk premia, each of which is a function of X_t . We define a general notation for the risk premium, $RP_{\nu_0 \nu_t}$, where the first subscript, ν_0 , is the historical state of the economy at the previous refinancing date and ν_t is the current state. Thus, in an economy with two states, there are four possible risk premia:

$$RP_{\nu_0 \nu_t} = \gamma \rho_{XC} \frac{\partial \ln S_{\nu_0 \nu_t}}{\partial \ln X} \sigma_{X, \nu_t}^s \sigma_{C, \nu_t} + (1 - \omega_{\nu_t}) \left(\frac{S_{\nu_0 \nu_0}}{S_{\nu_0 \nu_t}} - 1 \right) \lambda_{\nu_t}, \quad \nu_0, \nu_t \in \{1, 2\}.$$

■

C Appendix: Simulation procedure

To generate the dynamic economy implied by our model, we assume that all 3,000 firms are at their refinancing points at date 0. We then simulate the model for 100 years at a quarterly frequency to ensure convergence to the long-run steady state. The last period in this simulation is reset as the starting period for any results reported in the paper. Each quarter, firms observe both idiosyncratic and systematic contemporaneous shocks as well as the state of the economy and make optimal decisions. If the earnings level reaches the state-conditional refinancing boundary, the firm refinances. If the earnings level reaches the state-conditional default boundary, the firm defaults. In the benchmark simulation exercise, we assume that the firm is liquidated and a new firm with the date-0 earnings level enters the economy. The total number of firms is therefore constant in the dynamic economy. If the earnings level does not reach either of the two boundaries, the firm optimally takes no action.

For every firm i and time t , the quasi-market leverage ratio, $QML_{i,t}$, is defined as

$$QML_{i,t} = \frac{B_{\nu_0}(X_{\nu_0}, c_{\nu_0}, \nu_0)}{B_{\nu_0}(X_{\nu_0}, c_{\nu_0}, \nu_0) + S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)}, \quad (\text{C40})$$

where $B_{\nu_0}(X_{\nu_0}, c_{\nu_0}, \nu_0)$ is the par value of debt issued at the start of the current refinancing period (in other words, the book value of debt).

Our simulation procedure is robust to various modifications. For example, in Section IV.B we match the average empirical sample rather than an empirical sample on each date for time-efficiency reasons. Alternatively, we could match an empirical sample at every date t for which the empirical data are available and average the results over time. This assumption, as well as the choice of a metric used to match empirical and simulated samples do not significantly alter our results. The results are also robust: (1) to various assumptions on what occurs at default as long as there is some replacement of defaulted firms (otherwise, for a sufficiently long horizon, all firms default); (2) when time intervals are less than a quarter; (3) and when longer horizons are allowed for convergence.

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Table I : Summary of models in the literature

	FHZ	Leland	GJL	HMM	CC	BY	CF	CDG	This paper
Structural model features									
default risky corporate debt	✓	✓	✓	✓				✓	✓
endogenous default boundary	✓	✓	✓	✓					✓
dynamic capital structure	✓		✓	✓					✓
stochastic earnings growth rates				✓					✓
dynamic cross-section ('true dynamics')									✓
Consumption-based model									
recursive preferences						✓	✓		✓
habit-formation preferences					✓			✓	
stochastic dividend growth rate						✓	✓		✓
stochastic consumption growth rate						✓	✓		✓
Pricing implications									
credit spread	✓	✓	✓	✓				✓	✓
equity risk premium						✓	✓	✓	✓
impact of default risk on equity risk premium									✓
cross-market predictability								✓	✓

This table compares features of structural models which are used to price corporate debt: Fischer et al. (1989) (FHZ), Leland (1994), Goldstein et al. (2001) (GJL), Hackbarth et al. (2006) (HMM), consumption-based models which are used to price the aggregate stock market: Campbell and Cochrane (1999) (CC), Bansal and Yaron (2004) (BY), Calvet and Fisher (2008) (CF), and models which are used to price both corporate debt and the aggregate stock market: Chen et al. (2008) (CDG) and this paper. The comparison table is divided into 3 panels. The first panel focuses on the features of structural models, the second on the features of consumption-based models, while the third focuses on the pricing implications of the various models.

Table II : Parameter estimates

This table reports the estimates of model parameters. To calibrate the model to the aggregate US economy, quarterly real non-durable plus service consumption expenditure from the Bureau of Economic Analysis and quarterly earnings data from Standard and Poor's, provided by Robert J. Shiller, are used. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. Panel A reports unconditional first and second moments of empirical variables. Panel B reports conditional estimates. The estimates of consumption growth rate and volatility, earnings growth rate and volatility, and correlation between earnings and consumption growth are obtained by maximum likelihood and based on quarterly log growth rates for the period from 1947 to 2005. All variables are given per annum and in per cent (0.01 means 1% p.a.)

