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Adverse Selection and Non-Exclusive Contracts^{*}

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Abstract

This paper studies the [Rothschild and Stiglitz \(1976\)](#) adverse selection environment, relaxing the assumption of exclusivity of insurance contracts. There are three types of agents that differ in their risk level, their riskiness is private information and known before any contract is signed. Agents can engage in multiple insurance contracts simultaneously, and the terms of these contracts are not observed by other firms. Insurance providers behave non-cooperatively and compete offering menus of insurance contracts from an unrestricted contract space. We derive conditions under which a separating equilibrium exists and fully characterize it. The unique equilibrium allocation consists of agents with a lower probability of accident purchasing no insurance and agents with higher accident probability buying the actuarially-fair level of insurance. The equilibrium allocation also constitutes a linear price schedule for insurance. To sustain the equilibrium allocation, firms must offer latent contracts. These contracts are necessary to prevent deviations by other firms; in particular they can prevent cream-skimming strategies. As in [Rothschild and Stiglitz \(1976\)](#), pooling equilibrium still fails to exist.

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1 Introduction

In this paper we address the question of what type of insurance contracts emerge when insurance providers compete among themselves. We are interested in environments where the insured have private information on their risk probability and can sign without being observed multiple insurance contracts with different insurance providers.

Insurance contracts are written to offset the risk associated with a wide variety of events. Examples of different types of insurance contracts include insurance against person-related events (medical, life, annuities), property events (car, home), and financial events (credit default swaps).¹ These insurance contracts share two common properties: the realization of uncertainty can be verified, and subscribers might have additional private information about the *probabilities* that an event realizes. However, due to different regulatory oversight, a feature that varies greatly amongst them is the ability of the insurer to enter into additional contracts with other insurance providers. This possibility of non-exclusive insurance holding, while rare in property insurance, is a definite possibility – for example, in the case of credit default swaps.²

Motivated by the above observations we investigate the restrictions on the equilibrium insurance contracts that arise once we dispense with the exclusivity assumption. We consider a variation of the standard [Rothschild and Stiglitz \(1976\)](#) (RS henceforth) environment. Agents are subject to uncertainty regarding their endowment realization, the endowment can be either high or low. We consider three types of agents, each type features different probability of the high realization of the endowment. This probability is private information of the agent. Differently from RS we allow agents to engage in multiple insurance contracts simultaneously with multiple insurance providers. The key assumption is that the terms of these contracts are not observed by other insurance providers. Insurance providers behave non-cooperatively and compete offering menus of insurance contracts from an unrestricted contract space. We derive parameter restrictions under which a separating equilibrium exists and fully characterize it. The unique equilibrium allocation consists of agents with the lowest probability of receiving the high realization of endowment (the bad type) buying the actuarially-fair level of insurance. The other two types with medium and high probability of high endowment realization (the medium and good type) purchase no insurance.

¹For a review, refer to [Duffie \(1999\)](#). For recent empirical pricing evidence refer to [Blanco, Brennan, and Marsh \(2005\)](#).

²Until early 2009, credit default swaps were issued in private bilateral trades without any intermediation by any clearing house. On March 10, 2009 ICE TrustTM began operating as a central counter party clearing house for credit default swaps in North America.

Similarly to our paper, the equilibrium allocation in RS features full insurance at the fair price for the low type. Differently from our paper in RS (see also [Wilson \(1977\)](#)) the medium and high type receive a positive amount of insurance. Another key difference with respect to RS are the condition required for existence. In our environment these conditions are stronger, this is due to the nature of non exclusive competition that allows for additional types of deviation by entrants that are not present in RS. When an equilibrium exists we find that latent contracts must be offered by insurance providers. These are contract offered in equilibrium but not chosen by any type. These contracts are necessary to prevent cream-skimming deviations by entrants and also deviations by incumbents. This highlights the dual role that non-exclusivity plays in our environment. First, by allowing agents to sign additional insurance contracts, it constitutes a constraint on what an insurance providers can offer hence limiting the availability of insurance and in certain cases leading to non existence of equilibrium. Second, non exclusivity enables insurance providers to sustain equilibrium contracts. Deviations from equilibrium can be prevented with the threat that any agent can combine latent contracts with the ones offered following a deviation. This behavior makes it impossible for an entrant to separate different agent types.

Related Literature

This paper is related to two large and growing literatures. The first one studies problems with adverse selection. It originates from the pioneering work of [Akerlof \(1970\)](#), [Rothschild and Stiglitz \(1976\)](#), [Wilson \(1977\)](#) and [Miyazaki \(1977\)](#).³ From this literature we take our basic setup where agents seek insurance and are privately informed on their risk type. The second one is a more recent literature focusing on non exclusive contracting, see for example the work of [Epstein and Peters \(1999\)](#), [Peters \(2001\)](#), [Martimort and Stole \(2002\)](#) and references therein. In these papers a principal cannot prevent an agent to contract with other competing principals. In addition contracts are private information of the agent and the counterparty. From this literature, as in [Biais, Martimort, and Rochet \(2000\)](#), we adopt the approach to equilibrium characterization. We consider the case where each individual insurance provider offers a set of menus of contracts and delegates the choice to the agent on which insurance contract to pick.

The closest paper to ours is [Attar, Mariotti, and Salanié \(2011a\)](#) independently developed during the same time as ours. The two papers share many similarities and some differences. There are three main differences in terms of modeling assumptions. First, their paper restricts the analysis to the two types case, while we consider agents with three privately observed types. Second, they consider the case without free entry while we consider

³For a review refer to [Dionne, Doherty, and Fombaron \(2001\)](#).

the case with free entry. Finally, although their preference specification is more general nesting both a pure trade environment and an insurance environment, the restriction on insurance purchase is different in the two papers. In our paper any insurance purchase that does not lead to negative consumption is allowed. On the other hand [Attar, Mariotti, and Salanié \(2011a\)](#) considers either the case either with arbitrary amount of insurance (without non-negativity constraint on consumption) or the case with only positive insurance (in the appendix). Both papers reach similar results: pooling equilibrium fails to exist, the agent with the highest risk reach full insurance, while for everybody else no insurance is provided. An equilibrium may fail to exist altogether. In the body of the paper we highlight additional similarities and differences.

This paper is also related to a series of papers that analyze the effect of non-exclusive contracting in the purchase of goods. One of the first papers to do so is [Biais, Martimort, and Rochet \(2000\)](#). The authors consider an environment where competing traders provide liquidity to a risk-averse agent who is privately informed on the value of an asset. As in this paper, the agent is not restricted in trading with only one trader. Moreover traders compete among each other using menus of possible trades. Differently from our paper, they consider an environment where goods are being traded rather than insurance and they consider the case where the privately observed type can take a continuum of values while we consider a finite (three) number of types. [Ales and Maziero \(2010\)](#) study a dynamic environment with private information (but unlike this paper, the realization of private information happens after agents sign the contract) where agents can engage in multiple non-exclusive contracts for both labor and credit relationships. The paper shows that a unique equilibrium always exists and that latent contracts are necessary. As in this paper, the equilibrium can be implemented using linear contracts for wages and bonds. In a recent paper, [Attar, Mariotti, and Salanié \(2011b\)](#) extend the environment of [Akerlof \(1970\)](#) to include non-exclusive contracting. They show, contrary to this paper, that a unique equilibrium always exists. Similarly to this paper the equilibrium involves linear prices, and is sustained by latent contracts. [Arnott and Stiglitz \(1991\)](#) and [Bisin and Guaitoli \(2004\)](#) study static moral hazard environments where agents trade in non exclusive relationships. In particular, the latter shows that latent contracts are necessary to sustain the equilibrium and lead to positive profit for the insurance providers. In our paper insurance provide will generate zero profits. Finally [Parlour and Rajan \(2001\)](#) and [Attar, Campioni, and Piaser \(2006\)](#) study the effect of non exclusive relationships for the provision of credit either under limited commitment (the former) or under moral hazard (the latter).

In spirit, this paper is also related to an earlier literature focused on modifying the equi-

librium concept first studied by [Rothschild and Stiglitz \(1976\)](#). In particular [Wilson \(1977\)](#) extends the equilibrium concept used in RS beyond static Nash equilibrium by allowing insurance providers to take into account how a change in their policy offers might affect the set of policies offered by other insurance providers. In our paper, latent contracts play a similar role to these non stationary expectations by enabling a reaction of insurance providers to deviations of other insurance providers. On a similar note are the papers that study inter-firm communication in insurance settings, such as [Jaynes \(1978\)](#) and [Hellwig \(1988\)](#).⁴ The first considers a static adverse selection economy and allows firms the choice to disclose or not information on who accepts the insurance contract. It shows that some firms will share information leading to a separating equilibrium (that always exists), while in the case where no information is shared, no equilibrium exists. Sharing information allows a firm to offer an insurance contract that is contingent on additional purchases of insurance an agent might accept. In our paper, firms gather information on insurance purchased by also offering latent contracts, which allows us, in contrast to Jaynes, to have an equilibrium even without any information being shared directly. Latent contracts have the same role as information sharing, since they enable firms to react to deviation of incumbent firms. [Hellwig \(1988\)](#) highlights that the ability to react is the key to equilibrium existence rather than inter-firm communication. In particular, inter-firm communication enables firms to react only if the equilibrium concept considered in [Jaynes \(1978\)](#) is implicitly assuming a non-stationary expectation similar to [Wilson \(1977\)](#). Along similar lines, [Picard \(2009\)](#) considers the case where the contracts offered by the insurance providers feature participating clauses so that the payout will be conditional on the profits of the insurance provider. In this setup it is shown that an equilibrium always exists and coincides with the Miyazaki-Spence-Wilson allocation.

This paper is organized as follows: Section 2 describes the environment. Characterization and implementation are studied respectively in Sections 3 and 4. In Section 4.1 we compare directly the current paper to the equilibrium allocation characterized in [Rothschild and Stiglitz \(1976\)](#). Section 5 concludes.

2 Environment

The economy is populated by a continuum of measure one of agents and countably many insurance providers. We assume free-entry in the insurance market.⁵ Agents are ex ante

⁴For an extension to a moral hazard environment, also look at [Hellwig \(1983\)](#).

⁵In [Attar, Mariotti, and Salanié \(2011a\)](#) the number of firms is fixed.

heterogeneous. We consider three types of agents. There is a fraction p_g of type g agents (the good type) a fraction p_b of type b (the bad type) and a remaining fraction $p_m = 1 - p_g - p_b$ of type m (the medium type).⁶ We assume that $p_b > 0$, $p_m > 0$ and $p_g > 0$.⁷ The economy lasts for 1 period. Agents' utility u is defined over consumption c . Assume $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a twice continuously differentiable, increasing, and strictly concave function. At time 1, an agent of type $j = b, m, g$ receives an endowment ω_H with probability π_j and ω_L with probability $1 - \pi_j$. Let $\omega_H > \omega_L$ and denote $\omega = (\omega_L, \omega_H)$. The realization of the endowment is publicly observed. Assume that $\pi_g > \pi_m > \pi_b$, that these probabilities are private information of the agent and that are known to the agent before signing any insurance contract. Define the market (average) probability of high realization as $\hat{p} = p_g\pi_g + p_m\pi_m + p_b\pi_b$. The probability of high endowment realization averaged across any two types is given by $\hat{p}_{i,j} = \frac{p_i\pi_i + p_j\pi_j}{p_i + p_j}$ with $i, j = b, m, g$.

