Logic, Methodology and Philosophy of Science IX
D. Prawitz, B. Skyrms and D. Westerståhl (Editors)

WHEN NORMAL AND EXTENSIVE FORM DECISIONS DIFFER

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0. Introduction and outline.

The "traditional" view of normative decision theory, as reported (for example) in chapter 2 of Luce and Raiffa's [1957] classic work, *Games and Decisions*, proposes a reduction of sequential decisions problems to non-sequential decisions: a reduction of extensive forms to normal forms. Nonetheless, this reduction is not without its critics, both from inside and outside expected utility theory.[1] It is my purpose in this essay to join with those critics by advocating the following thesis.

THESIS: *Sequential decisions, in* extensive form, *may lead to different outcomes than their non-sequential,* normal form *versions, in a variety of problems where the normal form fails to eliminate some "future" options that will not be chosen.*

My plan for this paper is to review the non-equivalence of extensive and normal forms in the following contexts and show how the thesis applies in each one:

In section 1, I rehearse the Harsanyi-Selten (1988) argument, applied to Game Theory. They use this thesis to distinguish "perfect" from "imperfect" equilibria in extensive forms and show that this distinction is lost in the reduction to normal forms. They appeal to a "trembling hands" model of players' options to salvage a modified version of the reduction.

In section 2, I address an ingenious argument, due to M. Goldstein (in his [1983] "Prevision of a Prevision") which uses the extensive-normal form reduction to constrain a coherent (Bayesian) agent's current beliefs about his/her future degrees of belief. In particular, I point out (§ 2.1)

[1] See LaValle and Fishburn [1987] for a useful review of the issue for problems involving one decision maker.

where Goldstein's result leads to excessive use of Bayes' rule for updating: Temporal Conditionalization.[2] And I point out (§ 2.2) where it precludes the use of Bayes' rule in updating finitely additive probabilities.

Last, in section 3, I report on some relevant consequences of using sets of probabilities: Robust Bayesian analysis. In collaborated work with L. Wasserman (Statistics, CMU) we investigate a phenomenon we call "dilation" of sets of probabilities. This occurs when the set of unconditional probabilities for an event are (properly) smaller than the set of conditional probabilities for that event (given each outcome of a partition). I illustrate how "dilation" leads to a violation of the reduction of extensive to normal forms. In § 3.1 and § 3.2 I report some of our work-in-progress indicating necessary and sufficient conditions for "dilation".

1. Harsanyi & Selten's "trembling hands"

John Harsanyi and Reinhard Selten (1988) question the adequacy of Nash's concept applied to the normal-form version of an extensive form game. They deny the equivalence of normal and extensive game forms. Instead, they advocate a refined equilibrium concept for extensive form games, based on a "trembling hands" model of choice.

An equilibrium for extensive forms is acceptable, according to their account, provided it is robust over small perturbations in choice. One of their examples from (1992) beautifully illustrates the difference between the two kinds of equilibria. Each player has two pure strategies: In the extensive form, player-1 had choice set $\{a, b\}$ and, provided his/her information set is reached (provided player-1 chooses a), player-2 has a choice set $\{c, d\}$. In the corresponding normal form, the strategies are $\{A, B\}$ for player-1 and $\{C, D\}$ for player-2. Payoffs are displayed in the next two figures.

[2] The analysis of § 2.1 addresses Goldstein's reasons. I. Levi [1987] successfully responds to a variety of arguments purporting to show that Bayes' rule is mandatory for updating beliefs.

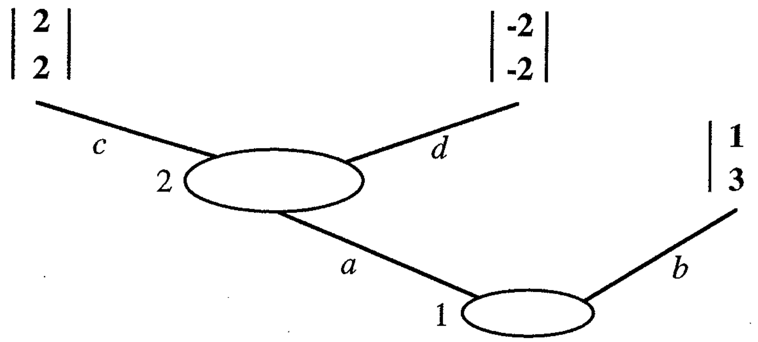

Figure 1.1 — the extensive form game
Player-1's payoff's are listed above Player-2's

	C	D
A	2 2	-2 -2
B	1 3	1 3

Figure 1.2 — Normal form of the game, above
Player-1's payoffs appear in the top-left corners.