Panel A: Unconditional estimates		
	Mean	Std. dev.
Real consumption growth	0.0333	0.0099
Real earnings growth	0.0343	0.1072

Panel B: Conditional estimates			
Parameter	Symbol	State 1	State 2
Consumption growth rate	g_i	0.0141	0.0420
Consumption growth volatility	$\sigma_{C,i}$	0.0114	0.0094
Earnings growth rate	θ_i	-0.0401	0.0782
Earnings growth volatility	$\sigma_{X,i}^s$	0.1334	0.0834
Idiosyncratic earnings growth volatility	σ_X^{id}	0.2258	0.2258
Correlation	ρ_{XC}	0.1998	0.1998
Actual long-run probabilities	f_i	0.3555	0.6445
Actual convergence rate to long-run	p	0.7646	0.7646
Annual discount rate	β	0.01	0.01
Tax rate	η	0.15	0.15
Bankruptcy costs	$1 - \alpha_i$	0.30	0.10
Debt issuance cost	ι_i	0.03	0.01

Table III : Credit risk implications

This table reports the credit risk implications of the model. Panel A gives results for the specification with exogenous default boundary chosen to match actual default probabilities for the BBB sample and exogenous leverage. Panel B shows results for the specification with optimal default boundary and exogenous leverage. Panels A and B give results at the refinancing point, with exogenous leverage fixed at 40%. In static (dynamic) capital structure, firms do not (do) refinance their liabilities. Panel C reports the results for a dynamic simulated cross-section of BBB firms implied by the model with optimal default and dynamic capital structure decisions. The simulation procedure is described in Appendix C. Statistics are averaged across all simulated economies. Quantiles for default probability are based on the distribution of default rates over all generated economies. Default probability is cumulative default rate over the horizon of five or ten years. Arrow-Debreu default claim is the price of one unit of consumption paid at default. Risk adjustment is the ratio of the risk-neutral to actual default probabilities. Time adjustment is the ratio of the Arrow-Debreu claim to the risk-neutral default probability. Credit spread is given in basis points and all other variables in per cent.

Sample	Capital Structure	Panel A: Calibrated Default Boundary					Panel B: Optimal Default Boundary					Panel C: True Dynamics				
		Refinancing Date					Refinancing Date					Dynamic Cross-Section				
		Static	Dynamic	Static	Dynamic	Static	Static	Dynamic	Static	Dynamic	Static	Dynamic	Dynamic	Dynamic	Dynamic	Dynamic
Maturity (years)	T	5	10	5	10	5	10	5	10	5	10	5	10	5	10	10
Credit spread	s	b.p.	43	74	45	75	10	49	8	42	75	115				
Default probability	p_D	%	2.00	5.00	2.00	5.00	0.38	3.14	0.32	2.78	3.47	8.38				
Average		%									2.24	7.07				
Median		%									1.12	3.56				
25% quantile		%									4.49	12.9				
75% quantile		%														
Arrow-Debreu																
default claim	q_D	%	4.28	13.66	3.97	13.40	0.61	5.79	0.49	5.26	4.33	12.11				
Risk adjustment	\mathcal{R}		1.85	2.38	1.92	2.48	2.11	2.63	2.10	2.65	1.83	2.31				
Time adjustment	\mathcal{T}		0.93	0.87	0.92	0.86	0.92	0.86	0.93	0.86	0.93	0.86				
Current leverage	$B/(B+S)$	%	40	40	40	40	40	40	40	40	Sample	Sample				

Table IV : Empirical default rates and credit spreads

Panel A reports average cumulative issuer-weighted annualized default rates for BBB debt over 5, 10, and 15 year horizons for US firms as reported by Cantor et al. (2008). The first row shows mean historical default rates for the period 1920–2007 and the second row for 1970–2007. Panel B reports the difference between average spreads for BBB and AAA corporate debt, sorted by maturity. Data from Duffee (1998) are for bonds with no option-like features, taken from the Fixed Income Dataset, University of Houston, for the period Jan 1973 to March 1995, where maturities from 2 to 7 years are short, 7 to 15 are medium, and 15 to 30 are long. For Huang and Huang (2003), short denotes a maturity of 4 years and medium of 10 years. The data used in David (2008) are taken from Moody’s and medium denotes a maturity of 10 years. For Davydenko and Strebulaev (2007), the data are taken from the National Association of Insurance Companies; short denotes a maturity from 1 to 7 years, medium – 7 to 15 years, and long – 15 to 30 years.

