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Abstract

In this paper we study the life cycle locational choices of heterogeneous households and characterize the dynamics of metropolitan areas. We develop an overlapping generations model for households in of a system of multiple jurisdictions. At each point of time households choose among the finite number of jurisdictions, pick consumption and housing plans, and vote over education expenditures and tax policies. A household's composition changes over the life cycle as children grow up and leave the household. These changes in turn impact the household's need for housing and for local public services, particularly education. Households face frictions in relocating. A household may relocate as needs change, bearing the associated financial and psychic costs of moving. Alternatively, the household may choose to remain in a community that suited its initial needs, finding the costs of relocation to be greater than the potential benefits of moving to a community better suited to its changed housing and public education. Our quantitative analysis shows that mobility costs are likely to have a large impact on household sorting patterns. Mobility costs also impact the political decisions that determine local tax and expenditure policies. Finally, we compare centralized and decentralized mechanisms for providing education and their impact on human capital accumulation and welfare.

JEL classification: D72, D91, H31, R12

1 Introduction

A fundamental premise in modeling local jurisdictions is that households make their location decisions taking account of the public good bundles available in alternative jurisdictions. This hypothesis, first proposed by Tiebout (1956), has been the subject of extensive formal modeling and empirical analysis. Early empirical work, pioneered by Oates (1969), investigated the extent to which differentials in housing prices across jurisdictions reflect differentials in quality of local public goods and property tax rates.[1] Much recent empirical work has focused directly on the extent to which households stratify based on differences in the quality of local public goods.[2] Both research on capitalization and research on stratification of households across jurisdictions supports the hypothesis that households do in fact take account of differences in local public good bundles in making location choices.

In research to date, both theoretical models and empirical research have largely focused on static equilibrium models or cross-sectional empirical studies.[3] However the same logic that suggests households sort based on tastes for local public goods implies that households have incentives to change location over the life cycle. For example, a household's consumption of local public education begins when its first child enters kindergarten and ends when its last child leaves high school. Thus, one would expect that households with school-age children would place weight on the quality of local

[1]Black (1999), Epple and Romano (2003), Figlio and Lucas (2004), Calabrese, Epple, and Romano (2006), and Bayer, Ferreira, and McMillan (2007) have extended this analysis to investigate whether differences within jurisdictions in the quality of local public goods are capitalized into house prices.

[2]See, for example, Epple and Sieg (1999), Epple, Romer, and Sieg (2001), Bayer, McMillan, and Reuben (2004), Bajari and Kahn (2004), Sieg, Smith, Banzhaf, and Walsh (2004), Ferreyra (2007), and Ferreira (2005).

[3]Examples of theoretical models using static frameworks are Ellickson (1973), Westhoff (1977), Epple, Filimon, and Romer (1984), Goodspeed (1989), Epple and Romer (1991), Nechyba (1997), Fernandez and Rogerson (1996), Henderson and Thisse (2001), and Rothstein (2006). Exceptions are Benabou (1996a, 1996b), Durlauf (1996), Fernandez and Rogerson (1998), Glomm and Lagunoff (1999), Benabou (2002) and Ortalo-Magne and Rady (2006). Farnham and Sevak (2006) investigate empirically the extent to which "empty-nest" households relocate to communities with lower property taxes and school expenditures.

public schools when considering location choices, but that those same households would place little weight on quality of local public schools when their children have left school. Indeed, one would expect households would tend to move to locations with good quality public schools when they have school-age children while moving to locations with lower housing prices and lower quality public schools when their children have left school. Departure of children from high school presages their departure from the household, and the associated decrease in need for housing reinforces the incentives for relocation that accompany the decrease in need for local public services. These incentives for relocation over the life cycle in turn create incentives for young households to make their initial location choices and housing purchases taking account of the likelihood that they will relocate in the future. A dynamic equilibrium model embodying household life cycle choices offers the potential to improve understanding both of community characteristics and of housing markets.

The main contribution of this paper is a new life cycle model of community formation with limited household mobility. Our model captures three important dimensions by which households differ: income, moving cost, and age or family structure. Income is clearly a key factor influencing a household's ability and willingness to pay the moving cost and housing price premium to relocate to a community with higher quality public services. Moving costs, both financial and psychic, are important factors in the decision process. In addition to transactions costs, relocation often entails costs associated with moving away from friends, neighbors, and familiar surroundings and the associated costs of becoming established in a new neighborhood. While financial costs will typically be roughly proportional to house value, psychic costs are likely to exhibit greater variation across households. Finally, our model also captures the fact that relocation incentives vary over the life cycle. These incentives are largely driven by the presence or absence of children at home at various points during the life cycle.

We derive the stationary equilibrium of our model and characterize its properties. In our model, adults live for two periods and thus can live in at most two different locations. One important property of equilibrium is that many community pairs that could be chosen over the life cycle are strictly dominated by other pairs in equilibrium.

This result is important since it reduces the dimensionality of the choice set. Restricting our attention to community pairs in the relevant choice set, we can provide conditions that guarantee that households stratify by wealth (conditional on moving costs) in equilibrium. Old households have few incentives to move to a community that has higher levels of public good provision than the community chosen when young. We show that this conjecture is correct if the relative weight placed on the local public good is higher when young than when old.

Equilibria do not have analytical solutions. Nevertheless, we can provide a general characterization of the partition of household types into residential plans. Moreover, we develop an algorithm that can be used to compute equilibria numerically. To illustrate some of the quantitative implications of our model, we calibrate our model and compute equilibria for economies with multiple communities. We find that our model can generate equilibria in which a reasonable fraction of households relocates to a different community when old in equilibrium. This property of equilibrium is consistent with evidence on turnover in local housing markets. This feature of our model has important implications for the political decisions made in the communities. We find that older households are typically in the majority in communities with low levels of public good provision, while young households dominate in communities with high levels of expenditures. Our model thus captures the intergenerational conflict in collective choice processes within municipalities.

We use our model to consider two policy experiments. First, we compare our baseline equilibrium with a model in which there is partial equalization in expenditures which is achieved by a foundation grant system. We find that that moderate levels of foundation grants – similar to average levels observed in reality – have small effects on the equilibrium sorting of households and public policies. Second we compare our decentralized equilibrium with a centralized model that fully equalizes expenditures and tax rates. We find that the decentralized equilibrium yields higher average expenditures and as a consequence higher steady state mean incomes than the centralized equilibrium. However, welfare is higher in the centralized equilibrium with a gain equivalent to approximately 1.1% of mean income relative to the decentralized equilibrium. Low income households gain from centralization while high income households are made worse off.

We also use our model to investigate the effects of changes in mobility costs. We find that changes in mobility costs have relatively dramatic effects on community composition and public good provision.

The rest of the paper is organized as follows. Section 2 develops our theoretical model. Section 3 defines equilibrium and discusses its properties. We discuss how to compute equilibria in Section 4 and introduce a parametrized version of our model. Section 5 examines the quantitative properties of our model. Section 6 offers conclusions.

2 An Overlapping Generations Model

We develop an overlapping generations model to study the life cycle location choices of heterogeneous households, the associated demographic composition and public good provision of communities, and the dynamics of metropolitan areas.

2.1 Communities

Consider a closed economy in which activity occurs at discrete points of time $t = 1, 2, ...$ The economy consists of J communities. At each point of time, each community provides a congested local public good g, which is financed by property taxes τ.[4] Each community has a fixed supply of land, and thus a housing supply that is not perfectly elastic. Let p^h denote the net of tax price of a unit of housing services, h, in a community; and $p = (1 + \tau)p^h$ the gross of tax price.

2.2 Households

There is a continuum of individuals each of whom lives for three periods, one period as a child and two periods as an adult. Thus at each point of time individuals in the

[4]We often suppress the time subscripts for notational simplicity. Subscripts have the obvious ranges unless we state otherwise.

economy consist of three over-lapping generations, denoted child (c), young (y), and old (o). Each household has one child who attends school and lives at home.[5] Children become young adults at the same time that young adults transition to old age. Hence, there are no children in old households. Each young adult household is characterized by a lifetime wealth level denoted by w and achievement of the household's child, denoted a. The achievement of the child, $a(g)$, is determined by government spending in the community occupied by the young household in which the child resides.

Assumption 1 *We assume that achievement $a(g)$ is increasing in g.*

Households have additive, time-separable utility. The period utility when young is defined over housing h, child achievement, and a numeraire good b. The period utility when old is defined over housing, the local public good, and the numeraire. Period utility for a young household is denoted $U^y(b, h, a(g))$ and $U^o(b, h, g)$ for an old household. For notational convenience we write $U^y(b, h, g) \equiv U^y(b, h, a(g))$.

Assumption 2 *The current period utility function of a young household $U^y(b, h, g)$ and the utility function of an old household $U^o(b, h, g)$ are increasing, twice differentiable, and concave in (b, h, g).*

Thus, the presence of children in young households induces different preferences for public goods and housing for young than for old households. Since there is no uncertainty about future equilibrium outcomes in this model, we make the following assumption:

Assumption 3 *Households behave as price takers and have perfect foresight about current and future equilibrium prices, tax rates, and levels of local public goods.*

2.3 Mobility Costs

When they finish school and leave their parents' homes, young households are not committed to any community. We assume that they can pick any community of residence in

[5]Since we assume that each generation has the same mass, we implicitly assume single parents. We could also assume two parent households that have two identical children. Variation in the ability and age of children in a household would add considerable complications to the model.

the first period without facing mobility costs. Old households have already established a residence when they were young adults. If they decide to relocate to another community as they enter old age, then they face mobility costs. Mobility costs are denoted by mc. We assume that households have heterogeneous mobility costs.

Assumption 4 *The joint distribution of lifetime income and mobility costs at time t, denoted by $F_t(w, mc)$, is continuous with support $S = R^2_+$ and joint density $f_t(w, mc)$ with $f_t(\cdot)$ everywhere positive on its support.*

The assumption that the $f_t(\cdot)$ are everywhere positive on their supports simplifies some of the arguments, but is not crucial for the main results.

