Allocation and taxation in uncommitted societies*

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First Version: January 2008. This Version: January 2009

Abstract

We analyze the properties of credible equilibria in dynamic settings with privately informed agents, capital accumulation and a lack of societal commitment. We show that a lack of commitment tilts the social tradeoff between equality and incentives towards the former and has ambiguous implications for capital accumulation. We isolate forces that promote and retard capital accumulation in these settings, derive the pattern of intertemporal wedges that characterize optimal credible allocations and show that these allocations solve the problem of a committed pseudo-planner with perturbed preferences and production possibilities. We obtain implementations of credible societal optima that feature progressive taxes on capital. Finally, we derive asset pricing implications of no commitment-private information models.

1 Introduction

A large body of recent research utilizes dynamic private information models. Such models underpin a growing normative literature on optimal policy design and have been the basis of positive analyses of risk sharing and asset pricing. We

*This paper previously circulated as "Output, inequality and taxation in uncommitted societies". We thank seminar participants at Princeton, UCSB and the FRB Richmond. We especially thank Marco Basetto for his comments.

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contribute to this literature by deriving properties of societal optima in dynamic settings with privately informed agents, capital accumulation and a lack of societal commitment. We describe a tax-market implementation for these optima and use it to derive prescriptions for optimal asset taxation. Additionally, we outline the positive content of such optima for asset pricing under the assumption that they are implemented by some tax-contracting arrangement.

The concrete environment we consider features a continuum of infinitely-lived agents whose effort is combined with capital to produce output. These agents receive private shocks to their disutility of effort. Future utility rewards and sanctions may be used to induce those with low effort costs to identify themselves and accept a larger current effort assignment. Consequently, the societies in our model confront two intertemporal tradeoffs. First, they must choose how much output they will collectively set aside as capital. Second, they must choose the extent to which future utility rewards and sanctions are used to provide incentives for current shock revelation. The latter tradeoff underpins a societal commitment problem since these rewards and sanctions translate into future inequality that a society may be tempted to undo ex post.

To model a lack of societal commitment, we suppose that allocations are implemented by an uncommitted planner playing a dynamic game with agents.[1] In this game, the planner is deterred from undoing inequality ex post by the disruption to future agent incentives that this will cause. In particular, if a planner defection leads agents to believe that future utility rewards and sanctions will not be honored, then incentives for truthful reporting are undermined. The implied equilibrium restrictions on allocations include conditions that ensure agents are motivated to be truthful and the planner is motivated to continue implementing the allocation. The latter translate into a sequence of *societal credibility constraints* that place lower bounds on continuation societal payoffs at each date. We call allocations that satisfy these conditions credible and focus on optimal (according to some societal criterion) credible allocations. Absent credibility constraints, optimal allocations imply ever increasing inequality; binding credibility constraints arrest this trend and dilute incentives.

The implications of these constraints for capital accumulation are more ambiguous. If additional capital at $t+1$ tightens

[1] Sleet-Yeltekin (2008a) consider an explicit political economy model with probabilistic voting over allocations. This model is equivalent to one with an uncommitted utilitarian planner: it gives rise to the same set of equilibrium allocations. To simplify the exposition, we directly assume such a planner in the current paper.

the $t+1$- credibility constraint, an additional shadow cost of capital accumulation is introduced at t. To restore credibility after such an accumulation, inequality at $t+1$ must be reduced, diluting incentives at t. Optimal (credible) capital accumulation is correspondingly suppressed. If, on the other hand, capital accumulation relaxes future credibility constraints, then it has an additional societal benefit and is increased.[2] We use a normalized measure of the marginal value of additional capital which we label the *credibility factor* to summarise these effects. This variable features prominently in the first order conditions that describe individual and societal tradeoffs at the social optimum; it appears in wedge, tax and asset pricing formulas.

We use three two-period economies to illustrate the main themes of the paper. The first features a committed *benchmark planner* who has the same preferences as the agents; the second a committed *paternalistic planner* who discounts the future less heavily than the agents. In the third, an *uncommitted planner* with a second period outside utility option $W(K_2)$ makes choices. This planner cannot commit to implementing a continuation allocation $\{c_2, e_2, K_2\}$ of consumption, effort and capital with a payoff below $W(K_2)$.

Since the paternalistic planner attaches greater weight to second period costs and benefits than the benchmark planner, she is correspondingly more willing to accumulate capital and less willing to use second period inequality to provide first period incentives. The uncommitted planner and the benchmark planner have the same preferences. However, we show that the former's choices coincide with those of a committed paternalist using a perturbed production function with a reduced marginal product of capital. We call the uncommitted planner's committed alter ego the *pseudo-planner*. The pseudo-planner's resolution of the incentives-inequality tradeoff resembles that of a paternalist (with unperturbed technology), but her lower effective marginal product of capital makes her less willing to substitute current for future aggregate consumption.

The optimal allocations in each of our examples satisfy a *conditional inverted Euler equation* (CIEE). These equations are reminiscent of traditional Euler equations, but they are expressed in terms of *reciprocals* of marginal utilities rather than the marginal utilities themselves. They hold for each agent at each date conditional on the agent's past history. In the

[2] There are indirect affects on capital accumulation that operate through the aggregate labor input. If credibility considerations impede incentive provision and supress this, then steady state capital is reduced.

benchmark commitment environment, binding incentive compatibility constraints and the CIEE imply a wedge between an agent's expected intertemporal marginal rate of substitution and the intertemporal shadow price of resources.[3] In the no commitment environment, this *private incentive effect* is supplemented with a *societal credibility effect*. We show that the latter has both an aggregate and a distributional component. If a small additional accumulation of capital is used to raise the utility of all agents equally in period 2, it increases both the second period utility of the planner and the planner's outside option. If it increases the latter by more than the former, the credibility constraint is tightened and the wedge increased. If, instead, the accumulation is rebated to a rich agent, the increase in the planner's second period utility is dampened and the credibility constraint further tightened. Consequently, saving by the rich incurs an additional shadow cost and is more likely to imply a positive intertemporal wedge. In contrast, saving by the poor tends to mitigate second period inequality, relax the credibility constraint and imply a smaller wedge.

We derive the *normative implications* of this pattern of wedges for taxes. In an environment with binding credibility constraints, capital taxation is progressive in the sense that an agent's expected marginal asset tax at $t+1$ is increasing in her consumption at t. Thus, consistent with our description of intertemporal wedges, the accumulation of poorer agents is subsidized, while that of richer agents is taxed. In the benchmark commitment case, conditional expected marginal asset taxes are zero for all agents and do not exhibit progressivity.[4] Farhi and Werning (2007) have previously shown that capital taxes are progressive in paternalistic settings and this is true in our second example. In addition, all agents in this setting face an expected capital subsidy. Since the paternalistic planner discounts the future less heavily than agents, implementation of its optimal allocation requires the promotion of saving via asset subsidies for all. In contrast in credibility-constrained environments, asset subsidies are only available for the poor.

Although most recent work on dynamic private information environments has been normative and prescriptive, some contributors (notably, Ligon (1998) and Kocherlakota-Pistaferri (2007a, 2007b)) have used such models to explain empirical patterns of risk sharing and asset prices. In this spirit, we derive asset pricing implications of the optimal allocations in

[3] Golosov et al (2003) provide a very general derivation of this result.

[4] See Albanesi-Sleet (2006) and Kocherlakota (2005) for earlier and, in the case of the latter, more general derivations of this result.

our environments. In each, the optimal allocation satisfies an *unconditional* inverted Euler equation (UIEE) that integrates out individual histories. These UIEEs provide joint restrictions on intertemporal shadow prices and population moments of consumption; in settings with aggregate public shocks, they deliver shadow asset pricing kernels. Kocherlakota-Pistaferri (2007b) derive such an asset pricing kernel in an environment with commitment and argue that it offers a potential resolution of the equity premium puzzle. We show that the asset pricing kernel from a credibility-constrained economy augments that from the commitment settings with a stochastic version of the credibility factor. We suggest ways in which this may help resolve asset pricing puzzles.

In the remainder of the paper, we extend the model to infinite period horizon settings and formalize credibility as an equilibrium concept in a repeated game played by an uncommitted planner and privately informed agents. This allows us to explicitly derive societal credibility constraints as equilibrium restrictions. The set of credible allocations typically excludes the (infinite horizon) optimal allocation with commitment. We provide an example in which this allocation features all agents ultimately being absorbed by either an immiserating state in which they consume nothing and work a maximal amount or a high utility state in which they do not work at all. Such limiting inequality is not credible.

As in the two period examples, we focus on optimal credible allocations and show that they solve the problem of a committed pseudo-planner whose preferences and technology are perturbed relative to those of the true planner. We obtain first order necessary conditions from this (concave) pseudo-planning problem and use them to generate expressions for intertemporal wedges, taxes and asset prices. These expressions are qualitatively the same as the two period case, but are now defined in terms of an endogenous outside utility option. In particular, we obtain an expression for the credibility factor in terms of allocations on and off the equilibrium path. We also derive a first order condition for steady state capital, contrast it with the corresponding expression for the commitment case and identify the channels via which credibility affects steady state capital and output. We conclude with a quantitative assessment of optimal taxes for the case with log preferences and Cobb-Douglas technology.

Literature In the large literature on credibility in Ramsey settings, the government is tempted to impose a tax on assets ex post because the private accumulation decision is sunk. Ex post asset taxation is an attempt to replicate lump sum taxation which is excluded by assumption. In contrast, in our dynamic Mirrleesian models, the credibility problem is distributional in nature. It stems from the society's temptation to undo inequality after its prospect has been used to motivate agents. The commitment friction in our paper has implications for the progressivity of taxation that by assumption are excluded from Ramsey analyses.

Our prior papers, Sleet-Yeltekin (2006, 2008a), consider related issues in the context of models without capital and, thus, incorporate only one of the two intertemporal societal tradeoffs present in the current paper. They do not provide implications for taxes and asset prices. In independent work, Farhi-Werning (2008) augment the political economy setup in Sleet-Yeltekin (2008a) with capital. They derive related results to those obtained in this paper.

Acemoğlu, Golosov and Tsyvinski (2008) considers similar questions to us, but arrives at very different answers. They focus on settings in which credibility constraints bind temporarily, whereas in our model the credibility constraints bind eventually (and in some cases always). For example, they consider situations in which a non-benevolent planner-government is tempted to abscond with as much current output as she can extract. Modulo planner rents, the optimal allocation they solve for is constrained Pareto efficient, but it may involve arbitrarily low continuation utilitarian payoffs. Such allocations would not be credible in our setup.

Hassler et al (2003) lies between our paper and the traditional Ramsey literature. It features private information and credibility frictions. The focus is on Markov perfect equilibria in which a distribution of utility promises serves as a state variable. This is made "payoff relevant" via exogenous restrictions on the tax system that are similar in spirit to the Ramsey literature. In particular, restrictions on taxes imply that a society cannot default on prior promises without disrupting current incentives. In contrast, our model imposes no exogenous restrictions on taxes that tie defaults on promises to the provision of current incentives. Instead a default is disruptive because it leads agents to anticipate future defaults.

2 Three simple economies

We use three simple examples to convey the essential themes of our paper.

2.1 Common elements

Shocks and Preferences Our economies are inhabited by a continuum of two period-lived agents. Agents consume and exert effort in each period of their life. They receive *private* shocks $\theta_t \in \Theta = \{\widehat{\theta}_1, \widehat{\theta}_2\}$, $\widehat{\theta}_1 < \widehat{\theta}_2$, that affect their disutility from leisure.[5] These shocks are assumed to be i.i.d. with distribution π. Probability distributions over histories of shocks $\theta^t = \{\theta_r\}_{r=1}^t$ are denoted π^t. We appeal to a law of large numbers (see, for e.g. Sun (2006)) and interpret $\pi^t(E)$ as the fraction of agents with shock history $E \subset \Theta^t$. Agent preferences over consumption-effort streams are given by:

$$\sum_{t=1}^{2} \beta^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi^t(\theta^t),$$

where $u : \mathbb{R}_+ \to \mathbf{D} \subset \mathbb{R} \cup \{-\infty\}$ and $v : [0, T] \to \mathbf{L} \subset \mathbb{R}$ are the agent's current utility from consumption and disutility from effort. $\mathbf{D}$ and $\mathbf{L}$ denote the ranges of u and v respectively; T is the agent's total per period time endowment. u and v are assumed to be continuously differentiable on the interior of their domains and strictly concave, u and $-v$ are assumed to be strictly increasing. The private preference shocks perturb disutility from effort. Agents prefer to work less if they draw the high shock value $\widehat{\theta}_2$ and a benevolent planner would prefer to work them less. $\beta \in (0, 1)$ is a discount factor.

Technologies and constraints In each period t, the aggregate labor input $L_t = \sum_{\Theta^t} e_t(\theta^t)\pi^t(\theta^t)$ is combined with aggregate capital K_t to produce output. The initial capital endowment K_1 is given. Output in each period is given by:

$$Y_t = F(K_t, L_t)$$

[5] θ may be interpreted as a health shock that makes work more or less costly or, with a slight respecification of the model, a productivity shock.

where $F : \mathbb{R}_+ \times [0,T] \to \mathbb{R}_+$ is smooth, strictly concave and increasing. We assume some $\widehat{K} > 0$ such that for $K > \widehat{K}$, $F(K,T) < K$ and for $K \leq \widehat{K}$, $F(K,T) \geq K$. An allocation $\{\{c_t, e_t\}_{t=1}^2, K_2\}$ is resource-feasible given K_1 if for $t = 1, 2$,

$$\sum_{\Theta^t} c_t(\theta^t)\pi^t(\theta^t) + K_{t+1} \leq F(K_t, L_t), \tag{1}$$

where $K_3 := 0$. Given that taste shocks are private and agents must be motivated to reveal them, allocations must satisfy the following temporary incentive constraints, for period 1 and each i and $j \neq i$,

$$\begin{aligned} u(c_1(\widehat{\theta}_i)) + \widehat{\theta}_i v(e_1(\widehat{\theta}_i)) + \beta \sum_{k=1}^{2} [u(c_2(\widehat{\theta}_i, \widehat{\theta}_k)) + \widehat{\theta}_k v(e_2(\widehat{\theta}_i, \widehat{\theta}_k))]\pi(\theta_k) \geq \\ u(c_1(\widehat{\theta}_j)) + \widehat{\theta}_i v(e_1(\widehat{\theta}_j)) + \beta \sum_{k=1}^{2} [u(c_2(\widehat{\theta}_j, \widehat{\theta}_k)) + \widehat{\theta}_k v(e_2(\widehat{\theta}_j, \widehat{\theta}_k))]\pi(\theta_k) \end{aligned} \tag{2}$$

and for period 2 and each k, i and $j \neq i$,

$$u(c_2(\widehat{\theta}_k, \widehat{\theta}_i)) + \widehat{\theta}_i v(e_2(\widehat{\theta}_k, \widehat{\theta}_i)) \geq u(c_2(\widehat{\theta}_k, \widehat{\theta}_j)) + \widehat{\theta}_i v(e_2(\widehat{\theta}_k, \widehat{\theta}_j)). \tag{3}$$

Condition (2) ensures that it is optimal for an agent to be truthful in the first period given that he is truthful in the second; (3) ensures that the agent is indeed truthful in the second period. We index these constraints by their dates and histories of shocks so that (2) is labelled the $(1, i)$-th constraint and (3) the $(2, k, i)$-th constraint.

2.2 Environment 1: Commitment

Optimal allocations and wedges To begin with, we assume that the planner is utilitarian and can commit to allocations. Given K_1, she selects an allocation $\{\{c_t, e_t\}_{t=1}^2, K_2\}$ to solve

$$\sup_{\{\{c_t,e_t\}_{t=1}^2, K_2\}} \sum_{t=1}^{2} \beta^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi^t(\theta^t) \tag{4}$$

subject to (1), (2), (3) and the constraints c_t, $K_2 \geq 0$ and $e_t \in [0,T]$. Denoting the shadow price of resources in period t by q_t, the Lagrange multiplier on the period (t, θ^t)-th incentive constraint by $\eta_t(\theta^t)$ and the optimal allocation with $*$'s, the first order conditions for consumption are given by:

$$q_1 = \left[1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right] u'(c_1^*(\widehat{\theta}_i)) \tag{5}$$

and

$$q_2 = \left[1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)} + \eta_2(\widehat{\theta}_i, \widehat{\theta}_k) - \eta_2(\widehat{\theta}_i, \widehat{\theta}_l)\frac{\pi(\widehat{\theta}_l)}{\pi(\widehat{\theta}_k)}\right] \beta u'(c_2^*(\widehat{\theta}_i, \widehat{\theta}_k)). \tag{6}$$

Anticipating our later recursive formulations, we interpret the planner as assigning all agents the Pareto weight $\gamma_1 = 1$ in the initial period and then updating this to the *effective Pareto weight* of $\gamma_2 = 1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}$ in the second period. In both periods the distribution of effective Pareto weights has a cross sectional mean of 1. However, in the second period, they are more dispersed capturing the inequality in continuation Pareto weights, utilities and consumption required for the optimal provision of incentives in period 1. This dispersion is desirable ex ante, but ex post it is costly for a utilitarian planner to provide. This is the basis of the commitment problem considered below.

The first order condition for K_2 is:

$$-q_1 + q_2 F_K(K_2^*, L_2^*) = 0. \tag{7}$$

Together (5), (6) and (7) imply the well known *conditional inverted Euler equation*:

$$E\left[\frac{1}{u'(c_2^*)}\middle|\widehat{\theta}_i\right] = \beta F_K(K_2^*, L_2^*)\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right], \tag{8}$$

where $E\left[\cdot|\widehat{\theta}_i\right]$ denotes an expectation conditional on $\theta_1 = \widehat{\theta}_i$. (8) implies, after an application of Jensen's inequality,

$$u'(c_1^*(\widehat{\theta}_i)) \leq F_K(K_2^*, L_2^*)\beta E\left[u'(c_2^*)|\widehat{\theta}_i\right],$$

with strict inequality if one or more of the period 2 incentive constraints are binding. Thus there is a wedge between intertemporal marginal rates of substitution $\beta\frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))}$ and the intertemporal marginal rate of transformation $\frac{1}{F_K(K_2^*,L_2^*)}$. Implementation of this allocation in a market economy requires an institution that prevents agents from equating their intertemporal marginal rates of substitution to $\frac{1}{F_K(K_2^*,L_2^*)}$. The tax code provides one such institution. However, as Albanesi-

Sleet (2006) and Kocherlakota (2005) (Collectively: ASK) have shown, there are some subtleties in moving between wedges and taxes. For benchmarking purposes, we briefly elaborate these.

