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Decision Theory without "Independence" or without "Ordering"

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DECISION THEORY WITHOUT "INDEPENDENCE" OR WITHOUT "ORDERING"

What Is the Difference?

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1. INTRODUCTION

It is a familiar argument that advocates accommodating the so-called paradoxes of decision theory by abandoning the "independence" postulate. After all, if we grant that choice reveals preference, the anomalous choice patterns of the Allais and Ellsberg problems (reviewed in Section 3) violate postulate P2 ("sure thing") of Savage's (1954) system. The strategy of making room for new preference patterns by relaxing independence is adopted in each of the following works: Samuelson (1950), Kahneman and Tversky's "Prospect Theory" (1979), Allais and Hagen (1979), Fishburn (1981), Chew and MacCrimmon (1979), McClennen (1983), and in closely argued essays by Machina (1982, 1983 [see the latter for an extensive bibliography]).

There is, however, a persistent underground movement that challenges instead the normative status of the "ordering" postulate for preference. Those whose theories evidence some misgivings about ordering

I have benefitted from discussions with each of the following about the problems addressed in this essay: W. Harper, J. Kadane, M. Machina, P. Maher, E. F. McClennen, M. Schervish; and I am especially grateful to I. Levi.

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include: Good (1952), C. A. B. Smith (1961), Levi (1974, 1980), Suppes (1974), Walley and Fine (1979), Wolfenson and Fine (1982), and Schick (1984). And abandoning ordering is a strategy that has been used to resolve group decision problems. For this see Savage (1954, section 7.2) and Kadane and Sedransk (1980), and see Kadane (1986) for an application to clinical trials. "Regret" models also involve a failure of ordering since choice-with-regret does not satisfy Sen's (1977) principle of "independence of irrelevant alternatives": Savage (1954, section 13.5), Bell and Raiffa (1979) and Bell (1982), Loomes and Sugden (1982), and Fishburn (1983) discuss regret.

The goal of this essay is to help sort out the differences between these two strategies for generalizing expected-utility theory (EU). I argue in Section 4 that abandoning independence leads to an incoherence in sequential decisions. As Levi (1986a, 1986b, 1987) asserts and as I outline reasons for in Section 5, one can avoid this incoherence while relaxing ordering. Thus, the criterion of avoiding a sure loss in sequential decisions provides the needed demarcation.

2. EXPECTED UTILITY FOR SIMPLE LOTTERIES – A REVIEW

For ease of exposition, let us adopt an axiomatization similar to the von Neumann and Morgenstern (1947) theory, as condensed by Jensen (1967). Let R be a set of β-many rewards (or payoffs), $R = (r_\alpha\colon \alpha \leq \beta)$. In the spirit of Debreu's (1959, chapter 4) presentation, we can think of R as including (infinitely divisible) monetary rewards. A (simple) lottery over R is a probability measure P on R with the added requirement that $P(X) = 1$ for some *finite* subset of rewards.

Lotteries are individuated according to the following (0^{th}) *reduction postulate:* Let L_1, L_2 be two lotteries with probabilities P_1, P_2 and let $R_n = (r_1, \ldots, r_n)$ be the finite set of the union of the payoffs under these two lotteries. A convex combination, $\alpha L_1 + (1-\alpha)L_2$ $(0 \leq \alpha \leq 1)$, of the two lotteries is again a lottery with probability measure $\alpha P_1 + (1-\alpha)P_2$ over R_n. Thus, the set of lotteries is a mixture set $M(R)$ in the sense of Herstein and Milnor (1953).

Three postulates comprise expected utility theory:

(1) An ordering requirement: preference, $\leq$, a relation over $M \times M$, is a weak-order. That is, $\leq$ is reflexive, transitive, and all pairs of lotteries are comparable under $\leq$. (Strict preference, $<$, and indifference, $\sim$, are defined relations.)

(2) An Archimedean requirement: If $L_1 < L_2$ and $L_2 < L_3$, there is a nontrivial convex combination of L_1 and L_3 strictly preferred (and another combination strictly dispreferred) to L_2. That is, there exist

$$0 < \alpha,\ \beta < 1 \text{ with } \alpha L_1 + (1-\alpha)L_3 < L_2 \text{ and } L_2 < \beta L_1 + (1-\beta)L_3.$$

The point in assuming that R includes (infinitely divisible) monetary payoffs is made clear by the additional stipulation that each lottery in M carries a sure-dollar equivalent (under $\leq$):

$$\forall L \in \mathbf{M}\ \exists \$x \in \mathbf{R}\ (L \sim L_{\$x}), \tag{*}$$

where $L_{\$x}$ is a degenerate lottery having only one prize, \$$x$. Then principle (2) deserves its title for, with (*) and the added stipulation that more is (strictly) better when it comes to money, (1) and (2) entail a real-valued utility representation for $\leq$ (continuous in \$).[1] To simplify still further, let us restrict attention to lotteries with none but monetary payoffs.

(3) The "independence" principle: For all L_i, L_j, and L_k, and for all α $(0 < \alpha \leq 1)$, $L_i \leq L_j \Leftrightarrow \alpha L_i + (1-\alpha)L_k \leq \alpha L_j + (1-\alpha)L_k$.

Let us examine these postulates for the special case of lotteries on three rewards: $R = (r_1 < r_2 < r_3)$, where the reward r_i is identified with the degenerate lottery having point-mass $P(r_i) = 1$ $(i = 1,2,3)$. Following the excellent presentation by Machina (1982), we arrive at a simple geometric account of what is permitted by expected-utility theory. Figure 1 depicts the consequences of postulates (1)–(3).

According to the postulates (1)–(3), indifference curves ($\sim$) over lotteries are parallel, straight lines of (finite) positive slope. L_i is (strictly) preferred to L_j, $L_j < L_i$, is just in case the indifference curve for L_i is to the left of the indifference curve for L_j.

Consider a lottery L_i, as in Figure 2. Stochastic dominance provides a (strict) preference for lotteries to the NW of L_i, whereas L_i is (strictly) preferred to lotteries to its SE.[2] Thus, the indifference lines must have

1. By assuming (*), we fix it that $M/\sim$ (no longer assumed to be a mixture set) has a countable dense subset in the $<$-order on $M/\sim$, e.g., the rational-valued sure-dollar equivalents. Then our first two postulates ensure a real-valued utility u on M with the property that $L_1 < L_2$ if and only if $u(L_1) < u(L_2)$. The point is that, without "independence," the usual Archimedean axiom is neither necessary nor sufficient for a real valued utility. See Fishburn (1970, Section 3.1) for details, or Debreu (1959, Chapter 4), who discusses conditions for u to be continuous. Debreu uses a "continuity" postulate in place of (2) that, in our setting, requires that if the sequence $[\![L_i]\!]$ converges (in distribution) to the lottery L_j, and $L_j < L_k$, then all but finitely many of the $L_i < L_k$. If we extend $\leq$ to general distributions over R, Debreu's continuity postulate entails countably additive probability. See Seidenfeld and Schervish (1983) for some discussion of the decision-theoretic features of finitely additive probability.
2. Recall, lottery L_2 (first order) stochastically dominates lottery L_1 if L_2 can be obtained from L_1 by shifting probability mass from less to more desirable payoffs. More precisely, L_2 stochastically dominates L_1 if, as a function of increasingly preferred rewards, the cumulative probability distribution for L_2 is everywhere less than (or equal to) the cumulative probability distribution for L_1. Of course, whenever L_2 stochastically dominates L_1, there is a scheme for payoffs, in accord with the two probability measures, where L_2 weakly dominates L_1.

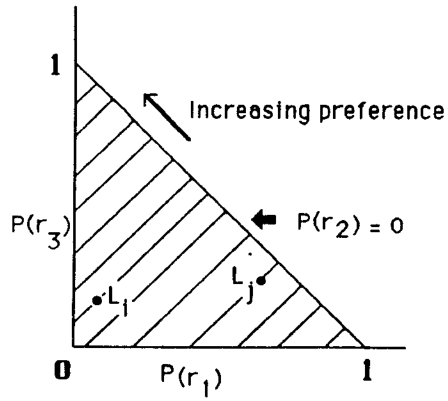


FIGURE 1.

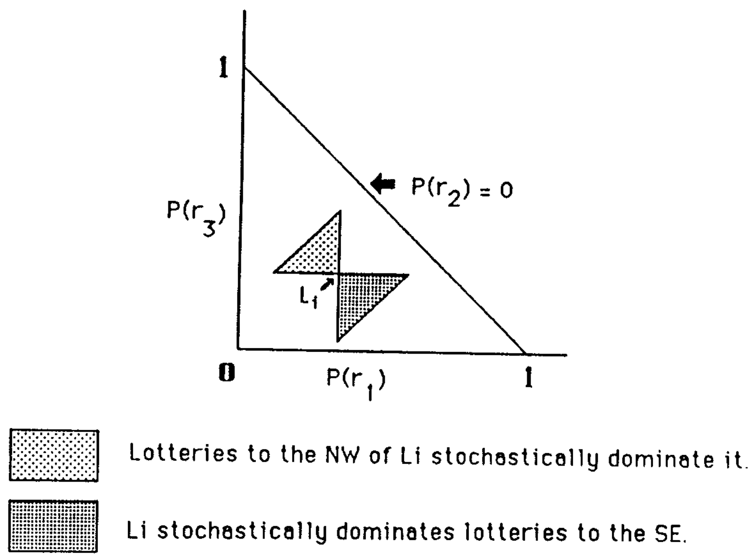


FIGURE 2.

positive slope. Hence, in this setting with lotteries over three rewards, expected-utility theory permits one degree of freedom for preferences, corresponding to the choice of a slope for the lines of indifference.

In a collaborated effort, Seidenfeld, Schervish, and Kadane (1987, Section 1), we apply this analysis to the selection of "sizes" (α-levels) for statistical tests of a simple null hypothesis against a simple rival hypothesis. The conclusion we derive is the surprising "incoherence" (conflict with expected-utility theory) of the familiar convention to

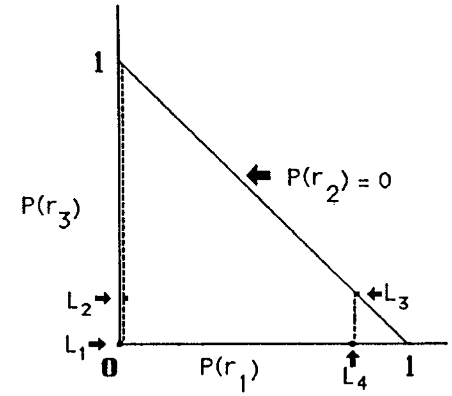


FIGURE 3.

choose statistical tests with a size, e.g., $\alpha = .01$ or $\alpha = .05$, independent of the sample size. This reasoning generalizes that of Lindley (1972, p. 14, where he gives his argument for the special case of "0–1" losses). In a purely "inferential" (nondecision-theoretic) Bayesian treatment for the testing of a simple hypothesis versus a composite alternative, Jeffreys (1971, p. 248) argues for the same caveat about constant α-levels.

3. REVIEW OF THE ALLAIS AND ELLSBERG "PARADOXES"

3.1

Allais (1953) poses the following question. For the three rewards, $r_1 = \$0$, $r_2 = \$1$ million, and $r_3 = \$5$ million (so $r_1 < r_2 < r_3$), what are your preferences, in the choice between lotteries L_1 and L_2, and in the choice between lotteries L_3 and L_4, where:

L_1 – with $P(r_2) = 1$ (\$1 million for certain),
L_2 – with $P(r_1) = .01$, $P(r_2) = .89$, and $P(r_3) = .10$,
L_3 – with $P(r_1) = .90$ and $P(r_3) = .10$, and
L_4 – with $P(r_1) = .89$ and $P(r_2) = .11$?

The common response, choose L_1 over L_2, and L_3 over L_4, violates EU theory (under the assumption that the choices reveal $<$). This is made evident by an application, Figure 3, of (Machina's) figure 1.

The lines connecting the pairs of lotteries in the two choices are parallel. Thus, regardless of the slope of the parallel, straight-line indifference curves (from Figure 1) imposed on lotteries over the three rewards, either L_2 and L_3 are preferred to their rivals, or else L_1 and L_4 are preferred. EU precludes the common answer to Allais' question.

3.2

Ellsberg's (1961) paradox of preference for lotteries with known risk ($\in M$) over uncertain lotteries ($\notin M$), bearing unknown risk, does not fit the simple mixture-set model, M. We can accommodate Ellsberg-styled problems by generalizing our concept of acts so that an act is a function f from states, a (finite, exhaustive) partition, to distributions on the reward set R. These more general acts are called "horse lotteries" by Anscombe and Aumann (1963). Denote by M' ($\supset M$) the generalized mixture-set for the class of horse lotteries.[3] Then lotteries of known risk belong to this enlarged (mixture) set M' as a special case: they are the "constant" acts. That is, the acts of known risk are those for which f^{-1} is a determinate probability measure.