In this environment, each agent seeks to insure himself against the uncertain realization of the endowment. Insurance will be provided by insurance providers (referred to as firms in the body of the paper). Denote by I the number firms active in equilibrium. Each firm $i \in \{1, \dots, I\}$ offers insurance contracts to agents. A contract prescribes consumption transfers conditional on the realization of the endowment. The key feature of our environment is that agents can simultaneously sign contracts with more than one firm and that the terms of the contract between an agent and any firm are not observed by other firms. As described in [Martimort and Stole \(2002\)](#) and [Peters \(2001\)](#), due to the delegation principle, we can restrict the analysis to menu games. In a menu game, each firm i offers a menu: a set C^i in $\mathcal{P}(\mathbb{R}^2)$ (the power set of \mathbb{R}^2).⁸ Elements of C^i are transfers pairs, $\tau^i = (\tau_{L,j}^i, \tau_{H,j}^i)$, conditional on a realization respectively of ω_L or ω_H . We do not impose any restriction on the type of menus offered by firms. We do require however that each firm offers the null transfer $(0, 0)$. Note that a menu might contain more alternatives than the number of types, given our focus on symmetric equilibria this will imply that some alternatives will not be chosen in equilibrium. We denote a contract as *latent* if it is offered in equilibrium by some firm and is not chosen in equilibrium by any agent. Let $(\tau_{L,j}, \tau_{H,j}) = (\tau_{L,j}^1, \tau_{H,j}^1) \times \dots \times (\tau_{L,j}^I, \tau_{H,j}^I)$ where $(\tau_{L,j}^i, \tau_{H,j}^i)$ is the pair of transfers chosen by an agent of type j within the menu offered by firm i . Let $C = C^1 \times \dots \times C^I$. The expected utility for an agent of type $j = b, m, g$ given

⁶The original insurance problem discussed in [Rothschild and Stiglitz \(1976\)](#) focuses on two types only. Refer to [Wilson \(1977\)](#) for the case with multiple types.

⁷The case with one of the $p_j = 0$ with $j = b, m, g$, reduces the environment to one with only two types of agents. This case was studied in a previous version of this paper. All of the results of this paper hold in that case also.

⁸We do not allow the use of random contracts.

menus C offered by firms, is defined as

$$U^j(C) = \max_{(\tau_{L,j}, \tau_{H,j}) \in C} \left[\pi_j u \left(\omega_H + \sum_{i=1}^I \tau_{H,j}^i \right) + (1 - \pi_j) u \left(\omega_L + \sum_{i=1}^I \tau_{L,j}^i \right) \right]. \quad (1)$$

We will focus on symmetric equilibria hence we require that agents of the same type will choose the same allocation. Firms are risk neutral. Their objective is to maximize profits by optimally choosing a menu. A firm takes as given the menus offered by other firms, denoted by C^{-i} and optimal choices of the agents. Profits from offering C^i are given by

$$\begin{aligned} \Pi(C^i, C^{-i}) &= \max_{C^i = (x \in \mathcal{P}(\mathbb{R}^2)) \cup (0,0)} - \sum_{j=b,m,g} p_j [\pi_j \tau_{H,j}^i + (1 - \pi_j) \tau_{L,j}^i] \\ \text{s.t.} \quad & (\tau_{L,j}^i, \tau_{H,j}^i) \in C^i, \quad \forall j \in \{b, m, g\} \end{aligned} \quad (2)$$

We can now define equilibrium in the menu game

Definition 1 (Equilibrium). *A pure strategy symmetric equilibrium of the menu game is a collection of menus C^i for all $i \in \{1, \dots, I\}$ and agents' choices $(\tau_{L,j}, \tau_{H,j})$, for all $j = b, m, g$ such that:*

1. For each $j = b, m, g$, $(\tau_{L,j}, \tau_{H,j})$ is a solution of the agent problem (1).
2. For each $i \in \{1, \dots, I\}$, taking as given C^{-i} and agents' choice $(\tau_{L,j}, \tau_{H,j})$ for each $j = b, m, g$, C^i solves (2).

For notation convenience we consider, in the body of the paper, utility and profits derived from consumption transfers rather than by menus. Let $c = (c_L, c_H)$ denote the consumption in the low and high state realization. For a given type $j = b, m, g$, with a slight abuse of notation denote by $U^j(c) = \pi_j u(c_H) + (1 - \pi_j) u(c_L)$. In this case $U^j(\omega)$ is the utility in autarky for agents of type j . Conditional on agents of type $j \in \{b, m, g\}$ accepting contract $\tau = (\tau_L, \tau_H)$, profits are equal to $\Pi^j(\tau) = -\pi_j \tau_H - (1 - \pi_j) \tau_L$. Similarly, if multiple types j, j' accept the contract we denote the profits with $\Pi^{j,j'}(\tau) = -\hat{p}_{jj'} \tau_H - (1 - \hat{p}_{jj'}) \tau_L$. If all three types accept the contract τ then we denote profits with $\Pi(\tau)$.

3 Characterization of Equilibrium

A first straightforward result is that in any equilibrium profits for the firms must be non-negative: for all i , $\Pi(C^i, C^{-i}) \geq 0$. To characterize equilibrium, we consider two cases. The

first case refers to a *pooling equilibrium*. In this case agents of all three types receive the same equilibrium allocation. In the next subsection we show that this type of equilibrium never exists. A second type of equilibrium, a *separating equilibrium*, occurs when at least two types receive a different consumption allocation. In subsection 3.2, we show that the unique equilibrium of the menu game is a separating equilibrium with the following characteristics: agents of type b (the bad type) receive full insurance against the realization of the endowment at their actuarially-fair price, while agents of types m and g , receive no insurance.

3.1 Pooling Equilibrium

We first determine necessary conditions that any pooling equilibrium must satisfy. In this subsection we let $c = (c_L, c_H)$ be the candidate pooling equilibrium level of consumption for the three types.

Lemma 1. *For any pooling equilibrium allocation for consumption $c = (c_L, c_H)$, the following conditions must hold:*

$$c_L \geq c_H, \tag{3}$$

$$\text{if } \pi_m \leq \hat{p}, \quad \frac{1 - \pi_g}{\pi_g} \frac{u'(c_L)}{u'(c_H)} \geq \frac{1 - \hat{p}}{\hat{p}}, \tag{4}$$

$$\text{if } \pi_m > \hat{p}, \quad \frac{1 - \pi_m}{\pi_m} \frac{u'(c_L)}{u'(c_H)} \geq \frac{1 - \hat{p}}{\hat{p}}. \tag{5}$$

Proof. In appendix A. □

Equation (3) is equivalent to the following relation

$$\frac{1 - \pi_b}{\pi_b} \frac{u'(c_L)}{u'(c_H)} \leq \frac{1 - \pi_b}{\pi_b},$$

the above implies that the marginal rate of substitution between consumption in the two states for the b agent is less than or equal to the actuarially-fair price for the insurance only if b agents accept it. This relation provides an intuition for the necessary condition (3). If it were not to hold, c cannot be an equilibrium since a profitable entry opportunity is always available. A firm can provide additional insurance for agents of type b charging a price slightly higher than the actuarially-fair one. Such deviation is always profitable for the firm and increases expected utility for the agents of type b . Similarly, the necessary conditions (4) and (5) require that the marginal rate of substitution between consumption in the two states for agents of type g and m respectively is greater than the price for insurance when

all agents accept the contract. If not, entrant insurance providers can profitably provide additional insurance to agents of type g and m . A direct implication of Lemma 1 is that there is no pooling equilibrium, since there is no allocation that simultaneously satisfies the conditions described in equations (3), and either (4) or (5).

Proposition 1. *There is no pooling equilibrium.*

Proof. Suppose there exists a pooling equilibrium $c = (c_L, c_H)$. The equilibrium allocation must satisfy conditions (3) and either (4) or (5). In the first case with $\pi_m \leq \hat{p}$, from (3) and (4) we have $\frac{1-\pi_g}{\pi_g} \geq \frac{1-\hat{p}}{\hat{p}}$. So that $\pi_g \leq \hat{p}$, this is a contradiction since $\pi_b < \pi_m < \pi_g$ implies $\pi_g > \hat{p}$. In the second case with $\pi_m > \hat{p}$, from (3) and (5) we have that $\frac{1-\pi_m}{\pi_m} \geq \frac{1-\hat{p}}{\hat{p}}$. This implies $\pi_m \leq \hat{p}$, a contradiction. \square

3.2 Separating equilibrium

We now study separating equilibria. For each type $j = b, m, g$ denote the equilibrium consumption allocation by $c^j = (c_L^j, c_H^j)$. The separating equilibrium allocation is denoted by $c = \{c^b, c^m, c^g\}$. Since c is a separating equilibria there must exist at least one j and j' such that $c^j \neq c^{j'}$. By revealed preferences, the following incentive constraints must hold

$$U^j(c^j) \geq U^j(c^{j'}), \text{ for } j, j' = b, m, g. \quad (6)$$

Before fully characterizing the equilibrium allocations we provide 3 Lemmas containing necessary conditions that any separating equilibrium must satisfy. Lemma 2 focuses on the magnitude of consumption for each type for each endowment realization. Lemma 3 provides characterization of the consumption allocation based on the profitability of particular combination of contracts. Finally, Lemma 4 provides characterization of the equilibrium allocation based on the marginal propensity of each type of agent for buying additional insurance.

Lemma 2. *Any separating equilibrium allocation $c = \{c^b, c^m, c^g\}$ must satisfy:*

$$c_H^g \geq c_L^g, \quad (7)$$

$$c_L^b \geq c_H^b, \quad (8)$$

$$c_H^g \geq c_H^m \geq c_H^b \quad \text{and} \quad c_L^g \leq c_L^m \leq c_L^b. \quad (9)$$

Proof. In appendix A. \square

The above lemma implies that the allocation for the g type must be in the under-insurance region, while the allocation of the b type must be in the over insurance region. The intuition for this result is as follows: if the agents of type g were in the over insurance region, an entrant might offer a small negative insurance contract at a price worse than the actuarially fair one. Agents of type g will accept this contract. This contract will be profitable even if any other type accept it. The intuition for the case when b is underinsured is similar. In addition, Lemma 2 states that consumption under the realization of the low endowment must be weakly decreasing as agent type increases, while must be weakly increasing in type upon the realization of the high endowment shock.⁹

Lemma 3. *Any separating equilibrium allocation $c = \{c^b, c^m, c^g\}$ must satisfy:*

$$\pi_b(\omega_H - c_H^b) + (1 - \pi_b)(\omega_L - c_L^b) \leq 0. \quad (10)$$

$$\pi_b(c_H^j - c_H^b) + (1 - \pi_b)(c_L^j - c_L^b) \leq 0, \quad \forall j = g, m. \quad (11)$$

$$\text{If } c_L^m > \omega_L, \quad \hat{p}_{b,m}(\omega_H - c_H^m) + (1 - \hat{p}_{b,m})(\omega_L - c_L^m) \leq 0, \quad (12)$$

$$\pi_g(c_H^m - c_H^g) + (1 - \pi_g)(c_L^m - c_L^g) \leq 0. \quad (13)$$

$$\hat{p}_{b,m}(c_H^g - c_H^m) + (1 - \hat{p}_{b,m})(c_L^g - c_L^m) \leq 0 \quad (14)$$

$$\Pi(c^i - \omega) \geq 0, \quad i = g, m. \quad (15)$$

Proof. In appendix A. □

The proofs of the previous conditions share a common theme. If any of the conditions in Lemma 3 is violated then an “entrant”, a firm currently making zero profits, can deviate and offer a contract that will make strictly positive profits independent of the type of agents accepting it. Depending on the type of condition violated, an entrant will find it profitable to offer either additional or substitute contracts. Additional contracts offer additional amounts of positive or negative insurance. These contracts are always chosen in combination with contracts already offered in equilibrium. Substitute contracts, as the name suggests, are accepted instead of the equilibrium allocation. As an example suppose that condition (10) is violated, in this case an entrant can provide a substitute contract to agents of type b that is welfare improving and always profitable. If condition (14) is violated, then an entrant can

⁹Lemma 2 applies to any equilibrium not only the separating one. For the pooling equilibrium, conditions (7) and (8) imply that $c_L = c_H$ this immediately implies a violation of condition (4) and (5) in Lemma 1 so that a pooling equilibrium does not exist.

provide an additional contract that agents of type m will accept together with the original allocation of agents of type g .