Normative implications for taxes Consider a two period market economy with taxes. In the first period, markets for effort and (pre-tax) riskless claims to period two consumption open. Prices in these markets are, respectively, denoted w_1 and Q. In the second period, an effort market with wage w_2 opens. Agents enter period 1 with an identical bundle of b_1 riskless claims to current consumption, they trade effort e_1 and period 2 claims b_2, pay taxes and consume. In the second period, they trade only effort e_2, pay taxes and consume. In periods $t = 1, 2$, the planner-government imposes a tax schedule:

$$T_t(e^t, b_t) = \tilde{\tau}_t^0(e^t) + \tilde{\tau}_t^1(e^t)b_t, \tag{9}$$

where $\tilde{\tau}_t^0$ and $\tilde{\tau}_t^1$ depend on the agent's history of efforts e^t. We seek a tax system, initial endowment of claims b_1 and prices $\{\{w_t\}_{t=1}^2, Q\}$ that implement the allocation $\{\{c_t^*, e_t^*\}_{t=1}^2, K_2^*\}$ as a competitive equilibrium at the initial capital stock K_1. In particular, we require that $\{c_t^*, e_t^*\}_{t=1}^2$ along with some $b_2^* : \Theta \to \mathbb{R}$ solves

$$\sup_{\{c_t,e_t\}_{t=1}^2, b_2} \sum_{t=1}^2 \beta^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi^t(\theta^t)$$

subject to:

$$\begin{aligned} b_1 + w_1 e_1(\theta_1) &= c_1(\theta_1) + T_1(e_1(\theta_1), b_1) + Q_2 b_2(\theta_1) \\ b_2(\theta_1) + w_2 e_2(\theta_1, \theta_2) &= c_2(\theta_1, \theta_2) + T_2(e_2(\theta_1, \theta_2), b_2(\theta_1)). \end{aligned}$$

We identify the period t wage with the period t marginal product of labor: $w_1 = F_L(K_1, L_1^*)$, $w_2 = F_L(K_2^*, L_2^*)$, and the asset price with the reciprocal of the marginal product of capital or, since the two are equal, the societal intertemporal shadow price: $Q = [F_K(K_2^*, L_2^*)]^{-1} = \frac{q_2}{q_1}$. As ASK have shown, to induce the desired effort and consumption choices with tax schedules of the form (9), it is essential that the period 2 marginal asset tax $\tilde{\tau}_2^1$ depends on period 2 effort. In particular, defining τ_2^1 according to:

$$\forall \theta^2, \qquad \tau_2^1(\theta^2) = \tilde{\tau}_2^1(e_2^*(\theta^2)),$$

it is necessary that:

$$\forall \theta^2, \qquad Qu'(c_1^*(\theta_1)) = \beta(1 - \tau_2^1(\theta^2))u'(c_2^*(\theta^2)). \tag{10}$$

Combining this expression with (8) implies that the conditional expectation of period 2 asset taxes is zero:

$$\forall \theta, \qquad \sum_{\Theta} \tau_2^1(\theta, \theta')\pi(\theta') = 0.$$

Positive implications for asset prices We now take a more agnostic approach to implementation and ask: if the allocation is implemented by some mixture of taxes and private contracts, what are the implications for the prices of assets that pay out conditional on public observables?[6]

We continue to identify the price of a (pre-tax) riskless asset with the societal intertemporal shadow price. The unconditional inverted Euler equation obtained by integrating our individual histories in (8) ties this price to moments of the reciprocal of marginal utilities:

$$QE\left[\frac{1}{u'(c_2^*)}\right] = \beta E\left[\frac{1}{u'(c_1^*)}\right]. \tag{11}$$

Applying an appropriate law of large numbers, this expression can be interpreted as a joint restriction on the shadow (or, in an implementation, market) price of riskless claims Q and the cross sectional distribution of consumption.

A richer set of asset pricing implications can be obtained by extending the above framework to include a publicly observable aggregate shock Z in period 2. These shocks could, in principle, be to productivity, government spending or they could be common and publicly observable effort taste shocks. Analogous to the case without aggregate shocks, we identify the market pricing kernel Q of Z-contingent assets with the shadow societal price $\frac{q_2(Z)}{q_1}$. Then:

$$Q(Z)E\left[\frac{1}{u'(c_2^*)}\middle| Z\right] = \beta E\left[\frac{1}{u'(c_1^*)}\right]\Lambda(Z), \tag{12}$$

[6] Golosov-Tsyvinski (2006) emphasize that with commitment the optimal allocation can be implemented with taxes or with private contracts. Kocherlakota-Pistaferri (2007b), in their analysis of the implications of private information for asset pricing, omit a description of the full implementation. Their only requirement is that markets for assets contingent on public observables exist. We follow them.

where $\Lambda(Z)$ is the probability of Z, $Q(Z) = \frac{\Lambda(Z)}{F_K(K_2^*, L_2^*(Z), Z)}$ and $F_K(K_2^*, L_2^*(Z), Z)$ is the marginal product of capital extended to depend on Z. The associated stochastic discount factor, $\mathcal{M}^C$, is:

$$\mathcal{M}^C(Z) = \frac{\beta E\left[\frac{1}{u'(c_1^*)}\right]}{E\left[\frac{1}{u'(c_2^*)}\middle| Z\right]}.$$

Kocherlakota-Pistaferri (2007b) derive such a stochastic discount factor in a commitment economy with more general shock processes and CRRA preferences. Extending the implementation to incorporate a market for stocks whose returns R^S are contingent on aggregate shocks and riskless bonds whose returns R^B are not, they derive the equity premium condition:

$$0 = E[(R^S - R^B)\mathcal{M}^C]. \tag{13}$$

Kocherlakota-Pistaferri ask whether the expression (13) can reconcile the observed equity premium and consumption data with plausible assumptions about the risk aversion of agents. They argue that it can.

2.3 Environment 2: Paternalism

Optimal allocations and wedges We now assume that the planner uses the discount factor $\widehat{\beta} = \beta(1+\phi)$ rather than β to evaluate future agent payoffs. "The planner knows best" and solves:

$$\sup_{\{\{c_t,e_t\}_{t=1}^2, K_2\}} \sum_{t=1}^{2} [\beta(1+\phi)]^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi^t(\theta^t)$$

subject to the same constraints as before.[7] The first order conditions from this planning problem are now (5), (7) and

$$q_2 = \left[\frac{\phi}{1+\phi} + \frac{1}{1+\phi}\left[1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right] + \eta_2(\widehat{\theta}_i, \widehat{\theta}_k) - \eta_2(\widehat{\theta}_i, \widehat{\theta}_l)\frac{\pi(\widehat{\theta}_l)}{\pi(\widehat{\theta}_k)}\right]\beta(1+\phi)u'(c_2^*(\widehat{\theta}_i, \widehat{\theta}_k)). \tag{14}$$

Only, the latter first order condition for period 2 consumption has changed. Again anticipating our later recursive formulation, we interpret $\frac{\phi}{1+\phi} + \frac{1}{1+\phi}\left[1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right]$ as an updated Pareto weight for period 2. Now the dispersion in period

[7] See Phelan (2006) and Farhi and Werning (2007) for an alternative interpretation in an integenerational context. In this, the planner attaches weight 1 to an initial generation of one period-lived agents who discount their period 2 descendants by β and a weight $\beta\phi$ to the descendants.

2 effective Pareto weights induced by the provision of period 1 incentives is moderated by the $\frac{1}{1+\phi}$ term. Intuitively, the paternalistic planner is more patient than the agents and more averse to inequality in period 2. She is willing to trade less inequality in period 2 off against less insurance in period 1.

The conditional inverted Euler equation in this setting is:

$$q_2 E\left[\frac{1}{u'(c_2^*)}\middle|\widehat{\theta}_i\right] = \beta(1+\phi)\left[\frac{\phi}{1+\phi} + \frac{1}{1+\phi}\left(\frac{q_1}{u'(c_1^*(\widehat{\theta}_i))}\right)\right]. \tag{15}$$

Combining the first order conditions for period 1 consumption, (7), (15) and Jensen's inequality yields:

$$1 \leq \beta(1+\phi)F_K(K_2^*, L_2^*)H(\widehat{\theta}_i)\frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))}, \tag{16}$$

where $H(\widehat{\theta}_i) = \frac{\phi}{1+\phi}E\left[\frac{1}{u'(c_1^*)}\right]\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right]^{-1} + \frac{1}{1+\phi}$. (16) has indeterminate implications for the sign of the intertemporal wedge $\beta F_K(K_2^*, L_2^*)\frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))} - 1$. We identify three distinct forces. First, the standard period 2 incentive effect present in Environment 1 is present here. This works in the direction of a positive wedge and ensures the inequality in (16). Second, the higher planner discount factor introduces the $1+\phi$ term into (16) and works in the opposite direction. Third, when the period 1 incentive constraints are binding and there is variation in agent consumption in period 1, $H(\widehat{\theta}_1) < 1 < H(\widehat{\theta}_2)$. For agents who receive the low taste shock $\widehat{\theta}_1$ in period 1, work harder and have higher consumption, the H term contributes to a positive wedge. For other agents, it contributes to a negative one. Intuitively, if the paternalistic planner was to decentralize her preferred allocation using markets and taxes, her concern with period 2 incentives, which are undermined by higher period 2 consumption, would work in the direction of deterring agent saving, while the greater value the planner attaches to period 2 utility would work in the direction of encouraging it. Her desire to avoid too much period 2 inequality works in the direction of encouraging saving amongst the period 1 poor (the $\widehat{\theta}_2$-shock agents) and discouraging it amongst the rich (the $\widehat{\theta}_1$-shock agents).

Normative implications for taxes Although the model has indeterminate implications for wedges, it has sharper ones for marginal asset taxes. In particular, the definition of market prices, (10) and (15) imply that:

$$E\left[\tau_2^1\middle|\,\widehat{\theta}_i\right] = -\phi\frac{u'(c_1^*(\widehat{\theta}_i))}{q_1}. \tag{17}$$

Hence, conditional expected marginal asset taxes are negative for all agents and are increasing in an agent's period 1 consumption. In this sense they are progressive. This result was first derived by Farhi and Werning (2007).

Positive implications for asset prices Once more we identify the price of a riskless claim Q with the societal intertemporal shadow price, $\frac{q_2}{q_1}$. (15) and the first order condition for period 1 consumption then imply the following *unconditional* inverted Euler equation:

$$QE\left[\frac{1}{u'(c_2^*)}\right] = \beta(1+\phi)E\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right]. \tag{18}$$

In the extended environment with public aggregate shocks considered by Kocherlakota-Pistaferri, the analogue of (18) is:

$$Q(Z)E\left[\frac{1}{u'(c_2^*)}\middle|\,Z\right] = \beta(1+\phi)E\left[\frac{1}{u'(c_1^*)}\right]\Lambda(Z), \tag{19}$$

where Q is now an asset pricing kernel. Thus, the stochastic discount factor in this case, $\mathcal{M}^P$, equals that from the commitment environment scaled by the constant $1+\phi$, i.e. $\mathcal{M}^P = (1+\phi)\mathcal{M}^C$. Consequently, this model places the same restrictions on the equity premium and consumption as does the commitment model, i.e.

$$0 = E[(R^S - R^B)\mathcal{M}^P] = E[(R^S - R^B)\mathcal{M}^C]. \tag{20}$$

Kocherlakota-Pistaferri's (2007b) assessment that $\mathcal{M}^C$ better reconciles equity premium and consumption data with plausible agent preferences than do standard stochastic discount factors extends to $\mathcal{M}^P$.

2.4 Environment 3: Credibility-constrained

Optimal allocations and wedges Instead of altering the planner's preferences as in the paternalistic case, we now alter her constraints relative to the initial commitment environment. In particular, we supplement the constraint set with the additional *credibility constraint*:

$$\sum_{\Theta^2} \left[u(c_2(\theta^2)) + \theta_2 v(e_2(\theta^2))\right] \pi^2(\theta^2) \geq W(K_2), \tag{21}$$

where W is a smooth, increasing and concave function. Later, in an infinite horizon setting, we explicitly derive the function W as the worst equilibrium payoff function in a dynamic game played by the planner and the agents. Conditions of the form (21) then emerge as equilibrium incentive constraints for *the planner.* For now, however, we take the function W as given and simply assume that the planner solves (4) subject to the additional constraint (21).

The next proposition connects the limited commitment planning problem to the previous planning environments. It allows us to decompose and interpret the impact of the credibility constraints.

Proposition 1 *Let $\{\{c_t^*, e_t^*\}_{t=1}^2, K_2^*\}$ be a solution to the planner's problem with credibility constraints. There is some $\phi \geq 0$ such that $\{\{c_t^*, e_t^*\}_{t=1}^2, K_2^*\}$ is the unique solution to:*

$$\max_{\{\{c_t, e_t\}_{t=1}^2, K_2\}} \sum_{t=1}^{2} [\beta(1+\phi)]^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi^t(\theta^t)$$

subject to the incentive constraints, the first period resource constraint and the modified second period resource constraint:

$$\widehat{F}\left(K_2, \sum_{\Theta^2} e_2(\theta^2)\pi^2(\theta^2)\right) - \sum_{\Theta^2} c_2(\theta^2)\pi^2(\theta^2),$$

where $\widehat{F}(K, L) = F(K, L) - \frac{\beta\phi}{q_2} W_K(K_2^*)(K - K_2^*)$.

Proof: Let $L_t^* = \sum_{\Theta^t} e_t^*(\theta^t)\pi^t(\theta^t)$. There exists a vector of multipliers $\Gamma = \{\beta\phi,\ q_1,\ q_2,\ \{w_t\},\ \{\eta_{1,i,j}\}_{i,j\in\mathbb{I}},\ \{\beta(1+\phi)\eta_{2,i,j}(\theta)\}_{\theta\in\Theta, i,j\in\mathbb{R}}\}$, $\mathbb{I} = \{(i,j) : i \in \{1,2\},\ j \in \{1,2\} \setminus \{i\}\}$ such that $\{\{c_t^*, e_t^*, L_t^*\}_{t=1}^2, K_2^*\}$ is a stationary point of the Lagrangian $\mathcal{L}(\{\{c_t, e_t, L_t\}_{t=1}^2, K_2\}, \Gamma)$ and that $\{\{c_t^*, e_t^*, L_t^*\}_{t=1}^2, K_2^*\}$ and Γ satisfy relevant complementary slackness conditions. The Lagrangian $\mathcal{L}$ is defined by:

$$\mathcal{L}(\{\{c_t, e_t, L_t\}_{t=1}^2, K_2\}, \Gamma) = \sum_{t=1}^{2} \beta^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi^t(\theta^t) \tag{22}$$

$$+\beta\phi \left[\sum_{\Theta^2} \left[u(c_2(\theta^2)) + \theta_2 v(e_2(\theta^2))\right] \pi^2(\theta^2) - W(K_2)\right]$$

$$+q_1 \left[F(K_1, L_1) - \sum_{\Theta} c_1(\theta)\pi(\theta) - K_2\right] + q_2 \left[F(K_2, L_2) - \sum_{\Theta^2} c_2(\theta^2)\pi^2(\theta^2)\right] + \sum_{t=1}^{2} w_t \left[\sum_{\Theta^t} e_t(\theta^t) - L_t\right]$$

$$+ \sum_{i,j\in\mathbb{I}} \eta_{1,i,j} \left[u(c_1(\widehat{\theta}_i)) + \widehat{\theta}_i v(e_1(\widehat{\theta}_i)) + \beta \sum_{\Theta} [u(c_2(\widehat{\theta}_i, \theta)) + \widehat{\theta}_k v(e_2(\widehat{\theta}_i, \theta))]\pi(\theta)\right.$$

$$\left. -u(c_1(\widehat{\theta}_j)) - \widehat{\theta}_i v(e_1(\widehat{\theta}_j)) - \beta \sum_{\Theta} [u(c_2(\widehat{\theta}_j, \theta)) + \widehat{\theta}_k v(e_2(\widehat{\theta}_j, \theta))]\pi(\theta)\right]$$

$$+\beta \sum_{\Theta} \sum_{i,j\in\mathbb{I}} (1+\phi)\eta_{2,i,j}(\theta) \left[u(c_2(\theta, \widehat{\theta}_i)) + \widehat{\theta}_i v(e_2(\theta, \widehat{\theta}_i)) - u(c_2(\theta, \widehat{\theta}_j)) - \widehat{\theta}_i v(e_2(\theta, \widehat{\theta}_j))\right] \pi(\theta)\pi(\widehat{\theta}_i)$$

The first order conditions from this Lagrangian include:

$$\begin{aligned} 0 &= q_2 F_K(K_2^*, L_2^*) - \beta\phi W_K(K_2^*) - q_1, \\ 0 &= q_t F_L(K_t^*, L_t^*) - w_t. \end{aligned}$$

Consequently, (K_2^*, L_2^*) solves the concave problem $\max F(K_2, L_2) - \frac{\beta\phi}{q_2} W_K(K_2^*)(K_2 - K_2^*) - \frac{q_1}{q_2} K_2 - \frac{w_2}{q_2} L_2$, L_1^* solves $\max F(K_1, L_1) - \frac{w_1}{q_1} L_1$ and $\{c_t^*, e_t^*\}_{t=1}^2$ solves the problem:

$$\sum_{t=1}^{2} [\beta(1+\phi)]^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi^t(\theta^t)$$

$$+q_1 \left[\frac{w_1}{q_1} \sum_{\Theta} e_1(\theta)\pi(\theta) - \sum_{\Theta} c_1(\theta)\pi(\theta)\right] + q_2 \left[\frac{w_2}{q_2} \sum_{\Theta^2} e_2(\theta^2)\pi^2(\theta^2) - \sum_{\Theta^2} c_2(\theta^2)\pi^2(\theta^2)\right]$$

$$+ \sum_{i,j\in\mathbb{I}} \eta_{1,i,j} \left[u(c_1(\widehat{\theta}_i)) + \widehat{\theta}_i v(e_1(\widehat{\theta}_i)) + \beta \sum_{\Theta} [u(c_2(\widehat{\theta}_i, \theta)) + \widehat{\theta}_k v(e_2(\widehat{\theta}_i, \theta))]\pi(\theta)\right.$$

$$\left. -u(c_1(\widehat{\theta}_j)) - \widehat{\theta}_i v(e_1(\widehat{\theta}_j)) - \beta \sum_{\Theta} [u(c_2(\widehat{\theta}_j, \theta)) + \widehat{\theta}_k v(e_2(\widehat{\theta}_j, \theta))]\pi(\theta)\right] \pi(\widehat{\theta}_i)$$

$$+\beta \sum_{\Theta} \sum_{i,j\in\mathbb{I}} (1+\phi)\eta_{2,i,j}(\theta) \left[u(c_2(\theta, \widehat{\theta}_i)) + \widehat{\theta}_i v(e_2(\theta, \widehat{\theta}_i)) - u(c_2(\theta, \widehat{\theta}_j)) - \widehat{\theta}_i v(e_2(\theta, \widehat{\theta}_j))\right] \pi(\theta)\pi(\widehat{\theta}_i).$$

This last problem can be rendered (strictly) concave by re-expressing it as a choice over utilities from consumption and effort. It follows that $\{\{c_t^*, e_t^*\}_{t=1}^2, K_2^*\}$ uniquely solves

$$\max \sum_{t=1}^{2} [\beta(1+\phi)]^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi^t(\theta^t)$$

subject to the incentive constraints, the first period resource constraint and the modified second period resource constraint:

$$\widehat{F}\left(K_2, \sum_{\Theta^2} e_2(\theta^2)\pi^2(\theta^2)\right) - \sum_{\Theta^2} c_2(\theta^2)\pi^2(\theta^2),$$

where $\widehat{F}(K,L) = F(K,L) - \frac{\beta\phi}{q_2} W_K(K_2^*)(K - K_2^*)$. ■

Thus, the optimal allocation with limited commitment is a solution to the problem of a *committed* "pseudo-planner" with perturbed preferences and a perturbed technology. The preferences of the pseudo-planner are *identical to those of a paternalistic planner who discounts the future with discount factor* $\beta(1+\phi)$, where the discount factor markup ϕ is derived endogenously from the credibility constraint shadow price. The perturbed second period technology $\widehat{F}$ used by the pseudo-planner corresponds to the true technology F pivoted and flattened around the optimal capital stock K_2^*. In the proof of the proposition the Lagrangian is used to decouple the continuation societal utility on the left hand side of the credibility constraint from the outside option on the right hand side. The former is absorbed into pseudo-planner's preferences. When the credibility constraint binds, the shadow value of continuation payoffs is raised and it is as if the societal discount factor has increased above the value applied by agents. The outside option function $W(K)$ can be replaced with the function $W(K_2^*) + W_K(K_2^*)(K_2 - K_2^*)$. This reduces the set of feasible allocations, but retains $\{\{c_t^*, e_t^*\}_{t=1}^2, K_2^*\}$ as a feasible and, hence, optimal choice. When the credibility constraint binds, the additional shadow cost of capital accumulation stemming from the $W_K(K_2^*)(K_2 - K_2^*)$ term is absorbed into the production function.

