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Mulan X. Wang
DTE Energy Trading

Nicola Secomandi
Carnegie Mellon University, ns7@andrew.cmu.edu

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Mulan X. Wang[1], Nicola Secomandi[2]
[1]DTE Energy Trading, 414 S. Main Street, Suite 200, Ann Arbor, MI 48104, USA
[2]Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213-3890, USA
wangx@dteenergy.com, ns7@andrew.cmu.edu
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Abstract

In the United States natural gas pipelines lease their transport capacity to shippers via contracts, which shippers manage as real options on differences between natural gas prices at different geographical locations. In practice it is common to value these real options using spread option valuation techniques, because they quickly compute both their values and, even more important, their sensitivities to parameters of interest (the greeks). Although fast, we show that in the general case of network contracts this approach is heuristic. Thus, we propose a novel and computationally efficient method that estimates the exact real option value of such a contract and computes unbiased estimates of its greeks, based on the application of linear programming, Monte Carlo simulation, and direct greek estimation techniques. We test this method on realistic instances modeled after contracts available on the Transco pipeline, using a reduced form model of the risk neutral evolution of natural gas prices calibrated on real data. Our main findings are that our method can significantly improve the practice based valuations of these contracts, by up to about 10%, and the application of direct greek estimation techniques is critical to make our method computationally efficient. Our work is relevant to natural gas shippers; a version of our model was recently implemented by a major international energy trading company. Potentially, our work has wider significance for the valuation and management of other commodity and energy real options, whose payoffs are determined by solving capacity constrained optimization models.

1 Introduction

Natural gas is a major energy source in the United States (U.S.) and other industrialized countries. In the U.S. it accounted for 24% of total energy consumption in 2008 (EIA [12]). The Energy Information Administration (EIA [11]) has projected that U.S. natural gas consumption will grow by 0.2% per year from 2009 to 2030. In North America natural gas is traded on both spot and forward markets at different geographical locations. Gas Daily, a widely circulated industry newsletter, includes more than 80 pricing points. These locational markets are connected by a web of about 160 interstate pipelines.

In North America the New York Mercantile Exchange (NYMEX) and the IntercontinentalExchange (ICE) trade financial contracts associated with about 40 locational markets. These contracts include futures with physical delivery at Henry Hub, Louisiana, basis swaps (forward financial contracts on price differences between Henry Hub and other locations), and put and call options on

Table 1: Points and capacities of a network transport contract on the Transco pipeline (Source: Transco web site; MMBtu abbreviates million British thermal units).

Point	Type	Capacity (MMBtu/day)
Zone 3	Delivery	1,816
Zone 4	Delivery	111,366
Zone 1	Receipt	18,932
Zone 2	Receipt	27,841
Zone 3	Receipt	21,160
Zone 3	Receipt	43,433
Zone 3	Receipt	1,816

NYMEX futures. There are also over the counter (OTC) markets. Together NYMEX, ICE, and OTC markets provide market participants with a high level of price transparency.

In the U.S., while natural gas wholesale markets are unregulated, interstate pipeline companies are regulated entities that act as common carriers, that is, they do not own the gas they transport. Shippers, that is, those who own the gas being transported by pipelines, must contract with pipeline companies for portions of their transport capacity to receive service. Pipeline companies sell their capacity through sealed bid auctions run on their web sites. Interstate pipeline *minimum* and *maximum* contract rates (prices) are regulated, and shippers need to form their own valuations of pipeline capacity when they bid for securing it. Thus, the valuation of pipeline capacity is an important problem in practice.

As discussed by Eydeland and Wolyniec [13], shippers can value exactly pipeline transport capacity between *two* locations as an option on the spread between the prices of natural gas at these locations (see also Deng et al. [9] for the related valuation of electricity transmission capacity). The argument here is that shippers employ pipeline capacity to support the following trade: they purchase natural gas at a receipt market and inject it into the pipeline, which in turn transports and delivers it to the delivery market, where shippers finally sell the gas on the spot market. Although this argument applies when the shipper is a merchant, Secomandi [32] shows that it also holds when the shipper is a natural gas producer, an industrial consumer, or a local distribution company. Secomandi [32] provides empirical evidence that shippers use spread option pricing methods to value point to point pipeline transport capacity.

But not all pipeline capacity contracts have this simple structure, as they can feature a *network* structure that includes multiple receipt and delivery points, that is, shippers can ship gas from several receipt markets to several delivery markets under the same contract. Table 1 shows the points and capacities of a real network transport contract on the Transco pipeline (the meaning of the point capacities is explained in §4.1).

Simple adaptations of the spread option pricing approach are widespread among shippers to value the real option embedded in network contracts. These approaches feature a spread option valuation model and a model to select a basket of such spread options to account for the network structure and capacity limits of a pipeline capacity contract. This approach quickly computes both a real option value for the network contract and, even more important, its sensitivities (greeks), which shippers use to support their financial trading (hedging) activities on NYMEX or ICE. The greeks include delta and gamma, the first and second partial derivatives of a contract value with respect to the current futures price at a given location, respectively, and vega and eta, the partial derivatives of the contract value with respect to the volatility of this futures price and the instantaneous correlation coefficient associated with the futures prices at two different locations, respectively. The greeks are key inputs to hedging the changes in the contract value between the contract inception and execution times (see, e.g., Duffie [10], Shreve [34]).

In this paper we show that the spread option approach to value network contracts is in general heuristic (it is exact in some special cases, discussed in §6). Thus, we propose a novel method, based on linear programming, Monte Carlo simulation, and direct sensitivity estimation techniques (Fu and Hu [14], Broadie and Glasserman [6], Boyle et al. [3]), which efficiently computes both an estimate of the exact (optimal) real option value of a network contract and unbiased estimates of its greeks. We derive direct estimators for several greeks based on a reduced form model of natural gas price evolution (Seppi [33]). This model generalizes to multiple locations models presented by Black [2] and Schwartz [30], as it features a spatial price correlation structure, a key driver of the value of natural gas pipeline capacity. Specifically, we derive delta estimators using the pathwise and likelihood ratio methods, gamma estimators using the likelihood ratio method and combining it with the pathwise method, and vega and eta estimators using the likelihood ratio method.

We test our proposed method on realistic instances, modeled after contracts available on the Transco pipeline. Here, we calibrate to real data and use a price model in which the natural logarithms of natural gas spot prices at different locations evolve as correlated mean reverting processes, a generalization of a model used by Secomandi [32]. Our method requires between 13.17-23.09 Cpu seconds to value these contracts and compute all the relevant greeks with 100,000 price samples. Thus, our method is computationally efficient. Using a greedy algorithm (GA) to solve each relevant linear program, which we show to be optimal in some cases but heuristic in general, further reduces this computational effort to 2.41-2.63 Cpu seconds, without materially affecting the quality of the contract valuations and greek estimates.

Moreover, we find that roughly at least 49% of the real option values of the considered contracts is attributable to price volatility, that is, is extrinsic. Thus, price uncertainty plays an important role in the valuation of contracts for pipeline capacity.

We also implement the practice based spread option approach by using a linear program to choose the basket of spread options used to value a network contract. We find that the valuations of our proposed method significantly outperform those of this approach, by up to about 10% on the considered instances. This suggests that there is significant benefit from adopting our method to support the real option valuation of network transport contracts in practice.

Resimulation is a straightforward greek estimation approach, but it is both biased and computationally expensive (Glasserman [17, §7.1]). We use it as a benchmark to investigate the performance of the direct greek estimators that we derive. With 100,000 price samples, we do not observe substantial differences between the direct and resimulation delta and gamma estimates, but the direct methods are considerably faster than resimulation. Moreover, both the pathwise and resimulation deltas have essentially the lowest standard errors, and combining the pathwise and likelihood ratio methods, as in Glasserman and Zhao [18] and Glasserman [17, §7.3.3], yields gamma estimates with much lower standard errors than those of the likelihood ratio and resimulation estimates.

For the most part, with 100,000 price samples, the likelihood ratio method estimates of the vegas and etas are similar to those obtained with the resimulation approach, but have higher standard errors. We substantially reduce these errors by increasing the number of samples from 100,000 to 1,000,000, but this increases the computational effort, which however remains lower than that taken by resimulation. To reduce this effort, we use the GA version of our method with 1,000,000 price samples; although the standard errors of the vegas and etas so estimated remain larger than those of the resimulation estimates obtained with 100,000 samples, the computational requirement of the GA based version of our method is only a fraction of that of resimulation. The main insights of this analysis are that the use of direct sensitivity estimation techniques is critical to make our model computationally efficient when computing the greeks, and that the GA based version of our method can be useful to obtain more precise estimates of the greeks in a faster fashion than the exact version of our method.

Our work is relevant to natural gas shippers: it suggests that the spread option based valuation approaches widespread among practitioners are likely significantly undervaluing network contracts for natural gas pipeline capacity, and we propose a method to improve upon this practice; a version of our model was recently implemented at a major international energy company. We also provide guidance on how to use our method for greek estimation purposes.

Our work is specific to one type of real option in the natural gas industry, but it has potential relevance beyond this application. For example, our work can be extended to value and manage other real options associated with the shipping, transportation, distribution, and refining of other energy sources, such as coal, oil, and biofuels, and commodities, such as metals and agricultural products. In other words, our method remains relevant, possibly in extended form, for the valuation and management of commodity and energy real options whose payoffs are obtained by solving a capacity constrained optimization model.

The remainder of this paper is organized as follows. We discuss the novelty of our contributions relative to the extant literature in §2. We describe the problem setting in §3. We present our valuation model in §4. In §5 we derive direct estimators for several greeks. We present the spread option approach in §6. We discuss our computational results in §7. We conclude in §8.

2 Literature Review

Our work falls within the real option literature that deals with applications in commodity and energy industries. Smith and McCardle [35] discuss the valuation of oil and gas investments, the books by Geman [15, 16] provide recent introductions to this field and several references, and Seppi [33] reviews the price evolution models typically used in this literature.

To the best of our knowledge, this literature has not considered the version of the problem that we study. We provide novel insights into the effectiveness of the spread option approach used in practice to deal with this problem. In this respect, our work complements that of Lai et al. [24], who benchmark practice based methods, including a spread option based method, for valuing natural gas storage contracts. Different from these authors, we find that the suboptimality of practice based methods for the problem considered here can be substantial, and we consider the estimation of the greeks.

Secomandi [32] finds that a large fraction of the value of point to point contracts for pipeline transport capacity is extrinsic. We confirm this result for network contracts.

Methodologically, our contract valuation problem can be interpreted as the valuation of an exotic European option, which is well studied in the financial engineering literature (Hull [19, Chapter 18], Broadie and Detemple [5]). But the payoff of a network transport contract is in general obtained by numerically solving a linear program, rather than being a closed form expression, the typical case in this literature. This generalizes several known exotic European options, including spread options (Carmona and Durrleman [7], Li et al. [25]) and rainbow options (Stulz [36], Johnson [20], and Boyle and Tse [4]), which do not capture the entire richness of the problem we study.