Observe that, corresponding to the normal form Figure 1.2, there are two equilibria: the pairs $\{A, C\}$ and $\{B, D\}$. However, the latter in "imperfect" in the extensive form of Figure 1.1, as that requires player-2 to (threaten to) play option d in case choice node **2** is reached. Of course, at node 2, player-2 maximizes by playing option c instead of d, and player-1 knows this fact. Thus, the normal form equilibrium, $\{B, D\}$, depends, in the extensive form, upon ignoring that option D will not be chosen by player-2 if player-1 chooses B. To put the point another way, the normal form fails to distinguish between the extensive form of figure 1.1 and a different game where both play simultaneously, i.e., where player-2's information set does not reflect whether or not player-1 chooses a or b.

In order to avoid "imperfect equilibria", Harsanyi and Selten alter the basic moves in a game so that an agent selects one from a set of distributions (on pure options). A player chooses a mixed strategy rather than a pure option. Figure 1.3 gives the normal form for the "trembling hands" perturbated game, where players may choose one of two mixed strategies in a perturbed extensive form game (not pictured).

In the perturbed game, the normal form options given in Figure 1.3 arise by using a two point distribution, with probabilities $(1 - \varepsilon)$ and ε assigned to each pure option in the corresponding perturbed extensive form.

	C*	D*
A*	$2-5\varepsilon+4\varepsilon^2$, $2+3\varepsilon+4\varepsilon^2$	$-2+7\varepsilon-4\varepsilon^2$, $-2+9\varepsilon-4\varepsilon^2$
B*	$1+\varepsilon-4\varepsilon^2$, $3-\varepsilon-4\varepsilon^2$	$1-3\varepsilon+4\varepsilon^2$, $3-5\varepsilon+4\varepsilon^2$

Figure 1.3

In the perturbed versions of the game, this difference between the two solutions pairs (which are in equilibrium in game form 1.2) is made evident. In the normal form 1.3, only the pair $\{A^*, C^*\}$ is in equilibrium. The $\{B^*, D^*\}$ pair is not in equilibrium since, when player-1 chooses B^*, player-2 improves his/her (expected) payoff by shifting from D^* to C^*, i.e., D^* is not player-2's best response to B^*.

The Harsanyi-Selten point is that "imperfect equilibria" are deficient because, in extensive game forms, they require a player to choose an outcome which fails to maximize his/her utility. Nonetheless, the suspect choice is justified by Nash's criterion of equilibrium in the corresponding normal form. In the extensive form of their game, player-2 does not maximize utility by choosing option d (if node 2 arises) — choice d is an *idle* threat. That move is inconsistent with the assumption that the players are utility maximizers and model each other that way. "Trembling hands", using sets of "ε-mixtures", is Harsanyi and Selten's ingenious way of reconstituting the reduction of extensive to normal forms in game theory. In section 3, I shall use sets of "ε-mixtures" of probabilities to defeat the extensive-to-normal form reduction!

2. The "prevision of a prevision" (M. Goldstein, 1983)

Goldstein's result concerns a coherent agent's currents beliefs about his/her future beliefs. It rests on the following, simple (yet suspect), lemma concerning sequential decisions.

LEMMA (Goldstein). *Let (terminal) decision D_1 lead to the "penalty" A. Suppose, also, there is a (sequential) option O to defer the choice between "penalties" A and B. Then, on pain of a sure loss, you may not now prefer D_1 over O.*

His proof (as summarized below), pivots on the extensive-to-normal form reduction.

" PROOF" (reductio): Suppose, now, you prefer D_1 to O by an amount greater than C. Then you are willing to pay amount C to receive D_1 over O. But then you suffer the sure loss C as you might just as well have only penalty A: first choice O (now), then A (later), rather than the larger penalty $A + C$.