Let us see how an Ellsberg-styled paradoxical choice violates postulate **3**, supposing **(1)** and **(2)** obtain, when the postulates are applied to M'. Imagine I have placed \$10 in one of two pockets, which are otherwise empty. Consider the following three lotteries:

L_{left} – take the contents of my left pocket,
L_{right} – take the contents of my right pocket, and
L_{mix} – take the contents of my left pocket if a "fair" coin lands tails up, and take the contents of my right pocket if the fair coin lands head up.

Lotteries L_{left} and L_{right} are uncertain prospects. Suppose you are indifferent ($\sim$) between these two, which you evaluate as having a sure-dollar equivalent of \$2.50. However, the third option, L_{mix}, is (under the "reduction" postulate) a lottery of known risk. That is, L_{mix} is a lottery with an equal (.5; .5) probability distribution on the two payoffs (\$0, \$10). In the spirit of the Ellsberg paradoxical choice, suppose you evaluate the fair gamble on these two payoffs as having, say, a sure-dollar equivalent of \$4.00. You (strictly) prefer the lottery of known risk, L_{mix}, to either of the two uncertain lotteries. Finally, as the coin flip gives you no relevant information about the location of the \$10, your conditional preferences over the two uncertain lotteries (and their \$2.50 equivalent) are unaffected by the outcome of the coin flip. Then, as L_{mix} is (under reduction) equivalent to the (α = .5) convex combination of L_{left} and

3. To define the generalized mixture-set M', it suffices to define the operation of convex-combination of two (generalized) lotteries. This is done exactly as in Anscombe and Aumann's (1963) treatment of "horse lotteries." Horse lotteries, the generalized postulates (1)–(3) for horse lotteries, and, with the addition of two minor assumptions (precluding a preference-interaction between payoffs and states), the subjective expected-utility theory that results, are discussed by Fishburn (1970, Chapter 13) and briefly in Section 5 here.

L_{right}, preference for "risk" over "uncertainty" violates the independence postulate **3**, assuming **(1)** and **(2)** hold.[4]

In fact, given **(1)** and **(2)**, this version of the Ellsberg paradox conflicts with a principle **(4)**, (strictly) weaker than principle **(3)**.

(4) Mixture dominance ("betweenness"): Of lotteries L_1 and L_2, if each is (weakly or strictly) preferred (or dispreferred) to a lottery L_3; so, too, each convex combination of L_1 and L_2 is (weakly or strictly) preferred (or dispreferred) to L_3.

In terms of (Machina's) figure 1, mixture dominance entails linear indifference curves. (This follows directly with **(4)**, as then indifference is preserved under convex combinations. In Figure 1, the set of convex combinations of two lotteries graphs as a straight line.) But the conjunction of **(1)**, **(2)**, and **(4)** does not entail **(3)**. Samuelson's (1950) "Ysidro" ranking, and the "weighted utility" theory of Chew (1981) satisfy **(1)**, **(2)** and **(4)** but fail **(3)**.[5] Chew (1983) shows that the Allais paradoxical choices are admitted by his theory. What we find here is that Ellsberg-styled preference for risk over uncertainty cannot be so easily absorbed. In order to admit the Ellsberg-styled paradoxical choices, mixture dominance, **(4)**, too, must fail.

4. These preferences are in conflict with Savage's (1954) "sure-thing" postulate P2. P2 is inconsistent with the following two preferences:

(i) $L_{\text{right}} < L_{\text{mix}}$.
(ii) $L_{\text{left}} \sim L_{\text{right}}$, given the coin lands heads up.

Consider the four-event partition generated by whether the coin lands heads (H) or tails (T), and whether the \$10 is in the left (L) or right (R) pocket. Then, by (i), the first row (below) is preferred to the second. Savage's theory uses "called-off" acts to capture conditional preference. Thus, by (ii), the agent is indifferent between the third and fourth rows.

	HL	HR	TL	TR
L_{mix}	\$10	\$0	\$0	\$10
L_{right}	\$0	\$10	\$0	\$10
$L_{\text{left}} \mid H$	\$10	\$0	\$0	\$0
$L_{\text{right}} \mid H$	\$0	\$10	\$0	\$0

5. Let u be a utility on payoffs and assume u is positive. Denote by $E_u(L_i)$ the expected utility of lottery L_i under utility function u. Denote by L_i^{-1} the lottery that has payoffs with (multiplicative) inverse utility to L_i. Samuelson's (1950) "Ysidro" ranking, $\leq_Y$, on lotteries is given by the function

$$Y(L_i) = [E_u(L_i)/E_u(L_i^{-1})]^{.5}.$$

Not only does $\leq_Y$ satisfy the ordering, Archimedean, and mixture dominance postulates while failing indepe[illegible], but in addition $\leq_Y$ respects stochastic dominance!

4. OBJECTIONS TO THE DENIAL OF "INDEPENDENCE"

4.1 On Failures of "Stochastic Dominance"

Kahneman and Tversky's (1979) intriguing alternative to EU, "Prospect Theory," gives a reconstruction of Allais' paradoxical choice behavior at the expense of the independence postulate. Call a simple lottery *regular* provided not all its payoffs are (strictly) preferred to "status quo." Recall, the ranking of a lottery by expected utility uses the formula:

$$\sum_i P(r_i)u(r_i).$$

For regular lotteries, the ranking of a lottery by prospect theory uses the formula:

$$\sum_i \pi[P(r_i)]v(r_i),$$

where v is a value-function for rewards (akin to the utility u), and π is some monotone-increasing function with $\pi(0) = 0$ and $\pi(1) = 1$. Again, let us consider (regular) lotteries on three rewards $r_1 < r_2 < r_3$, where we may take r_1 as status quo. If (and only if) π is linear do we have agreement between prospect theory and EU (for then $\pi(x) = x$ with the scalar constant absorbed into the utility u, defined up to positive linear transformations). If π is not linear, so that prospect theory violates independence, stochastic dominance fails too.[6]

I do not know whether this aspect of prospect theory has been subjected to test for its descriptive accuracy. (I find it hard to believe that subjects would prefer a stochastically dominated lottery when the comparison involves just three rewards and the two lotteries involved assign identical probability to the status quo reward r_1.) In any event, it is normatively unacceptable to mandate a violation of stochastic dominance. What, after all, is left of the concern to avoid unnecessary losses (with respect to payoffs) when a theory *requires* a strict preference for an option dominated (on a set of positive probability)?

Thus, we shall examine only those violations of independence that induce preferences consistent with the partial order imposed by stochastic dominance. To that end, with an eye on the anticipated exchange between theories that abandon ordering versus those that abandon independence, I elevate respect for stochastic dominance to the status of a coherence condition.

6. The result is elementary and has been noted by many, including Kahneman and Tversky (1979, p. 283–284). Suppose π is not linear so that $\pi(p + q) > \pi(p) + \pi(q)$. Then by letting the value $v(r_2)$ approach the value $v(r_3)$, the agent is required (strictly) to prefer L_1: $P_1(r_1) = (1 - [p + q])$, $P_1(r_2) = p + q$, and $P_1(r_3) = 0$ – over L_2: $P_2(r_1) = P_1(r_1)$, $P_2(r_2) = p$, $P_2(r_3) = q$, even though L_2 stochastically dominates L_1. The argument for the other case is similar: $\pi(p + q) < \pi(p) + \pi(q)$.

DEFINITION:

A decision rule is coherent *if (i) admissible choices under the rule are stochastically undominated, and (ii) admissibility is preserved under substitution (at choice points) of "indifferent" options.*

In Section 5, I summarize choice-based generalizations of "preference over rewards" (to explicate stochastic dominance) and "indifference over options" without assuming that choice induces a weak-order. However, in terminal decisions, when a choice rule induces an ordering $\leq$, condition (ii) adds nothing to (i). (See Sen's [1977] excellent discussion relating properties of choice rules to ordering.) That is, suppose lottery L_2 stochastically dominates L_1. By (i), L_1 is inadmissible when L_2 is available for choice, and $L_1 < L_2$. If, moreover, $L_1 \sim L_3$ and $L_2 \sim L_4$ then $L_3 < L_4$ by properties of $\leq$; so that inadmissibility of dominated options is preserved under substitution of indifferents, (ii). Therefore, in nonsequential decisions, and depending upon how indifference is defined without ordering, clause ii only serves as an added restriction on the coherence of choice rules that relax ordering. In sequential decisions the situation is rather different. As shown in Section 4.2, even though a choice rule induces a weak-order and respects stochastic dominance in nonsequential decisions, it may fail to be sequentially coherent.

The point of clause (ii) is to help identify a standard for evaluating decision rules predicated on the supposition that the agent's values for rewards (and for lotteries over those rewards) are stable over time. That is, this standard of coherence is offered for assessing the performance of a choice rule in sequential decisions when basic values are unchanging. Clause (ii) is not cogent, I would argue, when basic values are subject to revision over time. Then there may be a current preference between two rewards that are (to be) judged indifferent relative to the future, changed values. Thus there is no reason to demand that substitution of "future" indifferents preserves the inadmissibility of what is, by current values, a dominated option.

Of course, the agent's knowledge of events, chance occurrences, and preceding choices inevitably changes in the course of a sequential decision. In fact, these changes in evidence are what makes valuable adaptive experimental designs.

4.2 Sequential Incoherence – An Example When Mixture Dominance Fails

Respect for stochastic dominance provides a safeguard that choice over lotteries attends to sure-gains in payoffs. Single stage (nonsequential) decisions are thereby protected from violations of weak dominance over rewards. That is not the case, however, when we attend to sequential

decisions. Specifically, coherence in nonsequential decisions, in choices over lotteries, does not entail the sequential version of coherence in choices over plans. This is illustrated by an example.

Consider what happens when mixture dominance (4) fails: *Example:* Let lotteries L_1 and L_2 be indifferent with a sure-dollar equivalent of \$5.00. Suppose, contrary to (4), that an equal ($\alpha = .5$) convex combination of them is strictly preferred with a sure-dollar equivalent of, e.g., \$6.00. Denote this by $L_3 = .5L_1 + .5L_2 \sim \$6.00$. Then, by continuity of preference for monetary payoffs, there is some fee, $\$\epsilon$, that can be attached to the *payoffs* of L_1 and L_2 (resulting in the lotteries denoted by "$L_1 - \epsilon$" and "$L_2 - \epsilon$") satisfying

$$L_4 = (L_3 - \epsilon) = .5(L_1 - \epsilon) + .5(L_2 - \epsilon) \sim \$5.75.$$

Also, we can find some dollar prize strictly dispreferred to both of the $\$\epsilon$-modifications of L_1 and L_2, e.g., suppose

$$\$4.00 < (L_1 - \epsilon), (L_2 - \epsilon).$$

Thus, we have the sequence:

$$\$4.00 < (L_1 - \epsilon), (L_2 - \epsilon) < L_1 \sim L_2 \sim \$5.00 < L_4 \sim \$5.75 < L_3 \sim \$6.00$$

and, by assumption, these preferences respect stochastic dominance in dollar payoffs. So nonsequential choices among these options, according to these preferences, result in no incoherence.

Imagine, however, that an agent with these same preferences faces the following, two-stage sequential decision. Initially (at choice point A), the agent has two (sequential) alternatives, plans 1 and 2. Under sequential plan 1, a fair coin is flipped; (a) if it lands heads up, the agent chooses between L_1 and a dollar prize of \$5.50; and (b) if it lands tails up, the choice is between L_2 and the dollar prize of \$5.50. Under sequential option 2, the fair coin is flipped; (c) if it lands heads up, the agent chooses between $L_1 - \epsilon$ and \$4.00; and (d) if it lands tails up, the choice is between $L_2 - \epsilon$ and \$4.00. (The problem is depicted by Figure 4, where $L_1 \sim L_2 \sim \$5.00 < .5L_1 + .5L_2 \sim \$6.00$, and where one finds the $\$\epsilon$ fee satisfying: $.5(L_1 - \epsilon) + .5(L_2 - \epsilon) \sim \$5.75$.)

How is the agent to choose between plans 1 and 2? It is clear, I think, that he should face up to what he knows his preferences are at choice nodes B, the choices he faces after the coin is flipped.[7] That is,

7. Hammond's (1976, Section 3.3) felicitous phrase is that the agent uses "sophisticated" versus "myopic" choice.

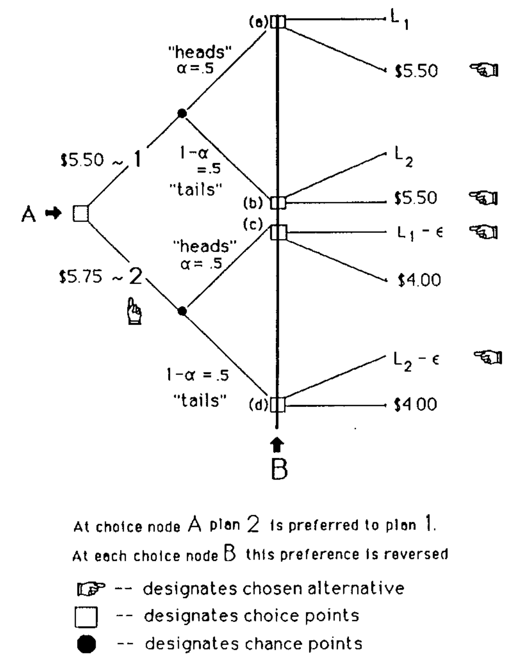


FIGURE 4. An illustration of sequential incoherence for a failure of mixture of dominance ("betweenness").

the agent should assess the two (sequential) plans 1 and 2 in light of what he knows they lead to.