The next Lemma, following similar steps to Lemma 1, characterizes equilibrium contracts by focusing on the marginal propensity of agents to buy additional positive or negative infinitesimal amounts of insurance.

Lemma 4. *Any separating equilibrium allocation must satisfy:*

$$\frac{1 - \pi_g}{\pi_g} \frac{u'(c_L^g)}{u'(c_H^g)} \leq \frac{1 - \underline{p}}{\underline{p}}, \quad \underline{p} = \min\{\widehat{p}, \widehat{p}_{b,g}\}. \quad (16)$$

$$\frac{1 - \pi_m}{\pi_m} \frac{u'(c_L^m)}{u'(c_H^m)} \leq \frac{1 - \widehat{p}_{b,m}}{\widehat{p}_{b,m}}. \quad (17)$$

Proof. In appendix A. □

We now complete the characterization of the unique equilibrium consumption allocation. Let the candidate equilibrium be $c^b = (\omega^b, \omega^b)$ with $\omega^b = \pi_b \omega_H + (1 - \pi_b) \omega_L$ and $c^m = c^g = (\omega_L, \omega_H)$. Notice that this allocation features pooling between agents of type m and of type g . In particular the allocation provides no insurance for agents of type m and g and instead provides full insurance at the actuarially-fair price to agents of type b . To aid intuition, for the following proposition, we display key steps of the proof graphically for the simpler two type case: $j = b, g$. Refer to figure 1(a). This figure displays in (c_L, c_H) space the full insurance line (the 45 degree line), and the zero-profit lines if only good agents buy the insurance, only bad and if all agents purchase the contract (market break even line). In addition the separating equilibrium (c^b, c^g) with associated indifference curves is displayed.

Proposition 2. *Any equilibrium allocation of the menu game satisfies: $c^b = (\omega_b, \omega_b)$, where $\omega_b = \pi_b \omega_H + (1 - \pi_b) \omega_L$; $c^m = c^g = (\omega_L, \omega_H)$.*

Proof. In appendix A. □

A key first step of this proof involves showing that neither the good or the medium types buy any insurance. In particular, the equilibrium allocation for these types cannot at the same time: i) be preferred by each agent relative to the endowment point; ii) deliver positive profits if only these agents buy it and iii) not be preferred by the bad type relative to his equilibrium. Graphically, for the two type case, it implies that it cannot lie in the green-shaded triangular area of figure 1(a). An illustration of this step is in figure 1(b). Suppose that the separating equilibrium is given by (c^b, c^g) in the figure, where c^g lies in the green-shaded area. In this case an entrant can offer a contract such as *Entry*, which is always

profitable no matter which agent accepts it. In the presence of such contract, it is welfare improving for agents of type b to choose the contract providing consumption allocation c^g , which contradicts c^b being an equilibrium allocation. Note that $c^g + \text{Entry}$ cannot be an equilibrium allocation for the consumption of agents of type b since c^g is profitable only if agents of type g select it. In addition in the proof we show that in equilibrium there cannot be cross subsidization across types, so that for example $\Pi^b(c^b) < 0$. The intuition is that if it were the case that $\Pi^b(c^b) < 0$, then it must also be true that the insurance providers are generating positive profits with either agents of type g or m (or both). In the proof it is shown that an entrant can always “cream skim” agents of this type and make a positive profit.¹⁰

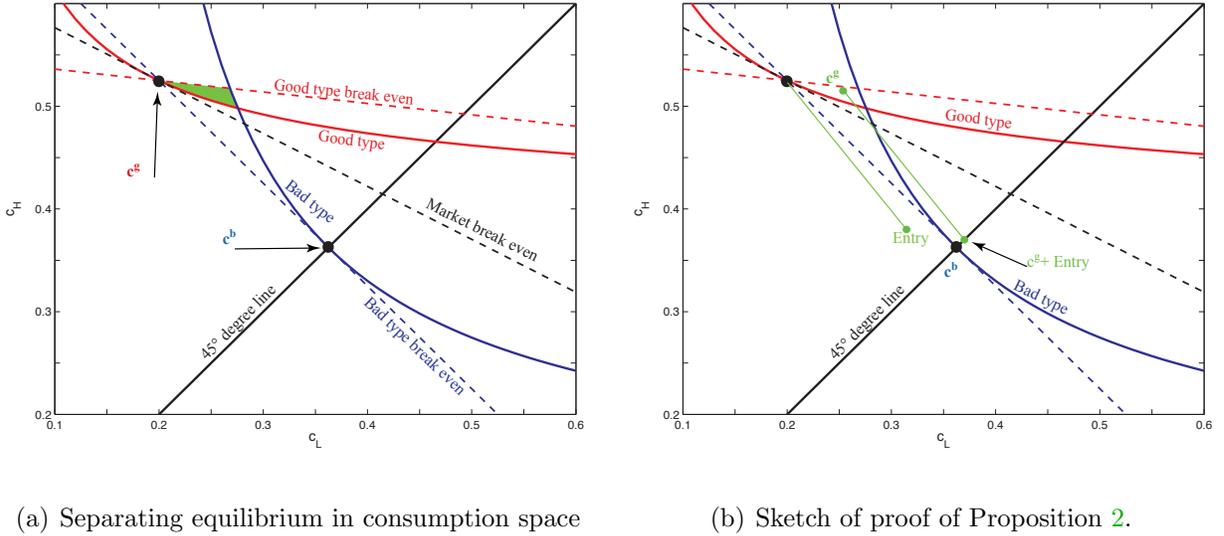


Figure 1: Separating Equilibrium Case with $p_m = 0$.

As in [Rothschild and Stiglitz \(1976\)](#), a separating equilibrium may fail to exist. In our environment, we require the following necessary condition on primitives to guarantee the existence of an equilibrium.¹¹

Assumption 1.

$$\frac{1 - \pi_g u'(\omega_L)}{\pi_g u'(\omega_H)} \leq \frac{1 - \hat{p}}{\hat{p}}, \quad (18)$$

¹⁰We thank the co-editor and an anonymous referee for helpful comments on this point.

¹¹An interesting extension left for future research is the characterization of equilibrium using random menus. See [Dasgupta and Maskin \(1986a,b\)](#) for the study of existence of equilibrium in the case with exclusive contracts. And also [Carmona and Fajardo \(2009\)](#) and [Monteiro and Page \(2008\)](#) for the case with non-exclusive contracts.

$$\frac{1 - \widehat{p}_{m,g}}{\widehat{p}_{m,g}} \leq \frac{1 - \pi_m}{\pi_m} \frac{u'(\omega_L)}{u'(\omega_H)} \leq \frac{1 - \widehat{p}_{b,m}}{\widehat{p}_{b,m}}. \quad (19)$$

The above conditions are satisfied if, for example, π_g is large relative to π_m and π_b or if the spread between ω_L and ω_H is sufficiently small.

Proposition 3. *If Assumption 1 does not hold, there is no equilibrium.*

Proof. Suppose (18) in Assumption 1 is violated. Consider an entrant firm offering a menu containing $\widehat{\tau} = (\varepsilon, -\alpha\varepsilon)$ with $\varepsilon > 0$ and small, and α satisfying $\frac{1-\pi_g}{\pi_g} \frac{u'(\omega_L)}{u'(\omega_H)} > \alpha > \frac{1-\widehat{p}}{\widehat{p}}$. As shown in the proof of Lemma 4, $\widehat{\tau}$ is accepted by agents of type g : $U^g(\omega + \widehat{\tau}) > U^g(\omega)$. This implies that $\pi_g[u(\omega_H - \alpha\varepsilon) - u(\omega_H)] + (1 - \pi_g)[u(\omega_L + \varepsilon) - u(\omega_L)] > 0$. Since $\varepsilon > 0$ and $\pi_m < \pi_g$ it follows that $\pi_m[u(\omega_H - \alpha\varepsilon) - u(\omega_H)] + (1 - \pi_m)[u(\omega_L + \varepsilon) - u(\omega_L)] > 0$, so that $U^m(\omega + \widehat{\tau}) > U^m(\omega)$. Given this, minimum profits are achieved when also agents of type b accept $\widehat{\tau}$. From the definition of α this deviation is always profitable. This implies that the consumption allocation for agents of type g and m is not $c^g = \omega$, which contradicts Proposition 2 completing the result.

Suppose that the first inequality in (19) in Assumption 1 is violated, so that $\frac{1-\widehat{p}_{m,g}}{\widehat{p}_{m,g}} > \frac{1-\pi_m}{\pi_m} \frac{u'(\omega_L)}{u'(\omega_H)}$. In this case there exists an α so that $\frac{1-\widehat{p}_{m,g}}{\widehat{p}_{m,g}} > \alpha > \frac{1-\pi_m}{\pi_m} \frac{u'(\omega_L)}{u'(\omega_H)}$. Consider an entrant firm offering a menu containing $\widehat{\tau} = (-\varepsilon, \alpha\varepsilon)$ with $\varepsilon > 0$ and small. Note that in this case $\widehat{\tau}$ consists of a negative form of insurance. In this case we have that both agents of type m and g pick contract $\widehat{\tau}$ since $\frac{1-\pi_m}{\pi_m} > \frac{1-\pi_g}{\pi_g}$. From the definition of α this contract is also profitable. Hence the allocation is at the same time profitable and preferred by agents of type m and g , implying that the equilibrium allocation is not $\widehat{c}^g = (\omega + \widehat{\tau}) \neq \omega$. This contradicts Proposition 2 and the result follows.

