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Endogenous Flows of Foreign Direct Investment and International Real Business Cycles

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Endogenous Flows of Foreign Direct Investment and International Real Business Cycles.

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Abstract

This paper models flows of foreign direct investment (FDI) in a two country, two sector DSGE framework. The allocation of capital to production capacity abroad is subject to a search-and-matching friction with endogenous capital reallocation. The model is calibrated on observed gross inflows and outflows of FDI and leads to dynamics of net foreign direct investment consistent with the empirical evidence documented in this paper: relative to domestic investment, FDI is more volatile, and inward and outward net flows of FDI are positively correlated. This contrasts with a standard International Real Business Cycle model with investment adjustment costs and its prediction of a negative correlation and low volatility. Moreover, the model solves the aggregate investment quantity puzzle as it generates cross-country correlations in-line with the data.

JEL classification: F41 E32 E22

Keywords: International business cycles; foreign direct investment; comovements.
1 Introduction

Generally foreign investment is welcomed for bringing new capital to an economy and increasing productivity through the arrival of new technologies. This has also been the main focus of the theoretical and empirical literatures concerned with foreign direct investment. Little attention has been paid, however, to the short and medium run behavior of foreign-controlled firms and, in general, to their importance in understanding the business cycle of open economies. This seems somewhat surprising, as a commonly used measure of the rate at which foreigners gain control over a domestic economy, flows of foreign direct investment (FDI), are large and very volatile. In Canada, for example, foreign-owned firms generate up to one third of employment and control over a fifth of all assets, a share that has been stable over the last four decades.¹

The bulk of FDI, among developed countries, involves the replication of production capacity abroad, or what is known as horizontal FDI, and in particular for the purpose of serving the host market (Brainard 1993, 1997). What is less well known, and is documented in detail for the case of Canada in Section 2, is that both gross inflows into, and outflows from, a host economy of FDI by foreigners increase during an upturn. Moreover, business cycle fluctuations in net FDI in Canada and net Canadian investment abroad are positively correlated. Thus periods of increased net inflows into an expanding economy are also periods of increased investment abroad by that same economy. The classic international real business cycle, however, generates a negative correlation between these flows.

The approach taken by this paper, in Section 3, is to model gross flows of horizontal foreign direct investment in a two-country, two-sector model,² in which the allocation of capital to production abroad is subject to a friction of the search and matching type: bringing to fruition a new investment project abroad is costly and time consuming and, once in place, faces an endogenous termination probability. The model therefore provides a theoretical framework with endogenous gross inflows and outflows of foreign direct investment.

Several considerations motivate the modeling strategy adopted here. First, as argued by Gordon and Bovenberg (1996), due to a lack of knowledge of the domestic economy foreign firms are at a

¹These figures are for the manufacturing sector, see Baldwin and Dhalwai (2001) and Baldwin and Gellatly (2005). The importance of FDI does not limit itself to the case of Canada. For example, the ratio of FDI to domestic investment in the US has risen from 6% in the 1970s to 15% in the 2000s. Lipsey (2000) reports ratios above 10% for many industrialized country over the period 1970 to 1995.

²This paper models horizontal FDI, treated in the trade theory literature by papers such as Markusen (1984), Markusen and Venables (2000), and Helpman, Melitz and Yeaple (2004). By opposition, vertical FDI refers to the "geographic distribution of production globally in response to the opportunities afforded by different markets." Models of the first category center around the "proximity-concentration" trade-off, while the second are models of factor proportions. See Markusen (2004) for a good overview of the multi-national firm literature.
disadvantage in setting up and running a firm. While these authors capture this idea by assuming that output at foreign firms is reduced by some fixed proportion, the search and matching framework yields two distinct sources of costs. First, foreign firms expend more than domestic firms in bringing a new investment project to fruition. Second, the probabilistic nature of the matching process captures the fact that foreigners incur the cost of more time in setting up a new production facility or acquiring information about a risky investment project as in Gopinath (2004).

The second consideration concerns the empirical reality of large and volatile gross FDI outflows. To incorporate this feature of the data, the model relies on endogenous capital reallocation resulting from the introduction of match specific idiosyncratic productivity shocks. Drawing from the labor market literature, starting with Mortensen and Pissarides (1994) and, den Haan, Ramey and Watson (2000), reallocation occurs for realizations of the idiosyncratic shock that yield a negative surplus, and is thus efficient from the point of view of the firm and the capital owner. Following an expansionary technology innovation in the host economy, the opportunity cost to the foreign capital lender of maintaining a unit of capital locked in with a foreign affiliate increases. This is because foreign affiliates increase their project initiations faster than foreign capital owners increase the pool of liquid capital available for allocation to production abroad (i.e. to the expanding host economy), increasing the probability for a given foreign capital lender of finding an appropriate investment project. Thus the rate of capital reallocation from foreign affiliates increases with the host’s business cycle.

Empirically, the model generates the high cyclical volatility of net FDI flows, and the positive correlation of net foreign direct investment inflows and outflows observed in the data. By contrast, a standard IRBC model with investment adjustment costs predicts a negative correlation, and lower volatilities of FDI flows, similar in magnitude to that of aggregate investment. As Section 4 elaborates in assessing the quantitative implications of the model, the allocation friction is central to explaining the positive correlation of net inflows and net outflows of FDI, the pro-cyclical reallocation raising the correlation even further. Following a positive technology shock in the host economy, whether in a standard IRBC model with investment adjustment costs or a search in FDI model with endogenous reallocation, flows of net inward FDI increase on impact. By simple arbitrage, gross flows of FDI from the host economy abroad decrease on impact, generating a negative correlation in the standard IRBC model. However, in the proposed model this same drop in the pool of capital

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3 Proximity-concentration models of FDI, e.g. Helpman, Melitz and Yeaple (2004), generally assume a fixed cost to setting up operations abroad, above the cost of entering the domestic market. As will be discussed below, foreign affiliates pay a cost per investment project initiation. Although allocation of capital to domestic firms will be frictionless, this is only a special case of the search environment when the initiation cost is nil.

4 This setup was explored extensively in a closed economy setting for the allocation of physical capital by Kurmann and Petry-Ok-Nadeau (2007). Gopinath (2004) models the difficulty in acquiring the information on investment projects in emerging projects as a time consuming search process.
goods available for allocation abroad increases the allocation probability for the capital owner in the short run, thus mitigating the drop in new allocations abroad. This same congestion in capital allocation increases the opportunity cost of terminating existing capital relationships, and thus the capital reallocation rate in the foreign country drops on impact. This change in the gross flow of capital from the expanding economy’s direct investment abroad further counters the drop in gross outflows, and the sum leads to the positive correlation of net inward and outward flows of FDI that is a feature of the data.