Under (1), if the coin lands heads up (a) he will choose \$5.50 over lottery L_1.[8] And if the coin lands tails up (b) again, he will choose the \$5.50 (over L_2). Thus, from the standpoint of (A), choosing plan 1 leads to a sure payoff of \$5.50.

8. McClennen (1986, 1988, forthcoming) sketches a program of "resolute" choice to govern sequential decisions when independence fails. I am not very sure how resolute choice works. Part of my ignorance stems from my inability to find a satisfactory answer to several questions.

As I understand McClennen's notion of resolute choice, the agent's preferences for basic lotteries change across nodes in a sequential decision tree. (Then, the premises of the argument in Section 4 do not obtain.) In terms of the problem depicted in Figure 4, at node A the agent resolves that he will choose L_1 at (a) of node B, and by so resolving increases its value at (a) of node B above the \$5.50 alternative.

There are several difficulties I find with this proposal. I suspect that the new value of L_1 at (a) will be fixed at \$6.00, and likewise for L_2 at (b). (The details of resolute choice are lacking on this point, but this suspicion is based on the observation that a minor variation in the sequential incoherence argument applies unless these two lotteries change their value from node A to node B as indicated. Just modify the construction so that the rejected cash alternative at B is $\$6.00 - \delta$.) Then the assessed

Under (2), if the coin lands heads up (c) he will choose the lottery $L_1 - \epsilon$ over the dollar reward of \$4.00. And if the coin lands tails up (d), the lottery $L_2 - \epsilon$ is preferred to a sure \$4.00. Hence, from the standpoint of (A), choosing plan 2 leads to an equal ($\alpha = .5$) convex combination of the two lotteries $L_1 - \epsilon$ and $L_2 - \epsilon$. That is, from the perspective of choice node (A), sequential option 2 yields the lottery L_4, which is valued at \$5.75.

We make this reasoning precise with the following principle.

DYNAMIC FEASIBILITY (DF):

To assess plan p *at a choice node* n_i, *anticipate how you will choose at its (potential) "future" choice nodes* n_j *and declare infeasible all future alternatives under* p *which are inadmissible at* n_j.

By this account, according to the principle DF, at (A) the agent prefers plan 2 over the rival plan 1. At (A) plan 2 is worth \$5.75 where plan 1 is worth only \$5.50. However, there is an embarrassment to these preferences. At choice nodes B, regardless of the fall of the coin, the agent prefers the choice he makes under plan 1 to what he chooses under plan 2.

If the coin lands heads up, the choice under (1), at (a), \$5.50 is preferred to the choice under (2), at (c), the lottery $L_1 - \epsilon$. Likewise, if the coin lands tails up, the choice under (1), at (b), \$5.50 is preferred to the choice under (2), at (d), $L_2 - \epsilon$. Therefore, though the agent prefers plan 2 to plan 1 initially [at (A)], he knows that this preference is reversed at (B), regardless of how the coin lands.

Under the indifferences of the ordering postulate and the preferences induced by stochastic dominance, at nodes B, the upshot is a contradiction in assessment of the sequential decision problem. The contradiction obtains as follows.

By postulate (1), the agent has a weak ordering of options at choice nodes B. There are some sure-dollar equivalents for the choices $L_1 - \epsilon$ and $L_2 - \epsilon$. By simple dominance, the sure-dollar equivalent for $L_1 - \epsilon$ (or for $L_2 - \epsilon$) is less than that for L_1. That is, each of $L_1 - \epsilon$ and $L_2 - \epsilon$ is worth less than \$5.00 and more than \$4.00. Since admissibility at choice nodes (such as nodes B) respects indifference, one's anticipation and knowledge [at (A)] of one's choices is only up to the level of indifferents. That is, the ordering postulate fixes choices only up to the equivalences of indifference, ~. Thus, substitution of ~-indifferents at choice nodes B leaves unchanged choices at those nodes. But plan 2 is dominated by plan 1 when, as dictated by postulate (1), the choices at nodes (c) and (d) [of (B)] are switched for their (indifferent) sure-dollar equivalents.

Of course, given the replacements of $L_1 - \epsilon$ and $L_2 - \epsilon$ by their sure-dollar equivalents at (A) plan 1 is preferred to plan 2 by dominance. (See Figure 5, where, as in Figure 4, $L_1 \sim L_2 \sim \$5.00 < .5L_1 + .5L_2 \sim \$6.00$, and where one finds the \$$\epsilon$ fee satisfying: $.5(L_1 - \epsilon) + .5(L_2 - \epsilon) \sim \$5.75$.) Hence, subject to DF, applications of postulate (1) – ordering – at nodes B lead to contradictory conclusions with decisions made at node A.

Let us call this contradictory assessment an episode of *sequential incoherence*. That is, subject to Dynamic Feasibility, respect for stochastic dominance is not preserved under substitution of indifferent alternatives at choice nodes. That is, clause ii of coherence fails with this choice rule.

It is important to understand that, at (A) (in the problem depicted in Figure 4), the agent has *no* "terminal" options, no choices of lotteries. In particular, at choice node A, the agent does not have the terminal option L_3. Nor does he have any of the terminal options corresponding to the other seven lotteries that arise from an equal ($\alpha = .5$) convex combination of the options available to him at nodes B. Thus, at (A), he does not have the choice of \$5.50 outright. What he does have as a choice at (A) is plan 1, which, by DF, he equates with a certain \$5.50. But plan 1 calls for decisions at nodes B, depending upon how the coin lands, and these subsequent choices are not to be ignored at (A). The principle of Dynamic Feasibility achieves a limited reduction of plans to terminal options.

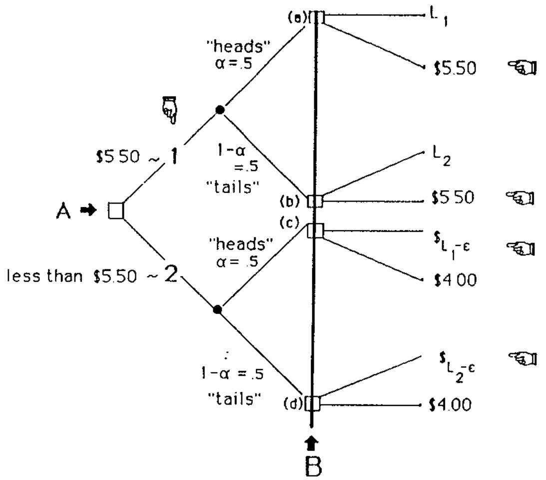
In the language of game theory (Luce and Raiffa, 1957, chapter 3), the sequential decision problem (above) is in *extensive* form. What we

value of \$6.00 for L_3 (a mixture of the lotteries L_1 and L_2, now valued at \$6.00 each) is in accord with postulate (2). Such resolutions mandate that changes in preferences agree, sequentially, with the independence postulate. In terms of *sequential* decisions, is it not the case that resolute choice requires changes in values to agree with the independence postulate?

A second problem with resolute choice directs attention at the reasonableness of these mandatory changes in values. For example, consider the Ellsberg-styled choice problem described in Section 3.2. Cast in a sequential form, under this interpretation of resolute choice, if L_{mix} is most preferred, then the agent is required to increase the value for the option "take the contents of the right pocket," given that the coin lands heads up, over the value it has prior to the coin flip.

But the coin flip is irrelevant to a judgment of where the money is. However uncertain the agent is prior to the coin flip, is he not just as uncertain afterwards? Concern with uncertainty in the location of the money is the alleged justification for a failure of independence when comparing the three terminal options: L_{left}, L_{right}, L_{mix}, and declaring L_{mix} (strictly) better than the other two. What justifies the preference shift, given the outcome of the coin flip, when L_{right} becomes equivalued with an even-odds lottery over \$10 and \$0 despite the same state of uncertainty about the location of the money before and after the coin flip?

Third, by what standards can the agent reassess the merits of a resolution made at an earlier time? In other words, how is the agent to determine whether or not to ignore an earlier judgment: the judgment to commit himself to a resolute future choice and thereby to change his future values. Once the future has arrived, why not instead frame the decision with the current choice node as the initial node without altering basic values? Unless this issue is addressed, the question of how to ratify a resolution is left unanswered, and the problem remains of how to make sense of the earlier resolution once the moment of choice is at hand.



At choice node A plan 1 is preferred to plan 2.

The tree results by replacing $L_i - \epsilon$ ($i = 1,2$) with \$-equivalents under $\lesssim$

☞ -- designates chosen alternative

□ -- designates choice points

● -- designates chance points

FIGURE 5. An illustration of sequential incoherence for a failure of mixture dominance ("betweenness").

learn from this problem is that, when mixture dominance fails, even with DF, sequential decisions in extensive form are not equivalent to the normal form one-stage (nonsequential) decisions that result by ignoring subsequent choice nodes like (*B*). In a normal form version of this sequential problem each of the two plans is represented by a set of four lotteries. In normal form, the decision is among the eight lotteries:

$$[L_3, (.5 \cdot \$5.50 + .5L_2), (.5L_1 + .5 \cdot \$5.50), \$5.50, \ldots, \$4.00].$$

Of these L_3 is most preferred, say. In normal form, then, plan 1 is chosen and is valued at \$6.00 ($\sim L_3$). However, the argument offered in this section (establishing sequential incoherence) does not presume the equivalence of extensive and normal forms.[9]

In the sequential problem, at node *A*, the agent knows L_3 is not available to him under plan 1. This is because he knows that (at nodes *B*) the dollar prize (\$5.50) is preferred to each of the lotteries L_1 and L_2.

9. By contrast, Raiffa's (1968, pp. 83–85) classic objection to the failure of independence in the Allais paradox depends upon a reduction of extensive to normal forms. Also, in his interesting discussion, Hammond (1984) requires the equivalence of extensive and normal forms through his postulate of "consequentialism" in decision trees. These

Under these preferences, the choice of plan 1 at (*A*) in the hope that L_1 will be chosen if heads and L_2 if tails is a pipe dream – mere wishful thinking that is brought up short by Dynamic Feasibility.[10]

4.3 Sequential Incoherence from Failures of Independence

What is the relation between failures of independence and episodes of sequential incoherence? An answer is given by the central critical result of this essay:

THEOREM:

If $\lesssim$ is a weak-order (1) of simple lotteries satisfying the Archimedean postulate (2) with sure-dollar equivalents for lotteries, and if $\lesssim$ respects stochastic dominance in payoffs ("a greater chance at more is better"), then a failure of independence, (3), entails an episode of sequential incoherence.

Proof:

The proof is given in two cases. (The argument for the second case uses the full assumption of stochastic dominance rather than the weaker assumption [used in Case 1] that $<$ respects simple dominance in dollar payoffs.)

Case 1:

Let $L_1 \lesssim L_2$, yet for some L_3 and α, $\alpha L_2 + (1-\alpha)L_3 < \alpha L_1 + (1-\alpha)L_3$. Let $\$X \sim \alpha L_2 + (1-\alpha)L_3$, $\$Z \sim \alpha L_1 + (1-\alpha)L_3$, and $\$U \sim L_3$, with $X < Z$. By our assumptions of a weak order for preference, continuity in dollar payoffs, and the strict preference for more (over less) money, there is some $\$2\epsilon$ fee and amount $\$Y$ for which:

$$\$X < \alpha(L_1 - 2\epsilon) + (1-\alpha)(L_3 - 2\epsilon) \sim \$Y < \$Z.$$

authors defend a strict expected-utility theory, in which the equivalence obtains. Likewise, LaValle and Wapman (1986) argue for the independence postulate with the aid of the assumption that extensive and normal forms are equivalent.

The analysis offered here does not presume this equivalence, nor does avoidance of sequential incoherence entail this equivalence, since, e.g., it is not satisfied in Levi's theory either – though his theory avoids such incoherence. Hence, for the purpose of separating decision theories without independence from those without ordering, it is critical to avoid equating extensive and normal forms of decision problems.

10. One may propose that, by force of will, an agent can introduce new terminal options at an initial choice node, corresponding to the "normal" form version of a sequential decision given in "extensive" form. Thus, for the problem depicted in Figure 4, the assumption is that the agent can create the terminal option L_3 at node *A* by opting for plan 1 at *A* and then choosing L_1 at (a) and L_2 at (b).

Whatever the merit of this proposal, it does not apply to the sequential decisions discussed here, since, by stipulation, the agent cannot avoid reconsideration at nodes *B*. There may be some problems in which agents can create new terminal options at will, but that is not a luxury we freely enjoy. Sometimes we have desirable terminal options and sometimes we can only plan. (See Levi's [1980, Chapter 17] interesting account of "using data as input" for more on this subject.)

