Carnegie Mellon University
Research Showcase @ CMU

Tepper School of Business

11-2009

Revenue Management with Bargaining

Atul Bhandari
SmartOps

Nicola Secomandi
Carnegie Mellon University, ns7@andrew.cmu.edu

Follow this and additional works at: http://repository.cmu.edu/tepper

Part of the Economic Policy Commons, and the Industrial Organization Commons

Recommended Citation
.

This Working Paper is brought to you for free and open access by Research Showcase @ CMU. It has been accepted for inclusion in Tepper School of Business by an authorized administrator of Research Showcase @ CMU. For more information, please contact research-showcase@andrew.cmu.edu.

Revenue Management with Bargaining

Atul Bhandari
SmartOps, 12 Federal Street, Pittsburgh, PA 15212, USA, abhandari@smartops.com
Nicola Secomandi
Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213-3890, USA, ns7@andrew.cmu.edu
Tepper Working Paper 2006-E61
Submitted: June 2008; Revised: March 2009, May 2009, November 2009

Abstract

Static game-theoretic models of bilateral bargaining assume that the seller knows his valuation for the item that is up for sale; that is, how the seller may determine this quantity is exogenous to these models. In this paper, we develop and analyze a stylized Markov decision process that endogenizes the seller's computation of his marginal inventory valuation in an infinite horizon revenue management setting when each sale occurs according to a given bilateral bargaining mechanism. We use this model to compare, both analytically and numerically, the seller's performance under four basic bilateral bargaining mechanisms with a tractable information structure. These comparisons provide insights on the seller's performance under the following trading arrangements: buyer and seller posted pricing, negotiated pricing, and rule based pricing.

1. Introduction

Static game-theoretic models of single item bilateral bargaining, such as those discussed by Myerson [10, 11], Myerson and Satterthwaite [13], and Chatterjee and Samuelson [3], assume that the seller knows his marginal valuation, also referred to as his type, for the item whose sale is being negotiated. In other words, how the seller might determine this quantity is exogenous to these models. In this paper, we interpret the seller's marginal valuation for an item as his optimal opportunity cost (OC) of selling this item. We consider a revenue management setting, with a single seller, multiple buyers, and multiple items, and develop and analyze a *stylized* Markov decision process (MDP) that endogenizes the seller's computation of his optimal OC.

Specifically, we consider a risk neutral seller who owns a finite inventory of a single product that can be sold during an infinite horizon divided into discrete time periods. The initial availability of this inventory is exogenously specified; that is, we abstract from the seller's initial stocking/production decision. At the beginning of each time period there is a positive probability of at most one buyer's purchase request (arrival) for a single unit of the seller's inventory; that is, demand is Bernoulli (Talluri and van Ryzin [19, §5.2.2.2]). Buyers are risk neutral and do not strategize on when they arrive. The seller's and each requesting buyer's valuations for a unit of product that is up for sale are their private information.

At the time of a buyer's arrival, the seller and this buyer bargain over the sale of the requested item. We model each such negotiation as a Bayesian bargaining problem whose *outcome* is governed

by a given direct and feasible mechanism for bilateral bargaining (Myerson and Satterthwaite [13]). In this modeling approach, which is rare in the revenue management literature (Talluri and van Ryzin [19]), it is common knowledge that the beliefs of the seller and each arriving buyer about each other's valuations for the item that is up for sale are exogenously specified independent probability distributions. Moreover, the bargaining outcome is a Bayesian Nash equilibrium. This feature allows us to compare the seller's performance under different trading formats in a consistent fashion.

In our MDP, the beliefs of the seller and each buyer are stationary, so that the seller faces a sequence of stationary Bayesian bargaining problems linked by his inventory availability, which changes dynamically as sales are made over time. We exploit the structure of the problem, in particular the incentive compatibility constraints, to provide a compact MDP formulation. This formulation is insightful, because it elucidates the role that the seller's inventory plays in determining his type in a revenue management setting with bilateral bargaining. It is general, in the sense that it does not depend on the specific (direct and feasible) mechanism used to determine the outcome of a bilateral bargaining problem. It is useful, because it allows one to optimally solve our MDP by solving a finite sequence of univariate fixed point problems, one for each inventory level.

We also compare in a dynamic setting any two direct and feasible mechanisms for which the seller's interim expected utility, his expected utility conditional on his type (Myerson and Satterthwaite [13]), is ordered in the static setting. We show analytically that when these two mechanisms are employed in our dynamic setting the seller's optimal value functions satisfy the same ordering relationship for each inventory level. This result is nontrivial, because in our MDP the seller's type with the *same* inventory availability is in general different under the two mechanisms, but the assumed static ordering between them relies on the seller having the same type in both cases.

We apply these methodological results to investigate the seller's performance in the context of the symmetric uniform trading problem (SUTP) studied by Chatterjee and Samuelson [3] under the four mechanisms investigated by Myerson [12]. These are the seller posted price (SPP), the buyer posted price (BPP), the neutral bargaining solution (NBS), and the split the difference (STD) mechanisms. We use these mechanisms as normative models of the following business situations: the seller transacts with each arriving buyer by posting a take it or leave it price (SPP mechanism); accepting or rejecting a take it or leave it price posted by this buyer (BPP mechanism); negotiating a price with this buyer (NBS mechanism); and splitting the difference between his offer price and that of this buyer (STD mechanism).

We find that the seller typically benefits by using the SPP mechanism. However, this is not always the case; he can be better off under the STD mechanism when his relative inventory avail-

ability is exceptionally high. Moreover, the seller's performance is surprisingly similar under the SPP and NBS mechanisms, even though the seller is always better off under the SPP mechanism than the NBS mechanism. Finally, the seller is always worse off under the BPP mechanism.

These findings are significant because they largely support the superiority of posting prices for the seller, which has been extensively studied in the revenue management literature (Talluri and van Ryzin [19, Chapter 5]), and provide additional insights, some of which are unexpected, on the relative performance of the seller under alternative trading arrangements. For example, most transactions in business commerce are negotiated (Karmarkar [6], Kleindorfer and Wu [8]); our numerical comparison of the SPP and NBS mechanisms provides normative insights on the relative merit of this practice. Moreover, despite the exceptionally high relative inventory availability required to obtain it, the finding that the seller can be better off under rule based pricing than by posting prices is significant in theory, because it brings to light a situation where price posting by the seller can be outperformed by another trading format.

Our modeling approach is novel with respect to the static game-theoretic bilateral bargaining literature (see, e.g., Kennan and Wilson [7]) because it endogenizes the computation of the seller's marginal inventory valuation. When bargaining occurs via the SPP mechanism, our MDP corresponds to the dynamic pricing model of Das Varma and Vettas [4]. Different from these authors, we compare the seller's performance under alternative mechanisms.