As in the paternalistic case, the effect of the perturbed effective discount factor is to suppress second period inequality at the expense of first period incentives. Given ϕ, the first order conditions for consumption are the same across the paternalistic and limited commitment environments. In particular, the conditional inverted Euler equation (15) holds in both cases. The impact of the technology perturbation is to suppress incentives for capital accumulation relative to the paternalistic problem (with discount $\beta(1+\phi)$). The joint impact of perturbed preferences and technology has an ambiguous effect on capital accumulation relative to the benchmark commitment problem that we started with. On the one hand, the higher effective patience promotes capital accumulation; on the other hand, the reduction in the effective marginal product of capital suppresses it. It is convenient to summarize these effects with the following *credibility factor:*

$$\mathcal{K} = 1 + \phi\left(\frac{W_K^*(K_2^*) - W_K(K_2^*)}{W_K^*(K_2^*)}\right) \leq 1 + \phi, \tag{23}$$

where $W^*(K_2^*)$ is the second period value function:

$$\begin{aligned} W^*(K_2) \quad = \quad & \max_{c_2, e_2} \sum_{i,j \in \mathbb{I}} \left(1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right) \sum_{\Theta} \left[u(c_2(\widehat{\theta}_i, \theta)) + \theta v(e_2(\widehat{\theta}_i, \theta))\right] \pi(\widehat{\theta}_i)\pi(\theta) \\ & + \phi \sum_{\Theta^2} \left[u(c_2(\theta^2)) + \theta_2 v(e_2(\theta^2))\right] \pi^2(\theta^2) + \frac{q_2}{\beta}\left[F\left(K_2, \sum_{\theta^2} e_2(\theta^2)\pi^2(\theta^2)\right) - \sum_{\Theta^2} c_2(\theta^2)\pi^2(\theta^2)\right] \\ & + \sum_{\Theta} \sum_{i,j \in \mathbb{I}} (1+\phi)\eta_2(\theta, \widehat{\theta}_i) \left[u(c_2(\theta, \widehat{\theta}_i)) + \widehat{\theta}_i v(e_2(\theta, \widehat{\theta}_i)) - u(c_2(\theta, \widehat{\theta}_j)) - \widehat{\theta}_i v(e_2(\theta, \widehat{\theta}_j))\right] \pi(\theta)\pi(\widehat{\theta}_i) \end{aligned}$$

and the inequality in (23) is strict if the credibility constraint binds. When additional capital relaxes the credibility constraint, $W_K^*(K_2^*) - W_K(K_2^*) > 1$ and $\mathcal{K}$ exceeds 1; when it tightens this constraint $W_K^*(K_2^*) - W_K(K_2^*) < 1$ and $\mathcal{K}$ is less than one 1. The credibility factor appears in the intertemporal wedge, asset pricing and asset tax formulas given below. In each case it relates these formula to the impact of capital accumulation on the credibility constraint.

The first order condition for capital in the credibility-constrained (or pseudo-planner) problem implies:

$$F_K(K_2^*, L_2^*) = \frac{q_1}{q_2}\frac{1+\phi}{\mathcal{K}} \geq \frac{q_1}{q_2}. \tag{24}$$

Combining (24) with (15) gives:

$$1 \leq \beta\mathcal{K}H(\widehat{\theta}_i)F_K(K_2^*, L_2^*)\frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))}. \tag{25}$$

Hence, the intertemporal wedge is positive if $\mathcal{K}H(\widehat{\theta}_i) \leq 1$ and indeterminate otherwise. We have previously noted that $H(\widehat{\theta}_1) \leq 1$ with strict inequality if $\phi > 0$. If capital accumulation (weakly) tightens the credibility constraint then $\mathcal{K} \leq 1$

and low effort cost agents face a positive intertemporal wedge. We now turn to a special case in which capital accumulation neither tightens nor relaxes the credibility constraint and $\mathcal{K} = 1$.

A special case In later sections, we consider an infinitely horizon dynamic game without planner commitment and assume that $F(K, L) = K^{\alpha}L^{1-\alpha}$, $u(c) = \ln c$ and $v(e) = \ln(T - e)$. In this case, we show that the planner's outside option function W is of the form $W(K) = \underline{W} + \alpha \ln K$. For now, we simply assume that F, u *and* W have these functional forms. This formulation allows a clean separation of aggregate choices from distributional ones. In this case, the credibility-constrained planner (and her alter ego pseudo planner) make the same aggregate choices as the benchmark commitment planner, but the same distributional choices as a paternalistic planner. In short, the credibility-constrained planner is a hybrid of the two. To see this, note that the first order conditions with respect to consumption in each period are:

$$\frac{1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}}{c_1^*(\widehat{\theta}_i)} = q_1 \quad \text{and} \quad \frac{\beta(1+\phi)}{c_2^*(\widehat{\theta}_i, \widehat{\theta}_k)}\left[\frac{\phi}{1+\phi} + \frac{1}{1+\phi}\left[1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right] + \eta_2(\widehat{\theta}_i, \widehat{\theta}_k) - \eta_2(\widehat{\theta}_i, \widehat{\theta}_l)\frac{\pi(\widehat{\theta}_l)}{\pi(\widehat{\theta}_k)}\right] = q_2$$

Rearranging these and integrating over shocks gives:

$$\frac{1}{q_1} = \sum_{i=1}^{2} c_1^*(\widehat{\theta}_i)\pi(\widehat{\theta}_i) = K_1^{\alpha}L_1^{*1-\alpha} - K_2^* \qquad \text{and} \qquad \frac{\beta(1+\phi)}{q_2} = \sum_{i,k=1}^{2} c_2^*(\widehat{\theta}_i, \widehat{\theta}_k)\pi(\widehat{\theta}_i)\pi(\widehat{\theta}_k) = K_2^{*\alpha}L_2^{*1-\alpha}.$$

Combining this with the first order condition for K_2, $q_2\alpha K_2^{*\alpha-1}L_2^{*1-\alpha} - \frac{\beta\phi\alpha}{K_2^*} - q_1 = 0$, implies:

$$K_2^* = \frac{\beta\alpha}{q_1} = \beta\alpha\left[K_1^{\alpha}L_1^{*1-\alpha} - K_2^*\right] = \frac{\beta\alpha}{1+\beta\alpha}K_1^{\alpha}L_1^{*1-\alpha}. \tag{26}$$

Assuming that for each i,

$$\frac{\widehat{\theta}_i\left[1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right]}{T} \leq q_1 w_1,$$ [8]

and the zero lower bound on effort does not bind, the first order conditions for e_1 and L_1 together with (26) imply:

$$E[\theta] = w_1(T - L_1^*) = q_1(1-\alpha)K_1^{\alpha}L_1^{*-\alpha}(T - L_1^*) = (1-\alpha)(1+\beta\alpha)\left(\frac{T - L_1^*}{L_1^*}\right).$$

[8] A sufficient condition for this is that for each i, $\widehat{\theta}_i\left[1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right] \leq E[\theta] + (1-\alpha)(1+\beta\alpha)$.

Thus, the aggregates L_1^*, K_2^* and period 1 aggregate consumption C_1^* are independent of the credibility constraint (and the incentive-compatibility constraints on agents).[9] Similarly, the aggregates L_2^* and C_2^* can be shown to be independent of these constraints. However, the credibility constraint has important distributional implications. In particular, the first order conditions for consumption imply

$$\frac{\sum_{k=1}^{2} c_2^*(\widehat{\theta}_i, \widehat{\theta}_k)\pi(\widehat{\theta}_k)}{C_2^*} = \frac{\phi}{1+\phi} + \frac{1}{1+\phi}\left[\frac{c_1^*(\widehat{\theta}_i)}{C_1^*}\right], \tag{27}$$

where $\frac{c_1^*(\widehat{\theta}_i)}{C_1^*}$ is an agent's period 1 consumption share conditional on the shock realization $\widehat{\theta}_i$, while $\frac{\sum_{k=1}^{2} c_2^*(\widehat{\theta}_i, \widehat{\theta}_k)\pi(\widehat{\theta}_k)}{C_2^*}$ is the agent's expected second period consumption share conditional on this shock realization. (27) implies that if the credibility constraint is non-binding and $\phi = 0$, then an agent's consumption share is a martingale; an innovation in this share in period 1 has a "permanent" effect on the period 2 conditional expected consumption share. If, on the other hand, the credibility constraint binds and $\phi > 1$, then the conditional expected consumption share reverts towards one in period 2. In this case, the impact of period 1 shocks on period 2 inequality is dampened.

Manipulation of the first order conditions and the resource constraint in this case gives $\mathcal{K} = 1$, so that (25) reduces to

$$1 \leq \beta F_K(K_2^*, L_2^*) H(\widehat{\theta}_i) \frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))}.$$

The lower bound on the wedge, $\frac{1}{H(\widehat{\theta}_i)} - 1$, is scaled up relative to the paternalistic case (where it equalled $\frac{1}{(1+\phi)H(\widehat{\theta}_i)} - 1$). Since $H(\widehat{\theta}_1) < 1$, the intertemporal wedge is positive for "rich" agents who receive the low taste shock in period 1.

Normative implications for taxes We now describe an implementation of the optimal credibility-constrained allocation in an economy with markets and taxes. The taxes are used both to provide incentives for truthful reporting and to induce agents to internalize the aggregate credibility constraints.

If the asset market price is set to $\frac{1}{F_K(K_2^*, L_2^*)}$ and taxes are set according to (10), then the inverted Euler equation implies:

$$1 - E\left[\tau_2^1 \middle| \widehat{\theta}_i\right] = \frac{\mathcal{K}}{1+\phi} + \left(1 - \frac{\mathcal{K}}{1+\phi}\right)\frac{u'(c_1^*(\widehat{\theta}_i))}{W_K(K_2^*)},$$

[9] Again, this is under the assumption that in none of the cases and in no state does the zero lower bound on effort bind.

or, equivalently,

$$E\left[\tau_2^1\middle|\widehat{\theta}_i\right] = \left(1 - \frac{\mathcal{K}}{1+\phi}\right)\left[1 - \frac{u'(c_1^*(\widehat{\theta}_i))}{W_K(K_2^*)}\right]. \tag{28}$$

As in the paternalistic case (17), we observe, that when the credibility constraint binds and $1-\frac{\mathcal{K}}{1+\phi} > 0$, the expected marginal asset tax is increasing in $c_1^*(\widehat{\theta}_i)$ and in this sense is progressive. Now however, the tax formula includes the additional positive constant $1 - \frac{\mathcal{K}}{1+\phi}$. Like her paternalistic counterpart, the credibility-constrained planner seeks to moderate ex post inequality, but in this case to relax the credibility constraint. This underpins the dependence of expected marginal asset taxes in period 2 on consumption in period 1. In contrast to the paternalistic case, however, capital in the second period confers an additional social cost: it raises the planner's outside option $W(K_2)$ and tightens the credibility constraint. Consequently, the credibility-constrained planner does not encourage capital accumulation to the degree that the paternalist does.[10]

In the special log-Cobb Douglas case, the conditional marginal asset tax is $E\left[\tau_2^1\middle|\widehat{\theta}_i\right] = \frac{\phi}{1+\phi}\left[1 - \frac{C_1^*}{c_1^*(\widehat{\theta}_i)}\right]$, so that the unconditional (i.e. population average) marginal tax rate is:

$$E\left[\tau_2^1\right] = \frac{\phi}{1+\phi}\left[1 - C_1^* \sum_{i=1,2} \frac{1}{c_1^*(\widehat{\theta}_i)}\pi(\theta_i)\right] < 0$$

In this case, capital accumulation is subsidized overall since the subsidies of the poor exceed the taxes of the rich. For there to be net positive asset taxation, $\frac{W_K(K_2^*)}{W_K^*(K_2^*)}$ must be sufficiently large.

We can nest the conditional expected marginal tax formulas from the three environments considered in the formula:

$$E\left[\tau_2^1\middle|\widehat{\theta}_i\right] = A_0^k - A_1^k u'(c_1^*(\widehat{\theta}_i)), \tag{29}$$

where $k = \{C, P, CC\}$ for commitment, paternalistic and credibility-constrained economies. We observe that in the commitment economy, $A_0^C = A_1^C = 0$, in the paternalistic economy $A_1^P > A_0^P = 0$ and in the credibility-constrained economy with

[10] In Environments 1 and 2, the planner's shadow intertemporal marginal rate of substitution and the intertemporal marginal rate of transformation coincided and it was natural to use a single tax function to induce agents to save and work the appropriate amounts. In Environment 3, the societal intertemporal marginal rate of substitution and the intertemporal marginal rate of transformation are different. An alternative decentralization would use two taxes at date 2, one applied to firms that induces them to equate the marginal product of capital to the relevant societal marginal rate of substitution and another applied to agents. Our decentralization absorbs these two taxes into a single tax applied to agents.

binding credibility constraint, $A_1^{CC} > 0$, $A_0^{CC} > 0$.

Positive implications for asset prices In the previous environments, we identified the market asset pricing kernel with the relevant societal intertemporal shadow prices. These in turn equalled the (possibly state-contingent) reciprocal of the second period marginal product of capital. In the current setting, we directly identify the market asset pricing kernel with the reciprocal of the second period marginal products of capital.[11]

Absent aggregate risk, the unconditional Euler equation and the first order condition for capital imply:

$$E\left[\frac{1}{u'(c_2^*)}\right] = \beta\mathcal{K}F_K(K_2^*, L_2^*)E\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right]. \tag{30}$$

Equating the risk free price with the marginal product of capital gives:

$$Q = \beta\mathcal{K}\frac{E\left[\frac{1}{u'(c_1^*)}\right]}{E\left[\frac{1}{u'(c_2^*)}\right]}. \tag{31}$$

Equation (31) augments the formula from the commitment environment (11) with the credibility factor $\mathcal{K}$. In the log-Cobb Douglas case, $\mathcal{K} = 1$ and the formulas are the same.[12] However, if capital tightens the credibility constraint, then the term $\beta\frac{E\left[\frac{1}{u'(c_1^*)}\right]}{E\left[\frac{1}{u'(c_2^*)}\right]}$ is discounted by an additional $\mathcal{K} < 1$. Kocherlakota-Pistaferri (2007b) explore the implications of the commitment model for asset pricing in settings with aggregate shocks (to which we turn below). They show that while this model can reconcile equity price and consumption data with reasonable specifications of preferences, it is less successful in explaining the risk-free rate. Specifically, under the assumption of CRRA utility and at a value of the coefficient of relative risk aversion large enough to explain the equity premium puzzle, the sample analogue of $\frac{E\left[\frac{1}{u'(c_1^*)}\right]}{E\left[\frac{1}{u'(c_2^*)}\right]}$ is large. Consequently, an implausibly small value for β is required to reconcile consumption and (near) risk-free asset price data. Equation (31) suggests that a credibility-constrained model might achieve this reconciliation if capital tightens the credibility constraints and $\mathcal{K} < 1$.

[11] We implicitly assume an implementation in which assets contingent on public information are traded and firm's are not taxed. As in the previous cases, we are otherwise not specific about the form of the implementation.

[12] In both cases, with $U(c) = \ln c$, $Q = \beta C_2^*/C_1^*$, i.e. Q equals discounted aggregate consumption growth which is common across environments.

To explore this idea further, consider introducing aggregate shocks. Then, the first order condition for capital implies:[13]

$$\sum_Z \left[\frac{q_2(Z)}{q_1} \left(\frac{1}{1 + \sum_Z \left[\phi(Z) \frac{W_K(K_2,Z)}{q_1} \right] \Lambda(Z)} \right) F_K(K_2^*, L_2^*(Z), Z) \right] \Lambda(Z) = 1,$$

so that a natural candidate pricing kernel is:

$$Q(Z) = \frac{q_2(Z)}{q_1} \left(\frac{\Lambda(Z)}{1 + \sum_Z \left[\phi(Z) \frac{W_K(K_2,Z)}{q_1} \right] \Lambda(Z)} \right). \tag{32}$$

Using $q_1 = E\left[W_K^* + \phi\left[W_K^* - W_K\right]\right]$ and the unconditional inverted Euler equation, (32) can be re-expressed as:

$$Q(Z) = \mathcal{M}^{CC}(Z) := \beta \mathcal{K}(Z) \frac{E\left[\frac{1}{u'(c_1^*)}\right]}{E\left[\left.\frac{1}{u'(c_2^*)}\right| Z\right]} \Lambda(Z),$$

where $\mathcal{M}^{CC}(Z)$ is the credibility-constrained stochastic discount factor and $\mathcal{K}(Z) = (1 + \phi(Z)) \left(\frac{E[W_K^*] + E[\phi(W_K^* - W_K)]}{E[(1+\phi)W_K^*]} \right)$ is the (stochastic) credibility factor. Clearly, $\mathcal{M}^{CC}(Z) = \mathcal{K}(Z)\mathcal{M}^C(Z)$, where $\mathcal{M}^C(Z)$ is the SDF from the commitment environment, and the risk-free price is

$$E[Q] = E[\mathcal{M}^{CC}] = E[\mathcal{K}]E[\mathcal{M}^C] + \text{Cov}\left[\mathcal{K}, \mathcal{M}^C\right].$$

$E[\mathcal{M}^{CC}] < E[\mathcal{M}^C]$ and the credibility-constrained model delivers a lower risk price off of the same consumption data and discount factor if $\frac{Cov[\mathcal{K}, \mathcal{M}^C]}{E[\mathcal{M}^C]} < (1 - E[\mathcal{K}])$. In the deterministic case, this inequality reduces to $0 < 1 - \mathcal{K}$ and the credibility factor is less than one if capital tightens the credibility constraint.

3 An infinite horizon environment

In the remainder of the paper, we consider infinite horizon versions of Environments 1 and 3. This allows us to derive credibility constraints explicitly as equilibrium restrictions in a dynamic game and to endogenize the outside option function W. We sharpen our characterizations by focussing on the log case and translate the observations of the preceding section to the infinite horizon setting. Full characterization of the optimal solution even in this case requires numerical analysis. Consequently, we seek recursive formulations of our problems that facilitate numerical implementation.

