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The Yield Curve: Terms of Endearment or Terms of Endowment

Chris I. Telmer
*Carnegie Mellon University, chris.telmer@cmu.edu*

Stanley E. Zin
*Carnegie Mellon University*

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The Yield Curve:  
Terms of Endearment or Terms of Endowment?*

Chris I. Telmer†    Stanley E. Zin‡

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Abstract

By solving an incomplete-markets model of multiperiod bond pricing backwards, we show that the mean and autocorrelation properties of the term premiums in the yield curve can be a reflection of the temporal distribution of uninsurable income shocks, i.e., the term structure of endowments. Moreover, agents can exhibit an equilibrium preferred habitat in bond maturities in the absence of binding portfolio restrictions.

1 Introduction

When markets are complete, idiosyncratic income risk plays no role in the observable properties of asset prices in general, and multiperiod bond yields in particular. A risk premium earned from holding a risky asset, e.g., a two-period discount bond, for one period rather than a safe asset, e.g., a one-period discount bond, can depend only on aggregate risk and agents’ preferences. However, when aggregate risk is calibrated to match that of the US economy, it is difficult to find reasonable preference specifications that result in term premiums in the model that have both the size and the time-series properties of the

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†GSIA, Carnegie Mellon University  
‡GSIA, Carnegie Mellon University and NBER
term premiums observed in US government bond markets. See, for example, Backus, Gregory, and Zin (1989).

This complete-markets puzzle for the term structure of interest rates parallels the equity-premium puzzle (Mehra and Prescott (1985)). In response to these types of puzzles, a growing literature in the field of intertemporal asset pricing focuses on the implications of idiosyncratic risk and incomplete markets for the prices of financial assets. Recent examples are Aiyagari and Gertler (1991), Constantinides and Duffie (1996) den Haan (1994), Heaton and Lucas (1996), Krusell and Smith (1995), Mankiw (1986), Marcet and Singleton (1991), Storesletten, Telmer, and Yaron (1996), Telmer (1993), Weil (1992), and Zhang (1995). The common feature of these models is that agents will have equilibrium consumptions that are related to the idiosyncratic portion of their endowments if markets are incomplete. Therefore, since asset prices depend on individual-level consumption, they will also depend on individual-level endowments. Naturally, the degree of this dependence varies with the specification of the economic environment.

The idea that the cross-sectional distribution of consumption, portfolio holdings, and endowments may have important affects on asset prices is a familiar one for those who study the term structure of interest rates. The Preferred Habitat Theory of Modigliani and Sutch (1966) is based on the idea that investors differ in the temporal distribution of their endowments. This gives rise to a preferred habitat: a preference for bonds with a maturity similar to the timing of one’s endowments. An $n$-period habitat describes an investor who “has funds which he will not need for $n$ periods and which, therefore, he intends to keep invested in bonds for $n$ periods” (p. 183). Of course in equilibrium, not every investor is able to borrow or lend at their preferred maturity. As a result, risk premiums in the term structure can be interpreted as the compensation which an investor demands in order to leave their natural, or preferred, habitat.

In this paper we explore the preferred habitat idea of Modigliani and Sutch in a general equilibrium model with incomplete markets. In simple terms, we seek to relate the “term structure of interest rates” to the “term structure of endowments” in an environment where one is forced to be explicit about who trades with whom and at what price, or equivalently who remains within their habitat and who is induced to leave.

The general equilibrium feature of our model forces us to discuss and interpret exactly what a meaningful habitat can (or cannot) be. For example, it is not possible in equilibrium for all lenders to exhibit a short-term habitat
while all borrowers exhibit a long-term habitat (as in Hicks (1939), pp. 138-46). A less obvious example involves financial market structure. Complete markets imply that a habitat in endowments will not result in a habitat in consumption and, hence, prices. Our interpretation of an equilibrium habitat, therefore, differs from related work such as Cox, Ingersoll, and Ross (1981) and Bakshi and Chen (1995).