Our paper is related to the literature on the effectiveness of different transaction formats for a seller. Riley and Zeckhauser [16] show the optimality of posted pricing by the seller, our SPP mechanism, over any other selling mechanism when the seller has only one item to sell. Gallien [5] extends this result to an arbitrary number of items. In contrast, we find that the seller can be better off under the STD mechanism than the SPP mechanism for exceptionally high relative inventory availability. Comparing the model and analysis of Gallien [5] to ours suggests that, with multiple items, this finding is due to our use of the Nash Bargaining equilibrium concept rather than the dominant equilibrium concept used by this author (the STD mechanism is not supported by a dominant equilibrium). The equilibrium concept that we use also allows us to compare the seller's performance under the SPP mechanism against his performance under other mechanisms not considered by this author.

Wang [22] and Roth et al. [17] compare posted pricing by the seller against price bargaining when the latter is modeled using the generalized Nash bargaining solution (Binmore et al. [2]; see Kuo et al. [9] for a related comparison in a finite horizon revenue management setting). Wang [22] and Roth et al. [17] find that selling through price bargaining is better than posting prices,

a conclusion that is opposite to our result that the seller is always better off under the SPP mechanism than the NBS mechanism. This discrepancy is likely to stem from the fact that these authors assume that customers honestly disclose their valuations to the seller *before* price bargaining occurs, which allows these authors to use the generalized Nash bargaining solution that assumes a complete information setting, while buyers retain their private information when the seller uses posted pricing. Instead, as in Myerson [12], in our model buyers maintain their private information in both cases. This modeling assumption allows us to perform a consistent comparison of the seller's performance under different bargaining mechanisms.

Our model is related to that of Vulcano et al. [21], who study auction mechanisms in revenue management when the seller faces many buyers in each of a finite number of time periods. Different from these authors, we study bilateral bargaining mechanisms in revenue management when the seller faces at most one customer in each of an infinite number of time periods.

The remainder of this paper is organized as follows. In §2, we introduce bargaining mechanism concepts in a static setting and illustrate them by discussing the four stated mechanisms for SUTP. We present and analyze our MDP in §3. In §4, we compare in a dynamic setting the four mechanisms for SUTP discussed in §2. We conclude in §5 by summarizing our work and by discussing its limitations and opportunities for further research.

2. Static Bargaining Mechanisms

In this section, we present the static bargaining mechanisms that form the building blocks for the MDP studied in §§3-4. We employ the basic direct mechanism framework of Myerson and Satterthwaite [13]. A seller has one unit of inventory that a buyer wishes to purchase. The seller is player 1 and the buyer is player 2. Both players are risk neutral. It is common knowledge that from the other player's perspective the valuation of player i for this inventory unit is random variable $\tilde{v}_i$, whose probability distribution function is $F_i(v_i)$ with support $\mathcal{V}_i := [a_i, b_i]$ $(0 \leq a_i \leq b_i)$, independently of the probabilistic beliefs of player i about the other player's valuation. That is, the buyer believes that the random variable seller's valuation $\tilde{v}_1$ is distributed according to $F_1(v_1)$ independently of the fact that the seller believes that the random variable buyer's valuation $\tilde{v}_2$ is distributed according to distribution $F_2(v_2)$. Each player $i \in \{1, 2\}$ knows his marginal valuation $v_i \in \mathcal{V}_i$ at the time of trading.

By the well known revelation principle (see, e.g., Myerson and Satterthwaite [13]), for modeling purposes one can restrict attention to direct and feasible mechanisms without loss of generality. In a direct mechanism, the two players simultaneously report their valuations to a mediator, who

Table 1: The SPP, BPP, NBS, and STD mechanisms for SUTP.

j	$p^j(v_1,v_2)$	$x^j(v_1,v_2)$
SPP	$1\{v_2 \geq (1+v_1)/2\}$	$p^{SPP}(v_1,v_2)(1+v_1)/2$
BPP	$1\{v_2/2 \geq v_1\}$	$p^{BPP}(v_1,v_2)v_2/2$
NBS	$1\{v_2 \geq 3v_1 \text{ or } 3v_2 - 2 \geq v_1\}$	$1\{v_2 \leq 1-v_1\}p^{NBS}(v_1,v_2)v_2/2$
		$+1\{v_2 > 1-v_1\}p^{NBS}(v_1,v_2)(1+v_1)/2$
STD	$1\{v_2 \geq v_1 + 1/4\}$	$p^{STD}(v_1,v_2)(v_1+v_2+1/2)/3$

Note: $1\{\cdot\}$ equals 1 if its argument is true and 0 otherwise.

determines whether the unit of inventory is transfered from the seller to the buyer and at what price. A direct mechanism j consists of two *outcome* functions: contingent on the seller and the buyer being of types v_1 and v_2, respectively, $p^j(v_1,v_2) : \mathcal{V}_1 \times \mathcal{V}_2 \to [0,1]$ determines the transfer *probability*, $x^j(v_1,v_2) : \mathcal{V}_1 \times \mathcal{V}_2 \to \Re$ the *price* paid (in general, the price is paid irrespective of whether trading occurs or not). A mechanism is feasible if it is both incentive compatible and individually rational (formal definitions are given below).

The seller's and buyer's utilities under mechanism j when their respective types are $v_1 \in \mathcal{V}_1$ and $v_2 \in \mathcal{V}_2$ are $u_1^j(v_1,v_2) := x^j(v_1,v_2) - v_1 p^j(v_1,v_2)$ and $u_2^j(v_1,v_2) := v_2 p^j(v_1,v_2) - x^j(v_1,v_2)$. Under mechanism j, the seller's and buyer's expected transfer probability and price received/paid conditional on these players' types being $v_1 \in \mathcal{V}_1$ and $v_2 \in \mathcal{V}_2$ are $\overline{p}_1^j(v_1) := \mathbb{E}[p^j(v_1,\tilde{v}_2)]$, $\overline{x}_1^j(v_1) := \mathbb{E}[x^j(v_1,\tilde{v}_2)]$, and $\overline{p}_2^j(v_2) := \mathbb{E}[p^j(\tilde{v}_1,v_2)]$, $\overline{x}_2^j(v_2) := \mathbb{E}[x^j(\tilde{v}_1,v_2)]$, where for any function $\cdot(v_1,v_2) : \mathcal{V}_1 \times \mathcal{V}_2 \to \Re$ we define $\mathbb{E}[\cdot(v_1,\tilde{v}_2)] := \int_{v_2 \in \mathcal{V}_2} \cdot(v_1,v_2) dF_2(v_2)$ and $\mathbb{E}[\cdot(\tilde{v}_1,v_2)] := \int_{v_1 \in \mathcal{V}_1} \cdot(v_1,v_2) dF_1(v_1)$. The seller's and buyer's *interim* expected utilities under mechanism j, that is, their expected utilities conditional on their respective types being $v_1 \in \mathcal{V}_1$ and $v_2 \in \mathcal{V}_2$, are