[13] The contrasts with: $\sum_Z \left[\frac{q_2(Z)}{q_1} F_K(K_2^*, L_2^*(Z), Z) \right] \Lambda(Z) = 1$, from our earlier cases.

3.1 Preferences and technologies

A continuum of infinitely-lived agents is initially partitioned into a measure space $(\mathbb{R}, \mathcal{B}, \Phi)$ of types w, where $\mathcal{B}$ is the Borel sigma algebra. At this point, we interpret a type as a public signal that the planner can condition individual allocations upon. Later, we identify types with initial Pareto weights that are assigned to agents a priori. For the moment, we treat Φ as a parameter. As before, agents receive taste shocks $\theta_t \in \Theta := \{\widehat{\theta}_i\}_{i\in\mathbf{I}}$, $\mathbf{I} = 1, 2, \cdots, I$ that affect their disutility from effort and that are i.i.d. across agents and time with distribution π. Agents receive an infinite sequence of consumption and efforts $\{c_t, e_t\}_{t=1}^{\infty}$ with for all t, $c_t : \Theta^t \to \mathbb{R}_+$ and $e_t : \Theta^t \to E \subset \mathbb{R}_+$. Assume that u and v are as before and denote their inverses by C and N. We additionally assume that u is unbounded below. The payoff from such a sequence is:

$$\widetilde{U}(\{c_t, e_t\}_{t=1}^{\infty}) = \lim \inf_{S\to\infty} \sum_{t=1}^{S} \sum_{\Theta^t} \beta^{t-1}[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))]\pi^t(\theta^t).$$

It is convenient to describe allocations directly in terms of the stream of utility they provide rather than stream of resources they use. An *individual allocation* (of utility) is a sequence $\{\psi_t, \upsilon_t\}_{t=1}^{\infty}$ where $\psi_t : \Theta^t \to \mathbf{D}$ and $\upsilon_t : \Theta^t \to \mathbf{L}$ give an individual's utility from consumption and disutility from effort at t as functions of past shocks. An agent's payoff from such a sequence $\{\psi_t, \upsilon_t\}_{t=1}^{\infty}$ is:

$$U(\{\psi_t, \upsilon_t\}_{t=1}^{\infty}) = \lim \inf_{S\to\infty} \sum_{t=1}^{S} \sum_{\Theta^t} \beta^{t-1}[\psi_t(\theta^t) + \theta_t \upsilon_t(\theta^t)]\pi^t(\theta^t).$$

Let **IA** denote the set of bounded individual allocations: $\mathbf{IA} := \{\{\psi_t, \upsilon_t\}_{t=1}^{\infty} | \ \{\psi_t, \upsilon_t\}_{t=1}^{\infty}$ is an individual allocation and $\lim_{S\to\infty} \sum_{t=1}^{S} \sum_{\Theta^t} \beta^{t-1} [|\psi_t(\theta^t)| + \theta_t|\upsilon_t(\theta^t)|]\pi^t(\theta^t) < \infty\}$. Given Φ, an *allocation* is a sequence $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$ where $\varphi_t : \mathbb{R} \times \Theta^t \to \mathbf{D}$ and $\upsilon_t : \mathbb{R} \times \Theta^t \to \mathbf{L}$ are measurable functions and $\{K_{t+1}\}_{t=1}^{\infty}$ is a sequence of capital stocks such that i) for all $w \in \mathbb{R}$, $\{\varphi_t(w, \cdot), \upsilon_t(w, \cdot)\}_{t=1}^{\infty} \in \mathbf{IA}$ and ii) $f(w) = U(\{\varphi_t(w, \cdot),\ \upsilon_t(w, \cdot)\}_{t=1}^{\infty})$ is Φ-integrable. These technical conditions ensure that allocations have well defined utilitarian payoffs. Let **PA** denote the set of allocations.

As before, effort is combined with capital to produce output. Given an initial capital amount K_1, type distribution Φ and allocation $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$, the amount of output produced in period t is:

$$Y_t = F(K_t, L(\upsilon_t)),$$

where $L(\upsilon_t) = \int_{\mathbb{R}} \sum_{\Theta^t} N(\upsilon_t(w,\theta^t))\pi^t(\theta^t)\Phi(dw)$ is the aggregate labor input.

3.2 Resource and incentive constraints

An allocation is feasible if it is resource-feasible and incentive-compatible. Given Φ and $K_1 > 0$, $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$ is *resource-feasible* if for all t,

$$\int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w,\theta^t))\pi^t(\theta^t)\Phi(dw) + K_{t+1} \leq F(K_t, L(\upsilon_t)). \tag{33}$$

Let $m = \{m_t\}_{t=1}^{\infty}$ denote a reporting strategy, where $m_t : \Theta^t \to \Theta$ gives an agent's t-th period shock report as a function of her t-th period history of current and past shocks. The strategy m induces functions $m^t : \Theta^t \to \Theta^t$ that map true shock histories to report histories. An allocation is *incentive-compatible* if for Φ-a.e. w and all m,

$$\sum_{t=1}^{\infty} \sum_{\Theta^t} \beta^{t-1}[\varphi_t(w,\theta^t) + \theta_t \upsilon_t(w,\theta^t)]\pi^t(\theta^t) \geq \sum_{t=1}^{\infty} \sum_{\Theta^t} \beta^{t-1}[\varphi_t(w,m^t(\theta^t)) + \theta_t \upsilon_t(w,m^t(\theta^t))]\pi^t(\theta^t). \tag{34}$$

To obtain recursive formulations of our problems we relax (34) and replace it with a sequence of *temporary incentive-compatibility constraints,* for Φ-a.e. w, all t, θ^{t-1}, i, $j \in \mathbf{I}$,

$$\begin{aligned}\varphi_t(w,\theta^{t-1},\widehat{\theta}_i) + \widehat{\theta}_i \upsilon_t(w,\theta^{t-1},\widehat{\theta}_i) + \beta \sum_{r=1}^{\infty} \sum_{\Theta^r} \beta^{t-1}[\varphi_{t+r}(w,\theta^{t-1},\widehat{\theta}_i,\theta^r) + \theta_{t+r}\upsilon_{t+r}(w,\theta^{t-1},\widehat{\theta}_i,\theta^r)]\pi^r(\theta^r) \geq \\ \varphi_t(w,\theta^{t-1},\widehat{\theta}_j) + \widehat{\theta}_i \upsilon_t(w,\theta^{t-1},\widehat{\theta}_j) + \beta \sum_{r=1}^{\infty} \sum_{\Theta^r} \beta^{t-1}[\varphi_{t+r}(w,\theta^{t-1},\widehat{\theta}_j,\theta^r) + \theta_{t+r}\upsilon_{t+r}(w,\theta^{t-1},\widehat{\theta}_j,\theta^r)]\pi^r(\theta^r).\end{aligned} \tag{35}$$

It is well known that (34) implies (35). In addition, if an allocation satisfies the tail condition:

$$\lim_{S\to\infty} \sup_m \beta^S \sum_{t=1}^{\infty} \sum_{\Theta^t} \beta^{t-1}[\varphi_{T-1+t}(w,\theta^{S-1},m^t(\theta^t)) + \theta_{S-1+t}\upsilon_{T-1+t}(w,\theta^{S-1},m^t(\theta^t))]\pi^t(\theta^t) = 0 \tag{36}$$

and (35), then it satisfies (34).

4 The infinite-horizon optimal commitment problem

As in Environment 1, the infinite-horizon commitment problem entails maximizing a societal payoff subject to the resource and incentive constraints. We generalize the earlier objective by supposing that a planner attaches a Pareto weight $\gamma_0(w)$ to members of the w-indexed subpopulation of agents. Assume that the weighting function γ_0 is positive and integrable with $\int \gamma_0(w)\Phi(dw) = 1$. Let Ψ_0 denote the distribution over weights implied by Φ and γ_0. These weights may be thought of as primitive elements of the planner's preferences or the result of rewards and penalties accrued in earlier unmodeled periods. To economize on notation, we rewrite allocations directly as functions of Pareto weights (since the planner would not choose to treat different w-populations of agents with the same Pareto weight differently). The infinite-horizon commitment problem at (K_1, Ψ_0) is:

$$\sup_{\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}} \int_{\mathbb{R}} \gamma_0 U(\{\varphi_t(\gamma_0, \cdot), \upsilon_t(\gamma_0, \cdot)\}_{t=1}^{\infty})\Psi_0(d\gamma_0) \text{ subject to (33) and (35).} \tag{37}$$

The literature on efficient allocations with private information has tended to focus on dual problems in which a planner minimizes a cost aggregate subject to implementing a given distribution of utilities rather than the primal problem considered here. This is largely because the dual problem can be disaggregated into a family of "component planner problems" that have a well known recursive formulation in terms of utility promises.[14] We disaggregate the primal problem and use an alternative recursive formulation in terms of Pareto weights developed by Marcet and Marimon (1999) and extended by Sleet-Yeltekin (2008b).

4.1 Component planner formulations

The next lemma describes how (37) can be disaggregated into a collection of component planner problems. We use the notation $\ell_+(\beta) = \{\{x_t\}_{t=1}^{\infty} : \forall t,\, x_t \geq 0,\, \sum_{t=1}^{\infty} \beta^{t-1} x_t < \infty\}$.

Lemma 2 *Assume* $\{q_t\}_{t=1}^{\infty} \in \ell_+(\beta)$ *and* $\{w_t\}_{t=1}^{\infty} \in [\underline{w}, \overline{w}]^{\infty}$, $0 < \underline{w} < \overline{w}$. *Let* $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}_{t=1}^{\infty}$ *be an allocation and let* $\{L_t^*\}_{t=1}^{\infty} \in [0, T]^{\infty}$. *If* $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}_{t=1}^{\infty}$ *satisfies C1-C3 at* (K_1, Ψ_0) *with* $K_1 > 0$, *then it solves the commitment problem (37) at* (K_1, Ψ_0).

[14] For examples of the dual approach see, inter alia, Atkeson and Lucas (1992, 1995).

C1) Ψ_0*-a.e.* γ_0, $\varphi_t^*(\gamma_0,\cdot)$ *and* $\upsilon_t^*(\gamma_0,\cdot)$ *solve:*

$$V_1(\gamma_0) = \sup_{\{\psi_t,\nu_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty}\sum_{\theta^t}\{\beta^{t-1}\gamma_0[\psi_t(\theta^t)+\theta_t\nu_t(\theta^t)] - q_t[C(\psi_t(\theta^t)) - w_t N(\nu_t(\theta^t))]\}\pi^t(\theta^t) \tag{38}$$

subject to $\{\psi_t,\nu_t\}_{t=1}^{\infty} \in \mathbf{IA}$ *and (35).*

C2) For $t>1$, K_t^* *and* L_t^* *solve*

$$\sup_{K_t,L_t} F(K_t,L_t) - \frac{q_{t-1}}{q_t}K_t - w_t L_t, \tag{39}$$

for $t=1$, L_1^* *solves*

$$\sup_{L_1} F(K_1,L_1) - w_1 L_1. \tag{40}$$

C3) Let $K_1^* = K_1$. *For all* t, φ_t^*, υ_t^*, L_t^*, K_t^* *and* K_{t+1}^* *satisfy*

$$\int \sum_{\theta^t} C(\varphi_t^*(\gamma_0,\theta^t))\pi^t(\theta^t)\Psi_0(d\gamma_0) + K_{t+1}^* \quad = \quad F(K_t^*,L_t^*), \tag{41}$$

$$\int \sum_{\theta^t} N(\upsilon_t^*(\gamma_0,\theta^t))\pi^t(\theta^t)\Psi_0(d\gamma_0) \quad = \quad L_t^*. \tag{42}$$

The proof of Lemma 2 is a similar to the proof of Atkeson-Lucas (1992) [Theorem 1, p.443] and is omitted. The Lemma can be interpreted as a first welfare theorem for a particular market economy in which a representative firm hires capital K_t and labor L_t on competitive markets in each period t at prices $\frac{q_{t-1}}{q_t}$ and w_t. Agents are assigned to *component planners* who maximize the Pareto weighted utility net of cost of their client agents subject to incentive constraints. They do this by trading claims to output and labor with the firm and one another. The lemma asserts that equilibrium allocations in this economy solve the planning problem (37).

4.1.1 The firm's problem

The firm's problem (39) gives rise to the standard first order conditions:

$$-q_{t-1} + q_t F_K(K_t^*,L_t^*) \quad = \quad 0,$$

$$-w_t + F_L(K_t^*,L_t^*) \quad = \quad 0.$$

4.1.2 Recursive component planner problem

It is readily verified that if an allocation solves a relaxed version of (38) with only the $(i, i+1)$ local upwards and the $(i, i-1)$ local downwards temporary incentive constraints imposed, then it satisfies all of the temporary incentive constraints. Thus, we impose only these local constraints and define the date t-continuation component planning problem by:

$$V_t(\gamma) = \sup_{\{\psi_r,\nu_r\}_{r=1}^{\infty}\in\Omega_1} \sum_{r=1}^{\infty}\sum_{\theta^r}\{\beta^{r-1}\gamma[\psi_r(\theta^r) + \theta_r\nu_r(\theta^r)] - q_{t+r-1}[C(\psi_r(\theta^r)) - w_{t+r-1}N(\nu_r(\theta^r))]\}\pi^r(\theta^r), \tag{43}$$

where $\Omega_S = \{\{\psi_r,\nu_r\}_{r=1}^{\infty} \in \mathbf{IA} \mid \{\psi_r,\nu_r\}_{r=1}^{\infty}$ satisfies the local upward and downward temporary incentive constraints from period S onwards$\}$. Associated with problem (43) is a (well defined) Lagrangian $\mathcal{L}_t : \mathbb{R}_+^{I(I-1)} \times \mathbf{IA} \to \mathbb{R}$ that incorporates *only* the t-th period local temporary incentive constraints with multipliers $\eta = \{\eta_{i,j}\}_{i,j\in\mathbb{K}} \in \mathbb{R}_+^{I(I-1)}$, where $\mathbb{K} = \{(i,j) : i,j \in \mathbf{I}$, $j = i-1$ or $j = i+1\}$:

$$\begin{aligned}
\mathcal{L}_t(\eta, \{\psi_r,\nu_r\}_{r=1}^{\infty};\gamma) \quad = \quad & \sum_{r=1}^{\infty}\sum_{\theta^r}\{\beta^{r-1}\gamma[\psi_r(\theta^r)+\theta_r\nu_r(\theta^r)] - q_{t+r-1}[C(\psi_r(\theta^r)) - w_{t+r-1}N(\nu_r(\theta^r))]\}\pi^r(\theta^r) \\
& + \sum_{i,j\in\mathbb{K}}\eta_{i,j}\Bigg[\psi_1(\widehat{\theta}_i) + \widehat{\theta}_i\nu_1(\widehat{\theta}_i) + \beta\sum_{r=1}^{\infty}\sum_{\Theta^r}\beta^{r-1}[\psi_{r+1}(\widehat{\theta}_i,\theta^r) + \theta_{r+1}\nu_{r+1}(\widehat{\theta}_i,\theta^r)]\pi^r(\theta^r) \\
& -\psi_1(\widehat{\theta}_j) - \widehat{\theta}_i\nu_1(\widehat{\theta}_j) - \beta\sum_{r=1}^{\infty}\sum_{\Theta^r}\beta^{r-1}[\psi_{r+1}(\widehat{\theta}_j,\theta^r) + \theta_{r+1}\nu_{r+1}(\widehat{\theta}_j,\theta^r)]\pi^r(\theta^r)\Bigg].
\end{aligned} \tag{44}$$

An application of the classical result of Luenberger (1969, [Theorem 1, p.217]) establishes that $V_t(\gamma)$ is the optimal payoff from a saddle point problem involving $\mathcal{L}_t(\cdot;\gamma)$ and that an optimal multiplier exists for this problem.

Proposition 3 *For each t and $\gamma \in \mathbb{R}_+$, V_t and $\mathcal{L}_t$ satisfy,*

$$V_t(\gamma) = \inf_{\eta\in\mathbb{R}_+^{I(I-1)}} \sup_{\{\psi_r,\nu_r\}_{r=1}^{\infty}\in\Omega_2} \mathcal{L}_t(\eta, \{\psi_r,\nu_r\}_{r=1}^{\infty};\gamma). \tag{45}$$

Additionally, there exists an $\eta^* \in \mathbb{R}_+^{I(I-1)}$ that attains the infimum in (45).

It then follows from (45) at t and $t+1$ and a rearrangement of the terms in (44) that:

$$V_t(\gamma) = \inf_{\Lambda(\gamma)} \sum_{i=1}^{I}\{J_i(\rho_i(\zeta;\eta), \lambda_i(\zeta;\eta); q_t, w_t) + \beta V_{t+1}(\gamma_i'(\gamma;\eta))\}\pi(\widehat{\theta}_i), \tag{46}$$

where the indirect current payoff function is given by $J_i(\rho, \lambda; q_t, w_t) = \sup_\psi\{\rho\psi - q_t C(\psi)\} + \sup_\upsilon\{\widehat{\theta}_i \lambda \nu + q_t w_t N(\nu)\}$, the current utility weights are

$$\begin{aligned}
\rho_i(\gamma; \eta) &= \gamma + \sum_{j:\{(i,j)\in\mathbb{K}\}} \eta_{i,j} - \sum_{j:\{(j,i)\in\mathbb{K}\}} \eta_{j,i} \frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}, \\
\lambda_i(\gamma; \eta) &= \gamma + \sum_{j:\{(i,j)\in\mathbb{K}\}} \eta_{i,j} - \sum_{j:\{(j,i)\in\mathbb{K}\}} \eta_{j,i} \frac{\widehat{\theta}_j}{\widehat{\theta}_i} \frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)},
\end{aligned}$$

the updated *effective Pareto weights* are:

$$\gamma_i'(\gamma; \eta) = \gamma + \sum_{j:\{(i,j)\in\mathbb{K}\}} \eta_{i,j} - \sum_{j:\{(j,i)\in\mathbb{K}\}} \eta_{j,i} \frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}, \tag{47}$$

and the constraint correspondence for current multiplier choices is $\Lambda(\zeta) = \{\eta \in \mathbb{R}_+^{I(I-1)} | \ \forall i, \ \gamma_i'(\zeta; \eta) \geq 0\}$. Thus, V_t is a value function in a dynamic programming problem that features the Lagrange multipliers η from the current incentive constraints as choice variables and effective Pareto weights as state variables. The optimal policy functions $\{\eta_t^*\}_{t=1}^\infty$ from (46) and the updating functions $\{\gamma_i'\}$ imply a stochastic process for effective Pareto weights $\{\gamma_t^*\}_{t=1}^\infty$, with $\gamma_t^* : \mathbb{R}_+ \times \Theta^{t-1} \to \mathbb{R}_+$. These together with the static optimizations that define the functions J_i can be used to obtain an individual allocation $\{\varphi_t^*(\gamma), \upsilon_t^*(\gamma)\}_{t=1}^\infty$. This allocation solves the component planner problem (38) at initial Pareto weight γ; we say that $\{\varphi_t^*(\gamma), \upsilon_t^*(\gamma)\}_{t=1}^\infty$ *is induced by* $\{\eta_t^*\}_{t=1}^\infty$ *from* γ. The following proposition formalizes this.[15]

Proposition 4 *1) There exists a unique sequence of continuous policy functions $\{\eta_t^*\}_{t=1}^\infty$, with $\eta_t^* : \mathbb{R}_+ \to \mathbb{R}_+^{I(I-1)}$, that attain the infima in the optimization problems (46). 2) If the non-recursive component planner problem (38) has a solution, then the allocation $\{\varphi_t^*(\gamma), \upsilon_t^*(\gamma)\}_{t=1}^\infty$ induced by $\{\eta_t^*\}_{t=1}^\infty$ from γ solves (38).*

It follows from (47) that the optimal process for effective Pareto weights $\{\gamma_t^*\}_{t=1}^\infty$ starting from some $\gamma \in \mathbb{R}_+$ evolves according to:

$$\gamma_{t+1}^*(\gamma, \theta^{t-1}, \widehat{\theta}_i) = \gamma_t^*(\gamma, \theta^{t-1}) + \varepsilon_{t+1}(\gamma, \theta^{t-1}, \widehat{\theta}_i), \tag{48}$$

[15] See Sleet-Yeltekin (2008) for a proof.

where the sequence of incentive shocks $\{\varepsilon_{t+1}\}_{t=1}^{\infty}$ is obtained from the optimal incentive multipliers. These shocks incorporate the future rewards and penalties for sending different current reports into the effective Pareto weight process. For example, if $I = 2$, only the upwards local constraint binds. Agents receive a non-negative shock $\varepsilon_{t+1}(\gamma, \theta^{t-1}, \widehat{\theta}_1) := \eta^*_{t,1,2}(\gamma^*_t(\gamma, \theta^{t-1})) \geq 0$ for reporting $\widehat{\theta}_1$ and a non-positive shock $\varepsilon_{t+1}(\gamma, \theta^{t-1}, \widehat{\theta}_2) := -\eta^*_{t,1,2}(\gamma^*_t(\zeta, \theta^{t-1})) \frac{\pi(\widehat{\theta}_2)}{\pi(\widehat{\theta}_1)}$ otherwise. This motivates agents who have a low preference for leisure, $\widehat{\theta}_1$, to report their type truthfully. The incentive shocks satisfy: $E[\varepsilon_{t+1}|\gamma, \theta^{t-1}] = 0$. Consequently, an agent's optimal effective Pareto weight follows a non-negative martingale and, by the martingale convergence theorem, almost surely converges. This is a force for continued spreading of agent effective Pareto weights and increasing inequality over time.