We adopt the backward-solving methodology of Sims (1989) to find the numerical example that leads to our conclusions. We fit vector autoregression to data on 6-month and 12-month US treasury yields. We then calibrate a 4-period, 3-state process for prices that capture the salient features of this VAR. We then fix each agent’s equilibrium holdings of 1- and 2-period bonds to capture a habitat in maturities. Given this exogenous specification for what are typically the endogenous variables in this economy, we solve for a set of endowments state-by-state, that satisfy each agents’ budget constraints, the aggregate resource constraint, market clearing, and the optimality conditions for bond holdings for each agent. In other words, we find a distribution for endowments that would result in equilibrium bond holdings and bond prices that are exactly as we have specified.

2 A Digression on “Preferred Habitats”

A cursory reading of the literature on the term structure of interest rates makes it clear that the term “preferred habitat” can have a number of different meanings. In this section we review some of these different uses with the hope of clarifying our own use of the term.

Most work on maturity habitats and interest rates starts with the expectations hypothesis then introduces some form of investor heterogeneity. Hicks (1939) provides the basic intuitive role of risk aversion in this setting. His argument was simple. If investors are concerned primarily with short-horizon payoffs on their portfolios, then they are likely to view long-term bonds as being riskier that short-term bonds. Hicks interpreted a larger expected return on holding a long-term bond over a short-term bond as compensation for bearing interest rate risk. As for habitat effects, Hicks (1939) suggested that factors other than expected future interest rates may affect the relative pricing of bonds of differing maturities. Through the Liquiditity Preference Model, Hicks (1939) (pages 138-46), argues that to “hedge their future supplies of loan capital ... [borrowers] will have a strong propensity to borrow long.” In addition, because of “the
desire to keep one's hands free to meet ... uncertainty ... most people (and institutions) would prefer to lend short.” Finally, “speculators” will offset this “constitutional weakness” in the supply of long funds by borrowing short and lending long; however, they must receive a premium return as “compensation for the risk they are incurring.” The idea that different types of investors are subject to interest rate risk at different maturities is similar in spirit to our findings below.

Culbertson (1957) took a more extreme position with the Market Segmentation Hypothesis. Institutional and legal considerations may force investors to ignore bonds outside of their specific habitat. Modigliani and Sutch (1966) took a less extreme position and modified Hicks’ expectations hypothesis by allowing some investors to take a longer perspective on their decisions and not focus solely on short-term, or end-of-period, wealth. An investor who needs (or, equivalently, has) funds some \( n \) periods from the current date may view short-term bonds as being riskier than long-term bonds: financing a long-term liability by rolling over short-term assets involves interest rate risk in exactly the same manner as Hicks’ original short-term investor. Modigliani and Sutch (1966) therefore suggest that risk premiums in the term structure may be either positive or negative, depending on the “extent to which the supply of funds with habitat \( n \) differs from the aggregate demand for \( n \) period loans forthcoming at that rate” (page 184). The idea is that, unlike Hicks’ Liquidity Preference model with “speculators” taking the opposing side of the market, some investors will be tempted out of their “natural habitat” by the lure of an expected excess return. The question as to which investors leave their habitat also arises in our model.

More recent attempts at incorporating these ideas in a more structural model of risk premiums can be found in Cox, Ingersoll, and Ross (1981) and Bakshi and Chen (1995) as well as a large literature on clientele effects driven primarily by differential tax treatment (see Dybvig and Ross (1986) and Dammon and Green (1987) for instance). Cox, Ingersoll, and Ross (1981) endow the representative agent in their economy with a strong preference for consumption in period \( t + \tau \). This \( \tau \)-period habitat generates a risk premium on a \( \tau \)-period bond that depends primarily on the risk aversion on the representative agent. Bakshi and Chen (1995) extend this work to allow for investor heterogeneity and portfolio constraints. The habitat in their model is determined by corner solutions, i.e., the investor with the highest intertemporal marginal rate of substitution at a given maturity holds all of the bond of that maturity and the price is given by that agents marginal valuation. We attempt to avoid both the representative
agent and the constrained-portfolio approaches to preferred habitat. As for
tax clienteles, differential tax treatments across agents leads to investors facing
different state prices. This introduces the possibility of unbounded arbitrage
unless there are constraints on portfolios. Clientele, or habitat, effects therefore
depend critically on these portfolio constraints.