2.4 The Decision Problem of Households

Households are forward-looking and maximize lifetime utility, which is time separable with constant discount factor β. Households recognize that locational and housing choices in the first period will have an impact on their well-being in the second period. Households must choose a community of residence in each period. Let $d^y_{jt} \in \{0, 1\}$ denote an indicator that is equal to one if a young household lives in community j at time t and zero otherwise. Similarly define $d^o_{jt} \in \{0, 1\}$ for old households.

Households also determine consumption choices for housing and the composite private good numeraire. A young household at date t with characteristics (w_t, mc_t) maximizes lifetime utility:

$$\max_{d^y_t, h^y_t, b^y_t, d^o_{t+1}, h^o_{t+1}, b^o_{t+1}} \sum_{k=1}^{J} d^y_{kt} U^y(b^y_{kt}, h^y_{kt}, g_{kt}) \ + \ \beta \sum_{l=1}^{J} d^o_{lt+1} U^o(b^o_{lt+1}, h^o_{lt+1}, g_{lt+1}) \tag{1}$$

subject to the lifetime budget constraint

$$\sum_{k=1}^{J} d^y_{kt} \, (p_{kt} \, h^y_{kt} + b^y_{kt}) + \sum_{l=1}^{J} d^o_{lt+1}(p_{lt+1} \, h^o_{lt+1} + b^o_{kt+1}) = w_t - \sum_{k=1}^{J} \sum_{l \neq k} 1\{d^y_{kt} = d^o_{lt+1} = 1\} \, mc_t \tag{2}$$

and residential constraints:

$$\begin{aligned} \sum_{k=1}^{J} d^y_{kt} &= 1 \\ \sum_{l=1}^{J} d^o_{lt+1} &= 1 \end{aligned} \tag{3}$$

where $1\{\cdot\}$ is an indicator function. The last two constraints in (3) impose the requirement that the household lives in one and only one community at each point of time. Also, w_t is the present value of lifetime income, thus assuming perfect capital markets.[6] Finally, we have abstracted from discounting of future prices just for simplicity of exposition.

It is often convenient to express this decision problem using a conditional indirect utility (or value) function. Given a household with wealth, w, moving cost, mc, and community choice k when young and l when old, we can solve for the optimal demand for housing and other goods in both periods. Substituting these demand functions into the lifetime utility function yields the indirect utility function which can be written:

$$V_{kl}^y = V(w - \delta_{kl} mc, g_k, p_k, g_l, p_l) \tag{4}$$

where $\delta_{kl} = 1$ if $k \neq l$ and zero otherwise. Similarly, the indirect utility function of an old household that occupied community k when young and is occupying community l when old is:

$$V^o(w_n^o, g_l, p_l) = \max_{h_{lt}} \; U^o(w_n^o - p_l h_l, h_l, g_l) \tag{5}$$

where $w_n^o = w - \delta_{kl} mc - p_k h_k^y - b_k^y$.

Define the set of young households living in community j at time t as follows:

$$C_{jt}^y \;\; = \;\; \Big\{(w_t, mc_t)\Big|\; d_{jt}^y = 1\Big\} \tag{6}$$

The number of young households living in community j at time t is given by:[7]

$$n_{jt}^y \;\; = \;\; \int\int_{C_{jt}^y} f_t(w_t, mc_t)\; dw_t\; dmc_t \tag{7}$$

Similarly define the set of old households living in community j at time t as follows:

$$C_{jt}^o \;\; = \;\; \Big\{(w_{t-1}, mc_{t-1})\Big|\; d_{jt}^o = 1\Big\} \tag{8}$$

The number of old households living in community j at time t is given by:

$$n_{jt}^o \;\; = \;\; \int\int_{C_{jt}^o} f_{t-1}(w_{t-1}, mc_{t-1})\; dw_{t-1}\; dmc_{t-1} \tag{9}$$

[6] Abstracting from uncertainties and liquidity constraints are obviously strong assumptions that should be relaxed in future research.

[7] Expressions (7) and (9) make use of the fact *almost every* household of a given type makes the same community choice.

2.5 Housing Markets

In this model all households are renters.[8] Housing demand functions $h_j^y(\cdot)$ and $h_j^o(\cdot)$ can be derived by solving the decision problems characterized above. Below we introduce subscripts t to indicate the dependence of housing demands on prices young and old households confront during their life.[9] Aggregate housing demand in community j at time t is then defined as the sum of the demand of young and old households:

$$H_{jt}^d = H_{jt}^y \ + \ H_{jt}^o \tag{10}$$

where

$$\begin{aligned} H_{jt}^y &= \int\int_{C_{jt}^y} h_{jt}^y(w_t, mc_t) \ f_t(w_t, mc_t) \ dw_t \ dmc_t \\ H_{jt}^o &= \int\int_{C_{jt}^o} h_{jt}^o(w_{t-1}, mc_{t-1}) \ f_{t-1}(w_{t-1}^y, mc_{t-1}) \ dw_{t-1} \ dmc_{t-1} \end{aligned}$$

To characterize housing market equilibria, we need to specify housing supply in each community.

Assumption 5 *Housing is owned by absentee landlords. Housing supply is stationary and exogenously given by $H_j^s(p_{jt}^h)$, a non-decreasing continuous function.*

This ownership assumption is primarily imposed for simplicity. The alternative would be to assign property rights over land. Households would then obtain revenues from rental income. The income effects from this would be very minor. This would, however, significantly complicate the public choice problem for households who happen to live where they own land.[10] We avoid the additional complexity by assuming absentee owners of land.

The housing market in community j is in equilibrium at time t if:

$$H_{jt}^d = H_j^s(p_{jt}^h) \tag{11}$$

[8]We discuss the implications of housing ownership in the conclusions.

[9]Housing demand when young and old solve (1) and thus depend on the vector $(p_{kt}^y, g_{kt}^y, p_{lt+1}^o, g_{lt+1}^o)$. Since these variables are predictable, we use the subscript t to indicate this dependence, thus greatly simplifying notation.

[10]Since households are atomistic and thus no one household affects voting equilibrium, residential choices would not be affected by land ownership.

2.6 Community Budget Constraints

We assume that each community provides a congested local public good g_{jt} that is financed by property taxes τ_{jt}. Community budgets must be balanced at each point of time. We assume that the public good primarily reflects expenditures per student on education. Hence we can express the community specific budget constrained as:

$$\tau_{jt}\, p_{jt}^{h}\, H_{jt} = g_{jt} n_{jt}^{y} \tag{12}$$

The right hand side of equation (12) equals public expenditure on young households, consistent with young households having one child in public education.

2.7 Voting

Next consider public choice of the tax rate and public good provision in a community. Households have the opportunity to vote twice in their lives, once when they are young and again when they are old. Households vote to maximize their lifetime utility from that point forward. To specify fully a voting model, we need a) to describe the set of alternatives that are considered to be feasible outcomes by the voters; b) define preference orderings over feasible outcomes; and c) define a majority rule equilibrium.

Defining the set of feasible outcomes requires specifying the timing of decisions. We assume the following timing structure.

Assumption 6 *Each young household chooses their initial community of residence and rents a home there. The young household also commits to their old-aged community of residence. Housing markets then clear. Young households then vote taking as given the net housing prices, which have already been established, but also g and p in their future community of residence. Numeraire and public good consumption take place. The old household then occupies the community planned when young and consumes housing. The old household votes, and, last, old-aged numeraire and public good consumption occur.*

The timing and voter beliefs incorporated in Assumption 6 make the problem tractable. The key simplification is that young voters take their future community choices and the variables that characterize their old-aged community as given when voting. Of course, these variables will be those that arise in equilibrium and the future choice of community will be optimal given the equilibrium values. For example, a household that has committed to move to another community when it becomes old will in fact find it optimal to do so. But voters will not account for changes that would become optimal out of equilibrium, tremendously simplifying the voting problem.

A majority voting equilibrium is a public good provision level g_{jt} that at least half the population (weakly) prefers in pairwise comparisons to all other feasible levels of provision. We have the following result:

Proposition 1 *A voting equilibrium exists in all communities.*

Proof of Proposition 1:

Consider a community j which is characterized by a pair of housing price and public good provision (p_{jt}, g_{jt}). Combining the equation relating net and gross housing prices, $p_{jt} = p^h_{jt}(1 + \tau_{jt})$, and the community budget constraint, we obtain:

$$p_{jt} = p^h_{jt} \ + \ \frac{g_{jt} n^y_{jt}}{H_{jt}} \tag{13}$$

Given our timing assumptions, all variables in this expression except (p_{jt}, g_{jt}) have been determined prior to voting. Thus the set of feasible alternatives yields a linear relationship between the choice of g_{jt} and the resulting gross-of-tax housing price p_{jt}.

In each community j, there are two types of voters, young and old. Given the correct beliefs of each voter about feasible alternatives in equation (13), we can characterize each voter's decision problem and thus characterize the voter's behavior.

First consider an old household that has chosen to live in community j after living in community i when young. The household's old age income is given by $w^o_{nt} = w_{t-1} - p_{it-1}h^y_{it-1} - b^y_{it-1} - \delta_{ij} mc_{t-1}$. The household's budget constraint when old is given by: $w^o_{nt} = p_{jt}h^o_{jt} + b^o_{jt}$. Let h^o_{jt} be the amount of housing the household has chosen. The quantity h^o_{jt} is then fixed at the time that voting occurs. Substituting the community budget constraint that prevails at the time of voting into the voter's budget constraint, we obtain:

$$w^o_{nt} \;=\; p^h_{jt} h^o_{jt} \;+ \frac{g_{jt} n^y_{jt}}{H_{jt}}\, h^o_{jt} \;+\; b^o_{jt} \tag{14}$$

The voter's utility function is $U^o(g_{jt}, h^o_{jt}, b^o_{jt})$. At the time of voting, all elements of the preceding budget constraint and utility function have been determined except (g_{jt}, b^o_{jt}). Quasi-concavity of the utility function and convexity of the budget constraint imply that the voter's induced preference over g_{jt} is single-peaked (Denzau and Mackay, 1976).