$$\overline{u}_1^j(v_1) := \overline{x}_1^j(v_1) - v_1 \overline{p}_1^j(v_1) \tag{1}$$

and $\overline{u}_2^j(v_2) := v_2 \overline{p}_2^j(v_2) - \overline{x}_2^j(v_2)$. Mechanism j is incentive compatible if $\overline{u}_1^j(v_1) \geq \overline{x}_1^j(\hat{v}_1) - v_1 \overline{p}_1^j(\hat{v}_1)$, $\forall v_1, \hat{v}_1 \in \mathcal{V}_1$, and $\overline{u}_2^j(v_2) \geq v_2 \overline{p}_2^j(\hat{v}_2) - \overline{x}_2^j(\hat{v}_2)$, $\forall v_2, \hat{v}_2 \in \mathcal{V}_2$; it is individually rational if $\overline{u}_1^j(v_1) \geq 0$, $\forall v_1 \in \mathcal{V}_1$, and $\overline{u}_2^j(v_2) \geq 0$, $\forall v_2 \in \mathcal{V}_2$.

As in Myerson [12], we illustrate these concepts in the static SUTP context, where $\mathcal{V}_i \equiv [0,1]$, $\forall i \in \{1,2\}$, and $F_i(v_i) \equiv v_i$, $\forall i \in \{1,2\}$ and $v_i \in \mathcal{V}_i$, with respect to mechanisms SPP, BPP, STD, and NBS (these mechanisms are numbered 1, 2, 3, and 4, respectively, in Myerson [12]; we use our own labeling in the remainder of this paper). With the exception of Lemma 1, all the remaining material in this section can be found in Myerson [12].

Table 1 summarizes the four mechanisms for SUTP. Mechanisms SPP and BPP, respectively, consist of a posted take it or leave it price set by the seller and the buyer to maximize their respective

interim expected utilities. With mechanism SPP, a sale occurs at the seller's optimal posted price $(1+v_1)/2$ if and only if the buyer's valuation v_2 is not below this price; with mechanism BPP, a sale occurs at the buyer's optimal posted price $v_2/2$ if and only if the seller's valuation v_1 does not exceed this price. The SPP mechanism is equivalent to the seller's posted price trading format, which is well studied in the price based revenue management literature (Talluri and van Ryzin [19, Chapter 5]). Instead, mechanism BPP has not been apparently considered in this literature; in this case, the optimal price posted by the type v_2 buyer depends on his beliefs about the seller's type v_1 through the distribution $F_1(v_1) \equiv v_1$, because this price solves the following optimization problem: $\max_{x\in[0,1]} \int_0^x (v_2 - x)dv_1$.

The NBS axiomatic solution concept is Myerson's [11] extension to an incomplete information setting of the celebrated Nash bargaining solution (Nash [14]), and hence incorporates fairness considerations. The corresponding NBS mechanism for SUTP combines mechanisms SPP and BPP in a nontrivial manner. A sale occurs if and only if the buyer's type v_2 is not below the minimum of $3v_1$ and $(2+v_1)/3$; conditional on there being a sale, if the buyer is in a stronger bargaining position than the seller ($v_2 \leq 1 - v_1$; that is, v_2 is closer to 0 than v_1 is to 1), then mechanism NBS employs the optimal buyer posted price; otherwise, it uses the optimal seller posted price.

Chatterjee and Samuelson [3] study a model where the buyer and the seller report to each other offer prices B and S, respectively, and agree to transact at a price equal to $K \cdot B + (1-K) \cdot S$, with $K \in [0,1]$, if and only if $B \geq S$. The cases $K = 0$ and $K = 1$ correspond to mechanisms SPP and BPP, respectively. The case $K = 1/2$ is of special interest in the SUTP context, because in this case Myerson and Satterthwaite [13] show that the linear equilibrium derived by Chatterjee and Samuelson [3] maximizes the ex ante expected total gains from trade. The STD mechanism is the direct and feasible mechanism version of this equilibrium. With this mechanism, a sale occurs at a price equal to the average of the buyer's type, the seller's type, and 1/2 if and only if the buyer's valuation exceeds that of the seller by at least 1/4.

For the ensuing development, Lemma 1 compares the SPP, BPP, NBS, and STD mechanisms from the seller's perspective in the static SUTP context; the proof of this result is easily established and is omitted for brevity.

Lemma 1 (Seller's interim expected utility in SUTP)**.** *For SUTP it holds that* $\overline{u}_1^{SPP}(v_1) \geq \overline{u}_1^{NBS}(v_1) \geq \overline{u}_1^{BPP}(v_1)$ *and* $\overline{u}_1^{STD}(v_1) \geq \overline{u}_1^{BPP}(v_1)$, $\forall v_1 \in \mathcal{V}_1$.

3. MDP Model and Analysis

In this section, we present and analyze our MDP. Different from the static case discussed in §2, where how the seller knows his valuation v_1 for the single unit of inventory that is up for sale is exogenous to the model, this MDP endogenizes the computation of the seller's optimal marginal OC when multiple units of inventory can be sold to a stream of buyers that arrive stochastically during an infinite time horizon. Specifically, the seller has a given number $Y \geq 1$ of inventory units available at the beginning of this time horizon, which is partitioned into discrete time periods each of equal length. We let $y \in \mathcal{Y} := \{1, \ldots, Y\}$ denote the units of remaining inventory at the beginning of any such time period. At each such time, the seller faces a buyer's arrival with probability λ, independently of the other periods. The seller's one period discount factor is $\beta \in [0, 1)$.

The model's information structure is as follows. At any point in time, the quantities λ, β, Y, and y are the seller's *private* information. Upon a buyer's arrival, trading between the seller and this buyer is governed by direct and feasible mechanism j under the following *common* knowledge assumption: the seller believes that the marginal valuation of the arriving buyer is distributed according to $F_2(v_2)$ with support $\mathcal{V}_2 := [0, 1]$, independently of the beliefs of this buyer and of any other buyer who arrived in the past or will arrive in the future (the results of this section continue to hold in the more general case when $\mathcal{V}_2 := [0 \leq a, 1]$); each arriving buyer believes that the seller's marginal valuation is distributed according to $F_1(v_1)$ with support $\mathcal{V}_1 := [0, 1]$ independently of the beliefs of the seller and of any other buyer. These distributions are independent of the demand process.