3 The Model

We construct a class of finite-period, two-agent economies using a textbook
incomplete markets setup (see, for example, Huang and Litzenberger (1988)).
These economies are characterized by the following three properties: (1) Mar-
kets are incomplete: agents are restricted to trade in long and short term bonds
only, rather than date and state contingent claims. (2) The statistical properties
of equilibrium long- and short-term yields match those of US treasury securi-
ties. (3) Agents are at interior solutions to their portfolio choice problems and
these solutions exhibit a preferred habitat. The last property is one important
dimension along which we differ from previous work. Agents in our economy
choose a habitat as opposed to being forced to one via portfolio constraints.

The details of the model are as follows. Two agents live for four periods,
$t = 1, 2, 3, 4$. Each agent receives a random endowment $\omega^{(i)}_t$, $i = 1, 2$, in each
period, and there is a binding aggregate resource constraint, $\omega^{(1)}_t + \omega^{(2)}_t = \bar{\omega}$,
where $\bar{\omega}$ is constant across states and dates, i.e., there is no aggregate risk.
Agents have access to competitive markets for 1-period and 2-period discount
bounds, trading at prices $b_t$ and $B_t$, respectively. The net supply of bonds is
zero. Provided the dimension of the uncertainty, e.g., the number of endowment
states, is not spanned by the traded securities, e.g. the two bonds, markets are
incomplete. Consumptions, $c^{[i]}_t$, can differ from endowments according to the
budget constraints

\begin{align*}
c^{[i]}_1 &= \omega^{[i]}_1 - q^{[i]}_1 b_1 - Q^{[i]}_1 B_1 \\
c^{[i]}_2 &= \omega^{[i]}_2 - q^{[i]}_2 b_2 - Q^{[i]}_2 B_2 + q^{[i]}_1 + Q^{[i]}_1 b_2 \\
c^{[i]}_3 &= \omega^{[i]}_3 - q^{[i]}_3 b_3 + q^{[i]}_2 + Q^{[i]}_2 b_3 \\
c^{[i]}_4 &= \omega^{[i]}_4 + q^{[i]}_3,
\end{align*}
where $q_t^{(i)}$ is agent $i$’s holdings in the 1-period bond at date $t$, and $Q_t^{(i)}$ is agent $i$’s holdings in the 2-period bond at date $t$.

Each agent seeks to maximize the expected value of the same utility index

$$\log(c_1^{(i)}) + \beta \log(c_2^{(i)}) + \beta^2 \log(c_3^{(i)}) + \beta^3 \log(c_4^{(i)}).$$

subject to the budget constraints. An equilibrium is a set of prices and bond holdings that are consistent with each agent’s utility maximization and with market clearing.

For the numerical examples below, the utility discount factor, $\beta$, is fixed at 0.925.

4 Calibration

We fit a first-order VAR to 6-month and 12-month US government bond data using the annualized zero-coupon yields computed by McCulloch and Kwon (1993). Using 470 monthly observations from 1952:1 to 1991:2, we get the following estimates:

$$\begin{bmatrix} - \log(b_t) \\ - \log(B_t)/2 \end{bmatrix} = \begin{bmatrix} 0.089 \\ 0.128 \end{bmatrix} + \begin{bmatrix} 0.875 \\ 0.079 \end{bmatrix} \begin{bmatrix} \log(b_{t-1}) \\ \log(B_{t-1})/2 \end{bmatrix} + \begin{bmatrix} \hat{e}_t^1 \\ \hat{e}_t^2 \end{bmatrix},$$

where standard errors of the point estimates are in parentheses. The standard deviation for $\epsilon^1$ is 0.568 and the standard deviation for $\epsilon^2$ is 0.546, and their correlation coefficient is 0.96. Using the conditional mean functions and the innovation standard errors, we construct a single-factor, finite-state tree for yields given in Figure 1. The value of the short yield at each node is given by the conditional mean and an equally likely innovation of $\{-0.568, 0, 0.568\}$. Likewise, the value of the long yield at each node is given by the conditional mean and an equally likely innovation of $\{-0.546, 0, 0.546\}$. Yields are, therefore, perfectly correlated.
Figure 1: Yield Tree