Suppose that the second inequality in (19) in Assumption 1 is violated, so that $\frac{1-\pi_m}{\pi_m} \frac{u'(\omega_L)}{u'(\omega_H)} > \frac{1-\widehat{p}_{b,m}}{\widehat{p}_{b,m}}$. In this case, as in the proof of Lemma 4, there always exists a profitable deviation from entrants that is accepted by agents of type m and is profitable also in the case when agent of type b accept it. The resulting consumption allocation for agents of type m is $\widehat{c}^m \neq \omega$; by Proposition 2 this allocation cannot be an equilibrium. The result follows. \square

The necessary conditions for existence of equilibrium in Assumption 1 are stronger than those found in Rothschild and Stiglitz (1976) and Wilson (1977). The previous proposition provides an intuition on why this is the case. Relative to the case with exclusive contracts, the non-exclusivity assumption introduces additional opportunities for profitable deviations. These deviations cannot be prevented and are severe enough that might induce profits for the incumbents to become strictly negative. The lack of existence result is also confirmed in Attar, Mariotti, and Salanié (2011a). Their environment features the same necessary

conditions (once adapted for two types and preferences) as in Assumption 1. In Section 4.1, we consider how latent contracts can shut down some of deviations that were leading to the lack of existence result in Rothschild and Stiglitz (1976). One fundamental issue that leads to non-existence result present in both our paper and RS is the lack of any capacity constraint. This issue has been raised by Inderst and Wambach (2001) where the standard RS environment is complemented with capacity constraints in the amount of insurance that an insurance can provide. In this case it is shown that an equilibrium exists.¹² In our environment (see also the discussion in Attar, Mariotti, and Salanié (2011a) section 3.5) each insurance provider can service the entire market. Hence, an entrant can exploit this by forcing an incumbent insurance provider to provide insurance to a larger number of types than originally planned for.¹³

4 Implementation of Equilibrium

We now show that whenever Assumption 1 holds an equilibrium exists. The following proposition shows, by construction, that the allocation (c^b, c^m, c^g) characterized in Proposition 2 can be sustained in equilibrium.

Proposition 4. *Let $\{\pi_g, \pi_m, \pi_b, \omega_h, \omega_l, u, p_g, p_m, p_b\}$ satisfy Assumption 1, then there exists an equilibrium of the menu game.*

The complete proof of Proposition 4 is provided in Appendix B. In what follows we show the result in the simpler case with two types.¹⁴ Set $p_m = 0$. In this case Assumption 1 reduces to condition (18) only. Consider the following strategies by firms. Without loss of generality, let firms $i = 1, 2$ offer the menu:

$$C^i = \left\{ \left(\frac{\tau_{L,b}}{2}, \frac{\tau_{H,b}}{2} \right), L_b, (0, 0) \right\},$$

where $\tau_{L,b} = \pi_b(\omega_H - \omega_L)$, $\tau_{H,b} = (1 - \pi_b)(\omega_L - \omega_H)$ and the set L_b is defined as follows:¹⁵

$$L_b = \left\{ x_L \geq 0, x_H \leq 0 \mid -\pi_b x_H - (1 - \pi_b)x_L = 0 \right\}.$$

¹²Similarly, in Guerrieri, Shimer, and Wright (2010) the competitive search environment introduces a sort of capacity constraint. In this paper an equilibrium always exists.

¹³In the pure trade environment with non exclusivity of Attar, Mariotti, and Salanié (2011b) an equilibrium always exists. In this case the capacity constraint is in the form of the limited supply of goods that the seller is endowed with.

¹⁴We thank an anonymous referee for providing insights in simplifying the proof in this case.

¹⁵Graphically, the transfer pair $(x_L, x_H) \in L_b$ is on the zero-profit line for agents of type b conditional on providing positive insurance.

All remaining firms $i \neq 1, 2$ offer the menu: $C^i = (0, 0)$. It is easy to show that under Assumption 1, there exist an equilibrium where agents of type b choose $(\frac{\tau_{L,b}}{2}, \frac{\tau_{H,b}}{2})$ from both firms 1 and 2 and $(0, 0)$ from remaining firms; type g chooses $(0, 0)$ from all firms. In this equilibrium, all firms make zero profits. Agents of type b and g receive allocations c^b and c^g respectively. Suppose that firm i deviates and offers the contracts $\{\tau_b, \tau_g\}$ respectively to agents of type b and g . We do not rule out that $\tau_b = \tau_g$ or that any of the two (but not both) might be the null contract. As notation let $\tau_i = (\tau_{L,i}, \tau_{H,i})$. Consider the case with $\tau_{L,g} < 0$ which means negative insurance is being offered to agents of type g . Since $U^g(\omega + \tau_g) > U^g(\omega)$, if $\tau_{L,g} < 0$, it follows that $\Pi^g(\tau_g) < 0$.¹⁶ Also, since agents of type b are fully insured at their actuarially fair price, they will only accept contracts for which $\Pi^b(\tau_b) < 0$. This implies that total profits from the deviation $\{\tau_b, \tau_g\}$ are negative, hence we rule out the case with $\tau_{L,g} < 0$. Consider the case with $\tau_{L,g} > 0$, i.e. positive insurance is being offered to agents of type g . Consider the following transfer:

$$\hat{\tau} = \left(\pi_b(\omega_H - \omega_L + \tau_{H,g} - \tau_{L,g}), -\frac{1 - \pi_b}{\pi_b}(\omega_H - \omega_L + \tau_{H,g} - \tau_{L,g}) \right).$$

By construction $\hat{\tau} \in L_b$ and can be chosen from either firm $\hat{i} = 1, 2$ (for this step is crucial to have L_b being offered by at least two firms in equilibrium, so that following deviation of any firm i , L_b is still available to agents of type b). When combined with τ_g , contract $\hat{\tau}$ provides full insurance for agents of type b . Consumption in both states is given by $\omega_b + \pi_b\tau_{H,g} + (1 - \pi_b)\tau_{L,g}$. We must have that $U^b(\omega + \tau_g + \hat{\tau}) \leq U^b(\omega + \tau_b + \tau')$ where τ' is a contract in L_b . From concavity of preferences, since under $\tau_g + \hat{\tau}$ agent b is fully insured, it must be the case that τ_b provides a higher expected value of transfers: $\Pi^b(\tau_b) \leq \Pi^b(\tau_g)$. Aggregate profits for the deviation are: $p_b\Pi^b(\tau_b) + p_g\Pi^g(\tau_g) \leq p_b\Pi^b(\tau_g) + p_g\Pi^g(\tau_g) = \Pi(\tau_g) < 0$. Where the last inequality follows from (18) in Assumption 1, hence there is no profitable deviation. This argument is displayed in Figure 2. The general proof for the case with $p_m \neq 0$ in Appendix B.

From the proof of existence, two key facts emerge. First in equilibrium at least two firms must offer contracts different than the null one. If this was not the case, for example, the single firm offering the non-null contracts would deviate offering additional insurance to agents of type g at a price lower than the actuarially fair one. Second, the allocation (c^b, c^m, c^g) can be implemented as an equilibrium only if latent contracts are offered by both firms. Failure to offer latent contracts in equilibrium, would allow entrants to profitably offer

¹⁶Under Assumption 1, it can be shown that $U^g(\omega + \tau_g) \geq U^g(\omega + \tau_g + \tau')$ for any $\tau' \in L_b$. See Appendix B for details.

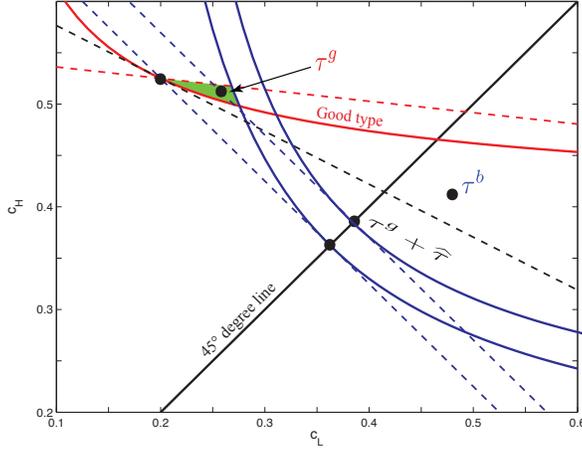


Figure 2: Sketch of proof of Proposition 4 for $p_m = 0$.

additional insurance to agents of type m and g .

4.1 The Rothschild-Stiglitz-Wilson Equilibrium

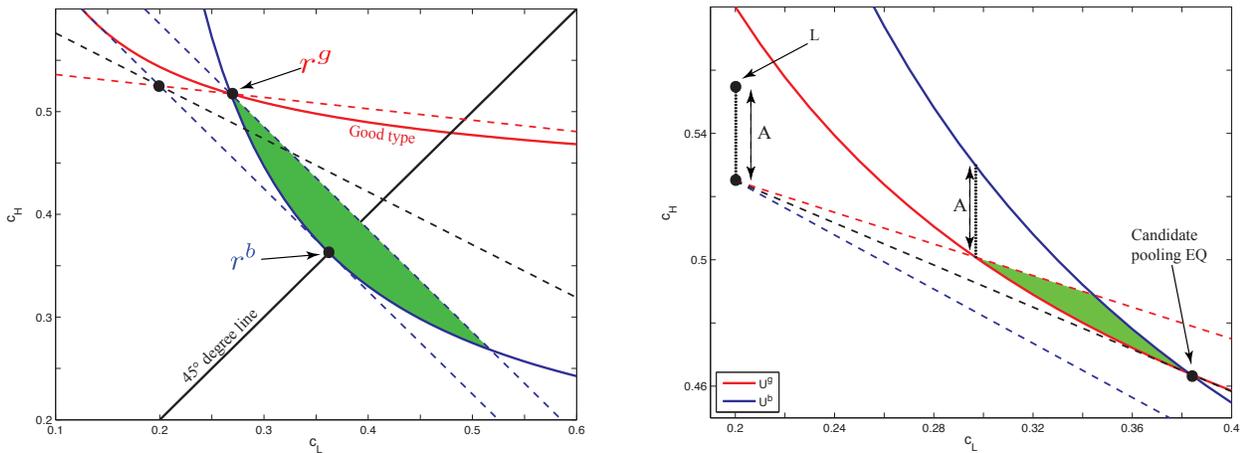
A natural benchmark to compare the results in this paper, is the classical environment with exclusive contracts characterized in [Rothschild and Stiglitz \(1976\)](#) and [Wilson \(1977\)](#). With exclusive contracts, under certain parameter restrictions, there exists a unique separating equilibrium (referred to as RSW from here onwards). The RSW equilibrium is defined as follows and is displayed in [Figure 3\(a\)](#).

Definition 2. *The RSW separating equilibrium consumption allocation is (r^b, r^g) where $r^b = (\omega^b, \omega^b)$ and $r^g = (r_L^g, c_H^g)$ such that $U^b(r^b) = U^b(r^g)$ and $\Pi^g(r^g - \omega) = 0$.*

Both equilibria with exclusive and non exclusive contracts feature agents of type b being fully insured. However with non-exclusive contracts agents of type g (and m) receive no additional insurance whereas in the RSW equilibrium agents of type g receive a positive amount of insurance at their actuarially fair price. The intuition for why no amount of insurance as in the RSW case can be offered with non-exclusive contract is straightforward. Consider the RSW equilibrium consumption allocation. In this case an entrant can offer additional insurance $\hat{\tau}$ for agents of type b at a price slightly worse than the actuarially fair price. Upon entry agents of type b accept this additional insurance together with the allocation r^g . Given the price charged for insurance, this entry is always profitable. The

shaded area in Figure 3(a) displays the set of consumption that can be achieved with an entrant offering $\hat{\tau}$.