Given that the seller's and each buyer's beliefs are stationary, so is the seller's optimal value function under mechanism j. This is denoted by $V^j(y)$ and is the seller's optimal total expected discounted revenue during the remaining time horizon, when the seller has $y \in \mathcal{Y}$ units of inventory at the beginning of any time period and the outcome of bargaining between the seller and each arriving buyer is determined by mechanism j.

To formulate our MDP, we define $s^j(v_1, v_2) : \mathcal{V}_1 \times \mathcal{V}_2 \to \{0, 1\}$ to be a function that is equal to 1 if one unit of inventory is sold to an arriving buyer under mechanism j when the seller's and a buyer's marginal valuations are v_1 and v_2, respectively. The seller's optimal value function solves the following Bellman's equations:

$$\begin{aligned} V^j(y) \quad = \quad & (1-\lambda)\beta V^j(y) + \lambda \max_{v_1 \in \mathcal{V}_1} \mathbb{E}\left[x^j(v_1, \tilde{v}_2) + \beta V^j(y-1) 1\{s^j(v_1, \tilde{v}_2) = 1\}\right. \\ & \left. + \beta V^j(y) 1\{s^j(v_1, \tilde{v}_2) = 0\}\right], \forall y \in \mathcal{Y}, \end{aligned} \tag{2}$$

with boundary condition $V^j(0) := 0$. In the optimization on the right hand side of (2), the decision variable v_1 is the marginal valuation to be reported by the seller to the mediator who applies mechanism j. By defining $V^j(0) := 0$ and $\Delta V^j(y) := V^j(y) - V^j(y-1)$, $\forall y \in \mathcal{Y}$, we can easily reformulate recursion (2) as follows:

$$V^j(y) \quad = \quad \frac{\lambda}{1-\beta} \max_{v_1 \in \mathcal{V}_1} \mathbb{E}\left[x^j(v_1, \tilde{v}_2) - \beta \Delta V^j(y) 1\{s^j(v_1, \tilde{v}_2) = 1\}\right], \ \forall y \in \mathcal{Y}. \tag{3}$$

Furthermore, it holds that $\mathbb{E}x^j(v_1, \tilde{v}_2) \equiv \overline{x}_1^j(v_1)$ and $\mathbb{E}\left[1\{s^j(v_1, \tilde{v}_2) = 1\}\right] \equiv \mathbb{E}\left[p^j(v_1, \tilde{v}_2)\right] \equiv \overline{p}_1^j(v_1)$. Therefore, we can equivalently rewrite (3) as

$$V^j(y) \quad = \quad \frac{\lambda}{1-\beta} \max_{v_1 \in \mathcal{V}_1} \left[\overline{x}_1^j(v_1) - \beta \Delta V^j(y) \overline{p}_1^j(v_1)\right], \ \forall y \in \mathcal{Y}. \tag{4}$$

It is tempting to conclude that $\beta \Delta V^j(y)$ is an optimal solution to the maximization on the right hand side of (4) by incentive compatibility of mechanism j. However, reaching this conclusion requires some care, because one must first show that $\beta \Delta V^j(y) \in \mathcal{V}_1$, $\forall y \in \mathcal{Y}$. Lemma 2 shows that this is in fact the case, so that we refer to this quantity as the seller's optimal OC for the y-th unit of inventory. Incidentally, part (a) of Lemma 2 establishes that the seller's optimal OC weakly decreases in inventory in addition to being nonnegative, or, equivalently, the seller's optimal value function is weakly concave in inventory in addition to being increasing in this quantity; these are well known properties when mechanism j is set equal to SPP (Talluri and van Ryzin [19, Chapter 5]), but we show that they persist in a more general setting. Moreover, part (b) of Lemma 2 is based on the natural assumption that mechanism j is normal for the seller in the sense of Myerson [12]; that is, the expected seller's gain from trading under this mechanism is zero when his type is 1 ($\overline{u}_1^j(1) = 0$).

Lemma 2 (Seller's optimal value function and marginal valuation)**.** *(a) Given direct and feasible mechanism j, the optimal value function $V^j(y)$ is weakly increasing and concave in inventory, $\forall y \in \mathcal{Y} \cup \{0\}$. Equivalently, the function $\beta \Delta V^j(y)$ is nonnegative and weakly decreases in inventory, $\forall y \in \mathcal{Y}$. (b) If direct mechanism j is feasible and normal for the seller then $\beta \Delta V^j(y) \leq 1$, $\forall y \in \mathcal{Y}$.*

Proof. (a) Since $V^j(0) \equiv 0$, it holds that $\Delta V^j(1) = V^j(1)$. Individual rationality of mechanism j, that is, $\overline{u}_1^j(v_1) \geq 0$, $\forall v_1 \in \mathcal{V}_1$, implies that $\overline{x}_1^j(v_1) \geq v_1 \overline{p}_1^j(v_1) \geq 0$, $\forall v_1 \in \mathcal{V}_1$, or that

$$\overline{x}_1^j(v_1) \geq 0, \ \forall v_1 \in \mathcal{V}_1. \tag{5}$$

Let $v_1^*(1) \in \arg\max_{v_1 \in \mathcal{V}_1} \left[\overline{x}_1^j(v_1) - \beta V^j(1) \overline{p}_1^j(v_1)\right]$. Notice that

$$V^j(1) \quad = \quad \frac{\lambda}{1-\beta} \left[\overline{x}_1^j(v_1^*(1)) - \beta V^j(1) \overline{p}_1^j(v_1^*(1))\right]$$

$$
\begin{aligned}
\Rightarrow V^j(1) &= \left[1+\frac{\lambda\beta\overline{p}_1^j(v_1^*(1))}{1-\beta}\right]^{-1}\left(\frac{\lambda}{1-\beta}\right)\overline{x}_1^j(v_1^*(1)) \geq 0; \text{by (5)} \\
\Rightarrow 0 &\leq \Delta V^j(1) \equiv V^j(1).
\end{aligned}
$$

Make the induction hypothesis that $\Delta V^j(\bar{y}) \geq 0$ for all $\bar{y} = 2, \ldots, y-1$, and consider inventory level y. Let

$$
\begin{aligned}
v_1^*(y) &\in \underset{v_1 \in \mathcal{V}_1}{\arg\max}\left[\overline{x}_1^j(v_1) - \beta\Delta V^j(y)\overline{p}_1^j(v_1)\right] \\
v_1^*(y-1) &\in \underset{v_1 \in \mathcal{V}_1}{\arg\max}\left[\overline{x}_1^j(v_1) - \beta\Delta V^j(y-1)\overline{p}_1^j(v_1)\right].
\end{aligned}
$$