This paper is related to the growing literature on international real business cycles, dating back to the seminal contribution of Backus, Kehoe and Kydland (1992), and to the transmission channels of international business cycles. One measure of the international transmission of business cycles, the cross country correlation of aggregate variables, poses a problem for standard IRBC models known of the quantity problem. That is, the ordering of output and consumption cross correlations in the model is opposite to that in the data. While many papers have made contributions in reducing or solving this problem, few address another quantity problem involving aggregate hours and investment. A result of focusing on net flows of FDI is that the model solves the investment quantity puzzle. That is, contrary to other international real business cycle model, the presence of endogenous reallocation of capital in the model generates a cross-country correlation of aggregate investment coherent with the data.

2 Flows of FDI and Canada - U.S. business cycles

This section reviews evidence on the cyclical characteristics of FDI flows outlined in the introduction. While the Canadian economy is of particular interest for this study because of the large and historically stable share of economic activity originating in the foreign sector, it is increasingly significant for other industrialized economies as they further integrate. Flows of foreign direct investment into Canada, and flows of Canadian direct investment abroad, concerning overwhelmingly the United States, the focus is placed on the similarities and interdependence of both countries.

2.1 Canadian and U.S. business cycles

Despite a large difference in absolute size, in per capita terms the Canadian and U.S. economies are remarkably similar. The evolution of hours worked (indexed), real output, investment and consumption per capita in both countries, over the period 1976-2006, have but for a few episodes

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5 This is because, in this context, new allocations are a function of both the pool of capital available and the allocation probability, or congestion on the foreign capital market.


7 See Crucini (2006) for an extensive survey.
followed each other closely. One example is the output per capita gap between the U.S. and Canada appearing during the 1990s, which also shows up as a gap in average hours worked.

While aggregate trends have been similar, Table 1 examines differences in cyclical fluctuations of prominent macroeconomic variables, measured as 2nd moments for Hodrick-Prescott filtered quarterly data over 1976:1 - 2005:4. The Canadian and U.S. economies display approximately the same business cycle characteristics of these variables, although there is evidence of less aggregate consumption smoothing in Canada, seen as the larger relative volatility of consumption.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>variable:</td>
<td>a</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.80</td>
<td>0.87</td>
<td>0.47</td>
</tr>
<tr>
<td>Hours</td>
<td>0.80</td>
<td>0.83</td>
<td>0.95</td>
</tr>
<tr>
<td>Investment</td>
<td>3.11</td>
<td>0.69</td>
<td>3.24</td>
</tr>
<tr>
<td>Output</td>
<td>1.53</td>
<td>1.42</td>
<td></td>
</tr>
</tbody>
</table>

a: Standard deviation relative to output;
b: Contemporaneous correlation with output;
All moments are Hodrick-Prescott filtered.

One indicator of business cycle synchronization, the cross-country contemporaneous correlation of prominent macroeconomic variables, is reported in the last column of Table 1. In their extensive study of international business cycles, Ambler, Cardia and Zimmermann (2004) find much lower, although positive, cross country correlations than those for the Canada - U.S. pair, suggesting a higher than average degree of integration of both economies. While both theoretical and empirical work have often followed trade as a vector of synchronization, the increasingly important channel of flows of foreign direct investment is explored in the next subsection.

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8 A data and technical appendix is available upon request. The time series for the mentioned variables are plotted in Figure 1 of said appendix.
9 See also Baxter and Crucini (1995), Ambler, Cardia and Zimmermann (2004) for similar observations.
10 Ambler, Cardia and Zimmermann (2004) consider a sample of 20 industrialized countries, and all pairwise cross-country correlations, on quarterly data over the period 1960:1-2000:4. Average output, consumption, investment and hours cross-correlations are, respectively, 0.22, 0.14, 0.18, 0.26.
11 Sales by multinational firms have outpaced the expansion of trade in manufactures over the last decades. See Markusen (2004). Kose and Yi (2001) explore and discuss the limitations of the trade approach to solving the quantity puzzles. Ambler, Cardia and Zimmermann (2002) explore the potential of a two country multi-sector model with trade in intermediate goods in addressing the same issue. Other avenues have been explored, such as variable capital utilization in Baxter and Parr (2001), or trade in capital goods in Boileau (1999). Iacoviello and Minetti (2006) explore the implications of imperfect cross-border credit relations for output cross-correlations. See also Schmitt-Grohé (1998) for an evaluation of various mechanisms.
2.2 Flows of FDI and foreign controlled firms in Canada

There are essentially two ways in which foreigners can access a domestic economy: (i) by establishing a branch or new business; (ii) through mergers and acquisitions of domestic firms. A commonly used measure of the rate at which foreigners access a domestic economy, flows of foreign direct investment, can further be categorized as either "horizontal" or "vertical". As described by Markusen (2002), horizontal FDI refers to the replication of capacity abroad, and vertical FDI to the division of the production process globally in order to exploit the benefits offered by different markets. As Brainard (1997) documents and argues, the majority of FDI between developed countries is horizontal. In addition, the large majority of foreign affiliate sales are destined to the host market. There remains, however, a debate over the principle mode of accessing an economy, although Helpman, Melitz and Yeaple (2004) argue that it occurs mainly through "greenfield" investment.

In order to assess the extent and effect of foreign control over the national economy, in 1962 the Canadian government passed the Corporations Returns Act (CRA), requiring firms doing business in Canada to report financial and ownership data. Of the 40 000 reporting firms in 2004, foreign controlled corporations accounted for 30.7% of total operating revenues and 28.5% of all assets held in Canada, shares that have historically remained stable (see Figure 1). The United States play a central role in the foreign control of the Canadian economy, generating 62.6% of the operating revenues of foreign controlled corporations. The closest behind are the United Kingdom and Germany with, respectively, 7% and 6.5% of operating revenues.

As seen by industrial sector, foreign control is most important in oil and gas, manufacturing and mining, and significant in wholesale trade, utilities, and transportation and warehousing. Manufacturing stands out as a sector with a large share of employment and high degree of foreign control, involving nearly one fifth of employment and where just over half of the revenues and assets

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12 As reported in Brainard (1993), approximately 92% a foreign affiliate production in the United States is destined for the host market.

13 By "greenfield" investment, one refers to the establishment of a branch or new business. The position taken by Helpman et al. (2004) differs from that of Graham and Krugman (1995) according to whom the evidence is less clear and leans rather towards a larger role for mergers and acquisitions. While this paper will follow Helpman et al. (2004), it worth noting a recent contribution by Nocke and Yeaple (2007). These authors investigate the theoretical determinants of FDI by M&A or greenfield investment.

14 "The notion of control encompasses both direct and effective control. Direct control is defined as a person, group or corporation holding, directly or indirectly, more than 50% of the voting equity. Effective control implies control through methods other than ownership of the majority voting equity, such as when more than 50% of the directors of a corporation are also directors of another corporation. A corporation is foreign controlled when either direct or effective control is held by a person, group or corporation not resident in Canada." For additional information, see "Corporations Returns Act, 2004," catalogue no. 61-220. Statistics Canada, vol XI E, p. 3.