Next consider a young voter that lives in community j at t and plans to live in community k in $t+1$. The development is analogous to that for old voters, and we thus summarize briefly. At the time of voting in community j, this household will have purchased housing h^y_{jt}. The budget constraint of the young voter is then:

$$w_t \;=\; p^h_{jt} h^y_{jt} \;+\; \frac{g_{jt}\; n^y_{jt}}{H_{jt}}\, h^y_{jt} \;+ b^y_{jt} \;+ p_{kt+1} h^o_{kt+1} + b^o_{kt+1} + \delta_{jk}\; mc_t \tag{15}$$

The young voters utility function is: $U^y(b^y_{jt}, h^y_{jt}, g_{jt}) \;+\; \beta U^o(b^o_{kt+1}, h^o_{kt+1}, g_{kt+1})$. At the time of voting, the community tax base, H_{jt}/n^y_{jt}, and the voter's housing consumption, h^y_{jt}, have been determined. The voter takes current and future prices (p^h_{jt}, p_{kt+1}) and future government provision, g_{kt+1}, as given. Quasi-concavity of the voter's utility function, $U^y + \beta U^o$, and convexity of the budget constraint then imply that induced preferences over g_{jt} are single-peaked (Slutsky, 1975).

The existence of a voting equilibrium follows from single-peakedness of preferences of all voters. Q.E.D.

In general, the pivotal voter will not be the voter with median income. Indeed, there will often be more than one household type that is pivotal. For example, a wealthy old household and a poor young household may both be pivotal, both having the same most-preferred tax rate and expenditure level.

3 Equilibrium Analysis

3.1 Definition of Equilibrium

Definition 1

An equilibrium for this economy is defined as an allocation that consists of a sequence of joint distributions of wealth and moving costs, $\{F_t(w, mc)\}_{t=1}^{\infty}$, a vector of prices, taxes, and public goods denoted by $\{p_{1t}, \tau_{1t}, g_{1t}, ..., p_{Jt}, \tau_{Jt}, g_{Jt}\}_{t=1}^{\infty}$, consumption plans for each household type, and a distribution of households among communities, $\{C_{1t}^y, ..., C_{Jt}^y, C_{1t}^o, ..., C_{Jt}^o\}_{t=1}^{\infty}$, such that:

1. *Households maximize lifetime utility and live in their preferred communities.*
2. *Housing markets clear in every community at each point of time.*
3. *Community budgets are balanced at each point of time.*
4. *There is a majority voting equilibrium in each community at each point of time.*

3.2 Intergenerational Income Transmission and Stationary Equilibrium

The last component of our model deals with the intergenerational income transmission process. We make the following assumption.

Assumption 7 *A child with achievement a_t, starts as a young adult with lifetime wealth w_{t+1}:*

$$\ln w_{t+1} \quad = \quad q(a_t, w_t, \epsilon_{t+1}) \tag{16}$$

where ϵ_{t+1} denotes an idiosyncratic shock. The dependence of children's income on parental income, w_t, captures intergenerational income persistence. Moreover, we assume that $q(\cdot)$ is increasing in all three elements.

We are then in a position to define a stationary equilibrium for our economy:

Definition 2

A stationary equilibrium is an equilibrium that satisfies the following additional conditions:

1. *Constant prices, tax rates and levels of public good provision, i.e. for each community j, $p_{jt} = p_j$, $\tau_{jt} = \tau_j$, and $g_{jt} = g_j$ $\forall t$.*
2. *A stationary distribution of households among communities, i.e. for each community j, we have $C^o_{jt} = C^o_j$ and $C^y_{jt} = C^y_j$ $\forall t$.*
3. *A stationary distribution of household wealth and moving costs, i.e. $F_t(w, mc) = F(w, mc)$ $\forall t$.*

The remainder of the paper focuses on properties of stationary equilibria.

3.3 Equilibrium Residential Choices

Upon entering adulthood, young households choose an initial and an old-age community of residence, correctly anticipating housing prices and local public good provision. Let k and l denote, respectively, the initial and old-age communities, $k, l \in \{1, 2, ..., J\}$. If $k \neq l$, then the household bears moving cost with present value of mc. We adopt the convention of numbering the communities so that $g_{j+1} > g_j$. Since households correctly anticipate g's and p's, gross housing prices will also ascend with the community number.[11]

We now place some restrictions on the form of the household utility function that greatly facilitate the analysis.

[11] We do not examine cases where communities have the same value of g and thus p. If two communities had the same values, then households would be indifferent between them, and we assume they would randomize so that the distributions of (w, mc) would be the same in the two communities. In turn, this would imply there is no difference between the two communities, so they could be treated as one community (with the usual aggregation of housing supplies). Thus there is no loss in generality in requiring that communities be different (as a case with two clone communities is equivalent to another case with one fewer distinct communities).

Assumption 8 *The utility function*

$$U^a(b,h,g) \;=\; u^a_g(g) \;+\; u^a(b,h),\;\; a \,\in\, \{y,o\}, \tag{17}$$

is separable and $u^a(b,y)$ is homogeneous of degree ψ.

Recall that $V^y(g_k, g_l, p_k, p_l, \tilde{w})$ denotes the indirect lifetime utility of a young household choosing residential plan kl, where $\tilde{w} \;=\; w - \delta_{kl} mc$ is lifetime wealth adjusted for any moving cost. Given Assumption 8, we show in Appendix A that the indirect utility can be written as:

$$V^y_{kl}(g_k, g_l, p_k, p_l, \tilde{w}) \;\;=\;\; G(g_k, g_l) \;+\; \tilde{w}^{-\psi} W(p_k, p_l); \tag{18}$$

with G an increasing function of (g_k, g_l) and W a decreasing function of (p_k, p_l).

Define $V^y_{kl} \equiv V^y(g_k, g_l, p_k, p_l, \tilde{w})$. The optimal residential choice plan of young adults maximizes V^y_{kl} over (k,l) taking anticipated p's and g's as given. It is also convenient to adopt a notation in which locational choices can be characterized by a single index subscript i. Let $i \,\in\, I_{kl}$, $I_{kl} \;=\; \{kl|\, k,l \;=\; 1,2,...,J\}$, indicate a residential plan. Let $P_i \equiv -W(p_k, p_l)$ for $i = kl$, which we refer to as the composite price of residential plan i. Note that P_i is increasing in (p_k, p_l). Using this definition, we have that indirect utility from residential plan i is given by:

$$V^y_i \;\;=\;\; G_i \;-\; (w - \delta_i mc)^{-\psi} P_i, \tag{19}$$

where $G_i \;\equiv\; G(g_k, g_l)$ for $i \;=\; kl$. As a final step, let $T \;\equiv\; mc/w$ denote proportional moving costs and again rewrite indirect utility using type-dependent price P^T_i.

$$V^y_i \;\;=\;\; G_i \;-\; w^{-\psi} P^T_i; \tag{20}$$

where

$$P^T_i \;\;\equiv\;\; \begin{cases} P_i \quad\;\; \text{if } i \text{ does not move } (k = l) \\ P_i(1-T)^{-\psi} \text{ if } i \text{ moves } (k \neq l) \end{cases}. \tag{21}$$

Household type (w,T) then chooses a residential plan i to maximize V^y_i in (20) taking (G_i, P^T_i), $i \,\in\, I_{kl}$, as given.

Household choices then satisfy the following three properties:

- (P1) Indifference curves V_i^y = const. in the (G_i, P_i^T) plane are linear with slope w^{ψ}.

- (P2) Indifference curve satisfy single crossing, with "slope increasing in wealth (SIW)."

- (P3) $dP_i^T/dT > 0 \; for \; k \neq l$; choices with moving are effectively more expensive as mc rises.

Properties (P1) – (P3) are intuitive and simply confirmed. (P1) will greatly simplify the analysis that follows. The single crossing property in (P2) means that the indifference curves defined in (P1) cross at most once, and with slopes increasing in wealth. (P2) and (P3) are keys to the character of sorting over communities over the life cycle.

With J communities, there are J^2 residential plans that could feasibly be chosen. Using properties of the choice problem, we can develop restrictions on the set of plans that are actually chosen and then develop an algorithm for mapping household types into their equilibrium residential plans. Let $B^0 \equiv \{G_i, P_i^T \big| i \in I_{kl}\}$ denote the set of bundles, corresponding to residential plans, that are feasible for households with $T = mc/w$. Let H^T denote the convex hull of B^0 and let $\underline{B}^0(T)$denote the set of residential plans (G_i, P_i^T) on the lower boundary of H^T. Formally, $\underline{B}^0(T)$is defined:

$$\underline{B}^0(T) \equiv \{ (G_i, P_i^T) \in B^0 \Big| \text{no distinct} \, (\tilde{G}_i, \tilde{P}_i^T) \in H^T \text{exist with} \, \tilde{G}_i \geq G_i \, \text{and} \, \tilde{P}_i^T \leq P_i^T \} \tag{22}$$

Figure 1 shows two examples from some of our computational analysis of these concepts for a case with J = 4. We have the following main result that characterizes the relevant choice set:

Proposition 2

Households with relative moving cost T choose in equilibrium any and all residential plans in $\underline{B}^0(T)$. As a consequence, we have:

(i) Any and all non-moving residential plans chosen by households with the maximum T comprise the non-moving residential plans chosen by all households.

(ii) Any and all moving plans chosen by households with the minimum T comprise the moving residential plans chosen by all households.

The proof of Proposition 2 and the remaining propositions are collected in an appendix.

We can also show that equilibrium satisfies an "ascending bundles" property and is characterized by a conditional wealth stratification property. Let $J_e \leq J^2$ denote the number of residential plans chosen by any household.[12] Number these plans $1, 2, \ldots, J_e$ such that $G_1 < G_2 < \ldots < G_{J_e}$. We make the following assumption:

Assumption 9 *The maximum T prohibits moving in equilibrium for all wealth types.*

The following proposition formalizes key properties of our model:

Proposition 3
(i) Ascending Bundles: Given residential plans chosen in equilibrium by household with T satisfying $G_i > G_j$, then $P_i^T > P_j^T$.
(ii) Conditional Wealth Stratification: For given T, if $w_2 > w_1$ and household with wealth w_2 chooses plan with G_i and household with wealth w_1 chooses plan with G_j ($j \neq i$), then $i > j$.