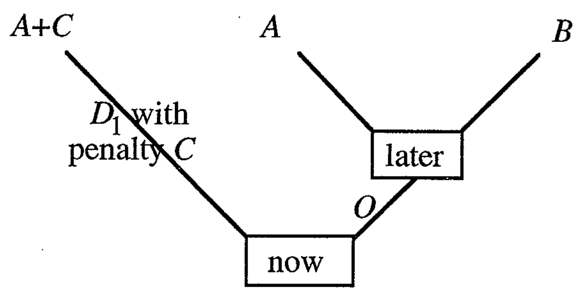

Figure 2.1 — the extensive version of Goldstein's argument

Goldstein's proof uses the reduction of the sequential option O to its normal form: a choice between penalties A and B. Goldstein compares $A + C$ and the better outcome A, without concern about what you know (now) you will choose "later". The "counterexamples" involve problems where you know (now) that were you to opt for O, then later you would choose B, which you now find inferior to $A + C$.

Next, let $P_t(E)$ denote your (currently unknown) probability for event E at the future time t. Let $P_{\text{now}}(E)$ be your current probability for E. And let $P_{\text{now}}(P_t(E))$ be your current expectation for the random variable $P_t(E)$. The result about your prevision of your (future) previsions is as follows.

THEOREM (Goldstein). $\quad P_{\text{now}}(P_t(E)) = P_{\text{now}}(E)$.[3]

PROOF: By the previous lemma on the value of deferred options.

Let us explore circumstances when this "theorem" fails, when the "lemma" fails, because extensive forms do not reduce.

[3]A related condition, called the "Principle of Reflection", is reported in van Fraassen's [1984] "Belief and the Will". See, e.g., Levi [1987] and Talbott [1991] for discussions.

2.1 Bayes' rule for updating — temporal conditionalization.

The dynamic version of Bayes' rule is this.

Suppose B summarizes the evidence acquired between (later) time t and now, then

$$P_t(\bullet) = P_{\text{now}}(\bullet \mid B).$$

If this temporal rule were mandatory then, as an extreme case: when you don't learn new evidence, you can't just change from one (coherent) distribution to another. Or, in a slightly different form using Goldstein's result, you aren't coherent if you now know that you are about to change your previsions from P to $P' \neq P$, though you will acquire no new evidence. However, in either of these cases the "lemma" does not apply as you are not prepared to equate the extensive and normal forms. The "lemma" fails to take into account that you know (now) certain choices will be rejected, yet you are asked to contrast such rejected (future) options with live current options.

The sequential argument offered on behalf of temporal conditionalization requires a questionable reduction to a normal form decision. The reduction is invalid because, by the agent's current lights, non-options are used in the normal form decision in order to show that violating the proposed dynamic rule leads to incoherent choices in the guise of a sure loss.

2.2 Non-conglomerability and the extensive to normal form reduction.

Next, I investigate where Goldstein's theorem precludes the use of Bayes' rule for updating. The case involves the use of probabilities which are finitely, but not countably additive. Let P be a f.a. probability defined on a σ-field of subsets of X. Let $E_p[\,]$ be the P-expectations for bounded, measurable functions f. And let $\pi = \{h_1, h_2, \ldots\}$ be a countable partition of X.

Definition (Dubins/de Finetti): Say that P is conglomerable in π provided that for each bounded, measurable function f, $\inf_\pi E_p[f \mid h] \leq E_p[f] \leq \sup_\pi E_p[f \mid h]$.

However, each P which is not σ-additive suffers a failure of conglomerability for some event E. (See Schervish et al, [1984].) That is, there exists an event E, a partition π and $\varepsilon > 0$ such that

$$P(E \mid h_i) < P(E) - \varepsilon \qquad (i = 1, \ldots)$$

Heuristic Example (Dubins, 1975). Figure 2.2 displays the finitely additive probabilities for "atoms". To help interpret P, assume that given

E, an integer 'i' is chosen "at random", $P(h = i \mid E) = 0$. Given E^c, a fair coin is flipped until a head appears and the number of flips determines i, $P(h = i \mid E^c) = 2^{-i}$. Also assume $P(E) = P(E^c) = 1/2$, leading to the values in Figure 2.2.

	h_1	h_2			h_i	
E	0	0			0	
E^c	2^{-2}	2^{-3}			$2^{-(i+1)}$	

Figure 2.2 — Dubins' example
$P(E) = P(E^c) = 1/2$, $P(h_i \mid E) = 0$ and $P(h_i \mid E^c) = 2^{-i}\,(i = 1, \ldots)$.

Thus, $P(h_i) = 2^{-(i+1)}$ and $P(E \mid h_i) = 0$. So, $0 = P(E \mid h_i) \ll P(E) = 1/2\,(i = 1, \ldots)$ and we see that P is not conglomerable in π.