15 This figure is reproduced from catalogue no. 61-220. Statistics Canada, vol XI E.

16 See Figure 2 of the appendix. The same figure also reports for these industries their share of total employment.
Figure 1: Share of assets and operating revenue under foreign control. Source: Statistics Canada

Figure 2: Flows of foreign direct investment receipts and payments, Canadian Balance of Payments. are under foreign control. In fact, Baldwin and Gellatly (2005) estimate the share of manufacturing employment originating in the foreign sector to be 30% of total sectoral employment. Together, sectors with more than 20% foreign control, in terms of assets, involve 55% of employment. Although these are not the ideal measures of aggregate activity generated in the foreign sector, they give a sense of the importance of foreign controlled firms for aggregate outcomes.

2.2.1 Flows of foreign direct investment

Flows of foreign direct investment in Canada (receipts) and flow of Canadian direct investment abroad (payments) from the Canadian Balance of Payments are large, historically around 20% of aggregate Canadian investment. The source and destination of these flows is overwhelmingly the U.S., generating a share of 44% of receipts and destination for 58% of payments. Except for a brief period in the early 1990s, payments have always exceeded receipts, leading to a persistent deficit offset only by Canada's historically positive trade balance (see Figure 2).

The business cycle component of net flows of FDI into Canada and flows of Canadian direct investments abroad, along with their cross-correlation are presented in Table 2.\textsuperscript{17} Both flows are

\textsuperscript{17}It is important to stress that flows of portfolio investment are excluded, keeping only flows of direct investment.
highly volatile and of similar magnitude, with H.-P. filtered standard deviations relative to output of 14.19 and 8.8, respectively. Table 2 further decomposes FDI in Canada and Canadian investment abroad into gross inflows and outflows.\footnote{All flows are described from the point of view of the Canadian economy. For example, gross outflows of Canadian direct investment abroad concern payments from Canada to the world. Gross inflows, on the other hand, correspond to flows returning to the Canadian economy.} Whether it be FDI in Canada, or Canadian investment abroad, gross flows exiting the host economy are more volatile than flows entering, and they are less correlated with the Canadian business cycle. It is particularly interesting to note here that gross outflows of FDI in Canada are slightly procyclical.

<table>
<thead>
<tr>
<th>1976:1 - 2005:4</th>
<th>Foreign Direct Inv. in Canada</th>
<th>Canadian Direct Inv. Abroad</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>inflows</td>
<td>outflows</td>
</tr>
<tr>
<td>std. dev rel output</td>
<td>14.83</td>
<td>17.39</td>
</tr>
<tr>
<td>contempor. corr. with output</td>
<td>0.48</td>
<td>0.12</td>
</tr>
<tr>
<td>Ratio of outward to inward net FDI flow standard deviations</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>Contemporaneous correlation, inward-outward net flows.</td>
<td>0.27</td>
<td></td>
</tr>
</tbody>
</table>

2nd moments were computed for Hodrick-Prescott filtered data. Data source: Statistics Canada

Both net inflows and net outflows, that is net FDI in Canada and net Canadian Direct Investment Abroad move strongly with the Canadian business cycle, with respective contemporaneous correlations with Canadian GDP of 0.36 and 0.4. Moreover, Table 2 also reveals that net inward and net outward flows are positively correlated. That is, periods of increased net inflows of FDI into Canada are also accompanied by increased net Canadian direct investments abroad. This fact has not received much attention, equilibrium models tending to predict that capital would simultaneously flow from high productivity and out of low productivity countries.

3 IRBC with search in FDI and endogenous reallocation

The model develops a framework with net inflows and net outflows of foreign direct investment in a two country, two sector DSGE model, where gross flows in both directions evolve endogenously with the business cycle. Each country is populated by domestic and foreign firms and a representative household. For simplicity the model abstracts from trade in consumption goods. Households decide on optimal consumption, an aggregate of goods produced by both types of firms, and allocation of investment goods to firms located at home or abroad. In order to initiate a new investment project abroad, foreign affiliates must disburse a flow cost $\kappa$. This cost is paid until the project is brought to fruition, a time consuming task abstracted as a search and matching process with investment
goods available for allocation abroad. No such friction applies to changing production capacity at
domestic plants located at in the home economy.\textsuperscript{19} Thus domestic firms rent capital on spot markets
while foreign affiliates choose the amount of new projects to initiate. Firms, domestic and foreign,
hire labor on competitive domestic markets.

As a matter of notation, the first country is referred to as the "Home" country and the second as
the "Foreign" country. Throughout, variables relating to the Foreign economy will be distinguished
by an asterisk. For example, $k^{f d i}$ denotes the stock of capital held by foreigners in the "Home"
economy while $k^{f d i*}$ denotes the stock of capital held by foreigners in the "Foreign" country, i.e.
held by residents of the Home country. We begin by describing the friction to allocating physical
capital abroad, domestic and foreign firms, and then examine the problem faced by the representa-
tive household in the Home economy, the problem in the Foreign economy being symmetrical.
Endogenous capital reallocation is introduced before closing the model.

\subsection*{3.1 Undertaking a foreign direct investment}

In order to form a unit of capital abroad, a new project, \( v \), must be initiated at a cost of \( \kappa \) by a foreign
affiliate.\textsuperscript{20} Meanwhile, a pool of liquid capital, \( l \), must be made available to be allocated abroad once
the right location has been found. This process of matching new projects and liquid capital is
abstracted by a constant returns to scale matching technology \( m(v, l) \). Denoting \( \theta = \frac{l}{v} \) as a relative
measure of capital liquidity, the probability for a given project initiation of becoming a productive
unit of capital in the current period is given by \( \frac{m(v, l)}{\theta} = m(1, \theta) = p(\theta) \), with \( \partial p(\theta) / \partial \theta > 0 \). The
equivalent probability for liquid capital is just \( \frac{m(v, l)}{\theta} = m(1/\theta, 1) = q(\theta) \), \( \partial q(\theta) / \partial \theta < 0 \).

Once in place, a particular unit of foreign capital faces a probability (the determination of which
is discussed later) \( s_t \) of being terminated. When this occurs the unit of capital returns to the pool
of liquid capital, net of depreciation, for reallocation. As a result, the total amount of liquid capital
available for allocation abroad in the current period is defined as

\[ l_t = i_t^{f d i} + (1 - \delta)s_t k_t + u_t, \]

where \( u_t \) is unmatched liquid capital from the previous period carried forward with no net return,
and \( i_t^{f d i} \) are new investment goods added to the pool of liquid capital.