It holds that

$$
\begin{aligned}
\Delta V^j(y) &= V^j(y) - V^j(y-1) \\
&= \frac{\lambda}{1-\beta}\Big[\overline{x}_1^j(v_1^*(y)) - \beta\Delta V^j(y)\overline{p}_1^j(v_1^*(y)) \\
&\qquad -\overline{x}_1^j(v_1^*(y-1)) + \beta\Delta V^j(y-1)\overline{p}_1^j(v_1^*(y-1))\Big] \\
&\geq \frac{\lambda}{1-\beta}\Big[\overline{x}_1^j(v_1^*(y-1)) - \beta\Delta V^j(y)\overline{p}_1^j(v_1^*(y-1)) \\
&\qquad -\overline{x}_1^j(v_1^*(y-1)) + \beta\Delta V^j(y-1)\overline{p}_1^j(v_1^*(y-1))\Big] \\
&= \frac{\lambda\beta\overline{p}_1^j(v_1^*(y-1))}{1-\beta}\left[\Delta V^j(y-1) - \Delta V^j(y)\right],
\end{aligned}
$$

with the inequality following from the optimality of $v_1^*(y)$. This implies that

$$
\Delta V^j(y) \geq \left[1+\frac{\lambda\beta\overline{p}_1^j(v_1^*(y-1))}{1-\beta}\right]^{-1}\left(\frac{\lambda\beta\overline{p}_1^j(v_1^*(y-1))}{1-\beta}\right)\Delta V^j(y-1) \geq 0,
$$

with the last inequality following from the induction hypothesis. By the principle of mathematical induction it holds that $\Delta V^j(y) \geq 0$, $\forall y \in \mathcal{Y}$.

Pick $y \in \mathcal{Y}$, and notice that

$$
\begin{aligned}
\Delta V^j(y) &= \frac{\lambda}{1-\beta}\Big[\overline{x}_1^j(v_1^*(y)) - \beta\Delta V^j(y)\overline{p}_1^j(v_1^*(y)) \\
&\qquad -\overline{x}_1^j(v_1^*(y-1)) + \beta\Delta V^j(y-1)\overline{p}_1^j(v_1^*(y-1))\Big] \\
&\leq \frac{\lambda}{1-\beta}\Big[\overline{x}_1^j(v_1^*(y)) - \beta\Delta V^j(y)\overline{p}_1^j(v_1^*(y)) \\
&\qquad -\overline{x}_1^j(v_1^*(y)) + \beta\Delta V^j(y-1)\overline{p}_1^j(v_1^*(y))\Big] \\
&= \frac{\lambda\beta\overline{p}_1^j(v_1^*(y))}{1-\beta}\left[\Delta V^j(y-1) - \Delta V^j(y)\right],
\end{aligned}
$$

with the inequality following from the optimality of $v_1^*(y-1)$. This implies that

$$\Delta V^j(y) \leq \left[1 + \frac{\lambda\beta\overline{p}_1^j(v_1^*(y))}{1-\beta}\right]^{-1} \left(\frac{\lambda\beta\overline{p}_1^j(v_1^*(y))}{1-\beta}\right) \Delta V^j(y-1) \leq \Delta V^j(y-1),$$

which establishes that $\Delta V^j(y)$ decreases in inventory, $\forall y \in \mathcal{Y}$.

(b) Let $v_1^*(1) \in \arg\max_{v_1 \in \mathcal{V}_1} \left[\overline{x}_1^j(v_1) - \beta V^j(1)\overline{p}_1^j(v_1)\right]$. If $\beta\Delta V^j(1) \equiv \beta V^j(1) > 1$ then

$$\begin{aligned} V^j(1) &= \frac{\lambda}{1-\beta}\left[\overline{x}_1^j(v_1^*(1)) - \beta V^j(1)\overline{p}_1^j(v_1^*(1))\right] \\ &< \frac{\lambda}{1-\beta}\left[\overline{x}_1^j(v_1^*(1)) - 1\overline{p}_1^j(v_1^*(1))\right] \\ &\leq \frac{\lambda}{1-\beta}\left[\overline{x}_1^j(1) - 1\overline{p}_1^j(1)\right]; \text{ by incentive compatibility of } j \\ &= 0; \text{ by normality of } j \text{ for the seller,} \end{aligned}$$

which contradicts the property that $V^j(1) \geq 0$ established in part (a) of this lemma. Thus, it must be that $\beta\Delta V^j(1) \leq 1$. The property holds for all other inventory levels in set $\mathcal{Y}$ because $\beta\Delta V^j(y) \leq \beta\Delta V^j(1)$, $\forall y \in \mathcal{Y}$, by part (a) of this lemma. □

Proposition 1 provides a compact formulation of our MDP; it follows from Lemma 2 and incentive compatibility of mechanism j.

Proposition 1 (MDP formulation)**.** *If direct mechanism j is feasible and normal for the seller then the seller's optimal value function satisfies the following conditions:*

$$V^j(y) = \frac{\lambda}{1-\beta}\overline{u}_1^j(\beta\Delta V^j(y)) = \frac{\lambda}{1-\beta}\left[\overline{x}_1^j(\beta\Delta V^j(y)) - \beta\Delta V^j(y)\overline{p}_1^j(\beta\Delta V^j(y))\right], \ \forall y \in \mathcal{Y}. \tag{6}$$

This result directly relates the seller's optimal value function in our MDP to expression (1) in §2; that is, it states that the seller's optimal value function with y units of inventory is proportional to his interim expected utility evaluated at his optimal OC. Although formulation (6) is directly related to expression (1) in §2, there is a fundamental difference between the two: model (6) is dynamic while expression (1) is static. The dynamic aspect of (6) is that in this formulation the seller's type is his optimal OC, which depends *endogenously* on his inventory availability and the bargaining mechanism employed in the current and remaining periods; instead, the seller's type is *exogenously* specified in (1).

As now shown, Proposition 1 has useful computational and analytical ramifications.