The trend to greater ex post inequality underpins a potential commitment problem. If the planner could revise allocations ex post, she would be tempted to remove this inequality. The following example illustrates that limiting inequality can be severe and the limiting utilitarian payoff arbitrarily low when agent utilities are unbounded below in consumption. The temptation to revise the allocation in these cases is correspondingly strong.

4.2 An example

Previous analyses of private information commitment economies (with infinitely-lived agents) have either assumed constant resource prices (see, for example, Thomas-Worrall (1990)) or sufficient homotheticity of agent preferences to facilitate aggregation (see, inter alia, Atkeson-Lucas (1992), Phelan (1994) or Khan-Ravikumar (2003)). Our environment does not fit into either of these categories: shadow resource prices are endogenously determined and our preference assumptions preclude straightforward aggregation. In Sleet-Yeltekin (2008c), we analytically characterize the equilibrium of a component planner economy with log preferences and a Cobb-Douglas technology. The equilibrium converges to a degenerate steady state in which all agents are either working 100% of the time and receiving no consumption or are not working and are sharing all output. We derive bounds for the fractions of agents in each state and lower bounds on steady state capital and labor supply. The following proposition summarizes these results.

Proposition 5 *Assume agents have per period preferences of the form* $\ln c + \ln(T-e)$ *and that* $F(K,L) = K^{\alpha}L^{1-\alpha}$*. Suppose that* $I = 2$ *and that* Ψ_0 *attaches unit mass to the point* 1*. Then there is a component planning equilibrium that converges to a limiting steady state. In this steady state, a fraction* $p \geq \frac{\frac{1-\alpha}{1-\alpha\beta}}{\widehat{\theta}_1 + \frac{1-\alpha}{1-\alpha\beta}}$ *of agents have effective Pareto weights that converge to* 0*, these agents exert limiting effort* T *and receive no consumption. A fraction* $1-p$ *of agents converge to a Pareto weight sufficiently large that they do not work at the limiting wage. In this equilibrium, the capital stock converges to a limiting value bounded below by* $(\alpha\beta)^{\frac{1}{1-\alpha}} \left[\frac{\frac{1-\alpha}{1-\alpha\beta}}{\widehat{\theta}_1 + \frac{1-\alpha}{1-\alpha\beta}}\right] T$ *and the aggregate labor supply converges to a value bounded below by* $\left[\frac{\frac{1-\alpha}{1-\alpha\beta}}{\widehat{\theta}_1 + \frac{1-\alpha}{1-\alpha\beta}}\right] T$*.*

Since a positive measure of agents are immiserated and agent utility is unbounded below in consumption, the associated utilitarian payoff converges to $-\infty$. As we show in the next section, a credible allocation must maintain continuation utilitarian payoffs above a lower bounding function $W(K)$ that depends on current capital. In the current example, the optimal allocation with commitment violates this constraint and is not credible.

5 A policy game

In this section we describe a policy game. As before, agent productivity shocks are not public information and the planner must induce agents to reveal information about them. The optimal allocation of the previous section relied on future allocations to do this. Now, however, we assume that the planner cannot commit to such allocations.

5.1 The stage game

In period t, each agent is publicly identified by a history of signals $w^{t-1} \in \mathbb{R}^{t-1}$ and a history of past reports $\theta^{t-1} \in \Theta^{t-1}$.[16] The ensuing stage game consists of three sub-periods. In the first, a planner chooses a mechanism $S_t = \{\xi_t, \widetilde{\varphi}_t, \widetilde{v}_t, K_{t+1}\}$, where $\xi_t : \mathbb{R}^{t-1} \times \Theta^{t-1} \to \mathbf{P}$ is a measurable function mapping histories of signals and reports to a lottery of current public signals and $\widetilde{\varphi}_t : \mathbb{R}^t \times \Theta^t \to \mathbb{R}_+$ and $\widetilde{v}_t : \mathbb{R}^t \times \Theta^t \to \mathbb{R}$ are a pair of measurable allocation functions that give each agent's utility

[16] We assume that the planner uses direct mechanisms to elicit information. Sleet-Yeltekin (2006) provide a revelation principle for a related environment. Their argument holds here.

from consumption and effort as a function of their histories (inclusive of their current signals and reports). $K_{t+1} \in \mathbb{R}_{++}$ is an aggregate capital choice.[17] In the second sub-period, agents receive a private shock, produce and send a message to the planner. In the third sub-period, the planner allocates utility according to $\widetilde{\varphi}_t$ and $\widetilde{\upsilon}_t$. The game then proceeds to the next period. The planner cannot commit in advance to a particular sequence of future mechanisms. However, having selected a mechanism in the first sub-period of the stage game, she must execute it in the third; she cannot deviate to some alternative current allocation after hearing the agents' messages. Thus, we assume planner commitment within, but not across periods.

5.2 Histories and strategies

This subsection introduces notation and definitions for the infinitely repeated game. Define an *aggregate history* $H_t = \{K_1, \{S_r\}_{r=1}^t\}$, $t \geq 1$, to be the initial capital stock and a sequence of current and past stage mechanisms chosen by the planner. Set H_0 equal to K_1. Let $\mathbf{H}^t$ denote the set of t-period aggregate histories and let $\mathbf{S}_t$ denote the set of t-period stage mechanisms. A planner strategy $\sigma = \{\sigma_t\}_{t=1}^\infty$ is a collection of functions that map from aggregate histories to a current mechanism, $\sigma_t : \mathbf{H}^{t-1} \to \mathbf{S}_t$.[18] Any given σ induces a sequence of aggregate histories recursively from an initial $H_0 = K_1$ according to: $H_t = (H_{t-1}, \sigma_t(H_{t-1}))$. Let $\sigma|H_r$ denote the continuation of σ after aggregate history H_r and, for $t \geq r$, let $H_t(\sigma|H_r)$ denote the period t aggregate history and $S_t(\sigma|H_r)$ the period t stage mechanism induced by $\sigma|H_r$ along its outcome path. Let $\widetilde{\varphi}_t(\sigma|H_r)$ and $\widetilde{\upsilon}_t(\sigma|H_r)$ denote the corresponding t-th period allocation functions.

The period t *individual public history* of an agent $\left(H_t, w^t, \theta^{t-1}\right) \in \mathbf{H}^t \times \mathbb{R}^t \times \Theta^{t-1}$ augments H_t with the agent's signal-report history. An agent's *reporting strategy*, $m = \{m_t\}_{t=1}^\infty$, where $m_t : \mathbf{H}^t \times \mathbb{R}^t \times \Theta^{t-1} \times \Theta \to \Theta$ gives the t-th period report of an agent contingent on his individual public history and current shock. Whenever they are well defined, let $W_t(\sigma, m|H_{t-1})$

[17] The lottery ξ_t gives the planner the flexibility to partition the population of agents arbitrarily at any date. In practice, the planner is tempted to choose allocation functions that ignore prior partitions of agents into histories, but it seems natural to allow the planner to adopt a finer partition at any date. This formulation also allows us to endogenize the initial distribution over agent types.

[18] As in Chari and Kehoe (1990), we consider an economy inhabited by a large strategic player, the planner, and a population of atomistic agents. This structure motivates our formulation of strategies; the planner's strategy is conditioned only on past histories of capital stocks and her own actions. The planner's strategy does not allow its treatment of an agent to depend on the past actions of other agents.

be the continuation payoff for a *utilitarian* planner induced by the strategy pair (σ, m) after the aggregate history H_{t-1} and let $U_t(\sigma, m|H_t, w^{t-1}, \theta^{t-1})$ be the continuation payoff for the agent induced by (σ, m) after $(H_t, w^{t-1}, \theta^{t-1})$. Finally, let $Q^t(m, H^t)$ denote the distribution over individual histories (w^t, θ^t) induced by m and H_t. We define a planner strategy σ to be *resource-feasible* given report strategy m if after each aggregate history the allocations induced by (σ, m) satisfy our earlier resource constraints. More formally, we have:

Definition 6 *Given a report strategy m, a mechanism $S = (\xi, \widetilde{\varphi}, \widetilde{\upsilon}, K)$ is resource-feasible at $H_{t-1} = (\ldots, K_t)$ if:*

$$\int_{\mathbb{R}^t} \sum_{\Theta^t} C(\widetilde{\varphi}(w^t, \theta^t)) Q^t(m, H^{t-1}, S) + K \leq F\left(K_t, \int_{\mathbb{R}^t} \sum_{\Theta^t} N(\widetilde{\upsilon}(w^t, \theta^t)) Q^t(m, H^{t-1}, S)\right).$$

Given m, a history $H_t = \{K_1, \{S_r\}_{r=1}^t\}$ is resource-feasible if each component S_r is resource-feasible at $H_{r-1} = \{K_1, \{S_s\}_{s=1}^{r-1}\}$. Let $\mathbf{H}^t(m)$ denote the corresponding set of t-period resource-feasible histories. Given m, a planner strategy σ is resource-feasible if $\forall t$ and $H_{t-1} \in \mathbf{H}^{t-1}(m)$, $\sigma_t(H_{t-1})$ is resource-feasible at H_{t-1}.

We impose the following bound as a primitive constraint on planner strategies.

Definition 7 *A planner strategy σ has well defined payoffs given a report strategy m if for all t and H_{t-1}, $W_t(\sigma, m|H_{t-1})$ is well defined and for all t, H_t and almost all (w^{t-1}, θ^{t-1}), $U_t(\sigma, m|H_t, w^{t-1}, \theta^{t-1})$ is well defined.*

Let $\mathbf{S}(m)$ denote the set of resource-feasible planner strategies with well defined payoffs given m.

5.3 Credible equilibria

A *credible equilibrium* is defined as follows.

Definition 8 *(σ, m) is a* ***credible equilibrium*** *if $\sigma \in \mathbf{S}(m)$ and,*

1. *(Agent optimality) $\forall t, H_t, w^{t-1}, \theta^{t-1}, \widehat{m}$,*

$$U_t(\sigma, m|H_t, w^{t-1}, \theta^{t-1}) \geq U_t(\sigma, \widehat{m}|H_t, w^{t-1}, \theta^{t-1});$$

2. *(Planner optimality)* $\forall t, H_{t-1} \in \mathbf{H}^{t-1}(m), \widehat{\sigma} \in \mathbf{S}(m)$,

$$W_t(\sigma, m|H_{t-1}) \geq W_t(\widehat{\sigma}, m|H_{t-1}).$$

The first of these conditions requires that the continuation of the agent's message strategy is optimal after all public individual histories given that the planner plays according to σ in the future. The second condition requires that after all resource-feasible aggregate histories, the planner is better off adhering to the strategy σ than deviating to some alternative resource-feasible strategy $\widehat{\sigma}$.

Worst credible equilibria We next define uninformed planner and uninformative message strategies and show that they constitute a worst utilitarian credible equilibrium. Under the uninformed planner strategy σ^{UI}, the planner makes no use of any information that the agents reveal and provides incentives for agents to "babble" in the aftermath of a defection.

σ^{UI} is constructed as follows. First, for each $K > 0$, define the *no insurance problem* (49) by

$$W(K) = \sup_{\{u_t, v_t, K_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \left[\beta^{t-1}(u_t + E[\theta]v_t)\right] \tag{49}$$

$$\text{subject to} \quad K_1 = K \quad \text{and } \forall t, \qquad C(u_t) + K_{t+1} \leq F(K_t, N(v_t)).$$

Let $\{u_t^*, v_t^*, K_{t+1}^*|K\}_{t=1}^{\infty}$ denote a solution to (49) and let $\{S_{r+t}^{UI}|K\}_{t=1}^{\infty}$ be a sequence of mechanisms that implements $\{u_t^*, v_t^*, K_{t+1}^*|K\}_{t=1}^{\infty}$ from period $r+1$ onwards. Set $r = 0$ and σ^{UI} so that $\{S_t(\sigma^{UI}|K_1)\}_{t=1}^{\infty} = \{S_t^{UI}|K_1\}_{t=1}^{\infty}$. Construct the remainder of the strategy recursively using the following procedure. Given a planner defection to mechanism $S' = (\xi', \varphi', v', K')$ at date r after history H_{r-1}, set $S_{r+1}(\sigma^{UI}|H_{r-1}, S') = \widehat{S}_{r+1}(S')$, where $\widehat{S}_{r+1}(S')$ is defined below, and $\{S_{r+t}(\sigma^{UI}|H_{r-1}, S', \widehat{S}_{r+1}(S'))\}_{t=2}^{\infty} = \{S_{r+t}^{UI}|K'\}_{t=2}^{\infty}$. In other words, σ^{UI} plays $\widehat{S}_{r+1}(S')$ in the period after a defection and then reverts to play of no insurance mechanisms. $\widehat{S}_{r+1}(S')$ augments the no insurance mechanism $S_{r+1}^{UI}|K'$ with a penalty for agents who did not "babble" in the defection period r. Define $\theta[w^r, \theta^{r-1}] := \arg\max_\theta \varphi'(w^r, \theta^{r-1}, \theta) + \widehat{\theta}_1 v'(w^r, \theta^{r-1}, \theta)$ to be the common "babbling" report for agents with history (w^r, θ^{r-1}). Then, the current utility loss for (w^r, θ^{r-1})-agents who do not send this report at r is bounded above by: $\Delta(w^r, \theta^{r-1}) = \max_{i,j}[\varphi'(w^r, \theta^{r-1}, \widehat{\theta}_j) + \widehat{\theta}_i v'(w^r, \theta^{r-1}, \widehat{\theta}_j) - \varphi'(w^r, \theta^{r-1}, \theta[w^r, \theta^{r-1}]) - \widehat{\theta}_i v'(w^r, \theta^{r-1}, \theta[w^r, \theta^{r-1}])] \geq 0$.[19] $\widehat{S}_{r+1}(S')$ is constructed from $S^{UI}_{r+1}|K'$ by depressing the date $r+1$-consumption utility of agents who failed to report $\theta[w^r, \theta^{r-1}]$ by $\frac{\Delta(w^r, \theta^{r-1})}{\beta}$. Formally, $\widehat{S}_{r+1}(S') = \{\xi^{UI}_{r+1}, \widehat{\varphi}^{UI}_{r+1}, v^{UI}_{r+1}, K^{UI}_{r+2}|K'\}$, where for $(w^{r+1}, \theta^{r+1}) = (w^{r+1}, \theta^{r-1}, \theta[w^r, \theta^{r-1}], \theta)$, $\widehat{\varphi}^{UI}_{r+1}(w^{r+1}, \theta^{r+1}) = \varphi^{UI}_{r+1}(w^{r+1}, \theta^{r+1})|K'$, otherwise $\widehat{\varphi}^{UI}_{r+1}(w^{r+1}, \theta^{r+1}) = \varphi^{UI}_{r+1}(w^{r+1}, \theta^{r+1})|K' - \frac{\Delta(w^r, \theta^{r-1})}{\beta}$. Since utility is unbounded below in consumption, the punishment $-\frac{\Delta(w^r, \theta^{r-1})}{\beta}$ is feasible. σ^{UI} is uninformed in the sense that the planner makes no use of any information it has to provide insurance.

The uninformative message strategy m^{UI} is defined for all H_t, w^t, θ^{t-1}, θ, by

$$m^{UI}_t(H_t, w^t, \theta^{t-1}, \theta) = \begin{cases} \theta \text{ if } H_t = (H_{t-1}, \sigma^{UI}_t(H_{t-1})) \\ \theta[w^t, \theta^{t-1}] \text{ otherwise.} \end{cases}.$$

Thus, m^{UI} is truthful if the planner chooses $\sigma^{UI}_t(H_{t-1})$ and commits not to use the truthful signals that it receives. Otherwise, m^{UI} requires that the agent babbles by sending out a common message $\theta[w^t, \theta^{t-1}]$.

Lemma 9 (σ^{UI}, m^{UI}) *constitutes a worst utilitarian credible equilibrium.*

Proof: To show that (σ^{UI}, m^{UI}) is a credible equilibrium first note that if the planner adheres to σ^{UI}, then agents receive the same payoff regardless of their current message. They may as well report their true shock. If the planner has defected in the current period, but is anticipated to adhere to σ^{UI} in the future, then agents anticipate a subsequent penalty for failing to send the babbling report. This penalty is large enough that it is in their interests to babble. Hence, m^{UI} is optimal for agents after all histories. m^{UI} is such that agents are truthful at t if the planner implements $\sigma^{UI}_t(H_{t-1})$, otherwise they are uninformative. In particular, no positive measure set of agents reveals their type following a planner defection. Given this, the planner might as well adhere to σ^{UI}, treat (almost) all agents equally, and punish any individual (measure zero) agent

[19] Feasibility of S' implies that $\varphi'(w^r, \theta^r) + \theta_r v'(w^r, \theta^r)$ is a.s. well defined and bounded above. If $\varphi'(w^r, \theta^{r-1}, \theta[w^r, \theta^{r-1}]) - \widehat{\theta}_1 v'(w^r, \theta^{r-1}, \theta[w^r, \theta^{r-1}]) = -\infty$, then $\varphi'(w^r, \theta^{r-1}, \widehat{\theta}_j) + \widehat{\theta}_i v'(w^r, \theta^{r-1}, \widehat{\theta}_j) = -\infty$ holds for all i and j and we define $\Delta = 0$. Hence, Δ is a. s. bounded above.

who fails to send the babbling message following a planner defection. Thus, σ^{UI} is optimal for the planner and (σ^{UI}, m^{UI}) is a credible equilibrium. Given an initial capital K, the planner can guarantee herself the payoff $W(K)$ by choosing $S_t^{UI}|K$ in each period independently of the agents' behavior. Hence, $W(K)$ is a lower bound on equilibrium utilitarian payoffs and since (σ^{UI}, m^{UI}) attains this payoff, it is a worst utilitarian equilibrium. ■

It follows from the definition of (σ^{UI}, m^{UI}) that after any history H_{t-1} culminating in capital K_t, the continuation payoff induced by (σ^{UI}, m^{UI}) is $W(K_t)$.