\textsuperscript{19}In fact, the frictionless capital market is a special case of the search environment with \( \kappa = 0 \). This extreme
assumption of no friction to allocating investment goods to domestic firms at home is made for simplicity. As long as
allocating investment goods abroad is relatively more costly than allocation at home the results go through.

\textsuperscript{20}This cost is reminiscent of Gordon and Bovenberg (1996) who assume that foreign investors, due to a lack
of knowledge of the domestic economy, are at a disadvantage in setting up and running a firm. They capture this
idea by assuming that output by foreign firms is reduced by some fixed proportion. Gopinath (2004) assumes that
investors in emerging markets must disburse a cost to acquire information on investment projects while the length of
the acquisition period is subject to search frictions.
These assumptions result in the following law of motion for the stock of foreign capital in the Home economy

\[ k_{t+1}^{fd} = (1 - \delta)(1 - s_t)k_t^{fd} + m(v_t, l_t). \]  

(2)

For ease of comparison with the Balance of Payments, it is useful to rewrite the law of motion as \( k_{t+1}^{fd} = (1 - \delta)k_t^{fd} + m(v_t, l_t) - (1 - \delta)s_t k_t^{fd} \). The expression \( m(v_t, l_t) \) corresponds to gross inflows of foreign direct investment while \( (1 - \delta)s_t k_t^{fd} \) corresponds to gross outflows of foreign direct investment, the difference being net flows of inward FDI. The Home economy’s direct investment abroad is likewise decomposed into gross outflows \( m(v^*_t, l^*_t) \) and gross inflows \( (1 - \delta)s_t^* k_t^{fd*} \) (i.e., returning from the Foreign country).

### 3.2 Domestic and foreign producers

Domestic firms produce with technology \( y_t^d = A_t(n_t^d)^{1-\alpha}(k_t^d)^{\alpha} \), hiring both factors of production from households on competitive markets. Optimization yields the following two first order conditions:

\[ (n_t^d) : \quad w_t^d = (1 - \alpha) \frac{y_t^d}{n_t^d}; \]

\[ (k_t^d) : \quad r_t^d = \alpha \frac{y_t^d}{k_t^d}; \]

where \( w_t^d \) and \( r_t^d \) are, respectively, the remunerations of labor and capital at domestic firms.

Foreign firms in the Home economy hire domestic labor, \( n_t^{fd} \), and make capital adjustment decisions by choosing the number of new projects to initiate, \( v \). This yields the following dynamic program:

\[
J(k_t^{fd}) = \max_{n_t^{fd}, v_t} \left[ A_t(n_t^{fd})^{1-\alpha}(k_t^{fd})^{\alpha} - w_t^{fd} n_t^{fd} - r_t^{fd} k_t^{fd} - \kappa v_t + \beta E_t \frac{\lambda_t^{*+1}}{\lambda_t} J(k_{t+1}^{fd}) \right]
\]

subject to \( n_t^{fd} = (1 - \delta)(1 - s_t)k_t^{fd} + p(\theta_t)v_t \),

where \( w_t^{fd} \) and \( r_t^{fd} \) are, respectively, the remunerations of labor and capital at foreign firms. The foreign affiliate’s discount rate is \( \beta E_t \frac{\lambda_t^{*+1}}{\lambda_t} \), because all profits are transferred to the foreign household.

Optimization yields the following two first order conditions:

\[ (n_t^{fd}) : \quad w_t^{fd} = (1 - \alpha) \frac{y_t^{fd}}{n_t^{fd}}; \]

\[ (v_t) : \quad \beta E_t \frac{\lambda_t^{*+1}}{\lambda_t} J_{k_t^{fd}} (k_{t+1}^{fd}) = \frac{\kappa}{p(\theta_t)}; \]

where \( J_{k_t^{fd}} (k_{t+1}^{fd}) \) is the marginal value of an additional unit of capital to the firm. While the first condition is quite standard, some interpretation of the optimality condition for project initiations is in order. This states that, at the margin, the discounted expected return to an additional unit of capital must be equal to the average cost of setting it up, \( \frac{\kappa}{p(\theta_t)} \). As such, this may be interpreted
as a “project creation” condition akin to the job creation condition in labor search and matching models. Differentiating the firm’s value function, the marginal value is defined as

\[ J_{k,t+1}(k_{t+1}^{di}) = \frac{y^{f,di}}{k_{t+1}^{di}} - r^{f,di} + (1 - \delta)(1 - s_t)\beta E_{t+1}^{\lambda_t+1} J_{k,t+1}(k_{t+1}^{f,di}) \]

In combination with the first order condition for project initiations, this yields the forward looking condition

\[ \frac{\kappa}{p(\theta_t)} = \beta E_{t}^{\lambda_t+1} \lambda_t \left\{ \frac{y^{f,di}}{k_{t+1}^{di}} - r^{f,di} + (1 - \delta)(1 - s_t)\frac{\kappa}{p(\theta_{t+1})} \right\} \]

(6)

3.3 Domestic households

Households choose a level of aggregate consumption (simply defined as the sum of both intermediate goods), hours to supply to both domestic and foreign employers, \( n_t^d \) and \( n_t^{f,di} \) respectively, and have two capital investment options: investing in firms at home, \( i_t^d \), or investing in capacity abroad, \( i_t^{f,di} \). In addition, there are convex cost to producing new investments goods, domestic and foreign.\(^{21}\) The resulting dynamic program for the representative household is thus

\[ V(k_t^d, k_t^{f,di}, u_t^*) = \max_{c_t, n_t^d, n_t^{f,di}, i_t^d, i_t^{f,di}} \left[ u(c_t, 1 - n_t) + \beta E_t V(k_{t+1}^d, k_{t+1}^{f,di}, u_{t+1}^*) \right] \]

subject to

\[ w_t^d n_t^d + w_t^{f,di} n_t^{f,di} + r_t^d k_t^d + r_t^{f,di} k_t^{f,di} + \Pi_t = c_t + q_t^d i_t^d + q_t^{f,di} i_t^{f,di} \]

and

\[ k_{t+1}^{f,di} = (1 - \delta)(1 - s_t) k_t^{f,di} + q(\theta_t^*) i_t^* \]

where \( n_t = n_t^d + n_t^{f,di} \), \( i_t^d = k_t^d + (1 - \delta) k_t^{f,di} \), and \( q_t^d \) and \( q_t^{f,di} \) are, respectively, the cost of new investment goods destined for plants at home and abroad. This cost is given by \( q_t^j = \left[ \Phi'(\theta_t^*) \right]^{-1} \), for \( j = d, f, di^* \), with \( \Phi'(\bullet) > 0 \) and \( \Phi''(\bullet) < 0 \), and such that in the steady state \( q = 1 \). New investment goods destined for foreign direct investment are defined as \( i_t^{f,di} = l_t^* - (1 - \delta) s_t k_t^{f,di} - u_t^* \), where \((1 - \delta) s_t k_t^{f,di} \) is capital recuperated from terminated operations abroad net of depreciation, and \( u_t^* \) are units of investment goods not yet allocated. Again, \( l_t^* \) is therefore the total amount of investment goods available for allocation to production abroad.