Rothschild and Stiglitz (1976) and Wilson (1977) show that there is no pooling equilibrium when contracts are exclusive. A key intuition for this is that there is always an alternative contract that can be offered by an entrant that is profitable and attracts only good types (a cream-skimming deviation). With non-exclusive contracts a pooling equilibrium also fails to exist, however the intuition is different. Indeed when agents sign non-exclusive contracts, cream-skimming deviations can be prevented by using latent contracts. We provide an example in Figure 3(b). The solid lines represent the indifference curves for agents of type b (the steeper curve) and g (the flatter curve) at the *best* pooling equilibrium.¹⁷ Any contract in the green shaded area in Figure 3(b) is preferred to the pooling equilibrium by agents of type g but not by agents of type b . In addition it is profitable for a firm as long as only agents of type g , hence will constitute a profitable deviation. This is not necessarily the case under non-exclusivity. A firm can offer a latent contract, as example point L in Figure 3(b). This latent contract makes any contract offered in the green shaded area unprofitable since any point in the shaded area when combined with L is strictly preferred to the pooling equilibrium also by agents of type b .



(a) R^g consumption for agents of type g , R^b consumption for agents of type b .

(b) The role of latent contracts.

Figure 3: The Rothschild-Stiglitz-Wilson Equilibrium.

¹⁷The pooling equilibrium that delivers the highest expected utility when agents are weighted equally.

5 Conclusion

In this paper we characterize the equilibrium of a standard adverse selection economy in which agents can sign simultaneous insurance contracts with more than one firm. We consider the case with three types of agents: a good a medium and a bad type. Worse types represent a higher probability of receiving the low endowment. Agents are privately informed on their own types prior to signing any insurance contract. In this environment we show that there is no pooling equilibrium and that under certain parameter restrictions there is a unique equilibrium consumption allocation. When those parameter restrictions are violated an equilibrium fails to exist. In the unique equilibrium, the bad type receives full insurance at his actuarially-fair price. The good and medium type receive no insurance. Overall in this environment, when an equilibrium exists, the amount of insurance provided in equilibrium is reduced when compared with the environment in which agents sign exclusive contracts as in [Rothschild and Stiglitz \(1976\)](#). An important message of this paper is that non-exclusivity of contracts imposes strong restrictions on the insurance contracts that are offered, reducing drastically the provision of insurance. The non-exclusivity friction discussed in this paper can then be viewed as a positive institutional foundation for the strong regulations against the multiplicity of insurance contracts observed in several insurance markets, such as property and health insurance.

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Appendix

A Proofs of Section 3

Proof of Lemma 1

Proof. Suppose a pooling equilibrium c exist where (3) does not hold. In this case we have that $c_H > c_L$. This implies:

$$\frac{1 - \pi_b}{\pi_b} \frac{u'(c_L)}{u'(c_H)} > \frac{1 - \pi_b}{\pi_b}. \quad (20)$$

Consider a firm not originally active in equilibrium, an entrant, deviating and offering a menu comprised of the null contract $(0, 0)$ and $\hat{\tau} = (\varepsilon, -\alpha\varepsilon)$ for some small $\varepsilon > 0$ and where α satisfies:

$$\frac{1 - \pi_b}{\pi_b} \frac{u'(c_L)}{u'(c_H)} > \alpha > \frac{1 - \pi_b}{\pi_b}. \quad (21)$$

From (20) such an α exists.¹⁸ The contract $\hat{\tau}$ is chosen by agents of type b together with the original pooling equilibrium. To see this:

$$U^b(c + \hat{\tau}) = \pi_b u(c_H - \alpha\varepsilon) + (1 - \pi_b) u(c_L + \varepsilon) \quad (22)$$

expanding for small values of ε we have

$$U^b(c + \hat{\tau}) = U^b(c) + \varepsilon \left[-\pi_b u'(c_H) \alpha + (1 - \pi_b) u'(c_L) \right] + \mathcal{O}(\varepsilon^2) \quad (23)$$

from the first inequality in (21), ε can be chosen small enough so that $U^b(c + \hat{\tau}) > U^b(c)$. Let Π be the profit of the entrant. Since $\varepsilon > 0$ minimum profits for the entrant occur when only agents of type b accept $\hat{\tau}$. So that $\Pi \geq \Pi^b(\hat{\tau}) = \pi_b \alpha \varepsilon - (1 - \pi_b) \varepsilon > 0$. Where the strict inequality follows from the second inequality of (21). Since a profitable deviation exists (trivially no latent contract can prevent this entry) we reach a contradiction with c being a pooling equilibrium.

Let $\pi_m \leq \hat{p}$. Suppose a pooling equilibrium c exist where (4) does not hold. This implies:

$$\frac{1 - \pi_g}{\pi_g} \frac{u'(c_L)}{u'(c_H)} < \frac{1 - \hat{p}}{\hat{p}}, \quad (24)$$

As in the previous case, consider an entrant offering $\hat{\tau} = (c_L - \varepsilon - \omega_L, c_H + \alpha\varepsilon - \omega_H)$ for some small $\varepsilon > 0$ and where α satisfies:

$$\frac{1 - \pi_g}{\pi_g} \frac{u'(c_L)}{u'(c_H)} < \alpha < \frac{1 - \hat{p}}{\hat{p}}. \quad (25)$$

¹⁸The parameter α can be interpreted as the slope of a line passing between the zero-profit line of the bad type and the slope of his indifference curve through $c = (c_L, c_H)$.

From (24) such an α exists. In this case, $\hat{\tau}$ is accepted by agents of type g ($\hat{\tau}$ will be a substitute transfer to the pooling equilibrium, rather than an additional transfer as in the previous case) to see this:

$$U^g(\omega + \hat{\tau}) = \pi_g u(c_H + \alpha\varepsilon) + (1 - \pi_g) u(c_L - \varepsilon)$$

expanding for small values of ε :

$$U^g(\omega + \hat{\tau}) = U^g(c) + \varepsilon\pi_g u'(c_H) \left[\alpha - \frac{1 - \pi_g}{\pi_g} \frac{u'(c_L)}{u'(c_H)} \right] + \mathcal{O}(\varepsilon^2) > U^g(c),$$

where the strict inequality follows from the first inequality in (25). Let $\bar{\Pi}$ be the profits of the entrant. Let \hat{p}_x the probability of receiving a high realization of the endowment given the types of agents that accept the entrant's menu. By definition $\bar{\Pi}$ can be rewritten as:

$$\begin{aligned} \bar{\Pi} &= \hat{p}_x (\omega_H - c_H - \alpha\varepsilon) + (1 - \hat{p}_x) (\omega_L - c_L + \varepsilon) \\ &= \hat{p}_x (\omega_H - c_H) + (1 - \hat{p}_x) (\omega_L - c_L) + \varepsilon(1 - \hat{p}_x - \hat{p}_x\alpha) \end{aligned}$$

Since agents of type g accept $\hat{\tau}$, \hat{p}_x will be equal to one of the following $\{\pi_g, \hat{p}, \hat{p}_{b,g}, \hat{p}_{m,g}\}$. From equation (3) it follows that $(\omega_H - c_H) > (\omega_L - c_L)$, this implies that for small enough ε , profits are decreasing in \hat{p}_x . From our assumption of $\pi_m \leq \hat{p}$ we have that $\hat{p} \leq \hat{p}_{b,g}$. Minimum profits will be achieved when $\hat{p}_x = \hat{p}$: all agents accept the entrant contract. This implies

$$\bar{\Pi} \geq \Pi(c - \omega) + \varepsilon(1 - \hat{p} - \hat{p}\alpha) > \Pi(c - \omega) \geq 0.$$

where the second inequality is given by the second inequality in equation (25) and the third inequality from the condition on aggregate equilibrium profits $\Pi(c - \omega)$ being non negative. Since a profitable deviation exists we reach a contradiction with c being a pooling equilibrium.

Let $\pi_m \leq \hat{p}$. Suppose a pooling equilibrium c exist where (5) does not hold. This implies

$$\frac{1 - \pi_m}{\pi_m} \frac{u'(c_L)}{u'(c_H)} < \frac{1 - \hat{p}}{\hat{p}}, \quad (26)$$

similarly to the previous case, consider an entrant offering $\hat{\tau} = (c_L - \varepsilon, c_H + \alpha\varepsilon)$ with $\varepsilon > 0$ and small. Given (26) and the fact that $\frac{1 - \pi_g}{\pi_g} < \frac{1 - \pi_m}{\pi_m}$ there exist an α satisfying equation (25). Proceeding as in the previous case we can show that the entrant contract will be accepted by both agents of type m and g . In this case minimum profits for the entrant are achieved when all agents accept $\hat{\tau}$. Hence, as in the previous case, the entrant always makes a strictly positive profit. Since a profitable deviation exists we reach a contradiction with c being a pooling equilibrium. □

Proof of Lemma 2

Proof. Suppose condition (7) does not hold, so that $c_H^g < c_L^g$ (the consumption of type g is

in the over-insurance region). If so, there exists an α so that

$$\frac{1 - \pi_g}{\pi_g} \frac{u'(c_L^g)}{u'(c_H^g)} < \alpha < \frac{1 - \pi_g}{\pi_g}. \quad (27)$$

Consider an entrant firm offering the menu of transfers $\hat{\tau} = (-\varepsilon, \alpha\varepsilon)$. This menu constitutes a form of negative insurance. For small enough ε , we have that $U^g(c^g + \hat{\tau}) > U^g(c^g)$, so that agents of type g accept the entrant's contract. Minimum profits from $\hat{\tau}$ occur when only agents of type g accept it. Profits from the deviation $\bar{\Pi}$, are such that $\bar{\Pi} \geq -\pi_g\alpha\varepsilon + (1-\pi_g)\varepsilon > 0$, where the strict inequality follows from the second inequality in (27). We thus reach a contradiction having found a profitable deviation. The proof of condition (8) follows the same steps of the proof of (3) in Lemma 1.

We next prove the condition in equation (9). We focus on the relation between quantities for the agents of type g and m . The proof for the relation between quantities for agents of type m and b is analogous. By contradiction suppose that $c_H^g < c_H^m$ from (6) it must also be the case that $c_L^m < c_L^g$. In this case we have that

$$u(c_H^m) - u(c_L^m) > u(c_H^g) - u(c_L^g) \quad (28)$$

From (6) we also have that $U^m(c^m) \geq U^m(c^g)$ and $U^g(c^g) \geq U^g(c^m)$, summing these two inequalities we get $(\pi_m - \pi_g)[u(c_H^m) - u(c_L^m) - (u(c_H^g) - u(c_L^g))] \geq 0$. Substituting (28) we get $\pi_m > \pi_g$ a contradiction. \square

Proof of Lemma 3

Proof. Suppose (10) does not hold, we then have $\pi_b(\omega_H - c_H^b) + (1 - \pi_b)(\omega_L - c_L^b) > 0$. This implies that profits from the consumption allocation for agents of type b are strictly positive. An entrant can offer the following contract $\hat{\tau} = (c_L^b + \delta - \omega_L, c_H^b - \omega_H)$. With $\delta > 0$ and small. We have that $U^b(\omega + \hat{\tau}) > U^b(c^b)$. Agents of type b will accept $\hat{\tau}$ to their original equilibrium allocation. In addition profits for the entrant $\bar{\Pi}$ are such that $\bar{\Pi} \geq \pi_b(\omega_H - c_H^b) + (1 - \pi_b)(\omega_L - c_L^b) - (1 - \pi_b)\delta > 0$ when δ is sufficiently small. We thus reach a contradiction having found a deviation that is always profitable.