Panel A: Historical BBB Default Probabilities				
Rating	Units	Year 5	Year 10	Year 15
1920 – 2007	%	3.142	7.061	10.444
1970 – 2007	%	1.835	4.353	7.601
Panel B: BBB/AAA Spreads				
Rating	Units	Short	Medium	Long
Duffee (1998)	b.p.	75	70	105
Huang and Huang (2003)	b.p.	103	131	–
David (2008)	b.p	–	96	–
Davydenko and Strebulaev (2007)	b.p	77	72	82

Table V : Path-dependence in credit risk

This table reports the state-conditional credit risk implications of the dynamic capital structure specification of the model. Column ν_0/ν_t reports the value of variables when the current state is ν_t and the state of the economy at the refinancing date was ν_0 . Default and restructuring boundaries are optimal, and the coupons set for the leverage ratios at refinancing dates in both states equal 40%. The earnings level is the same for all cases. All values are reported relative to the first column (1/1: the states of refinancing and the current state are 1) which is reset to be equal to 100%, and so units are in per cent. The actual default probability is a cumulative default rate over the horizon of five, ten, or fifteen years. The Arrow-Debreu default claim is the price of one unit of consumption paid at default. The risk adjustment is the ratio of the risk-neutral to actual default probabilities. Time adjustment is the ratio of the Arrow-Debreu claim to the risk-neutral default probability. Leverage is the ratio of the market value of debt to the sum of the market values of debt and equity.

		1/1	1/2	2/1	2/2
CREDIT SPREAD	s				
PERPETUITY		100.0	92.9	110.6	101.1
15-YEAR		100.0	75.7	129.5	97.1
10-YEAR		100.0	66.9	144.0	95.9
5-YEAR		100.0	48.7	202.3	95.7
ARROW-DEBREU DEFAULT CLAIM	q_D				
15-YEAR		100.0	74.6	129.4	97.0
10-YEAR		100.0	64.9	145.5	96.3
5-YEAR		100.0	44.2	208.6	97.0
ACTUAL DEFAULT PROBABILITY	p_D				
15-YEAR		100.0	72.5	133.7	98.0
10-YEAR		100.0	61.0	151.6	95.3
5-YEAR		100.0	39.4	220.0	93.2
RISK ADJUSTMENT	\mathcal{R}				
15-YEAR		100.0	105.9	95.8	101.1
10-YEAR		100.0	108.8	95.5	102.9
5-YEAR		100.0	113.7	94.7	105.3
TIME ADJUSTMENT	\mathcal{T}				
15-YEAR		100.0	97.2	101.0	98.0
10-YEAR		100.0	97.8	100.5	98.2
5-YEAR		100.0	98.8	100.1	98.8
LEVERAGE	$B/(B + S)$	100.0	87.7	113.3	100.0

Table VII : Comovement between debt and equity markets

This table reports the extent of comovement between equity and debt markets implied by the dynamic capital structure specification of the model. Reported are: regression coefficients of credit spreads on the jump-component of stock return volatility (Panel A), credit spreads on total stock return volatility (Panel B), credit spread volatility on total stock return volatility (Panel C), actual default probabilities on total stock return volatility (Panel D). For each model-implied result, 1,000 panels each containing 3,000 firms are simulated for 100 years. The simulation procedure is discussed in Appendix C. Credit spread volatility is based on the equally-weighted average credit spread. The jump component of levered equity return volatility is based on the value-weighted average levered equity return. Historical actual default probabilities are taken from Moody's Global default rates. Standard errors are reported in parenthesis. The same units (% per annum) are used for both dependent and independent variables. For example, 0.125 in Column 1 of Panel A means that in the data, a 1% increase in the jump-component of stock return volatility is associated with a 0.125% increase in the BBB-AAA credit spread.

Data	Model	25%-quantile	75%-quantile
Panel A: Credit spreads and jump return volatility			
0.125 (0.011)	0.105 (0.066)	0.090	0.123
Panel B: Credit spreads and total stock return volatility			
0.068 (0.011)	0.065 (0.007)	0.055	0.076
Panel C: Credit spread vol. and total stock return vol.			
0.062 (0.020)	0.031 (0.006)	0.016	0.040
Panel D: Act. default prob. and total stock return vol.			
0.138 (0.014)	0.147 (0.015)	0.101	0.185

Figure 1: Distribution of leverage

The figure shows the distribution of the quasi-market leverage ratios for the empirical BBB sample of corporate bonds over the period from 1997 to 2004. The quasi-market leverage ratio for each firm is given by $B_{\nu_0}(X_{\nu_0}, c_{\nu_0}, \nu_0) / (B_{\nu_0}(X_{\nu_0}, c_{\nu_0}, \nu_0) + S_{\nu_t}(X_t, c_{\nu_0}, \nu_0))$, where $B_{\nu_0}(X_{\nu_0}, c_{\nu_0}, \nu_0)$ is the book value of debt and $S_{\nu_t}(X_t, c_{\nu_0}, \nu_0)$ is the market value of equity.