Denoting the multiplier on the budget constraint \( \lambda_t \), the optimality conditions are

\[ (c_t) : u_c(c_t, 1 - n_t) = \lambda_t \]

(7)

\[ (n_t^d) : u_{n^d}(c_t, 1 - n_t) = \lambda_t w_t^d \]

(8)

\[ (n_t^{f,di}) : u_{n^{f,di}}(c_t, 1 - n_t) = \lambda_t w_t^{f,di} \]

(9)

\[ (k_{t+1}^d) : \lambda_t q_t^d = \beta E_t \lambda_{t+1} \left[ r_t^d + q_t^d (1 - \delta) \right] \]

(10)

\[ (l_t^*) : \lambda_t q_t^{f,di} = \beta E_t \left[ q(\theta_t^*) V_{k,f,di}^*(k_{t+1}^d, k_{t+1}^{f,di}, u_{t+1}^*) + (1 - q(\theta_t^*)) V_{u^*}^*(k_{t+1}^d, k_{t+1}^{f,di}, u_{t+1}^*) \right] \]

(11)

\(^{21}\)It is well known (see Backus, Kehoe and Kydland [1992], Baxter and Crucini [1995]) that without an adjustment cost to the production of new capital goods the volatility of new investment would be much too large in this setting.
The Euler equation for allocation of investment goods to domestic firms, equation (10), has the usual interpretation of equating the opportunity cost of the investment, in terms of current period foregone consumption, to the expected return net of depreciation. The Euler equation governing foreign investment decisions, equation (11), has a similar interpretation. The expected return, however, is an average of the marginal values of matched \((V_{k_{t+1}, u_t}^d(k^d_{t+1}, k^{dis}_{t+1}, u^*_t))\) and unmatched \((V_{u_t}^d(k^d_{t+1}, k^{dis}_{t+1}, u^*_t))\) capital, weighted by the matching probability \(q(\theta^*_t)\). The marginal values of allocated and non-allocated investment goods are given by

\[
V_{u_t}(k^d_{t+1}, k^{dis}_{t+1}, u^*_t) = \lambda_t q_t^{\text{dis}}; \\
V_{k_{t+1}, u_t}^d(k^d_{t+1}, k^{dis}_{t+1}, u^*_t) = \lambda_t \left[r_t^{\text{dis}} + q_t^{\text{dis}}(1-\delta)s_t^d + (1-\delta)(1-s_t^d)\beta E_t V_{k_{t+1}, u_t}^d(k^d_{t+1}, k^{dis}_{t+1}, u^*_t)\right].
\]

Since unmatched liquid capital yields not net return, its marginal value is simply the opportunity cost of funds. The marginal value of matched capital consists of the earnings on the unit, \(r_t^{\text{dis}}\), and the value of capital separated for reallocation net of depreciation. The last term captures the continuation value if reallocation does not occur.

### 3.4 Repayment on foreign capital

Each unit of capital allocated abroad generates a surplus for the foreign affiliate and the capital lender. The repayment on capital allocated abroad is determined by Nash bargaining over the total surplus generated by the relationship, defined as \(S_t = J(k^{dis}_t) + \frac{V_{k_{t+1}, u_t}^d(k^d_{t+1}, k^{dis}_{t+1}; u^*_t) - V_{u_t}^d(k^d_{t+1}, k^{dis}_{t+1}, u^*_t)}{\lambda_t}\). This results in the following repayment rule:\(^{22}\)

\[
\frac{r_t^{\text{dis}}}{k_t^{\text{dis}}} = \eta \alpha \frac{\theta_t}{k_t^{\text{dis}}} + (1-\eta)q_t^{\text{dis}} \delta + \eta(1-\delta)(1-s_t^d)\frac{\kappa}{\theta_t}. \tag{12}
\]

By the first term, the repayment is increasing in the marginal product of capital. The second term captures the loss of value due to physical depreciation, measured by the price of investment goods, the cost of which is split according to the lender’s bargaining weight \(\eta\). The long-term nature of the relationship is captured by the final term. It represents the initiation costs saved by the firm in the continued operation of the unit of capital. By changing the relative threat point of the firm in negotiations, a rise in \(\kappa\) puts upward pressure on the repayment.

### 3.5 Endogenous reallocation and profits

As in Mortensen and Pissarides (1994) and den Haan et al. (2000) for the labor market, the existence of a random idiosyncratic productivity to the match is assumed, the realization of which occurs after production decisions are made and factor price equilibria are established. Denote this realization

---

\(^{22}\)The appendix provides details on the derivation of this repayment rule.
$a_t > 0$, where $a$ is independently distributed over time with probability density $h(a)$, cumulative density $H(a)$ and mean $E(a) = 1$, and follows a lognormal distribution $\log(a) \sim N(-\frac{\sigma_{log(a)}^2}{2}, \sigma_{log(a)}^2)$.

The surplus generated by the relationship between a foreign affiliate and the capital lender (i.e. the household) is then an increasing function of this shock, $S(a_t)$. Once the shock is observed, both parties discontinue the match for realizations of $a_t < a_\tau$ where $a_\tau$ is define as $S(a_\tau) = 0$. Using a result of Nash bargaining, the separation threshold is defined by \(^{23}\)

$$r_t^{fdi} - \frac{\alpha_t u_t^{fdi}}{k_t^{fdi}} (1-\delta) (1-s_t) \frac{\kappa}{p(\theta_t)} = 0 \quad (13)$$

In effect, the match is discontinued if the realized marginal product of capital $a_t \alpha_t u_t^{fdi} / k_t^{fdi}$ plus the search cost saved by maintaining the current unit of capital is inferior to the negotiated repayment. An increase in the average search cost $\frac{\kappa}{p(\theta_t)}$, for example, by increasing the opportunity cost of exiting the match, lowers the separation probability.

Finally, an insurance mechanism funded out of profits from continuing relationships is assumed in order to insure that ex-post the household receives the full ex-ante return to foreign capital, and that the full wage bill and costs of project initiations are covered.\(^{24}\) Thus aggregate profits returned to the household are

$$\Pi_t = \int_0^\infty [a y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t] dH(a) + \int_0^\infty [a y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t] dH(a)$$
\[ \Pi_t = y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t. \] \quad (14)

Equilibrium is defined by the system of equations (1)-(14), production technologies for domestic and foreign firms, and the definition for the separation rate, $s_t = H(\underline{\omega})$, in both the Home and Foreign countries. The model is solved for the rational expectations equilibrium of the log-linear system of equations with the algorithm developed by King and Watson (1998).