Proposition 1 implies that $V^j(y)$ can be computed by means of a forward recursion in $y \in \mathcal{Y}$, which at each step involves the solution of a univariate fixed point problem. To see this, pick

$y \in \mathcal{Y}$, suppose that $V^j(y-1)$ is known, and let $V^j(y)$ be the unknown z, which must lie in set $\mathcal{Z}^j(y) := [V^j(y-1), 1/\beta + V^j(y-1)]$ because $\beta \Delta V^j(y) \in [0,1]$ by Lemma 2. Define function $f^j(z;y) : \mathcal{Z}^j(y) \to [0, \lambda \overline{x}_1^j(0)/(1-\beta)]$ as

$$f^j(z;y) := \frac{\lambda}{1-\beta} \left\{ \overline{x}_1^j(\beta(z - V^j(y-1))) - \beta \left[z - V^j(y-1) \right] \overline{p}_1^j(\beta(z - V^j(y-1))) \right\},$$

where the specified range of $f^j(z;y)$ follows because this function weakly decreases on its domain (this holds because $\overline{u}_1^j(v_1)$ weakly decreases in $v_1 \in \mathcal{V}_1$, as shown in the proof of Theorem 1 in Myerson and Satterthwaite [13], and mechanism j is normal for the seller). The fixed point problem involves solving equation $z = f^j(z;y)$ on the domain $\mathcal{Z}^j(y)$ of $f^j(z;y)$, a problem that we denote by FP $\left(f^j, \mathcal{Z}^j, y\right)$. A unique solution to this problem exists and can be found by standard methods. (Existence and uniqueness of this solution follow from well known results in Stokey and Lucas [18, Chapter 9], but also from noticing that $f^j(V^j(y-1);y) = \lambda \overline{x}_1^j(0)/(1-\beta) > V^j(y-1)$, which is straightforward to establish, $f^j(1/\beta + V^j(y-1);y) = 0$, and $f^j(z;y)$ weakly decreases in $z \in \mathcal{Z}^j(y)$, $\forall y \in \mathcal{Y}$.) Thus, the following forward recursion algorithm solves (6):

Step 0. Let $V^j(0) \leftarrow 0$ and $y \leftarrow 1$.

Step y. If $y > Y$ stop, else solve FP $\left(f^j, \mathcal{Z}^j, y\right)$. Denote its solution by z^*. Let $V^j(y) \leftarrow z^*$, $y \leftarrow y+1$, and repeat this step.

We use this algorithm in our SUTP based numerical study reported in §4.

Proposition 1 is also useful to establish Theorem 1, which analytically compares in the dynamic setting of this section the seller's performance under any two direct and feasible mechanisms that are are known to be ordered in a certain manner in a static setting (under the additional assumptions that they are normal for the seller and satisfy the information structure of this section). We apply this result in §4.

Theorem 1 (Optimal value function comparison)**.** *Suppose that direct and feasible mechanisms j and k are normal for the seller, are defined on set $\mathcal{V}_1 \times \mathcal{V}_2 \equiv [0,1]^2$, and are such that $\overline{u}_1^j(v_1) \geq \overline{u}_1^k(v_1)$, $\forall v_1 \in \mathcal{V}_1$. Then it holds that $V^j(y) \geq V^k(y)$, $\forall y \in \mathcal{Y}$.*

Proof. By the stated assumptions, Proposition 1 applies to mechanisms j and k. Pick $y = 1$. The assumption that $\overline{u}_1^j(v_1) \geq \overline{u}_1^k(v_1)$, $\forall v_1 \in \mathcal{V}_1$, implies that

$$\begin{aligned} V^k(1) &= \frac{\lambda}{1-\beta} \left[\overline{x}_1^k(\beta V^k(1)) - \beta V^k(1) \overline{p}_1^k(\beta V^k(1)) \right] \\ &\leq \frac{\lambda}{1-\beta} \left[\overline{x}_1^j(\beta V^k(1)) - \beta V^k(1) \overline{p}_1^j(\beta V^k(1)) \right] \\ \Rightarrow A^{j,k}(1) V^k(1) &\leq \overline{x}_1^j(\beta V^k(1)), \end{aligned} \tag{7}$$

where we define $A^{j,k}(y) := (1-\beta)/\lambda + \beta\overline{p}_1^j(\beta\Delta V^k(y))$, $\forall y \in \mathcal{Y}$. Proposition 1 and incentive compatibility of mechanism j imply that

$$\begin{aligned} V^j(1) &= \frac{\lambda}{1-\beta}\left[\overline{x}_1^j(\beta V^j(1)) - \beta V^j(1)\overline{p}_1^j(\beta V^j(1))\right] \\ &\geq \frac{\lambda}{1-\beta}\left[\overline{x}_1^j(\beta V^k(1)) - \beta V^j(1)\overline{p}_1^j(\beta V^k(1))\right] \\ \Rightarrow A^{j,k}(1)V^j(1) &\geq \overline{x}_1^j(\beta V^k(1)). \end{aligned} \tag{8}$$

Combining inequalities (7)-(8) yields that $V^j(1) \geq V^k(1)$.

Make the induction hypothesis that the inequality $V^j(\bar{y}) \geq V^k(\bar{y})$ holds for all inventory levels $\bar{y} = 2, \ldots, y-1$, and consider inventory level y. Proposition 1 and the assumption on the relationship between the seller's interim expected utilities under mechanisms j and k imply that

$$\begin{aligned} V^k(y) &= \frac{\lambda}{1-\beta}\left[\overline{x}_1^k(\beta\Delta V^k(y)) - \beta\Delta V^k(y)\overline{p}_1^k(\beta\Delta V^k(y))\right] \\ &\leq \frac{\lambda}{1-\beta}\left[\overline{x}_1^j(\beta\Delta V^k(y)) - \beta\Delta V^k(y)\overline{p}_1^j(\beta\Delta V^k(y))\right] \\ \Rightarrow A^{j,k}(y)V^k(y) &\leq \overline{x}_1^j(\beta\Delta V^k(y)) + \beta\overline{p}_1^j(\beta\Delta V^k(y))V^k(y-1). \end{aligned} \tag{9}$$

Proposition 1 and incentive compatibility of mechanism j imply that

$$\begin{aligned} V^j(y) &= \frac{\lambda}{1-\beta}\left[\overline{x}_1^j(\beta\Delta V^j(y)) - \beta\Delta V^j(y)\overline{p}_1^j(\beta\Delta V^j(y))\right] \\ &\geq \frac{\lambda}{1-\beta}\left[\overline{x}_1^j(\beta\Delta V^k(y)) - \beta\Delta V^j(y)\overline{p}_1^j(\beta\Delta V^k(y))\right] \\ \Rightarrow A^{j,k}(y)V^j(y) &\geq \overline{x}_1^j(\beta\Delta V^k(y)) + \beta\overline{p}_1^j(\beta\Delta V^k(y))V^j(y-1) \\ &\geq \overline{x}_1^j(\beta\Delta V^k(y)) + \beta\overline{p}_1^j(\beta\Delta V^k(y))V^k(y-1), \end{aligned} \tag{10}$$

with the last inequality following from the induction hypothesis. Combining inequalities (9)-(10) yields that $V^j(y) \geq V^k(y)$. The principle of mathematical induction implies that $V^j(y) \geq V^k(y)$, $\forall y \in \mathcal{Y}$. □