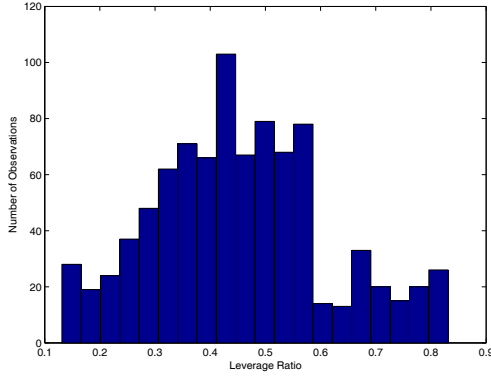


Figure 2: Distribution of cash flows

The figure shows the distribution of normalized firm cash flow levels for the model-implied data matched with the empirical BBB sample at the initial date as well as at five and ten years. Cash flows are normalized so that the most recent refinancing point is 1.

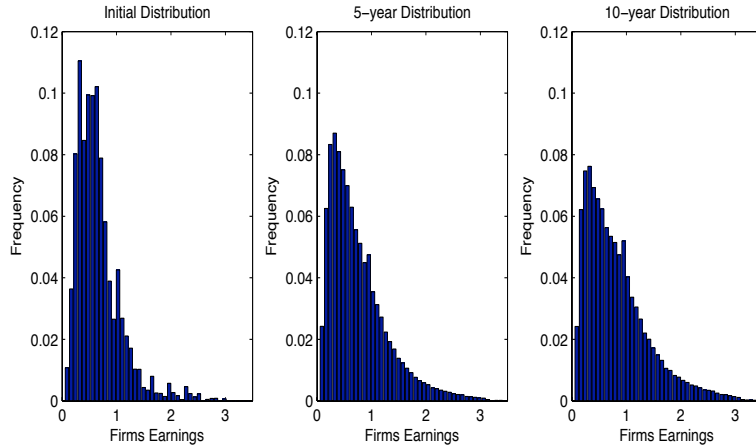


Figure 3: Time-variation in credit spreads and default rates

The figure shows the realized time variation in five-, ten-, and fifteen-year credit spreads (left figure) and actual annual default rates (right figure) for the model-generated simulated economy. The starting sample are the firms in the BBB rating category. The simulation procedure is described in Appendix C. Five-, ten-, and fifteen-year credit spreads are plotted using a solid line, dashed line and dashed line with dots, respectively. Credit spreads are given in basis points. Actual default probabilities are per annum and given in per cent. Grey bars represent periods when the economy is in state 1 (recession).

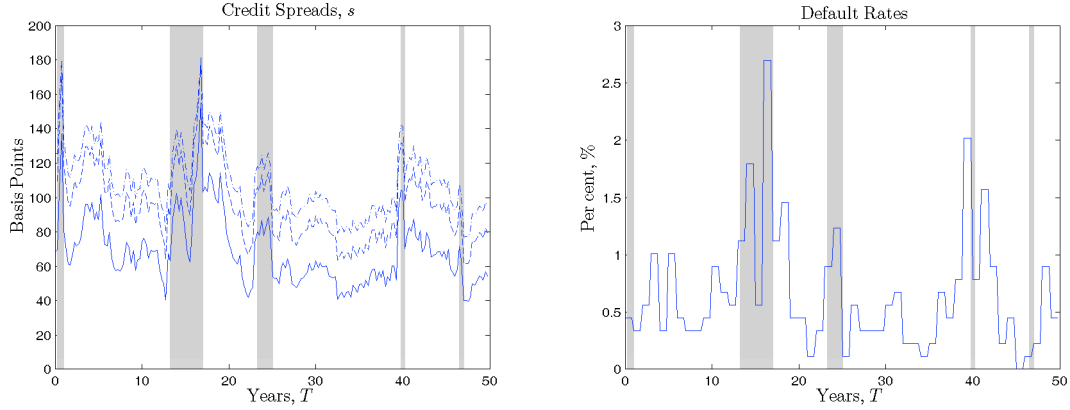


Figure 4: Levered equity risk premium and leverage

The figure shows the levered equity risk premium as a function of economy-wide leverage, for the dynamic capital structure specification of the model. The levered equity risk premium is the mean (across economies) of the value-weighted cross-sectional average risk premium. Economy-wide leverage is based on the ratio of value-weighted cross-sectional averages of debt and levered firm value.