In this paper we apply known direct greek estimation techniques (Glasserman [17, Chapter 7]) to a novel real option problem. Our results on the relative performance of these techniques and resimulation are consistent with those of Broadie and Glasserman [6] and Glasserman [17, §7.3]. We also provide novel evidence for the usefulness of combining the pathwise and likelihood ratio methods for gamma estimation, as in Glasserman and Zhao [18] and Glasserman [17, §7.3.3] but in a different application domain. An element of novelty in our application is showing that the pathwise method is applicable to our problem, where the option payoff is the optimal objective function value of a linear program. Unique to our work, we also investigate the tradeoff between precision of the greek estimates and computational effort required to obtain them using different optimization algorithms.

3 Problem Setting

Pipeline managers market their transport capacity to shippers in the form of contracts. There exist two basic types of transport contracts: *firm* and *interruptible*. Interruptible contracts are generally short term (1 month or less) best effort contracts, that is, pipelines are not obligated to provide interruptible transportation services to shippers. In contrast, firm contracts are, typically, longer term guaranteed contracts that give shippers the right to transport up to a given quantity of natural gas during each period in their term. Shippers pay a per unit premium (demand charge) to reserve transport capacity and a per unit execution fee (commodity charge) to use it. The majority of pipeline capacity is sold in advance on a firm basis. In this paper we focus on the valuation of firm contracts, which amounts to determine the reservation component of a contract value, its demand charge, for a given commodity fee structure.

Specifically, a network transport contract specifies a collection of time periods (term), two sets of receipt and delivery points, the capacity limits at each of these points, and the set of links that connect the receipt points to the delivery points. The point capacities indicate how much natural gas a shipper owning such a contract can inject/withdraw at each receipt/delivery point in each time period in the contract term (natural gas flows from receipt points to delivery points). These contracts do not specify link capacities, which are instead implicitly expressed by the point capacities. Thus, the logistic structure of a network contract can be represented as a bipartite graph with node capacities. We denote by $\mathcal{R}$ and $\mathcal{D}$ the sets of receipt and delivery points (nodes), respectively, and by $\mathcal{P} := \mathcal{R} \cup \mathcal{D}$ the set of all points.

Shippers purchase transport contracts several months before their usage (see, e.g., Knowles and Wirick [23]). We denote by 0 the time when a contract is transacted. Two typical contract terms

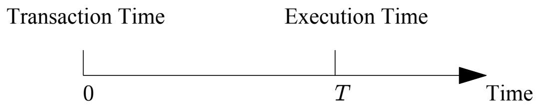

Figure 1: The contract transaction and execution timeline.

are the November-March (heating season) and April-October terms, but multiyear contracts are also common. A contract can be interpreted as a collection of subcontracts, one for each period in its term, because during each such period shippers have the option to ship natural gas up to the contracted capacity, that is, capacity unutilized in a given period cannot be utilized at a later time. Thus, without loss of generality, we only consider contracts with a single period term, which we denote by T. By a slight abuse of notation, we also denote by T the time that corresponds to the beginning of this period.

The contract receipt and delivery points are associated with natural gas market hubs. During the contract term, the contract owner buys natural gas at one or more receipt points and ships it to one or more delivery points, where the delivered natural gas is sold. Purchases and sales occur during the same time period because natural gas is received and delivered simultaneously by the pipeline (but, clearly, the received natural gas is not the same delivered natural gas).

Figure 1 illustrates the relevant timeline: the contract is transacted at time 0; shipping decisions are made at time T and operationally executed during the ensuing time period.

The problem studied in this paper is that of computing at any time $t \in [0, T]$ the real option value of a given network contract for usage of pipeline transport capacity during time period T, as well as the greeks associated with this value at any time $t \in [0, T)$. The contract value at time 0 is used to support a shipper's decision on how much to bid for the contract demand charge. If the contract is transacted, shippers can use the greeks, which are computed at each time $t \in [0, T)$, to support their hedging decisions during the time interval $[0, T)$. To avoid clutter, in this paper we only discuss the computation of the contract value and greeks at time 0, but all the results of this paper hold when time 0 is replaced with a later time.

4 Valuation

The valuation problem discussed in §3 can be tackled via risk neutral valuation techniques (see, e.g., Luenberger [26, Chapters 8-9], Duffie 2001 [10, Chapters 2 and 6]). This requires that a futures market exists at each point of a given network transport contract. In the U.S., the main financial contracts relevant for valuation purposes are NYMEX natural gas futures, whose delivery point is

Henry Hub, Louisiana. In addition, NYMEX and ICE trade basis swaps, which are locational price differences relative to Henry Hub. Basis swaps are purely financial contracts that do not entail physical delivery. While futures exist only at Henry Hub, by definition of basis swap it is clear that the sum of the Henry Hub futures price for a given maturity and the basis swap price for the same maturity yields a futures price for the basis swap location. Hence, valuation by risk neutral methods is possible for contracts that involve a large number of markets.

We take time T to correspond to the time when natural gas futures and basis swaps are settled, which is three business days prior to the beginning of a given month. We assume that this is when shippers decide how much natural gas to ship during the ensuing month, which we interpret as time period T. This is realistic: in practice shippers "nominate" their monthly shipping decisions to pipelines during the so called *bid week*, which is the week prior to each shipping month (Eydeland and Wolyniec [13, Chapter 1]). Thus, replication of the contract cash flows can be performed by trading futures and basis swaps at each of its receipt and delivery points.

We point out that shippers can modify their monthly nominations within the shipping month, e.g., in response to changes in the spot prices relative to the settled futures prices. We do not model these daily nomination updates, but our model can be modified in a straightforward manner to account for them, by dividing the monthly time period T into weekly or daily subperiods. In this case, financial replication could be performed by also using balance of the month/week or Gas Daily options (see Eydeland and Wolyniec [13, Chapter 4]).

4.1 Model

We introduce some additional notation to formulate our model. The capacities of each receipt point $i \in \mathcal{R}$ and delivery point $j \in \mathcal{D}$ are denoted by C_i and C_j, respectively, and are measured in MMBtu per month. We let sets $\mathcal{D}(i)$ and $\mathcal{R}(j)$ include the delivery points connected to receipt point $i \in \mathcal{R}$ and the receipt points connected to delivery point $j \in \mathcal{D}$, respectively. The per unit usage charge to ship natural gas during time period T on link i-j, $i \in \mathcal{R}$, $j \in \mathcal{D}$, is denoted by K_{ij}; this is the commodity charge of link i-j and is measured in \$/MMBtu.

The fuel coefficient associated with shipping natural gas on link i-j during time period T is $\phi_{ij} \in [0, 1]$; this coefficient is used to compute the fuel required by the compressor stations to ship one unit of natural gas on this link, which is $\phi_{ij}/(1-\phi_{ij})$, and is procured by the shipper at location i. Although the actual fuel burned by the compressor stations obviously depends on how natural gas is shipped by the pipeline, in practice this level of detail is not captured at the contract level; this is why the fuel coefficient only depends on the receipt and delivery points.

The risk free interest rate is r; we employ continuous compounding in this paper so that the risk free discount factor from time T back to time 0 is $\exp(-rT)$, which we define as δ.

The futures price at time $t \in [0, T]$ with maturity at time T for point $\ell \in \mathcal{P}$ is $F_\ell(t, T)$; if $t = T$ then this is a spot price. Since T is fixed and we focus on computing the contract value and its sensitivities at time 0, we simplify our notation by using F_ℓ to denote $F_\ell(0, T)$ and f_ℓ to denote $F_\ell(T, T)$. We define the time T value of shipping one unit of natural gas from receipt point i to delivery point j net of the commodity rate and fuel requirement as the following spot price spread:

$$s_{ij} := f_j - \frac{f_i}{1 - \phi_{ij}} - K_{ij}, \ \forall i \in \mathcal{R}, \ j \in \mathcal{D}(i). \tag{1}$$

In this definition, the commodity rate K_{ij} plays the role of a strike price, and the fuel adjustment factor $1/(1 - \phi_{ij})$ is the sum of one unit of gas shipped from point i to point j *and* the amount of fuel purchased at point i, that is, $1/(1 - \phi_{ij}) \equiv 1 + \phi_{ij}/(1 - \phi_{ij})$.

Let v denote the value of the contract cash flows at contract execution time T. This quantity is the optimal value of the following linear program:

$$\max_{w} \quad \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} s_{ij} w_{ij} \tag{2}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{D}(i)} w_{ij} \leqslant C_i, \ \forall i \in \mathcal{R} \tag{3}$$

$$\sum_{i \in \mathcal{R}(j)} w_{ij} \leqslant C_j, \ \forall j \in \mathcal{D} \tag{4}$$

$$w_{ij} \geqslant 0, \ \forall i \in \mathcal{R}, j \in \mathcal{D}(i). \tag{5}$$

In model (2)-(5) we assume that all cash flows incurred during time period T are accounted for at time T. Each decision variable w_{ij} represents the flow of natural gas from receipt point i to delivery point j during time period T, net of fuel. That is, the fuel needed to ship one unit of natural gas from i to j is assumed to be consumed upon receipt at point i. Thus, each receipt point capacity C_i is also expressed net of fuel.

As of time 0, the optimal objective function value of model (2)-(5) is a random variable that depends on the uncertain prices at the receipt and delivery points that will prevail at time T. We denote by $\mathbb{E}$ time 0 conditional expectation with respect to the risk neutral joint probability distribution of random vector $f := (f_\ell, \ \ell \in \mathcal{P})$ given price vector $F := (F_\ell, \ \ell \in \mathcal{P})$. This distribution exists under standard assumptions that we suppose to hold here (see, e.g., Duffie [10]). The contract value at time 0 is $V := \delta\mathbb{E}[v]$.

4.2 Computation

In general, it is difficult to express v in closed form. Even if this were possible, it is in general impossible to obtain a closed form expression for V. For example, when there are only one receipt point and one delivery point, v reduces to the payoff of a spread option for which no closed form expressions are known under common reduced form models of commodity price dynamics when its strike price is positive (Carmona and Durrleman [7], Li et al. [25]).

Thus, given a model that represents the risk neutral evolution of the futures prices at the receipt and delivery points, we numerically estimate V using linear programming and Monte Carlo simulation. This approach is exact aside from simulation error. Moreover, it can accommodate a variety of models of futures price dynamics because it only uses samples of the futures prices at the contract execution time. Specifically, we generate a large number of such price samples, at least 10,000, we compute the value v corresponding to each such sample, and estimate V by discounting back to time 0 and averaging all these v values.

The computation of the v values requires solving to optimality a large number of small linear programs. In our software implementation we use the free linear programming solver Clp of COIN-OR (www.coin-or.org) and take advantage of its re-solving capability. That is, we construct the constraint matrix (3)-(5) once, and use the `initialSolve` method to optimally solve model (2)-(5) on the first price sample; for all the remaining samples, we only update the objective function of this model using the `setObjective` method, and reoptimize the resulting model using the `resolve` method. This makes our implementation efficient.