This result is nontrivial because even if mechanisms j and k are ordered in a static setting, that is, $\overline{u}_1^j(v_1) \geq \overline{u}_1^k(v_1)$, $\forall v_1 \in \mathcal{V}_1$, and the seller's optimal value function is proportional to his optimal interim expected utility as in expression (6), it is unclear how the seller's optimal OCs under mechanisms j and k would compare. In fact, these OCs could relate to each other in the "wrong" way. For example, it could happen that $\beta\Delta V^j(y) < \beta\Delta V^k(y)$ for some inventory level y while at the same time $\overline{u}_1^j(v_1)$ and $\overline{u}_1^k(v_1)$ decrease in v_1; this can indeed occur, as discussed after Proposition 2 in §4. In this case, it is not at all clear that Theorem 1 should hold.

4. Comparisons

In this section, we compare the seller's performance under mechanisms SPP, BPP, NBS, and STD by focusing on SUTP in a dynamic setting; that is, we assume that both the seller and each arriving buyer believe that each other's marginal valuations are independently and uniformly distributed between 0 and 1.

We first establish some analytical comparisons of the seller's performance in Proposition 2, which holds by an application of Theorem 1 based on Lemma 1.

Proposition 2 (Optimal value function comparisons with SUTP)**.** *Suppose that* $\mathcal{V}_i \equiv [0,1]$, $\forall i \in \{1,2\}$, *and* $F_i(v_i) \equiv v_i$, $\forall i \in \{1,2\}$ *and* $v_i \in \mathcal{V}_i$. *Then it holds that* $V^{SPP}(y) \geq V^{NBS}(y) \geq V^{BPP}(y)$ *and* $V^{STD}(y) \geq V^{BPP}(y)$, $\forall y \in \mathcal{Y}$.

The comparisons established in Proposition 2 are nontrivial for the following reason. It can be verified that the seller's interim expected utility (weakly) decreases in his type under all the mechanisms studied in this paper. Moreover, given an inventory level, it can also be verified that the seller's marginal valuation in our MDP (optimal OCs) can strictly decrease when the mechanism changes in the following order: SPP, NBS, STD, and BPP. However, the relevant optimal OC differences are never "large enough" to invalidate the orderings stated in Proposition 2. This illustrates in a concrete setting the situation discussed after the proof of Theorem 1 in §3.

We now numerically quantify whether the seller's performance differences implied by Proposition 2 are significant or negligible. This numerical investigation also allows us to compare the seller's performance under mechanism STD and mechanisms SPP and NBS, respectively (these comparisons are not part of Proposition 2 because a complete ordering between these mechanisms in the static case does not exist).

We divide the infinite horizon into daily periods. Thus, the seller faces at most one request per day. Our numerical study encompasses the following parameter levels (with one exception discussed later): annual interest rate r in set $\{0.05, 0.10\}$ (the discount factor β is related to the interest rate via the expression $\beta = 1/(1 + r/365)$); arrival probability λ in set $\{0.3, 0.6, 0.9\}$; and initial inventory level Y equal to 90 units, which implies that the seller's relative inventory availability, Y/λ, corresponding to each considered arrival probability is 300, 150, and 100, respectively. Our results are fairly similar across the various interest rate and arrival probability levels. Thus, for brevity, the ensuing discussion pertains to the case of $r = 0.05$ and $\lambda = 0.30$.

Figure 1 displays the optimal value functions under mechanisms BPP, NBS, and STD, respectively, relative to that of mechanism SPP. (This figure is a line chart even if inventory is a discrete

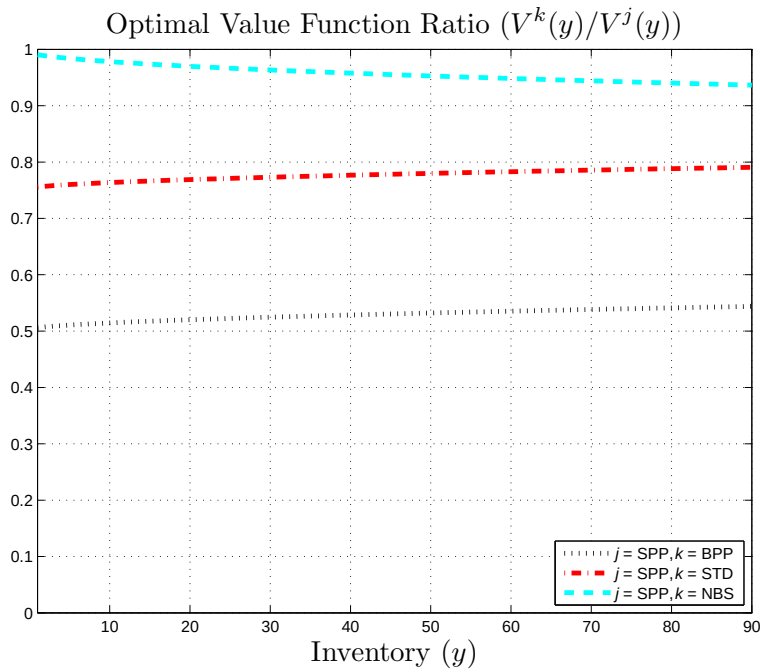

Figure 1: Ratios of the optimal value functions under mechanisms BPP, NBS, and STD to the optimal value function under mechanism SPP for $r = 0.05$ and $\lambda = 0.30$.

quantity, because using a chart format that emphasizes the discrete nature of inventory would render it extremely clumsy.) In addition to what stated in Proposition 2, we make the following observations. Surprisingly, the seller's performance under mechanism NBS is not very different from his performance under mechanism SPP, even though the relevant performance gap widens somewhat for larger inventory levels. In other words, for the range of parameters considered, the seller's performance does not seem to dramatically degrade by negotiating the price with each arriving buyer rather than posting prices in a take it or leave it fashion. In contrast, the seller performs significantly better under mechanisms SPP than mechanism STD. Put differently, he performs significantly better by making a first and final price offer than splitting the difference between his price offer and that of each arriving buyer. As discussed below, this does not always happen. The seller's performance gap is even wider when comparing his performance under mechanisms SPP and BPP. Moreover, the seller's performance is substantially better under mechanisms NBS and STD than under mechanism BPP. Thus, there is significant benefit for the seller to be in a position to influence, at least to some extent, the transacted price rather than accepting or rejecting that requested by each buyer. It is interesting that the seller performs significantly better under mechanism NBS than mechanism STD; that is, by negotiating the price with each arriving buyer rather by splitting the difference between his offer and that of each arriving buyer.

