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ANALYTICS I: A Conceptual Framework for The SYMBOL, TRUTH, and ARGUE Programs, Apple II Plus Versions

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ANALYTICS I

A Conceptual Framework for
The SYMBOL, TRUTH, and ARGUE Programs

Apple II Plus Versions

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1. What the ANALYTICS Package Helps Teach

1.1. Introduction

The ANALYTICS package of computer-assisted instruction programs for the Apple II Plus presently consists of the following three programs: SYMBOL, which generates symbolization exercises (with answers) in sentential logic; TRUTH, which generates exercises (and guidance) in the truth-functional analysis of sentential connectives; and ARGUE, which provides guidance in formal derivations and the reconstruction of English arguments in valid deductive form. Each of these programs provides guided practice with tools and techniques of formal logic that are essential not only to learning the apparatus of elementary symbolic logic but also to understanding the formal dimensions of argument reconstruction and analysis. This manual attempts to provide a conceptual framework for understanding the curricula of the programs and the applications of what they help teach. ANALYTICS II provides instructions for running the SYMBOL and TRUTH programs, with illustrative interactions. ANALYTICS III does the same for the ARGUE program.

A word of explanation is in order regarding the title of the set of computer-based tutorials, ANALYTICS, for which this manual is a companion. The computer tutorials are designed primarily to exercise you in the use of basic tools and techniques of logical analysis. The science of formal logic and the systematic study of what makes reasoning good or bad were invented by Aristotle over twenty centuries ago. Modern logic has made considerable progress on Aristotle's monumental beginnings. But the nature of logical analysis as we pursue it today still owes a great deal to Aristotle's original inspiration and definition. Any history of philosophy or logic will attest to this. The following brief description of Aristotle's basic enterprise from W. T. Jones' History of Western Philosophy (Vol. I, p. 224) is fitting for our course of study:

Aristotle was the inventor of formal logic in the sense that he was the first person to draw up precise rules for distinguishing valid from invalid thinking. Suppose I know that all Greeks are mortal and that Aristotle is a Greek. It follows that Aristotle is mortal. . . . [Now], why would the conclusion that Aristotle is a man not follow from these premises? Of course there are premises from which the latter conclusion could be drawn—for instance, "All Greeks are men" and "Aristotle is a Greek." But even though it is true that Aristotle is a man, this proposition does not follow from the facts that he is a Greek and that all Greeks are mortal.

Thus, as Aristotle saw, we must distinguish between truth and validity. Truth is a characteristic of individual propositions: An individual proposition is true if it correctly classifies things and false if it does not. Thus "Aristotle is a Greek" is true, and "Aristotle is a Turk" is false. Validity is not a characteristic of individual
propositions. It is the logical relation between premises ... and the conclusion that follows from these premises. Thus, although the proposition "Aristotle is a man" is true, it follows validly from some premises but not from others.

The two chief questions Aristotle set himself to answer ... were: (1) When we have true propositions, what are the rules of inference by which a conclusion can be [validly] drawn? (2) How can we know that the premises we start with are true?

In formal logic today we still distinguish (1) the logical FORM and VALIDITY of an argument from (2) the CONTENT or TRUTH of its premises and conclusion. We, like Aristotle, are interested in both of the following questions:

1. What are the rules of inference by which conclusions can be validly drawn from given premises?

2. How can we determine whether the premises of an argument are true or sufficiently plausible to justify assent?

We will also be concerned with how these two questions are inevitably and usefully related: with how valid logical form is usefully related to the pursuit of truth (in philosophy, in particular).

This concern with the distinction and relation between the FORM and CONTENT of reasoning remains very much in the spirit of Aristotle's pioneering study of logic. This interaction is reflected throughout the computer programs in logic and argument analysis. This is why the package of programs is entitled ANALYTICS, reminiscent of the title of Aristotle's major treatises on logical analysis, the Prior and Posterior Analytics.

As you probably know and as you will see, you can't argue well about the truth of anything without logic; but logic, while necessary, is hardly sufficient for determining the truth in any dispute. The relation between formal logic and the pursuit of truth in ANALYTICS becomes especially conspicuous in the formal reconstruction and analysis of arguments. For this job we need some special tools. The programs in the ANALYTICS package provide practice with some of the very basic tools of formal logic needed to analyze the logical form of arguments, to assess their validity, and to reconstruct them in expressly valid form.
1.2. Formal Deductive Validity

An ARGUMENT, for present purposes, is a set of statements, some of which (the PREMISES) purportedly support or imply some other statement (the CONCLUSION).

An argument purportedly provides some sort and degree of CONDITIONAL WARRANT for its conclusion: If its premises are evidently true or credible, then one has some reason to accept the conclusion. Some EVIDENTIARY CONNECTION is posited between the premises and conclusion: This means that such credibility as the premises possess is somehow passed along or 'lent' to the conclusion. Just how do credible premises 'lend' credibility to a conclusion? What kinds of evidentiary or logical connections are there?

One very special kind of evidentiary connection that can obtain between the premises and conclusion of an argument is called, variously: deductive VALIDITY, logical IMPLICATION, logical CONSEQUENCE.

An argument that is deductively VALID provides the strongest possible conditional warrant for its conclusion: IF the premises are true, the conclusion is not just metaphorically 'lent' some 'support'; it is absolutely GUARANTEED to be true.

A useful converse relation also obtains: If a set of statements (say, a theory) LOGICALLY IMPLIES a false or otherwise unacceptable consequence, then at least one of the statements must be false or likewise unacceptable.

This CONNECTION between a set of statements (say, the premises of an argument) and some LOGICAL CONSEQUENCE (say, the conclusion of the argument) has nothing to do with the content or actual truth or falsity of the statements.

DEDUCTIVE VALIDITY is accountable rather to LOGICAL FORM, to certain skeletal or structural features of the statements in question.

By analogy: whether the human body stands or falls depends in large part on its skeleton, as well as on its musculature; whether a bridge stands or falls depends in large part on its structural design, as well as on what it's made of. Likewise, whether an argument stands or falls, whether it supports its conclusion or not, depends in large part on its LOGICAL FORM, its skeletal structure, as well as on the truth or credibility of its premises. Examples follow.
The following argument (A) is **valid**, and this is by virtue of its having a certain skeleton or **Logical Form**, for example (A'), depicted to its right.

(A)  
(1) If you are illiterate  
(1') If I, not R  
you are not reading this  

(2) You are illiterate  
(2') I  

Therefore, (3) you are NOT  
(3') Not R  
reading this  

The fact that statements (2) and (3) are false does not affect the **validity** of the argument: If (2) as well as (1) were true, (3) would have to be true. (A) obviously is not seriously intended as an argument in the sense of an attempt to convince you of its conclusion. Whether we regard it as a serious or interesting argument does not change the **Logical Connection** between statements (1) and (2) and statement (3): (1) and (2) together logically **imply** (3). Moreover, what's of interest about this connection is that any statements having the logical forms (1') and (2') would together logically **imply** a statement of the form (3').

In the following argument, (B), the premises and conclusion all happen to be true:

(B)  
(1) If you are illiterate  
(1') If I, not R  
you are NOT reading  

But, (4) you are NOT  
illiterate  
(4') Not I  

So, (5) you are reading  
(5') R  

Close your eyes and the conclusion, statement (5), is false, while the premises remain true. Hence, this argument is **invalid**. But not just because of any accident or fact about the world that momentarily renders the conclusion false while the premises are true. It is invalid because the **Logical Form** of the argument (B') fails to **guarantee** a true conclusion, given true premises. We can know and prove this about the argument form (B') irrespective of anything we may know about the particular statements asserted in argument (B).

An argument skeleton of the form (B') fails to guarantee the truth of its conclusion, given true premises, if any argument of that form can have true premises **but** a false conclusion. Knowing nothing about you or the truth of statements (4) and (5), I know that argument (B) is invalid so far as its logical form, (B'), is the same in relevant respects as the following argument's, (C'):
(C) (6) If I'm on the moon I'm not on Venus (True)
(C') (6') If M, not V

(7) I'm not on the moon (True) (7') Not M

Therefore, (8) I'm on Venus (False!) (8') V

Demonstrate the fact as you will, any argument whose relevant FORM is the same as (B') or (C') is INVALID so far as it is possible for an argument of that form to have true premises and a false conclusion: An INVALID ARGUMENT FORM can lead us from truth into falsehood. A VALID argument form cannot.
2. What the SYMBOL Program Helps Teach

2.1. Formal Symbolic Logic: Why Symbolize Arguments?

Deductive logic is FORMAL insofar as it typically attributes validity/invalidity to LOGICAL FORM: it seeks to discover rules governing the use of those logical elements of our language that make arguments valid or invalid.

For example: the crucial elements of logical form singled out in arguments (A)–(C) were the sentential connective 'IF' and the negation term 'NOT.' Connectives like IF are crucial parts of the skeletons of arguments. The validity or invalidity of arguments (A)–(C) can be accounted for by the way the statements of the argument were constructed and combined using skeletal parts like 'IF' and 'NOT.' To make the skeleton or form of these arguments stand out clearly, it was convenient to symbolize (let single letters stand in for) the component statements that make up the arguments.

Formal logic is typically SYMBOLIC so far as it is convenient (for purposes, say, of easy pattern recognition and formal manipulation) to depict the statements and crucial logical elements of natural language (like 'if,' 'unless,' 'not') in some standard notation. It is often convenient to reduce the logical form and import of the variety of logical expressions found in ordinary language (e.g., 'if,' 'only if,' 'unless') to some standard symbolic form for purposes of easily construing the validity or invalidity of arguments. It can be very useful to examine the pure logical form and import of statements, quite apart from knowing their truth or falsity—especially when the matter at hand is controversial, the truth of the matter is elusive, and we are not sure what to believe.

For example: Suppose a person is wondering whether a human fetus can be shown to have a right to life. She's not at all sure what to believe on this issue. But she does think that, unless a fetus has no right to life, abortion is wrong. In any case, she can't help feeling that abortion is just not right. A quarrelsome friend then claims that she's effectively committed to a position on the right-to-life issue after all, and had better face up to it. That is, he claims that propositions (9) and (10) logically commit her to (12), as follows:

\[(D) \ (9) \text{ UNLESS it's the case that fetuses have no right to life, abortion is NOT right} \]

\[(D') \ (9') \text{ Unless not R} \]

\[(10) \text{ Abortion isn't right} \]

\[(10') \text{ Not A} \]
So, (11) it's NOT the case that fetuses have no right to life—
(11') Not not R

(12) fetuses do have a right to life
(12') R

Is it true that she is logically committed to believe (12) if she believes (9) and (10)? Do (9) and (10) logically imply (12)? Is (D) a valid argument? How can we tell?

Suppose her friend, while trying to argue for abortion and raise doubts in her mind about (10) by capitalizing on her doubts about (12), holds that abortion is not wrong unless fetuses have a right to life. But, he must confess, he thinks fetuses do have a right to life. She presses the point that he must, then, logically, concede that abortion is wrong, on the following deduction:

(E) (13) UNLESS fetuses have a right to life, abortion is NOT wrong
(E') (13') Unless R, not W
But (12) fetuses do have a right to life
(12') R

So, (14) abortion is wrong
(14') W

Is she right? Does (14) follow logically from (12) and (13)? This may be unclear; the logic of the matter may get lost in the verbiage. This is not uncommon. This is why we try to simplify or clarify the logical form of an argument by reducing it to a standard symbolic schema, by stripping away the verbiage and focusing on the crucial logical connections.

The foregoing hypothetical dispute is not about the TRUTH OR FALSITY of beliefs (about the rights of fetuses or the rights and wrongs of abortion). It is rather about the LOGICAL CONNECTIONS among the propositions in question. The dispute hangs in part on some LOGICAL CONNECTION, (9) or (13), that each disputant posited between the rights of fetuses and the rights and wrongs of abortion.

In fact, each party is incorrect about what the other is logically committed to concede: Arguments of the form (D') and (E') are, on one account, clearly INVALID. This may or may not be clear from the 'sound' or logical 'ring' of the arguments as given in ordinary language. The crux of the matter here is how we interpret the precise logical force of the ordinary conditionalizing connective 'UNLESS.' Symbolic logic can legislate the dispute and make the issue more transparent as follows.
Conditionals of the form (9') 'Unless not R, not A' have the same LOGICAL FORCE, the same LOGICAL MEANING as statements of the form 'A only if not R,' 'If A, not R' and 'If R, not A.' Why this should be so will require some study and justification, but the point can be illustrated by the following deductive sequence of LOGICALLY EQUIVALENT statements, any of which 'follows logically' from any other:

(15) IF it's raining out, it's not dry out
(16) It's raining out ONLY IF it's not dry out
(17) It's NOT raining out UNLESS it's not dry out
(18) UNLESS it's NOT dry out, it's not raining out
(19) IF it's NOT the case that it's not dry out, it's not raining out
(20) IF it's dry out, it's NOT raining out

(15') IF R, not D
(16') R ONLY IF not D
(17') Not R unless not D
(18') Unless not D, not R
(19') If not not D, not R
(20') If D, not R

Whatever actual sentences the sentence symbols 'R,' 'A,' 'D' stand for makes no difference to the logical force of these equivalent conditional statements, to the logical relation posited between the sentences connected by 'if,' 'only if' or 'unless.' At bottom, then, statement (9) may be seen to have the equivalent logical force of statements (4), (6) and statements (15)–(20).

It is convenient to symbolize the logical force of these diverse but logically equivalent connections in a standard way, with a single symbol, say, an arrow '=>' . The logical form of arguments (B) and (D) may then be readily represented as, at bottom, the same:

(B') If I, not R
      Not I
      _________
      R

(B'') I => Not R
      Not I
      _________
      R

(D') Not A unless not F
      Not A
      _________
      F

(D'') A => Not F
      Not A
      _________
      F

Arguments of the form (B') are not valid. Neither, then, is any argument of the form (D').
since the underlying logical form of (B') and (D') are equivalent, as represented by (B'') and (D''), above. That arguments of the forms (B') or (D') are invalid is readily seen from the following example, which has the the same logical form:

You're not in New York unless you're not in France (True)
You're not in New York (True)
Therefore, you're in France (False!)

Can you symbolize the logical form of this argument to show that it has the same logical form as (B') and (D')?

Arguments of the following form are also INVALID:
(E') Not W unless F
(E ' ) W => F

F

F

W

W

(E') is INVALID because an argument with the same underlying logical form (E'') can have true premises but a false conclusion, can lead us from truth into falsehood, as follows:
IF you're a whale, you're a mammal (True)
You are a mammal (True)
So, you're a whale (False!)

Or, equivalently:
You're NOT a whale UNLESS you're a mammal (True)
But you are a mammal (True)
So, you're a whale (False!)

Deductive logic, formal and symbolic, is concerned with discovering and demonstrating various sorts of LOGICAL FORM and LOGICAL CONNECTEDNESS, such as define the validity/invalidity of arguments, the relations of logical implication or consequence, logical equivalence and familiar derivative properties such as logical consistency/inconsistency. Judgments about these sorts of formal logical relations play an important role in everyday reasoning and a crucial role in philosophic argument.

These logical connections are conveniently defined and studied with the aid of symbolic notation. The advantage of formal symbolic logic is analogous to that of an x-ray device:
it allows us to depict and scan the supporting skeleton of an argument, and to isolate distinctively structural flaws, apart from the often obscuring verbal flesh and musculature. As you become practiced in depicting the LOGICAL FORM of an argument symbolically, you will develop a kind of 'x-ray vision' into the structural strengths and weakness in the skeletons of arguments—and you will not be confounded by logical disputes like those over arguments (D) and (E) above.
2.2. Logical Form: Sentential Logic

Deductive logic studies those logical terms or skeletal elements of language (like 'if,' 'not') that are crucial to determining the validity or invalidity of arguments. You now know that whether an argument is valid or not depends on its LOGICAL FORM. But how do we determine the logical form of an argument?

There are many logical elements of language that could be crucial to the logical form of a sentence or argument. We are going to study, for starters, the most basic ones, the most basic building blocks and connective tissue of arguments. These are logical terms like 'not,' 'and,' 'or,' 'if' by which we connect or negate sentences. At the most basic level, arguments are built up by putting simple sentences together by means of SENTENTIAL CONNECTIVES. The way in which sentences are connected (using words like 'and,' 'or,' 'if') or the way sentences are negated (using 'not'), when these sentences are constructed into arguments, determines the validity/invalidity of these arguments. You've seen examples of this phenomenon of language already.

Argument (A) below is valid. But argument (B-1) is not. The logical form of (B-1) is (B'), depicted to its right. The ARGUMENT FORM (B') is invalid because it's possible for an argument with this form to have true premises and a false conclusion: This is shown by argument (B-2), which has the same form as (B-1). Look closely at ARGUMENT FORM (A') and ARGUMENT FORM (B') to be sure you see how the LOGICAL FORMS of arguments (A) and (B-1)/(B-2) are different.
(A) (1) If you're in Pittsburgh then you're in Pennsylvania
(2) You're not in Pennsylvania
(3) Therefore, you're not in Pittsburgh

(A') If P, then Q

Not Q

Not P

(B-1) (1) If you're in New York City then you're in New York
(2) You're not in New York City
(3) Therefore, you're not in New York

(B') If P, then Q

Not P

Not Q

(B-2) (1) If you're in Philly, then you're in Pennsylvania (True)
(2) You're not in Philly (True)
(3) Therefore, you're not in Pennsylvania (False!)

(B') If P, then Q

Not P

Not Q

The logical form of argument (A) is different from the logical form of arguments (B-1)/(B-2) in one crucial respect: premise (2) of argument (A) negates the 'THEN'-clause of premise (1), and (A) concludes with the negation of the 'IF'-clause; whereas premises (2) of arguments (B-1)/(B-2) negate the 'IF'-clauses of their respective premises (1), and (B-1)/(B-2) conclude with the negation of the 'THEN'-clause.