Linear programming is needed to optimally solve model (2)-(5) because, in general, this model does not admit a simple optimal solution; in particular, it can be optimal to ship natural gas along the least valued link but not along the highest valued link in the contract network (this is due to the interplay between the price spreads and the capacity constraints). However, we also investigate solving model (2)-(5) heuristically in a faster fashion, using a simple greedy algorithm (GA).

GA algorithm. Given a sample realization of prices at time T:

Step 1. Set $v^{GA} \leftarrow 0$, $\mathrm{RC}_i \leftarrow C_i$, $\forall i \in \mathcal{R}$, and $\mathrm{DC}_j \leftarrow C_j$, $\forall j \in \mathcal{D}$.

Step 2. Compute the positive part of the price spread coefficients $s_{ij}^+ := \max\{s_{ij}, 0\}$, $\forall i \in \mathcal{R}$, $j \in \mathcal{D}(i)$, sort them in decreasing order, and store them in a stack.

Step 3. If the stack is empty or its first element is zero, stop and return v^{GA}. Otherwise, remove the first element $s_{ij}^{+,(1)}$ from the stack. Set $w_{ij} \leftarrow \min\{\mathrm{RC}_i, \mathrm{DC}_j\}$ and $v^{GA} \leftarrow v^{GA} + s_{ij}^{+,(1)} w_{ij}$.

Step 4. Set $\mathrm{RC}_i \leftarrow \mathrm{RC}_i - w_{ij}$ and $\mathrm{DC}_j \leftarrow \mathrm{DC}_j - w_{ij}$. Go to Step 3.

In general, GA is heuristic, but in §7 we observe that GA generates essentially the same valuations obtained by optimally solving model (2)-(5) using Clp, but in a faster fashion. Proposition 1, whose proof is omitted for brevity, states one case in which GA is provably optimal.

Proposition 1 (GA optimality)**.** *If the fuel and commodity rates are constants that do not depend on a specific link, then GA optimally solves model (2)-(5).*

4.3 Intrinsic and Extrinsic Values

The intrinsic value of a financial option "is defined as the maximum of zero and the value it would have if it were exercised immediately" Hull [19, p. 154]. This definition can be applied to compute the time 0 intrinsic value of the real option that we consider in this paper, which we denote by V^I. We define $S_{ij} := F_j - F_i/(1-\phi_{ij}) - K_{ij}$ and compute V^I as the following value:

$$\max_{w} \delta \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} S_{ij} w_{ij} \text{ s.t. (3)-(5).} \tag{6}$$

This is the value of making shipping decisions based on the price information available at time 0. Proposition 2 establishes that the intrinsic value of a transport contract does not exceed its real option value.

Proposition 2 (Intrinsic value)**.** *It holds that $V^I \leqslant V$.*

Proof. The martingale property of futures prices under the risk neutral measure implies that $S_{ij} = \mathbb{E}[s_{ij}]$, $\forall i \in \mathcal{R},\ j \in \mathcal{D}(j)$. Let w^I be an optimal solution to the intrinsic value model (6); w^I is a feasible solution to model (2)-(5) for any realization of the relevant spot prices at time T. Let w^* be an optimal solution to model (2)-(5); w^* is a random quantity as of time 0. It follows that

$$V^I = \delta \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} S_{ij} w^I_{ij} = \delta \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} \mathbb{E}[s_{ij}] w^I_{ij} \leqslant \delta \mathbb{E}[\sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} s_{ij} w^*_{ij}] = \delta \mathbb{E}[v] = V. \square$$

The intrinsic value of a transport contract, $V^E := V - V^I$, captures the value of price uncertainty.

5 Sensitivities

In this section we apply direct estimation techniques to compute the contract value greeks. Applying these methods requires specifying a risk neutral probability model of the relevant futures prices. We employ the following model:

$$f_\ell = F_\ell \exp(\alpha_\ell + Y_\ell),\ \forall \ell \in \mathcal{P}; \tag{7}$$

here Y_ℓ is the ℓ-th element of an n dimensional multivariate normal random vector Y with mean vector 0 and variance covariance matrix Σ, that is, $Y \sim N(0, \Sigma)$; the quantities α_ℓ and Σ can depend on T and other relevant parameters, but not on any of the prices F_ℓ. It will also be assumed throughout that Σ is positive definite. This matrix captures the spatial structure of price correlations, a key driver of the value of pipeline transport capacity (see §7.3).

Model (7) captures different models available in the literature. It includes as a special case a multilocation version of the celebrated Black [2] model of commodity futures prices, whereby the futures prices for a given maturity at different locations evolve, in the risk neutral world, as driftless and correlated geometric Brownian motions. This model is consistent with a version of Black's model widespread among practitioners to value the real option to store natural gas, which requires modeling the futures price dynamics at the same location for different maturities; in this case practitioners use an extended Black model with a separate volatility for each maturity and a separate correlation for each pair of maturities. From this perspective, one obtains model (7) by restricting attention to a single maturity and by considering multiple locations.

Model (7) also includes the case where the natural logarithms of the spot prices at each location follow correlated mean reverting processes, an extension of a well known model of commodity price evolution discussed, among others, by Schwartz [30]. Although this is a one factor model when one considers each location in isolation, so that it is limited in the type of correlation term structure that it can represent at a single location, it is quite flexible in the types of spatial price correlations structure that it can accommodate for a given maturity. Hence, it captures a key driver of the value of natural gas pipeline transport capacity. We use this specification of model (7) in Proposition 7 in §5.2, and in the numerical study reported in §7 where we consider a single maturity.

We investigate the computation of the deltas and gammas in §5.1 and etas and vegas in §5.2.

5.1 Deltas and Gammas

The deltas and gammas are the first and second partial derivatives of the contract value with respect to the futures prices at time 0: $\text{Delta}_\ell := \partial V / \partial F_\ell$, $\text{Gamma}_\ell := \partial^2 V / \partial F_\ell^2$. Before discussing their estimation, we sign them in Proposition 3, proved in Appendix A.

Proposition 3 (Deltas and gammas). *Under model (7) each receipt point delta is negative, each delivery point delta is positive, and every gamma is positive.*

This result can be equivalently stated as follows: the value V is convex and decreasing in each time 0 receipt point futures price, and convex and increasing in each time 0 delivery point futures price (these properties are in the weak sense).

We first apply the pathwise method to obtain unbiased estimates of the deltas by Monte Carlo simulation. The basic idea behind the pathwise estimation technique is to view $v(\theta)$ as a function of a parameter θ, in this case F_ℓ, obtain the pathwise derivative $\partial v(\theta)/\partial\theta$, and estimate the relevant greek by leveraging the expression $\partial V/\partial\theta = \delta\mathbb{E}[\partial v(\theta)/\partial\theta]$. The theoretical underpinning here is the interchange of expectation and differentiation (a limit) by an application of the dominated convergence theorem (Broadie and Glasserman [6], Glasserman [17, §7.2]). Proposition 4 presents pathwise expressions for the deltas.

Proposition 4 (Pathwise deltas)**.** *Let w^* be an optimal solution vector to linear program (2)-(5). Under model (7) the contract deltas can be expressed as follows:*

$$\begin{aligned} \mathit{Delta}_i &= -\delta\mathbb{E}[f_i \sum_{j\in\mathcal{D}(i)} w^*_{ij}/(1-\phi_{ij})]/F_i,\ \forall i \in \mathcal{R} \qquad (8) \\ \mathit{Delta}_j &= \delta\mathbb{E}[f_j \sum_{i\in\mathcal{R}(j)} w^*_{ij}]/F_j,\ \forall j \in \mathcal{D}. \qquad (9) \end{aligned}$$

Proof. Expressions (8)-(9) follow easily if conditions A1-A4 in Proposition 1 of Broadie and Glasserman [6] are satisfied. We show that they are below. We let $\theta = F_\ell$, and denote the dependence of the price f_ℓ on θ by $f_\ell(\theta)$.

(A1) Under model (7) the quantity $\partial f_\ell(\theta)/\partial\theta$ exists with probability 1 because it is equal to $\exp(\alpha_\ell + Y_\ell)$.

(A2) Interpret v as a function of the vector of prices $f := (f_\ell,\ \ell \in \mathcal{P})$, and let D_v denote the set of such price vectors where v is differentiable. Since v is the optimal objective value of a linear program, it follows that under model (7) it holds that $\Pr(f(\theta) \in D_v) = 1$, $\forall\theta \in \Re_+$.

(A3) Define $n^{\mathcal{R}} := |\mathcal{R}|$, $n^{\mathcal{D}} := |\mathcal{D}|$, and $n := n^{\mathcal{R}} + n^{\mathcal{D}}$. It is shown below that the function v is Lipschitz continuous in f with constant $\overline{C} := \max\{n^R, n^D\} \max_{i\in\mathcal{R}, j\in\mathcal{D}} \max\{C_i, C_j\}/(1-\phi_{ij})$:

$$|v(f^2) - v(f^1)| \leqslant \overline{C}||f^2 - f^1||,\ \forall f^1, f^2 \in \Re^n_+,$$

where $||f^2 - f^1|| \equiv \sum_{\ell\in\mathcal{P}} |f^2_\ell - f^2_\ell|$. Let w^1 and w^2 be *optimal* solution vectors to linear program (2)-(5) corresponding to price vectors f^1 and f^2, respectively. If link i-j does not exist, we define w^1_{ij}, $w^2_{ij} := 0$. Also, let s^1 and s^2 be the spread vectors corresponding to f^1 and f^2.