Suppose equation (11) does not hold. We then have

$$\pi_b(c_H^j - c_H^b) + (1 - \pi_b)(c_L^j - c_L^b) > 0 \quad (29)$$

Consider an entrant deviating and offering $\hat{\tau} = (c_L^b - c_L^j, c_H^b - c_H^j + \delta)$ with $\delta > 0$ and small. In this case we have $U^b(c^j + \tau) = \pi_b u(c_H^b + \delta) + (1 - \pi_b)u(c_L^b) > U^b(c^b)$ so that agents of type b will pick $\hat{\tau}$ together with the allocation originally chosen by agents of type j . As in the proof of Lemma 1 let \hat{p}_x be the probability of receiving a high realization given the types of agents that accept the entrant's menu. Profits for the entrant are

$$\bar{\Pi} = \hat{p}_x(c_H^j - c_H^b - \delta) + (1 - \hat{p}_x)(c_L^j - c_L^b)$$

From (9) we have that for small enough δ profits are increasing in \hat{p}_x hence minimum profits are achieved when only agents of type b accept $\hat{\tau}$: $\bar{\Pi} \geq \pi_b(c_H^j - c_H^b - \delta) + (1 - \pi_b)(c_L^j - c_L^b) > 0$. Where the last inequality follows from (29) and δ sufficiently small. We thus reach a contradiction having found a deviation that is always profitable.

Suppose that by contradiction (12) is violated, then

$$\hat{p}_{b,m}(\omega_H - c_H^m) + (1 - \hat{p}_{b,m})(\omega_L - c_L^m) > 0, \quad (30)$$

Consider an entrant deviating and offering the contract $\hat{\tau} = (c_L^m + \delta, c_H^m)$ with $\delta > 0$ and small. This contract will be accepted by agents of type m since it provides strictly greater utility than the original consumption allocation c^m . Profits from $\hat{\tau}$ are given by

$$\bar{\Pi} = \hat{p}_x(\omega_H - c_H^m) + (1 - \hat{p}_x)(\omega_L - c_L^m - \delta).$$

With \hat{p}_x as above. Since $c_L^m > \omega_L$, profits are decreasing in \hat{p}_x . Since m agents accept $\hat{\tau}$, minimum profits are reached when agent of type m and b accept the contract. Hence $\bar{\Pi} \geq \hat{p}_{b,m}(\omega_H - c_H^m) + (1 - \hat{p}_{b,m})(\omega_L - c_L^m - \delta) > 0$. Where the last inequality follows from (30) and δ sufficiently small. We thus reach a contradiction having found a deviation that is always profitable.

Suppose (13) does not hold:

$$\pi_g(c_H^m - c_H^g) + (1 - \pi_g)(c_L^m - c_L^g) > 0. \quad (31)$$

in this case, consider the following contract offered by an entrant $\hat{\tau} = (c_L^g - c_L^m, c_H^g - c_H^m + \delta)$ with $\delta > 0$ positive and small. Since $U^g(c^m + \hat{\tau}) = \pi_g u(c_H^g + \delta) + (1 - \pi_g)u(c_L^g) > U^g(c^g)$, agents of type g accept the entrant's contract. Profits for the entrant are given by

$$\bar{\Pi} = \hat{p}_x(c_H^m - c_H^g - \delta) + (1 - \hat{p}_x)(c_L^m - c_L^g)$$

with \hat{p}_x as above. From (9) we have that $c_H^g \geq c_H^m$ and $c_L^g \leq c_L^m$ together with (31), it implies that profit are decreasing in \hat{p}_x , minimum profits are achieved when only the g type accepts $\hat{\tau}$, so that $\bar{\Pi} > \pi_g(c_H^m - c_H^g - \delta) + (1 - \pi_g)(c_L^m - c_L^g) > 0$. Where the last inequality follows from equation (31) and a sufficiently small value of δ . We reach a contradiction having found a profitable entry.

Suppose that by contradiction (14) is violated, then

$$\hat{p}_{b,m}(c_H^g - c_H^m) + (1 - \hat{p}_{b,m})(c_L^g - c_L^m) > 0, \quad (32)$$

Consider an entrant deviating and offering the contract $\hat{\tau} = (c_L^m - c_L^g, c_H^m - c_H^g + \delta)$ with $\delta > 0$ and small. This contract will be accepted by m types together with c^g in the original allocation. By (32) for δ sufficiently small, profits are positive for any additional type that also accepts the contract. We then reach a contradiction having found a deviation that is always profitable.

We now show condition (15). Aggregate profits Π are given by

$$\begin{aligned}\Pi &= p_b \left[\pi_b(\omega_H - c_H^b) + (1 - \pi_b)(\omega_L - c_L^b) \right] + \\ &+ p_m \left[\pi_m(\omega_H - c_H^m) + (1 - \pi_m)(\omega_L - c_L^m) \right] + \\ &+ p_g \left[\pi_g(\omega_H - c_H^g) + (1 - \pi_g)(\omega_L - c_L^g) \right].\end{aligned}\tag{33}$$

Suppose (15) is violated for $i = g$: $\Pi(c^g - \omega) < 0$. Rewrite aggregate profits in (33) as

$$\begin{aligned}\Pi &= p_b \left[\pi_b(c_H^m - c_H^b) + (1 - \pi_b)(c_L^m - c_L^b) \right] + \\ &+ (p_m + p_b) \left[\hat{p}_{b,m}(c_H^g - c_H^m) + (1 - \hat{p}_{b,m})(c_L^g - c_L^m) \right] + \\ &+ \left[\hat{p}(\omega_H - c_H^g) + (1 - \hat{p})(\omega_L - c_L^g) \right],\end{aligned}\tag{34}$$

The above can be interpreted as the profits originating from agents of type g choosing c^g both agents of type b and m choosing c^m and finally agents of type b choosing the transfer $c^b - c^m$. Using condition (11) and (14) in the above together with $\Pi(c^g - \omega) < 0$ it follows that $\Pi < 0$ a contradiction. Suppose (15) is violated for $i = m$: $\Pi(c^m - \omega) < 0$. Rewrite (33) as

$$\begin{aligned}\Pi &= p_b \left[\pi_b(c_H^m - c_H^b) + (1 - \pi_b)(c_L^m - c_L^b) \right] + \\ &+ \left[\hat{p}(\omega_H - c_H^m) + (1 - \hat{p})(\omega_L - c_L^m) \right] + \\ &+ p_g \left[\pi_g(c_H^m - c_H^g) + (1 - \pi_g)(c_L^m - c_L^g) \right],\end{aligned}\tag{35}$$

Using condition (11) and (13) in the above together with $\Pi(c^m - \omega) < 0$ it follows that $\Pi < 0$ a contradiction. \square

Proof of Lemma 4

Proof. Suppose condition (16) is violated. If so, there exists an α so that

$$\frac{1 - \pi_g}{\pi_g} \frac{u'(c_L^g)}{u'(c_H^g)} > \alpha > \frac{1 - \underline{p}}{\underline{p}}.\tag{36}$$

Where $\underline{p} = \min\{\hat{p}, \hat{p}_{b,g}\}$. Consider an entrant firm offering a menu containing $\hat{\tau} = (\varepsilon, -\alpha\varepsilon)$ with $\varepsilon > 0$ and small and α as in (36). Given small enough ε together with the first inequality in (36) it follows that $\hat{\tau}$ is accepted by agents of type g since $U^g(c^g + \hat{\tau}) > U^g(c^g)$. Since $\hat{\tau}$ constitute positive insurance, minimum profits are decreasing in the probability of the realization of a low endowment. Hence since agents of type g accept $\hat{\tau}$ and from the definition of \underline{p} we have that profits for the entrant are given by $\bar{\Pi} \geq \underline{p}\alpha\varepsilon - (1 - \underline{p})\varepsilon > 0$. Where the last

inequality follows from the second inequality in (36). Having found a profitable deviation we reach a contradiction.

Suppose condition (17) is violated. If so, there exists an α so that

$$\frac{1 - \pi_m u'(c_L^m)}{\pi_m u'(c_H^m)} > \alpha > \frac{1 - \hat{p}_{b,m}}{\hat{p}_{b,m}}. \quad (37)$$

As in the previous step consider an entrant firm offering a menu containing $\hat{\tau} = (\varepsilon, -\alpha\varepsilon)$ with $\varepsilon > 0$ and small. $\hat{\tau}$ is accepted by agents of type m . Minimum profits are achieved when only agents of type b and m accept the contract. In this case, from the second inequality in (37), we have that the entrant makes positive profits. Having found a profitable deviation we reach a contradiction. \square

Proof of Proposition 2

Proof. Step 1

We begin by showing that

$$\Pi^b(c^b - c^m) = \pi_b(c_H^m - c_H^b) + (1 - \pi_b)(c_L^m - c_L^b) = 0. \quad (38)$$

Suppose not, from (11) for $j = m$, it implies that $\Pi^b(c^b - c^m) < 0$. Consider an entrant offering a menu containing transfers $\tau = \{\tau_g, \tau_m\} = \{c^g + (\varepsilon, 0), c^m + (\varepsilon, 0)\}$. Suppose agents of type g accept τ_g . From (14) it follows that minimum profits are achieved when agents of type b and m accept τ_m . Rewriting aggregate profits as in (34) we have that total profits for the entrant $\bar{\Pi}$ satisfy the following: $\bar{\Pi} \geq \Pi - p_b \Pi^b(c^b - c^m) - (1 - \hat{p})\varepsilon > \Pi \geq 0$ where the second inequality follows from $\Pi^b(c^b - c^m) < 0$ and ε sufficiently small.¹⁹ Having found a profitable entry we reach a contradiction and indeed (38) holds.

We next show that if $c^m, c^g \neq \omega$ and $\Pi^{b,m}(c^m - c^g) < 0$, then

$$\frac{1 - \pi_m u'(c_L^m)}{\pi_m u'(c_H^m)} = \frac{1 - \hat{p}_{b,m}}{\hat{p}_{b,m}}. \quad (39)$$

Suppose not. From (17) there exist an α such that $\frac{1 - \pi_m u'(c_L^m)}{\pi_m u'(c_H^m)} < \alpha < \frac{1 - \hat{p}_{b,m}}{\hat{p}_{b,m}}$. Given such α , define $\bar{c}^m = c^m - (\varepsilon, -\alpha\varepsilon)$ with $\varepsilon > 0$ and small enough so that, $\Pi^{b,m}(\bar{c}^m - c^g) < 0$. We have that $\Pi^{b,m}((\varepsilon, -\alpha\varepsilon)) = \hat{p}_{b,m}\alpha\varepsilon - (1 - \hat{p}_{b,m})\varepsilon < 0$, where the last inequality follows from the above definition of α . Consider an entrant deviating and offering a menu containing the following contract $\tau = \{\tau_g, \tau_m\} = \{c^g + (\varepsilon_1, 0) - \omega, \bar{c}^m - \omega\}$ with $\varepsilon_1 > 0$ and small. We have that $U^g(\omega + \tau_g) > U^g(c^g)$ and $U^m(\omega + \tau_m) > U^m(c^m)$. If agents of type m accept τ_m and agent of type g accept τ_g , since $\Pi^{b,m}(\bar{c}^m - c^g) \leq 0$, minimum profits occur when agents of type b accept τ_m . Profits from the deviation in this case are given by²⁰ $\bar{\Pi} \geq \Pi + \Pi^g((\varepsilon_1, 0)) - \Pi^{b,m}((\varepsilon, -\alpha\varepsilon))$. This implies that for a given ε , ε_1 can be chosen

¹⁹The case with agents of type g accepting τ_m follows from (15).