Differences in how sentences are combined into argument patterns using 'IF-THEN-' and 'NOT' can make all the difference as to the validity or invalidity of arguments.

Note: The fact that premise (2) of argument (A) is false makes no difference to the VALIDITY of the ARGUMENT FORM, (A'): If (2) as well as (1) were true, if you were indeed not in Pennsylvania, then you would not be in Pittsburgh. The ARGUMENT FORM is VALID because if the premises were true, the conclusion would be guaranteed to be true.

Likewise, the fact that the premises and conclusion of argument (B-1) are all true does not make the argument valid: the ARGUMENT FORM (B') is INVALID because it fails to GUARANTEE that EVERY argument of that form with true premises will have a true conclusion. Argument (B-2) has the form (B'): (B-2) has true premises but a false conclusion. INVALIDITY can lead us from truth into falsehood—this is why we want to be able to discern and avoid it.
You have now seen examples of how the validity of an argument can depend on the way sentences are combined using SENTENTIAL CONNECTIVES like 'IF' and 'NOT.' SENTENTIAL LOGIC determines the rules that govern the validity or invalidity of arguments so far as validity depends on how sentences are combined using the SENTENTIAL CONNECTIVES.

Terms like 'not,' 'never,' 'it is not the case that' are not really used to connect sentences so much as to negate them; but negation expressions like 'not' are crucial to the patterns in which sentences are combined into arguments. So, for convenience, we will refer to negation expressions like 'not' as sentential connectives.

The CONNECTIVES studied by SENTENTIAL LOGIC represent five basic types of logical operation that we can perform on sentences:

1. NEGATION, by means of terms like 'NOT':
   Given any arbitrary sentence, say
   
   You are reading
   
   we can form its negation thus:
   
   You are NOT reading
   
   It is not the case that you are reading

2. CONJUNCTION, by means of terms like 'AND':
   Given any two arbitrary sentences, say,
   
   You are reading
   You are bored
   
   we can conjoin them in a conjunction:
   
   You are reading AND you are bored.
   
   We can, of course, negate and conjoin sentences:
   
   You are reading AND you are NOT bored.

3. DISJUNCTION, by means of terms like 'OR' we form disjunctions: You are reading OR you are bored.

   You are reading OR you are NOT reading.

4. CONDITIONALIZATION, by means of expressions like 'IF' or 'IF-THEN': We can form conditionals like

   (a) IF you are reading, THEN you are bored.
(b) IF you are bored, THEN you are reading.

(c) IF you are NOT bored, THEN you are reading.

(d) IF you are reading, THEN you are NOT bored.

Depending on how we combine certain sentences (like 'You are reading,' 'You are bored') into more complex sentences (like (a) or (b) using a connective like 'IF,') the meanings of the more complex sentences that result are different. For example, the meaning of sentence (a) is different from that of sentence (b); likewise for sentences (c) and (d). Notice that these differences in meaning are accountable to differences in LOGICAL FORM: The component sentences ('You are reading,' 'You are bored') are the same in (a) and (b); but the 'IF'-clause in sentence (a) is the 'THEN'-clause in sentence (b), and vice versa. (We will study what these differences mean in sections 2.4 - 2.5 on the sentential connectives and their logical force.)

5. BICONDITIONALIZATION, by means of 'IF AND ONLY IF'

We form biconditionals like

(e) You are bored IF AND ONLY IF you are reading.

Notice that a BICONDITIONAL SENTENCE like (e) above, is, in effect, a CONJUNCTION of two CONDITIONAL sentences (f) and (g), as follows:

(f) You are bored IF you are reading

AND

(g) You are bored ONLY IF you are reading.

The logical force or meaning of the biconditional is clearly different from the meaning of either conditional even though the component sentences of each are the same. The logical force or meaning of (f) is also different from that of (g). The logical force of (f) is the same as (a) above. And the logical force of (g) is the same as (b) above. How and why this is the case will be explained in section 2.4. But you might want to try to reason it out for yourself with the following examples, having the same logical forms as examples (a), (f), (b) and (g), respectively.

(a') IF it's raining, THEN streets are wet.

(f') The streets are wet IF it's raining.

(b') IF the streets are wet, THEN it's raining.

(g') The streets are wet ONLY IF it's raining.

Note that (a') and (f') are true; whereas, (b') and (g') are false. Thus, the way in which complex sentences like (a'), (b'), (f'), (g') are constructed out of simpler sentences (like 'It's raining,' 'The streets are wet') using sentential connectives like 'IF' or 'ONLY IF' can make a difference to whether the resulting complex sentences are true or false. Notice that (h) has the same LOGICAL FORCE (the same Logical Meaning) as (a'), and, of course, both are true:
(a') IF it's raining, THEN the streets are wet

(h) It's raining ONLY IF the streets are wet.

Because sentences like (a') and (h) have the same logical meaning, it is convenient to be able to symbolize their logical form in the same way. Sentences that look different in ordinary language can have the same logical force and the same underlying logical form. So it's useful to be able to depict this fact by symbolizing them in the same way. For example:

(a') IF it's raining, THEN streets are wet.  \((a'') R \Rightarrow W\)

(h) It's raining ONLY IF the streets are wet.  \((h') R \Rightarrow W\)

(b') IF the streets are wet, THEN it's raining.  \((b'') W \Rightarrow R\)

(g') The streets are wet ONLY IF it's raining.  \((g'') W \Rightarrow R\)

Symbolization exhibits the fact that the logical force and underlying logical form of sentences (a') and (h) are the same. As are those of (b') and (g').

2.3. Symbolizing Sentential Logical Form

For purposes of depicting the LOGICAL FORM of sentences in sentential logic (i.e., depicting the crucial LOGICAL CONNECTIONS between the component sentences combined by SENTENTIAL CONNECTIVES to form more complex sentences), it will be convenient to resort to the following conventions.

We will define an ATOMIC SENTENCE as a sentence in which no sentential connectives occur as logical operators. Thus, the following are all atomic sentences:

- It's raining.
- You are reading.
- 'Not' is a sentential connective.
- All the streets are wet.

We will define a MOLECULAR SENTENCE as a sentence in which at least one sentential connective occurs as a logical operator. Thus, the following are molecular sentences:

- It's NOT raining.
- You are bored IF you are reading.
'Not' occurs in this sentence twice AND it is NOT a logical operator the first time it occurs.

IF the word 'not' is merely mentioned in a sentence AND it is NOT used as a sentential connective to negate anything, THEN 'not' does not occur in that sentence as a logical operator.

The sentential connectives 'not,' 'and,' 'but,' 'or,' 'if,' 'only if,' etc. are logical operators with which we construct molecular sentences out of atomic sentences AND they are logical operators which are studied in sentential logic BUT they are NOT the only logical operators we will study SINCE there are other logical terms (like 'all,' 'none,' 'some') that are important, which are studied in what's called quantificational logic.

Sentential logic AND quantificational logic are both concerned with the deductive validity of arguments BUT they are NOT both concerned with the same logical operators, SINCE sentential logic studies the logical terms that operate as sentential connectives AND quantificational logic studies logical terms that operate as quantifiers.

From these examples you can see that there are a variety of terms that operate as sentential connectives besides 'not,' 'and,' 'or,' and 'if.' 'But,' 'only if,' 'since' and a host of others operate as sentential connectives as well. Nonetheless, there are still only five basic logical operations performed on sentences that are important in sentential logic. These are the five ways in which MOLECULAR sentences can be constructed out of ATOMIC sentences: NEGATION, CONJUNCTION, DISJUNCTION, CONDITIONALIZATION, BICONDITIONALIZATION.

When a sentential connective is used to perform one of these logical operations, it is used as a logical operator. When it is merely mentioned (between quotation marks) it is not used as a logical operator.

The use of any one of the vast variety of connective expressions in ordinary language can be reduced to one of the five basic operations studied in sentential logic. It is the rules governing these LOGICAL OPERATIONS that it is important to understand in sentential logic: The ways in which MOLECULAR sentences are constructed by these operations and then combined into ARGUMENTS is what determines the VALIDITY or INVALIDITY of those arguments in sentential logic.

In sentential logic, the most basic units of CONTENT, the most basic elements of arguments are ATOMIC SENTENCES. What determines the VALIDITY of arguments is how the premises
and conclusion of arguments are constructed out of ATOMIC SENTENCES by means of any of the five kinds of SENTENTIAL CONNECTIVE. SENTENTIAL CONNECTIVES are the logical mortar used to build MOLECULAR SENTENCES out of ATOMIC SENTENCES. The sentential LOGICAL FORM of a sentence is determined by the way its ATOMIC component sentences are combined using sentential connectives. The logical form of an ARGUMENT is determined by the logical form of the SENTENCES that are its premises and conclusion.

In sentential logic, we can represent the LOGICAL FORM of a sentence as follows. We will let capital letters like P, Q, R, etc. stand for SENTENCES. These letters are called SENTENTIAL VARIABLES. (Their function is similar to that of the variables x, y, z, etc. in algebra, which stand for numbers). We let certain symbols stand for the five logical operations of negation, conjunction, disjunction, conditionalization, and biconditionalization. These symbols are called LOGICAL OPERATORS: They represent the logical operations that we perform on sentences when we negate or connect them by means of any of the five kinds of SENTENTIAL CONNECTIVES. (The function of these symbols is similar to that of mathematical symbols like '×', '=' ', ' etc., which represent various mathematical operations that we perform on numbers.) The symbols we will use for the five basic logical operations that we perform on sentences in sentential logic are:

1. - (the minus sign) for NEGATION
2. & (the ampersand) for CONJUNCTION
3. v (a 'v' for the Latin 'vel' for 'or') for DISJUNCTION
4. => (an arrow) for CONDITIONALIZATION
5. <=> (a two-way arrow) for BICONDITIONALIZATION

Different sentential connections in English (e.g., 'and,' 'but') are represented by the same logical operator (e.g., '&') when they represent the same logical operation (e.g., conjunction). With a single symbol to represent each of the basic logical operations we can conveniently represent the logical force of various connective expressions used in ordinary language. For example:
(a) It's raining AND it's sunny. (a') It's raining BUT it's sunny.
(b) IF it's raining, THEN it's NOT sunny.
(b') It's raining ONLY IF it's NOT sunny.

(a'') R & S
(b'') R & S
(b'') R => -S
(b'') R => -S

The above symbolizations readily represent the fact that the same logical operations are being performed in sentences (a) and (a'); likewise in sentences (b) and (b'); despite the fact that different conjunctive or conditionalizing expressions are used in each pair of examples. The situation here is again similar to that in mathematics where different expressions in ordinary language are used to denote one and the same mathematical operation; for example: 'Two plus two makes four' and 'Two and two equals four' are both rendered as '2 + 2 = 4.' In symbolically depicting the logical form of sentences (and the premises and conclusions of arguments) we will follow these conventions:

1. Assign a sentential variable to each and every distinct atomic sentence. E.g., the sentence 'It's not raining' should be depicted as '-P,' where P stands for the atomic sentence 'It's raining' (even though there's nothing logically incorrect with letting sentential variables stand for molecular sentences like 'It's not raining.') This assures that all of the relevant sentential logical structure of any sentence or argument is explicitly depicted.

2. Be sure to assign the same sentential variable to the same atomic sentence wherever that sentence occurs in a given argument. E.g., the argument 'It's raining only if the streets are wet. The streets aren't wet. Therefore, it's not raining' should be depicted as follows, where P stands for 'It's raining' and Q stands for 'The streets are wet':

\[
\begin{align*}
(1) & \quad P \Rightarrow Q \\
(2) & \quad \neg Q \\
(3) & \quad \neg P
\end{align*}
\]

3. Separate the conclusion of an argument from the premises or previous lines of an argument by a line (as in the example above) to indicate which sentence form represents the conclusion.

4. Number the lines representing the premises and conclusion of an argument, as above, for ease of reference.

When we depict the logical form of a sentence (as above), the formal or symbolized representation is called a SENTENCE FORM. Thus: (a') is a sentence form representing the
logical form of sentence (a); (a'') is the completed and standardized symbolization of that sentence form, where we let P stand for 'It's raining' and Q stand for 'The streets are wet'; we have, for example, the following:

(a) The streets are wet IF it's raining.

(a') Q IF P

(a'') P => Q

Examples (a') and (A'') represent two 'levels' of sentence form, partially symbolized and fully symbolized, respectively. Any number of sentences can have the sentence (A') symbolized by (A''). A SENTENCE FORM in sentential logic is just a schema or formula that can represent the logical form of any number of sentences, which, like the above, represent the same logical operation.

We give an INTERPRETATION of a sentence form when we assign actual sentences to the sentential variables. For example, we can give an interpretation of the sentence form

If P, then Q (equivalent to (a'), above)

by letting P stand for 'The streets are wet' and letting Q stand for 'It's raining.' In this case, we have given an interpretation of the sentence form that is false. Sentence forms themselves are neither true nor false. Any sentence may be substituted for any sentential variable. Until some substitution or interpretation is given, the sentence form by itself refers to no proposition that we can discern to be true or false.

There are six kinds of sentence forms in sentential logic. Any sentence has one of the following basic forms. Any sentence is either

(1) An atomic sentence (containing no operative connective), represented by a single sentential variable: P

or else it is a molecular sentence of one of the following forms:

(2) A negation: -P

(3) A conjunction: P & Q

(4) A disjunction: P v Q

(5) A conditional: P => Q

(6) A biconditional: P <=> Q

Molecular sentences can contain more than one operative connective:

(b) If you study hard and pay attention, you will
Sentence (b) has the form: (b') If P and Q, R.
It is symbolized: (b'')

(P & Q) => R

Sentence (b) is, at bottom, a CONDITIONAL whose 'IF'-clause is CONJUNCTION. Notice that in the English sentence itself and in its schematization (b''), this is intuitively clear. In the symbolization of (b) we use PARENTHESES to show the same effect. Were we to symbolize (b) without parentheses, as follows:

P & Q => R

does not mean the same thing. It would mean (b') if interpreted as a CONDITIONAL in which the IF-clause is a CONJUNCTION. This sentence form would be ambiguous and might be read in either of two ways:

1. As the DISJUNCTION of an atomic sentence (P) and a conditional (Q => R):

   P & (Q => R)

2. Or as a CONDITIONAL whose 'IF'-clause is a conjunction:

   (P & Q) => R

In symbolizing sentences we will sometimes need to use parentheses to indicate unambiguously which of two or more connectives is the MAJOR CONNECTIVE of the sentence. A few examples will serve to show how parentheses are used to disambiguate sentence forms and make clear which of the basic five molecular forms a sentence has:

P & Q v R might be read as either of the following:

(P & Q) v R: the DISJUNCTION of a conjunction (P & Q) with R

P & (Q v R): the CONJUNCTION of P and a conjunction (Q v R)

P & Q v R => S might be read as any of the following:

P & (Q v (R => S)): a conjunction

(P & Q) v (R => S): a disjunction

((P & Q) v R) => S: a conditional, with a disjunctional IF-clause

(P & (Q v R)) => S: a conditional, with a conjunctive IF-clause

Notice that PARENTHESES must be BALANCED; that is, counting out from the innermost parenthesized unit, the number of left-handed parentheses '(' must be equal to the number of right-handed parentheses ')'; for example:
Note that the sentence form

\[-P \& Q\]

is always to be read as the CONJUNCTION of the negation \((-P)\)

with Q, rather than as the negation of a conjunction:

\[-(P \& Q)\]

All molecular sentence forms which contain two or more connectives of the same kind, for example

- \(P \& Q \& R\)
- \(P \vee Q \vee R\)
- \(P \Rightarrow Q \Rightarrow R\)

must also be disambiguated to indicate which of the connectives is the major one, thus:

- \((P \& Q) \& R\) or \(P \& (Q \& R)\)
- \((P \vee Q) \vee R\) or \(P \vee (Q \vee R)\)
- \((P \Rightarrow Q) \Rightarrow R\) or \(P \Rightarrow (Q \Rightarrow R)\)

(You will see what difference these different groupings make later in the computer programs. Logically speaking, which grouping is chosen is a matter of indifference.)

Unambiguous sentential formulae (sentence forms) that employ only the legal symbols and contain only balanced parentheses are called WELL FORMED FORMULAE or WFF’s. When symbolically depicting the logical form of molecular sentences:

1. Correctly identify the MAJOR CONNECTIVE that determines the basic form of the sentence (and parse the sentence accordingly with parentheses where needed)

2. Assign sentential variables to each atomic sentence.

3. Depict all the operative connectives in the sentence by the appropriate symbols.

4. Be sure that your symbolized formula is WELL FORMED (with parentheses BALANCED and only the legal symbols for connectives)

The procedure for depicting the logical form of sentences symbolically will become second-nature after a bit of practice with the computer lessons on the sentential connectives and the computer tutorials on symbolization in the SYMBOL program.
The purpose of learning to symbolize sentences and sentential connectives is to be able better to perceive the LOGICAL FORM of arguments and to determine their VALIDITY or INVALIDITY. In particular, you will learn better to understand the LOGICAL FORCE and meaning of the variety of conditionalizing expressions ('if,' 'only if,' 'unless,' etc.) that play a crucial role in the construction of arguments.

The next section begins to apply the concept of LOGICAL FORM to the assessment of VALIDITY.
3. What the TRUTH Program Helps Teach

The TRUTH program provides practice in analyzing the 'logical' or, strictly speaking, truth-functional force of a variety of sentential connectives that occur in English. Its exercises rehearse you in the truth-functional analysis of sentential connectives that is crucial to proving, or understanding the proof of the validity of argument forms. This chapter outlines the truth-functional analysis of sentential connectives and its application in the proof of formal, deductive validity. It thereby provides the bare context necessary to understand the utility of the TRUTH program, and explains the notational and translation conventions employed by the program. Where these conventions are controversial (as in the case of the symbolization and truth-functional interpretation of unless), explanation or rationale is provided for the way in which the issue is legislated in the TRUTH (and SYMBOL) program.