For each link i-j, the feasibility of w^1 and w^2 implies that

$$|s^2_{ij}w^2_{ij} - s^1_{ij}w^1_{ij}| \leqslant |s^2_{ij} - s^1_{ij}| \max\{w^1_{ij}, w^2_{ij}\} \leqslant |s^2_{ij} - s^1_{ij}| \max\{C_i, C_j\}. \qquad (10)$$

Moreover, for each link i-j, we have

$$|s^2_{ij} - s^1_{ij}| = |(f^2_j - f^1_j) + (f^1_i - f^2_i)/(1-\phi_{ij})| \leqslant (|f^2_j - f^1_j| + |f^1_i - f^2_i|)/(1-\phi_{ij}). \qquad (11)$$

Inequalities (10)-(11) imply that

$$|s_{ij}^2 w_{ij}^2 - s_{ij}^1 w_{ij}^1| \leqslant (|f_j^2 - f_j^1| + |f_i^1 - f_i^2|) \underbrace{\max\{C_i, C_j\}/(1-\phi_{ij})}_{\overline{C}_{ij}:=}, \ \forall i \in \mathcal{R}, \ j \in \mathcal{D}. \tag{12}$$

It follows from (12) that

$$\begin{aligned} |v(f^2) - v(f^1)| = |\sum_{i\in\mathcal{R}}\sum_{j\in\mathcal{D}}(s_{ij}^2 w_{ij}^2 - s_{ij}^1 w_{ij}^1)| &\leqslant \sum_{i\in\mathcal{R}}\sum_{j\in\mathcal{D}} |s_{ij}^2 w_{ij}^2 - s_{ij}^1 w_{ij}^1| \\ &\leqslant \sum_{i\in\mathcal{R}}\sum_{j\in\mathcal{D}} (|f_j^2 - f_j^1| + |f_i^1 - f_i^2|)\overline{C}_{ij} \\ &\leqslant \overline{C}\sum_{\ell\in\mathcal{P}} |f_\ell^2 - f_\ell^1| = \overline{C}||f^2 - f^1||. \end{aligned}$$

(A4) Each random variable f_ℓ is almost surely Lipschitz with integrable modulus $\exp(\alpha_\ell + Y_\ell)$, because $||f_\ell(\theta^2) - f_\ell(\theta^1)|| = \exp(\alpha_\ell + Y_\ell)|\theta^2 - \theta^1|$, $\forall \theta^1$, $\theta^2 \in \Re_+$, and $\mathbb{E}[\exp(\alpha_\ell + Y_\ell)] = 1$. $\square$

Expressions (8)-(9) can be easily used in a Monte Carlo simulation to compute unbiased estimates of the deltas as sample averages, using the same price samples used to estimate V. Analogous expressions persist when linear program (2)-(5) is solved by using the GA heuristic, as conditions A1 and A4 in this proof hold unchanged, the analogous of condition A3, with v replaced by v^{GA} and w^1 and w^2 interpreted as feasible solutions to (2)-(5) computed by GA, holds by feasibility of each GA based solution, which then implies that the analogous of condition A2 remains true.

To apply the likelihood ratio we define

$$X_\ell := \ln f_\ell = \ln F_\ell + \alpha_\ell + Y_\ell \tag{13}$$

and $\mu_\ell := \ln F_\ell + \alpha_\ell$, so that $X \sim N(\mu, \Sigma)$. We denote by $g(x)$ the risk neutral probability density function of X. The basic idea behind the likelihood ratio method is to compute the greeks by exploiting the dependence of $g(\cdot)$ on a parameter of interest θ (this is not restricted to be F_ℓ). Following Broadie and Glasserman [6] and Glasserman [17, §7.3], expressing this dependence as $g_\theta(\cdot)$ and the dependence of v on a realization x of X as $v(x)$, we can write $V = \delta \int_{\Re^n} v(x) g_\theta(x) dx$. Assuming that exchanging differentiation and integration is warranted and defining $\dot{g}_\theta(x) := \partial g_\theta(x)/\partial\theta$ and $\ddot{g}_\theta(x) := \partial \dot{g}_\theta(x)/\partial\theta$, it holds that

$$\begin{aligned} \partial V/\partial\theta &= \delta \int_{\Re^n} v(x)[\dot{g}_\theta(x)/g_\theta(x)] g_\theta(x) dx \equiv \delta\mathbb{E}[v(X)\dot{g}_\theta(X)/g_\theta(X)] \\ \partial^2 V/\partial\theta^2 &= \delta \int_{\Re^n} v(x)[\ddot{g}_\theta(x)/g_\theta(x)] g_\theta(x) dx \equiv \delta\mathbb{E}[v(X)\ddot{g}_\theta(X)/g_\theta(X)]. \end{aligned}$$

The delta and gamma associated with each location ℓ can be computed by Monte Carlo simulation by letting $\theta = F_\ell$ and finding expressions for the ratios $\dot{g}_\theta(x)/g_\theta(x)$ and $\ddot{g}_\theta(x)/g_\theta(x)$. Lemma

1 presents such expressions: (14) is available in Glasserman [17, Example 7.3.4], (15) extends it based on simple linear algebra. Notationally, $\dot{\mu}(\theta)$ and $\ddot{\mu}(\theta)$, respectively, are the vectors of first and second derivatives with respect to θ of the elements of μ, each interpreted as a function of θ. We represent X as $\mu(\theta) + AZ$, with $Z \sim N(0, I)$ and A a lower triangular matrix such that $AA^\top = \Sigma$ (here $\cdot^\top$ denotes transposition). We define $b := Z^\top A^{-1}$; below, we denote its ℓ-th element as b_ℓ.

Lemma 1. *Under model (13), if μ depends on parameter θ but Σ does not, it holds that*

$$\dot{g}_\theta(x)/g_\theta(x) = b\dot{\mu}(\theta) \tag{14}$$

$$\ddot{g}_\theta(x)/g_\theta(x) = [b\dot{\mu}(\theta)]^2 - \dot{\mu}(\theta)^\top \Sigma^{-1} \dot{\mu}(\theta) + b\ddot{\mu}(\theta). \tag{15}$$

Proposition 5 follows from Lemma 1 with $\theta = F_\ell$. Results analogous to this and those established in the remainder of this section hold when one uses the GA version of our method.

Proposition 5 (Likelihood ratio deltas and gammas)**.** *Under model (13), it holds that*

$$Delta_\ell = \delta\mathbb{E}[v(X)b_\ell]/F_\ell, \ \forall \ell \in \mathcal{P} \tag{16}$$

$$Gamma_\ell = \delta\mathbb{E}[v(X)(b_\ell^2 - \Sigma_{\ell\ell}^{-1} + b_\ell)]/(F_\ell)^2, \ \forall \ell \in \mathcal{P}. \tag{17}$$

It is possible to combine the pathwise and likelihood ratio methods to obtain a hybrid gamma estimator, as in Glasserman and Zhao [18] and Glasserman [17, §7.3.3]. Proposition 6, stated without proof, presents the result of applying the likelihood ratio method to the pathwise expression for a delta. To streamline the exposition, we define $G_i := -\delta f_i[\sum_{j \in \mathcal{D}(i)} w_{ij}^*/(1 - \phi_{ij})]/F_i$, $\forall i \in \mathcal{R}$, and $G_j := \delta f_j[\sum_{i \in \mathcal{R}(j)} w_{ij}^*]/F_j$, $\forall j \in \mathcal{D}$, that is, we can equivalently express the right hand sides of (8) and (9) as $\mathbb{E}[G_i]$ and $\mathbb{E}[G_j]$, respectively.

Proposition 6 (Hybrid gamma)**.** *Under model (13), applying the pathwise and the likelihood ratio methods in this order yields*

$$Gamma_\ell = \mathbb{E}[G_\ell(b_\ell - 1)]/F_\ell, \ \forall \ell \in \mathcal{P}. \tag{18}$$

5.2 Etas and Vegas

The likelihood method can be applied to compute other greeks. Lemma 2 provides the relevant formulas: (20) extends (19), available in Glasserman [17, Example 7.3.4], based on simple linear algebra (tr($\cdot$) denotes the trace of $\cdot$). When Σ and hence A depend on θ, $b(\theta)$ denotes $Z^\top A^{-1}(\theta)$.

Lemma 2. *Under model (13), if Σ depends on parameter θ but μ does not then*

$$\dot{g}_\theta(x)/g_\theta(x) = -tr(A^{-1}(\theta)\dot{\Sigma}(\theta)A^{-1}(\theta)^\top)/2 + b(\theta)\dot{\Sigma}(\theta)b(\theta)^\top/2. \tag{19}$$

If both μ and Σ depend on θ then

$$\begin{aligned} \dot{g}_\theta(x)/g_\theta(x) &= -tr(A^{-1}(\theta)\dot{\Sigma}(\theta)A^{-1}(\theta)^\top)/2 + b(\theta)\dot{\Sigma}(\theta)b(\theta)^\top/2 \\ &\quad +2\mu(\theta)^\top\Sigma^{-1}(\theta)\dot{\mu}(\theta) + Z^\top A(\theta)^\top\Sigma^{-1}(\theta)^\top\mu(\theta). \end{aligned} \tag{20}$$

Lemma 2 can be used to derive expressions for the greeks $\text{Eta}_{\ell\ell'} := \partial V/\partial\rho_{\ell\ell'}$, $\forall\ell,\ \ell' \in \mathcal{P}$ and $\ell \neq \ell'$, and $\text{Vega}_\ell := \partial V/\partial\sigma_\ell$, $\forall\ell \in \mathcal{P}$, for given specifications of model (13). Proposition 7 presents expressions for these greeks when α and Σ in model (13) are specified according to a multilocation mean reverting process (see Secomandi [32] for the two location case), in which case it holds that

$$\alpha_\ell = (\sigma_\ell)^2[1-\exp(-2\kappa_\ell T)]/(4\kappa_\ell),\ \forall\ell \in \mathcal{P} \tag{21}$$

$$\Sigma_{\ell\ell'} = \rho_{\ell\ell'}\sigma_\ell\sigma_{\ell'}\{1-\exp[-2(\kappa_\ell+\kappa_{\ell'})T)]\}/(\kappa_\ell+\kappa_{\ell'}),\ \forall\ell,\ell' \in \mathcal{P}; \tag{22}$$

here κ_ℓ and σ_ℓ, respectively, are the speed of mean reversion and volatility of the natural logarithm of the spot price at location ℓ, $\rho_{\ell\ell'}$ is the instantaneous correlation associated with locations ℓ and ℓ', and $\Sigma_{\ell\ell'}$ is the element of Σ in position (ℓ,ℓ'). Proposition 7 follows from Lemma 2.

Proposition 7 (Eta and Vega)**.** *Consider model (21)-(22). It holds that*

$$Eta_{\ell\ell'} = \delta\mathbb{E}[v(X)\dot{\Sigma}_{\ell\ell'}(\rho_{\ell\ell'})[b_\ell(\rho_{\ell\ell'})b_{\ell'}(\rho_{\ell\ell'}) - H_{\ell\ell'}(\rho_{\ell\ell'})]],\ \forall\ell,\ \ell' \in \mathcal{P},\ \ell \neq \ell' \tag{23}$$

$$Vega_\ell = \delta\mathbb{E}[v(X)\{b_\ell(\sigma_\ell)[\sum_{k\in\mathcal{P},k\neq\ell} b_k(\sigma_\ell)\dot{\Sigma}_{\ell k}(\sigma_\ell) + b_\ell(\sigma_\ell)\dot{\Sigma}_{\ell\ell}(\sigma_\ell)/2] + b_\ell(\sigma_\ell)\dot{\mu}_\ell(\sigma_\ell) + K_\ell(\sigma_\ell)\}],\ \forall\ell \in \mathcal{P}, \tag{24}$$

where

$$\begin{aligned} \dot{\Sigma}_{\ell\ell'}(\rho_{\ell\ell'}) &= \sigma_\ell\sigma_{\ell'}\{1-\exp[(\kappa_\ell+\kappa_{\ell'})T]\}/(\kappa_\ell+\kappa_{\ell'}),\ \forall\ell,\ \ell' \in \mathcal{P},\ \ell \neq \ell' \\ H_{\ell\ell'}(\rho_{\ell\ell'}) &:= \sum_{k\in\mathcal{P},k\geqslant\max\{\ell,\ell'\}} A^{-1}_{k\ell}(\rho_{\ell\ell'})A^{-1}_{k\ell'}(\rho_{\ell\ell'}),\ \forall\ell,\ \ell' \in \mathcal{P},\ \ell \neq \ell' \\ K_\ell(\sigma_\ell) &:= 2\dot{\mu}_\ell(\sigma_\ell)\sum_{k\in\mathcal{P}}\mu_k(\sigma_\ell)\Sigma^{-1}_{k\ell}(\sigma_\ell) \\ &\quad - \sum_{k\in\mathcal{P},k\geqslant\ell}[2\dot{\Sigma}_{\ell\ell}(\sigma_\ell)A^{-1}_{k\ell}(\sigma_\ell) + \sum_{m\in\mathcal{P},m\neq\ell,m\leqslant k}\dot{\Sigma}_{\ell m}(\sigma_\ell)A^{-1}_{km}(\sigma_\ell)],\ \forall\ell \in \mathcal{P} \\ \dot{\Sigma}_{\ell\ell'}(\sigma_\ell) &= \rho_{\ell\ell'}\sigma_\ell\{1-\exp[(\kappa_\ell+\kappa_{\ell'})T]\}/(\kappa_\ell+\kappa_{\ell'}),\ \forall\ell,\ \ell' \in \mathcal{P} \\ \dot{\mu}_\ell(\sigma_\ell) &= \sigma_\ell[1-\exp(-2\kappa_\ell T)]/(2\kappa_\ell),\ \forall\ell \in \mathcal{P}. \end{aligned}$$