Suppose the agent has P for his/her current personal probability, will learn which element of π obtains at t, and plans to use temporal conditionalization to update at t. Then, $P_{\text{now}}(E) = 1/2$ and $P_t(E) = 0$. Thus, $P_{\text{now}}(P_t(E)) = 0 \neq P_{\text{now}}(E) = .5$, and the "prevision of a prevision" theorem fails. Once again, Goldstein's "lemma" is false as the extensive form does not reduce to the normal form for decisions involving the random variable $P_t(E)$.

3. Dilation of sets of probabilities (work with Larry Wasserman)[4]

In this section, I report on a phenomenon we call "dilation", which leads in a different way to a non-equivalence of extensive and normal form decisions.

Let $\mathcal{P}$ be a (convex) set of probabilities on algebra $\mathcal{A}$. For an event E, denote by $P_*(E)$ the "lower" probability of E: $\inf_{\mathcal{P}} \{P(E)\} = P_*(E)$ and denote by $P^*(E)$ the "upper" probability of E: $\sup_{\mathcal{P}} \{P(E)\} = P^*(E)$. Let $\pi = (B_1, \ldots, B_n)$ be a (finite) partition.

DEFINITION: The set of conditional probabilities $\{P(E \mid B_i)\}$ *dilate* if

$$P_*(E \mid B_i) < P_*(E) < P^*(E \mid B_i) \qquad (i = 1, \ldots, n).$$

[4] An illustration of what we here call "dilation" was reported by Levi and Seidenfeld to I. J. Good in connection with Good's [1966] argument about the value of new evidence. That communication prompted Good's [1974] reply. Additional rebuttal is found in my [1981], where I link "dilation" with randomization in experimental design. A recently published example of dilation, relating to the worth of new evidence, appears on pp. 298–299 of P. Walley's [1991].

That is, dilation occurs provided that, for each event (B_i) in a partition π, the conditional probabilities for an event E, given B_i, properly include the unconditional probabilities for E. Dilation of conditional probabilities is the opposite phenomenon to the more familiar "shrinking" of sets of options with increasing shared evidence.[5]

HEURISTIC EXAMPLE OF DILATION

Suppose A is a highly "uncertain" event. That is $P^*(A) - P_*(A) \approx 1$. Let $\{H, T\}$ indicate the flip of a fair coin whose outcomes are independent of A. That is, $P(A, H) = P(A)/2$ for each $P \in \mathcal{P}$. Define the event E by, $E = \{(A, H), (A^c, T)\}$. It follows, simply, that $P(E) = .5$ for each $P \in \mathcal{P}$.

Then $\quad 0 \approx P_*(E \mid H) < P_*(E) = P^*(E) < P^*(E \mid H) \approx 1$

and $\quad 0 \approx P_*(E \mid T) < P_*(E) = P^*(E) < P^*(E \mid T) \approx 1$.

Thus, regardless how the coin lands, the conditional probability for event E dilates to a large interval, increasing from a "determinate" value .5.

This example mimics Ellsberg's (1961) "paradox", where the mixture of two "uncertain" events has a "determinate" probability. In different terms, event E is "pivotal" over the set $\mathcal{P}$.

Next, I indicate by example, that extensive forms do not reduce to normal forms when dilation is present.

HEURISTIC EXAMPLE (continued): Consider a sequential (that is, extensive form) choice between:

terminal option D_1 — Win \$.75 if E; Lose \$1.25 if E^c.

Or, sequential option O — observe the coin flip and choose between

D_2 — an even money \$1 bet on E

and D_3 — a "fee" of \$.50.

Thus, option $D_1 = D_2$ (an even money \$1 bet on E)+\$.25 "fee". Figure 3.1 illustrates the extensive form problem. [For convenience, hereafter, assume dollars are linear in utility.]

[5] For discussion of different senses in which a set of conditional probabilities may "shrink" with increasing evidence, see Schervish and Seidenfeld [1990].