3.1. Can We Prove Validity?

You know already that the validity or invalidity of an argument depends on the argument's logical form. To review examples of this fact and the way in which logical form is determined and depicted in sentential logic, see sections 2.1 and 2.2. What follows are definitions and a review of the question of how we can prove whether or not an argument is valid.

The allegation that an argument is valid or invalid is a claim about the argument's logical form. To be precise, when talking about validity or invalidity, we are, at root, talking about ARGUMENT FORMS—it is the logical form attributed to an argument that is valid or invalid.

For any given ARGUMENT FORM, there are an infinite number of possible arguments that have that form. So, an allegation that an argument form is valid or invalid is, in effect, a claim about every possible argument that could have that form: an allegation that an argument form is valid or invalid is a statement about the infinite number of possible arguments that have that form.

If an ARGUMENT FORM is valid, then EVERY of the possible infinity of arguments that have that form is valid. If an ARGUMENT FORM is invalid, then EVERY of the possible infinity of arguments whose relevant form is the same is likewise invalid. Do you see why a statement about validity, a statement about an ARGUMENT FORM, is a statement about an
INFINITE number of possible arguments which have that form?

Consider the now familiar argument form (called Modus Ponens):

(MP) (1) If \( P \), then \( Q \)  
\( P \rightarrow Q \)
(2) \( P \)  
(3) Therefore, \( Q \)  
\( Q \)

Other arguments that have this same form are:

(MP-1) (1) If you're in New York City, then you're in New York

(2) You're in New York City

(3) So, you're in New York

(MP-2) (1) If Nork is in Tolway, then Nork is in Orslik

(2) Nork is in Tolway

(3) So, Nork is in Orslik

(MP-3) (1) If barricroves are mimsy, landaus sweat

(2) Barrigroves are mimsy

(3) So, landaus sweat

The argument below (MP-4) also has in effect the same relevant form as (MP), because, although it contains a piece of logical structure ('\( \not\)') that is not present in the other examples, the basic form of the argument consists in (1) a conditional (2) whose antecedent is affirmed and (3) whose consequent is then concluded. We traditionally call this argument form Modus Ponens (MP), a Latin term that is short for affirming the antecedent (2) of a conditional statement (1) in order to be able to affirm its consequent (3):

(MP-4) (1) If you are illiterate, you are not reading

(2) You are illiterate

(3) So, you are not reading

By giving an infinite number of interpretations to the sentential variables \( P \) and \( Q \) in the argument form (MP), we could produce an infinite number of arguments with that form. The
number of possible arguments having any given logical form is clearly infinite: between the meaningful and nonsensical sentences that we could substitute for any sentential variables in any argument form, the possibilities are surely infinite.

The fact that for any ARGUMENT FORM there are an infinite number of possible arguments having that form may be seen to pose a problem for determining whether any ARGUMENT FORM is valid. Consider the following various definitions of validity:

An ARGUMENT (A) is VALID if, and only if:

- NO argument with the same ARGUMENT FORM (A') has true premises but a false conclusion. Or, equivalently:

- its ARGUMENT FORM (A') is valid

An ARGUMENT FORM (A') is valid if, and only if:

- NO argument with the same form (A') has true premises but a false conclusion; or

- Every argument with the same form (A') that has true premises also has a true conclusion.

Notice that the allegation that either an argument or an argument form is valid is a statement about ALL POSSIBLE arguments having the same form. Recall that the number of possible arguments having any given form is potentially INFINITE.

This would seem to pose a problem for knowing or PROVING that any argument or argument form is indeed valid: How, after all, can we possibly examine EVERY POSSIBLE argument of a given form to see whether the ones with true premises also have true conclusions? This seems an endless, inconclusive and impossible task, given that the possibilities we would have to examine are infinite. How can we ever know that any argument or argument form is valid? Let's first consider one way to show that an argument form is invalid.

The Counter-Example Technique

For Proving Argument Forms Invalid

The allegation that an argument is valid is tantamount to a universal generalization (like 'All swans are white'), to the effect: All arguments of this form that have true premises also have a true conclusion. We know that to refute a universal generalization about all things of a kind (e.g., all swans, all arguments with a certain logical form), all we need to do is find
one case in point (e.g., a swan) that is an exception to the generalization (e.g., a case of a black swan, a swan that is not white).

So, to try to refute the allegation that an argument is valid, to show that an argument is not valid, we have the same procedure open to us: The allegation that an argument is valid is a generalization about ALL arguments with the same logical form; the allegation is that EVERY argument with the same logical form that has true premises also has a true conclusion. To refute this allegation, to show that an argument or ARGUMENT FORM is INVALID, we need to find an argument with the SAME LOGICAL FORM that has obviously true premises but an obviously false conclusion.
For example, the following argument form (DA) is invalid. This may not be obvious from argument (DA-1), whose premises and conclusion are all true (invalid arguments can ring with truth and, so, deceive). That the argument form (DA) is invalid is obvious from the second example (DA-2), which shows that an argument with the logical form (DA) can have true premises but a false conclusion: An invalid argument form can lead us from truth into falsehood. Consider:

\[
\text{(DA)} \quad \begin{align*}
(1) \ & \text{If } P, \text{ then } Q \\
(2) \ & \text{Not } P \\
(3) \ & \text{Therefore, not } Q
\end{align*} \quad \text{(DA')} \quad \begin{align*}
(1') \ & \text{ } P \implies Q \\
(2') \ & \text{ } -P \\
(3') \ & \text{ } -Q
\end{align*}
\]

\[
\text{(DA-1)} \quad \begin{align*}
(1) \ & \text{If you're in Moscow, you're in Russia} \\
(2) \ & \text{You're not in Moscow} \\
(3) \ & \text{Therefore you're not in Russia}
\end{align*}
\]

\[
\text{(DA-2)} \quad \begin{align*}
(1) \ & \text{If you're in Alaska, you're in North America (True)} \\
(2) \ & \text{You're not in Alaska (True)} \\
(3) \ & \text{Therefore you're not in North America (False!)}
\end{align*}
\]

This invalid argument form is called 'the fallacy of denying the antecedent' because on the basis of denying the antecedent (2) of a conditional statement (1), the conclusion of the argument (3) then denies the consequent of the conditional. This is logically fallacious: (3) does not follow from (1) and (2), as example (DA-2) clearly shows.
A similarly invalid argument form is 'the fallacy of affirming the consequent' where, on the basis of (2 affirming the consequent) of (1) a conditional, the argument then invalidly (3) affirms the antecedent:

\[
(\text{AC}) \quad (1) \text{ If } P, \text{ then } Q \\
(2) \quad Q \\
(3) \quad \therefore P
\]

\[
(\text{AC'}) \quad (1) \quad P \Rightarrow Q \\
(2) \quad Q \\
(3) \quad \therefore P
\]

(AC-1) 1) If you're on the moon, you're in our solar system (True)  
(2) You're in our solar system (True)  
(3) Therefore, you're on the moon (False!)

While example (AC-1) clearly proves that ARGUMENT FORM (AC) [and any argument whose relevant logical form is the same as (AC)] is invalid, invalid arguments of this form may deceive us with the ring of truth:

(AC-2) 1) If you're on earth, you're in our solar system  
(2) You're in our solar system (True)  
(3) Therefore, you're on earth

The logical form of (AC-2) is invalid [as proven by (AC-1)], even though its premises and conclusion all happen to be true: true though they all be, the conclusion (3) does not follow from premises (1) and (2).

The technique we just used to prove that a given ARGUMENT FORM is invalid is called the 'COUNTER-EXAMPLE TECHNIQUE.' This technique is familiar to you in the case of refuting (finding exceptions to) generalizations about all things of a certain kind. The generalization 'All swans are white' is disproven by the counter-evidence, the counter-instance or counter-example of a black swan. The generalization that all philosophers smoke pipes could be refuted by a counter-example: a case of a philosopher who did not smoke a pipe.
Likewise, the allegation or presumption that an argument's form is valid can be defeated by finding a counter-example to the proposition 'Every argument with the same form that has true premises also has a true conclusion':

A counter-example would be an argument with the same form that had true premises but a false conclusion—like (DA-2) and (AC-1) above.

Remember: The proposition that an argument or ARGUMENT FORM is valid is a UNIVERSAL GENERALIZATION to the effect that EVERY argument with the same form that has true premises also has a true conclusion. A COUNTER-EXAMPLE that would defeat the claim of validity would be any argument of the SAME FORM that had obviously TRUE PREMISES but an obviously FALSE CONCLUSION.

(Why, for purposes of PROVING the invalidity of an argument form, is it important that the counter-example have OBVIOUSLY true premises and an OBVIOUSLY false conclusion? Be sure you can answer this question, or discuss it with your instructor.)

So, as you see, we have a technique for proving that invalid argument forms are in fact invalid. But does this technique bring us any closer to being able to prove that any argument form is VALID? Not a bit.

Disproving a universal generalization is a good deal easier than proving one. To prove a universal generalization to be true, in effect, requires us to examine every case in the universe to which the generalization applies.

In the case of 'All swans are white' we would need to examine ALL swans to know that they all are white. Failure to find a counter-instance does not prove that a universal generalization is true; it only allows some probability that the generalization is true and only proves that we have failed to find any counter-evidence. In fact, the proposition 'All swans are white' is false, although it took quite some time for the western world to discover this fact (by finding counter-examples, black swans, in Australia).
The failure to find a counter-example to prove that an argument form is invalid does not prove that the argument form is valid. It only proves that we have failed to find otherwise. This failure may be accountable to a failure to search thoroughly enough, a failure in imagination, or the complexity of the argument form, or the fact that the argument form is after all valid!

Is the following argument form invalid?

\begin{align*}
P & \lor \lnot Q \\
Q & \lor \lnot R \\
R & \text{only if } (J \lor K) \\
K & \text{only if } M \\
M & \lor P \\
\end{align*}

Therefore, P

How long would you care to spend looking (in your imagination) for a counter-example to try to prove it invalid? Care to try?

It would be nice to have a straightforward decision procedure for determining the invalidity of argument forms. The counter-example technique can require a lot of work and imagination. Moreover, it can not provide us with any proof that an argument is VALID.

To prove that an argument or argument form is VALID, we would have to examine, in effect, EVERY argument of that form to see whether ALL the arguments with that form that had true premises also had true conclusions, such that NO argument of that form could lead from truth to falsehood.

Sentential logic provides a technique for doing just that, a decision procedure for, in effect, examining ALL the possibilities in order to determine whether any argument of a certain form can possibly have true premises and a false conclusion. So far as the validity or invalidity of arguments depends on how their premises and conclusions are constructed by means of SENTENTIAL CONNECTIVES, an analysis of the sentential connectives can provide a straightforward PROOF of validity or invalidity within this domain of logic.

To see how this proof technique works, we look next at what's called the truth-functional analysis of the five basic logical operations that we can perform on atomic sentences (when we
combine them into more complex propositions and arguments by use of sentential connectives).
3.2. Sentential Connectives and Truth-Functionality

You already, at least implicitly, know about truth-functionality from your commonsense understanding of the sentential connectives 'not' and 'and' (and their cognates, like 'hardly' and 'but,' etc.).

A sentential connective is TRUTH-FUNCTIONAL so far as the truth-value of any molecular sentence formed with it is a unique function of the truth-values of its component or atomic sentences.

For example, you know that the truth-value (truth or falsity) of the NEGATION of a sentence P is a unique function of the truth-value of P: If P is true, then its NEGATION (¬P) is false. If P is false, then its NEGATION (¬P) is true.

You know that the truth-value of a CONJUNCTION of two sentences (P & Q) is a unique function of the truth-values of its component atomic sentences P and Q:

(P & Q) says, in effect:

Both P AND Q are true; or
P is true AND Q is true.

So where both conjuncts are true, their conjunction is true; otherwise, the conjunction is false. This truth-functional rule for conjunction (or any truth-functional operation) can be represented schematically in what's called a TRUTH-TABLE. We take it that any sentence P is either true or else it's false—it has one of two possible truth-values. In tabular form, where 'T' stands for 'true' and 'F' for 'false':

<table>
<thead>
<tr>
<th>P</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P is either true,</td>
<td>or else it is false</td>
</tr>
</tbody>
</table>
Any two sentences P and Q will then have four possible combinations of truth-values; in tabular form, as follows:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>(1) where both are true</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>(2) where one is true and the other false</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>(3) where the one is false and the other true</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>(4) where both are false</td>
</tr>
</tbody>
</table>

The truth-functional rule for conjunction counts as conjunction (P & Q) true where both conjuncts are true [represented by line (1) below]; otherwise [where one or more conjuncts are false—lines (2)–(4) below], a conjunction is counted as false. This is, of course, nothing more than our common truth-functional sense. In tabular form:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P &amp; Q</th>
<th>The Rule:</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>Where both conjuncts true, the conjunction as a whole is true;</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>otherwise, it is false.</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

On the basis of the truth-functional analysis of the sentential connectives, we can prove the validity or invalidity of argument forms whose crucial logical components are sentential connectives.

For lessons on the truth-functional analysis of each of the basic sentential connectives, you are referred to the review lessons in the TRUTH program. For practice in analyzing the truth-functional meaning of the basic connectives, you are referred to the exercises in the TRUTH program.
3.2.1. Non-Truth-Functional Interpretations of Connectives

Truth-functional analysis does not capture everything that we may mean to say by use of sentential connectives, even in the case of the simple conjunction term 'and.'

Connectives like 'and,' which can be analyzed truth-functionally, can also bear other dimensions of meaning. For example, 'and' may carry a temporal or causal meaning, where it's not merely asserted by a conjunction (P & Q) that both are true, but where it's also asserted that the state of affairs described by P occurred before the state of affairs described by Q. For example, the following conjunctions can mean different things, and (1) can be true whereas (2) can be false, even though both conjuncts might be true in each case:

1. Sally got married and Sally got pregnant

2. Sally got pregnant and Sally got married

The order in which Sally did these things can be important to whether we count these conjunctions true or false: the conjunction term 'and' can be intended to convey not only that Sally did both things, but also that she did one before or because of the other. We can make these additional meanings of 'and' explicit as follows:

(1') Sally got married and then Sally got pregnant

(2') Sally got pregnant and because of this Sally got married

Note also that conjunctive terms like 'but,' 'however,' 'nevertheless' carry a certain sense of reservation and contrast not carried by the simple 'and'—a sense, like the temporal or causal sense of 'and,' that goes beyond the purely truth-functional analysis of CONJUNCTION. When we represent conjunction by the truth-functional operator '&,' we are representing the most basic sense of any conjunction of two sentences (P & Q), the bottom line, as it were, namely: BOTH conjuncts are asserted as true—never mind whatever other contrasts or (causal or temporal) relationships are imputed between the states of affairs that P and Q describe.
Conditional connectives perhaps best illustrate the variety of function that connectives can ordinarily enjoy in everyday usage. For example, statements of the form 'If P then Q' often refer to temporal or causal relations between states of affairs:

If there's lightning then there'll be thunder (afterwards)
When there's lightning, there's thunder
Where there's smoke, there's fire
If there's smoke, there's fire (as its cause)

These dimensions of meaning of ordinary conditional statements are clearly important in everyday life. In truth-functional logic we put them aside in order to concentrate on the 'bottom-line' interpretation of the various sentential connectives, namely, the truth-functional interpretation that allows us to account for the validity or invalidity of the most basic of argument forms.
The standard conjunctive expression is 'and.'

Its 'bottom-line,' truth-functional interpretation is represented by the truth-table for '&'

The conjunction of any two sentences, P and Q, symbolized 'P & Q,' asserts, at bottom: Both P AND Q are true

Other conjunctive expressions, which can give different colorations to conjunctions in ordinary language, also assert, as a matter of their 'bottom line' truth-functional interpretation, that both their conjuncts are true. For example,

I'm tired; however I will help you

asserts, however begrudgingly, that

It's true that I'm tired AND it's true that I will help you

A conjunction of P and Q, truth-functionally interpreted, differs in what it asserts as a matter of its 'bottom-line' (that is, in what it asserts about the truth-values of its component sentences) from the disjunction of P with Q:

An Inclusive Disjunction of P with Q, symbolized P v Q, asserts: either one disjunct or the other is true---at least one is true---or both are true. An inclusive disjunction does not insist that both its components are true (unlike a conjunction of P and Q, which asserts, in effect, that both P and Q are true).

An Exclusive Disjunction of P with Q, is really a conjunction of the inclusive disjunction (P v Q) and the negation -(P & Q), symbolized:

(P v Q) & -(P & Q)

[P or Q but not both P and Q]

An exclusive disjunction asserts, in effect: at least one disjunct is true, but not both of them. (Unlike an inclusive disjunction, an exclusive disjunction excludes the possibility of both disjuncts' being true. While a conjunction insists that both P and Q are true, an exclusive disjunction insists that it is not the case that both are true). A typical case of the exclusive use of 'or' would be:

You can have your cake, or you can eat it

[but you can't both have and eat it]
When we wish to avoid the ambiguity of 'or' in ordinary language, we can simply spell out what we mean by adding BUT NOT BOTH, as in:
You can either have a good time tonight or pass the exam tomorrow, but you can't do both.

There are a variety of conjunctive expressions in ordinary language that have the same 'bottom line' truth-functional force as 'and' in sentences of the form:
P and Q, symbolized: $P \& Q$
The following are all conjunctions with the same logical force

as 'P and Q' and all of them would be symbolized 'P & Q':

- $P$ but $Q$
- $P$, however $Q$
- Although $P$, $Q$
- $P$; nevertheless $Q$
- While $P$, $Q$
- $P$; $Q$
- $P$, yet $Q$
- $P$, albeit $Q$
- Whereas $P$, $Q$
- $P$, even if $Q$

Notice the last example: $P$, even if $Q$. This conjunctive expression contains an 'if' and to that extent looks like a conditional expression. But the following example of this form

I'm damn well going to finish the race even if I am tuckered out.

is clearly a conjunction by which I'm asserting both that I am tuckered out but that I'm going to finish the race nonetheless. This statement is not a conditional: it is not asserting that my being tuckered out is a condition (sufficient or otherwise) of my finishing the race; it does not mean that if I am tuckered out, then I will finish the race; nor that I am not tuckered out unless I'm going to finish the race. (The fact that I am now tuckered out is presumably in no way conditional upon either my determination to finish the race or the fact that I will finish it.)