The terms $H_{\ell\ell'}(\rho_{\ell\ell'})$ and $K_\ell(\sigma_\ell)$ in this proposition are deterministic. Hence, they can be computed before performing a Monte Carlo simulation. A result analogous to Proposition 7 can also be established for the stated multilocation version of Black's model.

6 Spread Option Based Approach

In this section we present an approach to value a network contract based on spread options and linear programming. We take this approach as representative of spread option based methods used in practice (see the discussion in §1). This model computes a value V^{SO} as the optimal objective function value of the linear program

$$\max_{w} \delta \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} S_{ij}^{+} w_{ij} \text{ s.t. (3)-(5),} \tag{25}$$

where $S_{ij}^{+} := \mathbb{E}[s_{ij}^{+}]$ is the undiscounted value of a spread option on s_{ij}^{+}. The value and the greeks of a spread option cannot be computed exactly when one uses common reduced form models of commodity prices, such as that used in §5. However, one could use closed form approximations, such as Kirk's [21] or those proposed by Carmona and Durrleman [7] and Li et al. [25].

The spread option based approach is extremely practical: first, one computes the values and greeks for all the relevant spread options; second, one determines a portfolio of spread options whose notional amounts are given by an optimal solution to (25). Once the portfolio composition is determined, given that the greeks of the relevant spread options are available, computation of the greeks of this portfolio is straightforward.

Proposition 8 establishes that in general this valuation approach outperforms the intrinsic valuation approach but is also suboptimal (this result assumes that V^{SO} is computed using exact spread option values); we assess the valuation performance of model (25) in §7.

Proposition 8 (Spread option valuation)**.** *It holds that* $V^{I} \leqslant V^{SO} \leqslant V$.

Proof. Let w^{SO} be an optimal solution to model (25); w^{SO} is deterministic and a feasible solution both to model (6) and model (2)-(5), in the latter case for every realization of the relevant prices at time T. Let w^{I} be an optimal solution to model (6). It follows from Jensen's inequality and the martingale property of futures prices under the risk neutral measure that $S_{ij} \leqslant S_{ij}^{+}$. Thus, it holds that

$$V^{I} = \delta \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} S_{ij} w_{ij}^{I} \leqslant \delta \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} S_{ij}^{+} w_{ij}^{I} \leqslant \delta \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} S_{ij}^{+} w_{ij}^{SO} = V^{SO}.$$

Let w^{*} be an optimal solution to model (2)-(5); w^{*} is a random quantity as of time 0. It follows that

$$V^{SO} = \delta \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} S_{ij}^{+} w_{ij}^{SO} = \delta \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} \mathbb{E}[s_{ij}^{+}] w_{ij}^{SO} \leqslant \delta \mathbb{E}[\sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{D}(i)} s_{ij} w_{ij}^{*}] = \delta \mathbb{E}[v] = V. \square$$

The intuition behind this result rests on the observation that model (25) only captures part of the optionality embedded in a network contract: once the optimal portfolio of spread option is determined at time 0, one does retain the option to transport natural gas at time T on link i-j if and only if the spread s_{ij} is positive, but this option is constrained by the decision made at time 0 on the maximum amount of gas to be shipped on this link at time T, that is, w^*_{ij}. Put differently, the spread option based valuation approach is optimal, provided that each spread option is valued exactly, when the maximum amount of natural gas that can be *optimally* transported along a link does not depend on knowledge of the prices realized at time T. This occurs when the receipt or the delivery point sets are singletons, because in this case this maximum for link i-j is $\min\{C_i, C_j\}$.

In this case, Monte Carlo simulation is relevant to compute unbiased estimates of the spread option values and greeks. In particular, computation of some of the greeks simplifies relative to the results presented in §5, and one can also use the pathwise method to compute unbiased estimates of the two vegas and the eta of each spread option (we do not show this here for brevity). Moreover, the eta of any two receipt (respectively, delivery) points of a single delivery (respectively, receipt) point contract are zero, because the spread options that represent this contract do not depend on the correlation between the prices at these locations.

7 Computational Results

In this section we present our computational results on the comparison of the valuations of our method and the spread option based approach, and the estimates of the greeks obtained with our method. Before discussing these results, we illustrate the data and the test instances used.

7.1 Data and Instances

We base our computational analysis on the Transco pipeline system, which extends from Texas to New York City, New Jersey, and Pennsylvania. It has six pricing zones. We focus on Henry Hub, Louisiana, and Zones 1-4; Zones 1 and 2 are in Texas, Zone 3 covers Louisiana, excluding Henry Hub, and Mississippi, and Zone 4 is in Alabama. For Henry Hub and these zones we obtained spot prices covering the period 1/2/2001-11/30/2006, and futures prices with December 2006 maturity traded on 6/1/2006, which we take as the contract valuation date (the December 2006 maturity is 11/28/2006, the closing date of the December 2006 NYMEX natural gas futures contract).

In this section we use the specification (21)-(22) of model (7), that is, we assume that the natural logarithms of the spot prices at these locations follow correlated mean reverting processes. As discussed in §5, this is a simple model that captures the spatial structure of the correlation

Table 2: Price model parameter estimates.

	Henry Hub	Zone 1	Zone 2	Zone 3	Zone 4
Mean Reversion Rate (κ_ℓ)	3.1856	4.1276	4.2477	3.5554	3.6016
Volatility (σ_ℓ)	1.0267	1.1130	1.1916	1.0922	1.1095
Instantaneous Correlation Coefficient ($\rho_{\ell\ell'}$)					
	Henry Hub	Zone 1	Zone 2	Zone 3	Zone 4
Henry Hub	1.0000	0.9118	0.9363	0.9581	0.9491
Zone 1	0.9118	1.0000	0.9011	0.9159	0.9149
Zone 2	0.9363	0.9011	1.0000	0.9329	0.9317
Zone 3	0.9581	0.9159	0.9329	1.0000	0.9814
Zone 4	0.9491	0.9149	0.9317	0.9814	1.0000

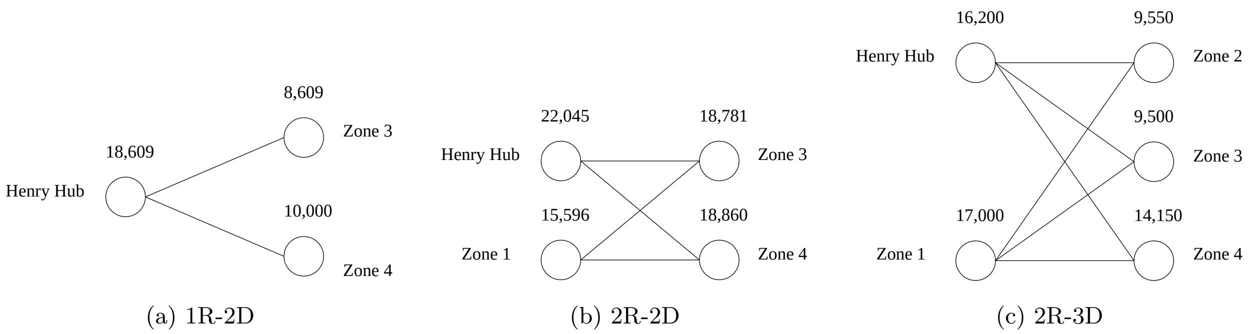

Figure 2: Contract networks used in our computational study.

among the prices at these locations, but is limited in the type of correlation term structure that it can represent at each location. Because we focus on a single maturity (December 2006), this modeling choice seems reasonable.

We calibrate the parameters of this model using standard methods (Clewlow and Strickland [8, §3.2.2]) applied to the described spot price data. The resulting root mean squared errors corresponding to Henry Hub and Zones 1-4 are 0.4372, 0.3911, 0.5278, 0.4466, and 0.4637, respectively (the corresponding minimum observed spot prices are 1.74, 1.385, 1.45, 1.51, and 1.575; the corresponding maximum prices are 18.60, 15.025, 19.50, 17.805, and 18.945, and the corresponding averages are 5.7003, 5.3815, 5.6112, 5.7634, and 5.8206; these prices are measured in \$/MMBtu). Because we have available the futures prices at each location of interest on the valuation date and we work with the risk neutral measure, the only parameters we need are κ_ℓ and σ_ℓ for each location $\ell \in \mathcal{P} := \{$Henry Hub, Zones 1-4$\}$, and $\rho_{\ell\ell'}$ for each pair of locations ℓ and $\ell' \in \mathcal{P}$ (if $\ell = \ell'$ then $\rho_{\ell\ell'} = 1$). Table 2 reports the estimates of these parameters, which we use in our computational study. Henry Hub has smaller mean reversion rate and spot volatility estimates that the other locations. The estimates of the instantaneous correlation coefficients are almost all above 90%, which is natural for natural gas prices (see, e.g., Secomandi [32]).

Table 3: Commodity and fuel rates (Source: Transco pipeline online information system).