²⁰Recall that $\Pi^b(c^b - c^m) = 0$.

small enough so that $\bar{\Pi} > 0$ hence the deviation is profitable and we reach a contradiction.²¹ Finally, suppose that agents of type m accept τ_m and agents of type g accept τ_m also. From (15), it must be the case that $\Pi(c^m - \omega) = 0$ else the deviation is immediately profitable for small enough ε . From (35) together with (13) it implies that $\Pi^g(c^g - c^m) = 0$ which implies $\Pi(c^g - \omega) < 0$ contradicting (15). Since a profitable deviation exists we have that condition (39) holds.

Step 2

We next show that $\Pi(c^g - \omega) = 0$. Suppose not, from (15) for $i = g$ it follows that $\Pi(c^g - \omega) > 0$. Consider the case where agents of type g purchase positive insurance: $c_L^g > \omega_L$; by Lemma 2 also agents of type m purchase positive insurance. In this case we have $\Pi^g(c^g - \omega) > 0$.²² We consider two sub-cases identified with the relation between \hat{p} and π_m . Suppose $\hat{p} \geq \pi_m$ so that agent of type m are worse than the average “market” type, it follows that $\Pi^{b,g}(c^g - \omega) \geq \Pi(c^g - \omega) > 0$. In this case an entrant can offer $\hat{\tau} = (c_L^g + \varepsilon - \omega_L, c_H^g - \omega_H)$ with $\varepsilon > 0$ and small. $\hat{\tau}$ will be accepted by the g type agents. Also this entry remains profitable for any additional type of agent that also accepts it, we thus reach a contradiction.

Suppose now that $\hat{p} < \pi_m$, so that agent of type m are better than the average “market” type. We show that an entrant adopting a “cream skimming” strategy with respect to the agents of type m will make positive profits. Since $\hat{p} < \pi_m$, from (15) for $i = m$, it follows that $\Pi^m(c^m - \omega) > 0$. We first show that it must be the case that aggregate profits from the allocation are zero: $\Pi = 0$. Suppose not, consider an entrant offering $\tau = \{\tau_g, \tau_m\} = \{c^g + (\varepsilon, 0), c^m + (\varepsilon, 0)\}$. From (14) it follows that minimum profits occur when agents of type b accept τ_m . If agents of type g accepts τ_m then from (15) for $i = m$ it follows that the deviation makes positive profits. If agents of type g accept τ_g then from (38) and $\Pi > 0$ it follows that ε can be chosen small enough so that the deviation is profitable. Hence $\Pi = 0$.

Since $\Pi = 0$, from (34) when $\Pi(c^g - \omega) > 0$, it also follows that $\Pi^{b,m}(c^m - c^g) < 0$. Hence From Step 1, (39) holds. Suppose an entrant offers the contract $\hat{\tau} = (c_L^m - \varepsilon - \omega_L, c_H^m + \alpha\varepsilon - \omega_H)$ where α satisfies $\frac{1 - \pi_b}{\pi_b} \frac{u'(c_L^m)}{u'(c_H^m)} > \alpha > \frac{1 - \pi_m}{\pi_m} \frac{u'(c_L^m)}{u'(c_H^m)}$. This condition on α implies that for small enough ε , $U^m(c^m + (-\varepsilon, \alpha\varepsilon)) > U^m(c^m)$, while $U^b(c^m + (-\varepsilon, \alpha\varepsilon)) < U^b(c^m)$. We show that agents of type b will not pick $\hat{\tau}$ implying that the entrant makes positive profits hence reaching a contradiction. Suppose by contradiction that agents of type b pick $\hat{\tau}$. Given the condition on α it must be the case that there exists an additional contract τ' available with the incumbents so that $U^b(c^m + (-\varepsilon, \alpha\varepsilon) + \tau') > U^b(c^b)$. We are going to show that the existence of such τ' leads to a contradiction, since it would have been chosen in the original equilibrium by agents of type b . As notation, let τ_m^i be the transfer that an agent of type m receives from incumbent firm i so that $c^m = \sum_{i \in I} \tau_m^i + \omega$. Suppose that $\hat{\tau}$ is offered by

²¹The case where agents of type m accept τ_g over τ_m follows from (15).

²²The case with $c_L^g < \omega_L$ under the contradicting assumption it implies $\Pi^g(c^g - \omega) < 0$ so that it must be the case that $\Pi^m(c^m - \omega) > 0$ and $c_L^m > \omega_L$, a case which is analyzed below when looking at entrant attracting agents of type m .

a single firm i' .²³ It must be the case that firm i' also offers transfers $\tau_m^{i'}$; if not it would be possible for agents of type b to achieve level of consumption $c^m + \tau'$ which is preferred to c^b by agents of type b .

We now show an important property that must hold in any separating equilibrium: it is always possible to reach consumption levels equal to c^m with firms other than i' .²⁴ Suppose firm i' deviates by withdrawing $\tau_m^{i'} = (\tau_{L,m}^{i'}, \tau_{H,m}^{i'})$ and instead offers $(\tau_{L,m}^{i'}, \tau_{H,m}^{i'} - \varepsilon)$, for this deviation not to increase profits, it must be the case that agents of type m do not accept $(\tau_{L,m}^{i'}, \tau_{H,m}^{i'} - \varepsilon)$ from i' .²⁵ This implies that is possible for agents of type m to pick alternative transfers with firms other than i' and reach a consumption level \bar{c}^m so that $U^m(c^m) = U^m(\bar{c}^m)$. It must be the case that $c^m = \bar{c}^m$. Suppose not, consider first the case in which $c_L^m < \bar{c}_L^m$ so that $\bar{c}_H^m < c_H^m$. In this case an entrant can offer the contract $\tau = (c_L^b - \bar{c}_L^m + \varepsilon, c_H^b - \bar{c}_H^m)$ with $\varepsilon > 0$ and small. This contract will be accepted by agents of type b together with \bar{c}^m . From (38) and the fact that $c_L^m < \bar{c}_L^m$, it follows that for ε sufficiently small this contract will be profitable for any additional type that accepts it. Suppose now that $c_L^m > \bar{c}_L^m$ in this case an entrant can offer the contract $\tau = (c_L^m - \bar{c}_L^m + \varepsilon, c_H^m - \bar{c}_H^m)$ with $\varepsilon > 0$ and small. This contract will be accepted by agents of type m together with \bar{c}^m . From (39) and ε sufficiently small the contract will be profitable for any additional type that accepts it.

Given the possibility of reaching a level of consumption c^m with firms other than i' , it follows that is possible for agents of type b to pick τ' together with c^m , this is a contradiction since it would have been chosen in equilibrium. Since no τ' exists, an entry offering $\hat{\tau}$ is always profitable and we reach a contradiction. So that it must be the case that $\Pi(c^g - \omega) = 0$.

Step 3

Given $\Pi(c^g - \omega) = 0$, we consider two sub-cases. Consider first the case with $\Pi^g(c^g - \omega) > 0$, from equation (14), (34) and (38) it follows that $\hat{p}_{b,m}(c_H^g - c_H^m) + (1 - \hat{p}_{b,m})(c_L^g - c_L^m) = 0$. It can be shown that equation (39) also holds in this case.²⁶ We can then proceed as in Step 2: in this case an entrant can “cream skim” agents of type m , hence the case with $\Pi^g(c^g - \omega) > 0$ leads to a contradiction.

The only remaining case is $\Pi(c^g - \omega) = \Pi^g(c^g - \omega) = 0$. This implies that $c^g = (\omega_L, \omega_H)$ and $c_L^m \geq \omega_L$. Since $\Pi^b(c^b - \omega) \leq 0$, we can have two sub-cases: either $\Pi^m(c^m - \omega) > 0$ or

²³The case in which multiple firms offer τ_i' so that $\sum_i \tau_i' = \tau'$ follows similarly. For this case the result follows by also imposing the requirement that for each firm offering τ_i' is never optimal to withdraw τ_i' and offer $\hat{\tau}$.

²⁴A similar results holds without free entry, see Attar, Mariotti, and Salanié (2011a) for details.

²⁵If agents of type b or type g accept this contract following the deviation it follows that they would have accepted also the original contract $(\tau_{L,m}^{i'}, \tau_{H,m}^{i'})$. Also, if following the deviation, agents of type m choose the contract that i' was offering to agents of type g : $\tau_g^{i'}$, it must be the case that agents of type m are indifferent in choosing $\tau_m^{i'}$ or $\tau_g^{i'}$. In this case an always profitable opportunity exist for entrants offering additional small amounts of insurance at a rate profitable if agents of type b and m accept it. From (39) it follows that this entry will be accepted by agents of type m together with $\tau_g^{i'}$ reaching a contradiction.

²⁶Note that the proof in Step 1 does not hold in this case. Instead to prove (39) proceed as follows. Suppose not, then there exists a \bar{c}^m preferred to c^m by agents of type m . Moreover, given that $\Pi^{b,m}(c^m - c^g) = 0$, \bar{c}^m has the property that $\Pi^{b,m}(\bar{c}^m - c^g) > 0$, following the steps used to prove (14) we reach a contradiction.

$\Pi^m(c^m - \omega) = 0$. In first sub-case, $\Pi^m(c^m - \omega) > 0$, we rewrite (33) as

$$\begin{aligned} \Pi &= (p_b + p_m) \left[\widehat{p}_{b,m}(\omega_H - c_H^m) + (1 - \widehat{p}_{b,m})(\omega_L - c_L^m) \right] + \\ &+ p_b \left[\pi_b(c_H^m - c_H^b) + (1 - \pi_b)(c_L^m - c_L^b) \right] + \\ &+ p_g \left[\pi_g(\omega_H - c_H^g) + (1 - \pi_g)(\omega_L - c_L^g) \right], \end{aligned} \quad (40)$$

Since $c_L^g = \omega_L$ and $\Pi^m(c^m - \omega) > 0$, from (9) it follows that $c_L^m > \omega_L$. Since $\Pi^g(c^g - \omega) = 0$ from (11), (12) and (40), it follows that $\Pi^{b,m}(c^m - c^g) = \Pi^b(c^b - c^m) = 0$. As shown before we have that condition (39) must hold. Hence we reach a contradiction by showing that an entrant can “cream skim” agents of type m . In the second sub-case, $\Pi^m(c^m - \omega) = 0$, from (33) it follows that $\Pi^b(c^b - \omega) = 0$. From (11) it follows that $c^m = (\omega_L, \omega_H)$. Finally if the agent of type b does not receive full insurance an entrant can provide a contract that agents of type b strictly prefer and is always profitable. The thesis follows. \square

B Proof of Proposition 4

Before proving the proposition we show the following Lemma.