The word 'if' by itself can serve as a conjunctive rather than as a conditionalizing expression,
as in:

It's desirable, if costly, to get the roof repaired

This statement, surely or probably, does not mean:

It's desirable to repair the roof IF it's costly to do so

Surely I don't mean to say that the costliness of the repair is a condition of its desirability. (If anything, the costliness by itself probably makes the repair undesirable. Who finds costliness desirable?) The statement is commonsensically interpreted as a conjunction:

It's desirable, although costly, to get the roof repaired

Just as some conditional-like expressions are sometimes conjunctive in their function, some conjunctive-like expressions can be conditional in their function. For example:

'Spare the rod and spoil the child'

presumably means:

If you spare the rod, then you will spoil the child

Another example from your text: You were given the following case of an invalid argument:

(1) If you are illiterate, you are not reading
But (2) You are not illiterate

Therefore (3) You are reading

Then it was observed:

Close your eyes and the conclusion (3) is false while the premises (1) and (2) remain true.

This statement could, and probably should, be logically parsed as a conjunction whose left conjunct is a conditional, as follows:

If/when you close your eyes, then the conclusion is false, and, even though you close your eyes, the premises remain true.

Here your closing your eyes is a sufficient condition for making the conclusion false: the falsity of the conclusion is conditional in some way on your closing your eyes (or otherwise paying it no notice). Here's another example:

Take a bad attitude and you'll surely fail

This statement probably means:

If you have a bad attitude, then you'll fail
not that both propositions are already true, i.e. that:
You (already) have a bad attitude and you will fail

As a piece of advice, the statement can be taken to contend that there is some conditional connection between your attitude and your success or failure, and that failure will follow upon your taking a certain attitude (to which you may not have yet succumbed). Far from asserting that you do in fact have a bad attitude, the statement may be meant to warn you against taking one.

3.2.3. Conditionals and Unless Expressions

The standard conditional expression is 'if' or 'if—then—'.

Its truth-functional interpretation is represented by the truth-table for the arrow '⇒'

A standard conditional of the form 'IF P, then Q.' truth-functionally construed and symbolized 'P ⇒ Q' asserts, variously and in effect:

If P is true, then Q is true
P's being true is sufficient for Q's being true
P is true only if Q is true
Q's being true is necessary for P's being true.
P is not true unless Q is true

Note that certain other expressions in ordinary language can assume conditional force, equivalent to 'if.' Sentences of the following forms can all have the same truth-functional or logical force, symbolized P ⇒ Q:

If P, Q
Q, provided that P
When P, Q
Q, in case P
Q, in the event that P
Notice that 'if--then--,' 'only if--' and 'not--unless--' expressions can be used to assert the same truth-functional or logical relation between the antecedent and consequent of a conditional:

<table>
<thead>
<tr>
<th>ANTECEDENT</th>
<th>=&gt;</th>
<th>CONSEQUENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>=&gt;</td>
<td>C</td>
</tr>
<tr>
<td>IF A</td>
<td>THEN</td>
<td>C</td>
</tr>
<tr>
<td>A</td>
<td>ONLY IF</td>
<td>C</td>
</tr>
<tr>
<td>NOT A</td>
<td>UNLESS</td>
<td>C</td>
</tr>
</tbody>
</table>

For now think about the translational key as a step-by-step translation of UNLESS expressions into a standard ONLY-IF conditional expression. Be sure to reason this procedure through with examples (e.g., 'It's NOT raining UNLESS it's wet': 'It's raining ONLY IF it's wet'), so that you understand intuitively the implicit negation operation involved, as follows:

- **STEP (1):** Think of the 'unless'-clause as the consequent, like the 'then'- or 'only if'- clause in conditionals: so, 'unless' is symbolized by the arrow '=>.' REFER TO THE SCHEMA PROVIDED ABOVE

- **STEP (2):** The antecedent of the 'unless' statement (the clause preceding 'unless') is negated.

- **STEP (3):** In 'not-unless' expressions, this is tantamount to eliminating the 'not' in 'not-unless' by double negation ('Not not P only if Q' is equivalent to 'P only if Q').
Notice that the translation of the 'if—then—' expression into an equivalent 'unless' expression entails the introduction of a NEGATION. Certain negation operations are implicit in conditionals. Consider the following series of logically equivalent expressions:

(1) If it's raining, then it's wet: \( R \Rightarrow W \)
(2) It's raining only if it's wet: \( R \Rightarrow W \)
(3) If it's not wet, then it's not raining: \( -W \Rightarrow -R \)

To assert a statement of the form:

(1) If P is true, then Q is true
or, equivalently:

(2') P is true only if Q is true

is to assert (implicity):

(3') If Q is not true, then P is not true.

Given the logical equivalence between conditionals and disjunctions stated by the Implication Rule (see ARGUE rule set in the Appendix to the ARGUE manual), we can translate 'unless' expressions either into a standard conditional or into a standard disjunction.

If you can understand the transformation of 'unless' expressions into only if conditionals, you can then see why 'unless' expressions can also be treated as disjunctions (and you can prove their truth-functional equivalence with a truth-table).

Treating 'unless' as if it were 'or' raises the question of whether 'unless statements, like 'or statements, might not be ambiguous. But therein lies another tale. For now, you will be safe translating 'unless statements into equivalent 'only if' conditionals on the model given above.
Biconditionals: Necessary and Sufficient Conditions

The standard biconditional expression is really conjunctive:

'--if AND only if--.'

Its truth-functional meaning is represented by the truth-table for the double arrow '↔'.

The expression 'just in case' is conventionally used in philosophical literature as an abbreviation for 'in case and just in case': 'P just in case Q' means the same as 'P if and only if Q.'

A biconditional P if and only if Q is at bottom a conjunction of two conditionals: P if Q and P only if Q. A biconditional of the form P if and only if Q asserts, in effect: P is a necessary and sufficient condition for Q (and Q is a necessary and sufficient condition for P). That is: P is true when, and only when, Q is true; Q is true when, and only when, P is true. The biconditional asserts, in short: P and Q have the same truth-value—i.e., P is true whenever Q is true and false whenever Q is false. The biconditional is itself true when in fact P and Q have the same truth-value (i.e., when in fact P and Q are either both true or both false). The biconditional is false when P and Q have different truth-values (i.e., when one is true and the other is false). The truth-functional rule for biconditionals is summarized in the truth-table in the TRUTH program (see the list of commands in TRUTH or type HELP) and it is also summarized in section 2.5.5 below.

Notice that the biconditional statement:

It's raining if and only if it's wet

is a much stronger statement than the conditionals:

It's raining only if it's wet.

If it's raining, then it's wet.

The above conditionals are true, whereas the above biconditional is false: Can you explain why in this case the biconditional is false? (Analyze it as the conjunction of two conditionals.)
Biconditionals are useful where we want explicitly to lay down both necessary and jointly sufficient conditions for some proposition's being true. For example, the principle

Something has a right to life IF AND ONLY IF . . .

would assert (in place of the elipsis' . . .') the conditions

on which something would be granted a right to life, each and every one of which purportedly must be satisfied for something to have a right to life. Have you any notion of what those necessary and jointly sufficient conditions conceivably might be? (You might try constructing such a principle.) The following principle is arguably false. Can you explain why?

Something has a right to life if, and only if, it is alive.

Do we wish to say that viruses have such a right? Would potentially fatal viruses not then have a right to life if merely being alive were a sufficient condition for having this right? (On the other hand, being alive seems a plausible necessary condition for having a right to life. So, one might argue that the biconditional is false since one of its conjuncts is false, namely, the conditional:

Something has a right to life if it is alive

Something has a right to life only if it is alive
3.2.4. The Story of *Unless* Continued

We would like to have a reliable rule of thumb for interpreting 'UNLESS' expressions — unless, of course, we don't care to be clear about exactly what to expect when people say things like the following:

UNLESS you fail the final, you will pass the course

What might this mean? Here are four possibilities:

1. You will NOT pass the course ONLY IF you fail the exam: $-P \Rightarrow F$
2. IF you fail the final, THEN you will NOT pass the course: $F \Rightarrow -P$
3. You will NOT pass IF AND ONLY IF you fail the final: $-P \iff F$
4. Either you will fail the final or you'll pass the course: $F \lor P$

Presumably, the conditions—necessary or sufficient—for passing the course are more clearly laid out in the conditional sentences 1–3. Because 'unless' statements so often imply that certain conditions are necessary or sufficient for some state of affairs to come about, we will translate them into some expressly conditional form. Notice that by the implication and transposition rules the disjunctive sentence 4 is equivalent to the conditional sentence 1, as follows:

$F \lor P : : -F \Rightarrow P : : -P \Rightarrow F$

Translating 'unless' as 'or' is easy, and makes for ease in symbolization, because no 'implicit' negations are involved as in the cases of sentences 1 through 3. But, since (a) translating 'unless' as 'or' does not often result in a clear statement of the conditional import of the 'unless' expression and since (b) not all 'unless' expressions are suitably rendered in the particular form of sentence 1 (logically equivalent to sentence 4), we will not translate 'unless' as 'or.' We will worry instead about which conditional form best captures the conditional import of 'unless' expressions.

Now, a good rule of thumb is to translate 'unless' expressions into the weaker conditional form of sentence 1 above, as follows:

(i) 'UNLESS' is replaced by 'ONLY IF' -- and thus

the 'UNLESS' clause becomes the CONSEQUENT of a conditional

(ii) The other clause is NEGATED

and becomes the ANTECEDENT of this conditional

THUS:
UNLESS you fail the final, you will pass the course

ONLY IF you fail the final, will you NOT pass the course

You will NOT pass the course ONLY IF you fail the final: \(-P \Rightarrow F\)

We say this is a 'weaker' interpretation of the conditional import of the 'unless' expression than sentence 2 above, because sentence 1 posits failing the final only as a necessary condition for not passing the course, and thus allows the possibility that you will still pass the course even in the event that you fail the final. Consider:

1. You will NOT pass the course ONLY IF you fail the final: \(-P \Rightarrow F\)
2. You will NOT pass the course IF you fail the final: \(F \Rightarrow -P\)

Suppose you were to fail the final: it would follow from 2 that you would not pass the course. But this would not follow from 1: here you still have a chance to pass the course anyway.

In general, given a statement of the form:

\[ P \text{ unless } Q \]

it can be crucial to know how to take its conditional import:

1. Not \(P\) only if \(Q\)
   - posits \(Q\) as a merely necessary condition for the negation of \(P\)

2. If \(Q\), then not \(P\)
   - posits \(Q\) as a sufficient condition for the negation of \(P\)

Consider, for example, where someone says to you:

I'll kill you UNLESS you hand over your money

It would be nice to know that the conditional import of this statement were the 'stronger':

IF you hand over your money, I will NOT kill you

rather than the 'weaker':

I will NOT kill you ONLY IF you hand over your money

It's always nice to be sure of what one's getting for one's money, especially when one's life is at stake.

Granted, interpreting 'unless' expressions is not likely to be a life-and-death matter. But how are we to know which interpretation to draw in any case?

We adopt the general rule of thumb that 'unless' be rendered in the weaker sense illustrated above if only because it the safer interpretation in many or most pragmatic contexts. If it is
not obvious in any given context what the conditional import of 'unless' is meant to be, we can ask for clarification: 'Do you mean that IF I hand over my money, then you WON'T kill me?'

Where there is no one to ask our rule of thumb may be safe more often than not — unless there are clear independent reasons for departing from it. Sometimes a little reflection will show that 'unless' is better rendered one way rather than the other. Consider the following cases:

(A) It's not raining unless the streets are wet

(B) It's raining unless the streets are dry

Consider now the alternative interpretations:

A-1. It's NOT not raining ONLY IF the streets are wet

\[ R \implies W \]

A-2. IF the streets are wet, then it's NOT not raining

\[ W \implies R \]

B-1. It's NOT raining ONLY IF the streets are dry

\[ \neg R \implies D \]

B-2. IF the streets are dry, then it's NOT raining

\[ D \implies \neg R \]

In these cases a little reflection will show that, if we want to interpret 'unless' in a way that will be sure to be in accord with what is true, our rule of thumb does not apply to sentence B, although it just as clearly does apply to sentence A: in general, it's true that it's raining ONLY IF the streets are wet; but it is not always true that IF it's NOT raining, then the streets are dry (because it may be that the streets are wet from a recent rain, or dew, or a street cleaner and that it is not presently raining).

Thus: the 'weaker' conditional interpretation of sentence A seems decidedly more plausible than the stronger; but the stronger interpretation of sentence B seems just as decidedly more plausible than the weaker.

You will encounter similar cases where the stronger interpretation of 'unless' seems called for in the context of what you know to be the case. In such cases you should be guided by your commonsense, linguistic intuitions and reflection. It's at least useful to be aware of the three
alternative interpretations of 'unless' for purposes of this kind of critical reflection on the conditional import of whatever is said.

We can now amend our rule of thumb for interpreting 'unless' expressions: Render sentences of the form 'P unless Q' as 'Not P only if Q' UNLESS a stronger interpretation is arguably indicated. Is this a good rule of thumb?

How do you think we should interpret the 'unless' in the statement of the rule?

Can you think of any cases where 'unless' should arguably be rendered using 'IF AND ONLY IF.' How would you write the rule for translating 'unless' into 'if and only if'? How would you translate a sentence of the form 'P unless Q' into a purportedly equivalent sentence form using 'if and only if'?

A good rule of thumb is to translate 'unless' expressions using 'only if.' Why?

This is the rule we shall follow in the SYMBOL and TRUTH programs when doing a truth-functional analysis of sentence schemas containing unless. The TRUTH and SYMBOL programs will exercise you in interpreting the logical force of unless in this way. Once you have facility in handling unless on this interpretation, it will be easier to sort out the other interpretations in a precise way.
3.2.5. Example and Note on the Relevance of Formal Logic: *Unless*

House Judiciary Committee Debate on "Unless"

This debate on the issue of impeachment proceedings against President Nixon took place in the morning session of the House Judiciary Committee on Friday, July 26, 1974; Peter W. Rodino (Chair) presiding. The numbers in brackets [ ] indicate each of eight different interpretations of Mr. McClory's original motion [•] — all of which hang on the logical interpretation of the key connective *unless*.

The questions for you are:

1. What are the formally/logically distinct options for Mr. McClory's motion: What are the logically exhaustive and distinct formulations of his motion, formulated without using *unless*?

2. Which of these logically exhaustive and distinct formulations does he intend?

3. Which of the formulations below are logically equivalent to what Mr. McClory intends?

Obviously, the first question to be settled is: What are the logically distinct possibilities for formulating Mr. McClory's motion without using *unless*? Formal logic can settle this question rather easily. The question of what Mr. McClory actually intended must, of course, be referred to him. But formal logic at least allows us to clearly and definitively sort out his options. See what you think:

The Transcript: An Injudicious Motion and Debate

Mr. McClory: I have a motion at the clerk's desk which I have distributed among the members, Mr. Chairman.

The Chairman: The clerk will read the motion.

The Clerk: Mr. McClory moves [•] to postpone for 10 days further consideration of whether sufficient grounds exist for the House of Representatives to exercise constitutional power of impeachment *unless* by 12 noon, EDT, on Saturday, July 27, 1974, the President *fails* to give his unequivocal assurance to produce forthwith all taped conversations subpoenaed by the committee which are to be made available to the district court pursuant to court order in *United States v. Mitchell*. ....

Mr. Latta: .... I just want to call (McClyor's) attention before we vote, to the wording of his
motion. You move to [*] postpone for 10 days unless the President fails to give his assurance to produce the tapes. So, [1] if he fails tomorrow, we get 10 days [postpone consideration of the grounds for impeachment]. [2] If he complies [does give assurance...], we do not [postpone]. The way you have drafted it, I would suggest that you correct your motion to say that [3] you get 10 days providing the President gives his unequivocal assurance to produce the tapes by tomorrow noon.

Mr. McClory: I think the motion is correctly worded, it has been thoughtfully drafted.

Mr. Latta: I suggest you rethink it. ....

Mr. Mann: Mr. Chairman, I think it is important that the committee vote on a resolution that properly expresses the intent of the gentleman from Illinois (Mr. McClory) and if he will examine his motion he will find [4] "fails to" needs to be stricken. . . .

Mr. McClory: If the gentleman will yield, the motion is correctly worded. It provides for [*] a postponement for 10 days unless the President fails tomorrow to give his assurance, so [5] there is no postponement for 10 days if the President fails to give the assurance, just [sic] 1 day. I think it is correctly drafted. I have had it drafted by counsel, and I was misled originally too, but it is correctly drafted. [*] There is a 10-day postponement unless the President fails to give assurance. [6] If he fails to give it, there is only a 24-hour or there is only a 23 1/2 hour day [sic].

Mr. Rangel: Mr. Chairman?

Mr. McClory: I think the members understand what they are voting on.

Mr. Dennis: Will the gentleman yield to me?

Mr. Rangel: Mr. Chairman—

Mr. Dennis: The gentleman yielded to me, Mr. Rangel. Excuse me. I know you did not realize that fact.

Mr. Rangel: No, I did not.