Commodity Rate ($/MMBtu)					
	Henry Hub	Zone 1	Zone 2	Zone 3	Zone 4
Henry Hub	n.a.	0.00652	0.00507	0.00268	0.01372
Zone 1	0.00652	0.00162	0.00401	0.00652	0.01756
Zone 2	0.00507	0.00401	0.00251	0.00507	0.01611
Zone 3	0.00268	0.00652	0.00507	0.00268	0.01372
Zone 4	0.01372	0.01756	0.01611	0.01372	0.01121
Fuel Rate (%)					
	Henry Hub	Zone 1	Zone 2	Zone 3	Zone 4
Henry Hub	n.a.	1.05	0.78	0.39	2.14
Zone 1	1.05	0.27	0.66	1.05	2.80
Zone 2	0.78	0.66	0.39	0.78	2.53
Zone 3	0.39	1.05	0.78	0.39	2.14
Zone 4	2.14	2.80	2.53	2.14	1.75

We consider the three contract networks shown in Figure 2, which we chose based upon examination of real contracts available on the Transco pipeline online information system. Network (a) models a contract with one receipt and two delivery points (1R-2D), network (b) a contract with two receipt and two delivery points (2R-2D), and network (c) a contract with two receipt and three delivery points (2R-3D). The numbers associated with each point are the point capacities, expressed in MMBtu/day. The term of each contract is the December 2006 month. We use the commodity and fuel rates reported in Table 3, which are available from the Transco pipeline online information system. We set the risk free rate equal to 4.75%, which is the one month U.S. Treasury rate on 6/1/2006, the contract valuation date for each of the test instances.

7.2 Valuation and Computational Performance

This subsection discusses the valuation and computational performance of the exact and GA versions of our method and of the spread option based method. All these methods were coded in C++ and compiled using the compiler `g++`. The reported Cpu times are obtained on a computer with one 2.20 GHz processor and 1.96 GB of RAM. We solve all the relevant linear programs using the Clp linear solver (see §4.2).

Table 4 displays the valuation and computational performance of the exact method on the three test instances, when varying the number of sample paths from 10,000 to 100,000 in increments of 10,000. For all these instances, the estimated valuations, indicated by $\hat{V}$, are at least 98.95% of their respective valuations obtained with 100,000 samples. The standard errors of these valuations decrease substantially when increasing the number of sample paths, dropping from roughly 0.91-

Table 4: Valuation and computational performance results; SE abbreviates standard error.

	Instance								
	1R-2D			2R-2D			2R-3D		
Number of Samples	$\hat{V}$	SE	Cpu Seconds	$\hat{V}$	SE	Cpu Seconds	$\hat{V}$	SE	Cpu Seconds
10000	9064.03	138.12	1.30	30870.31	320.56	2.00	31618.43	287.82	2.31
20000	9066.06	98.77	2.70	30938.46	226.31	4.06	31647.96	202.97	4.74
30000	9032.51	80.98	3.94	31007.03	185.49	5.98	31486.65	167.06	6.94
40000	9027.54	70.05	5.30	30978.28	162.32	7.97	31570.51	144.82	9.23
50000	9046.92	63.13	6.58	30943.67	144.92	10.00	31579.31	129.86	11.55
60000	9096.53	57.82	7.89	31001.80	132.57	11.97	31610.41	118.62	13.84
70000	9109.25	53.67	9.19	31003.96	122.41	13.98	31617.72	110.01	16.19
80000	9116.45	50.36	10.53	31087.91	115.02	15.97	31613.84	102.76	18.48
90000	9112.70	47.42	11.86	31106.55	108.56	17.94	31638.82	96.89	20.77
100000	9123.09	45.08	13.17	31128.10	103.20	19.94	31635.80	91.84	23.09

Table 5: Comparative performance of the GA based method with 100,000 samples.

		SE		CPU Seconds	
Instance	$\hat{V}^{GA}/\hat{V}$	$\hat{V}^{GA}$	$\hat{V}$	GA	Exact
1R-2D	1.0000	45.08	45.08	2.41	13.17
2R-2D	0.9987	103.01	103.20	2.49	19.94
2R-3D	0.9974	91.57	91.84	2.63	23.09

1.52% of their valuations with 10,000 samples to 0.41-0.70% and 0.29-0.49% of their valuations with 50,000 and 100,000 samples, respectively.

Of course, obtaining more accurate valuations, in the sense of reduced standard errors, by increased sampling requires more Cpu seconds. For the 1R-2D instance, the Cpu seconds increase from 1.36 with 10,000 samples to 6.77 and 13.38 with 50,000 and 100,000 samples, respectively; for the 2R-2D instance these figures are 2.02, 10.13, and 20.30; for the 2R-3D instance they are 2.34, 11.58, and 23.47. Notice that these Cpu seconds include the computation of all the greeks discussed in §7.3, as well as the intrinsic, extrinsic, and spread option based contract values. Despite the additional computational effort needed to obtain more accurate valuations, it seems fair to conclude that the proposed method is computationally efficient.

With the same 100,000 samples, the valuation estimates of the GA version of our method, each denoted by $\hat{V}^{GA}$, as well as their accuracies, essentially match those of the exact method on the three considered instances, but can be computed with a much lower computational effort (see Table 5). Thus, the GA version of our method is a fast and accurate heuristic.

We now discuss the intrinsic and extrinsic values. Table 6 reports these values as fractions of the valuations obtained by the exact method with 100,000 samples. The 1R-2D instance exhibits

Table 6: Intrinsic and extrinsic values (fraction of $\hat{V}$ computed with 100,000 samples).

	Instance		
	1R-2D	2R-2D	2R-3D
$\hat{V}^I$	0.0683	0.4855	0.5108
$\hat{V}^E$	0.9317	0.5145	0.4892

Table 7: Valuation performance of the spread option heuristic (fraction of $\hat{V}$ computed with 100,000 samples).

Instance		
1R-2D	2R-2D	2R-3D
1.0000	0.9497	0.9078

a very large extrinsic value (93.17%). This is due to the two spread options that represent this contract being almost at the money at contract inception, which implies that there is significant uncertainty as to whether these spreads would expire in or out of the money. The extrinsic values of the 2R-2D and 2R-3D contracts are lower than those of the 1R-2D contract but are nevertheless significant, being 51.45% in the 2R-2D case and 48.92% in the 2R-3D case. This is due to the spread corresponding to the Zone 1-4 link being deep in the money at each contract inception time, so that this link will be used with high probability at the expiration time. Thus, the contract intrinsic values play a larger role in the 2R-2D and 2R-3D cases than in the 1R-2D case.

Table 7 compares the valuation performance of the spread option heuristic to that of the exact method. We estimate the relevant spread option valuations by Monte Carlo simulation using the same samples used by the exact method. We focus on the case of 100,000 samples. The spread option heuristic is obviously optimal in the 1R-2D case. This heuristic captures 94.97% and 90.78% of the valuations of the exact method in the 2R-2D and 2R-3D cases, respectively. Thus, methods used in practice can substantially undervalue network contracts relative to our exact method.

As remarked in §6, this valuation performance gap occurs because the spread option heuristic is unable to fully capture the extrinsic value embedded in these contracts. To illustrate, consider the 2R-2D contract. The spread option heuristic assigns zero and positive flows, respectively, to the Zone 1-3 and Zone 1-4 spread options at contract inception. If the Zone 1-4 spread is out of the money at contract expiration, no injection is made in Zone 1. However, if in this case the Zone 1-3 spread is in the money at this time, it may be optimal to ship natural gas from Zone 1 to Zone 3, which is feasible for the exact model but impossible for the given solution of the spread option heuristic. Moreover, even when both the Zone 1-3 and Zone 1-4 spread options expire in the money, it may be optimal to ship natural gas from Zone 1 to Zone 3, rather than from Zone

Table 8: Computational requirements of the resimulation and direct approaches for estimating all the considered sensitivities; the times for the direct methods include those needed to compute the greeks, as well as the total (intrinsic plus extrinsic) and spread option based valuations.

		Cpu Seconds		
		Instance		
Approach	Number of Samples	1R-2D	2R-2D	2R-3D
Direct Exact	100,000	13.17	19.94	23.09
Resimulation Exact	100,000	233.14	551.45	903.61
Direct Exact	1,000,000	131.58	198.00	229.24
Resimulation GA	1,000,000	387.09	619.22	918.41
Direct GA	1,000,000	24.41	25.00	26.23

1 to Zone 4. By not fully capturing these features, the spread option heuristic underestimates the contract value.

7.3 Sensitivity Estimates

This subsection discusses the computation of the greeks discussed in §5. We use resimulation to benchmark the direct approaches. We implement the resimulation approach by employing central difference approximations with common random numbers for all the considered greeks, with relevant increments equal to 0.01 for the deltas and gammas and 0.001 for the vegas and etas. We use these same random numbers when estimating the greeks with the direct approaches. Table 8 displays the computational requirements of all the approaches on the three test instances for different sample sizes (the times of the direct approaches include the computation of the greeks using the pathwise, likelihood ratio, and combined pathwise and likelihood ratio methods, the intrinsic and extrinsic values of the contract, and its spread option based value; the reason for considering 1,000,000 samples is explained below). This table shows that direct estimation of the greeks is crucial to make the proposed method computationally efficient.

Tables 9-11 report the estimated greeks for the three test instances obtained by the exact method with 100,000 samples; here and in some of the subsequent tables the labels PW, LR, and RS abbreviate pathwise, likelihood ratio, and resimulation, respectively, and PWLR refers to the combined pathwise and likelihood ratio method.

Consider the results pertaining to the 1R-2D instance, that is, Table 9. All the methods yield similar deltas, but the pathwise and the resimulation methods feature standard errors that are very similar and smaller than those of the likelihood ratio method. There are more noticeable differences among the different methods in terms of their gamma estimates, e.g., compare the Henry Hub gamma estimated by resimulation to the other estimates. The likelihood ratio method displays

Table 9: Sensitivity estimates for the 1R-2D instance obtained by the exact model with 100,000 samples (SEs in parenthesis).

	Henry Hub	Zone 3	Zone 4
Delta			
PW	-8875.82	4600.32	5057.73
	(29.46)	(16.41)	(19.26)
LR	-8871.67	4569.15	5094.42
	(69.57)	(83.69)	(79.44)
RS	-8875.04	4599.92	5057.35
	(29.43)	(16.38)	(19.23)
Gamma			
LR	5993.74	2788.26	3067.36
	(120.44)	(184.72)	(163.54)
PWLR	6036.63	2867.27	3020.57
	(30.87)	(30.85)	(31.62)
RS	6154.26	2949.58	3018.13
	(208.80)	(138.20)	(149.45)
Vega			
LR	2319.53	2130.72	3729.19
	(290.10)	(364.13)	(373.08)
RS	2500.12	2353.03	3398.38
	(160.72)	(89.76)	(104.00)
Eta			
LR			
Zone 3	-43660.07		
	(1908.19)		
Zone 4	-48827.70		
	(1728.88)		
RS			
Zone 3	-44748.40		
	(281.97)		
Zone 4	-48122.34		
	(305.59)		

the largest gamma standard errors. However, using this method in combination with the pathwise method yields the smallest gamma standard errors; the reduction in these errors is noteworthy. These results are consistent with those reported by Broadie and Glasserman [6], Glasserman and Zhao [18], and Glasserman [17, §7.3.3]. We mention in passing that the signs of all the delta and gamma estimates are consistent with Proposition 3.