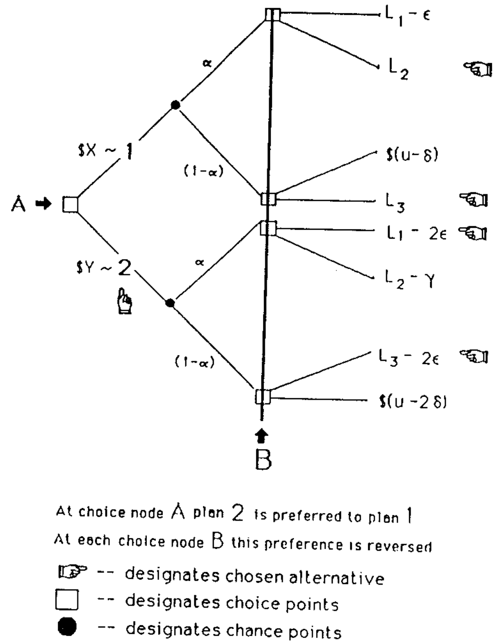


FIGURE 6. Sequential incoherence for failures of "independence": Case 1.

Next, choose fees $\$\gamma$ and $\$\delta$ so that:

$$L_2 - \gamma < L_1 - 2\epsilon \text{ and } \$(U - 2\delta) < L_3 - 2\epsilon.$$

If we consider the sequential decision problem whose "tree" is depicted in Figure 6 for Case 1, we discover by the same reasoning we used in the example above:

At node *A*, plan 1 is valued at $\$X$, whereas plan 2 is valued at $\$Y$. Thus, at node *A*, plan 2 is the preferred choice.

But at nodes *B*, regardless of which "chance event" occurs, the favored option under plan 1 is preferred to the favored option under plan 2. Thus, the preferences leading to a failure of independence in Case 1 succumb to sequential incoherence. An application of indifferences (from the ordering postulate 1 at nodes *B* leads to an inconsistent evaluation, at (*A*), of the sequential plans 1 and 2.

Case 2:

$L_1 < L_2$, yet there are L_3 and $\alpha > 0$ with $\alpha L_1 + (1 - \alpha)L_3 \sim \alpha L_2 + (1 - \alpha)L_3$. Let $\$U \sim L_3$ and $\$Z \sim \alpha L_1 + (1 - \alpha)L_3$. Then choose an $\$\epsilon$ fee so that $L_1 < L_2 - \epsilon$. Let $\$X \sim \alpha(L_2 - \epsilon) + (1 - \alpha)L_3$. Then by stochastic dominance, $X < Z$. Next, by continuity, choose a $\$\delta$ fee to satisfy $\$Y \sim \alpha(L_1 - \delta) + (1 - \alpha)(L_3 - \delta)$, where $X < Y < Z$. Choose an integer n so that $L_2 - n\epsilon < L_1 - \delta$. Finally, find any $\$\gamma$ fee and choose an integer m so that $\$(u - m\gamma) < L_3 - \delta$.

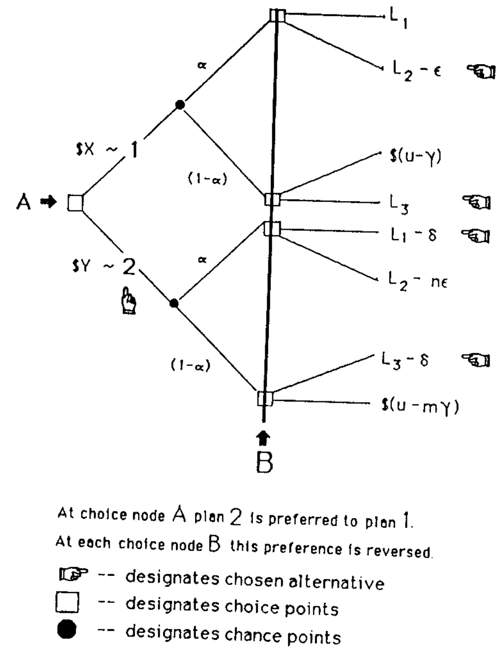


FIGURE 7. Sequential incoherence for failures of "independence": Case 2.

Consider a sequential decision problem whose "tree" is depicted in Figure 7 for Case 2. Once again we find an episode of sequential incoherence as:

> At (*A*), plan 1 is valued at $\$X$, whereas plan 2 is valued at $\$Y$. Thus, at node *A* plan 2 is the preferred choice.
> At node *B*, regardless of which chance event occurs, the favored option under plan 1 is preferred to the favored option under plan 2. Thus, the preferences leading to a failure of independence in Case 2 succumb to sequential incoherence.

4.4 Concluding Remark

Can familiar "Dutch Book" arguments (de Finetti, 1974; Shimony, 1955) be used to duplicate the results obtained here? Do the considerations of book establish sequential incoherence when independence fails? I think they do not.

The book arguments require an assumption that the concatenation (conjunction) of favorable or indifferent or unfavorable gambles is, again, a favorable or indifferent or unfavorable gamble. That is, the book arguments presume payoffs with a simple, additive, utility-like structure. The existence of such commodities does not follow from a (subjective) expected-utility theory, like Savage's. And rivals to EU, such as Samuelson's (1950) "Ysidro" ranking, can fail to satisfy this assumption though they respect stochastic dominance in lotteries. Thus, in light of this assumption about combinations of bets, the Dutch Book argument is not neutral to the dispute over coherence of preference when independence fails. (Of course, that debate is not what Dutch Book arguments are designed for.)[11]

This objection to the use of a book argument does not apply to the analysis presented here. The argument for sequential incoherence is not predicated on the Dutch Book premise about concatenations of favorable gambles. That assumption is replaced by a weaker one, to wit: $\lesssim$ respects stochastic dominance in \$-rewards. There is no mystery why the weakening is possible. Here, we avoid the central question addressed by the Dutch Book argument: When are betting odds subjective probabilities? The book arguments pursue the representation of coherent belief as probabilities, given a particular valuation for combinations of payoffs. Instead, the spotlight here is placed on the notion of coherent sequential preference, given a preference (a weak ordering) of lotteries with canonical probabilities.

4.5 Summary

Attempts to generalize EU by denying independence, while retaining the ordering and Archimedean postulates, fail the test of coherence in simple sequential choices over lotteries with dollar rewards.

5. SEQUENTIAL COHERENCE OF LEVI'S DECISION THEORY

A detailed analysis of Levi's Decision Theory (LDT), a theory without the ordering postulate, is beyond the scope of this essay. Here, instead, I shall merely report the central results that establish coherence of LDT in sequential choices over horse lotteries, a setting where both values (utility/security) and beliefs (probability) may be indeterminate. (I give proofs of these results in a technical report [1987].)

To begin with, consider the following choice-based generalizations of the concepts: indifference, preference, and stochastic dominance.

11. See Frederic Schick's "Dutch Bookies and Money Pumps" (1986) for discussion of the import of this concatenation assumption in the Dutch Book argument. Its abuse in certain "intertemporal" versions of Dutch Book is discussed in Levi (1987).

These generalizations are intended to apply in the domain of horse-lottery options, regardless of whether or not a decision rule induces a weak-ordering of acts.

The notational abbreviations I use are these. An option is denoted by o_i and, since acts are functions from states to outcomes, also by the function on states $o_i(s)$. Sets of feasible options are denoted by O, and the admissible options (according to a decision rule) from a feasible set O are denoted by the function C[O].

Call two options $\approx$-*indifferent*, if and only if, whenever both are available either both are admissible or neither is.

DEFINITION:

$o_1 \approx o_2 \Leftrightarrow \forall(O)\ (\{o_1, o_2\} \subset O \Rightarrow (o_1 \in C[O \Leftrightarrow o_2 \in C[O]))$.

When a choice rule induces a weak-order, denoted by $\lesssim$, then $\approx$ is just the symmetrized $\sim$ relation: $(o_1 \sim o_2) \Leftrightarrow (o_1 \lesssim o_2)$ *and* $(o_2 \lesssim o_1)$.

Next, define a choice-based relation of categorical preference over rewards using a restricted version of Anscombe and Aumann's (1963, p. 201) "Assumption 1," modified to avoid the ordering postulate. [This assumption is part of the value-neutrality of states. See Drèze's (1985) monograph for a helpful discussion of related issues.] Let o_1 and o_2 be two horse lotteries that differ only in that o_2 awards reward r_2 on states where o_1 awards reward r_1. Then reward r_2 is categorically preferred to reward r_1 just in case o_1 is inadmissible whenever o_2 is available. In symbols,

DEFINITION:

Reward r_2 *is* categorically preferred *to reward* $r_1 \Leftrightarrow \forall(O)\ \forall(o_1 \neq o_2)\ (\forall(s)[o_1(s) = o_2(s) \vee (o_1(s) = r_1\ \&\ o_2(s) = r_2)]\ \&\ o_2 \in O \Rightarrow o_1 \notin C[O])$.

Third, we need to make precise a suitable generalization of stochastic dominance among lotteries. Recall, when there is an ordering ($\lesssim$) of rewards, lottery L_2 stochastically dominates lottery L_1 if L_2 can be obtained from L_1 by shifting some distribution mass from less to more preferred rewards. Thus, L_2 stochastically dominates L_1 just in case $\alpha L_2 + (1 - \alpha)L_3$ stochastically dominates $\alpha L_1 + (1 - \alpha)L_3$ $(0 < \alpha \leq 1)$. Rely on this biconditional to formulate the following $<$-*dominance* relation over horse lotteries, defined for an arbitrary choice function C.

DEFINITION:

$o_1 < o_2 \Leftrightarrow \forall(O)\forall(o)\forall(\alpha > 0)\ ((\alpha o_2 + (1 - \alpha)o) \in O \Rightarrow (\alpha o_1 + (1 - \alpha)o) \notin C[O])$. Trivially, if o_2 $<$-*dominates* o_1, *then (let* $\alpha = 1$*)* o_1 *is inadmissible whenever* o_2 *is available.*

Sequential coherence for a decision rule requires, then, that

i – shifting distributional mass from categorically less to categorically more preferred rewards produces an $<$-dominating option, and
ii – the inadmissibility of $<$-dominated options is preserved under substitution (at choice nodes) of $\approx$-indifferents.

To see why LDT is sequentially coherent, recall that admissibility in Levi's (1974, 1980) decision theory is determined by a two-tiered lexicographic rule. The first tier is "*E*-admissibility." An option is *E*-admissible in the set O of feasible options, provided it maximizes expected utility (over O) for some pair (P,U) – where P is a personal probability in the (convex) set **P** that represents the agent's beliefs about the states, and where U is a (cardinal) utility in the (convex) set **U** that represents one aspect of the agent's values over the rewards.

DEFINITION:
o *is* E-*admissible* $\Leftrightarrow \exists(P,U)\ \forall(o' \in O)\ E_{PU}[o] \geq E_{PU}[o']$.[12]

The second tier in admissibility requires that an option be "*S*-admissible." This condition demands of an option that it be *E*-admissible and maximize a "security" index among the *E*-admissible options. The security of an option reflects yet another aspect of the agent's value structure. For purposes of this section, the notion of security is illustrated with three (real-valued) varieties:

$sec_O[o] = O$ – a vacuous standard, where all options have equal security;

$sec_1[o] = \text{infemum}_{U,R_o} U[r]$, where $\mathbf{R}_o$ is the set of possible outcomes (rewards) for option o and $r \in \mathbf{R}_o$. Thus, sec_1 is security indexed by the "worst" possible reward, a maximin consideration;

$sec_2[o] = \text{infemum}_{P \times U} E_{P,U}[o]$ – security indexed by the least expected utility, also called the "Γ-minimax" level for option o. (I avoid the minor details of defining sec_2 when events have probability O and expectations are determined by rule†, as reported in note 12.)

Thus, an option is admissible in LDT just in case it is both *E*-admissible and maximizes security among those that likewise are *E*-admissible. As a special case, when security is vacuous or is indexed by sec_2, and when both **P** and **U** are unit sets, $C[O]$ is the subset of options that maximize subjective expected utility: the strict Bayesian framework. Then admissibility satisfies the ordering and independence postulates (and the Archimedean axiom, too, provided events have positive probability or rule† is not used).

12. Levi (1980, Section 5.6) offers a novel rule, here called rule†, for determining expectation-inequalities when the partition of states, π, is finite but when events may have subjective probability 0. The motive for this emendation is to extend the applicability of "called-off" bets (Shimony, 1955) to include a definition of conditional probability given an event of (unconditional) probability 0. Also, it extends the range of cases where a weak-dominance relation determines a strict preference.

Given a probability/utility pair (P,U), maximizing †-expected utility (with rule†) includes a weak-order that satisfies the independence axiom, though, †-expectations may fail to admit a real-valued representation, i.e., the "Archimedean" axiom is not then valid. Under rule†, given a pair (P,U), †-expectations are represented by a lexicographic ordering of a vector-valued quantity.

The next few results, stated without proof, provide the core of the argument for sequential coherence of LDT. Condition i of coherence in nonsequential decisions, and more, is shown by the following.

THEOREM 5.1:
If o_2 *can be obtained from* o_1 *by shifting distribution masses from categorically less to more preferred rewards, then* $o_1 < o_2$, *and thus* o_1 *is inadmissible whenever* o_2 *is available.*

The theorem follows directly from a useful lemma about < and categorical preference in LDT.

LEMMA:
$o_1 < o_2 \Leftrightarrow \forall(P,U) E_{PU}(o_1) < E_{PU}(o_2)$. *Thus,* r_2 *is categorically preferred to* r_1 $\Leftrightarrow \forall(U)\ U(r_1) < U(r_2)$.