Mr. Dennis: He did not. I realize that. What Mr. Mann says and what Mr. Latta says is true, in my opinion. It would be much better drafted if you said [?] "provided that" or [?] "unless he does not", or something, but I think nevertheless, the gentleman from Illinois (Mr.
McClory) is correct, that although this is a very backhanded way of stating it, it does in fact state it because it says [7] he gets 10 days if he does not—well, it is a backhanded way of stating what the gentleman is trying to state. It could be improved but what he is doing is nevertheless there.

Mr. Mann: I guess we can settle for it as long as we all understand it, Mr. Chairman.

The Chairman: Will the gentleman yield?

Mr. Rangel: Mr. Chairman, I think this motion itself has provided sufficient delay and I move the question.

The Chairman: The question is on the motion of the gentleman from Illinois.

The Clerk: Mr. Chairman, 11 members have voted aye, 27 members have voted no.

The Chairman: And the motion is not agreed to.

Note on the Relevance of Formal Logic:

A House Judiciary Committee debate about possible impeachment proceedings against the President of the United States is obviously of tremendous importance. The transcript above demonstrates that the House Judiciary Committee deliberating then-President Nixon's possible impeachment was hopelessly crippled by its inability to agree on the logical meaning of unless.

At bottom, there are three and only three logically different interpretations that can be given to Mr. McClory's motion. Each of these three formally distinct interpretations has several logically equivalent expressions in English.

Formal logic can clarify what three logical options are possible for the interpretation of unless. It is convenient to symbolize these three possibilities, in order to clearly depict the formal logical differences among them. Mr. McClory could then chose which of the three options he truly intended.

If the committee members were better schooled in formal logic, they could then readily decide which of the several formulations bandied about in the debate above are, in point of logical fact, equivalent to McClory's intended motion.

Absent this understanding of formal logic, lawmakers are unable to conduct important debates either intelligently or intelligibly. This is just one example where a lack of formal logical
tools can cripple important public business and waste our tax money. Unfortunately, formal logical incompetence is not limited to our esteemed congress.
Logic Exercise on "Unless"

A. Where Logic May Mean Life-or-Death, Unless...
   Consider: [A*] I'll shoot you unless you hand over your money

   Circle the number of any of the following that is equivalent to [A*]:

   [I-a] I will not shoot you only if you hand over your money [I-b] if you do not hand over your money, I will shoot you

   [II-a] I will shoot you or you hand over your money [II-b] You hand over your money or else I'll shoot you

   [III-a] if you hand over your money, I will not shoot you [III-b] I will shoot you only if you do not hand over your money

   [IV] I will not shoot you if and only if you hand over your money

B. The Injudicious Judiciary Committee Debate

   The following is an elliptical version of Mr. McClory's original motion:

   [B*] The Committee postpones [...] further consideration of [...] impeachment] unless the President fails to give assurance ...

   The following are elliptical statements of the committee members' versions of the original motion [B*]. Circle the number [1] ... [7] of any that is, as a matter of logical fact, equivalent to [B*].

   [1] if President fails to give assurance, Committee postpones
   [2] if President gives assurance, Committee does not postpone
   [3] Committee postpones providing President gives assurance
   [4] Committee postpones unless President gives assurance
   [5] Committee does not postpone if President fails to give assurance
   [6] if President fails to give assurance, Committee does not postpone
   [7] Committee postpones if President does not give assurance
Consider: How would you go about settling the committee members debate and decisively resolving any doubts about what the motion really meant? Is this not a straightforward and objectively decidable matter?
Answers for Logic Exercise on “Unless”

A. [A-\*] I’ll shoot you unless you hand over your money

Can be reformulated and symbolized in the following ways.

Let:  

\[ S = \text{I will shoot you} \]  
\[ H = \text{You hand over your money} \]

\[ \text{[I-a]} \quad \text{I will not shoot you only if you hand over your money} \quad -S \Rightarrow H \]

\[ \text{[I-b]} \quad \text{if you do not hand over your money, I will shoot you} \quad -H \Rightarrow S \]

[I-a] and [I-b] are logically equivalent (by the Transposition rule).

\[ \text{[II-a]} \quad \text{I will shoot you or you hand over your money} \quad S \lor H \]

\[ \text{[II-b]} \quad \text{You hand over your money or else I will shoot you} \quad H \lor S \]

[I-a] is equivalent to [II-a], [I-b] is equivalent to [II-b] (by Implication).

[II-a] and [II-b] are equivalent (by Commutation)

\[ \text{[III-a]} \quad \text{if you hand over your money, I will not shoot you} \quad H \Rightarrow -S \]

\[ \text{[III-b]} \quad \text{I will shoot you only if you do not hand over your money} \quad S \Rightarrow -H \]

[III-a] and [III-b] are equivalent (by Transposition).

[III-a] and [III-b] are not logically equivalent to [I-a] or [I-b].

\[ \text{[IV]} \quad \text{I will not shoot you if and only if you hand over your money} \quad -S \Leftrightarrow H \]

B. [B-\*] The Committee postpones unless the President fails to give assurance

Let:  

\[ P = \text{The Committee postpones} \]  
\[ G = \text{The President gives assurance} \]

The seven formulations of this motion are symbolized as follows:

\[ \text{[1]} \quad \text{If President fails to give assurance, Committee postpones} \quad -G \Rightarrow P \]

\[ \text{[2]} \quad \text{If President gives assurance, Committee does not postpone} \quad G \Rightarrow -P \]

\[ \text{[3]} \quad \text{Committee postpones providing President gives assurance} \quad G \Rightarrow P \]

\[ \text{[4]} \quad \text{Committee postpones unless President gives assurance} \quad \text{Ambiguous (like B-\*)}: \]

\[ \text{[I]} \quad -P \Rightarrow G : : -G \Rightarrow P \]

\[ \text{[II]} \quad P \lor G : : G \lor P \]

\[ \text{[III]} \quad G \Rightarrow -P : : P \Rightarrow -G \]

\[ \text{[IV]} \quad -P \Leftrightarrow G : : P \Leftrightarrow -G \]

Options 4 [I] - 4 [IV] are the same as options [I] - [IV] for A-\*.

But formulation [4] is not equivalent to the original motion B-\*.

\[ \text{[5]} \quad \text{Committee does not postpone if President fails to give assurance} \quad -G \Rightarrow -P \]

\[ \text{[6]} \quad \text{if President fails to give assurance, Committee does not postpone} \quad -G \Rightarrow -P \]

\[ \text{[7]} \quad \text{Committee postpones if President does not give assurance} \quad -G \Rightarrow P \]

So, which formulation(s) did McClory intend? Which of the above are equivalent?
3.3. The Truth-Functional Rules for Interpreting Connectives

3.3.1. Negations: Not

The TRUTH-VALUE (truth or falsity) of a DENIAL or NEGATION, for example,

You are NOT reading

depends upon, is a direct FUNCTION of the truth-value of its COMPONENT sentence

You are reading

Where its COMPONENT sentence is TRUE, as in the example above, the NEGATION is FALSE, and conversely. Commonsensically:

To DENY a TRUE statement is, in effect, to make a FALSE statement.

And, conversely, to DENY a FALSE statement is, in effect, to make a TRUE statement.

The logical form of a negation

Not R

is symbolized

-R

where '-' stands for NEGATION and 'R' stands for 'You are reading'

Then:

It is NOT the case that you are NOT reading

(a NEGATION of a negation) is symbolized:

--R

The general rule by which we decide the truth-value of the NEGATION of any arbitrary sentence P is:

Where P is TRUE, NOT P is FALSE; and

Where P is FALSE, NOT P is TRUE.

This rule states the TRUTH-CONDITIONS of NEGATION: the conditions under which a negation (Not P) is TRUE (i.e., where P is FALSE) and the conditions under which a negation (Not P) is FALSE (i.e., where P is TRUE).

The TRUTH-CONDITIONS for NEGATION -- for any TRUTH-FUNCTIONAL SENTENTIAL CONNECTIVE -- can be displayed briefly in a TRUTH-TABLE.
NEGATION is considered TRUTH-FUNCTIONAL because the truth-value of a negation (Not P) varies as a direct FUNCTION of the truth-value of its COMPONENT sentence (P), as the following truth-table will show.

Let 'T'='true' and 'F'='false'. Any sentence P has one of TWO POSSIBLE truth-values: P is either TRUE or else P is FALSE:

<table>
<thead>
<tr>
<th>P</th>
<th>Not P</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

(And the truth-value of the negation will vary according to our general rule.)

So, a truth-table summarizes ALL POSSIBLE truth conditions for NEGATION and it shows how NEGATION is TRUTH-FUNCTIONAL, that is, how the truth-value of a negation (¬P) varies as a direct FUNCTION of the truth-value of its COMPONENT (P):

<table>
<thead>
<tr>
<th>P</th>
<th>¬P</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

This truth-table is a summary of the rule for deciding the truth-value of negations.
3.3.2. Conjunctions: *And*

The logic of conjunction concerns the TRUTH-FUNCTIONALITY of 'AND', that is, it concerns how the truth-value of the CONJUNCTION of any two sentences

\[ P \text{ AND } Q \]

varies as a direct FUNCTION of the truth-values of its COMPONENTS

\[ P \]
\[ Q \]

Suppose we know that the following is true about any arbitrary sentence \( P \),

It is not the case that \( P \)

which, for convenience, we abbreviate as

\( \text{Not } P \)

and symbolize as

\( -P \)

where, equivalently, we could say that

\( P \) is false

What then do we know, what can we infer or deduce about a statement of the following form (where \( Q \) stands for any arbitrary sentence other than \( P \))?  

\[ P \text{ and } Q \]

Is it true or false?

What if we KNOW that \( Q \) is true? What would be the truth-value of the conjunction

\[ P \text{ and } Q \]

where, as we've already supposed, \( P \) is false?

GIVEN that at least one conjunct (\( P \)) is false, any conjunction of the form

\[ P \text{ and } Q \]

MUST be false. This is commonsense.

Only ONE conjunct's being true is not sufficient to count the conjunction as a whole true.

A conjunction of any two arbitrary sentences
P and Q asserts, in effect, the following:

'P' is true AND 'Q' is true

or, more emphatically,

BOTH 'P' AND 'Q' are true.

A conjunction of any two arbitrary sentences P and Q is counted as TRUE, commonsensically:

If, and ONLY if, BOTH component sentences (CONJUNCTS) are TRUE.

A conjunction is FALSE:

If AT LEAST ONE component sentence (CONJUNCT) is FALSE.

The rule for deciding the truth-value of any CONJUNCTION is, simply

The conjunction is true ONLY on condition that BOTH conjuncts are true; otherwise, it is false.

We can summarize the rule for deciding the truth-value of the conjunction of any two arbitrary sentences P and Q in a TRUTH-TABLE.

The truth-table for conjunction displays the TRUTH-CONDITIONS for conjunction: the conditions under which a conjunction is counted as true or false.

A truth-table summarizes ALL POSSIBLE COMBINATIONS of truth-values for the component sentences of a conjunction, as follows.

The POSSIBLE COMBINATIONS of truth-values for any TWO sentences, P and Q, are FOUR, as follows:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

The conjunctive 'AND' is considered a TRUTH-FUNCTIONAL connective because the truth-value of any CONJUNCTION is a direct FUNCTION of the truth-value of its COMPONENT sentences.

A truth-table shows how the truth-value of a conjunction varies as a direct function of the truth-values of its components, according to our simple rule:
The rule states -- and the above table displays -- the TRUTH-
FUNCTIONALITY of the sentential connective 'AND'.

You should be aware that there are several CONJUNCTION terms in English
besides 'AND'. The following are all equivalent:

P and Q
P but Q
Although P, Q
Q, however P
P, nevertheless Q

All are symbolized

P & Q

and assert, as their 'bottom line' logical 'cash value':

P is true and Q is true.
3.3.3. Disjunctions: *Or*

The use of the disjunctive connectives

```
  OR
  EITHER _ OR _
```

in ordinary language is AMBIGUOUS.

That is, a sentence of the form

```
P or Q
```

may mean either of the two following different things:

1. Either P is true, or Q is true --
   AT LEAST ONE is true --
   but NOT BOTH.

   This is called the EXCLUSIVE sense of 'or'.

2. Either P is true, or Q is true --
   AT LEAST ONE is true --
   OR BOTH are true.

   This is called the INCLUSION sense of 'or'.

We let the letter

```
v
```

stand for

```
or
```

in the INCLUSION sense.

We will understand disjunctions of the form

```
P or Q
```

in the INCLUSION sense, symbolized

```
P v Q
```

and meaning

```
Either P or Q OR BOTH.
```

When an EXCLUSIVE disjunction is intended (where the possibility of both alternatives' being true is meant to be excluded) we will SPELL OUT this exclusive sense explicitly, as follows:

```
P or Q [in the EXCLUSIVE sense]
```

means

```
Either P or Q BUT NOT BOTH.
```
So, we can spell out the sense of

EXCLUSIVE 'OR'

in terms of a combination of

INCLUSIVE 'OR'

CONJUNCTION ('and'/ 'but')

NEGATION ('not')

An EXCLUSIVE disjunction of any two arbitrary sentences P and Q, once spelled out fully, may be analyzed as

A CONJUNCTION of

an INCLUSIVE DISJUNCTION and the NEGATION of a conjunction:

\[(P \lor Q) \land \neg(P \land Q)\]

Either P or Q BUT Not both P and Q

A truth-table will show us under which of all the possible truth conditions inclusive and exclusive disjunction differ in truth-value.

Recall: We analyze the EXCLUSIVE DISJUNCTION of P and Q as the

CONJUNCTION: \[(P \lor Q) \land \neg(P \land Q)\]

P or Q but not both P and Q

consisting of the

INCLUSIVE DISJUNCTION: \(P \lor Q\)

and the

NEGATION: \(\neg(P \land Q)\)

denying the conjunction: \(P \land Q\)

Let's see how the truth-value of

the INCLUSIVE \(P \lor Q\)

and

the EXCLUSIVE \((P \lor Q) \land \neg(P \land Q)\)

vary as a function of the truth-values of their component sentences.

P & Q P v Q P & Q -(P & Q) -(P & Q)

---!---------!--------!--------!---------

(1) T ! T ! T ! F ! F
(2) T ! F ! T ! F ! T ! T
(3) F ! T ! T ! F ! T ! T
(4) F ! F ! F ! F ! T ! F

Lines (2), (3), and (4) represent the fact that INCLUSIVE and EXCLUSIVE disjunctions are true if AT LEAST ONE disjunct is true and false if both disjuncts are false.

Line (1) shows how INCLUSIVE and EXCLUSIVE disjunctions differ: INCLUSIVE allows both alternatives to be true. EXCLUSIVE disjunction excludes the possibility of both alternatives being true.

AGAIN: We will interpret disjunctions like 'P or Q' in the INCLUSIVE sense and count them as true whenever AT LEAST ONE disjunct is true and also when both disjuncts are true.

We symbolize the inclusive disjunction of P with Q as: P v Q

When we wish to posit an EXCLUSIVE disjunction between two statements where we wish to say that AT LEAST ONE but NOT BOTH are true, we will simply spell this intention or interpretation out in the following explicit formulation: P or Q but not both P and Q, which we symbolize: (P v Q) & -(P & Q) using the more elementary logical operations of inclusive disjunction ['v'], conjunction ['&'], and negation ['\(^{-}\)'].
3.3.4. Conditionals: The Big If

Conditionals are expressed variously as follows.

All the conditional expressions below are symbolized:  $P \Rightarrow Q$

- If $P$ then $Q$  ($P$ is a sufficient condition for $Q$)
- $Q$, if $P$
- $P$ only if $Q$  ($Q$ is a necessary condition for $P$)
- Not $P$ unless $Q$

In the standard form conditional:

If $P$ then $Q$

we call the 'if _' clause the ANTECEDENT.

We call the 'then _' clause the CONSEQUENT.

We will symbolize an 'if _ then _' conditional as follows

ANTECEDENT $\Rightarrow$ CONSEQUENT

The ANTECEDENT is the 'if _' clause. Notice that 'then' is not always stated:

$P$ if $Q$

is the logical equivalent of

If $Q$ then $P$

or

If $Q$, $P$.

which would be symbolized as follows:

$Q \Rightarrow P$

All conditionals are symbolized in this format

ANTECEDENT $\Rightarrow$ CONSEQUENT

regardless of the order in which the antecedent and consequent occur in English.  In 'P if Q', Q is the antecedent and 'P' is the consequent.

For any conditional of the form:

If $P$ then $Q$

symbolized:

$P \Rightarrow Q$
there are four possible combinations of truth-values for its
ANTECEDENT (in this case P) and CONSEQUENT (in this case Q):

\[ P \rightarrow Q \]

---!

(1) T ! T ! where BOTH antecedent and consequent are TRUE
(2) T ! F ! where the ANTECEDENT is TRUE but the CONSEQUENT FALSE
(3) F ! T ! where the ANTECEDENT is FALSE and the CONSEQUENT TRUE
(4) F ! F ! where BOTH antecedent and consequent are FALSE

But what in each case, (1)-(4), is the truth-value of the CONDITIONAL
P \( \rightarrow \) Q? Let's take it line by line.

Consider. A conditional of the form

\[ P \rightarrow Q \]

asserts, in effect

IF P is true, THEN Q is true.

When P IS true AND Q is also true, how do we count the conditional?

Well, P is true in this case.

So, the conditional 'speaks' truly if the consequent is also true.

In this case, the consequent, Q, IS also true. So the conditional
is true in this case:

\[ P ! Q ! P \rightarrow Q \]

(1) T ! T ! T

Suppose now P is TRUE but Q is FALSE.

What is the truth-value of the following conditional?

If P, then Q

It is FALSE, of course. Consider, again, what a conditional asserts:

The conditional

If P then Q

asserts that

\[ Q \] is true when \[ P \] is true.

So, where P IS true, but Q fails to be true,

the conditional assertion is assuredly false, as shown in line (2) below:
\[ P \land Q \rightarrow P \rightarrow Q \]

\[
\begin{array}{ccc}
(1) & T & T & T \\
(2) & T & F & F \\
\end{array}
\]

Consider the following concrete examples:

If I, Charlie Conditional, promise you

If Carter wins re-election, I'll eat my hat

and It's true that Carter wins, but I do NOT eat my hat

then is my promise to you not false?