The likelihood ratio vega estimates appear to be somewhat different from the resimulation vega estimates, and the latter display a significantly smaller standard error. It is important to recall that the resimulation approach is in general biased, whereas the likelihood ratio method is not. We will see later that there is reason to believe that the bias of the displayed resimulation estimates should be small. The signs of the vegas are all positive, which indicates that an increase in each relevant volatility parameter leads to a higher valuation. This need not be the case, as it is well known (see, e.g., Eydeland and Wolyniec [13, pp. 342-349]) that the vega of an exchange option, a spread option with zero strike price, can be negative for some combinations of volatility parameters and instantaneous correlation coefficient. However, this cannot occur when this correlation coefficient is sufficiently high. Our results are consistent with the latter case.

The likelihood ratio and resimulation eta estimates are roughly similar, with the former exhibiting larger standard errors than the latter. All the eta estimates are negative, which is intuitive, as increased correlation leads to prices being more likely to move in the same direction, and hence to price spreads being less likely to take on large positive values (the eta corresponding to Zones 3 and 4 is zero, as discussed at the end of §6, and is not displayed in Table 9). More interesting, the absolute values of most of the etas are much larger than those of the other greeks. This shows that the correlation structure is a key driver of the value of pipeline transport capacity.

The results for the 2R-2D and 2R-3D instances, reported in Tables 10-11, are substantially similar to those of the 1R-2D instance. For brevity, we do not discuss them here. We only make two remarks. First, there are noticeable differences between some of the eta estimates obtained by the likelihood and resimulation methods. Second, whereas in the 1R-2D instance the eta corresponding to the two delivery points is zero, this need not be the case for the two other network structures; similarly, the eta of the two receipt points of a hypothetical 2R-1D contract would also be zero, while this is not so for the eta of the two receipt points in the considered 2R-2D and 2R-3D instances.

In summary, considering the direct estimation techniques, the pathwise method is the most accurate method for estimating the deltas; combined with the likelihood ratio method, it also yields the most accurate estimates of the gammas. Compared to the vega and eta estimates obtained with the resimulation method, which are in general biased, those of the likelihood ratio method,

Table 10: Sensitivity estimates for the 2R-2D instance obtained by the exact method with 100,000 samples (SEs in parenthesis).

	Henry Hub	Zone 1	Zone 3	Zone 4
Delta				
PW	-9729.80	-11006.01	11879.19	10619.64
	(35.87)	(27.18)	(34.25)	(35.67)
LR	-9681.39	-11008.02	11956.81	10510.93
	(156.07)	(120.24)	(227.12)	(210.82)
RS	-9729.77	-11005.68	11878.81	10619.49
	(35.82)	(27.14)	(34.18)	(35.62)
Gamma				
LR	7488.84	3839.01	7573.40	7400.37
	(241.49)	(157.32)	(502.06)	(425.65)
PWLR	7435.05	3902.28	7891.83	7482.54
	(37.70)	(24.69)	(66.89)	(59.49)
RS	7104.51	3775.57	7675.32	7401.32
	(275.66)	(204.40)	(295.84)	(280.73)
Vega				
LR	3872.76	1911.59	4848.29	7732.76
	(645.24)	(405.20)	(1032.84)	(1024.33)
RS	3963.58	1454.85	5730.52	7464.97
	(184.57)	(141.80)	(206.24)	(203.09)
Eta				
LR				
Zone 1	-10497.85			
	(1751.77)			
Zone 3	-52730.57	-17575.38		
	(4217.55)	(2690.29)		
Zone 4	-51705.42	-17558.07	-49345.00	
	(3694.72)	(2587.17)	(6291.85)	
RS				
Zone 1	-9636.82			
	(499.65)			
Zone 3	-51794.88	-20346.97		
	(713.42)	(502.71)		
Zone 4	-51954.38	-17202.20	-51162.97	
	(690.74)	(486.74)	(1165.21)	

Table 11: Sensitivity estimates for the 2R-3D instance obtained by the exact method with 100,000 samples (SEs in parenthesis).

	Henry Hub	Zone 1	Zone 2	Zone 3	Zone 4
Delta					
PW	−6421.67	−12004.31	5685.17	6170.17	8366.39
	(24.02)	(27.58)	(18.17)	(17.17)	(25.91)
LR	−6392.15	−12094.96	5713.78	5966.62	8571.22
	(149.02)	(120.74)	(119.22)	(208.60)	(196.21)
RS	−6422.45	−12003.79	5685.82	6170.15	8366.16
	(23.99)	(27.56)	(18.16)	(17.15)	(25.88)
Gamma					
LR	5584.34	4207.54	2605.58	3659.07	5111.38
	(232.17)	(164.64)	(139.08)	(444.74)	(389.06)
PWLR	5371.86	4278.98	2652.77	3824.88	5167.50
	(28.85)	(27.53)	(17.68)	(34.39)	(46.76)
RS	5237.55	4321.30	2536.41	3537.35	4966.77
	(172.90)	(168.60)	(113.24)	(128.94)	(164.57)
Vega					
LR	4711.89	2056.15	3043.71	1579.60	6358.92
	(639.33)	(397.25)	(489.01)	(937.35)	(879.43)
RS	4103.42	1986.70	3255.41	2915.40	5748.96
	(119.66)	(148.47)	(95.37)	(104.86)	(152.94)
Eta					
LR					
Zone 1	−15744.76				
	(1778.82)				
Zone 2	−16058.98	−8572.60			
	(2092.32)	(1521.65)			
Zone 3	−18476.55	−12896.89	−4757.22		
	(3693.14)	(2646.45)	(2939.30)		
Zone 4	−34176.62	−12995.45	−11288.74	−22890.29	
	(3429.25)	(2510.37)	(2814.36)	(5767.57)	
RS					
Zone 1	−14027.33				
	(441.03)				
Zone 2	−15802.27	−8637.26			
	(563.49)	(313.75)			
Zone 3	−20647.73	−12195.18	−4424.59		
	(390.31)	(267.72)	(333.63)		
Zone 4	−31066.45	−16444.79	−12291.19	−23376.46	
	(563.86)	(372.70)	(463.62)	(885.09)	

which are provably unbiased, exhibit higher standard errors.

With the motivation of obtaining more precise likelihood ratio estimates of the vegas and the etas in a faster fashion than the resimulation approach, we increase the number of samples to 1,000,000 and use GA to heuristically solve each relevant linear program. Table 8 displays the Cpu requirement of doing so; these are much smaller than those required by resimulation with GA or the direct and exact method with the same number of samples. Tables 12-14 display the resulting estimates for each of the three test instances. For completeness, these tables report the estimates and standard errors of all the considered greeks, not only those of the vegas and etas. For brevity we do not report the results pertaining to the resimulation and direct and exact methods with 1,000,000 samples, but these are very similar to those obtained with the direct and GA version of our method with the same number of samples.

The likelihood ratio vega and eta sections of these tables reveal that the standard errors of these greeks significantly decrease when compared to their analogous errors reported in Tables 9-11, but they are not as small as those obtained with the resimulation method with 100,000 samples. However, in almost all the cases the reported estimates appear to be consistent with each other. This is also true of the estimates of the other greeks, which suggests that the resimulation approach delivers very good estimates of the greeks. However, this also suggests that despite its very good estimation performance, the dramatically higher computational requirement of the resimulation approach puts it at a marked disadvantage with respect to the direct approaches. In contrast, the GA based direct method strikes a very good compromise between the accuracy of its greek estimates, expressed by the size of their standard errors, and its Cpu requirement, while also computing nearly optimal valuations.

8 Conclusions

In this paper we study the problem of computing the real option value of network contracts for natural gas pipeline capacity and its sensitivities, an important problem faced by natural gas shippers. We show that models employed in practice, although fast, are in general suboptimal. Hence, we propose a novel and computationally efficient method, based on linear programming, Monte Carlo simulation, and direct greek estimation techniques, to compute unbiased estimates of the value of a network contract and its sensitivities. Our main findings, based on using real price data and realistic contract instances modeled after contracts available on the Transco pipeline, are as follows: our method can significantly improve the practice based valuations of these contracts,

Table 12: Sensitivity estimates for the 1R-2D instance obtained by the GA based method with 1,000,000 samples (SEs in parenthesis).

	Henry Hub	Zone 3	Zone 4
Delta			
PW	-8899.61	4611.63	5071.63
	(9.31)	(5.18)	(6.08)
LR	-8922.69	4648.33	5055.29
	(22.14)	(26.75)	(24.94)
Gamma			
LR	6104.17	2847.27	2932.05
	(38.88)	(59.88)	(51.01)
PWLR	6061.41	2874.28	2999.47
	(9.78)	(9.73)	(9.99)
Vega			
LR	2554.13	2408.16	3278.15
	(91.30)	(119.53)	(116.27)
Eta			
LR			
Zone 3	-45640.63		
	(618.50)		
Zone 4	-48332.50		
	(543.75)		

by up to about 10%; the application of direct greek estimation techniques is critical to make our method computationally efficient.

Our work is relevant to natural gas shippers; a major international energy trading company recently implemented a version of our method. Our work can be extended to deal with other features of the problem, such as primary/secondary point and contingent capacity rights that give shippers different priorities and more flexibility, respectively, in their use of contracted pipeline capacity. Our work can be further extended by testing our method on different data sets, or by using multilocation extensions of multifactor models of the evolution of commodity and energy prices, such as those of Schwartz and Smith [31] and Routledge et al. [28, 29].

Our work has potential relevance beyond the specific application of this paper. For example, applications and extensions beyond the natural gas industry include the real option valuation and hedging of shipping, transportation, distribution, and refining capacity for other energy sources and commodities, such as coal, oil, biofuels, metals, and agricultural products, including corn and soybean. These are examples of a more general area of research and applications dealing with business-to-business commerce and contracting in capital intensive industries (Kleindorfer and Wu [22]), which exhibits important real option capacity valuation issues (Birge [1]). In these settings,

Table 13: Sensitivity estimates for the 2R-2D instance obtained by the GA based method with 1,000,000 samples (SEs in parenthesis).

	Henry Hub	Zone 1	Zone 3	Zone 4
Delta				
PW	−9752.68	−10955.89	11873.12	10589.40
	(11.36)	(8.57)	(10.82)	(11.26)
LR	−9706.45	−10943.00	11829.88	10566.64
	(49.04)	(38.19)	(71.81)	(65.95)
Gamma				
LR	7379.97	3878.59	7492.09	7249.30
	(75.37)	(49.85)	(158.11)	(133.06)
PWLR	7444.00	3865.54	7835.73	7485.81
	(11.89)	(7.78)	(21.11)	(18.78)
Vega				
LR	4231.98	1674.23	5496.27	6830.83
	(204.99)	(127.84)	(330.95)	(307.29)
Eta				
LR				
Zone 1	−10016.47			
	(558.25)			
Zone 3	−49659.61	−20281.99		
	(1320.71)	(860.73)		
Zone 4	−53146.28	−16111.23	−47779.46	
	(1153.47)	(811.58)	(1986.11)	

Table 14: Sensitivity estimates for the 2R-3D instance obtained by the GA based method with 1,000,000 samples (SEs in parenthesis).