Note that the hierarchical ordering of residential plans is consistent across types T though the composite prices differ and the subset of the J_e plans chosen by different T types vary. Figure 2 shows an example from our computational analysis with $J = 4$ of the partition of young households by type (w, T) across residential plans kl. In this example, only five of the residential plan entailing moving arise in equilibrium. There are four no-moving plans and thus $J_e = 9$.

In our computational analysis below, we adopt the following lifetime utility function:

$$U = a + \frac{1}{\rho}[\alpha_h h_k^\rho + \alpha_b b_k^\rho + \beta_g g_l^\rho + \beta_h h_l^\rho + \beta_b b_l^\rho], \ \ \rho < 0; \tag{23}$$

and the following achievement function:

$$a = \frac{\alpha_g}{\rho_a} g_k^{\rho_a}; \tag{24}$$

The specification in (23) and (24) is a variant of a CES specification that satisfies our general assumptions and is tractable while retaining substantial flexibility. Note that if $\rho = \rho_a$, the standard CES case arises.

[12] Later we show that $J_e < J^2$ under reasonable restrictions.

Substituting the achievement function into the utility function, we obtain:

$$U = [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \frac{\beta_g}{\rho} g_l^{\rho}] + \frac{1}{\rho}[\alpha_h h_k^{\rho} + \alpha_b b_k^{\rho} + \beta_h h_l^{\rho} + \beta_b b_l^{\rho}], \ \ \rho < 0; \tag{25}$$

After some manipulation one obtains indirect utility:

$$V_i^y = G_i - (w - \delta_i mc)^{\rho} P_i; \tag{26}$$

where:

$$\begin{aligned} P_i &= -\frac{1}{\rho} z_{kl}^{-\rho} \left[\alpha_h (\frac{\alpha_b}{\alpha_h} p_k)^{-\frac{\rho}{1-\rho}} + \alpha_b + \beta_h (\frac{\alpha_b}{\beta_h} p_l)^{-\frac{\rho}{1-\rho}} + \beta_b (\frac{\alpha_b}{\beta_b})^{-\frac{\rho}{1-\rho}} \right]; \\ z_{kl} &= [p_k (\frac{\alpha_b}{\alpha_h} p_k)^{-\frac{1}{1-\rho}} + 1 + p_l (\frac{\alpha_b}{\beta_h} p_l)^{-\frac{1}{1-\rho}} + (\frac{\alpha_b}{\beta_b})^{-\frac{1}{1-\rho}}]; \\ G_i &= [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \frac{\beta_g}{\rho} g_l^{\rho}]. \end{aligned} \tag{27}$$

and where we have assumed again residential plan $i = kl$.

Keeping in mind that $\rho < 0$, one can see that all the properties of the preceding more general case are satisfied. In particular the composite public good G_i is increasing in the g's and the composite price P_i is increasing in the p's.

Another type of restriction on equilibrium residential plans derives from limits on the relative values of the parameters of the utility function. We can provide conditions such that no household will move when old to a community with higher g. We assume that:

Assumption 10 *The utility function satisfies the following parameter restrictions:*

$$\alpha_g (\alpha_h/\alpha_b)^{1/(\rho-1)} > \beta_g (\beta_h/\beta_b)^{1/(\rho-1)} \text{ and } \rho_a \geq \rho. \tag{28}$$

Proposition 4

No household will choose a community with higher (p, g) pair when old than when young in a stationary equilibrium.

The willingness to pay a higher housing price to live in a community with higher g increases with the coefficient on g in the period utility function and decreases with the coefficient on housing. While the presence of children when young indicates that both

$\alpha_g > \beta_g$ and $\alpha_h > \beta_h$ are to be expected, the condition of Proposition 4 implies that the relatively stronger preference for g when young outweighs the relatively stronger preference for housing so that moving to a higher (p, g) community when old would not result.

4 Quantitative Analysis

Equilibria of this model can only be computed numerically. We next turn to the quantitative part of the analysis. We first present an algorithm that can be used to compute equilibria. To implement the algorithm, we must fully specify the model choosing functional forms and assigning parameter values.

4.1 Computation of Stationary Equilibria

Given a stationary distribution of wealth and moving costs, a stationary equilibrium in this model is determined fully by a vector $\{p_j, g_j, \tau_j\}_{j=1}^{J}$. Computing an equilibrium is, then, equivalent to finding a root to a system of $3 * J$ nonlinear equations. For each community, the three equations of interest are the housing market equilibrium in (11), the balanced budget requirement in (12), and the majority rule equilibrium requirement.

The full algorithm, therefore consists of an outer loop that searches over admissible distributions of wealth and moving costs and an inner loop that computes a stationary equilibrium holding the joint distribution fixed. The algorithm in the inner loop finds a root of $3 \times J$ dimensional system of linear equations. More specifically, the algorithm can be describes as follows:

1. Fix the joint distribution of wealth and moving costs.

2. Compute equilibrium for that distribution:

    (a) Given a vector (p_j, τ_j, g_j) we can compute p_j^h from the identity $p_j = (1+\tau_j)p_j^h$.

    (b) For each young household type (w, mc), we can compute the optimal residential choices for both time periods. Hence we can characterize household sorting across the J communities.

(c) Given the residential decisions, we can characterize total housing demand, as well as total government revenues for each community.

(d) Given p_j^h, we can compute housing supply for each community, and check whether the housing market clears in each community.

(e) Given g_j, we can check whether the budget in each community is balanced.

(f) For each young household and each old household living in community j determine whether the household prefers lower expenditures than the status quo and check whether g_j is a majority rule equilibrium

(g) Iterate over (p_j, τ_j, g_j) until convergence obtains.

3. Update the joint distribution of wealth and moving costs using the law of motion in (16).

4. Check for convergence of wealth and moving cost distributions.

4.2 Parametrization and Calibration

We calibrate the eight parameters of the utility function as follows. We set $\rho_a = \rho$ as explained below, leaving 7 parameters. The strategy is then to set parameters to match predictions of the baseline model to empirical estimates of expenditure shares and demand elasticities. The α's and β's are set so that equilibrium predictions approximately conform to empirical values of: (i) relative expenditure of lifetime wealth while young versus old; (ii) the housing expenditure shares while young and old; (iii) proportional expenditure on local public goods; and (iv) a constant share of expenditure on the numeraire good while young and old. Since the ordinality of utility makes one parameter free, calibrating to the latter five conditions pin down the α's and β's. We employ data from the Consumer Expenditure Survey to obtain the shares in (i) and (ii), treating the data as if it pertains to a single cohort moving through the life cycle. We take households aged 35-44 as typical of young households in our model, and households aged 65-74 as typical of old households who have relocated.[13] We then find that households spend 60% of lifetime wealth when young and 40% when old. We also find that approximately 26 percent of expenditures at each life stage are for housing services.

While we have emphasized education as a key factor influencing household location choices, we include in local government expenditure the other components that potentially influence location choices in estimating (iii); specifically expenditures for public safety (police and corrections), fire, sanitation, health, transportation, debt expense, and government administration. These totaled \$901.8 billion in 2004. Personal income in 2004 was \$9,731 billion, implying local government expenditure equal to 8.7% of income. Of this total, \$474 billion (52.5%) was for education. (Sources: Statistical Abstract, 2008, Table 442. Local Governments Expenditure and Debt by State: 2004). Using this strategy, we obtain: $\alpha_h = 0.096$, $\beta_h = 0.053$, $\alpha_g = 0.075$, $\beta_g = 0.028$, $\alpha_b = 1.00$, and $\beta_b = 0.57$. We then choose $\rho = -.4$ as this yields price elasticities between -.7 and -.8 for all goods.[14]

Our algorithm requires that we specify an initial distribution of household income. We approximate the initial income distribution using a log-normal. In 2005, U.S. mean and median incomes were \$63,344 and \$46,326. These imply that $\mu_{\ln y} = 10.743$ and $\sigma^2_{\ln y} = .626$. We treat each of the two periods of adult life in our model as "representative years." This implies that wealth equals twice annual income, $w = 2y$, and hence $\ln(w) = \ln(2)+\ln(y)$. This and the distribution of $\ln(y)$ imply $\ln(w) \sim N(11.436, .626)$. The mean and standard deviation of w are then \$112,638 and \$78,018. Calibrating wealth as twice annual income is convenient in then permitting us to interpret the equilibrium values of variables as typical annualized values for a young and an old household respectively.

Our achievement function is given by equation (24). We assume that the logarithm of wealth when an adult for a child with achievement a is given by

$$\ln w_{t+1} \;=\; \gamma_p \, a_t \, + \gamma_w \ln w_t \, + \, \epsilon_{t+1} \tag{29}$$

[13]We opt for a somewhat older group of households in calibrating consumption than in calibrating mobility (as described below) so as to obtain expenditure data typical of households who have completed relocation.

[14]Note that these parameters are consistent with Assumption 10 so that no household will "move up" as they age in equilibrium.

where ϵ_{t+1} is normally distributed with mean μ_ϵ and variance σ^2_ϵ. To calibrate the intergenerational income transmission function, we consider the stationary equilibrium in the one community-case. In stationary equilibrium, the distribution of wealth is invariant across generations. Moreover, we require that the transmission function generates an income distribution that is log-normal with mean and variance reported above. This provides two moment conditions for the four parameters to be calibrated. The other two moments are obtained using the correlation of parent and child earnings and the elasticity of spending on educational outcomes. The literature suggests that the correlation of parent and child earnings is approximately .4 (Solon, 1992).

The effect of spending on educational outcomes is more difficult to establish since there is a lack of agreement in the empirical literature about the magnitude of this effect. Fernandez and Rogerson (2003) adopt a utility function that also has education spending entering the utility function is the same way as our function above. Fernandez and Rogerson (1998) review evidence regarding the elasticity of earnings with respect to education spending, concluding that the evidence suggests a range of 0 to .2. We choose an elasticity, .1, in the middle of this range.[15]

We choose parameters of the income transmission function which, in equilibrium, satisfy the four moment conditions discussed above. It is then straight-forward to show that there is closed-form solution that maps the moment conditions into the parameter estimates. We obtain the following estimates $\mu_\epsilon = 7.11$, $\sigma_\epsilon = .573$, $\gamma_p = 49.32$, and $\gamma_w = .4$.