Thus, both <-dominance and categorical preference are strict partial orders, being irreflexive and transitive. In addition, following obviously from its definition, <-dominance satisfies the independence axiom. [Note the term "categorical preference" is borrowed from Levi (1986c, p. 91). The lemma provides the collateral for this conceptual loan.]

Condition ii of coherence for LDT in nonsequential decisions is shown by a result that two options are ≈-related exactly when they have the same security index and are indiscernible by expectations:

THEOREM 5.2:
$o_1 \approx o_2 \Leftrightarrow (sec[o_1] = sec[o_2]\ \&\ \forall(P,U)(E_{PU}[o_1] = E_{PU}[o_2])$. *Thus,* ≈ *is an equivalence relation and, in nonsequential decisions, admissibility is preserved under substitution of* ≈*-indifferent options.*

In order to extend the analysis to sequential decisions, the notion of ≈-indifference is generalized to include conditional assessments, conditional upon the occurrence of either "chance" or "event" outcomes. The next corollary is elementary and indicates how conditional ≈-indifference applies when for instance, choice nodes follow chance nodes.

COROLLARY 5.1:
≈ *is preserved under chance mixtures,*

$$\forall(i,j,k \text{ and } \alpha)\ o_i \approx o_j \Rightarrow (\alpha o_i + (1-\alpha)o_k \approx \alpha o_j + (1-\alpha)o_k).$$

Under the principle of Dynamic Feasibility (which provides a limited reduction of sequential plans to nonsequential options), these findings combine to support the desired conclusion.

Sequential coherence for LDT

i – Admissible options are <-undominated among the dynamically feasible alternatives, and
ii – Provided the agent updates his beliefs by Bayes' rule and does

not change his value structure, admissibility is preserved under the substitution of ≈-indifferent alternatives at choice nodes.

6. CONCLUSIONS ABOUT RELAXING INDEPENDENCE OR ORDERING

I have argued that decision theories that relax only the independence postulate succumb to sequential incoherence. That is, such programs face the embarrassment of choosing stochastically dominated options when, in simple two-stage sequential decisions, dollar equivalents are substituted for their indifferent options at terminal choice nodes. Moreover, the criticism does *not* presume an equivalence between sequential decisions (in extensive form) and their normal form reductions; instead, all decisions are subject to a principle of Dynamic Feasibility.

In Section 5, I generalize sequential coherence to choice rules that may not induce an ordering of options by preference. Also, I outline reasons for the claim that Levi's Decision Theory is sequentially coherent (in a setting where both belief and value are subject to indeterminacy). Since Levi's theory is one that fails the ordering postulate, the combined results establish a demarcation between these two strategies for relaxing traditional (subjective) expected-utility theory. The difference is that only one of the two approaches is sequentially coherent.

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DISCUSSIONS

EDITORS' NOTE

Subjective expected-utility theory provides simple and powerful guidance concerning how to make rational decisions in circumstances involving risk. Yet actual decision making often fails, as has been well known for decades, to conform to the theory's recommendations. If subjective expected-utility theory represents the ideal of rational behavior, these failures may simply show that people often behave irrationally. Yet if the gap between ideal and actual behavior is too wide, or if behavior that on the best analysis we can make is rational but not consistent with subjective expected-utility theory, then we may come to doubt some of the axioms of the theory. Two main lines of revision have been suggested: either weakening the "ordering" axiom that requires preferences to be complete or surrendering the so-called independence principle. Although the issues are highly abstract and somewhat technical, the stakes are high; subjective expected-utility theory is critical to contemporary economic thought concerning rational conduct in public as well as private affairs.

In the preceding article, "Decision Theory without 'Independence' or without 'Ordering': What Is the Difference?" Teddy Seidenfeld argued for the sacrifice of ordering rather than independence by attempting to show that abandoning the latter leads to a kind of sequential incoherence in decision making that will not result from one specific proposal (Isaac Levi's) for abandoning ordering. In their comments in this section, Edward McClennen, who supports surrendering the independence postulate rather than ordering, and Peter Hammond, who argues against any weakening of subjective expected-utility theory, discuss Seidenfeld's argument from their quite different theoretical perspectives.

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ORDERLY DECISION THEORY

A Comment on Professor Seidenfeld

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1. DEPENDENCE AND ESSENTIAL INCONSISTENCY

The preceding paper by Seidenfeld (1988) is a valid defense of the following independence axiom originally formulated by Samuelson (1952). For all finite lotteries L_i, L_j and L_k and all α satisfying $0 < \alpha \leq 1$, the "weak dispreference" relation $\preccurlyeq$ satisfies

$$L_i \preccurlyeq L_j \Longleftrightarrow \alpha\, L_i + (1 - \alpha)\, L_k \preccurlyeq \alpha\, L_j + (1 - \alpha)\, L_k.$$

As Seidenfeld states, violations of this independence axiom can be of two kinds:

$$L_1 \preccurlyeq L_2 \text{ yet } \alpha L_1 + (1 - \alpha)L_3 \succ \alpha L_2 + (1 - \alpha)L_3; \qquad (1)$$

$$L_1 \prec L_2 \text{ yet } \alpha L_1 + (1 - \alpha)L_3 \succcurlyeq \alpha L_2 + (1 - \alpha)L_3. \qquad (2)$$

Here $\prec$ denotes the strict dispreference relation, $\succ$ the strict preference relation, and $\succcurlyeq$ the weak preference relation. After increasing the prizes in L_2 very slightly in Case 1, or decreasing them in Case 2, continuity of preferences leads to a *strict violation of independence*, with:

$$L_1 \prec L_2 \quad \text{yet} \quad \alpha L_1 + (1 - \alpha)L_3 \succ \alpha L_2 + (1 - \alpha)L_3.$$

Now it is not unreasonable to suppose that there is another lottery L_0 such that $\alpha L_1 + (1 - \alpha)L_3 \succ L_0 \succ \alpha L_2 + (1 - \alpha)L_3$. For example, L_0 may be $\alpha L_1' + (1 - \alpha)L_3$ where the prizes in L_1' are slightly less than

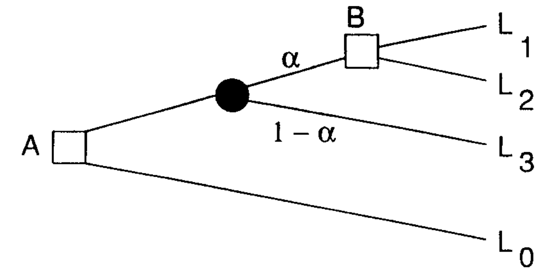

FIGURE 1.

those in L_1, or it may be $\alpha L_2' + (1 - \alpha)L_3$ where the prizes in L_2' are slightly greater than those in L_2. Either way, we have:

$$L_1 \prec L_2 \quad \text{yet} \quad \alpha L_1 + (1 - \alpha)L_3 \succ L_0 \succ \alpha L_2 + (1 - \alpha)L_3.$$

This construction has yielded an *essential inconsistency*, to use the terminology of Hammond (1976) in a slightly different context. For consider the decision tree illustrated in Figure 1.

As Strotz (1956) and Pollak (1968) have pointed out for related decision problems under certainty, there are two ways of behaving in this tree. A *naive* person at A evaluates the following three strategies and their consequent lotteries:

(a) go to the chance node, then choose L_1 if B is reached, getting lottery

$$\alpha L_1 + (1 - \alpha)L_3;$$

(b) go to the chance node, then choose L_2 if B is reached, getting lottery

$$\alpha L_2 + (1 - \alpha)L_3;$$

(c) go to L_0 directly, getting lottery L_0.

Evidently the preferences imply that (a) is preferred to (c), which is preferred to (b). So the naive agent plans (a) and goes to the chance node. If the result is lottery L_3 there is no problem. But if the agent finds himself at B, the choice is between L_1 and L_2. Since the agent prefers L_2 to L_1 he selects L_2. The result is $\alpha L_2 + (1 - \alpha)L_3$, the *worst* of the three outcomes available at A.

A *sophisticated* agent, on the other hand, realizes that if he reaches B he will choose L_2. That makes strategy (a) unavailable, so, as in the

"classical example" when Odysseus heeded the goddess Circe's advice and took precautions against the Sirens (cf. Strotz, 1956), the sophisticated agent realizes that his only real choice is between (b), with consequent lottery $\alpha L_2 + (1 - \alpha)L_3$, and (c) with consequent lottery L_0. Thus he chooses (c) and goes directly to get L_0.

This shows that Seidenfeld's "sequential incoherence" is indeed a troubling phenomenon. In the example of Figure 1, the naive agent's plan to choose L_1 if B is reached will not actually be carried out. So the agent has to be prepared for the risk that his future behavior will depart from his plans. This risk is usually present unless the independence axiom is satisfied.

Violations of the independence axiom need to be interpreted carefully, however. They may indeed be real violations that could even lead to essential inconsistencies. Or they may be only apparent violations that would disappear if everything of relevance to decision making were included in the description of each random consequence. This I have discussed elsewhere (Hammond, 1988).

2. ADMISSIBILITY AND INCONSISTENCY

After successfully defending independence, Seidenfeld argues in his section 5 that the ordering axiom is not required to ensure sequential coherence. This can be shown, of course, by providing an example of a decision criterion that is not a preference ordering but is sequentially coherent. The particular criterion presented is due to Levi (1974, 1980).

It is true that Levi's criterion leads neither to that kind of dynamic preference reversal which Seidenfeld calls "sequentially incoherent," nor to an essential inconsistency of the kind discussed in Section 1. Nevertheless, it is generally inconsistent in a weaker sense that is also rather troubling. This will be illustrated with an example. The argument that inconsistencies of this weaker kind can be avoided only by having a preference ordering is given in Hammond (1977, 1988).

Suppose that the set **U** is a unique cardinal equivalence class of von Neumann–Morgenstern utility functions, but that **P** consists of all probability distributions on the pair $\{s',s''\}$ of states of the world. Let there be four acts a, b, c, d that, for a representative utility function $U \in \mathbf{U}$, yield utility levels that depend upon the states s' and s'' as indicated in Table 1. With these assumptions, Levi's E-admissibility criterion is equivalent to being undominated. So only a and c are E-admissible in the set $\{a, b, c, d\}$ because b is strictly dominated by c and d is strictly dominated by a.

Consider next the decision tree in Figure 2.

At A, where the feasible set is $\{a, b, c, d\}$, both a and c are E-admissible, so both moves to B and C are acceptable. But at B, where the new feasible set is $\{a, b\}$, *both* a and b are E-admissible. This is because c,

Table 1.

Act	s'	s''
a	3	1
b	0	2
c	1	3
d	1	0

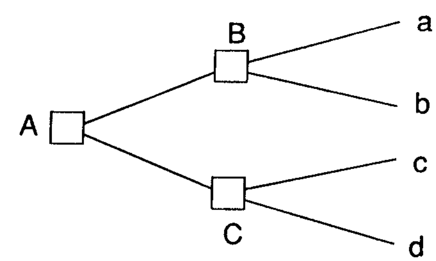

FIGURE 2.

which used to dominate b, is no longer available. Similarly both c and d are E-admissible at C. So there is a real danger that an agent who uses Levi's criterion naively will be inconsistent and choose b or d, even though neither is E-admissible at A, and both are dominated by other acts. The example, of course, is just like that used to discuss the "inconsequentialism" of the Pareto rule in Hammond (1986).

This inconsistency is not quite so devastating as the earlier fundamental inconsistencies. After all, somebody who was just a little "resolute" (cf. McClennen, 1986, 1988, 1989) could remember not to choose b at B, where the feasible set is $\{a, b\}$, or d at C, where the feasible set is $\{c, d\}$. But then we should start wondering what this resolute person should be remembering not to choose at A, where the feasible set is $\{a, b, c, d\}$. For example, there may have been an earlier opportunity, before A is reached, to have chosen between the two alternatives of either going to A or of performing a fifth act e that yields immediate utility levels 2 and 4 in the two states of the world s' and s'' respectively. Then, in the set $\{a, b, c, d, e\}$, only the two acts a and e are E-admissible. So the agent must be presumed to reach node A with the resolution to choose a. If so, the act c, which is E-admissible in the set $\{a, b, c, d\}$, really is not acceptable any more. Thus, behavior has to depend upon past resolutions, and these have to be added to our description of the decision problem – the usual decision tree formulation is only part of the complete description.

To me, the obvious way of allowing for such resolutions is to include them among the consequences of behavior in any decision tree, allowing

as well for the fact that behavior either has or has not been in accord with past resolutions. Such "resolution consequences" extend those that we are more accustomed to considering. Yet if resolution consequences are included among the relevant consequences, the arguments I have presented in Hammond (1988) lead us fairly directly to a preference ordering over probability distributions of such extended consequences, and to the independence axiom. Only continuity need then be added to reach expected utility once again, at least for objective probabilities. The only novelty will be the inclusion of resolution consequences as arguments of the von Neumann–Morgenstern utility function.