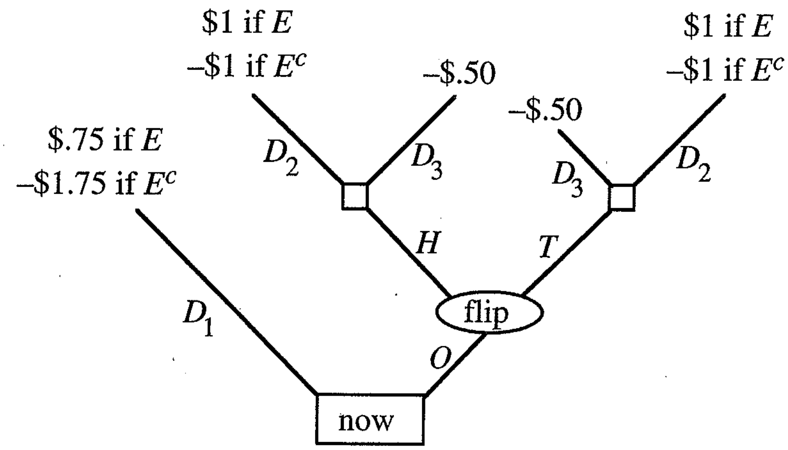

Figure 3.1 — Sequential Decision associated with the heuristic example of dilation.

Observe that in a pairwise choice between D_1 and D_2, option D_2 (simply) dominates option D_1. Therefore, in the normal version of this problem D_1 is not admissible. (D_1 fails to maximize expected utility for each $P \in \mathcal{P}$.) However, in the sequential (extensive form) problem above, after having seen the coin flip, conditional upon either H or T, both choices D_2 and D_3 are (pairwise) admissible according to expected utility considerations. That is, for some $P \in \mathcal{P}$ D_2 has higher expected utility than D_3 and for other probabilities this inequality reverses. But D_3 maximizes "security": D_3 has a better "worst" payoff, (-\$.50 versus -\$1.00)
or D_3 has a higher, minimum expected value (Γ-minimax).[6]

Thus, anticipating choices that will be made if the sequential option is taken, D_3 is the result of choosing O "now". Then, to complete the analysis, compare the two "live" options available "now": a choice between D_1 and D_3. But, between these two options D_3 fails to maximize expected utility for each $P \in \mathcal{P}$. Hence, D_1, which is inadmissible in the normal form, is the (sole) admissible option in the extensive form decision.[7]

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[6] I allude, here, to decision theories like Levi's [1980] where an option is admissible from a choice set provided (i) it maximizes expected utility for some probability/utility in the agent's set of probabilities and utilities, and (ii) it maximizes a "security" index among those options passing the first condition. In the example here, "security" may be indicated by a maximum value or by a Γ-minimax value.

As an aside, I note that Γ-minimax requires an extraneous stipulation when sets of utilities are used. Specifically, depending upon how a set of utilities is standardized, i.e., depending upon which consequences are assigned 0 and 1, different options may be declared Γ-minimax.

[7] Of course, even when extensive forms do not reduce to normal forms, "backward

Using this example as a template, non-equivalence of extensive and normal forms can be manufactured whenever dilation occurs. In the following two sub-sections, I report on necessary and sufficient conditions for dilation.

3.1 Independence and dilation.

Independence is sufficient for dilation.

Let $\mathbb{Q}$ be a convex set of probabilities on an algebra $\mathcal{A}$ and suppose we have access to a "fair" coin which may be flipped repeatedly: coin-flip events are confined to algebra $\mathcal{C}$. Assume the coin flips are independent and, with respect to $\mathbb{Q}$, also independent of events in $\mathcal{A}$. Let $\mathcal{P}$ be the resulting convex set of probabilities on $\mathcal{A} \times \mathcal{C}$.[8]

THEOREM. *If $\mathbb{Q}$ is not a singleton, there is a 2×2 table of the form $(E, E^c) \times (H, T)$ where both:*

$$P_*(E \mid H) < P_*(E) = .5 = P^*(E) < P^*(E \mid H)$$
$$P_*(E \mid T) < P_*(E) = .5 = P^*(E) < P^*(E \mid T)$$

That is, dilation occurs.

PROOF (sketch): Let $A \in \mathcal{A}$ be "uncertain" with respect to $\mathbb{Q}$. Use the "fair" coin to form event F where $P_*(F) < .5 < P^*(F)$. Then mimic the "Ellsberg" heuristic example, above. □

Independece is necessary for dilation.

Let P be a convex set of probabilities on an algebra $\mathcal{A}$. the next result is formulated for subalgebras of 4 atoms: (p_1, p_2, p_3, p_4)

	B_1	B_2
A_1	p_1	p_2
A_2	p_3	p_4

Figure 3.2 — the case of 2×2 tables.