Remark In the equilibrium (σ^{UI}, m^{UI}), the threat of a future punishment is used to induce agents to babble following a planner defection. Although, the planner cares about the collective well-being of agents, she is willing to punish individual agents who deviate from m^{UI} and fail to babble because they are measure zero. Such equilibria would not emerge in economies with a finite number of agents, nor are they likely to be the limit of a sequence of equilibria in economies with a finite, but increasing number of agents. Despite this, we continue to use the credible equilibrium concept defined above. Imposing the additional refinement that equilibria be limits of those in finite agent economies would tighten credibility constraints and complicate the task of finding worst credible equilibria. Qualitatively, it would not overturn our results; quantitatively, it would only sharpen them. We leave consideration of this refinement to later work.[20]

5.4 Credible allocations

Credible allocations are the outcomes of credible equilibria in which the allocation functions do not condition on public signals after the initial period. From a welfare point of view, restricting attention to such equilibria is without loss of generality. Formally, we define a distribution-allocation pair $(\Phi, \{\varphi_t, \upsilon_t, K_{t+1}\})$ to be credible at K_1 if there is a credible equilibrium (σ, m) with $\Phi = \xi_1(\sigma|K_1)$ and $\forall$ t, w, w^{t-1}, $\varphi_t(w, \theta^t) = \widetilde{\varphi}_t(\sigma|K_1)(w, w^{t-1}, \theta^t)$, $\upsilon_t(w, \theta^t) = \widetilde{\upsilon}_t(\sigma|K_1)(w, w^{t-1}, \theta^t)$ and $K_{t+1} = K_{t+1}(\sigma|K_1)$. The following proposition uses Lemma 9 to characterize credible allocations.

[20] We thank Marco Bassetto for pointing this issue out to us.

Proposition 10 *Given an initial capital stock K_1, a distribution and allocation pair $(\Phi, \{\varphi_t, \upsilon_t, K_{t+1}\})$ is credible if and only if it satisfies (33), (35) and*

$$\forall t, \qquad \int \sum_{s=0}^{\infty} \beta^s \sum_{\Theta^{t+s}} \left[\varphi_{t+s}(w, \theta^{t+s}) + \theta_{t+s}\upsilon_{t+s}(w, \theta^{t+s})\right] \pi(\theta^{t+s})\Phi(dw) \geq W(K_t). \tag{50}$$

We sketch the proof (see Sleet-Yeltekin (2006) for a formal argument). If an allocation satisfies the conditions in the proposition, then a supporting credible equilibrium is constructed as follows. The planner implements (Φ, $\{\varphi_t, \upsilon_t, K_{t+1}\}$) from K_1 and agents are truthful provided there has been no prior planner defection. Following a defection, agents switch to the play of m^{UI} and the planner switches to the play of σ^{UI}. Given the play of agents, the best continuation payoff available to the planner from defection at r is $W(K_r)$ and since the allocation's continuation payoffs exceed this, the planner would never wish to defect. Planner optimality along the equilibrium path is ensured, (33) and (35) ensure resource-feasibility and agent optimality along the path. Following a planner defection, (σ^{UI}, m^{UI}) is played. Since this is a credible equilibrium, agent and planner optimality and resource-feasibility hold in all periods subsequent to a defection. Hence, the conditions in the proposition are sufficient for credibility. Conversely, if the planner reaches period r with capital K_r, she can guarantee herself a continuation payoff of $W(K_r)$ by playing the no insurance mechanisms $\{S^{UI}_{r+t}|K_r\}_{t=0}^{\infty}$. Therefore, no credible allocation can give a continuation payoff below $W(K_r)$, implying the necessity of (50). Necessity of the other conditions follows from the requirement that agent behavior is optimal and strategies are resource-feasible in a credible equilibrium.

It follows from Proposition 10 that the credibility constraints on allocations *assumed* in the two period examples of Section 2 are equilibrium restrictions in the infinitely repeated game. A Pareto optimal credible allocation can then be obtained by supplementing (37) at a given (K_1, Ψ_0) with the sequence of constraints (50), i.e. by solving:[21]

$$\sup \int_{\mathbb{R}_+} \gamma_0 U(\{\varphi_t(\gamma_0, \cdot), \upsilon_t(\gamma_0, \cdot)\}_{t=1}^{\infty})\Psi_0(d\gamma_0) \tag{51}$$

subject to (33), (35), (50), $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty} \in \mathbf{PA}$ and $K_1 > 0$ given.

[21] The interpretation here is that agents are assigned Pareto weights conditional on their initial period signals. Notation is adjusted accordingly.

6 Optimal infinite-horizon credible allocations

6.1 Pseudo-planner problem

To begin with, we establish an equivalence between the optimal credible allocation problem (51) and the problem of a committed "pseudo-planner". The latter allows us to decompose and interpret the impact of the (now endogenous) credibility constraints. Proposition 11 below summarizes the equivalence.

Proposition 11 *Let $K_1^* > 0$ and Ψ_0 be a distribution on $\mathbb{R}_{++}$ with unit mean. Let $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$ be a solution to (51) at (K_1^*, Ψ_0) that satisfies for all t, $\sup_t |W(K_t^*)| < \infty$. Given a non-negative multiplier sequence $\{\phi_t, q_t\}$, construct the discounting scheme $B_1^1 = 1$ and, for $t > 1$, $B_1^t = \beta^{t-1}\prod_{r=2}^{t}(1+\phi_r)$, the Pareto weighting scheme $\gamma^1(x) = \frac{x+\phi_1}{1+\phi_1}$ and $\gamma^{t+1}(x) = \frac{\phi_{t+1}}{1+\phi_{t+1}} + \frac{\gamma^t(x)}{1+\phi_{t+1}}$ and the sequence of modified production functions $\widehat{F}_t(K_t, L_t) := F(K_t, L_t) - \frac{\phi_t}{q_t}\frac{B_1^{t-1}}{\beta^{t-1}} W_K(K_t^*)\ (K_t - K_t^*)$. The pseudo-planner's problem at $(K_1^*, \Psi_0, \{\phi_t, q_t\})$ is defined as:*

$$\sup \int \sum_{t=1}^{\infty} B_1^t \gamma_1^t(\gamma_0) \sum_{\theta^t} \left[\varphi_t(\gamma_0, \theta^t) + \theta_t \upsilon_t(\gamma_0, \theta^t)\right] \pi^t(\theta^t)\Psi_0(d\gamma_0) \tag{52}$$

*subject to the incentive-constraints, $\{\varphi_t, \upsilon_t, K_{t+1}\} \in$ **PA** and the modified resource constraints*

$$\widehat{F}_t(K_t, L_t(\upsilon_t)) - \int \sum_{\theta^{t+r}} C(\varphi_t(\gamma_0, \theta^t))\pi^t(\theta^t)\Psi_0(d\gamma_0) - K_{t+1} \geq 0,$$

where $L_t(\upsilon_t) = \int \sum_{\theta^t} N(\upsilon_t(\gamma_0, \theta^t))\ \pi^t(\theta^t)\Psi_0(d\gamma_0)$.

There exists a non-negative sequence $\{\phi_t^*, q_t^*\}$ such that $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$ solves the pseudo-planner's problem at $(K_1^*, \Psi_0, \{\phi_t^*, q_t^*\})$. Conversely, if an allocation $\{\varphi_t', \upsilon_t', K_{t+1}'\}$ solves the pseudo-planner problem (52) at $(K_1^*, \Psi_0, \{\phi_t^*, q_t^*\})$, where $\sum \phi_t^* \prod_{r=1}^{t-1}(1+\phi_r^*) < \infty$, $\sum q_t^* < \infty$ and $\{\varphi_t', \upsilon_t', K_{t+1}'\}$ and $\{\phi_t^*, q_t^*\}$ satisfy appropriate complementary slackness conditions, then $\{\varphi_t', \upsilon_t', K_{t+1}'\}$ equals $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$.

Proof: See Appendix. ■

The equivalence between the planner problem with credibility constraints (51) and the pseudo-planner problem (52) is obtained by replacing the constraint set in (51) with a convex subset that includes $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$ and manipulating the resulting Lagrangian. The credibility constraint terms in this Lagrangian are used to perturb the planner's objective and technology to obtain those of the pseudo-planner. The resulting pseudo-planner objective incorporates a modified discounting scheme; the t-th discount factor $B_1^t/B_1^{t-1} = \beta(1+\phi_t^*)$ exceeds the agents' when the t-th credibility constraint binds. It also incorporates a modified Pareto weighting sequence $\gamma^t(\gamma_0)$ that converges towards 1; any initial rewards and penalties implied by the period 1 cross sectional distribution Ψ_0 are washed out. Underpinning this convergence is the fact that the uncommitted planner's *equally weighted* utilitarian payoff must be maintained above a lower bound. The pseudo-planner's technology is also modified; the marginal product of capital is depressed in those periods in which the credibility constraints bind.

The pseudo-planner's modified discount factor and technology permit a decomposition of the impact of the credibility constraints on intertemporal tradeoffs. Relative to the benchmark commitment case, the higher pseudo-planner discount factor tilts the tradeoff between current incentives and future equality. It dilutes the former, while encouraging the latter. The implications of the pseudo-planner for capital accumulation are more ambiguous. On the one hand, the higher pseudo-planner discount factor encourages capital accumulation, on the other, the depressed marginal product of capital discourages it. This ambiguity is a reflection of the indeterminate impact of capital on the credibility constraints since higher capital levels raise both the planner's continuation payoff and her outside option.

6.2 Wedges and taxes without commitment

The wedge and tax results from the two period examples with limited commitment extend to the infinite horizon credibility setting with little alteration. However, with W endogenized, we can now give an explicit expression for the credibility factor in terms of allocations on and off the equilibrium path. We assume the existence of an optimal allocation $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$ at given (K_1^*, Ψ_0) satisfying $\sup_t |W(K_t^*)| < \infty$. Then by the proof of Proposition 11, this allocation maximizes a Lagrangian that incorporates resource and (modified) credibility constraints over the set of bounded incentive-compatible allocations. The optimal multipliers on the resource and credibility constraints are denoted $\{q_t^*\}$ and $\{\phi_t^*\}$ respectively.

6.2.1 An intertemporal wedge

The first order necessary conditions for the Lagrangian problem and Jensen's inequality imply the following bound for the intertemporal wedge:

$$\text{Wedge} = \beta F_K(K^*_{t+1}, L^*_{t+1}) \frac{\sum_\theta u'(c^*_{t+1}(\gamma_0, \theta^t, \theta))\pi(\theta)}{u'(c^*_t(\gamma_0, \theta^t))} - 1 \geq \frac{1}{\mathcal{K}_{t+1}} \frac{1}{H_{t+1}(\gamma_0, \theta^t)} - 1, \tag{53}$$

where $\mathcal{K}_{t+1} := 1 + \phi_{t+1} \left(\frac{W^*_K(K^*_{t+1}) - W_K(K^*_{t+1})}{W^*_K(K^*_{t+1})} \right)$ is the credibility factor at $t+1$, $W^*_K(K^*_{t+1})$ is the marginal value of additional capital on the equilibrium path at $t+1$ and $H_{t+1}(\gamma_0, \theta^t) := \frac{\phi^*_{t+1}}{1+\phi^*_{t+1}} E\left[\frac{1}{u'(c^*_t(\gamma_0, \theta^t))}\right] \left[\frac{1}{u'(c^*_t(\gamma_0, \theta^t))}\right]^{-1} + \frac{1}{1+\phi^*_{t+1}}$.

Inequality (53) isolates the roles that individual incentive and societal credibility constraints play in generating the intertemporal wedge. As in Section 2, the substitution of consumption between periods t and $t+1$ tightens the incentive constraints, introducing a shadow cost above that of foregone period t consumption. This results in the inequality in (53) and works to increase the wedge. The effect of the substitution on the $t+1$-societal credibility constraint is more ambiguous. The right hand side of (53) decomposes this effect into an aggregate component $\mathcal{K}_{t+1}$ and an individual specific one $H_{t+1}(\gamma_0, \theta^t)$. The former measures the impact on the constraint of a small additional accumulation of capital that is used to raise the utility of agents equally; the latter adjusts this value to capture the impact of an accumulation whose proceeds are diverted to the (γ_0, θ^t)-th agent. If a capital accumulation that benefits all agents equally raises the planner's continuation payoff by less than her outside option ($\mathcal{K}_{t+1} < 1$) and if the delivery of its proceeds to the (γ_0, θ^t)-th agent exacerbates period $t+1$ inequality ($H_{t+1}(\gamma_0, \theta^t) < 1$), then the consumption substitution tightens the credibility constraint and incurs a further social cost. In this case, the lower bound on the intertemporal wedge is raised.

In the infinite horizon setting, we can obtain the following alternative expression for the credibility factor:

$$\mathcal{K}_{t+1} = 1 + \phi^*_{t+1} - \phi^*_{t+1} \frac{F_K\left(K^*_{t+1}, L'_{t+1}\right)}{F_K\left(K^*_{t+1}, L^*_{t+1}\right)} E\left[\frac{u'(c'_{t+1})}{u'(c^*_{t+1})}\right], \tag{54}$$

where L'_{t+1} and c'_{t+1} are the effort and consumption allocated to all agents at $t+1$ if the no insurance equilibrium is played from that date onwards. Consequently, additional capital tightens the credibility constraint and $\mathcal{K}_{t+1} < 1$, if the marginal product of capital is higher off the equilibrium path than on and if the distributions of consumption on and off the equilibrium path are such that $E\left[\frac{1}{u'(c^*_{t+1})}\right] > \frac{1}{u'(c_{t+1})}$.

6.2.2 A tax-market implementation

A tax-market implementation is available in which agents can trade pre-tax riskless claims to capital and the tax system is of the form $\{T_t(\gamma_0, y^t, b_t)\}_{t=1}^{\infty}$ where $T_t(\gamma_0, y^t, b_t) = T^0_t(\gamma_0, y^t) + \tau_t(\gamma_0, y^t)b_t$. The implications for marginal asset taxes are identical to those of the two period implementation. Using the Lagrangian from the proof of Proposition 11 and following the derivation in Section 2, we obtain:

$$E\left[\tau_{t+1}|\,\gamma_0, \theta^t\right] = A_0 - A_1 u'(c^*_t(\gamma_0, \theta^t)),$$

where $A_0 = 1 - \frac{\mathcal{K}_{t+1}}{1+\phi^*_{t+1}} \geq 0$ and $A_1 = \frac{1}{W_K(K^*_{t+1})}\left(1 - \frac{\mathcal{K}_{t+1}}{1+\phi^*_{t+1}}\right) \geq 0$ and these inequalities are strict if the $t+1$-th credibility constraint binds.

6.2.3 Asset pricing

Extending the model to incorporate public, aggregate productivity shocks $\{Z_t\}$ delivers similar formulas for asset prices to those in Section 2. As before, there is a class of decentralizations for the optimal allocation in this setting in which assets contingent on $\{Z_t\}$ are traded. For these decentralizations, the date t price of a claim paying out after history Z^{t+1} is naturally defined as:

$$Q_t(Z^{t+1}) = \beta \mathcal{K}_{t+1}(Z^{t+1}) \frac{E\left[\frac{1}{u'(c^*_t)}\,\middle|\, Z^t\right]}{E\left[\frac{1}{u'(c^*_{t+1})}\,\middle|\, Z^{t+1}\right]} \Lambda(Z_{t+1}|Z_t), \tag{55}$$

where $\mathcal{K}_{t+1}(Z^{t+1}) = (1 + \phi_{t+1}(Z^{t+1}))\left(\frac{E[W^*_K(K^*_{t+1})|Z^t] + E[\phi_{t+1}\left(W^*_K(K^*_{t+1}) - W_K(K^*_{t+1})\right)|Z^t]}{E[(1+\phi_{t+1})W^*_K(K^*_{t+1})|Z^t]}\right)$ is the (stochastic) credibility factor[22] and Λ is a Markov transition for $\{Z_t\}$. Equivalently, $\mathcal{M}^{CC}_{t+1}(Z^{t+1})$, the SDF in this setting, satisfies $\mathcal{M}^{CC}_{t+1}(Z^{t+1}) =$

[22] The alternative expressions $W^*_K(K^*_{t+1}, Z_{t+1}) = \frac{F_K\left(K^*_{t+1}, L^*_{t+1}, Z_{t+1}\right)}{E\left[\frac{1}{u'(c^*_{t+1})}|Z^{t+1}\right]}$ and $W_K(K^*_{t+1}, Z_{t+1}) = \frac{F_K\left(K^*_{t+1}, L_{t+1}, Z_{t+1}\right)}{E\left[\frac{1}{u'(c_{t+1})}|Z^{t+1}\right]}$, for the marginal values of capital on and off the equilibrium path are available.

$\mathcal{K}_{t+1}(Z^{t+1})\mathcal{M}^{C}_{t+1}(Z^{t+1})$, where $\mathcal{M}^{C}_{t+1}(Z^{t+1}) = \frac{E\left[\frac{1}{u'(c^*_t)}\middle| Z^t\right]}{E\left[\frac{1}{u'(c^*_{t+1})}\middle| Z^{t+1}\right]}$ is the commitment SDF. As noted, Kocherlakota-Pistaferri (2007b) show that $\mathcal{M}^{C}_{t+1}$ can provide a resolution of the equity premium puzzle.[23] However, at the coefficient of relative risk aversion required to explain the equity premium puzzle, the sample mean of $\mathcal{M}^{C}_{t+1}$ is high, and an implausibly low level of the discount factor is required to explain the returns on very low risk assets. When $E[\mathcal{K}_{t+1}(Z^{t+1})] < 1$, $\mathcal{M}^{CC}_{t+1}(Z^{t+1})$ may better reconcile consumption and asset price data with plausible discount factor values.[24] We leave an empirical investigation of asset pricing kernels in credibility-constrained settings to future work.

7 Optimal credible steady states

7.1 General case

In the remainder of the paper, we consider steady state optimal credible allocations. These allocations solve a problem of the form (51) or its pseudo-planner variant (52), and have time invariant distributions for consumption, labor and utility, and constant aggregate capital amounts. In addition, the corresponding optimal multipliers on the (modified) credibility and resource constraints satisfy $\phi^*_t = \phi^*$ and $q^*_t = q^* B^{t-1}$ where $B = \beta(1+\phi^*)$ and the marginal product of labor is time invariant at some value w^*.