If I say

If you get an A on the exam, you'll get an A in the course

and You A the exam but you do NOT get an A in the course

would you say I had told you a falsehood or gave you a false hope?

I rather imagine.

Thus far, the rule for conditionals is pretty much common sense.

In truth-tabular form:

\[
\begin{array}{ccc}
(1) & T & T & T \\
(2) & T & F & F \\
\end{array}
\]
But what of the two other possibilities?

\[ P \land Q \land P \Rightarrow Q \]

(3) \( F \land T \land \text{How are we to account the truth-value} \]

(4) \( F \land F \land \text{of the conditional in these cases?} \]

Consider, again, the analogy of a conditional promise.

I, Charlie Conditional, promise

If you A the exam, you'll A the course.

Suppose: (if it's possible)

You fail to A the exam but you get an A in the course anyway

Would you say I made a FALSE claim or promise?

I claimed that

You'd get a A in the course

IF you get an A on the exam

NOT that

You wouldn't get on A in the course anyway, even if you FAILED to get an A on the exam.

In fact, I made NO CLAIM WHATEVER about what would be the case in the event that you failed to A the exam; I did not claim that failure to A the exam would result in failure to A the course.

In the following case:

\[ P \land Q \land P \Rightarrow Q \]

(3) \( F \land T \land \text{How shall we count the conditional?} \]

It would be odd to count the conditional as false just because \( P \) is false.

Consider:

(i) The conditional really makes no claim about the case where \( P \) is false. It claims rather that WHERE \( P \) IS TRUE, \( Q \) is true.

(ii) The conditional does not claim that \( Q \) is false when \( P \) is false or that \( Q \) is true ONLY if \( P \) is true.

So, the conditional is NOT FALSE in this case.

Where I claim:

If you fail the course, I'll eat my hat

and
You do NOT fail the course

and yet

I eat my hat anyway

we could hardly say I made a FALSE claim.

I never said I would NOT eat my hat if you did NOT fail the course. I made NO CLAIM at all about what I would do if you did NOT fail the course.

So, my conditional claim

If you fail the course, I'll eat my hat

is NOT FALSE when the antecedent is false (You do NOT fail the exam) and the consequent is nonetheless true (I eat my hat).

A conditional, which in effect asserts

IF P is true, Q is true

is NOT a FALSE claim where P is false. Because it makes NO CLAIM WHATEVER about the case where P is false, it makes no FALSE claim where P is false.

By parity of reasoning, how do we decide the following case?

Suppose I claim:

If you pass the exam, you pass the course.

Suppose, also:

You do NOT pass the exam

and:

You do NOT pass the course.

Can I be accused of a FALSE claim, of lying to you?

I did not claim you WOULD pass the course even if you failed the exam; indeed, I said nothing at all about the case where you fail the exam.

So, it seems wrong to count my merely conditional 'IF-FY' claim as false.

In sum, conditionals with FALSE ANTECEDENTS are always counted as TRUE because, and in the sense that, they are NOT FALSE claims.

For this and other reasons the rule for conditionals is as follows:

\[
\begin{array}{ccc}
P & Q & P \rightarrow Q \\
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
\]

* A conditional is FALSE IF, and ONLY IF, it has a TRUE ANTECEDENT and a FALSE CONSEQUENT; otherwise, it is counted as true.

REMEMBER:
If P, then Q
P only if Q
Q, if P
Not P unless Q

are all symbolized:  \( P \rightarrow Q \)

Make note of this! They are all equivalent expressions. You need to master the translation of these expressions into STANDARD FORM.

For a discussion of how to translate 'UNLESS' expressions into a standard conditional form, see section 2.4.3 above.
3.3.5. Biconditionals: *if and only if*

A biconditional of the form

\[ P \text{ if and only if } Q \]

and symbolized

\[ P \iff Q \]

asserts, in effect, the following:

- \( P \) is true if but only if \( Q \) is true

and

- \( Q \) is true if but only if \( P \) is true

In other words, the biconditional asserts, in effect, that

- \( P \) has the same truth-value as \( Q \):
  - \( P \) is true IF \( Q \) is true

  AND

  - \( P \) is true ONLY IF \( Q \) is true (so, \( P \) is false if \( Q \) is false)

So, the truth-functional rule for biconditionals is simple:

A biconditional of the form \( P \iff Q \) is true

Whenever both \( P \) and \( Q \) have the same truth-value

(whenever both are true or both are false)

A biconditional \( P \iff Q \) is false

Whenever \( P \) and \( Q \) have different truth-values

(whenever one is true but the other is false)

This truth-functional rule for biconditionals is summarized in the following truth-table:
\[
\begin{array}{ccc}
P \leftrightarrow Q \\
\hline
\text{T} & \text{T} & \\ 
\text{T} & \text{F} & \\ 
\text{F} & \text{T} & \\ 
\text{F} & \text{F} & \\ 
\end{array}
\]

NOTICE that where both \( P \) and \( Q \) are false (and so have the SAME truth-value) the biconditional is counted as TRUE [on line (4) of the table above].

This is because a biconditional asserts that \( P \) is true if BUT ONLY if \( Q \) is true (that \( P \) is false if \( Q \) is false): It does NOT assert that either \( P \) or \( Q \) is actually true — only that IF one is true THEN the other is also true.
3.4. Proving Validity in Sentential Logic

You know that an argument form is valid if, and only if, it is impossible (e.g., impossible according to the truth-functional meanings of the sentential connectives) for any argument having that form to have true premises but a false conclusion.

Argument forms can be proven valid (or invalid) within the domain of truth-functional sentential logic by means of truth-tables: truth-tables allow us to examine all possible combinations of truth-values among the premises and conclusion of an argument form (truth-functionally construed), so that we can conclusively determine whether there is any case (any assignment of truth-values to the component atomic sentences of the argument form) in which the conclusion can be false while all the premises are true.

The truth-functional test of validity comes to this: if there is any interpretation, any assignment of truth-values to the atomic sentential variables of an argument form that renders the conclusion false and all the premises true, then the argument form in question is invalid. If there is no case, no possible assignment of truth-values to the component atomic sentences of the argument form in which all the premises are true but the conclusion false, then the argument form is valid.

3.4.1. The Truth-Tabular Proof of Validity

To show that an argument form is invalid as depicted in truth-functional sentential logic, we show that we can assign truth-values so as to make the conclusion false and all the premises true. If there is no possible truth-value assignment that both makes the conclusion false and all the premises true, then the argument form is valid: any argument having that form is valid because it is not possible that any argument of that form have all true premises and yet a false conclusion.
We wish to see if, by virtue of an argument's truth-functional logical form, it is possible for all the premises of the argument to be true while its conclusion is false. Once an argument's logical form is depicted in sentential logic we can survey all possible interpretations of that argument form by surveying all possible truth-value assignments to the atomic components of its premises and conclusion. Since we wish to know whether one particular pattern of truth-values is possible, we can narrow our search: we consider only those truth-value assignments which render the conclusion of the argument false.

By use of a truth-table we can summarize all the conditions under which the premises and conclusion of an argument are rendered either true or false. Consider: any sentence P is either true or false. For any two sentences P and Q there are four possible combinations of truth-values, namely, where:

\[
\begin{array}{c|c|c|c}
P & Q & & \\
\hline
T & T & Both are true. & \\
T & F & One is true and the other is false. & \\
F & T & The one is false and the other is true. & \\
F & F & Both are false. & \\
\end{array}
\]

(We know this fact about any two sentences without knowing anything about their content, their actual truth or falsity or the state of the world. Why is this so?) For any three sentences P, Q, and R there are eight possible combinations of truth-values:

\[
\begin{array}{c|c|c|c|c}
P & Q & R & Note: For any sentence, there are 2 possible truth-values (true or false). For any number of sentences N, the possible combinations of truth-values are \(2^N\). \\
\hline
T & T & T & For two sentences: \(2^2 = 4\). \\
T & T & F & \\
T & F & T & \\
T & F & F & \\
F & T & T & \\
F & T & F & \\
F & F & T & For three sentences: \(2^3 = 8\), etc. \\
F & F & F & \\
\end{array}
\]
Consider the argument forms below. We wish to know, for each one, whether it is valid or not. We would then know whether any argument having one of these forms was valid or invalid within the framework of truth-functional sentential logic.

(A) \( P \rightarrow Q \\
    P \)

(B) \( P \rightarrow Q \\
    Q \)

(C) \( P \lor Q \\
    -P \)

(D) \( P \lor Q \\
    P \)

We wish to know whether there are any conditions under which the premises of an argument having one of these forms could be true while its conclusion is false. If so, the argument form—and any argument having that form—is invalid. If there are no such conditions, then the argument form—and any argument having that form—is valid, such that, if the premises are true, the conclusion cannot possibly be false. A truth-table will serve to survey all possible conditions under which the premises or conclusion of any argument having the above forms are true or false. For example, for argument form (A) we have:

<table>
<thead>
<tr>
<th>Form (A)</th>
<th>! Conclusion</th>
<th>! Premises</th>
</tr>
</thead>
</table>
| P \( \rightarrow \)
| Q ! P ! Q | ! ! P \( \rightarrow \) Q ! P |

(1) T ! T ! T !
(2) T ! F ! *F ! F* ! T
(3) F ! T ! *F !
(4) F ! F ! *F ! T ! F*

- We will concentrate on all and only those conditions, those possible truth-value assignments to the constituent sentences P and Q, which render the conclusion false (those lines marked by asterisks '*'). All the possible combinations of truth-value assignments to the components P and Q under which the conclusion would be false are represented by lines (2) and (4) of the truth-table above. The question then is: Is it possible for all of the premises of an argument of form (a) to be true under any of the truth-value assignments that render the conclusion false? Clearly not. No more than one of the premises can be true under the assignments which render the conclusion false. The argument form is valid: no argument of this form can have a false conclusion and true premises. Remember: A VALID argument form will never lead us from truth into falsehood.

We could as well have concentrated on all and only those truth-value assignments which
rendered all the premises true and then inspected the truth-table to see whether under any of those conditions the conclusion was rendered false. Those conditions are represented by lines (1) and (3) in the truth-table for argument (A) above. By filling out those lines, you can see for yourself that there are no conditions under which the conclusion is rendered false while all the premises are true. No argument of this form can possibly have all true premises and a false conclusion. And so for argument form (C): wherever the conclusion is false, at least one premise must also be false.*

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Conclusion</th>
<th>Premises</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>*F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>*F</td>
<td>T</td>
</tr>
</tbody>
</table>

By contrast, arguments form (B) and (D) are invalid: it is possible for the conclusion of an argument of either form to be false while its premises are all true. This is readily shown by a truth-table survey of all the possible combinations of truth-value assignments to the component variables of the argument form. If under some combination of assignments the conclusion is rendered false while the premises are true, the argument form—and any argument having that form—is invalid as an argument of that form fails to guarantee the truth of its conclusion given the truth of all its premises. An invalid argument form is unreliable because it can lead us from true premises to a false conclusion. Proof of the invalidity of argument forms (B) and (D) by the truth-table method is left to you as an exercise.

The nice thing about the truth-table method is that it is conclusive, exhaustive, and purely mechanical. Of course, you could also use the counter-example method (discussed in the last chapter) to prove argument forms (B) and (D) invalid. But whereas the counter-example method for proving invalidity requires ingenuity and imagination, the truth-table method does not: the latter is an infallible, purely mechanical decision procedure. And, whereas the counter-example method cannot be used to prove an argument form valid, the truth-table method provides an unerring decision-procedure for determining validity in sentential logic.

The section that follows illustrates how to use truth-functional sentential logic and the technique of truth-table analysis to PROVE that certain argument forms are VALID.

VALID ARGUMENT FORMS, in effect, provide us with RULES OF VALID INFERENCE—rules for constructing and evaluating arguments.
An inference rule allows us to draw CONCLUSIONS from given PREMISES by a series of valid INTERMEDIATE STEPS.

The ARGUMENT FORMS whose VALIDITY we prove are used as INHERENCE RULES that you may use in constructing valid derivations in the ARGUE program, which checks each step in your derivation for validity according to these rules.
3.5. Proving Rules of Inference Valid

3.5.1. The Modus Ponens Rule

The simple deductive schema (argument form) that represents the MODUS PONENS inference rule is:

\[ P \rightarrow Q \]
\[ P \]
\[ \underline{Q} \]

We can give a reading of the reasoning here as follows:

\[ P \rightarrow Q \quad \text{Assume that IF } P \text{ is true, } Q \text{ is true} \]
\[ P \quad \text{Now suppose } P \text{ IS true} \]
\[ \underline{Q} \quad \text{Then, it follows that } Q \text{ MUST be true.} \]

Another way to read (1) \( P \rightarrow Q \)

(2) \( P \)

(3) \( \underline{Q} \) is as follows:

The conditional (1) says that IF you have \( P \) that's SUFFICIENT to get \( Q \). (2) affirms that you have \( P \). So, given (1) and (2), you may conclude \( Q \). IF (1) and (2) are true, \( Q \) MUST be true.
A truth-functional CONDITIONAL of the form 'P ⇒ Q' asserts two things in effect:

(1) P's being true is a SUFFICIENT CONDITION for Q's being true:
   IF P is true, that suffices to make Q true.

(2) Q's being true is a NECESSARY CONDITION for P's being true:
   P is true ONLY IF Q is true
   P is NOT true UNLESS Q is true.

Think about these two basic senses and variant translations of the truth-functional analysis of conditionals and REMEMBER them. If you understand what a CONDITIONAL asserts about the TRUTH-VALUES of its ANTECEDENT and CONSEQUENT, then it's easier to understand the reasoning behind both the MODUS PONENS and the MODUS TOLLENS rules.

The two truth-functional interpretations of what a conditional says are like two sides of the same coin. 'P⇒Q' says, in effect:

(1) If the ANTECEDENT (P) is true, that is SUFFICIENT to make the CONSEQUENT (Q) true and

(2) For the ANTECEDENT (P) to be true it's NECESSARY that the CONSEQUENT (Q) be true:
   P is true ONLY IF Q is true
   P is NOT true UNLESS Q is true
   If Q is NOT true, then P is not true
3.5.2. The Modus Tollens Rule

From this second reading (2) of a conditional 'P => Q' we see the simple sense of the MODUS TOLLENS rule. Consider:

P => Q  
P is true ONLY IF Q is true
       (Or: P is NOT true UNLESS Q is true)
-Q    
Well, Q is NOT true
---
-P    
So, P is not true

This rule simply says:
Given any CONDITIONAL sentence and the DENIAL of its CONSEQUENT
You may deny/negate its ANTECEDENT

In sum, reading 'P=>Q' in the first sense:
(1) IF P is true, THEN Q is true

we can make sense of the MODUS PONENS rule:

P => Q  
IF you have the antecedent, you can have the consequent
P     
We have the antecedent
Q     
So, we can have the consequent

Reading 'P=>Q' in its other sense:

(2) P is true ONLY IF Q is true :: P is NOT true UNLESS Q is true
we can make sense of the MODUS TOLLENS rule:

P => Q  
The antecedent is true ONLY IF the consequent is true
-Q     
But we DENY the consequent
-P     
So, we can DENY the antecedent
REMEMBER: The rules MP and MT are talking about logical connections that are posited between the ANTECEDENTS and CONSEQUENTS of conditionals.

MODUS PONENS: \( P \rightarrow Q \)

\[
\begin{align*}
P & \\
\hline \\
Q
\end{align*}
\]

If you can AFFIRM the ANTECEDENT of any conditional, then you can AFFIRM its CONSEQUENT.

This goes for conditionals with complex antecedents and consequents. Keep in mind not all conditionals are as simple as \( P \rightarrow Q \). For example, the following argument forms correspond to the rule MODUS PONENS and, so, are valid deductions:

\[
\begin{align*}
(P & \land Q) \rightarrow -R \\
(P \land Q) & \\
\hline \\
-R
\end{align*}
\]

\[
\begin{align*}
-(P \leftrightarrow Q) & \rightarrow (R \lor S) \\
-(P \leftrightarrow Q) & \\
\hline \\
R \lor S
\end{align*}
\]

MODUS TOLLENS: \( P \rightarrow Q \)

\[
\begin{align*}
-Q & \\
\hline \\
-P
\end{align*}
\]

says

If you DENY the CONSEQUENT of any conditional, then you can DENY the ANTECEDENT.

This rule goes for conditionals with complex antecedents and consequents. For example, the following all correspond to the MODUS TOLLENS rule:

\[
\begin{align*}
-P & \rightarrow -Q \\
\hline \\
--Q
\end{align*}
\]

\[
\begin{align*}
(P & \rightarrow Q) \rightarrow R \\
-P & \\
\hline \\
-R
\end{align*}
\]

\[
\begin{align*}
((P & Q) \leftrightarrow R) & \rightarrow (S \lor F) \\
-(S \lor F) & \\
\hline \\
-(P \rightarrow Q)
\end{align*}
\]

\[
\begin{align*}
-((P & Q) \leftrightarrow R) & \\
\hline \\
-(P \rightarrow Q)
\end{align*}
\]
3.5.3. Review: Necessary vs. Sufficient Conditions

To understand what sense the inference rules MODUS PONENS and MODUS TOLLENS make, it's helpful to understand the two senses in which 'P=>Q' can be taken. Here's an example to help you remember the two senses of any conditional—the two ways in which ANTECEDENT and CONSEQUENT are logically related:

(1) IF P is true, THEN Q is true: P => Q

The antecedent is a SUFFICIENT CONDITION of the consequent

E.g. IF it's raining, THEN it's wet out.

(2) P is true ONLY IF Q is true: P => Q

The consequent is a NECESSARY condition of the antecedent

E.g. It's raining ONLY IF it's wet out.