3. CONCLUSION

So violations of the independence axiom usually lead to especially damaging "fundamental inconsistencies" in certain decision trees. Whereas decision criteria such as Levi's that are not based upon a preference ordering may lead to rather weaker inconsistencies, at worst. But to avoid dynamic inconsistencies altogether, while retaining the basic postulate of decision theory that acts should be judged only by their consequences, it is necessary to maximize a preference ordering over random consequences that satisfies the independence axiom. As I see it, then, the most natural justification of either axiom – ordering or independence – is a justification of the other as well, so the two stand or fall together. Moreover, with a suitably liberal interpretation of the concept of consequence, enriched to include everything that is or should be of relevance to decision making, neither axiom seems restrictive. Nevertheless, to the extent that essential inconsistency (or sequential incoherence) is clearly even more troubling than (weak) inconsistency, Seidenfeld is right – the independence axiom is even easier to justify than the ordering axiom.

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ORDERING AND INDEPENDENCE

A Comment on Professor Seidenfeld

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1. INTRODUCTION

In "Decision Theory without 'Independence' or without 'Ordering': What Is the Difference?" Seidenfeld (1988) makes three claims with which I must take issue. The first is that those who adopt a "sophisticated" approach to sequential choice and who are prepared to relax the strong independence axiom will find themselves in certain cases making seriously "incoherent" choices. The second is that those who relax certain "consistency" conditions on choice functions – such as Alpha and Beta – do not face a parallel problem. The third is that the "resolute" approach to sequential choice, which I have recently advocated, and which would incidentally provide an escape from the "problem" he has posed for would-be violators of the independence axiom, is not open to a rational agent.

With regard to the first of these claims, I believe that the problem he poses for would-be violators of the independence condition is imaginary. As to the second, I think that if he were to succeed in establishing the first claim, it would be possible to show that those who, like Isaac Levi, are prepared to violate the consistency conditions, will also end up making "incoherent" choices. And with regard to the third thesis, my own view is that a resolute choice is not only open to any agent who is fully rational, but is mandated for anyone who is committed to maximizing with respect to certain continuing preferences.

I shall not argue here for the last of these claims. Its defense would require considerably more than the space here allotted to me. The interested reader is invited to consult McClennen (1986, 1988, 1989), where I have sought to explain and defend the notion of a resolute approach to sequential choice problems.

2. INTERPRETING SEIDENFELD'S FIRST CLAIM

The essence of the first issue between Seidenfeld and myself can be captured by considering the argument he constructs against those who would violate the mixture dominance (MD) version of the independence axiom. As he tells the story, the problem that arises for one who wants to relax MD turns on there being a difference between the assessment of plans when, as given in his Figure 4, plan 2 terminates in the lotteries $L_1 - e$ and $L_2 - e$, and when plan 2 is modified into plan 2*, which terminates in the sure-dollar equivalences of $L_1 - e$ and $L_2 - e$, designated by "$\$(L_1 - e)$" and "$\$(L_2 - e)$". In the problem as presented in Figure 4, plan 2 is preferred to plan 1. In the altered problem, his Figure 5, in which plan 2 is replaced with plan 2*, plan 1 is the preferred plan. This, we are informed, means that clause (ii) of what he characterizes as a "coherence" condition will be violated. The coherence condition in question reads as follows:

> *Coherence:* (i) admissible choices under the rule are stochastically undominated; and (ii) admissibility is preserved under substitution (at choice points) of "indifferent" options. (p. 275)

Now, my problem begins with uncertainty as to what Seidenfeld takes clause (ii) of his "coherence" condition to imply *in a dynamic choice context*. In his gloss on this condition, Seidenfeld suggests that choice rules do not have to induce orderings but that when they do so with regard to "terminal decisions" (by which, I presume, he means static choices), condition (ii) adds nothing to condition (i) (p. 275). This appears to speak to the proper interpretation of clause (ii) in static (nondynamic) settings. Seidenfeld's point here, I take it, is that insofar as static alternatives are preferentially comparable, and some lottery L_1 is judged inadmissible by reference to some other lottery L_2, which stochastically dominates it, then anything else that is judged indifferent to (the rejected) option L_1 will also be judged inadmissible. That is, the assumption that the agent has an ordering over various alternatives does the work. As the rest of the argument makes clear, however, the case at issue between us is one involving a sequences of choices to be made. In that context, clause (ii) is acknowledged to add something to clause (i). But what?

By way of trying to understand the import of clause (ii) in a dynamic context, it will prove useful, I think, to take the sense of certain other features of his argument. Looking over the argument as a whole, some things do seem clear. I take it that Seidenfeld invokes (if only implicitly) an important principle when he remarks that "substitution of ~-indifferents at choice nodes (B) leaves unchanged choices at those nodes" (p. 279). Call this principle, REPLACE. Applied to the problem at hand,

REPLACE requires that if one were to substitute the dollar equivalent of the best-option lottery at a choice node, that should not alter the choice to be made at that node, i.e., the dollar equivalent will now be the best option. Thus if $L_1 - e$ is replaced with $\$(L_1 - e)$ at (c), and $L_2 - e$ is replaced with $\$(L_2 - e)$ at (d), in Figure 4, then $\$(L_1 - e)$ will be chosen at (c) and $\$(L_2 - e)$ will be chosen at (d). On the basis of my theories about the possibility of resolute choice, I think this principle must be rejected. I am prepared for the moment to grant Seidenfeld this step, however, for I think it is true that the kind of agent he describes – one who adopts a sophisticated approach to sequential choice – will accept this principle.[1]

Seidenfeld also invokes (although again only implicitly) what can be characterized as a best-option valuation principle (BOV). This requires that the value of a choice node (*qua* opportunity to make a certain choice) be treated as equal to the value of a best option available there. I have no quarrel with this principle.

However, these two principles, REPLACE and BOV, as tacitly employed by Seidenfeld, speak only to the manner in which the agent must view choice at the "B" nodes – nodes (a) through (d). What can be said about how the agent must proceed from the perspective of the "A" node? As Seidenfeld correctly points out, conclusions can be drawn about the ordering of certain of the prospects in the altered version of the problem, involving plan 2*, and hence, by implication, about the ordering of the plans that access those prospects. The prospects are: L_6 = (\$5.50, .5; \$5.50, .5) = \$5.50 – that is the prospect the agent faces if he adopts plan 1; and L_5 = ($\$(L_1 - e)$, .5; $\$(L_2 - e)$, .5) – this is the prospect he faces if he adopts plan 2. By hypothesis, \$5.50 is strictly greater than both $\$(L_1 - e)$ and $\$(L_2 - e)$, and hence L_5 itself is just an even-chance lottery over two smaller amounts of money. By straightforward appeal to clause (i) of his coherence condition – First Order Stochastic Dominance (FOSD) – the agent must strictly prefer \$5.50 to L_5, and, correspondingly, strictly prefer plan 1 to plan 2*.

Having carried the argument this far, however, Seidenfeld is still one step away from his desired conclusion. To be sure, the principles to which he has implicitly or explicitly appealed, as noted above, establish that the agent must strictly prefer plan 1 to plan 2*, and it is also clear that plan 2 can be transformed into plan 2*, by substitution of "indifferents." But the fact that one can get from the former to the latter by such a substitution does not yet establish that plan 2 must be indifferent to plan 2*. And that, it would seem, is what Seidenfeld must show, if his argument is to go through.

1. A resolute chooser will not accept the principle REPLACE. A substitution at some subsequent choice point can make some overall plan unattractive, and this, in turn, can have implications, within a resolute choice perspective, for the choices one will make at those subsequent nodes. See footnote 4 on page 273.

Of course, the argument as reconstructed up to this point has yet to make appeal to clause (ii). It is plausible to suppose, then, that it is clause (ii), as it applies to dynamic choice situations, that requires that L_4 be indifferent to L_5, and correspondingly, that plan 2 be indifferent to plan 2*.[2] But what is it about clause (ii), as it applies to dynamic choice situations, that requires this conclusion? Seidenfeld originally introduces his coherence condition by remarking that he proposes to "elevate respect for stochastic dominance to the status of a coherence condition" (p. 275). Clause (i), to be sure, does clearly formulate a stochastic dominance condition, but clause (ii) speaks to something else. To repeat: the principle of stochastic dominance surely does require that \$5.50 must be preferred to L_5; but it offers no basis for the conclusion that L_6 must be indifferent to L_4.

I suggest that if Seidenfeld's argument is to go through, we must suppose that clause (ii) invokes the following distinct principle:

> *Dynamic Substitution* (DYN-SUB): In a dynamic choice context, substitution of component prospects with other prospects indifferent to them preserves indifference with respect to plans.

Such a principle, as its name is intended to suggest, constitutes a dynamic choice version of the standard substitution principle (SUB).[3] To see this, consider the variation on Seidenfeld's example in Figure 1. In this instance, the agent has only a single choice up front – all subsequent moves are by nature, and as before $\$(L_1 - e)$ and $\$(L_2 - e)$ designate the dollar-equivalences for the lotteries $L_1 - e$ and $L_2 - e$, respectively. DYN-SUB does not apply to this problem, but SUB does. That is, if the agent who confronts this problem is committed to SUB, he must rank plan 3 and plan 4 indifferently, since each can be derived from the other by substitution of indifferent components. However, with a simple modification of this problem, one can capture the import of DYN-SUB. Suppose, for example, that after nature makes its move, the agent then has a second choice to make (see Figure 2). Here \$4 is dispreferred to any of the other prospects. DYN-SUB now applies, and what it requires is that the agent must rank plan 3* indifferently to plan 4*.

2. To be sure, if one could show that L_4 must be indifferent to L_5, by appeal to some independent principle, one could then reach the desired conclusion by appeal to clause (ii), as it is intended to apply to nonsequential choices. More specifically, since \$5.50 first-order stochastically dominates L_5, if there were an independent way to show that L_4 must be indifferent to L_5, one could then conclude, by clause (ii), as it applied to nonsequential choices, that \$5.50 must be preferred to L_4. But it seems most doubtful that this is the route that Seidenfeld intends his argument to take. Were such reasoning followed, there would be no need to invoke the implications of clause (ii) for dynamic choice situations. Seidenfeld writes, however, as if he supposes that the argument hinges on an appeal to clause (ii) as it applies to sequential choice.
3. See, for example, Luce and Raiffa (1957, p. 27).

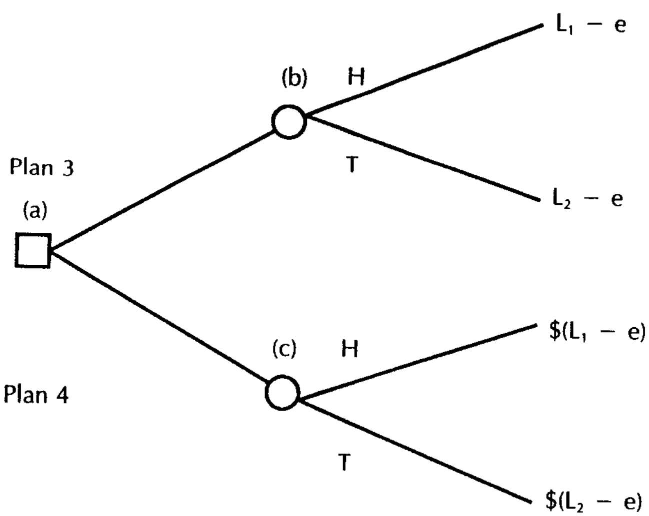

FIGURE 1.

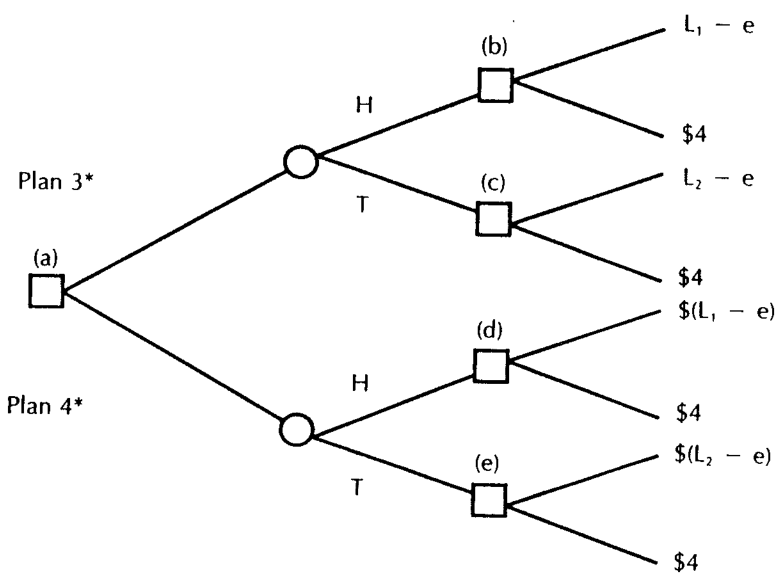

FIGURE 2.

It is clear, from the very example that Seidenfeld has constructed, that the agent who is prepared to violate MD can be confronted with a situation in which he will end up violating DYN-SUB. Thus, holding everything else constant, the agent cannot at the same time accept DYN-SUB *and* order the corresponding plans in the manner discussed above, i.e., rank plan 2 over plan 1 in the original problem, but rank plan 1 over plan 2* in the modified version. On the best reconstruction of Seidenfeld's argument that I have been able to devise, then, DYN-SUB would seem to spell out what is implicit in clause (ii) of his coherence condition – insofar as it applies to dynamic choice situations.