	Henry Hub	Zone 1	Zone 2	Zone 3	Zone 4
Delta					
PW	−6344.01	−11970.76	5628.14	6157.35	8308.94
	(7.58)	(8.76)	(5.75)	(5.44)	(8.20)
LR	−6305.81	−11948.06	5629.24	6086.00	8313.24
	(46.60)	(38.22)	(37.51)	(65.67)	(61.82)
Gamma					
LR	5317.18	4212.70	2672.85	3902.06	5056.90
	(71.49)	(51.94)	(43.98)	(140.49)	(123.49)
PWLR	5323.48	4243.81	2658.16	3837.88	5153.35
	(9.09)	(8.68)	(5.58)	(10.88)	(14.69)
Vega					
LR	4074.97	2080.21	3264.70	2820.35	5584.64
	(199.30)	(126.92)	(155.50)	(292.85)	(279.57)
Eta					
LR					
Zone 1	−15098.01				
	(560.36)				
Zone 2	−14888.08	−8164.85			
	(653.69)	(482.41)			
Zone 3	−21178.56	−11655.40	−5402.18		
	(1178.95)	(829.80)	(920.26)		
Zone 4	−29661.67	−15319.60	−13076.97	−23351.40	
	(1066.34)	(799.01)	(886.01)	(1834.16)	

extensions of our model may include the addition of lead times, inventory, and the consideration of market frictions, such as transaction costs and market power. For example, in a very recent paper, Martínez-de-Albéniz and Vendrell Simón [27] study the point to point version of the problem studied in this paper, that is, the problem considered by Secomandi [32], from the point of view of a trader with market power.

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A Proof of Proposition 3 (Deltas and gammas)

Define $\beta_\ell := \exp(\alpha_\ell + Y_\ell)$, so that $f_\ell = F_\ell \beta_\ell$, $\forall \ell \in \mathcal{P}$. To show that each delivery point delta is positive, fix arbitrary $\bar{j} \in \mathcal{D}$ and pick $F^2_{\bar{j}} > F^1_{\bar{j}}$. Given realizations of each random variable β_ℓ, denote by $v(F^1_{\bar{j}})$ and $v(F^2_{\bar{j}})$ the contract values at time T, conditional on the time 0 futures prices of delivery location $\bar{j}$ being $F^1_{\bar{j}}$ and $F^2_{\bar{j}}$, respectively, and the time 0 futures price at each other location being F_ℓ; analogously, let w^1 and w^2 be the corresponding optimal solutions to model (2)-(5). It holds that

$$\begin{aligned}
v(F^2_{\bar{j}}) &= \sum_{i \in \mathcal{R}(\bar{j})} [F^2_{\bar{j}} \beta_{\bar{j}} - \frac{f_i}{1-\phi_{i\bar{j}}} - K_{i\bar{j}}] w^2_{i\bar{j}} + \sum_{j \in \mathcal{D} \setminus \{\bar{j}\}} \sum_{i \in \mathcal{R}(j)} s_{ij} w^2_{ij} \\
&\geqslant \sum_{i \in \mathcal{R}(\bar{j})} [F^2_{\bar{j}} \beta_{\bar{j}} - \frac{f_i}{1-\phi_{i\bar{j}}} - K_{i\bar{j}}] w^1_{i\bar{j}} + \sum_{j \in \mathcal{D} \setminus \{\bar{j}\}} \sum_{i \in \mathcal{R}(j)} s_{ij} w^1_{ij} \\
&\geqslant \sum_{i \in \mathcal{R}(\bar{j})} [F^1_{\bar{j}} \beta_{\bar{j}} - \frac{f_i}{1-\phi_{i\bar{j}}} - K_{i\bar{j}}] w^1_{i\bar{j}} + \sum_{j \in \mathcal{D} \setminus \{\bar{j}\}} \sum_{i \in \mathcal{R}(j)} s_{ij} w^1_{ij} \\
&= v(F^1_{\bar{j}}).
\end{aligned}$$

Denote by $V(F^1_{\bar{j}})$ and $V(F^2_{\bar{j}})$ the contract values at time 0 given that the futures prices of delivery location $\bar{j}$ at this time are $F^1_{\bar{j}}$ and $F^2_{\bar{j}}$, respectively. Because this inequality holds for all realizations of random vector $\beta := (\beta_\ell,\ \ell \in \mathcal{P})$, it follows that $V(F^2_{\bar{j}}) = \delta \mathbb{E}[v(F^2_{\bar{j}})] \geqslant \delta \mathbb{E}[v(F^1_{\bar{j}})] = V(F^1_{\bar{j}})$, that is, V increases in $F_{\bar{j}}$.

To show that each delivery point gamma is positive, pick $\varphi \in (0,1)$, and define $F^\varphi_{\bar{j}} := \varphi F^1_{\bar{j}} + (1-\varphi) F^2_{\bar{j}}$. Analogous to $v(F^k_{\bar{j}})$ and w^k, $k = 1, 2$, let $v(F^\varphi_{\bar{j}})$ and w^φ be the optimal objective function value and an optimal solution of model (2)-(5) corresponding to $F_{\bar{j}}^{\varphi}$, respectively. Notice that

$$
\begin{aligned}
v(F_{\bar{j}}^{\varphi}) &= \sum_{i\in\mathcal{R}(\bar{j})}[F_{\bar{j}}^{\varphi}\beta_{\bar{j}}-\frac{f_i}{1-\phi_{i\bar{j}}}-K_{i\bar{j}}]w_{i\bar{j}}^{\varphi}+\sum_{j\in\mathcal{D}\backslash\{\bar{j}\}}\sum_{i\in\mathcal{R}(j)}s_{ij}w_{ij}^{\varphi}\\
&= \varphi\left\{\sum_{i\in\mathcal{R}(\bar{j})}[F_{\bar{j}}^{1}\beta_{\bar{j}}-\frac{f_i}{1-\phi_{i\bar{j}}}-K_{i\bar{j}}]w_{i\bar{j}}^{\varphi}+\sum_{j\in\mathcal{D}\backslash\{\bar{j}\}}\sum_{i\in\mathcal{R}(j)}s_{ij}w_{ij}^{\varphi}\right\}\\
&\quad+(1-\varphi)\left\{\sum_{i\in\mathcal{R}(\bar{j})}[F_{\bar{j}}^{2}\beta_{\bar{j}}-\frac{f_i}{1-\phi_{i\bar{j}}}-K_{i\bar{j}}]w_{i\bar{j}}^{\varphi}+\sum_{j\in\mathcal{D}\backslash\{\bar{j}\}}\sum_{i\in\mathcal{R}(j)}s_{ij}w_{ij}^{\varphi}\right\}\\
&\leqslant \varphi\left\{\sum_{i\in\mathcal{R}(\bar{j})}[F_{\bar{j}}^{1}\beta_{\bar{j}}-\frac{f_i}{1-\phi_{i\bar{j}}}-K_{i\bar{j}}]w_{i\bar{j}}^{1}+\sum_{j\in\mathcal{D}\backslash\{\bar{j}\}}\sum_{i\in\mathcal{R}(j)}s_{ij}w_{ij}^{1}\right\}\\
&\quad+(1-\varphi)\left\{\sum_{i\in\mathcal{R}(\bar{j})}[F_{\bar{j}}^{2}\beta_{\bar{j}}-\frac{f_i}{1-\phi_{i\bar{j}}}-K_{i\bar{j}}]w_{i\bar{j}}^{2}+\sum_{j\in\mathcal{D}\backslash\{\bar{j}\}}\sum_{i\in\mathcal{R}(j)}s_{ij}w_{ij}^{2}\right\}\\
&= \varphi v(F_{\bar{j}}^{1})+(1-\varphi)v(F_{\bar{j}}^{2}).
\end{aligned}
$$

Analogous to $V(F_{\bar{j}}^{k})$, $k=1,2$, let $V(F_{\bar{j}}^{\varphi})$ be the contract value corresponding to $F_{\bar{j}}^{\varphi}$. This then implies that $V(F_{\bar{j}}^{\varphi})=\delta\mathbb{E}[v(F_{\bar{j}}^{\varphi})]\leqslant\delta\{\varphi\mathbb{E}[v(F_{\bar{j}}^{1})]+(1-\varphi)\mathbb{E}[v(F_{\bar{j}}^{2})]\}=\varphi V(F_{\bar{j}}^{1})+(1-\varphi)V(F_{\bar{j}}^{2})$, that is, V is convex in $F_{\bar{j}}$.

The claimed properties of each delivery point delta and gamma hold because $\bar{j}$ is arbitrary in $\mathcal{D}$.

To show that each receipt point delta is negative, fix arbitrary $\bar{i}\in\mathcal{R}$, and pick $F_{\bar{i}}^{2}>F_{\bar{i}}^{1}$. Interpret $v(F_{\bar{i}}^{k})$, w^k, and $V(F_{\bar{i}}^{k})$, $k=1,2$, in the obvious way. It holds that

$$
\begin{aligned}
v(F_{\bar{i}}^{2}) &= \sum_{j\in\mathcal{D}(\bar{i})}[f_j-\frac{F_{\bar{i}}^{2}\beta_{\bar{i}}}{1-\phi_{\bar{i}j}}-K_{\bar{i}j}]w_{\bar{i}j}^{2}+\sum_{i\in\mathcal{R}\backslash\{\bar{i}\}}\sum_{j\in\mathcal{D}(i)}s_{ij}w_{ij}^{2}\\
&\leqslant \sum_{j\in\mathcal{D}(\bar{i})}[f_j-\frac{F_{\bar{i}}^{1}\beta_{\bar{i}}}{1-\phi_{\bar{i}j}}-K_{\bar{i}j}]w_{\bar{i}j}^{2}+\sum_{i\in\mathcal{R}\backslash\{\bar{i}\}}\sum_{j\in\mathcal{D}(i)}s_{ij}w_{ij}^{2}\\
&\leqslant \sum_{j\in\mathcal{D}(\bar{i})}[f_j-\frac{F_{\bar{i}}^{1}\beta_{\bar{i}}}{1-\phi_{\bar{i}j}}-K_{\bar{i}j}]w_{\bar{i}j}^{1}+\sum_{i\in\mathcal{R}\backslash\{\bar{i}\}}\sum_{j\in\mathcal{D}(i)}s_{ij}w_{ij}^{1}\\
&= v(F_{\bar{i}}^{1}).
\end{aligned}
$$

This then implies $V(F_{\bar{i}}^{2})\leqslant V(F_{\bar{i}}^{1})$, that is, V decreases in $F_{\bar{i}}$. One can show that V is convex in $F_{\bar{i}}$ in a manner analogous to the proof of the convexity of V in $F_{\bar{j}}$. The claimed properties of each receipt point delta and gamma hold because $\bar{i}$ is arbitrary in $\mathcal{R}$.

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