The necessary optimality conditions from the pseudo-planner's problem imply the following steady state first order condition for capital:

$$1 = \beta \mathcal{K} F_K(K^*, L^*), \tag{56}$$

where K^* and L^* are steady state capital and labor, $\mathcal{K} = 1 + \phi^* - \phi^* \frac{F_K\left(K^*, L'\right)}{F_K(K^*, L^*)} E\left[\frac{u'(c')}{u'(c^*)}\right]$ is the steady state credibility factor, $E\left[\frac{u'(c')}{u'(c^*)}\right]$ is the expected ratio of marginal utility reciprocals at the optimum and in the first period of the no insurance

[23] They assume CRRA preferences, but place few restrictions on the process for individual level shocks.

[24] The optimal consumption allocations in our commitment economy featured ever increasing inequality and the immiseration of a positive measure of agents. Immiseration results characterize many dynamic private information models. To reconcile empirical cross sectional consumption distributions with the implications of models, it seems necessary to eliminate this force. Credibility constraints do this.

equilibrium and L' is aggregate labor in the first period of the no insurance equilibrium. The corresponding expression from the steady state of a commitment economy is

$$1 = \beta F_K(K^{**}, L^{**}), \tag{57}$$

where K^{**} and L^{**} are steady state capital and labor values for this case.[25] Comparing the expressions, we see that $F_K(K^*, L^*) \gtreqless F_K(K^{**}, L^{**})$ if $\mathcal{K} \lesseqgtr 1$ or, equivalently, if capital accumulation tightens/relaxes the credibility constraint. Even when $\mathcal{K} \geq 1$, K^* may still be less than K^{**} if L^* may still be less than L^{**}. This can happen if the credibility constraints disrupt the provision of incentives for effort.[26]

Recursive component planner problems The pseudo-planner's problem can be decentralized into recursive component planner problems in much the same way that the commitment planner's problem can. In particular, given an optimal steady state multiplier-wage combination $\{\phi^*, q^*, w^*\}$ and the corresponding discount $B = \beta(1 + \phi^*)$, the optimal individual allocation of the γ_0-th agent $\{\varphi_t^*(\gamma_0, \cdot), \upsilon_t^*(\gamma_0, \cdot)\}$ solves the concave component planner problem:

$$V(\gamma^1(\gamma_0)) = \sup_{\{\psi_s, \nu_s\}_{s=1}^{\infty}} \sum_{s=1}^{\infty} \sum_{\theta^s} B^s \{\gamma^s(\gamma_0)[\psi_s(\theta^s) + \theta_s \nu_s(\theta^s)] - q^*[C(\psi_s(\theta^s)) - w^* N(\nu_s(\theta^s))]\} \pi^s(\theta^s) \tag{58}$$

subject to $\{\psi_s, \nu_s\}_{s=1}^{\infty} \in \mathbf{IA}$ and (35). This problem resembles that for the commitment case except that it incorporates the modified discount B and modified Pareto weighting scheme $\{\gamma^s(\gamma)\}_{s=1}^{\infty}$ with $\gamma^s(x) = \frac{\phi^* + \gamma^{s-1}(x)}{1+\phi^*}$ and $\gamma^0(\gamma) = \gamma$. Rearranging a Lagrangian for (58) that incorporates the initial incentive constraint and substituting for V delivers the dynamic programming problem:

$$V(\gamma) = \inf_{\Lambda(\gamma)} \sum_{i=1}^{I} \{J_i\left(\rho_i(\zeta;\eta), \lambda_i(\zeta;\eta); q_t^*, w_t^*\right) + BV\left(\gamma_i'(\gamma;\eta)\right)\} \pi(\widehat{\theta}_k), \tag{59}$$

where J_k is defined as in the commitment case and

$$\gamma_i'(\gamma;\eta) = \frac{\phi^*}{1+\phi^*} + \frac{1}{1+\phi^*}\left(\gamma + \sum_{j:\{(i,j)\in\mathbb{K}\}} \eta_{i,j} - \sum_{j:\{(j,i)\in\mathbb{K}\}} \eta_{j,i} \frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right). \tag{60}$$

[25] We assume that (K^{**}, L^{**}) and, implicitly, that the steady state moment $E\left[\frac{1}{u'(c^{**})}\right]$ are well defined and, in the latter case, positive.

[26] This disruption can be direct (the credibility constraints raise the shadow cost of future inequality) or indirect (it is more costly to elicit effort at the less dispersed distribution of utilities in the credibility case).

The law of motion for effective Pareto weights (60) corresponds to that obtained in the commitment case only if the credibility constraints do not bind and $\phi^* = 0$. When these constraints do bind, $\phi^* > 0$ and the law of motion incorporates a force for mean reversion. As in the commitment case, effective Pareto weights are augmented with "incentive shocks" derived from the multipliers on the incentive constraints. We have the following (steady state) analogue of Proposition 4.

Proposition 12 *1) There exists a continuous policy function η^*, with $\eta^* : \mathbb{R}_{++} \to \mathbb{R}_+^{I(I-1)}$, that attains the infima in (59). 2) If the non-recursive problem (58) has a solution, then the individual allocation $\{\varphi_t^*(\gamma), v_t^*(\gamma)\}_{t=1}^\infty$ induced by η^* from γ solves (58).*

It follows from (60) that the optimal process for effective Pareto weights $\{\gamma_t^*\}_{t=1}^\infty$ starting from γ evolves according to:

$$\gamma_{t+1}^*(\gamma, \theta^{t-1}, \widehat{\theta}_k) = (1-\omega) + \omega\gamma_t^*(\gamma, \theta^{t-1}) + \varepsilon_{t+1}(\gamma, \theta^{t-1}, \widehat{\theta}_k), \tag{61}$$

where the sequence of incentive shocks $\{\varepsilon_{t+1}\}_{t=1}^\infty$ has a conditional expectation of 0 and $\omega = \frac{1}{1+\phi^*}$. As before, the incentive shocks are a source of dispersion in effective Pareto weights, and, hence, in consumption and utility. Now, binding credibility constraints are a source of mean reversion for these weights; the planner cannot credibly implement allocations that feature too much ex post inequality.

The optimal steady state distribution for effective Pareto weights Ψ^* is then an invariant measure implied by η^* and π (or, equivalently, (61) and π). The corresponding policy functions and invariant measures for consumption and effort can then be recovered from Ψ^*, η^* and the optimizations underpinning the cost functions J_k. In short, an optimal steady state can be summarized by a tuple $\{q^*, \phi^*, w^*, \eta^*, \zeta^*, c^*, e^*, \Psi^*, K^*, L^*\}$ where $\{q^*, \phi^*, w^*\}$ are the aggregate multipliers, $\{\eta^*$, ζ^*, c^*, $e^*\}$ are optimal time invariant policy functions for multipliers, effective Pareto weights, consumption and effort implied by (59), Ψ^* is an invariant measure for Pareto weights implied by ζ^* and π and K^* and L^* are a constant optimal aggregate capital stock and labor supply.

7.2 An example

We now turn to an example with log preferences and a Cobb-Douglas technology ($K^\alpha L^{1-\alpha}$). Lemma 13 gives *sufficient* conditions for a steady state optimal credible allocation in this case.

Lemma 13 *If $\{q^*, \phi^*, w^*, \eta^*, \zeta^*, c^*, e^*, \Psi^*, K^*, L^*\}$ satisfies the following conditions, then it is an optimal steady state allocation. 1) $L^* = T\left[\frac{\frac{1-\alpha}{1-\alpha\beta}}{E[\theta]+\frac{1-\alpha}{1-\alpha\beta}}\right]$, $K^* = \frac{1}{q^*}\frac{\alpha\beta}{1-\alpha\beta}$, $K^{*\alpha}L^{*1-\alpha} = \frac{1}{q^*}\frac{1}{1-\alpha\beta}$ and $w^* = (1-\alpha)K^*L^{*-\alpha}$, 2) η^* is the policy function that solves (59), 3) (ζ^*, c^*, e^*) are implied by η^*, q^* and w^*, 4) Ψ^* is the invariant measure over effective Pareto weights implied by ζ^* and π, 5) $\frac{1}{1-\beta}\int\sum_\Theta[u(c^*(\gamma,\theta)) + \theta\upsilon(e^*(\gamma,\theta))]\ \pi(\theta)\Psi^*(d\gamma)\ -W(K^*) = 0$, and 6) for almost all $\gamma \in$ supp Ψ and $\widehat{\theta}_i \in \Theta$, the upper bound on leisure is non-binding:*

$$\widehat{\theta}_i\left[\gamma + \sum_{j:\{(i,j)\in\mathbb{K}\}}\eta^*_{i,j}(\gamma) - \sum_{j:\{(i,j)\in\mathbb{K}\}}\eta^*_{j,i}(\gamma)\frac{\widehat{\theta}_j\pi(\widehat{\theta}_j)}{\widehat{\theta}_i\pi(\widehat{\theta}_i)}\right] \leq q^*w^*T. \tag{62}$$

Proof: See Appendix. ■

Lemma 13 implies that the task of finding the resource aggregates L^* and K^* and the multiplier q^* can be split from that of finding the distribution of effective Pareto weights and ϕ^* (subject to verification of (62)). The first condition in the lemma can be used to determine candidate values for K^*, L^*, q^* and steady state aggregate consumption $C^* = K^{*\alpha}L^{*1-\alpha} - K^*$. Then, given q^* and a value for ϕ, an optimal component planner policy function η^*_ϕ can be obtained from a time invariant version of the dynamic programming problem (59). Such a policy function implies an optimal law of motion for Pareto weights that induces an invariant measure, Ψ_ϕ (see, Sleet-Yeltekin (2008b)). If a multiplier ϕ^* can be found such that:

$$\frac{1}{1-\beta}\int\sum_\Theta[\varphi^*_{\phi^*}(\gamma,\theta) + \theta\upsilon^*_{\phi^*}(\gamma,\theta)]\pi(\theta)\Psi_{\phi^*}(d\gamma) - W(K^*) = 0,$$

and if Condition (62) holds, then the implied steady state allocation is optimal.

In the log-Cobb Douglas case, with non-binding bounds on leisure, the steady state first order condition for capital is:

$$1 = \beta\alpha K^{*(\alpha-1)}L^{*(1-\alpha)},$$

where $L^* = T\left[\frac{\frac{1-\alpha}{1-\alpha\beta}}{E[\theta]+\frac{1-\alpha}{1-\alpha\beta}}\right]$. Steady state capital, labor supply and output are below their corresponding values in the commitment case because in the latter the upper bound on leisure is binding for a positive measure of agents. For these agents, labor supply is constrained to be non-negative and aggregate labor supply is raised.

7.2.1 Numerical Implementation

We fix the parameters $\{\beta, \alpha, T\}$ at $\{0.9, 0.5, 1\}$ and assume that there are 4 shock values uniformly spaced between 0.9 and 1.1 that occur with equal probability. We implement the procedure sketched above numerically and compute the optimal multiplier ϕ^* to be 0.02 and B, the pseudo-planner's discount factor, to be 0.92. The following graphs describe key aspects of the steady state allocation and its tax-market implementation. Figure 1 shows the optimal adjustment of effective Pareto weights $\gamma_i^{'*}(\gamma) - \gamma$ as a function of γ and $\hat{\theta}_i$. In this and all subsequent figures, the support of the stationary distribution Ψ_{ϕ^*} corresponds to the shaded region of γ values. To the left of this region, the optimal adjustment in the Pareto weight is positive for all shocks; to the right, it is negative for all shocks. Hence, γ-values outside of the shaded region are transient.

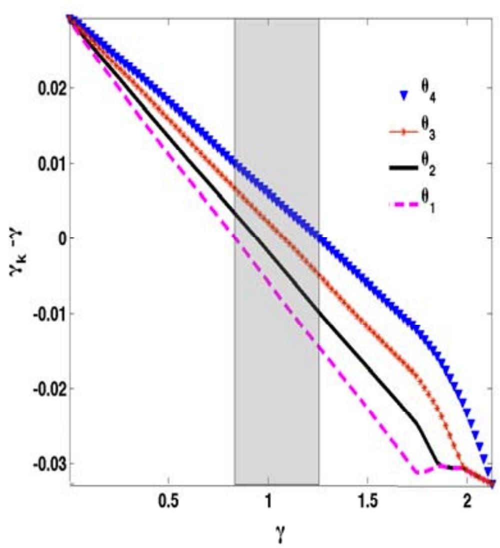

Figure 1: Effective Pareto weights

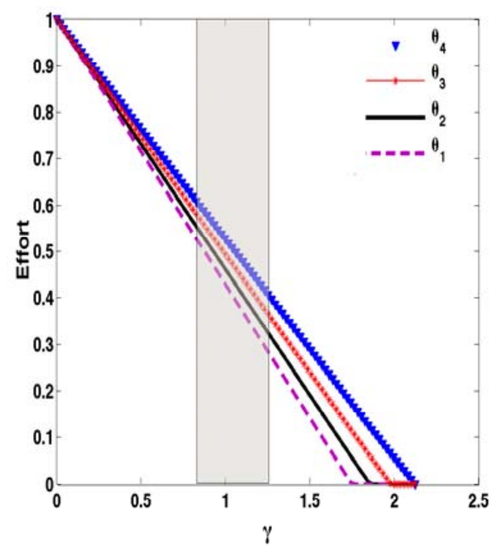

Figure 2: Effort choices

Figure 2 shows the optimal effort levels at each γ value. These are falling in γ and the shock $\hat{\theta}_i$. The figure confirms that the upper bound on leisure (or lower bound of 0 on effort) does not bind for any γ in the support of Ψ_{ϕ^*}. Figure 3 illustrates expected marginal asset taxes at $t+1$ conditional on an agent's effective Pareto weight and shock at t. We express these expected marginal asset taxes as taxes on asset income rather than wealth. i.e. they show $E_t[\widehat{\tau}(\gamma, \theta_t, \theta_{t+1})]$, where $\widehat{\tau}(\gamma, \theta_t, \theta_{t+1})$ satisfies $\widehat{\tau}(\gamma_t^*(\gamma, \theta^{t-1}), \theta_t, \theta_{t+1}) = \frac{\tau_t(\gamma, \theta^{t+1})}{1-Q}$ and Q is the steady state riskless claims price. Over much of the Pareto weight state space, and over the support Ψ_{ϕ^*} in particular, these tax rates are fairly small, between -2% and 2%. However, they are neither all zero (as in Kocherlakota (2005)) or all negative (as in Farhi-Werning (2007)). They become large and negative at low γ values. Poor agents are given large subsidies to accumulate more wealth; rich agents are lightly taxed to deter too much accumulation.

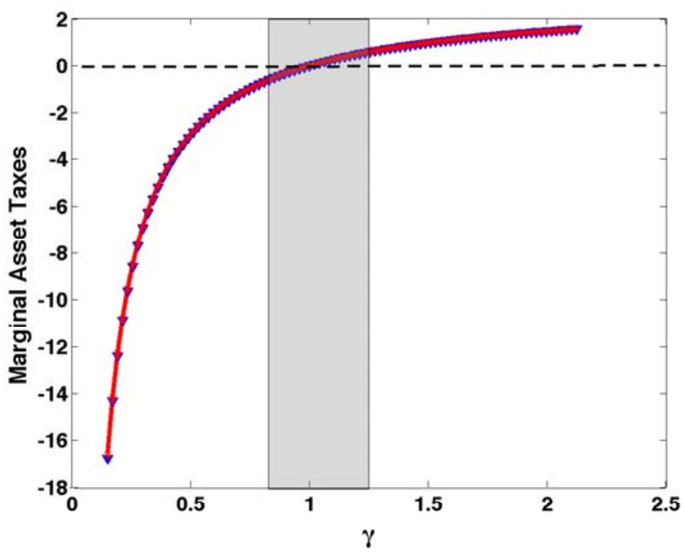

Figure 3: Marginal asset taxes

8 Conclusion

Most analyses of policy credibility use Ramsey models. In these, credibility frictions stem from exogenous restrictions on the tax system. Lump sum taxation avoids tax distortions, but is excluded by assumption. The government is tempted to replicate it by reneging on past pledges and taxing privately accumulated assets. In contrast, in our model, restrictions on policies and allocations stem from primitive frictions. The credibility problem is distributional in nature and arises from a society's desire to undo inequality after its prospect has been used to motivate privately informed agents.

We consider a game played by an uncommitted planner and privately informed agents and derive implications for societal optima, optimal tax policy and asset pricing. Individual-level incentive-compatibility and societal credibility constraints emerge as equilibrium restrictions in our game. We show that the latter arrest the upward trend in inequality present in dynamic private information models with commitment and have ambiguous implications for capital accumulation. We isolate the forces that promote and retard capital accumulation in these settings, derive the pattern of intertemporal wedges that characterize optimal credible allocations and show that these allocations solve the problem of a committed pseudo-planner with perturbed preferences and production possibilities. We identify the joint implications of private information and credibility for optimal tax policy. In particular, we show that the optimal asset taxes are progressive with subsidies for the poor and taxes for rich. Under the assumption that the optimal allocation is implemented by some arrangement of taxes, markets and contracts, we show that asset pricing formulas derived in private information settings with commitment are augmented with a stochastic credibility factor and suggest ways in which this may help resolve asset pricing puzzles.

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9 Appendix

Proof of Proposition 11 Define the modified credibility constraints, for all $t \geq 1$,

$$\int \sum_{s=0}^{\infty} \beta^s \sum_{\Theta^{t+s}} \left[\varphi_{t+s}(\gamma, \theta^{t+s}) + \theta_{t+s} \upsilon_{t+s}(\gamma, \theta^{t+s})\right] \pi(\theta^{t+s}) \Psi_0(d\gamma) \geq W(K_t^*) + W_K(K_t^*)(K_t - K_t^*). \tag{63}$$

Define $\Omega(K_1^*)$ to equal all incentive-compatible allocations at K_1^* that satisfy the additional restrictions: $\sup_t |R_t| < \infty$, $R_t := F(K_t, L_t(\upsilon_t)) - \int_{\mathbb{R}_+} \sum_{\theta^t} C(\varphi_t(\gamma, \theta^t)) \; \pi^t(\theta^t) \Psi_0(d\gamma) - K_{t+1}$, $\sup_t |W_t| < \infty$, $W_t = \int_{\mathbb{R}_+} \sum_{\theta^{t-1}} \gamma_0 \; U(\{\varphi_{t+r}(\gamma_0, \theta^{t-1}, \cdot), \upsilon_{t+r}(\gamma, \theta^{t-1}, \cdot)\}_{r=0}^{\infty}) \; \Psi_0(d\gamma)$ and $\sup_t |W(K_t)| < \infty$. We compactly state the resource and modified credibility constraint using the mapping $G(K_1^*; \Omega)$ where $G(K_1^*; \{\varphi_t, \upsilon_t, K_{t+1}\}) = \{G_t(K_1^*; \{\varphi_t, \upsilon_t, K_{t+1}\})\}_{t=1}^{\infty}$, $G_t(K_1^*; \{\varphi_t, \upsilon_t, K_{t+1}\}) = (R_t, W_t - W(K_t^*) - W_K(K_t^*)(K_t - K_t^*))$, and R_t and W_t are evaluated at $(K_1^*; \{\varphi_t, \upsilon_t, K_{t+1}\})$. Define the *modified credibility problem* *at* K_1^*,

$$\sup_{\Gamma} \int \gamma \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\theta^t} \left[\varphi_t(\gamma,\theta^t) + \theta_t \upsilon_t(\gamma_0,\theta^t)\right] \pi^t(\theta^t)\Psi_0(d\gamma). \tag{64}$$

where $\Gamma = \{\{\varphi_t, \upsilon_t, K_{t+1}\} \in \Omega | G(K_1^*; \{\varphi_t, \upsilon_t, K_{t+1}\}) \geq 0\}$. We note that $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$ is in Γ and that Γ is a convex subset of the constraint set in (51) so that $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$ solves (64). $G(K_1^*; \cdot)$ is convex and $G(K_1^*; \cdot) : \Omega(K_1^*) \to \ell_\infty$. For $K_1^* > 0$, it is possible to find an allocation $\{\widehat{\varphi}_t, \widehat{\upsilon}_t, \widehat{K}_{t+1} | K\} \in \Omega(K_1^*)$ such that $G(K_1^*; \{\varphi_t, \upsilon_t, K_{t+1}\}) > 0$, by perturbing the no insurance allocation at K_1^*. Then by Luenberger (1969, [Theorem 1, p.217]), $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$ solves

$$\sup_{\Omega(K_1^*)} \int \gamma \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\theta^t} \left[\varphi_t(\gamma,\theta^t) + \theta_t \upsilon_t(\gamma,\theta^t)\right] \pi^t(\theta^t)\Psi_0(d\gamma) + \langle G(K_1^*; \{\varphi_t, \upsilon_t, K_{t+1}\}), \boldsymbol{\lambda}^* \rangle$$

where $\boldsymbol{\lambda}^*$ belongs to the positive cone of the dual space of ℓ_∞ and $\left\langle G(K_1^*; \{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}), \boldsymbol{\lambda}^* \right\rangle = 0$.