To calibrate moving costs, we take moving costs as a share of income, T, to be log-normally distributed. To gain some insights into the importance of moving costs, we consider the age distributions in metropolitan areas. Figure 3 plots the ratio of old to young households as a function of median community income for the 92 municipalities in the Boston SMSA in 1980. We define cohorts representative of our young and old households. For the former, we choose age 35 to 49 and, for the latter, age 55 to 69.[16]

[15]They also review the evidence, concluding that the exponent on expenditure is in the range from 0 to -3. The value $\rho_a = -.4$ that we have chosen for the other component (ρ) of utility falls within this range.

[16]The metropolitan population in the former cohort is 7% larger than the metropolitan population

We find that the proportion of old to young households, young and old as defined above, is inversely related to community income. In our framework, this arises as households locate in municipalities with high school quality and high housing price premia when they have children of school age, and then relocate to municipalities with lower school quality and lower housing price premia when their children exit school.[17] It is also of interest to note how the pattern observed for 1980 compares to the pattern in a non-stationary environment. With the emergence of the baby boom generation, the cohort aged 35 to 49 in the Boston SMSA was 69% larger than the cohort aged 55 to 69. The ratio of old to young by community income for 1990, illustrated in the lower panel of Figure 3, exhibits a less pronounced pattern of variation by age

The plots in Figure 3 suggest a calibration of the distribution of moving cost so that our model can replicate the observed age ratios. To formalize this procedure, we combined the 92 communities by income into four groups with population proportions approximately equal to those in our four-community equilibrium. Next, we calculated the ratio of old to young households in each of these groups. The results are in column 2 of Table 1. One might argue that households will typically be in the age range 30 to 44 when their first child enters school. Hence, as a second calculation, we treated the young as cohort 30 to 44. The results are in column 3 of Table 1. It is important to note that the 30 to 44 cohort in 1980 is substantially larger than the 35 to 49 cohort, the former being heavily influenced by the baby boom generation.[18] Thus, while we present it for completeness, the 3rd column is of questionable value for calibration of our stationary equilibrium. One might also argue that households do not contemplate relocating until their children have completed college. Hence, as a third calculation we defined ages 60 to 74 as the old cohort, with results in column 4 of Table 1.

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in the latter. Since our model presumes equal cohort sizes, we increase all community populations in the 55 to 69 cohort by 7%.

[17] A natural concern is that the pattern exhibited in Figure 3 might arise because wealthier households exit the metropolitan area when their children complete school. If fact, however, the proportion of old to young households in the Boston metropolitan area in 1980 (93%) was higher than the nationwide ratio (86%), suggesting, if anything, some migration by older households into the metropolitan area.

[18] It is for this reason that we focus on data for 1980. The data for 1990 and 2000 would be even more strongly impacted by the baby boom.

It is useful to note that the cross-sectional comparisons in Table 1 tend to understate the extent of mobility. For example if an old household moves from community 4 to 3, another moves from 3 to 2, and still another from 2 to 1, these three moves would generate a change in the cross-section that would be indistinguishable from movement by a single household from community 4 to community 1. Aggregating into four groups also may lead to understatement of mobility since a relocation from one community to another within a group would not be reflected in the aggregate numbers. The cross-sectional analysis may also overstate mobility. As we noted above, the cohort aged 35 to 49 is larger than the cohort aged 55 to 69.[19] If this smaller cohort size reflects migration out of the metropolitan area by wealthier old households, our procedure might then overstate the extent of intra-metropolitan mobility. This latter concern is somewhat ameliorated by our previous calculating showing substantial intra-metropolitan relocation by older households. Despite these limitations, we view the calculations in Table 1 as providing useful guidance for calibrating our model.

Table 1: Cohort Ratios

1980					
1	2	3	4	5	6
Community	(55-69)/(35-49)	(55-69)/(30-44)	(60-74)/(35-49)	Foundation Grant Model	No Foundation Grant Model
1	1.128	1.025	1.155	1.179	1.178
2	1.184	1.181	1.231	1.104	1.123
3	0.926	0.956	0.910	0.966	0.989
4	0.742	0.804	0.683	0.707	0.777

We chose parameters of our moving cost distribution to generate an equilibrium with cohort ratios roughly in accord with those summarized in columns 2 through 4 of Table 1. With some experimentation, we settled on $\ln(T) \sim N(.00925, .0026)$ with a correlation of $\ln(T)$ and $\ln(w)$ equal to zero. This yields the cohort ratios in column 5 of Table 1.

[19]The low ratio in community 1 is because young households tend to locate in the city (Boston) for reasons not fully captured in our model. If we remove Boston from community 1, the ratio is much higher–which can be seen by inspection of Figure 3.

Finally, we assume that the housing supply has constant elasticity θ and is given by

$$H_{jt}^{s} = [p_{jt}^{h}]^{\theta} \tag{30}$$

Thus we assume that the four communities have the same housing supply function and in this sense are of "equal size." We set the supply elasticity, θ, equal to 3.[20]

4.3 Persistence of Community Characteristics

Finally it is useful to consider the persistence of community characteristics since most of our analysis is based on a stationary equilibrium concept. To gain some insights about the persistence of community characteristics, we again consider again the Boston Metropolitan Area. Our analysis is based on data for the census years 1970, 1980, 1990 and 2000. A distinctive feature of the Boston metropolitan area is that the population was virtually the same in 1970, 1980 and 1990.[21] Thus, the Boston metropolitan area allows us to investigate interjurisdictional mobility in an environment in which the overall community population was unchanging. Of course, real incomes were growing over this period of time and family size was declining. Another striking feature of the data is that individual community populations also show much persistence, despite the growth in income and the decline in family size.[22] We find that the correlation of the logarithms of community populations in 1970 and 2000 was .981, revealing an extraordinary degree of stability of sizes of municipalities over that thirty-year period.

In addition to stability in population size, the data also reveal strong persistence in community incomes. The median community incomes in 1970 and median community incomes in the year 2000 have a correlation of .93. We thus conclude that community level incomes are highly persistent across decades. We can also analyze the persistence

[20]This is consistent with empirical evidence as discussed in detail in Epple, Gordon, and Sieg (2009).

[21]From 1990 to 2000, the Boston metropolitan area population grew by approximately 8%.

[22]There are 92 (119) municipalities as part of the greater Boston metropolitan area in 1980 (1990). This list of municipalities includes almost all municipalities that were considered to be part of the MSA using the definitions for 1970, 1980, and 1990. The Census drastically changed the definition of the Boston MSA in 2000 and we do not include the municipalities that were added in 2000. A list of all municipalities in our sample is available from the authors.

of community compositions by age and family size. We have calculated the correlation between median community income and the fraction of the population aged 19 or younger in each of the four census years that we study. We find the following correlations: .33, .42, .22, and .54. These correlations might simply reflect variation in family size with income. However, data for the U.S. population reveal that the proportion of individuals under age 19 is relatively constant across household incomes. This suggests that the positive correlations between income and population below age 19 reported above are a result of household sorting across municipalities based on the presence of children and their income levels. The data also reveal much persistence in the population age distribution over time. The correlation of fraction of community populations aged 19 or younger between 1970 and 2000 is 0.66.

Finally, we analyzed the persistence of public policies. We have assembled a comprehensive data base that contains the main fiscal and tax variables for all municipalities in the Boston MSA during the past 25 years. We find that government policies are more volatile than income sorting patterns. There was a decrease in property tax taxes in the first part of the 1980's which was a direct consequence of Proposition 2 1/2. This law, which was passed in 1981, limited property tax rates to two-and-a-half percent. Since many jurisdictions had property taxes in the period leading up to 1981 that were higher than the limits set in Proposition $2\frac{1}{2}$, the law imposed for all practical purposes a binding constraint on these municipalities. Both tax levies and educational expenditures are more stable than tax rates and largely track income increases during that time period.

5 Quantitative Properties of Equilibrium

Having fully parametrized and calibrated the model, we can solve the model numerically. Table 2 illustrates two different equilibria with four communities. In the upper panel we consider our baseline equilibrium with a foundation grant that is financed by a given proportional income tax. In the equilibrium with a foundation grant, the proceeds from the income tax revenues finance a constant economy-wide expenditure on g, and jurisdictional property tax revenues provide a local supplement. Since income is exogenous in this model, an income tax is equal to a lump sum tax. For simplicity we assume that an income tax is paid by young adult households and the tax base is total lifetime income. In 2006, state and local government revenues for primary and secondary education were approximately equal. Thus, we choose the foundation grant to equalize state and local expenditures on education. As we noted above, education expenditures are 52.5% of local expenditures. With state funding equal to half this amount in our calibrated equilibrium, we obtain a foundation grant of $2,600 per young household. In the lower panel we consider an equilibrium without a foundation.

Table 2 reports expenditures, tax rates, and housing prices in the stationary equilibria. First consider the equilibrium with a foundation grant. Expenditures range from $4,012 in the low income community to $17,996 in the high-income community. Property tax rates range from 0.124 to 0.366. This finding is consistent with the observation that households in the higher income communities prefer much higher levels of expenditures than the threshold level guaranteed by the foundation grant.

Table 2 also report the fraction of young and old households in each community. We find that there are more old households in communities 1 and 2 than young households. This ratio reverses for the higher income communities 3 and 4. This finding reinforces the notion that households tend to downsize when old and move from high to low amenity communities. These mobility patterns are also reflected in the average lifetime wealth of young and old households in each community. In general, we find that old households that live in a given community have a higher wealth than young households. This finding is due to the fact that old households that move to a lower community typically have a higher lifetime wealth than the household that always live in this community.

We also report the fraction of young and old households that prefer lower expenditures than the equilibrium level for each community. Here we find the expected generational divide in voting patterns. Young household typically prefer higher expenditure levels while the vast majority of older household prefer lower expenditure levels. This finding holds for all four communities and is especially pronounced for the high income communities in which almost all old households prefer lower expenditure levels.