It's NOT raining UNLESS it's wet out.

REMEMBER: Any conditional can be taken in either sense (1) or (2). Both senses are depicted: P => Q
3.5.4. Proving the Validity of MP and MT

While it's important to understand the SENSE of the rules, we can also PROVE that the argument schemas

$$\text{P} \implies \text{Q} \quad \text{P} \implies \text{Q}$$

$$\text{P} \quad \neg \text{Q}$$

are valid: in NO CASE whatsoever can an argument with either form have true premises and a false conclusion.

The strategy for testing the validity of an argument form is:

1. Construct a truth-table for the premises and conclusion of the argument form in question.

2. Start with the CONCLUSION: find those truth-value assignments (lines of the truth-table) which render the conclusion false.

3. Then proceed, premise by premise, to check whether any of the truth-value assignments that renders the conclusion false also allows ALL of the premises to be true.

IF there is NO truth-value assignment to the component sentential variables of the argument form that renders the conclusion false and all of the premises true,

THEN the argument form is VALID.

IF there is SOME truth-value assignment to the component sentential variables of the argument form that allows all premises to be true and the conclusion to be false,

THEN the argument form is INVALID.
We prove any argument form valid in sentential logic by constructing a truth-table to show that there is no case (no line in the truth-table) where an argument of the given form has all true premises when the conclusion is false.

The following tables prove the validity of argument forms corresponding to MODUS PONENS and MODUS TOLLENS.

**MODUS PONENS:**

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Conclusion</th>
<th>Premises</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>P=&gt;Q</td>
</tr>
</tbody>
</table>

* (1) T ! T | (2) T ! F | (3) F ! T | (4) F ! T


**MODUS TOLLENS:**

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Conclusion</th>
<th>Premises</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>P=&gt;Q</td>
</tr>
</tbody>
</table>

* (1) T ! T | (2) T ! F | (3) F ! T | (4) F ! F

* NOTE: You need look at only those lines of the truth-table where the conclusion is false; you need proceed only until you find that at least one premise must be false when the conclusion is false.
3.6. Proving Validity by Derivation

Once we have proven to our satisfaction that certain argument forms are valid, we can use those argument patterns as schematic models which, in effect, provide us with rules of valid inference. Inferences that formally conform to these valid argument schemas are themselves valid.

For example, inferences that formally conform to the following schemas are valid:

(MP) (1) \( P \implies Q \)  
(2) \( P \)  
------  
(3) \( Q \)

(DS) (1) \( P \lor Q \)  
(2) \( \neg P \)  
------  
(3) \( Q \)

These schemas enunciate, in effect, rules of valid inference, as follows.

(MP) Modus Ponens says, in effect:
(1) Given a conditional of the form: \( P \implies Q \)  
(2) If you can affirm the antecedent: \( P \)  
(3) You may then affirm/derive the consequent: \( Q \)

(DS) Disjunctive Syllogism says, in effect:
(1) Given a disjunction, which says that either one disjunct (\( P \)) or the other (\( Q \)) is true: \( P \lor Q \)  
(2) If it's not the one that's true: \( \neg P \)  
(3) You may then infer that the other is: \( Q \)
When we come upon more complex argument forms or longer sequences of inference, like:

(1)  \( P \lor Q \)
(2)  \( \neg P \)
(3)  \( Q \Rightarrow R \)

Therefore:  \( R \)

we can show that such argument forms are valid by deriving the CONCLUSION from the given PREMISES in a series of valid INTERMEDIATE STEPS according to previously validated inference rules. The argument form above is proven to be valid by deriving its conclusion (\( R' \)) from its given premises (1)–(3) according to the rules Modus Ponens and Disjunctive Syllogism, as follows.

(1)  \( P \lor Q \) / PREMISE
(2)  \( \neg P \) / PREMISE
(3)  \( Q \Rightarrow R \) / PREMISE
(4)  \( Q \) / INTERMEDIATE STEP:

[Justified by the application of the DS rule to lines (1) and (2)]

(5)  \( R \) / CONCLUSION:

Justified by the MP rule applied
to lines (3) and (4)

(Note: As in the computer-assist programs, a slash ' / ' indicates that what follows is the justification of the given line in the derivation.)

Once you become familiar with certain basic valid argument forms or valid inference rule schemas, you will be able to analyze the pertinent logical form of ordinary language arguments and prove arguments valid by deriving their conclusions from their given premises in a sequence of valid intermediate steps. Sometimes additional premises must be supplied in order to make an argument valid. You will get better at perceiving where an argument needs additional premises for validity as you become practiced in the application of valid rule schemas. While the TRUTH program exercises you in the truth-functional analysis of sentential connectives whereby we prove the validity of argument forms, the ARGUE program will give you guidance and practice in reconstructing arguments in valid logical form and proving those arguments valid by derivation. The agenda of formal argument reconstruction is illustrated in the next chapter.
4. What the ARGUE Program Helps Teach

Formal Logic in the Service of Argument Analysis

4.1. Four Tasks of Formal Argument Analysis

This section contains brief illustrations of four basic tasks in the analysis of arguments, particularly philosophic arguments about normative issues. Normative issues are issues about the norms, rules or principles which are the logical bases for our arguments and judgments about what is right and what is wrong.

The following sections offer an extended illustration of these four basic tasks of argument analysis applied to a sample argument about social policy, the policy of preferential hiring. These illustrations demonstrate how the formal logic you are learning can be applied in the systematic reconstruction of arguments. Careful reconstruction of arguments is a prerequisite for their analysis; and it provides a guiding framework for philosophic analysis, the analysis of the normative principles that are crucial premises of our arguments about what is right and what is wrong. These illustrations will demonstrate how the formal reconstruction of arguments is relevant to philosophic analysis and will introduce you to basic tools and techniques of philosophic analysis, in particular, the concept of plausibility and the use of counter-examples in testing the plausibility of normative principles. In the sections that follow you will begin to see how formal validity can serve as a guide in the pursuit of truth and justified belief in philosophy.

1. ANALYZING LOGICAL FORM: CHECKING FOR VALIDITY

One way to get clear about a given line of reasoning, about the assumptions that must be made for a conclusion to follow logically, is to reconstruct the line of reasoning in the precise form of a deductively valid argument. This enables one to identify key assumptions, to see the precise logical form these must take in order to support a given conclusion, to uncover tacit assumptions (unstated premises needed to make the argument valid), and to single out inessential assumptions (logically superfluous premises, premises not needed for the argument to be valid).

In many of the arguments we encounter, especially in philosophic disputes, there will be tacit premises (assumptions that are not stated but that must be made explicit for the argument to be valid). Before we can adequately evaluate any argument, we must make any tacit premises
(unstated but necessary assumptions) explicit. The first step in our reconstruction of any argument will be to analyze and represent the logical form of the argument's stated premises and conclusion: this provides an important clue to the form and content that any tacit premises must have in order to make the argument valid.

Consider the following argument.

As the first step in reconstruction, I've represented the logical form of the argument symbolically to its right:

If you have ambition, you're in for a lot of frustration. And, you're apt to be miserable if you're often frustrated. So, your life will take on purpose only if you're apt to be miserable.

Once the logical form of the argument is represented as above we can tell by inspection that the argument is invalid: There's no way to derive the conclusion from the given premises. But, in this case, it should also be clear from inspection of the argument's logical form that a premise of the following logical form would be sufficient to make the argument valid:

Once you've depicted the logical form of an argument in some clear and explicit (symbolic) way, it's easier to inspect the argument for validity and to see what forms of additional premise(s) will make it valid. This is why our first task in reconstructing and analyzing an argument will be to analyze and represent (symbolize) its logical form: to check the argument for validity and unstated premises needed for validity.

This first step in the analysis of arguments (ANALYZING LOGICAL FORM and CHECKING
FOR VALIDITY) will help us with one of the most crucial tasks of PHILOSOPHIC ANALYSIS, which is the second basic task of argument analysis: identifying underlying principles and 'hidden' or tacit (unstated but necessary) premises, illustrated below.

2. IDENTIFYING UNDERLYING PRINCIPLES AND TACIT PREMISES

To identify the general principle behind a position on some issue, it helps to try various formulations of the likely candidate principles as explicit premises in a valid deductive argument whose conclusion is the position in question. Consider, for illustration, the following widely held position:

[P] We owe it to future generations to control population size.

A likely principle behind this position [P] would be the following:

[O] We have an obligation to future generations to bequeath to them the means for the best possible life.

An additional tacit premise in support of [P] might be:

[O] => [P] IF we have an obligation to future generations to bequeath them the means for the best possible life,

THEN we owe it to future generations to control population size.

We can see by inspection, or prove by the derivation on the right below, that the resulting argument form is valid:

O        1 O  /  Premise [Unstated Principle]

O => P  2 O => P  /  Premise [Unstated]

P  3 P  /  1, 2 MP
3. SORTING OUT AMBIGUITIES IN PRINCIPLES AND PREMISES

There is usually more than one way of construing the general principle behind a given position. We want to beware, in particular, of ambiguous principles, principles that can be taken to mean either or both of two different things. Once we've identified a likely candidate principle (like [O]) behind a position (like [P]) and reconstructed an argument for that position with the likely principle and other likely assumptions needed for validity stated as explicit premises (as above), we need to be aware of other possible formulations of the principle in question and, in particular, to beware of any ambiguity that may be lurking in the principle we've chosen in reconstructing the argument. This is the next task in argument analysis: sorting out ambiguities in underlying principles and premises.

Can you think of at least two different things that the principle [O] (Premise 1 below) might be taken to mean?

1. O
   We have an obligation to future generations to bequeath them the means for the best possible life

2. O => P
   IF so, THEN we owe it to future generations to control population size

3. P
   So, we owe it to future generations to control population size

Here are three things Premise 1 might be taken to mean:

[O1] We have an obligation to future generations to ensure that the future population, however it be constituted, has the means for the best possible life of which it is capable

[O2] We have an obligation to future generations to ensure that the future population is so constituted as to be capable of enjoying the best possible life

[O1 & O2] We have BOTH these obligations to future generations
WHICH of the above versions of the original principle [O] we incorporate into the argument for the call for population control [P] will make a decided difference to the SOUNDNESS or PLAUSIBILITY of the argument, as follows.

Using [O2] as a premise (or the conjunction of [O2] and [O1]), it's pretty easy to construct an argument with [P] as a conclusion that is valid. But it is not so easy—it may well be impossible—to construct an argument using [O2] that BOTH (1) is VALID and (2) has all true or PLAUSIBLE premises, for this reason: [O2] (or any premise containing [O2]) is objectionable/IMPLAUSIBLE because it has objectionable logical consequences: Not only can [O2] be used to support a policy controlling the SIZE of any future population, but it can also be used to support a number of other quite objectionable policies, such as policies of genetic manipulation, selective breeding and sterilization of humans, consumer preference conditioning, and the like.

On the other hand, while [O1] by itself may be perfectly plausible (and have no untoward consequences if adopted), [O1] by itself (without additional premises) will not constitute a VALID argument in support of the control of population SIZE [P]. It will be difficult—if not impossible—to use [O1] along with additional PLAUSIBLE premises to construct a VALID argument in support of the control of population SIZE [P].

TRY IT! Try to construct an argument for [P] using [O1] (but not [O2]) that BOTH (1) is VALID and (2) all of whose premises are true or PLAUSIBLE.

How, actually can we assess the PLAUSIBILITY or [TRUTH] of the premises (normative or factual) of philosophic arguments. This is the next crucial task in argument analysis, and a basic task in all philosophic analysis: testing the truth or plausibility of general principles and premises. An important tool for this task is the counter-example, illustrated in the next sections.
4. TESTING THE PLAUSIBILITY OF PRINCIPLES AND PREMISES
THE USE OF COUNTER-EXAMPLES IN PHILOSOPHIC ARGUMENT

Scientists refute empirical hypotheses by citing counter-evidence—instances in which the hypotheses are false. Somewhat similarly, philosophers standardly refute putative definitions of important concepts (justice, knowledge, etc.), as well as general normative principles, by citing counter-examples. Arriving at a satisfactory definition by a series of successive formulations and counter-examples is the most characteristically philosophical of reasoning techniques. Attributed to Socrates, it is an important feature of philosophic analysis.

To illustrate the philosophic use of counter-examples, consider the distinction commonly cited by medical personnel between killing a patient and letting him die. Some try to explain it thus:

To let someone die, as opposed to killing him, is to be in a position to save his life but deliberately to refrain from doing so.

This formulation may be shown to be incorrect by the following counter-example. Suppose someone smothers you by pressing a pillow to your face for a period of several minutes. Once the pillow is in place, he is in a position to do something that would save your life—viz., lift the pillow—but he deliberately refrains from doing so. Yet, contrary to the proposed explanation, it would be natural to say that such a person killed you, not that he merely let you die.

Sometimes it is said that the distinction between killing and letting die is an instance of the 'active/passive' distinction. Putative counter-examples to this proposal are ready to hand. Removing a respirator from a critically ill patient is surely 'active' rather than 'passive.' Yet such an action could well be described as 'letting die' rather than 'killing.' Or suppose an anesthesiologist deliberately fails to make the necessary adjustments in certain life-support and monitoring systems attached to a patient undergoing surgery, thereby deliberately ensuring that the patient dies. Although such failures to act are 'passive,' it would be natural to accuse the anesthesiologist not merely of letting the patient die but of killing him.
The four basic tasks illustrated in the analysis of the sample argument below are:

1. The Reconstruction of an Argument in Valid Deductive Form.

2. The Explication or Revision of the Underlying Normative Principles that are Crucial Premises of the Argument.

3. The Analysis of any Ambiguity in the Crucial Premises of the Argument.

4. Testing the Plausibility or Truth of the Crucial Premises of the Argument Against Putative Counter-Example

These tasks are not exhaustive of what—all is involved in the analysis of arguments, but they are basic and important. Tasks (3)–(4) often require a reformulation of an argument, which in turn requires the repetition of tasks (1) and (2) in order to preserve the validity of the argument and the plausibility of its crucial premises.

The analysis of arguments requires the coordination of logical analysis and philosophic analysis.

Logical analysis primarily concerns the logical form the premises must take in order to support conclusion and maintain validity.

One function of philosophic analysis is to test the plausibility of the normative principles underlying our judgments and arguments; thereby, to articulate and develop the various issues underlying our arguments by adducing pertinent objections to the crucial premises of an argument (counter—examples or problem—cases), possible replies to those objections, possible rejoinders to the replies. The procedure, in rough outline, is: (1) to try to capture in explicit premises the 'intuitions' (and tacit principles) to which we appeal in our particular judgments by (2) abstracting those principles from clear 'paradigm' cases (cases, precedents or common practice where we are especially confident in our judgments); then (3) to adduce the logical consequences of our principles and test our premises against conflicting intuitions about putative counter—examples and (4) to reformulate these premises in order to overcome refutation by counter—example, to adjudicate or explain away problem—cases; and, so, (5) to repeat this process of careful formulation, testing and reformulation . . . until, ideally, we have made our arguments clearly valid and rendered our principles (a) perfectly explicit, (b) logically consistent among themselves, (c) evidently immune to further refutation by counter—example and (d) sufficient for adjudicating or explaining away problem—cases.

The analysis of the sample argument that follows is an illustration of logical and philosophical analysis coordinated in the service of the four basic tasks outlined above.
4.2. A Sample Argument: Preferential Hiring

Justice surely demands that someone unjustly deprived of something to which he had a right be compensated. Of course, normally, it's wrong to discriminate among job applicants on the basis of racial or sexual characteristics. But there are exceptions (as to any general rule). Blacks and women, for example, have a right to equal opportunity for advancement in education and employment. Yet both have been unjustly discriminated against in these areas. Not only does justice require that victims of such discrimination and right-violation be compensated, but by hiring blacks and women in preference to white males we do not thereby discriminate in a morally objectionable way. We rather compensate the victims of job discrimination as justice demands.

The foregoing is an argument from alleged requirements of justice. People often appeal to considerations of compensatory justice in defense of preferential hiring. One point we want to make as explicit and precise as possible by constraining the argument in valid form is exactly what justice is supposed to require.

Reconstructing this line of argument in deductively valid form will not produce a single argument or principle of justice: many valid reconstructions are possible employing any one of several possible formulations of the alleged requirements of justice.

But by constraining the argument in deductively valid form we force ourselves to specify some principle explicitly connected to the policy in question. This begins the dialectical program of successive reconstructions of the principle to take account of objections to it, and successive reconstructions of the argument providing the logical connection between the principle and the policy of preferential hiring.
4.3. An Initial Reconstruction

Any simple, plausible formulation of the argument will do for starters. Whatever formulation we begin with, it can be made progressively more precise under the fire of counter-examples and within the constraint of deductively valid form.