Note: (1) The first number in parentheses is the 1-period yield, and the second number is the 2-period yield. (2) The innovations to the conditional mean are symmetric around zero. (3) Each node is equally likely.
5 A Preferred Habitat Example

We next specify bond holdings for one of the agents. The second agent’s equilibrium bond holdings are simply the opposite of this agent’s, since the bonds are in zero net supply. We imbue a strong habitat in maturities for these bond holdings. In particular, the agent will always be short, i.e., borrowing, 2 of the long bond, and will be long, i.e., saving, 1 of the short bonds. Naturally, there is no long bond available in the third period, in which case the agent maintains his long position of 1 short bond. This is an extreme case of a habitat: the agent has an identical portfolio across all dates and states of nature. There is no aggregate uncertainty in the economy, and the aggregate endowment is constant over time. Figure 2 depicts the distribution of endowments that is consistent with these equilibrium bond holding at the prices in Figure 1.

Initial consumption is arbitrarily set to 3.0 and the aggregate endowment is 6.0. The period-1 endowment is 2.17 and the unconditional mean of the endowments for periods 2, 3, and 4 are 3.06, 4.83, and 2.0, respectively. This agent exhibits the classic life-cycle shape of income: steadily increasing on average throughout the agent’s life, then falling abruptly at the end. Average consumption is 3.0 for all periods, hence, the agent is able to perfectly smooth the mean of consumption over time through his bond-market transactions. However, relatively little risk sharing can be accomplished through these markets. The standard deviation of the endowment and consumption in period 2 are roughly equal at 0.4. Consumption is slightly less variable than the endowment in the third period with a standard deviation of 0.56 compared to 0.57. No risk sharing is possible in period 4.

To get a rough idea of the utility value of this bond trading, we can calculate the decrease in per period consumption the agent would be willing to undertake to achieve a greater level of smoothing/risk sharing. To move from autarky to the equilibrium level of consumption given in Figure 2, the agent would be willing to forego 13.48% of his consumption in each of the four periods. To move from the equilibrium to perfect risk sharing, i.e., a consumption of 3.0 at each date and state, the agent would be willing to forego an additional 1.34% of his per period consumption. The greatest utility gains, therefore, derive from temporal substitution, rather than risk hedging. Of course, these results are sensitive to the parameter values of the utility function, in particular the relatively modest amount of risk aversion.

This maturity habitat resembles that of a typical homeowner who borrows at a long maturity, e.g., a mortgage, which is frequently refinanced, yet saves
Figure 2: Endowment/Consumption Tree (1)

Note: (1) The first number in parentheses agent’s endowment at that node and the second number in parentheses is the agent’s equilibrium consumption. (2) As in the tree for yields, each node is equally likely.
using short-maturity bank deposits. Given the life-cycle nature of endowments, and the risks associated with long-term investments, this turns out to be an efficient method of smoothing consumption over time (if not over states).

[Insert paragraph relating this example to the discussion in Section 2.]

Figure 3 depicts the opposite side of the economy to this life-cycle consumer/investor. This agent has equilibrium bond holdings that are the opposite of the other agent. That is, he is always long 2 of the long bonds (when they are available) and short 1 of the short bonds. Note the opposite pattern of the term structure of endowments: income is steadily falling throughout the agent’s lifetime then it takes a large jump in the final period.

6 Conclusion

We have found an example of a simple incomplete-markets economy in which idiosyncratic income risk drives the prices of bonds of differing maturities. Bond yields in the model exhibit the dynamic properties of US government bond data. Moreover, agents have preferred maturity habitats in equilibrium, even though they do not face binding portfolio constraints. Our example demonstrates that this joint behavior of prices and portfolios reflects the temporal distribution of endowments, or the “term structure of endowments.”
Figure 3: Endowment/Consumption Tree (2)

Note: (1) The first number in parentheses agent’s endowment at that node and the second number in parentheses is the agent’s equilibrium consumption. (2) As in the tree for yields, each node is equally likely.
References