## 4 Quantitative Results

The model’s quantitative implications are evaluated through impulse responses and unconditional second moments. The results for flows of FDI are discussed first, before looking at aggregate variables and cross-country correlations. In all instances the results are contrasted with those for a standard IRBC with investment adjustment costs, and a model with exogenous reallocation.\(^{25}\)

\(^{23}\)See appendix for details.

\(^{24}\)Gordon and Bovenberg [1996] use a similar assumption about the realization of an idiosyncratic productivity shock, and use the law of large numbers to argue that there is no aggregate uncertainty. Here, an insurance funded out of aggregate profits is used to address the issue.

\(^{25}\)The details these models are presented in the appendix. It is important to note that the first corresponds to the search model without allocation frictions, i.e. a model with $\kappa = 0$. 

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4.1 Shocks and Calibration

4.1.1 Extraction of a Solow Residual.

The underlying exogenous processes for technology, as in Backus Kehoe and Kydland (1992), is assumed stationary and to follow a VAR(1) process with possible cross-country spill-overs. Parameter estimates are obtained by extracting Solow residuals for the Canadian and U.S. economies and then estimating the following bivariate VAR(1):26

\[
\begin{bmatrix}
A_{t}^{\text{can}} \\
A_{t}^{\text{us}}
\end{bmatrix} = 
\begin{bmatrix}
\rho_{c} & \rho_{c,us} \\
\rho_{us,c} & \rho_{us}
\end{bmatrix}
\begin{bmatrix}
A_{t-1}^{\text{can}} \\
A_{t-1}^{\text{us}}
\end{bmatrix} + 
\begin{bmatrix}
\epsilon_{t}^{\text{can}} \\
\epsilon_{t}^{\text{us}}
\end{bmatrix}
\]

The results of the estimation are presented below for the period 1976:1 - 2005:4, with t-statistics presented below coefficient estimates. As is usual in this sort of estimation, the persistence parameter is very high. Also, as can be gleaned from the covariance matrix, Canadian and U.S. innovations to the exogenous process for technology are positively correlated. In a subsection below, the sensitivity of the quantitative results to the specification of the exogenous process for technology will be examined.

\[
\begin{bmatrix}
\rho_{c} & \rho_{c,us} \\
\rho_{us,c} & \rho_{us}
\end{bmatrix} = 
\begin{bmatrix}
0.988 & 0.037 \\
-0.104 & 0.979
\end{bmatrix}
\text{Residuals Covariance matrix : } 
\begin{bmatrix}
0.11 & 0.045 \\
0.045 & 0.07
\end{bmatrix}
\]

4.1.2 Calibration

The discount factor is set to $\beta = 0.99$ which corresponds to an average annual real yield on a risk-less bond of 4.1%. Preferences are separable in consumption and leisure, and take the form $u(c_{t},1-n_{t}) = \log(c_{t}) + \frac{\varpi(1-n_{t})^{1-\xi}}{1-\xi}$. The parameter $\varpi$ is fixed such that the average fraction of total hours worked equals $n = 0.2$. Together with $\xi = 4$ this results in a Frisch elasticity of labor supply of 1. Furthermore, the share of hours worked in the foreign sector is set to 1/4, which is in the range of employment shares reported earlier. The share of capital in the production function is set to $\alpha = 1/3$, and the rate of depreciation of capital to $\delta = 0.025$, which corresponds to an annual decline of productive use of capital of 10%. The elasticity of the investment adjustment cost is 0.075, in the middle of a range of values used in different studies (e.g., Baxter and Crucini, 1995, Ambler, Cardia and Zimmermann, 2002, and Baxter and Farr, 2005), and is chosen to match the relative standard deviation of aggregate investment in the data.

To calibrate parameters relative to foreign direct investment, it is useful to let the theory shed

\footnote{Disembodied productivity is measured as a residual for a Cobb-Douglas production function: $\log(A_{t}) = \log(y_{t}) - \alpha \log(n_{t}) - (1 - \alpha) \log(k_{t})$. The quarterly series of an aggregate capital stock for both economies is estimated using the perpetual inventory method.}
some light on the data. Recall the foreign capital accumulation equation

\[ k_{t+1}^{fdi} = (1 - \delta)k_t^{fdi} + \text{inflow}_t - \text{outflow}_t. \]

As the Balance of Payments provide information on foreign direct investment gross inflows and outflows, given a rate of capital depreciation one can compute the implied foreign capital stock in the host economy, using the steady state property \( k^{fdi} = [\text{inflow} - \text{outflow}] / \delta \) to initiate the capital stock. It is then possible, using the time series on outflows, to obtain a time series for the reallocation rate as

\[ s_t = \frac{\text{outflow}_t}{(1 - \delta)k_t^{fdi}}, \]

resulting in a mean rate of \( s = 0.0602 \), an H.-P. filtered standard deviation relative to output of 1.45 and contemporaneous correlation with output of 0.16.

Next it is assumed that it takes on average a little more than a quarter before liquid capital is allocated and becomes productive; i.e. \( q(\theta) = 0.75 \), and we set the household's bargaining weight to \( \eta = 0.5 \). The latter is chosen such that the profit margins at foreign establishments is relatively low, i.e. 2.75%. The final parameter left to calibrate is the elasticity of the matching function, which is of the form \( m(v_t, l_t) = (v_t)^\epsilon(l_t)^{1-\epsilon} \). This parameter only influences the dynamics of the model but does not affect the steady state, and is therefore selected such that the relative volatility of the reallocation rate \( s \) is close to the data, leading to a choice of \( \epsilon = 0.66 \). A sensitivity analysis of results to variations in these parameters is performed below.

With these calibrations there is sufficient information to endogenously determine the rest of the parameters (i.e. \( \theta, \kappa, \sigma \)) such that the system of steady state equations is satisfied.\(^{27}\) The resulting long-run ratios of interest are the following: the consumption-output ratio equals 76.57\% in line with King and Rebelo (1999); the labor share of income amounts to 0.67, which lies in the range reported by Gollin (2002); Furthermore, this calibration implies that the steady state ratio of net FDI to aggregate investment is 23\%, that the average initiation cost relative to output equals \( \nu k / y^{fdi} = 1.52\% \) and that the standard deviation of the idiosyncratic productivity shock equals \( \sigma_a = 0.27 \).

### 4.2 Flows of foreign direct investment

Figure 3 plots the impulse responses to a persistent technology shock in the Home economy of net inward and outward foreign direct investment for that expanding economy. The significant difference between the responses of the proposed model (Panel A) and an IRBC model with investment adjustment costs (Panel B), beyond their magnitude, is the behavior of net outward flows (see circled line of panels A and B). In the search model, outward flows drop progressively, whereas in the standard

\(^{27}\) The details concerning the procedure for computing the steady state are available in the appendix.
model the drop occurs on impact. It is this difference that generates the positive cyclical correlation of net inward and outward flows that is a characteristic of the data.