The crucial question now is this: does DYN-SUB provide any independent leverage to be used against the agent who is disposed to violate MD? One can begin by recalling that since both MD and SUB are versions of the independence principle, those who are disposed to relax MD will typically also be disposed to relax SUB. This, of course, doesn't require us to answer in the negative the question just posed, since DYN-SUB is logically distinct from SUB. SUB applies only to replacements at chance nodes in a decision tree, while DYN-SUB applies to replacements at choice nodes. However, within the frame in which Seidenfeld works, the extension of SUB to choice nodes seems plausible enough. On Seidenfeld's account, the agent must face up to the fact that he foreknows – that is, he must determine how he will choose in the future, by reference to the preferences he will then have for the prospects still open to him, and from there reason back to his present situation.[4] I presume, then, that Seidenfeld's agent, when confronted

4. Seidenfeld here makes heavy use of a separability assumption for dynamic contexts. All preferences at subsequent choice points, on this account, are simply to be read off the basic ordering of the original set of (static) lotteries. Note in particular the wording of the crucial claim: "If the coin lands heads up (a) he will choose $5.50 over lottery L_1. And if the coin lands tails up (b), again, he will choose the $5.50 (over L_2)" (p. 277). My appeal to the notion of a resolute approach – see McClennen (1986, 1988, 1989) – is motivated by a desire to explore what choice might be like in contexts where this sort of separability does not hold – where a prior commitment to a plan of action can determine the preference ordering at a given choice point, even though, were the agent to approach that sort of choice point *de novo* – i.e., not against the background of some plan previously decided upon, his preference ordering would be different. A resolute chooser will be unimpressed, on the account I have offered, by Seidenfeld's insistence that in the problem in Figure 4 the agent must expect that he will choose $5.50 if at (a) and $5.50 if at (b), and that he must, then, accept that the only feasible plan associated with heading towards (a) and (b) is plan 1, with its sure payoff of $5.50. To the contrary, such an agent will suppose that there is another feasible plan, 1', which calls for him to move towards (a) and (b) and select L_1 if chance takes him to (a) and L_2 if chance takes him to (b). Moreover, given the hypothesis concerning the agent's preferences (preferences that violate MD), the agent will regard plan 1' as clearly superior to plan 2 (by appeal to FOSD) – indeed, it is the best plan of all, and hence the one he will adopt and resolutely implement. This is why, even if Seidenfeld's argument went through, it would present no problem as such for the resolute chooser, but only for one who was committed to a sophisticated approach.

with the problem given in Figure 2, will anticipate that if and when node (b) is reached he will choose $L_1 - e$; this expectation will naturally express itself by his assigning a probability of 1 (or something closely approximating it) to his choice of $L_1 - e$, conditional upon his reaching node (b). And similar assignments can be made with respect to the various propositions regarding what he will do, conditional upon reaching the other possible choice nodes. That is, evaluation of present options proceeds by anticipation of future (independently motivated) choices, use of that information to reinterpret subsequent choice nodes as chance nodes, and evaluation of the various prospects that can be defined by reference to that reinterpretation.[5]

In effect, then, the agent will treat the problem in Figure 2 as if it were the problem given in Figure 1. But for such an agent, committing himself to SUB in nonsequential choice situations will imply choosing in sequential choice situations as if he were committed to DYN-SUB. And, turning this point around, a disposition to relax SUB will imply a disposition to choose in certain dynamic choice situations as if he were not committed to DYN-SUB.

This being the case, however, it seems plain enough that an appeal to clause (ii) (i.e., DYN-SUB) cannot provide any independent leverage against violations of SUB (or MD). Independent leverage would be possible only if it made sense to suppose that an agent would not want to abandon DYN-SUB, even if he were prepared to relax MD (or SUB). I suggested some years ago that a much discussed argument of Raiffa (1970) begged the issue against violations of independence.[6] While it is clear from the interpretation just offered of Seidenfeld's argument that it differs from the one employed by Raiffa (appealing to very different assumptions), the fact nevertheless remains that for him to argue against violations of MD by appeal to DYN-SUB is to proceed in a fashion that comes uncomfortably close to begging the issue once again.

To be sure, there might be some independent argument that Seidenfeld could mount in favor of DYN-SUB. But if DYN-SUB is to count as a rationality condition, the case for this needs to be made out. This does not seem to be something that Seidenfeld has done.[7] I conclude, then, that despite the use of the word "incoherence," Seidenfeld really has not shown that the sophisticated agent who violates independence is guilty of anything as serious as this term suggests. As far as I can see,

5. When subsequent options are equally acceptable (or indifferent to one another) matters become somewhat more complicated. I go into this matter in McClennen (1989, Chapter 8).

6. See McClennen (1983, pp. 119–20).

7. Recall again that in introducing his coherence condition, Seidenfeld announces that he proposes to "elevate respect for stochastic dominance to the status of a coherence condition" (p. 275). That certainly does serve to motivate clause (i); but what motivates clause (ii)?

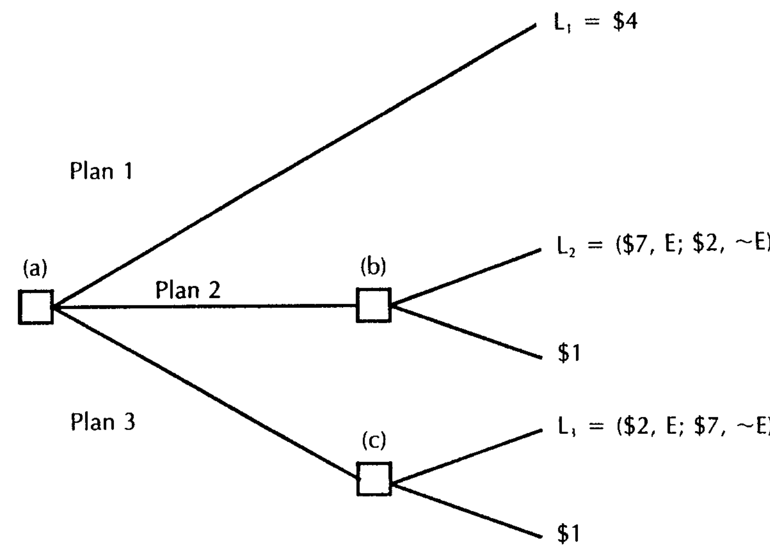

FIGURE 3.

really all he has shown is that an agent who rejects the independence condition (or SUB, or MS) will evaluate certain sequential choice situations differently from those agents who accept that condition.

3. SEIDENFELD'S SECOND THESIS

Suppose, contrary to what I have suggested above, that Seidenfeld is correct that those who violate MD face a real problem. Consider now the decision tree in Figure 3 and an agent who is committed to Levi's E- and S-admissibility rules. With respect to the "terminal" outcomes, if $P(E)$ is maximally indeterminate, then E-admissibility calls for the rejection of L_1 when both L_2 and L_3 are also available, although L_1 would be chosen in pairwise comparison with either (by appeal to S-admissibility).[8] Reasoning backwards from choices that such an agent can project he would make at nodes (b) and (c), it is clear that an agent committed to E-admissibility confronts the following feasible plans at (a):

Plan 1: select L_1, i.e., \$4, outright.
Plan 2: move to (b) and select L_2.
Plan 3: move to (c) and select L_3.

Hence, the agent who is committed to E- and S-admissibility must elect to move either to (b) or to (c), i.e., reject plan 1. Now suppose that plans

8. See, for example, Levi (1974, 1986a, 1986b).

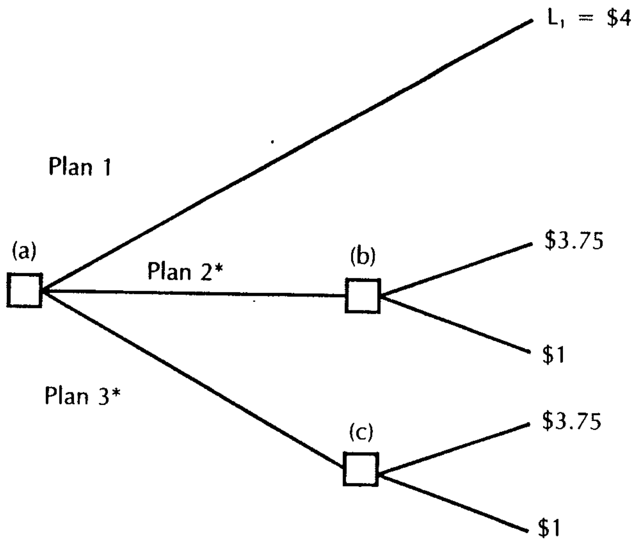

FIGURE 4.

2 and 3 are modified in the following manner: L_2 is replaced by a dollar amount that is less than $4, but still one that the agent would choose over L_2 in pairwise comparison; and L_3 is replaced similarly with a dollar amount that is less than $4, but still one that the agent would choose over L_3 in pairwise comparison. Thus, for example, the agent who is committed to both E- and S-admissibility will choose $3.75, and judge L_2 unacceptable, when just those two options are available, and will choose $3.75 and judge L_3 unacceptable, when just those two are available, but still reject $3.75 in favor of either L_2 or L_3, when both of these are available.[9] The decision problem now looks like Figure 4.

I take it we can expect, then, that a person who is committed to Levi's E- and S-admissibility rules will reject plans 2* and 3* in favor of plan 1. Recall, however, that in the problem given in Figure 3, such an agent will reject plan 1 in favor of either plan 2 or plan 3. Now what is odd here – or at least what should be regarded as odd on the sort of account that Seidenfeld has offered – is that plans that were formerly judged to be acceptable are now to be rejected – in favor of a plan that was itself previously rejected – even though the transformations in question amount to replacing best options down the road (as it were) with

9. I shift here to speak in terms of acceptable choice, rather than preference, so as to acknowledge – at least for the sake of this argument – Levi's point that in the face of uncertainties, the agent may not have well-defined preferences, even though he still has to make choices. However, as Levi himself insists, in such situations (rational) choices may yet be subject to certain requirements. E- and S-admissibility express two such requirements.

options that are judged – from that perspective (in terms of S-admissibility – to be even better. This pattern of choice violates what can be characterized as a strict ordering version of DYN-SUB:

> *Dynamic Substitution with "Improved" Options* (DYN-SUB*): In a dynamic choice context, if a particular plan is acceptable, and the agent substitutes for one of its component prospects another prospect judged even more acceptable (in pairwise comparison), then he or she should judge the modified plan at least as acceptable as the original, unmodified one.[10]

In short, Levi's agent, who is committed to E- and S-admissibility, may not violate DYN-SUB, but he will violate DYN-SUB*. If, however, a rational agent must accept DYN-SUB, then it seems to me that he must also accept DYN-SUB*. That is, orienting myself within the frame in which Seidenfeld has chosen to work, I cannot understand why such violations of DYN-SUB* do not pose just as serious a problem for any theory which calls for relaxing the "consistency" conditions on choice functions, as violations of DYN-SUB pose for those who are prepared to relax the independence axiom.

Seidenfeld (and Levi) may, of course, want to insist that DYN-SUB* fails to take account of the contextural nature of judgments of acceptability: there are substitutions that the agent would be prepared to accept within the context of pairwise choice but this says nothing about the acceptability of various plans when other plans are also available. To this, however, my resolute response is simply that a similar line of reasoning can be used to reject DYN-SUB (and SUB) as well: what Allais, Ellsberg, and a host of other critics have been arguing for years is precisely that *context makes a difference* – that just because the agent takes some lottery L_1 to be indifferent to some other lottery L_2, it does not follow that replacement of the former with the latter in the context of some compound lottery (or sequential choice situation) will leave the agent indifferent between the original option (or plan) and the modified option (or plan).

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10. Again, this principle is framed as a constraint on acceptable choice, rather than preference, for the reasons cited in footnote 9.

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REJOINDER

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I thank Professors Hammond (1988) and McClennen (1988) for their insightful comments on sequential decision making. No doubt readers will appreciate the variety of conclusions these two draw from the same arguments. Professor Hammond endorses the claim that sequential incoherence accompanies failures of the independence postulate (provided the agent's preferences for monetary lotteries are weakly ordered, respect stochastic dominance, and are stable across choice nodes). Professor McClennen demurs from this conclusion. Both think, although for different reasons, that sequential considerations can be used to justify the ordering postulate, agreeing with each other and disagreeing with me. Let me reply to these remarks, trusting that along the way I do not seriously misunderstand their contributions.

1.

I welcome Hammond's ratification of the central conclusions of my essay (Seidenfeld, 1988):

(1) that violations of the independence axiom entail what he describes as "especially damaging, 'fundamental inconsistencies' in certain decision trees"; and
(2) that such inconsistencies are absent in the rival decision theory (of I. Levi) that I analyze in section 5, which relaxes ordering instead.