Table 2: Quantitative Properties of Equilibrium

Decentralized Equilibrium with $2600 Foundation Grant: 4 Communities									
	Housing	Government		Populations		Wealth		Voting	
community	p	t	g	fraction young	fraction old	young	old	lower g young	lower g old
1	9.117	0.120	4,012	0.280	0.330	43,626	55,601	0.058	0.798
2	10.935	0.236	6,959	0.249	0.275	78,771	89,795	0.001	0.952
3	12.391	0.313	10,406	0.238	0.230	121,499	125,054	0.017	0.999
4	14.600	0.366	17,996	0.233	0.164	224,382	249,981	0.152	0.982
Centralized Equilibrium with $2600 Foundation Grant: 1 Community									
1	11.732	0.237	8,353	1.000	1.000	112,638	112,638	0.131	0.869

Decentralized Equilibrium without Foundation Grant : 4 Communities									
	Housing	Government		Populations		Wealth		Voting	
community	p	t	g	fraction young	fraction old	income young	income old	lower g young	lower g old
1	9.979	0.429	3668	0.209	0.246	38,713	49,569	0.059	0.874
2	11.828	0.422	6259	0.242	0.272	68,958	80,022	0.001	0.945
3	13.334	0.440	9297	0.261	0.258	107,833	110,407	0.006	0.999
4	15.901	0.428	17099	0.288	0.224	209,096	226,496	0.119	0.989
Centralized Equilibrium without Foundation Grant : 1 Community									
1	12.821	0.371	8510	1.000	1.000	112,909	112,909	0.131	0.869

Comparing the equilibrium with a foundation grant with the one obtained without the foundation grant, we find that the same qualitative properties of the equilibrium carry over to the case without a foundation grant. However, there are also some pronounced quantitative difference. One obvious consequence of the lack of the foundation grant is that property taxes are uniformly higher since property tax revenues are being substituted for income tax revenues in all communities. On average expenditures in all communities are slightly lower without the foundation grant. Less obvious is the finding that the high income community is larger and thus appears less "selective" under a pure property tax system. There is a stronger incentive to choose a richer community than under a foundation grant system since the latter provides somewhat higher expenditures in relative less wealthy communities.

Table 3 provides some additional insights into the nature of the household sorting process in both equilibria. As we have discussed in the previous section, not all possible residential plans are used in equilibrium. For the equilibria in Table 2, there are only nine residential plans that are optimal for households. There include four plans that involve no re-locations and five plans with re-locations. We find that households that initially choose communities 2 and 3 either stay in these communities or move to community 1. In contrast, households from community 4 move to all three other communities. We also find that 8 % of all households find it optimal to relocate in the baseline (foundation grant) equilibrium. About 22 percent of the households that choose community 4 when young move to one of the three other communities as they enter old age.

Comparing the equilibrium with a foundation grant to the equilibrium without a foundation grant, we find similar mobility patterns. We find that mobility decreases slightly as we move to a foundation grant equilibrium. This is because there are fewer households moving from communities 3 and 2 to community 1

Figure 2 shows the boundaries of adjacent communities that correspond to the equilibrium choice plans with a foundation grant. Figure 2 illustrates several general properties. First, of course, lower moving costs induce more households to follow lifetime plans where they will move to lower (g, p) communities when old. Second, the propensity to move rises with wealth until the household is sufficiently wealthy that living in the

Table 3: Quantitative Properties of Residential Plans

Community	Community	No Foundation Grant		Foundation Grant	
Young	Old	Population	Wealth	Population	Wealth
1	1	0.208	38,713	0.280	43,626
2	1	0.002	48,353	0.001	56,096
2	2	0.240	68,975	0.248	78,801
3	1	0.006	75,596	0.011	85,502
3	3	0.255	108,606	0.228	123,195
4	1	0.031	117,577	0.039	133,258
4	2	0.030	169,183	0.026	193,621
4	3	0.003	246,919	0.003	281,539
4	4	0.224	226,496	0.165	249,981

highest g community throughout adult life is preferred. Third, there is wealth stratification across plans for given moving cost. Fourth, the measure of households indifferent between plans is zero, corresponding to those households on the boundaries that partition the type space into residential plans. Those boundaries are horizontal in the (T, w) plane, i.e., independent of moving cost, when the relevant alternative plans both entail either moving or not moving. These boundaries are upward (downward) sloping when one plan involves moving and the other does not if the higher G plan is the moving plan since maintaining indifference as wealth rises implies higher (lower) moving costs.

To assess the effects of moving costs on equilibrium, we calculate equilibrium with very low moving costs (having distributional mean equal to .16 of the mean in the benchmark equilibrium) and with a virtually prohibitive moving cost distribution. Table 4 presents the results, where we include the benchmark values for ease of comparison. In the alternative cases of the moving cost distribution, we keep the economy's income distribution as in the benchmark steady state to isolate the effects of moving costs by abstracting from long run effects on the income distribution.[23] These changes in the distribution of moving costs have massive effects. When moving costs are very low, 37%

[23]Effects on the steady state are available on request.

Table 4: The Impact of Moving Costs: Sensitivity Analysis

Benchmark Equilibrium (no foundation grant)						
Community	g	t	p	Fraction of young	Fraction of old	Fraction of population
1	3,668	.43	9.98	.21	.25	.23
2	6,259	.42	11.83	.24	.27	.25
3	9,297	.44	13.33	.26	.26	.26
4	17,099	.43	15.90	.29	.22	.26
Low Moving Cost Equilibrium						
1	6,279	.31	11.72	.24	.54	.39
2	7,342	.43	12.27	.24	.23	.24
3	9,495	.49	13.31	.25	.15	.20
4	15,003	.50	15.42	.28	.09	.18
High Moving Cost Equilibrium						
1	2,813	.39	9.05	.19	.19	.19
2	5,586	.44	11.49	.24	.24	.24
3	8,039	.39	12.91	.27	.27	.27
4	16,980	.41	16.02	.30	.30	.30

move over the life cycle as compared to 8% in the benchmark.[24] Examining the fractions of old and young that make up communities, we see that the old population would dominate in the lowest-g community and the young would dominate in the two higher-g communities. Associated with this increased relocation, the lowest-g community would increase markedly in household population relative to the benchmark from 23 to 39 %, while the highest-g community would shrink from having 26 to 18 % of the household population. Most interesting is that the increased relocation would substantially lower the variability in g levels across communities, increasing public provision in the lowest-g community and lowering public provision in the highest-g community. While the older

[24]Here all downward moving plans arise in equilibrium except from community 3 to 2.

households that move to lower-g communities place less weight on g in their utility functions, they are relatively wealthy and increase the tax base in poorer communities. In short, the effects of increased moving on tax bases outweigh the political economy effects. This finding persists when one examines the no-moving case. It has the highest variability in public provision and the lowest expenditure in the low-g community among the cases examined. Note, too, that the changes in variability in the g's is in the same direction as the changes in variability in gross housing prices. We find, perhaps surprisingly, that facilitating moving of older households has an equalizing effect.

Last we provide a comparison between the decentralized equilibrium with four communities and a centralized equilibrium with only one community. Table 2 reports the corresponding allocations for the models with and without a foundation grant. We find that the decentralized equilibrium yields higher average expenditures than the centralized equilibrium: \$9,524 (\$9,639) versus \$8,353 (\$8,510). As a consequence there is a higher steady state mean income in the decentralized equilibrium than the centralized equilibrium: \$113,006 (\$113,229) versus \$112,638 (\$112,909). The welfare analysis implies that lower income households gain in the centralized equilibrium and higher income households prefer the decentralized equilibrium. Following Fernandez and Rogerson (1998), we compare the steady state equilibria and adjust the lifetime income distributions so that average utility is equalized. We find that households prefer on average the centralized equilibrium over the decentralized equilibrium. In the foundation grant case, the welfare gains are approximately 1.1 percent of lifetime income. Sources of the gain in steady state average welfare from centralization are diseconomies in the education production function, reduced variability in income, and avoidance of moving costs.[25]

[25]Benabou (1996b, 2002) provides a general analysis of the trade-offs in educational finance and taxation that affect growth. Calabrese, Epple, and Romano (2009) investigate the static inefficiencies that arise in Tiebout equilibrium with property taxation and show that centralization dampens them.

6 Conclusions

We have provided a new overlapping generations model for studying the life cycle locational choices of heterogeneous households and the associated dynamics of metropolitan areas. Understanding household and community dynamics is an important research area, and there is ample scope for future research. One interesting avenue for future research is to analyze the differences between families with and without children. In our model we have assumed that all families have children when young. However, a substantial fraction of households never have children. The life cycle incentives of these individuals are different, since they do not have reason to pay the housing price premia to locate in areas with high quality public education. The presence of such households in the model can be expected to affect the age composition of communities as well as the outcomes that arise from voting over public good levels. Another important generalization is introduction of peer effects. There is some econometric evidence that supports the view that income stratification and associated stratification of peers influences expenditure policies, with poorer jurisdictions taxing heavily in order to provide expenditures to compensate for schools that have relatively disadvantaged peer groups. This is an important issue from the perspective of providing a fuller characterization of equilibrium.

We also need more research that captures more fully the incentives affecting voting for public education. Households with grown children have an incentive to support high provision of education to maintain property values. These incentives depend on whether the household owns or rents, and on the household's beliefs about the way in which quality of public services impacts rental prices or the value of the home. Property owners have different preferences over public good provision than renters since owners are affected by capital gains or losses that may arise from changes in public policies. The key complication in such a generalization is in characterizing voting equilibrium.[26] Introducing ownership into our dynamic framework is a challenging but important task for future research.

[26] Owner-occupants who anticipate capital gains and losses when voting have been incorporated in static models (Epple and Romer, 1991), and those investigations reveal that ownership substantially affects voter incentives and equilibrium outcomes.

References

Bajari, P. and Kahn, M. (2004). Estimating Housing Demand with an Application to Explaining Racial Segregation in Cities. *Journal of Business and Economic Statistics*, *23(1)*, 20–33.

Bayer, P., Ferreira, F., and McMillan, R. (2007). A Unified Framework for Measuring Preferences for Schools and Neighborhoods. *Journal of Political Economy*, *115 (4)*, 588–638.

Bayer, P., McMillan, R., and Reuben, K. (2004). The Causes and Consequences of Residential Segregation: An Equilibrium Analysis of Neighborhood Sorting. Working Paper.

Benabou, R. (1996a). Equity and Efficiency in Human Capital Investments: The Local Connection. *Review of Economic Studies*, *63(2)*, 237–64.