To detail the response of net outward direct investment flows, it is useful to recall its definition as the difference between gross outflows and gross inflows from the Home to the Foreign economy:

$$Net \ outward : l^*_t q(\theta^*_t) - (1 - \delta) s^*_t k^{fdi}_t$$

The second column of Figure 3 decomposes the response of net outward flows into these new allocations (dashed line) and reallocation (solid line) of capital abroad.28

Consider now what matters for the initial response of net outward FDI: the response of gross outflows $l^*_t q(\theta^*_t)$ and the reallocation rate $s^*_t, k^{fdi}_t$ being predetermined. As the opportunity cost of capital abroad increases, households diminish their pool of liquid capital $l^*$, shifting resources to domestic firms, causing a drop in the Home country’s pool of capital available for investment abroad. The is the only source of change in net outflows in the model without allocation frictions, and therefore the drop in outward FDI is immediate. When allocation frictions are present, however, the decline in the pool of liquid capital is larger than the initial decline in project initiations at foreign affiliates for two periods after the shock, leading to a short lived increase in the capital allocation probability $q(\theta^*_t)$. This counters some of the drop in $l^*$ upon impact, as seen by a slightly muted initial decline in new allocations abroad (see the upper right quadrant of Figure 3). As $q(\theta^*_t)$ declines thereafter, new allocations attain their lowest at 6 quarters following the shock.

The second distinction comes from the effects on reallocation of capital already abroad. As illustrated in the second quadrant of Figure 4, a positive innovation in the Home country causes the

\[ \text{Figure 3: IRFs to a persistent "Home" sourced technology shock.} \]
Figure 4: IRFs of the separation rates to a Home country technology shock.

reallocation rate in the foreign country to drop, reducing the gross flow \((1 - \delta)s_t^* k_t^{dis}\) on impact.\(^{29}\) The drop pulls net outward flows from the Home the Foreign country upward, such that on impact net outward flows change very little (again, see Figure 3). Thus the key to understanding the response of net foreign direct investment flows are the time-varying congestion and reallocation rates, effects that are absent in the standard model.

Making this point clearer, Table 3 presents unconditional second moments for flows of foreign direct investment. The standard IRBC model with investment adjustment costs, for the reasons just outlined, generates a negative contemporaneous correlation between net inflows and outflows of FDI. A model with constant, exogenous reallocation goes a long way in improving this correlation. The contribution of endogenous reallocation is to increase the correlation even further. Indeed, the proposed model comes remarkably close to the empirical evidence, with a contemporaneous correlation of 0.32 compared to 0.27 in the data. Contrasting this result, the IRBC model generates a correlation of -0.88.

Another feature of the data concerns the relative volatility of net FDI outflows and inflows of approximately two thirds, and that net outflows are as procyclical as net inflows. The model IRBC fails on both these counts. The ratio of H.-P. filtered standard deviations is only 0.20, and the correlation of net outflows with the source country’s business cycle is strongly negative at -0.96. On the other hand, the proposed model of search in FDI performs very well on the relative volatility of net inflows and outflows, the ratio being 0.67 compared to 0.62 in the data. In addition, the model raises the correlation of net outflows with the business cycle, but not enough under this calibration to be in-line with the data.

\(^{29}\)The contemporaneous correlation with the host economy’s business cycle is 0.85 and its standard deviation relative to aggregate output 1.13. These numbers relate to Hodrick-Prescott filtered moments.
Table 3: 2nd moments for flows of foreign direct investment.

<table>
<thead>
<tr>
<th>1976:1 - 2005:4</th>
<th>Canadian data</th>
<th>Search in FDI model</th>
<th>IRBC with FDI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>Net FDI in Canada</td>
<td>14.19</td>
<td>0.36</td>
<td>8.42</td>
</tr>
<tr>
<td>Net Canadian Direct Inv abroad</td>
<td>8.8</td>
<td>0.40</td>
<td>5.69</td>
</tr>
<tr>
<td>Net outward / Net inward FDI</td>
<td>c</td>
<td>d</td>
<td>c</td>
</tr>
</tbody>
</table>

a: Standard deviation relative to output; b: Contemporaneous correlation with output; c: Ratio of outward to inward FDI flow standard deviations; d: contemporaneous corr., inward-outward net flows. All moments are Hodrick-Prescott filtered.

4.3 Robustness of results

This section performs two sets of sensitivity checks, first to the values of search related parameters, second to the specification of the exogenous technological process.

4.3.1 Sensitivity to search parameters

Given the lack of direct evidence on the mean allocation rate \( q(\theta) \), the effects of variations in this parameter on the main results, along with the consequence of varying the mean reallocation rate \( s \), are presented in Table 4.\(^\text{30}\) Beginning with the latter, the results are very robust to changes in the reallocation rate \( s \), the volatility of net inward FDI, and the relative volatility of net inward and outward flows changing little. The main effect is to increase the correlation, from 0.18 when \( s = 0.04 \) to 0.42 when \( s = 0.08 \). This is to be expected as the mechanism generating a positive correlation that is endogenous reallocation becomes more important.

Table 4: Sensitivity to search parameters

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>( s = 0.04 )</th>
<th>( s = 0.08 )</th>
<th>( q(\theta) = 0.5 )</th>
<th>( q(\theta) = 0.9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(\text{net inward}) )</td>
<td>14.67</td>
<td>13.72</td>
<td>15.73</td>
<td>13.11</td>
<td>15.52</td>
</tr>
<tr>
<td>( \sigma(\text{net outward})/\sigma(\text{net inward}) )</td>
<td>0.67</td>
<td>0.58</td>
<td>0.74</td>
<td>0.79</td>
<td>0.60</td>
</tr>
<tr>
<td>( \text{corr(inward, outward)} )</td>
<td>0.32</td>
<td>0.18</td>
<td>0.42</td>
<td>0.27</td>
<td>0.34</td>
</tr>
</tbody>
</table>

It is reassuring that the same can be said of changes in the mean allocation rate, it’s principle effect being changing the relative standard deviation of gross foreign direct investment flows. The change to the correlation between net inward and outward flows of FDI when decreasing the degree

\(^{30}\)The mean rate \( s \) may be affected by the initial foreign capital stock. It is therefore worth while exploring the sensitivity of the results to its calibration.
of congestion in the allocation of capital abroad (i.e., increase the mean \( q(\theta) \)) between approximately a year and a half and just over a quarter is as expected, from 0.27 to 0.34, and very small.