Finally, we investigate whether the seller can be better off under the STD mechanism than mechanisms SPP and NBS in our dynamic setting. It can be verified that in a static setting the seller can indeed be better off in terms of interim expected utility with mechanism STD than with mechanisms SPP and NBS, respectively, when his marginal valuation is sufficiently low. Lowering λ to 0.006 while keeping $Y = 90$ and $r = 0.05$ makes the seller fare better under mechanism STD than under both mechanisms SPP and NBS when his inventory level is sufficiently high; that is, his optimal OC is sufficiently low. We do not claim that this combination of parameter values is realistic in terms of the ratio Y/λ; it simply allows us to emulate a situation where the seller's marginal valuation can be very low.

5. Conclusions

In this paper, we study a traditional price based revenue management problem with the twist that transactions between the seller and each arriving buyer are governed by a mechanism for bilateral bargaining. The novelty of our modeling approach relies on embedding static bargaining mechanisms within an MDP formulation, which we simplify based on exploiting basic structural properties. From this perspective, our model endogenizes the computation of the seller's marginal inventory valuation, which is assumed to be known to the seller in static game-theoretic models of bargaining. We compare, both analytically and numerically, the seller's performance in a tractable information setting under four foundational bargaining mechanisms. Our findings largely support the superiority of posting prices as a selling format, but also show that negotiating prices is only slightly inferior to this format, and that posting prices is not always the best selling arrangement.

There are limitations in our work. Although we endogenize the seller's computation of his marginal valuation, his and each buyer's beliefs about each other's marginal valuations remain exogenous to our model, the probability distributions representing these beliefs are stationary and known, and buyers do not strategize over the timing of their purchase requests. Moreover, we only deal with the infinite horizon case and our comparisons among bargaining mechanisms are only based on the SUTP setting. These limitations could be addressed by further research.

Our model is stylized when compared to real business situations. Yet, it bridges two separate streams of literature, bilateral bargaining in microeconomics and revenue management in operations research, by endogenizing the computation of a seller's optimal opportunity cost for his inventory when sales are governed by a given bargaining mechanism. The concept of optimal opportunity cost is related to the seller's value for his best alternative to a negotiated agreement, which is widely used in the largely prescriptive negotiation analysis literature (Raiffa et al. [15]). This literature

does not typically discuss how a seller might systematically compute such value. We hope that our work might stimulate the development of prescriptive revenue management models for this purpose. From this perspective, it would be interesting to investigate the benefit of using descriptive models of the negotiation process, for example, the model of Balakrishnan and Eliashberg [1] (see also Terwiesch et al. [20]), rather than normative models of the outcome of a bilateral negotiation.

Acknowledgments

The comments and suggestions of an anonymous associate editor and three anonymous referees led to an improved version of this paper and are gratefully acknowledged.

References

[1] P. V. Balakrishnan and J. Eliashberg. An analytical process model of two-party negotiations. *Management Science*, 41:226–243, 1995.

[2] K. Binmore, A. Rubinstein, and A. Wolinski. The Nash bargaining solution in economic modeling. *Rand Journal of Economics*, 17:176–188, 1986.

[3] K. Chatterjee and W. Samuelson. Bargaining under incomplete information. *Operations Research*, 31:835–851, 1983.

[4] G. Das Varma and N. Vettas. Optimal dynamic pricing with inventories. *Economics Letters*, 72:335–340, 2001.

[5] J. Gallien. Dynamic mechanism design for online commerce. *Operations Research*, 54:291–310, 2006.

[6] U. S. Karmarkar. Integrative research in marketing and operations management. *Journal of Marketing Research*, 33:125–133, 1996.

[7] J. Kennan and R. Wilson. Bargaining with private information. *Journal of Economic Literature*, 31:45–104, 1993.

[8] P. R. Kleindorfer and J. Wu. Integrating long- and short-term contracting via business-to-business exchanges for capital-intensive industries. *Management Science*, 49:1597–1615, 2003.

[9] H.-W. Kuo, H.-S. Ahn, and G. Aydin. Dynamic pricing of limited inventories when customers negotiate. Working Paper, University of Michigan, Ann Arbor, MI, USA, 2009.

[10] R. B. Myerson. Mechanism design by an informed principal. *Econometrica*, 51:1767–1798, 1983.

[11] R. B. Myerson. Two-person bargaining problems with incomplete information. *Econometrica*, 52:461–487, 1984.

[12] R. B. Myerson. Analysis of two bargaining problems with incomplete information. In A. E. Roth, editor, *Game-theoretic Models of Bargaining*, pages 115–147. Cambridge University Press, Cambridge, UK, 1985.

[13] R. B. Myerson and M. A. Satterthwaite. Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 29:265–281, 1983.

[14] J. F. Nash. The bargaining problem. *Econometrica*, 18:155–162, 1950.

[15] H. Raiffa, J. Richardson, and D. Metcalfe. *Negotiation Analysis: The Science and Art of Collaborative Decision Making*. Harvard University Press, Cambridge, MA, USA, 2003.

[16] J. Riley and R. Zeckhauser. Optimal selling strategies: When to haggle, when to hold firm. *The Quarterly Journal of Economics*, 98:267–289, 1983.

[17] S. Roth, H. Woratschek, and S. Pastowski. Negotiating prices for customized services. *Journal of Service Research*, 8:316–329, 2006.

[18] N. L. Stokey and R. E. Lucas. *Recursive Methods in Economic Dynamics*. Harvard University Press, Cambridge, MA, USA, 1989. With E. C. Prescott.

[19] K. T. Talluri and G. J. van Ryzin. *The Theory and Practice of Revenue Management*. Kluwer Academic Publishers, Boston, MA, USA, 2004.

[20] C. Terwiesch, S. Savin, and I.-H. Hann. Online haggling at a name-your-own-price retailer: Theory and application. *Management Science*, 51:339–351, 2005.

[21] G. Vulcano, G. J. van Ryzin, and C. Maglaras. Optimal dynamic auctions for revenue management. *Management Science*, 48:1388–1408, 2002.

[22] R. Wang. Bargaining versus posted-price selling. *European Economic Review*, 39:1747–1764, 1995.