Consider the following generalized reconstruction of the argument. Replacing 'Xs' by 'blacks' or 'women' and 'Ys' by 'white males' will render the intended conclusion of the original argument. In brackets I will assign a variable letter to each statement in the argument so that its sentential-logical form can be readily depicted.

\[(A)\] (1) If [0] Xs have been unjustly deprived of something (e.g., equal employment opportunity) to which they had a right, then [J] justice demands that Xs be compensated

(2) [0] Xs have been unjustly deprived of something (i.e., equal employment opportunity) to which they had a right

(3) [M] Preferential hiring of Xs over Ys is on balance morally permissable if [S] preferential hiring of Xs serves to compensate them as victims of job discrimination

Therefore: [P] Justice demands preferential hiring of Xs over Ys

There are at least three problems with the argument as stated that come out in the course of reconstruction. First, whatever quarrel one might have with the accuracy of the initial reconstruction (A) (anyone may try his own), one can see that the logical form of the argument is, in any case, not manifestly valid: the conclusion does not follow. There are important unstated assumptions. Sentential logic will suffice, for starters, to find and fill the gross logical gaps in the argument. Second, the argument is rife with ambiguity. Third, once one has sorted out some of the ambiguity, it turns out to be remarkably difficult to reconstruct the argument so that it both is valid and has all true or plausible premises, premises at least immune to obvious counter-example. From these lessons of reconstruction philosophic lessons are also to be learned. I will deal with them in turn.
4.4. The First Problem: Invalidity and Unstated Premises

The argument as it stands is invalid. This is easily seen by inspection of its logical form, abstracted symbolically:

\[(A') \ (1') \text{If } O \text{ then } J \quad \text{(A'')} \ (1'') \text{ } O \Rightarrow J \]
\[(2') \text{ } O \quad \text{(2'')} \text{ } O \]
\[(3') \text{ } M \text{ if } S \quad \text{(3'')} \text{ } S \Rightarrow M \]

Therefore: \[P \quad P\]

One advantage of being able to depict the logical form of an argument in abbreviated notation is analogous to the advantage of having an x-ray device: it allows us to look at the bare skeletal structure apparently supporting the conclusion, to detect distinctly structural flaws underneath the enveloping verbal flesh and musculature. From our x-ray of argument (A) it's clear that the conclusion P is in no way explicitly connected to any of the stated premises. Nor is any explicit connection between premises (1) and (2) and premise (3) yet apparent: from (1) and (2) we can conclude J; but what connections are presumed to exist among J, premise (3) and the conclusion P? These connections, in some form, must be made explicit, so as to make explicit use of the stated premises of the argument and also render it valid. Here we need to consider the content as well as the form of the argument.

Symbolic logical form and validity serve, respectively, as clues and guiding constraints in the search for tacit premises; but they are not sufficient grounds for generating sensible additional premises, or for deciding among competing premises where any number might make an argument valid. It is necessary to introduce other guiding constraints in the reconstruction of an argument. Validity remains a powerful minimal condition of the enterprise nonetheless: insisting on manifest validity keeps us honest about what exactly is or must be assumed and exactly what follows from what. In this case it requires us to produce some further assumptions on which the conclusion tacitly rests. Once laid out explicitly, these assumptions are open to question; and they may force us to change the shape of the argument or even abandon it. One obvious tacit assumption is:

[R] Ys (white males) have received undue preferential treatment over Xs (blacks or women) in hiring practice.

Without assuming at least some such condition it would make no sense to assert that it is morally permissible to compensate Xs at the expense of Ys. Moreover, without the addition of some such condition as R to premise (3), this premise is open to obvious counter-example and, so, is false. That is, the truth of S is not always a sufficient condition for the truth of M, for surely the following interpretation of premise (3) is false:
If preferential hiring of Mexican-Americans (X's) compensates them . . . then preferential hiring of Mexican-Americans (X's) over blacks (Y's) is morally permissible.

The logical form of our further reconstruction now looks like this:

\[(A') (1') \text{ If } O \text{ then } J \]
\[(2') O \]

Therefore: J

\[(3')* \text{ If } S \text{ and } R, \text{ then } M \]
\[(4') R \]

Therefore: P

Explicitly assuming the condition S as an additional premise

\[(5') S \]

we may draw the further intermediary conclusion

Therefore: M

from (3'), (4') and (5'). We have now gotten so far as to conclude that [J] justice demands compensation and that [M] preferential hiring is on balance a morally permissible way to compensate. We have yet explicitly to complete the connection to the ultimate conclusion [P] to the effect that justice in turn demands preferential hiring as the mode of compensation. Any of the following additional premises connecting the conclusion to the foregoing results would render the argument valid:

\[(6') (a) \text{ If } J \text{ and } M, \text{ then } P \]
\[(b) \text{ If } J \text{ then } P \]
\[(c) \text{ If } M \text{ then } P \]

Both (b) and (c) are objectionable, on similar grounds. That [J] justice demands compensation is not sufficient grounds for asserting that [P] justice demands that compensation take a particular form, namely, preferential hiring. That [M] preferential hiring (or anything else, say, singing in the shower) is on balance morally permissible is not sufficient grounds for holding that [P] justice requires it. So, (b) and (c) are implausible or false. Moreover, their addition to the argument, while making it valid, would be to cast adrift other presumably relevant premises as logically superfluous.
(6') (a) seems the best of the three alternatives. Choosing it has been an exercise in the reconstruction of a normative principle, an attempt to specify sufficient grounds on which justice would require and, so, justify a particular policy. The reconstruction of principles and the reconstruction of arguments go hand-in-hand in the moral-philosophic forum, because general normative principles are always among the (stated or tacit) assumptions of a moral-philosophic argument. Hence, the reconstruction of arguments can play a heuristic role in the explication and analysis of the normative principles underlying our reasonings.

Once the argument, with its tacit underlying principles, has been reconstructed in valid form, we are at least assured that if the premises are acceptable, so must be the conclusion. But are they?
4.5. Further Problems: Ambiguity and Vulnerability to Counter Example

For purposes of illustration, I will focus on the first premise of the argument only. Where ambiguities are discerned or counter-examples found premises must be reformulated, jettisoned, or added. Premise revision involves further, alternative reconstructions of the argument to preserve its validity. This may be difficult, but to that extent instructive.

Consider: The principle of compensatory justice to which argument (A) appeals, premise (A-1), can of course be applied quite generally. So, the original line of reasoning and policy based on this principle can be applied quite generally. How generally? To whoever can be counted among the X's. Who might be counted among the X's? On grounds of premise (A-1), anyone who has ever been deprived of something to which she had a right, say, 'equal' employment opportunity. (He could well be a highly competent white bank executive from a wealthy family who has been denied 'equal' consideration for jobs many times because of his religious or political views.)

There is a serious ambiguity in premise (A-1). How are we to interpret the demand for compensation of Xs? There are at least two possibilities where Xs are members of some identifiable group:

**A distributive interpretation:** a person is to be compensated if he is an X (black, woman, atheist . . .) and he has himself been unjustly discriminated against . . . :)

**A collective interpretation:** a person is to be compensated if he is an X and Xs (blacks, women, Irish-Catholics, communists . . . ) have in general been unjustly discriminated against . . .

The collective interpretation of the demand for compensation for Xs does not require that any given X have been unjustly discriminated against, but rather that other Xs as a group have been unjustly deprived: under this interpretation Xs as such are to be compensated.

A wealthy Jewish or Irish Catholic businessman who had never himself been deprived of anything could qualify under the collective interpretation for compensation where X's were Irish Catholics or Jews. A wealthy white male who had himself been unjustly discriminated against because of his atheism could qualify for compensation under a distributive interpretation. Presumably the purpose of the preferential hiring policy in question is neither to compensate wealthy people nor to compensate just anybody for any unjust deprivation she may have suffered. Under either interpretation the general demand for compensation could be applied to practically anybody; whereas the specific demand for compensatory preferential
hiring is on behalf of certain presently and unfairly disadvantaged groups, namely, certain racial minorities and women.

We need to specify the conditions of premise (A–l) so as to justify compensatory treatment in the form of preferential hiring for all and only those whom the policy is meant to compensate. For a sense of these two possibly conflicting constraints—the justice and purpose of the policy in question—we need appeal to our intuitions, our tacit conceptions of both, as yet imperfectly captured in premise (A–l) (and yet to be tested against limiting counterexamples).

We need first to specify more precisely the grounds on which justice demands compensation. We will consider five candidate criteria. These will be sufficient to delineate some of the major ambiguities of our original principle of compensatory justice.

(a) Membership in a group whose members have been widely and unjustly discriminated against and thereby deprived of something to which they had a right.

This criterion would qualify blacks and women, but would it qualify all and only those actually deserving compensatory treatment? As already suggested, it would not. The criterion is too inclusive. Who would not qualify for compensation? Consider the cases of well-to-do Catholics, Protestants or Jews who have never been unjustly deprived of anything but who are members of groups which have (somewhere) suffered great injustice. Does justice require that they be compensated?

It is evidently not mere membership in some identifiable class of persons many of whose members have suffered injustice at some time in the past that recommends a given member for compensation. Yet the policy we are seeking to justify on the basis of the requirements of justice designates its beneficiaries according to racial or sexual characteristics.

Perhaps it is rather the likelihood of having herself suffered injustice, given effective, recent and widespread prejudice and discrimination against Xs as such, that recommends any given X for compensatory treatment.

(b) Likelihood of having suffered unjust deprivation oneself because of membership in a group whose members have been recently and widely discriminated against.

This criterion would include blacks and women and exclude consideration of white Catholics or Jews for compensation. But against this suggestion stands the case of any well-to-do black woman who has never been deprived of anything. Does justice demand that a person who has
never suffered any injustice be compensated? What is it for which she would be compensated? Analogously, should courts award compensatory damages to a person on the likelihood that he suffered defamation of character when in fact he hasn’t—or because it is established that he was actually wronged and harmed in a way penalizable by law? The latter case is more problem-case than counter-example. Consider then:

(c) Having in fact been unjustly discriminated against and thereby deprived of something to which one had a right.

Whatever justice demands in the way of compensation, it would seem that justice demands it only for persons who have themselves been unjustly harmed or deprived, not for persons who haven’t in fact been wronged but who happen to have certain characteristics (e.g., race) in common with others who have.

But while actually having been wronged oneself may be a necessary condition for claiming compensation on grounds of justice, this requirement of justice would not justify preferential hiring of blacks or women as such. On the other hand, a principle stipulating criterion (c) as a sufficient condition for compensatory treatment would not justify preferential hiring of all and only those (certain minority groups and women) whom the policy seems intended to benefit. Such a principle would justify compensatory treatment of a person irrespective of her race, sex or socio-economic status. A policy of preferential hiring based on such a criterion, designating beneficiaries according to their personal histories rather than race or sex, would seem impracticable.

There are further ambiguities in the position regarding the qualifications for compensatory treatment. Some of these are made explicit in the multiple-choice reconstruction, argument (E), below.

One instructive difficulty with the line of argument under consideration has clearly emerged in our reconstructive effort: the problem of fitting (logically connecting) the desired policy (compensatory preferential hiring of certain racial minorities and women) to the requirements of justice that seemed initially to demand such a policy.

The grounds on which justice might demand and distribute compensation are not obviously the grounds on which the policy in question would distribute preferential treatment.

A fairly superficial examination of the ambiguities of our initial formulation of what justice requires has produced a fair array of questions. Never mind objections to other premises for now. We find that the alleged requirements of justice are themselves clearly questionable. We
can map out the issues and strategic options confronting us by reformulating the original deductive argument to take account of the ambiguities and objections raised. We might consider this endeavor a kind of game, a game of argumentative reconstruction and counter-example.

Attempting to reconstruct the argument in expressly valid form, while taking account of ambiguities and counter-examples, makes the philosophic problem of fitting the desired policy to the demands of justice more acute. While this reconstructive exercise may well make the argument less persuasive, the exercise is nonetheless instructive regarding some of the philosophic issues underlying the policy in question. This is one educational objective of the task of reconstructing arguments in deductively valid form, and one rationale for the CAI programs in argument construction and reconstruction, which enforce the constraint of validity while allowing the student to view and manipulate both the logical form and the content of an argument side-by-side.
4.6. The Game of Formal Argument Reconstruction and Counter-Example

The object of this game is to construct an evidently valid and plausible argument supporting the policy of preferential hiring in question from the requirements of justice.

The first phase of the game is to construct a deductive argument whose conclusion is the position on the policy in question by selecting those premises required to make manifest the validity of the argument.

The second phase of the game is to test the plausibility of the selected premises, by explicating ambiguities and adducing putative counter-examples or problem-cases. A given premise remains plausible only so far as it is at least immune to obvious counter-examples.

Our two concerns are the logical connection between premises and conclusion, a matter of logical form, and the plausibility of the premises, a function of their actual content and the argumentative context. As we shall see, these concerns are not unrelated in the game of argumentative strategy that follows. It is often very difficult to satisfy both constraints at once. In this respect deductive validity, a matter of logical form, is indeed related to the pursuit of truth. Where the task is apparently impossible, we have good reason to abandon an argument and seek alternative strategies.

Consider now a multiple-choice reformulation of the original argument from justice, argument (B), below. Premises may be constructed by selecting one (or more) of the lettered options (and providing suitable logical connectives). The options (a)–(e) given under each premise are intended to take account of the ambiguities already detected in the principle of justice employed in the original argument (A). At this stage of reconstruction it is useful to have recourse to quantificational logic. The logical form of each premise is symbolized to its right so that validity can be readily assessed by inspection or derivation.
(1) If a person

(a) is a member of a group whose members have been widely and unjustly discriminated against and thereby deprived of something to which they had a right

(b) is likely himself to have been unjustly discriminated against and thereby deprived of something to which he had a right

(c) has in fact himself been unjustly deprived of something to which he had a right

(d) has himself suffered harm or serious disadvantage as a result of having been unjustly discriminated against

(e) presently is suffering harm or serious disadvantage as a result of having been unjustly discriminated against

THEN justice demands that person be compensated.

(2) ALL women

(a) are members of some group whose members have been widely and unjustly discriminated against and thereby deprived . . .

(b) are likely to have been unjustly deprived

(c) have in fact been unjustly discriminated against and thereby deprived . . .

(d) have suffered harm or serious disadvantage as a result of having been unjustly discriminated against

(e) are presently suffering harm or serious disadvantage as a result of having been unjustly discriminated against

Therefore: Justice demands that women be compensated
We are considering now just the first stage of the original argument, to the first intermediate conclusion that justice demands compensation for, say, women. If we can't construct a valid and plausible argument to this intermediate conclusion, we can hardly justify the policy in question on the basis of premise (A-1).

There are at least five different sets of premises, consisting of alternative versions of premises (A-1) and (A-2), that will each provide a valid argument to the first conclusion. They are: (A-1a), (A-2a); (A-1b), (A-2b); (A-1c), (A-2c); (A-1d), (A-2d); (A-1e), (A-2e). The list could be lengthened by including sets of compound premises, such as: (A-1e or d), (A-2e). However, nothing would be gained thereby. Keeping in mind the questions raised previously regarding criteria (a) through (e), consider the arguments resulting from these sets of premises. Which of the optional arguments is most evidently sound? As it happens, none is sound (i.e., both is valid and has all its premises immune to obvious counter-example).

Conditions (1c), (1d) and (1e) seem to provide the most plausible grounds for justice to demand compensation, namely: that a person herself has suffered an injustice, or harm as a result of injustice, in order that she actually have something to be compensated for. The difference between (1c) and (1d) or (1e) concerns whether we wish to compensate persons who were in fact treated unjustly but who suffered no harm or disadvantage on that account. Is it the mere fact of injustice or rather the resultant harm that demands compensation? If the latter, is it present or past harm?

By contrast, condition (1a) seems an implausible basis for justice to demand anything, let alone compensation. Mere membership in a group does not suffice to establish that a person suffered any injustice. If a person suffered no injustice, what is she to be compensated for? Some further condition seems necessary to establish an evidentiary connection between group membership and injustice. (A-1b) makes this condition explicit, asserting a probable connection. The implausibility of (A-1a) can be shown by an appeal to the untoward consequence that would result from its general application: cases where justice demanded compensation but where there was no victim of injustice to be compensated. The same objection could be lodged against (A-1b), with counter-examples.

Whereas premises (A-1c), (A-1d) and (A-1e) provide plausible grounds for justice to demand compensation, the factual assumptions respectively required to entail the desired conclusion are very likely false. On the other hand the factual assumptions (A-2a) and (A-2b) while true, seem to provide insufficient grounds for justice to demand compensation, as shown by counter-examples to (A-1a) and (A-1b) above.
If we revise this stage of the argument in order to demand compensation, discriminately, only for those (blacks, women or Xs) who qualify on conditions (1c), (1d) or (1e), we would vitiate the validity of the argument to the final conclusion (which calls for compensation for ALL blacks or women as such). If we revise the final conclusion, to preserve validity, and thereby discriminately demand preferential hiring for only those who qualify on conditions (1c), (1d), or (1e), we cannot justify compensatory hiring of blacks and women as such. We would then be arguing for a very different, and probably impracticable, policy. The difficulty is to provide a manifestly valid argument with plausible premises to the specified conclusion. This difficulty would be compounded if we were to examine other premises in the argument. More subtle refinement or reformulation of the premises will not eliminate the basic difficulty.

We have reached an apparent impasse in our game of argument reconstruction. There are some lessons of argumentative strategy to be gained at this impasse.

It is not obvious that an appeal to the requirements of compensatory justice is, after all, viable in behalf of preferential hiring of blacks, women or other minority members as such. This has been shown by making certain alternative connections between the policy and the presumed requirements of justice explicit in valid deductive form. Making the supposed logical connections between policies and principles explicit in this way forces us to clarify our normative assumptions and provides us with clear departure points for further dialectical analysis of policy issues and matters of principle.

We also get clearer on exactly what position we want or need to hold on the policy in question. When we reach an impasse such as we have in our appeal to compensatory justice, we would be well-advised at least to consider other lines of argument. Perhaps mere compatibility with the requirements of justice and an appeal to 'social utility' would suffice to support the policy in question. Perhaps the correction of certain social ills (disproportionate poverty, unfair competitive disadvantage or unemployment among certain minorities) or the provision of certain social benefits (positive role models and career incentives) is the proper aim of the policy in question. Perhaps these ends, if achievable with negligible infractions of justice, would justify preferential hiring of blacks or women as such. Perhaps. This, in any case, is a strategy different from the one with which we began. To get clear on exactly what would have to be true in the way of both normative and factual assumptions in order to support preferential hiring along these lines we would do well to make those assumptions explicit within the frame of a valid deductive argument.

And the game of argument reconstruction and counter-example would resume. A game that
is, after all, a serious form of philosophic inquiry within strict logical constraints. The ARGUE program enforces the constraint of validity, while hints in the stored problems provide suggestions regarding the plausibility of alternative premises, in order to rehearse you in the game of argument reconstruction.