4.3.2 Sensitivity to the specification of the technological process

An alternative specification of the exogenous process for technology cuts off cross-country spill-overs and reduces the cross-country correlation from 0.045 to 0.015, while fixing identical persistence parameters.

\[
\begin{bmatrix}
\rho_c & \rho_{c,us} \\
\rho_{us,c} & \rho_{us}
\end{bmatrix} = \begin{bmatrix}
0.988 & 0 \\
0 & 0.988
\end{bmatrix}
\]

Covariance matrix: \[
\begin{bmatrix}
0.11 & 0.015 \\
0.015 & 0.07
\end{bmatrix}
\]

The results are presented in Table 5. Changing the specification of the exogenous process has little effect on the relative volatilities of net inward and outward flows of FDI. However, the proposed model now generates more volatile flows by a factor of two compared to the IRBC model with investment adjustment costs. The ratio of volatilities of inward and outward flows hardly changes except for the IRBC with investment adjustment costs model, for which the ratio becomes close to the data. However, the contemporaneous correlation of net inward of outward flows remains strongly negative, at -0.31, while for the proposed model it almost exactly matches the data.

Table 5: Sensitivity specification of exogenous process

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Endogenous real.</td>
<td>Exogenous real.</td>
<td></td>
</tr>
<tr>
<td>Net FDI in Canada</td>
<td>a 8.42 b 0.84</td>
<td>a 8.60 b 0.60</td>
<td>a 8.50 b 0.70</td>
</tr>
<tr>
<td>Net Canadian Direct Inv abroad</td>
<td>5.69 0.08 d 4.70 c 0.14</td>
<td>4.61 0.15 b 4.60 c 0.15</td>
<td>2.25 0.08 c 2.20 d 0.07</td>
</tr>
<tr>
<td>Net outward / Net inward FDI</td>
<td>0.67 0.32 c 0.65 d 0.29</td>
<td>0.65 0.27 c 0.64 d 0.27</td>
<td>0.64 0.31 c 0.63 d 0.30</td>
</tr>
</tbody>
</table>

a: Standard deviation relative to output; b: Contemporaneous correlation with output; c: Ratio of outward to inward FDI flow standard deviations; d: contemporaneous corr., inward-outward net flows. All moments are Hodrick-Prescott filtered.

4.4 Aggregate variables

Figure 5 plots the impulse response functions of output, hours and capital, at domestic firms, foreign firms and in the aggregate, to a Home sourced persistent expansionary technology shock. Panel A presents results for the proposed search model with endogenous reallocation, while Panel B reports results for a standard IRBC with investment adjustment costs.

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Figure 5: IRFs to a persistent "Home" country technology shock.

The first observation is that the responses of aggregate variables are quite similar for both models. The differences arise in the response of foreign firms when investing in capacity abroad is subject to a time consuming search process. On impact, hours at foreign firms rise more than at domestic firms in both models. However, the ensuing additional increase in hours at foreign firms is more pronounced in the proposed model, and stems from the different capital stock dynamics: the stock of foreign capital rises more quickly than in the standard case, pushing up further the labor demand of foreign establishments. The model thus implies that hours at foreign firms are more volatile than at domestic firms over the business cycle. There is recent empirical evidence from Europe that foreign controlled firms tend to make larger and more frequent employment adjustments (Checchi et al. 2003), but no direct evidence of systematic differences in the response of hours to the business cycle.

A second dimension along which the model’s performance is evaluated is a series of H.-P. filtered second moments. Table 6 presents the 2nd moments of prominent macroeconomic variables for the three models and the data. Both in term of standard deviations and correlations with output, all three models are similar in being close to the data, with the well known exception of the volatility of hours. Thus, the ability of the model to generate high volatility in flows of FDI does not come at the expense of creating too much volatility in aggregate investment.
Table 6: 2nd moments for prominent macro variables.

<table>
<thead>
<tr>
<th>1976:1 - 2005:4</th>
<th>Canadian data</th>
<th>Search in FDI model</th>
<th>IRBC with FDI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>variable:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.80</td>
<td>0.87</td>
<td>0.60</td>
</tr>
<tr>
<td>Hours</td>
<td>0.80</td>
<td>0.83</td>
<td>0.63</td>
</tr>
<tr>
<td>Investment</td>
<td>3.11</td>
<td>0.69</td>
<td>0.45</td>
</tr>
<tr>
<td>Output</td>
<td>1.53</td>
<td>0.75</td>
<td>1.74</td>
</tr>
</tbody>
</table>

a: Standard deviation relative to output; b: Contemporaneous correlation with output

A well know deficiency of standard IRBC models, the quantity problem, concerns the ordering of cross country correlations of consumption, output, investment and hours. The problem of the ordering of consumption and output cross correlations is the most known of the quantity problems in the IRBC literature, as raised in the work of Backus, Kehoe and Kydland (1995), while the shortcomings related to the cross correlation of hours and investment have been raised in papers such as Ambler, Cardia and Zimmermann (2004). Table 6 shows the performance of the search in FDI model with this respect. All models get the ordering of a higher output than consumption cross-correlations right, although the cross-correlation of consumption is lower than in the data. This time on labor markets the three models perform quite well. However, where aggregate investment is concerned, the search model with endogenous reallocation clearly outperforms the other two. The investment cross-correlation is very close to the data, at 0.4 against 0.45, while a model with exogenous reallocation generates a cross-correlation of 0.60 and the IRBC with investment adjustment costs 0.67. It is clear that incorporating endogenous gross-outflows is essential for getting the aggregate investment quantity puzzle right.

5 Conclusion

A commonly used measure of the rate at which foreigners gain control over a domestic economy, flows of foreign direct investment (FDI), represent increasingly important share of aggregate investment in industrialized economies as they further integrate. Given the importance of the foreign sector for aggregate outcomes and the relatively high volatility of direct investment flows, quantitative models of open economies need to be consistent with their dynamics.

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31 This is due, essentially, to the correlation structure to innovations and the presence of investment adjustment costs. This was first pointed out by Backus, Kehoe and Kydland (1992), but made more explicit in Baxter and Crucini (1995).
As this paper has shown, a combination of frictions in the allocation of physical capital to production abroad, and allowing for the endogenous reallocation of this capital, can replicate the positive correlation between net inflows and outflows of FDI that is a feature of the data. In addition the model can generate the higher volatilities of inward and outward net FDI, while the implication for prominent macroeconomic variables are similar to a standard IRBC model with investment adjustment costs. However, there are important sectoral differences worth mentioning in conclusion. The model implies that, for example, hours worked at foreign establishments are more volatile than hours worked at domestic establishments. An interesting question, and most relevant for economic policy, is whether this is the case in the data. In particular, if one considers the extensive margin of labor adjustments, are jobs at foreign establishments more elastic to the business cycle? If so, this might offer a rationalization for the public’s skepticism toward the benefits of increased foreign control of a domestic economy as employment at these firms would be more fragile.
References