Benabou, R. (1996b). Heterogeneity, Stratification and Growth: Macroeconomic Effects of Community Structure. *American Economic Review*, *86*, 584–609.

Benabou, R. (2002). Tax and Education Policy in a Heterogeneous-Agent Economy: Maximize Growth and Efficiency?. *Econometrica*, *70(2)*, 481–517.

Black, S. (1999). Do Better Schools Matter? Parental Valuation of Elementary School Education. *Quarterly Journal of Economics*, *114(2)*, 577–600.

Calabrese, S., Epple, D., and Romano, R. (2006). Non-Fiscal Residential Zoning. In Fischel, W. (Ed.), *The Tiebout Model at Fifty: Essays in Public Economics in Honor of Wallace Oates*, pp. 211–230. Lincoln Institute of Land Policy.

Calabrese, S., Epple, D., and Romano, R. (2009). Inefficiencies from Metropolitan Political and Fiscal Decentralization: Failures of Tiebout Competition. Working Paper.

Denzau, A. and Mackay, R. (1976). Benefit Shares and Majority Voting. *American Economic Review*, *66*, 69–79.

Durlauf, S. (1996). A Theory of Persistent Income Inequality. *Journal of Economic Growth*, *1*, 75–93.

Ellickson, B. (1973). A Generalization of the Pure Theory of Public Goods. *American Economic Review*, *63 (3)*, 417–432.

Epple, D., Filimon, R., and Romer, T. (1984). Equilibrium Among Local Jurisdictions: Toward an Integrated Treatment of Voting and Residential Choice. *Journal of Public Economics*, *24*, 281–304.

Epple, D., Gordon, B., and Sieg, H. (2009). A New Approach to Estimating the Production Function for Housing. *American Economic Review*, *forthcoming*.

Epple, D. and Romano, R. (2003). Public School Choice and Finance Policies, Neighborhood Formation, and the Distribution of Educational Benefits. In Hoxby, C. (Ed.), *The Economics of School Choice*. Univesity of Chicago Press.

Epple, D. and Romer, T. (1991). Mobility and Redistribution. *Journal of Political Economy*, *99(4)*, 828–858.

Epple, D., Romer, T., and Sieg, H. (2001). Interjurisdictional Sorting and Majority Rule: An Empirical Analysis. *Econometrica*, *69*, 1437–1465.

Epple, D. and Sieg, H. (1999). Estimating Equilibrium Models of Local Jurisdictions. *Journal of Political Economy*, *107 (4)*, 645–681.

Farnham, M. and Sevak, P. (2006). State Fiscal Institutions and Empty-Nest Migration: Are Tiebout Voters Hobbled. *Journal of Public Economics*, *90*, 407–27.

Fernandez, R. and Rogerson, R. (1996). Income Distribution, Communities, and the Quality of Public Education. *Quarterly Journal of Economics*, *111 (1)*, 135–164.

Fernandez, R. and Rogerson, R. (1998). Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform. *American Economic Review*, *88(4)*, 813–833.

Fernandez, R. and Rogerson, R. (2003). Equity and Efficiency: An Analysis of Education Finance System. *Journal of Political Economy*, *111*, 858–97.

Ferreira, F. (2005). You Can Take It with You: Proposition 13 Tax Benefits, Residential Mobility, and Willingness to Pay for Housing Amenities. Working Paper.

Ferreyra, M. (2007). Estimating the Effects of Private School Vouchers in Multi-District Economies. *American Economic Review*, *97*, 789–817.

Figlio, D. and Lucas, M. (2004). What's in a Grade? School Report Cards and House Prices. *American Economic Review*, *94*, 591–604.

Glomm, G. and Lagunoff, R. (1999). A Dynamic Tiebout Theory of Voluntary vs Involuntary Provision of Public Goods. *Review of Economic Studies*, *66*, 659–678.

Goodspeed, T. (1989). A Reexamination of the Use of Ability-to-Pay Taxes by Local Governments. *Journal of Public Economics*, *38 (3)*, 319–342.

Henderson, V. and Thisse, J. (2001). On Strategic Community Development. *Journal of Political Economy*, *109(3)*, 546–569.

Lau, L. (1970). Duality and the Structure of Utility Functions. *Journal of Economic Theory*, *1*, 374–396.

Nechyba, T. (1997). Local Property and State Income Taxes: the Role of Interjurisdictional Competition and Collusion. *Journal of Political Economy*, *105 (2)*, 351–84.

Oates, W. (1969). The Effects of Property Taxation and Local Spending on Property Values: An Empirical Study of Tax Capitalization and the Tiebout Hypothesis. *Journal of Political Economy*, *77*, 957–971.

Ortalo-Magne, F. and Rady, S. (2006). Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints. *Review of Economics Studies*, *73*, 459–85.

Rothstein, J. (2006). Good Principals or Good Peers? Parental Valuation of School Characteristics, Tiebout Equilibrium, and the Incentive Effects of Competition Among Jurisdictions. *American Economic Review*, *96*, 1333–1350.

Sieg, H., Smith, V. K., Banzhaf, S., and Walsh, R. (2004). Estimating the General Equilibrium Benefits of Large Changes in Spatially Delineated Public Goods. *International Economic Review*, *45 (4)*, 1047–1077.

Slutsky, S. (1975). Majority Voting and the Allocation of Public Goods. Dissertation, Yale University.

Solon, G. (1992). Intergenerational Income Mobility in the United States. *American Economic Review*, *82*, 393–408.

Tiebout, C. (1956). A Pure Theory of Local Expenditures. *Journal of Political Economy*, *64 (5)*, 416–424.

Westhoff, F. (1977). Existence of Equilibrium in Economies with a Local Public Good. *Journal of Economic Theory*, *14*, 84–112.

A Additional Proofs

The indirect utility is given by:[27]

$$\begin{aligned} V^y &= \underset{h_k,h_l}{Max}\,[u_g^y(g_k) + u_g^o(g_l) + u^y(b_k,h_k) + u^o(b_l,h_l)] \\ &\quad s.t.\ p_k h_k + b_k + p_l h_l + b_l \leq \tilde{w} \\ &= G(g_k,g_l) + \underset{h_k,h_l}{Max}\,[u^y(b_k,h_k) + u^o(b_l,h_l)] \\ &\quad s.t.\ p_k h_k + b_k + p_l h_l + b_l \leq \tilde{w}\,; \end{aligned} \tag{31}$$

where $G(g_k, g_l) \equiv u_g^y(g_k) + u_g^o(g_l)$ is an increasing function of (g_k, g_l). Since $u^a(b, h)$ is homogeneous of degree ψ, it follows from Theorem I in (Lau, 1970) (p. 376) that the maximand in the lower line of (31) equals $\tilde{W}\left(\frac{p_k}{\tilde{w}}, \frac{p_l}{\tilde{w}}\right)$, a function homogeneous of degree $-\psi$ and decreasing in its arguments . Then: $V^y = G(p_k, p_l) + \tilde{w}^{-\psi} W(p_k, p_l)$.

Proof of Proposition 2:

Households with T maximize V_i^y as defined in (20) – (21). Since V_i^y is increasing in G_i and decreasing in P_i^T, households choose among the residential plans in $\underline{B}^0(T)$. Since w ranges from 0 to ∞, the slope of an indifference curve in the (G_i, P_i^T) plane ranges from 0 to ∞ as well, implying all plans in $\underline{B}^0(T)$ are chosen by some households with T.

(i) Obviously all non-moving residential plans chosen by households with the maximum T are in the set of chosen residential plans by all households. To confirm that only these non-moving plans are equilibrium ones, observe from (21) that, since P_i^T is increasing in T for moving plans and independent of T for non-moving plans, lowering T can eliminate but cannot add non-moving plans to $\underline{B}^0(T)$. From the result above it follows that no households with lower T than the maximum choose a non-moving residential plan not chosen by a household with the maximum T.

(ii) Let T_m denote the minimum T. Obviously all moving plans chosen by such households are in the equilibrium set of moving plans. To confirm only such moving plans are in the equilibrium set of all households, suppose household "2" with (w_2, T_2), $T_2 > T_m$, chooses a moving plan lk in equilibrium that is not chosen by any households with T_m. Consider household "1" with $(w_1, T_1) = (w_2 \frac{1-T_2}{1-T_m}, T_m)$. Note that $w_1 < w_2$.

[27] The discount factor β is subsumed in the old age utility function with no loss of generality.

Households 1 and 2 obtain the same level of utility from all moving plans (by (20) – (21)). Household 1 obtains lower utility from all non-moving plans than does household 2, since household 1 has lower wealth (and moving costs are irrelevant). But then household 1 would share household 2's preference for moving plan lk, a contradiction. Q.E.D.

Proof of Proposition 3: First we show that the plan with $G = G_1$ corresponds to $lk = 11$ and the plan with $G = G_{J_e}$ corresponds to $lk = JJ$.

The residential plans on the lower boundary of the convex hull of all feasible plans corresponds to just non-moving plans for any types with T that will never move in equilibrium. Plans $lk = 11$ and $lk = JJ$ are the endpoints of the lower boundary of the convex hull for all of these types. The result then follows from Assumption 9.

(i) If $P_j^T \geq P_i^T$, then choice of plan j would contradict maximization of V^y (recall (20)).

(ii) Using that households chose residential plans to maximize V^y, wealth stratification follows from the ascending bundles property and SIW. Q.E.D.