Define the quantity $S_P(A_1, B_1) = p_1 / (p_1 + p_2)(p_1 + p_3) = P(A_1, B_1) / P(A_1) P(B_1)$. Thus, $S_P(A_1, B_1) = 1$ iff A and B are independent under P.

induction" remains a valid sequential decision rule! See my [1988] discussion of this issue in connection with decision rules that abandon the "independence" postulate.

[8]The condition involving $\mathcal{C}$ is similar to, e.g., DeGroot's [1970] assumption of an extraneous continuous random variable, and is similar to the "fineness" assumptions in the theories of Savage [1954], Ramsey [1926], Jeffrey [1965], etc.

LEMMA. *If $\mathcal{P}$ displays dilation in this sub-algebra, then*

$$\inf_{\mathcal{P}} \{S_P(A_1, B_1)\} < 1 < \sup_{\mathcal{P}} \{S_P(A_1, B_1)\}.$$

PROOF: Direct calculation.

THEOREM. *If $\mathcal{P}$ displays dilation in this subalgebra, then there exists $P^\# \in \mathcal{P}$ such that*

$$S_{P\#}(A_1, B_1) = 1.$$

PROOF: By the lemma, there exists P_1 and P_2 such that $S_{P_1}(A_1, B_1) < 1 < S_{P_2}(A_1, B_1)$.

Write $P_x = xP_1 + (1-x)P_2$ and note that $S_{P_x}(A_1, B_1)$ is a continuous (quadratic) function of x, with coefficients involving $P_1(A_1), P_1(B_1)$, $P_2(A_1)$ and $P_2(B_1)$. By the mean value theorem, for some $0 < x < 1$, $S_{P_x}(A_1, B_1) = 1$.

3.2 Dilation and ε-contamined models.

In this subsection, I report additional details about dilation for a particular (convex) set of distributions, known as the ε-contaminated model.

Given a probability P and $1 > \varepsilon > 0$, define the convex set

$$\mathcal{P}_\varepsilon = \{(1-\varepsilon)P + \varepsilon Q : Q \text{ an arbitrary probability}\}.$$

This "model" is popular in studies of Bayesian Robustness. (See, e.g., Huber, 1981.) As before, the following result applies to sub-algebras of 4 atoms.

THEOREM $\mathcal{P}_\varepsilon$ experiences dilation iff

case 1: if $S_P(A_1, B_1) > 1$,
$$\varepsilon > [S_P(A_1, B_1) - 1] \bullet \max\{P(A_1)/P(A_2); P(B_1)/P(B_2)\}$$

or

case 2: if $S_P(A_1, B_1) < 1$,
$$\varepsilon > [1 - S_P(A_1, B_1)] \bullet \max\{1; P(A_1)P(B_1)/P(A_2)P(B_2)\}$$

or

case 3: if $S_P(A_1, B_1) = 1$,
P is internal to $\mathcal{P}$.

(I omit the proof of this theorem.)

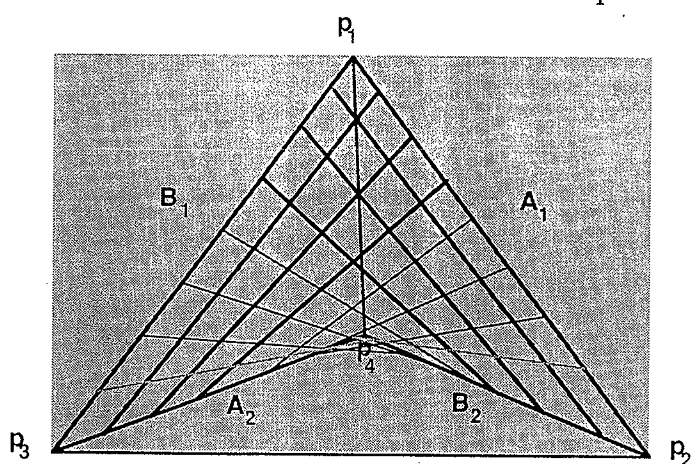
Thus, dilation occurs in the ε-contaminated model if and only if P is close enough (in the tetrahedron of distributions) to the saddle-shaped surface of distributions which make A and B independent.[9] The Figure 3.3 illustrates the "saddle" of probabilities satisfying $P(A, B) = P(A)P(B)$.