What difference persists, I suspect, is over the question of so-called "weaker inconsistency," which Hammond rightly conjectures is present in Levi's theory. Let me discuss this, briefly. In what follows, the linchpin in the analysis is "Dynamic Feasibility" (p. 278), a principle for limiting the live options in a sequential decision problem. The rebuttal will serve

to explain why I think weak consistency is not a compelling norm on rational choice.

Consider the revealing sequential decision Hammond introduces with his figure 2. Under the conditions of this problem, he rightly points out that acts ***a*** and ***b*** are both E-admissible in a pairwise choice between them; likewise, ***c*** and (a modified) ***d*** are pairwise E-admissible.[1] But only ***a*** and ***c*** are E-admissible in a nonsequential choice among all four options. What is the agent facing the sequential decision of Figure 2 to do? The answer depends upon the "security" index (p. 286) employed.

If security is indexed by worst outcomes, what I label "sec_1" in my sketch of Levi's theory, then act ***a*** alone is admissible in the pairwise choice with ***b***, and similarly ***c*** alone is admissible in the pairwise choice with ***d***. Thus, the agent can anticipate choosing ***a*** at choice node *B* and can anticipate choosing ***c*** at choice node *C*. By the rule of Dynamic Feasibility, options ***b*** and ***d*** are not feasible at choice node *A*. There the decision maker assesses the two sequential plans as a choice between ***a*** and ***c***. (Nonfeasible options ***b*** and ***d*** are irrelevant to the considerations at *A*.) In the pairwise choice between ***a*** and ***c*** both are admissible, so that the agent is free to choose either plan available at *A* knowing that the result will be ***a*** or ***c***. There is no need for any additional "resolutions" to avoid ***b*** and ***d***. (The same conclusion follows if security is indexed by Γ-minimax, the infemum of expected utilities, what I label "sec_2."[2])

The analysis for this problem is more intricate if security is vacuous, "sec_0," so that E-admissibility determines admissibility. This is the more interesting case and the one, I believe, that Hammond has in mind. Then, at node *A* the agent cannot anticipate choices at *B* and *C*. In keeping with the program of representing uncertainty through a *set* of probabilities, at node *A* the agent has uncertain beliefs about his or her future choices at nodes *B* and *C*. Suppose the deciding agent is maximally uncertain which admissible choices will be taken. Then, at choice node *A*, to plan on *B* yields an uncertain prospect over the pair {***a***,***b***} with a maximally indeterminate (personal) probability of choosing act ***a*** or choosing act ***b*** (each of which is, itself, an indeterminate option involving uncertainty about the states s' and s''). The agent represents this uncertainty with a (maximal) convex set of personal probabilities over the two states ("choose ***a***", "choose ***b***"). Likewise, the plan at node *A* for node *C* yields a maximally uncertain prospect of choosing between ***c*** and ***d***. The upshot of all this uncertainty is that, with the vacuous security index, at choice node *A* both sequential plans are E-admissible; hence, both are admissible.

1. To get the effect Hammond wants, modify his act ***d*** so that, e.g., it awards 2 utiles in state s' and 0 utiles in the complementary state s''. Otherwise, ***d*** is E-inadmissible in a pairwise choice with act *c* according to rule † for weak dominance (as indicated in my footnote 12).

2. The sketch of Levi's theory, given in my section 5, provides real-valued versions of the "vacuous" and (nonlexicographic) minimax security rules, which can be found in Levi's book (1980, chapter 7). However, my proposal to use "Γ-minimax" as a security index extends what Levi calls "Wald's method" (1980, 7.4). I am unsure what constitutes necessary and sufficient conditions of adequacy for indices of security. Thus, I do not know whether "sec_2" is a legitimate security index.

It is important to see that on either reading of this problem, i.e., no matter which security index is used, and subject to Dynamic Feasibility, at *A* the agent never faces a choice among the four terminal options {***a***, ***b***, ***c***, ***d***}. In the first case, the choice at *A* reduces (according to Dynamic Feasibility) to a choice between ***a*** and ***c***. In the second case, the choice at *A* reduces to a choice between two, doubly indeterminate options: choose at *A* between the indeterminate ***a-b*** option and the indeterminate ***c-d*** option.

Of course, as Levi's theory satisfies neither Sen's (1977) property α nor property γ, there are so-called "inconsistencies" which follow just from this fact in static, nonsequential decisions. That is, with the right security index, act *e* may be uniquely admissible from each of the pairs {*e*, *f*} and {*e*, *g*}. Yet *f* alone may be admissible from the triple of options {*e*, *f*, *g*}. (This constitutes a violation of both-named properties and, obviously, does not involve sequential decisions.) But, as I argue in section 5 of my paper, the failure of ordering does not produce failures of stochastic dominance even under substitution of indifferent options.

There is yet another aspect of the concern for ordering which emerges in sequential decisions and which violates Professor Hammond's condition that choice be "consequentialist." In Levi's theory, there is no general equivalence between decisions in extensive and normal forms. In section 4 (and especially in footnote 9) of my essay, I point out that the argument alleging sequential incoherence in failures of independence does *not* presume an agreement between a sequential decision in extensive form and in its nonsequential, normal version.

In Levi's theory, the nonequivalence of extensive and normal forms is the direct by-product of relaxing ordering in nonsequential decisions. Perhaps this, then, is the weak inconsistency that Hammond correctly suspects is present in Levi's theory. We see this phenomenon in Hammond's useful example.

Suppose, as before, the agent uses the vacuous security index in the decision schematized by Hammond's figure 2: admissibility is E-admissibility. Then the agent may end up choosing any one of the four {***a***, ***b***, ***c***, ***d***}, as the preceding analysis indicates, though he or she never faces the four of them simultaneously in a single set of feasible options. Now recast this sequential decision problem into its normal form. Then the agent confronts a single terminal choice among the four options {***a***, ***b***, ***c***, ***d***}. As Hammond rightly observes, the admissible options here are

just the $\{a, c\}$ pair; thus, b and d are inadmissible in the normal form of the decision. Hence, decisions are not the same in extensive and normal form.

This is the weak inconsistency that occurs in Levi's theory. But what reason is there to require equivalence between decisions in extensive and normal forms? Surely that equivalence *simplifies* the analysis of many sequential decision problems. However, provided the agent is careful to respect Dynamic Feasibility, what is the normative error in denying the equivalence?

2A.

Professor McClennen raises a similar question (in section 3 of his comments): Do not sequential considerations also compel ordering? To show they do, he proposes a substitution rule "Dynamic Substitution with 'Improved' Options" [DYN-SUB*] that is not satisfied by Levi's theory when particular security indices are used.[3] Unfortunately, the rule McClennen offers is not neutral regarding the ordering principle in static, nonsequential decisions; so that its application in sequential decisions serves to mask the point of the debate.

The relation he introduces, *e is even more acceptable than f*, is defined by choice in pairwise comparisons. It holds between two options provided that *e* alone is admissible in a pairwise choice between them. A simple application of DYN-SUB* in nonsequential decisions establishes that the newly defined relation is transitive: If *e is even more acceptable than f*, and *f is even more acceptable than g*, then (by DYN-SUB* of *e* for *f*) *e is even more acceptable than g*.

The use of security indices that lead (in Levi's theory) to sequential violations of DYN-SUB* are the very ones that fail to make "is even more acceptable than" a transitive relation. The issue with DYN-SUB* is in what it requires for static, nonsequential decisions in Levi's theory – the sequential aspect is a digression.

This is to be contrasted with Hammond's argument for ordering. His defense of ordering rests on an equivalence between extensive and normal forms. That *is* a sequential consideration which does not prejudice the question about ordering in nonsequential decisions, which is not the case in McClennen's account. McClennen's argument is in contrast also with the one in section 4 of my essay. The conditions I introduce there are neutral between failures of independence and ordering in nonsequential decisions. It is only in sequential cases that the differences emerge.

If Professor McClennen objects to failures of ordering because a particular binary choice relation is not transitive, let him say so. That has nothing yet to do with sequential decisions. It seems to me to beg the question on ordering to suppose that "is even more acceptable than" is transitive (as the irreflexive, transitive, ordinary English "more than" requires), and then to adduce sequential failures of DYN-SUB* in a program that does not satisfy the antecedent condition. If, on the other hand, we delete the supposition that the new choice relation is transitive, then the DYN-SUB* rule is invalid even in static, nonsequential decisions.

In my (section 5) sketch of Levi's theory, I offer a choice-based generalization of the strict preference relation (over rewards) for decision rules that may fail the ordering postulate. That is the relation of "categorical preference." This is shown to be a transitive relation in Levi's theory (see the Lemma, p. 287), and the substitution of categorically more preferred rewards preserves admissibility, as a corollary to Theorem 5.1. Thus, when McClennen's principle of DYN-SUB* is corrected to apply with a generalized preference relation, neutral about ordering in static decisions, the conclusion he seeks is forthcoming in Levi's theory.

2B.

In part 2, McClennen asks for a justification of clause (ii) to the coherence condition (p. 275). Clause (ii) requires that admissibility is to be preserved under substitution (at choice nodes) of indifferent options. The "indifference" relation is generalized for choice rules which may fail ordering (see p. 285). And as noted there, when ordering obtains, this generalized indifference relation is equivalent to the familiar one: that is, where two alternatives are indifferent if and only if both are admissible in *some* decision. McClennen's question is an important one and I thank him for the opportunity to answer.

It is unnecessary to introduce new principles, e.g., REPLACE, BOV, or DYN-SUB, in order to defend clause (ii) for the section 4 argument that a failure of independence results in sequential incoherence. Dynamic Feasibility and ordering suffice, provided that values are stable over time.

CLAIM:

For dynamically stable values, Dynamic Feasibility and ordering yield clause (ii) of coherence.

Proof:

Under these three assumptions, clause (ii) is demonstrated by showing that plans which result from a substitution of indifferents at choice nodes

3. For example, the rule DYN-SUB* fails with security indexed by "worst outcomes," sec_1. It does not fail with the vacuous security index, sec_0, for which McClennen's "is even more acceptable than" is a transitive admissibility relation in Levi's theory. This phenomenon is related to the fact that Sen's property α fails with sec_1 but not with sec_0. However, property γ fails on either security index. Thus, Levi's choice rules are not "normal," in Sen's jargon. (I discuss this in Seidenfeld, 1985.)

are themselves indifferent. Then, by ordering, since admissibility is invariant over the equivalence class of plans indifferent to a given plan, clause (ii) obtains.

Let *o* and *o'* be indifferent options. (Stability of values insures this relation is well defined over choice nodes.) Consider two sequential plans *p* and *p'* that differ solely in that, at one designated choice node, *p* selects *o* and *p'* selects *o'*. The two plans are identical otherwise. First, we establish that *p* and *p'* are indifferent, as follows.

Imagine a simple sequential decision that has an initial choice between the *p*/*p'* plans and another option *q*, which is dispreferred to each of *p* and *p'*. Since *o* and *o'* are indifferent, both are admissible at the designated node where there is a choice between them. Since *p* and *p'* are identical otherwise, by Dynamic Feasibility, both plans *p* and *p'* are admissible at the initial choice node, and *q* is inadmissible there. Then, by ordering, *p* and *p'* are indifferent; both are admissible in this decision.

Second, since ordering requires that indifference is transitive, it is preserved under iterated substitutions of indifferents at (other) choice nodes. That is, if plans *p'* and *p"* differ solely by a substitution of indifferents at some (other) choice node, then *p'* and *p"* are indifferent (as above), and by transitivity *p* and *p"* are indifferent.

Last, by ordering, admissibility is invariant over elements of an equivalence class of indifferent options. Hence, clause (ii) obtains.[4]

Thus, in answer to McClennen's question, clause (ii) is a consequence of the other assumptions used in the section 4 argument that a failure of independence produces sequential incoherence.

2C.

I remain unclear how McClennen's program of resolute choice is to avoid the quandary of sequential incoherence that results from denying independence. If clause (ii) of coherence is voided, since he advocates the ordering principle, McClennen shall have to develop his program so that either Dynamic Feasibility fails, or basic values are dynamically unstable across choice nodes (or both).

Will resolute choice oppose the following rule? It asserts, if you know now that you will reject option *o* at a future choice node, then the plan to choose *o* at that node is not a feasible alternative now. In this regard, Dynamic Feasibility requires only that you look before you leap.

In footnote 8 of my essay, I raise some questions about resolute choice under the interpretation that it mandates sequential changes in the agent's preferences for lotteries. My view is that the mandated changes in values are unreasonable.

4. Sequential decisions are needed for this simple argument. Moreover, this proof does not require an equivalence between decisions in extensive and normal forms beyond that which follows from Dynamic Feasibility.

Last, in footnote 4 of his commentary, McClennen states that resolute choice creates "another feasible plan," e.g., corresponding to the choices of lottery L_1 at node ***a*** and lottery L_2 at node ***b*** in the decision of Figure 4. If "resolutions" are to be understood as providing, cost free, new terminal options, then I find myself repeating the criticism of footnote 10: As rational agents, we are not generally at liberty to create, cost free, terminal options corresponding to the normal form of a sequential decision. Even the force of a rational will is an insufficient means to that end, as Ulysses knows all too well. His problem and ours is *not* weakness of will. Sometimes we just cannot avoid reconsidering yesterday's plans tomorrow.

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