Proof of Proposition 4: The proof is by contradiction, so suppose a household makes such a choice. Then that choice solves the program:

$$\begin{aligned} \max_{h_k,b_k,h_l,b_l} \quad U &= [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \frac{\beta_g}{\rho} g_l^{\rho}] + \frac{1}{\rho}[\alpha_h h_k^{\rho} + \alpha_b b_k^{\rho} + \beta_h h_l^{\rho} + \beta_b b_l^{\rho}] \\ s.t. \; w - mc &= p_k h_k + b_k + p_l h_l + b_l \end{aligned} \tag{32}$$

with $(p_k, g_k) < (p_l, g_l)$. Let:

$$\begin{aligned} L^* \equiv \; & [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \frac{\beta_g}{\rho} g_l^{\rho}] + \frac{1}{\rho}[\alpha_h h_k^{\rho} + \alpha_b b_k^{\rho} + \beta_h h_l^{\rho} + \beta_b b_l^{\rho}] \; + \\ & \lambda[w - mc \; - \; p_k h_k - b_k - p_l h_l - b_l] \end{aligned} \tag{33}$$

denote the Lagrangian function at the household's optimum, where λ denotes the multiplier on the budget constraint. Thus, $V_{kl}^y(p_k, g_k, p_l, g_l) \equiv L^*(p_k, g_k, p_l, g_l)$. Using the latter and (33), compute, respectively, slopes of the indifference curves over (p, g) pairs while young and (p, g) pairs while old:

$$\left.\frac{dp_k}{dg_k}\right|_{V_{kl}^y = const.} = -\frac{\partial V_{kl}^y / \partial g_k}{\partial V_{kl}^y / \partial p_k} = -\frac{\partial L^* / \partial g_k}{\partial L^* / \partial p_k} = \frac{\alpha_g g_k^{\rho_a - 1}}{\lambda h_k}; \tag{34}$$

and

$$\left.\frac{dp_l}{dg_l}\right|_{V^y_{kl}=const.} = -\frac{\partial V^y_{kl}/\partial g_l}{\partial V^y_{kl}/\partial p_l} = -\frac{\partial L^*/\partial g_l}{\partial L^*/\partial p_l} = \frac{\beta_g g_l^{\rho-1}}{\lambda h_l}; \tag{35}$$

where the last equality in each of (34) and (35) uses the Envelope Theorem. Using the first-order conditions from (33), one obtains:

$$h_k = \frac{w - mc}{z_{kl}} \left(\frac{\alpha_b}{\alpha_h} p_k\right)^{1/(\rho-1)} \tag{36}$$

$$h_l = \frac{w - mc}{z_{kl}} \left(\frac{\beta_b}{\beta_h} p_l\right)^{1/(\rho-1)} . \tag{37}$$

Substituting (37) into (35) and (36) into (34) and evaluating slopes at a common (p, g) point, one finds that the indifference curve over (p, g) pairs while young are everywhere steeper than the indifference curve over (p, g) pairs while old if $\alpha_g(\alpha_h/\alpha_b)^{1/(\rho-1)} g^{\rho_a-\rho} > \beta_g(\beta_h/\beta_b)^{1/(\rho-1)}$. This condition holds under Assumption 10.[28] In a stationary equilibrium, the (p, g) pairs available in each period of life are the same.

The steeper curve in the Figure 4 shows the indifference curve of the young household that chooses community k while young given community l is available (with $(p_k, g_k) < (p_l, g_l)$). This curve must pass below the point (p_l, g_l) as shown, or the household would prefer community l while young. (The fact that the household would save moving costs by choosing l while young, while the indifference curves assume moving costs are paid, only reinforces the claim.) The flatter indifference curve shows that of the household through (p_k, g_k) when old, which implies the household would prefer to choose community k when old while paying moving costs. The fact that the household would not have to pay moving costs (since it resides initially in community k) implies a stronger yet preference for community k when old, hence a contradiction. Q.E.D.

[28] Here we are presuming $g \geq 1$. In economies with realistic wealth levels, the presumption that equilibrium spending in all communities is more than a dollar per student is innocuous.

B Figures

Figure 1: The Relevant Choice Set

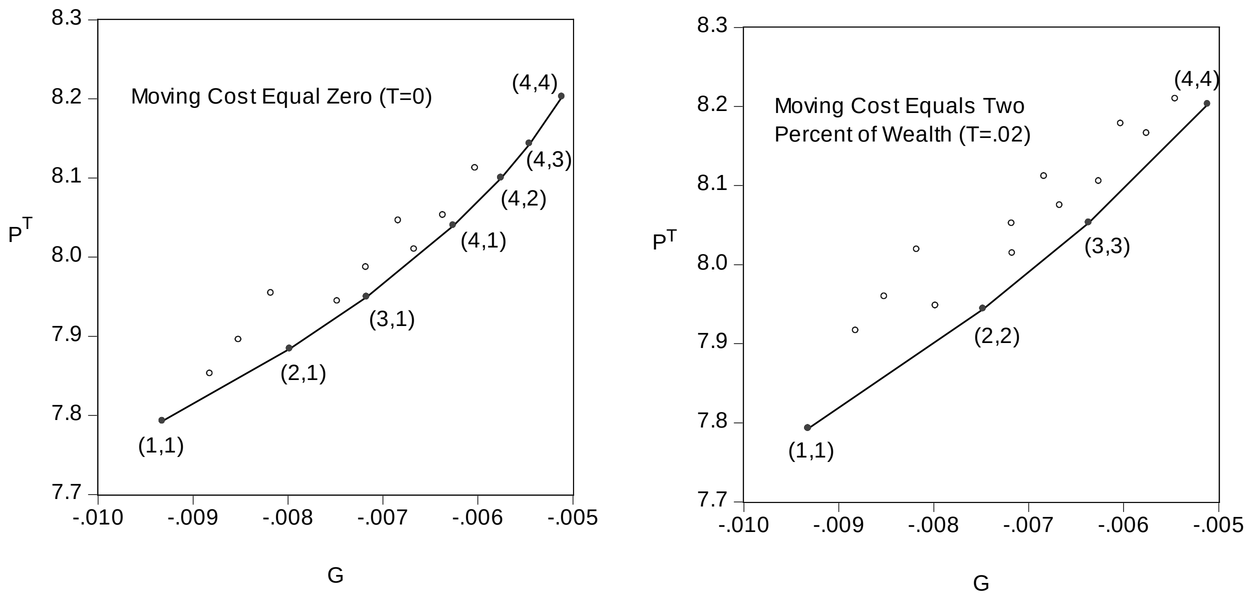

Figure 2: Residential Plans in Equilibrium

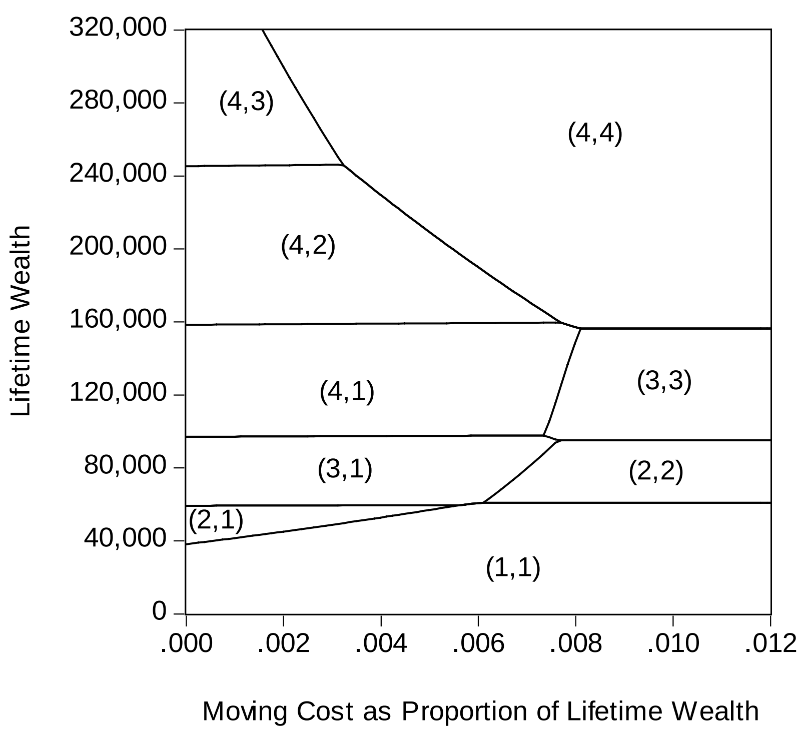

Figure 3: Ratio of Old to Young Households by Community Income in 1980 and 1990

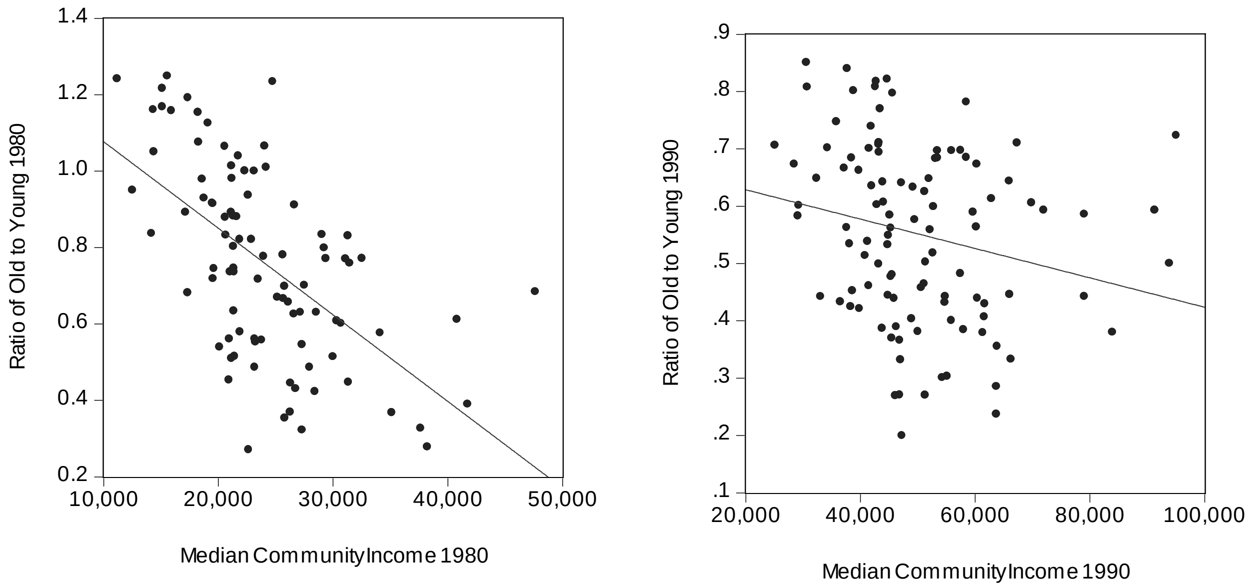

Figure 4: Indifference Curves by Cohort