Lemma 5. *Suppose Assumption 1 holds. Let $c = (c_L, c_H) \neq (\omega_L, \omega_H)$, we have that*

1. *If $c_L > \omega_L$ and $\Pi^g(c - \omega) \geq 0$ or $c_L < \omega_L$ and $U^g(c) \geq U^g(\omega)$ then*

$$\frac{1 - \pi_g}{\pi_g} \frac{u'(c_L)}{u'(c_H)} < \frac{1 - \widehat{p}}{\widehat{p}}, \quad (41)$$

and

$$\frac{1 - \pi_m}{\pi_m} \frac{u'(c_L)}{u'(c_H)} \leq \frac{1 - \widehat{p}_{b,m}}{\widehat{p}_{b,m}}, \quad (42)$$

2. *If $c_L > \omega_L$ and $\Pi^g(c - \omega) \geq 0$ or $c_L < \omega_L$ and $U^m(c) \geq U^m(\omega)$ then*

$$\frac{1 - \pi_m}{\pi_m} \frac{u'(c_L)}{u'(c_H)} \leq \frac{1 - \widehat{p}_{b,m}}{\widehat{p}_{b,m}}, \quad (43)$$

Proof. For all $c = (c_L, c_H)$, define the function

$$f_j(c_L, c_H) = \frac{1 - \pi_j}{\pi_j} \frac{u'(c_L)}{u'(c_H)}.$$

We have that $\frac{\partial f_g(c_L, c_H)}{\partial c_L} < 0$ and $\frac{\partial f_g(c_L, c_H)}{\partial c_H} > 0$. We begin by showing equation (41). Consider the case with $c_L > \omega_L$, since $\Pi^g(c - \omega) \geq 0$ it follows that $c_H < \omega_H$. From property of f_j it follows that $f_g(c_L, c_H) < f_g(\omega_L, \omega_H) \leq \frac{1 - \widehat{p}}{\widehat{p}}$. Where the last inequality follows from (18) in

Assumption 1. Suppose now that $c_L < \omega_L$ in this case since $U^g(c) \geq U^g(\omega)$ it must be the case that $c_H > \omega_H$ in which case the result follows as before. The proof of equations (42) and (43) are analogous. \square

We now move to the proof of Proposition 4.

Proof.

The proof is by construction. We first describe strategies adopted by firms, and choices of agents then we show that no incumbent or entrant wishes to deviate from the proposed equilibrium. Consider the following strategies by firms. Let firms $i = 1, 2$ offer the menu:

$$C^i = \left\{ \left(\frac{\tau_{L,b}}{2}, \frac{\tau_{H,b}}{2} \right), L_b, (0, 0) \right\},$$

where $\tau_{L,b} = \pi_b(\omega_H - \omega_L)$, $\tau_{H,b} = (1 - \pi_b)(\omega_L - \omega_H)$ and the set L_b is defined as follows

$$L_b = \left\{ x_L \geq 0, x_H \leq 0 \mid -\pi_b x_H - (1 - \pi_b)x_L = 0 \right\}.$$

All remaining firms $i \neq 1, 2$ offer the menu: $C^i = \{(0, 0)\}$. It is easy to show that under Assumption 1, the agents strategies are (without loss of generality): agents of type b choose $\left(\frac{\tau_{L,b}}{2}, \frac{\tau_{H,b}}{2}\right)$ from both firms 1 and 2 and $(0, 0)$ from remaining firms; types m and g choose $(0, 0)$ from all firms. In this equilibrium, all firms make zero profits, and agents of type b, m and g receive allocations c^b, c^m and c^g respectively.

The proof proceeds in three steps. We consider in turn the case where a deviating firm attracts one type (implying that at least one type chooses a contract different than the null contract $(0, 0)$), two types and finally the case where a deviating firm attracts all three types.

Step 1

Consider firm i deviating and offering the menu $C^i = \{\tau, (0, 0)\}$. Suppose that only one type of agent picks $\tau = (\tau_H, \tau_L)$. Agents of type b are fully insured at the actuarially fair price. Hence if τ is accepted by agents of type b it follows that $\Pi^b(\tau) < 0$, the deviation results in negative profits for firm i , hence we reach a contradiction. Suppose that τ is accepted only by agents of type $j = m, g$. Consider the case with $\tau_L > 0$. Consider the following choice from agents of type b : accept τ from firm i and pick an additional transfer $\hat{\tau} \in L_b$ from either $i = 1, 2$ equal to

$$\hat{\tau} = \left(\pi_b(\omega_H - \omega_L + \tau_H - \tau_L), -\frac{1 - \pi_b}{\pi_b}(\omega_H - \omega_L + \tau_H - \tau_L) \right). \quad (44)$$

Under this choice the consumption for agents of type b is given by $\{\omega_b + \pi_b\tau_H + (1 - \pi_b)\tau_L, \omega_b + \pi_b\tau_H + (1 - \pi_b)\tau_L\}$:²⁷ full insurance together with additional consumption in each state. This implies $U^b(\omega + \tau + \hat{\tau}) > U^b(c^b)$. Hence, upon entry, τ is chosen by agents of type b and of type j reaching a contradiction with the assumption that τ is accepted by only one type.

²⁷Recall that $\omega_b = \pi_b\omega_H + (1 - \pi_b)\omega_L$. In addition, from Assumption 1, and $U^j(\omega + \tau + \tau') > U^j(\omega)$ for some $\tau' \in L_b$, it follows that $\pi_b\tau_H + (1 - \pi_b)\tau_L > 0$.

For the case with $\tau_L < 0$ it follows that if $U^j(\omega + \tau + \tau') > U^j(\omega)$ for some $\tau' \in L_b$ then $\Pi^j(\tau) < 0$.

Step 2

Suppose that firm i deviates and offers a menu C^i and that only two types of agent pick elements in C^i different than $(0,0)$. Suppose that agents of type b and of type m choose non-null contracts in C^i , denoted by τ_b and τ_m respectively. Profits are given by $\bar{\Pi} = p_b \Pi^b(\tau_b) + p_m \Pi^m(\tau_m)$, as before $\Pi^b(\tau_b) < 0$. Since agent of type m choose τ_m over $(0,0)$, it must be the case that τ_m provides positive insurance else $\Pi^m(\tau_m) < 0$. Suppose this is the case, from (19) in Assumption 1, it must be the case that $\Pi^{b,m}(\tau_m) < 0$. As in the previous step, consider the deviation of agents of type b , $\hat{\tau} \in L_b$, with $\hat{\tau}$ defined in (44). It must be the case that $U^b(\omega + \tau_b + \tau') \geq U^b(\omega + \tau_m + \hat{\tau})$ where τ' is any contract in L_b . Since $\tau_m + \hat{\tau}$ provides full insurance and $\tau' \in L_b$, due to concavity of the utility function, the expected transfers from τ_b must be weakly higher than transfers from $\tau_m + \hat{\tau}$ i.e. $\Pi^b(\tau_m) = \Pi^b(\tau_m + \hat{\tau}) \geq \Pi^b(\tau_b + \tau') = \Pi^b(\tau_b)$. Total profits for the entrant are:

$$\bar{\Pi} = p_b \Pi^b(\tau_b) + p_m \Pi^m(\tau_m) \leq p_b \Pi^b(\tau_m) + p_m \Pi^m(\tau_m) = \Pi^{b,m}(\tau_m) < 0$$

We thus reach a contradiction since the entrant is making negative profits. Using a similar strategy as in the previous step for agents of type b , it can be shown that the case where only agents of type m and g accept the non-null contracts (τ_m, τ_g) leads to a contradiction since agents of type b will strictly prefer τ_m combined to $\hat{\tau}$ as in (44) with respect to $(0,0)$. As a final case suppose that only agents of type b and g accept the non-null contract (τ_b, τ_g) . In this case it immediately follows that agents of type m prefer τ_g to $(0,0)$. Hence an entrant must necessary attract all three types, a case we consider next.

Step 3

Finally, suppose that firm i deviates and offers a menu C^i . The only remaining case is to consider when all three types of agents b , m and g , pick contracts different than $(0,0)$ with a deviating firm i . Denote these contract respectively by τ_b , τ_m and τ_g (we don't exclude the case that any two might be equal). For firm i , profits are given by $\bar{\Pi} = p_b \Pi^b(\tau_b) + p_m \Pi^m(\tau_m) + p_g \Pi^g(\tau_g)$. We first show that $\Pi^g(\tau_g) > 0$. Suppose not, we then have $\bar{\Pi} \leq p_b \Pi^b(\tau_b) + p_m \Pi^m(\tau_m)$. Following Step 2 it follows that $\bar{\Pi} < 0$ reaching a contradiction with $\Pi^g(\tau_g) \leq 0$. Let $\tau_g = (\tau_{L,g}, \tau_{H,g})$. Since $\Pi^g(\tau_g) > 0$ it must be the case that $\tau_{L,g} > 0$. Rewrite $\bar{\Pi}$ as:

$$\bar{\Pi} = \Pi(\tau_g) + (p_m + p_b) \Pi^{b,m}(\tau_m - \tau_g) + p_b \Pi^b(\tau_b - \tau_m) \quad (45)$$

From (18) in Assumption 1 it follows that $\Pi(\tau_g) < 0$. As before $\Pi^b(\tau_b - \tau_m) \leq 0$. To complete the proof we show that $\Pi^{b,m}(\tau_m - \tau_g) \leq 0$. If $\tau_m = \tau_g$ we are done. Suppose $\tau_m \neq \tau_g$. From equation (42) in Lemma 5 and $\pi_b < \hat{p}_{b,m}$ we have that

$$\frac{1 - \pi_m}{\pi_m} \frac{u'(\omega_L + \tau_{L,g})}{u'(\omega_H + \tau_{H,g})} \leq \frac{1 - \pi_b}{\pi_b},$$

so that $U^m(\omega + \tau_g) \geq U^m(\omega + \tau_g + \tau')$ for any $\tau' \in L_b$.²⁸ Similarly, it must be the case that $U^g(\omega + \tau_g) > U^g(\omega + \tau_g + \tau')$ for any $\tau' \in L_b$. Let $\tau_m = (\tau_{L,m}, \tau_{H,m})$. It must be the case that $0 < \tau_{L,g} < \tau_{L,m}$ if not $U^g(\omega + \tau_g) \geq U^g(\omega + \tau_m + \tau')$ for any $\tau' \in L_b$ implies $U^m(\omega + \tau_g) > U^m(\omega + \tau_m + \tau')$, for any $\tau' \in L_b$. Consider the case with $\Pi^g(\tau_m) \geq 0$. From (43) we have that $U^m(\omega + \tau_m) \geq U^m(\omega + \tau_m + \tau')$ for any $\tau' \in L_b$. Given the choice of agent of type m it must be then the case that $U^m(\omega + \tau_m) \geq U^m(\omega + \tau_g)$, from equation (42) it then follows that $\Pi^{b,m}(\tau_m - \tau_g) < 0$.

Given the assumption of Lemma 5, a case that needs to be proved separately is the case where $\Pi^g(\tau_m) < 0$. This case implies that $\Pi^m(\tau_m) < 0$, it is immediate that $\Pi^g(\tau_m - \tau_g) < 0$ which implies $\Pi^{b,m}(\tau_m - \tau_g) < 0$

Since $\Pi(\tau_g) < 0$, $\Pi^{b,m}(\tau_m - \tau_g) \leq 0$ and $\Pi^b(\tau_b - \tau_m) \leq 0$, from equation (45) it follows that $\bar{\Pi} < 0$. Since there is no profitable deviation the result follows. \square

²⁸Intuitively this is because the slope of the indifference curve at $\omega + \tau_g$ is flatter than the slope of L_b .