4. Summary

I have discussed three decision contexts where extensive forms do not reduce to normal forms:

1. Game theory — The Harsanyi-Selten argument about "imperfect" equilibria.
2. Denying "The Prevision of a Prevision" (M. Goldstein's argument)
   2a — involving failures of temporal conditionalization
   2b — involving non-conglomerability of finitely additive probability
3. Dilation of Sets of Probabilities.

The common reason why there is no reduction for these cases is that particular "future" options, which the agent knows (in advance) will *not* be chosen in the sequential decision are, nonetheless, used as though they were feasible options in the normal form. That is, an option which is inadmissible in the normal form may be admissible in an extensive form (generating that normal form). Rival choices which defeat that choice in the normal form turn out to be not feasible in the sequential form.



Tetrahedron showing "saddle" surface of distributions which make events A and B independent

Figure 3.3

[9]As a contrast, the Density Ratio model is immune to dilation. Let P be a fixed probability defined on the atomic algebra $\mathcal{A}$, with "atomic" probabilities denoted p_i. The Density Ratio model on $\mathcal{A}$, for $k \geq 1, \mathcal{R}(P, k) = \{Q : q_i/q_j \leq k \bullet p_i/p_j\}$.

REFERENCES

DEFINETTI, B. (1972) *Probability, Induction and Statistics.* New York: Wiley.

DEGROOT, M. (1970) *Optimal Statistical Decisions.* New York: Wiley.

DUBINS, L. (1975) "Finitely Additive Conditional Probabilities, Conglomerability and Disintegrations", *Annals of Prob.* 3. 89–99.

ELLSBERG, D. (1961) "Risk, Ambiguity, and the Savage axioms", *Q. J. Economics* 75, 643–669.

GOLDSTEIN, M. (1983) "The Prevision of a Prevision", *J. A. S. A.* 78, 817–819.

GOOD, I. J. (1966) "On the Principle of Total Evidence", *B. J. P. S.* 17, 319–321.

GOOD, I. J. (1974) "A Little Learning Can Be Dangerous", *B. J. P. S.* 25, 340–342.

HARSANYI, J. (1992) "Game Solutions and the Normal Form", in *Knowledge, Belief, and Strategic Interaction*, C. Biccieri and M. Della Chiara (eds.), Cambridge Univ. Press, 355–376.

HARSANYI, J. and SELTEN, R. (1988) *A General Theory of Equilibrium Selection in Games.* Cambridge: MIT Press.

HUBER, P. (1981) *Robust Statistics.* New York: Wiley.

JEFFREY, R. C. (1965) *The Logic of Decision.* New York: McGraw-Hill.

LAVALLE, I. and FISHBURN, P. (1987) "Equivalent Decision Trees and their Associated Strategy Sets", *Theory and Decision* 23, 37–63.

LEVI, I. (1980) *The Enterprise of Knowledge.* Cambridge: MIT Press.

LEVI, I. (1987) "The Demons of Decision", *The Monist* 70, 193–211.

LUCE, R. D. and RAIFFA, H. (1957) *Games and Decisions.* New York: Wiley.

RAMSEY, F. P. (1926) "Truth and Probability" in his Philosophical Papers [1990] Cambridge: Cambridge University Press (H. Mellor, ed.), pp. 52–109.

SAVAGE, L. J. (1954) *The Foundations of Statistics.* New York: Wiley.

SCHERVISH, M. and SEIDENFELD, T. (1990) "An approach to consensus and certainty with increasing evidence", *J. Stat. Planning and Inference* 25, 401–414.

SCHERVISH, M., SEIDENFELD, T., and KADANE, J. (1984) "The Extent of Non-conglomerability of Finitely Additive Probabilities", *Z. Wahr.* 66, 205–226.

SEIDENFELD, T. (1981) "Levi on the Dogma of Randomization", in R. Bogdan (ed.) *Henry E. Kyburg, Jr. & Isaac Levi.* Dordrecht: D. Reidel, pp. 263–291.

SEIDENFELD, T. (1988) "Decision Theory Without 'Independence' or Without 'Ordering': What is the Difference", *Economics and Philosophy* 4, 267–290.

TALBOTT, W.J. (1991) "Two Principles of Bayesian Epistemology", *Phil. Studies* 62, 135–150.

VAN FRAASSEN, B. C. (1984) "Belief and the Will", *J. Phil.* 81, 235–256.

WALLEY, P. (1991) *Statistical Reasoning with Imprecise Probabilities.* St.Edmunds, Suffolk: St. Edmundsbury Press.