Let $\alpha^i = \{\varphi_t^i, \upsilon_t^i, K_{t+1}^i\}$, $i = 1, 2$, be two allocations in $\Omega(K_1^*)$ and let $\alpha_T(\alpha^1, \alpha^2) = \{\{\varphi_t^1, \upsilon_t^1, K_{t+1}^1\}_{t=1}^T, \{\varphi_t^2, \upsilon_t^2, K_{t+1}^2\}_{t>T}\}$. Then, $\int \gamma U(\alpha_T(\alpha^1, \alpha^2)(\gamma))\Psi_0(d\gamma) \to \int \gamma U(\alpha^1(\gamma))\Psi_0(d\gamma)$. Also, $G_t(\alpha_T(\alpha^1, \alpha^2)) = \{\int \gamma U(\alpha_T(\alpha^1, \alpha^2)(\gamma, \theta^{t-1}))\pi^{t-1}(\theta^{t-1})\Psi_0(d\gamma)$ $-W(K_t^*)$ $-W_K(K_t^{i(T,t)})(K_t^{i(T,t)} - K_t^*)$, $F(K_t^{i(T,t)}, L_t^{i(T,t)})$ $-K_{t+1}^{i(T,t)}$ $-C_t^{i(T,t)}\}$ where $i(T,t) = 1$ if $t \leq T$ and 2 otherwise and $L_t^{i(T,t)}$ and $C_t^{i(T,t)}$ are the corresponding aggregate consumptions and labor supplies. We observe that $\lim_T G_t(\alpha_T(\alpha^1, \alpha^2))$ $= G_t(\alpha^1)$, for all T, $\sup_t \left\|G_t(\alpha_T(\alpha^1, \alpha^2))\right\| \leq M < \infty$ and, for all T, $\lim_t \left\|G_t(\alpha_T(\alpha^1, \alpha^2)) - G_t(\alpha^2)\right\| = 0$. Then applying an argument in Le Van-Sağlam (2006, [Theorem 2]), we have that the multiplier $\boldsymbol{\lambda}^*$ can be represented as an element of ℓ_1. Consequently, there exist multipliers $\{\phi_t, q_t\}$ such that $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$ maximizes the Lagrangian:

$$\begin{aligned}
\mathcal{L}(\{\varphi_t, \upsilon_t, K_{t+1}\}; \{\phi_t, q_t\}) &= \int \gamma \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\theta^t} \left[\varphi_t(\gamma,\theta^t) + \theta_t \upsilon_t(\gamma,\theta^t)\right] \pi^t(\theta^t)\Psi_0(d\gamma_0) \\
&+ \sum_{t=1}^{\infty} \phi_t \beta^{t-1} \prod_{r=1}^{t-1}(1+\phi_r) \left[\int \sum_{r=0}^{\infty} \beta^r \sum_{\theta^{t+r}} \left[\varphi_{t+r}(\gamma,\theta^{t+r}) + \theta_{t+r}\upsilon_{t+r}(\gamma,\theta^{t+r})\right] \pi^{t+r}(\theta^{t+r})\Psi_0(d\gamma) - \underline{W}(K_t^*) - \underline{W}_K(K_t^*)(K_t - K_t^*)\right] \\
&+ \sum_{t=1}^{\infty} q_t \left[F(K_t, L_t(\upsilon_t)) - \int \sum_{\theta^{t+r}} C(\varphi_t(\gamma,\theta^t))\pi^t(\theta^t)\Psi_0(d\gamma) - K_{t+1}\right]
\end{aligned} \tag{65}$$

subject to $\{\varphi_t, \upsilon_t, K_{t+1}\} \in \Omega(K_1^*)$ and $K_1 = K_1^*$ given. Appealing to Abel's Lemma (Rudin (1976) [Theorem 3.41, p.70]) and the definitions of $\Omega(K_1^*)$ and $\mathcal{L}$ implies that (65) can be rearranged to give:

$$\mathcal{L}(\{\varphi_t, \upsilon_t, K_{t+1}\}; \{\phi_t, q_t\}) = \int B_1^t \gamma_1^t(\gamma) \sum_{\theta^t} \left[\varphi_t(\gamma_0, \theta^t) + \theta_t \upsilon_t(\gamma, \theta^t)\right] \pi^t(\theta^t)\Psi_0(d\gamma_0) - \sum_{t=1}^{\infty} \phi_t B_1^{t-1} \underline{W}(K_t^*)$$
$$+ \sum_{t=1}^{\infty} q_t \left[F(K_t, L_t(\upsilon_t)) - \frac{\phi_t}{q_t} \frac{\beta B_1^{t-1}}{\beta^{t-1}} \underline{W}_K(K_t^*)(K_t - K_t^*) - \int \sum_{\theta^{t+r}} C(\varphi_t(\gamma, \theta^t))\pi^t(\theta^t)\Psi_0(d\gamma) - K_{t+1} \right],$$

where $B_j^0 = 1$ and for $t \geq 1$, $B_j^t = \beta^{t-1} \prod_{r=0}^{t-1}(1+\phi_{j+r})$, $\gamma_j^1(x) = \frac{x}{1+\phi_j}$, $\gamma_j^{t+1}(x) = \frac{\phi_{t+j}}{1+\phi_{t+j}} + \frac{\gamma_j^t(x)}{1+\phi_{t+j}}$. We infer that $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$ solves the pseudo-planner problem at $\{q_t, \phi_t, K_1^*, \Psi_0\}$.

Conversely, if $\{\varphi_t', \upsilon_t', K_{t+1}'\}$ maximizes (65), at some sequence of non-negative multipliers $\{\phi_t, q_t\}$ and if these multipliers satisfy the bounds in the proposition and, together with the allocation, complementary slackness conditions, then $\{\varphi_t', \upsilon_t', K_{t+1}'\}$ solves the modified credibility problem. The strict concavity of the latter problem implies that it cannot admit multiple solutions and so $\{\varphi_t', \upsilon_t', K_{t+1}'\} = \{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$. We infer that if an allocation $\{\varphi_t', \upsilon_t', K_{t+1}'\}$ solves the *pseudo-planner problem* at such multipliers it coincides with the optimal allocation $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}$. ■

Proof of Proposition 13 (Sketch) In the log-Cobb Douglas case, (51) may be mapped into a concave problem with a simple change of variables. Let $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$ be a resource-feasible allocation at some fixed $K_1 > 0$ and Ψ_0. Define, for all t, $\chi_t = \varphi_t - \alpha \ln K_t$ and $\kappa_{t+1} = K_{t+1}/K_t^{\alpha}$. The definition of an allocation (in particular, the boundedness condition, for Ψ_0-almost every γ, $\lim_{S\to\infty} \sum_{t=1}^{S} \sum_{\Theta^t} \beta^{t-1} [|\varphi_t(\gamma, \theta^t)| + \theta_t|\upsilon_t(\gamma, \theta^t)|]\pi^t(\theta^t) < \infty$), ensures that $K_t > 0$ and that these new variables are well defined. Hence, the allocation $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$ can be represented by $(K_1, \{\chi_t, \upsilon_t, \kappa_{t+1}\}_{t=1}^{\infty})$, where the latter inherits boundedness and measurability conditions from $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$. Conversely, any $(K_1, \{\chi_t, \upsilon_t, \kappa_{t+1}\})$ satisfying such boundedness and measurability conditions implies an allocation. We call a sequence $(K_1, \{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\})$ satisfying these conditions a *transformed allocation*, where $\{\widetilde{L}_t\}$, $\widetilde{L}_t \in (0, T]$, is a sequence of aggregate labor inputs. The objective and constraints from (51) can be re-expressed in terms of transformed allocations. The transformed resource constraints are: $G_1(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}) = \{G_{1t}(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\})\}_{t=0}^{\infty} = \{\widetilde{L}_t^{1-\alpha} - \int \exp \chi_t(\gamma, \theta^t)\pi^t(\theta^t)\Psi_0(d\gamma) - \kappa_{t+1}\}_{t=0}^{\infty} \geq 0$, the transformed credibility constraints are: $G_2(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}) = \{G_{2t}(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\})\}_{t=0}^{\infty} = \{\int \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\theta^t}[\chi_{r+t}(\gamma, \theta^{r+t})$

$+\ \theta_{r+t}\upsilon_{r+t}(\gamma, \theta^{r+t})]\pi^{r+t}(\theta^{r+t})\Psi_0(d\gamma)\ + \frac{\beta\alpha}{1-\beta\alpha}\sum_{t=1}^{\infty}\beta^t \ln \kappa_{r+t+1}\ - W_0\}_{r=0}^{\infty} \geq 0$ and we add supplementary aggregate labor constraints $G_3(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}) = \{G_{3t}(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\})\}_{t=0}^{\infty} = \{T - \widetilde{L}_t - \int \exp \upsilon_t(\gamma, \theta^t)\pi^t(\theta^t)\Psi_0(d\gamma)\}_{r=0}^{\infty} \geq 0$. (51) is transformed into the problem:

$$\frac{\alpha}{1-\beta\alpha}\ln K_1 + \sup_{\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}} \int \gamma \sum_{t=1}^{\infty}\beta^{t-1}\sum_{\theta^t}[\chi_t(\gamma, \theta^t) + \theta_t \upsilon_t(\gamma, \theta^t)]\pi^t(\theta^t)\Psi_0(d\gamma) + \frac{\beta\alpha}{1-\beta\alpha}\sum_{t=1}^{\infty}\beta^{t-1}\ln \kappa_{t+1} \tag{66}$$

subject to the constraints described above, the incentive constraints re-expressed in terms of $\{\chi_t, \upsilon_t\}$ and boundedness and measurability conditions. Let Ω' denote the set of incentive-compatible transformed allocations$\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}$ with each $G_i(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}) \in \ell_\infty$, i = 1,2,3. From Luenberger (1969, [Theorem 1, p. 220]), it follows that if there is a multiplier sequence $\{q'_t, \phi'_t, w'_t\} \in \ell_1$ and a transformed allocation $\{\chi^*_t, \upsilon^*_t, \widetilde{L}^*_t, \kappa^*_{t+1}\}$ such that A) $\{\chi^*_t, \upsilon^*_t, \widetilde{L}^*_t, \kappa^*_{t+1}\}$ solves

$$\begin{aligned}\sup_{\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}\in\Omega'} &\int \gamma \sum_{t=1}^{\infty}\beta^{t-1}\sum_{\theta^t}[\chi_t(\gamma, \theta^t) + \theta_t \upsilon_t(\gamma, \theta^t)]\pi^t(\theta^t)\Psi_0(d\gamma) + \frac{\beta\alpha}{1-\beta\alpha}\sum_{t=1}^{\infty}\beta^{t-1}\ln \kappa_{t+1} \\ &+\sum_{t=1}^{\infty} q'_t G_{1t}(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}) + \sum_{t=1}^{\infty} w'_t G_{2t}(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}) + \sum_{t=1}^{\infty} \phi'_t G_{3t}(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}),\end{aligned} \tag{67}$$

and B) for each i, $G_i(\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}) = 0$, then $\{\chi^*_t, \upsilon^*_t, \widetilde{L}^*_t, \kappa^*_{t+1}\}$ solves (66) at Ψ_0 and any K.

Suppose that $\phi'_t = B^{t-1}\phi$, $q'_t = B^{t-1}q' = B^{t-1}q^{1-\alpha}\left(\frac{\beta\alpha}{1-\beta\alpha}\right)^{\alpha}$ and $w'_t = q'_t w$ for $B = \beta(1+\phi)$, $\phi \in (0, \frac{1-\beta}{\beta})$, $q > 0$ and $w > 0$. Let $z = (q', w, \phi)$. Rearranging problem (67) at these multipliers gives:

$$\begin{aligned}\sup_{\{\chi_t, \upsilon_t, \widetilde{L}_t, \kappa_{t+1}\}\in\Omega'} &\int \sum_{t=1}^{\infty} B^{t-1}\gamma^t(\gamma)\sum_{\theta^t}[\chi_t(\gamma, \theta^t) + \theta_{r+t}\upsilon_t(\gamma, \theta^t) - q'\{\exp \chi_t(\gamma, \theta^t) - w\{T - \exp \upsilon_t(\gamma, \theta^t)\}\}]\pi^t(\theta^t)\Psi_0(d\gamma) \\ +&\sum_{t=1}^{\infty} B^{t-1}\left[\frac{\beta\alpha}{1-\beta\alpha}\ln \kappa_{t+1} - q'\kappa_{t+1}\right] + \sum_{t=1}^{\infty} B^{t-1}q'\left[\widetilde{L}_t^{1-\alpha} - w\widetilde{L}_t\right].\end{aligned} \tag{68}$$

A sufficient condition for $\{\chi^*_t, \upsilon^*_t, \widetilde{L}^*_t, \kappa^*_{t+1}\}$ to solve (68) at $z = (q', w, \phi)$ is that for Ψ_0-a.e. γ, $\{\chi^*_t(\gamma, \cdot), \upsilon^*_t(\gamma, \cdot)\}$ maximizes:

$$\sum_{t=1}^{\infty} B^{t-1}\gamma^t(\gamma)\sum_{\theta^t}[\chi_t(\gamma, \theta^t) + \theta_t \upsilon_t(\gamma, \theta^t) - q'\{\exp \chi_t(\gamma, \theta^t) - w\{T - \exp \upsilon_t(\gamma, \theta^t)\}\}]\pi^t(\theta^t) \tag{69}$$

over the set of incentive-compatible and appropriately measurable and bounded individual transformed allocations and that each $\kappa^*_{t+1} = \kappa^*_z := \frac{1}{q'}\frac{\beta\alpha}{1-\beta\alpha}$ and $\widetilde{L}^*_t = \widetilde{L}^*_z := \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}$.

(69) can be reformulated recursively and an optimizing sequence $\alpha_z^* = \{\chi_{z,t}^*, \upsilon_{z,t}^*\}$ recovered from the policy function η_z^* that solves the associated Bellman equation (see main text). Let ζ_z^*, χ_z^* and υ_z^* denote the law of motion for effective Pareto weights and the policy functions for utilities implied by η_z^*. Suppose that (ζ_z^*, π) induces an invariant measure over effective Pareto weights Ψ_z. If $\Psi_0 = \Psi_z$, then the corresponding (transformed) time invariant utilitarian payoff is: $\widehat{W}(z) = \frac{1}{1-\beta}\int \sum_\theta [\chi_z^*(\gamma,\theta) + \theta\upsilon_z^*(\gamma,\theta)]\pi(\theta)\Psi_z(d\gamma) + \frac{\beta\alpha}{1-\beta\alpha}\frac{\beta}{1-\beta}\ln\kappa^*$. The implied aggregates for normalized consumption and labor supply are given by: $\widehat{C}_z = \int \sum \exp(\chi_z^*(\zeta,\theta))\pi(\theta)\Psi_z(d\zeta)$ and $\widehat{L}_z^* = \int\sum[T - \exp(\upsilon_z^*(\zeta,\theta))]\pi(\theta)\Psi_z(d\zeta)$. The first order conditions from (69) (or its recursive version) imply that $\widehat{C}_z = \frac{1}{q'}$ and, if $\exp(\upsilon_z^*(\zeta,\theta)) \in (0,T)$ for ζ in the support of Ψ_z and all θ, $\widehat{L}_z = T - \frac{E[\theta]}{q'w}$. $\exp(\upsilon_z^*(\zeta,\theta)) \in (0,T)$ for ζ in the support of Ψ_z and all θ, corresponds to the condition

$$\widehat{\theta}_i\left[\gamma + \sum_{j:\{(i,j)\in\mathbb{K}\}}\eta_{z,i,j}^*(\gamma) - \sum_{j:\{(i,j)\in\mathbb{K}\}}\eta_{z,j,i}^*(\gamma)\frac{\widehat{\theta}_j\pi(\widehat{\theta}_j)}{\widehat{\theta}_i\pi(\widehat{\theta}_i)}\right] \leq q'wT, \tag{70}$$

on the underlying policy functions. It remains to find a value for z such that the optimal transformed resource, aggregate labor and credibility constraints hold at Ψ_z.

The expressions for $\widehat{C}_z$, $\widehat{L}_z$, κ_z^* and $\widetilde{L}_z^*$ imply that the aggregate labor and transformed resource constraints hold with equality if $q' = q'^* := \frac{1}{1-\beta\alpha}\frac{1}{L^{*1-\alpha}}$, $w = w^* := \frac{(1-\alpha)}{\widetilde{L}^{*\alpha}}$ and $\widehat{L}_z = \widetilde{L}_z^* = \widetilde{L}^* := T\left[\frac{\frac{1-\alpha}{1-\beta\alpha}}{E[\theta]+\frac{1-\alpha}{1-\beta\alpha}}\right]$. The corresponding value for κ_z^* is $\kappa^* := \frac{1}{q'}\frac{\beta\alpha}{1-\beta\alpha}$. These conditions hold independently of the value of ϕ (provided this value is such that (70) holds). Finally, if $z^* = (q'^*, w^*, \phi^*)$ is such that $\widehat{W}(z^*) = W_0$, then the implied transformed allocation satisfies the credibility conditions with equality. This allocation (and the multiplier sequence implied by z^*) satisfy the sufficient conditions A and B and the allocation solves (66). If $K_1 = K^* = \kappa^{*\frac{1}{1-\alpha}}$, then the capital stock is at steady state. It remains only to restate the sufficient conditions above in terms of the untransformed variables ϕ^*, c^*, q^*, e^* and K^* to obtain those in the proposition. ■