

# Stock Market Pressure on Inventory Investment and Sales Reporting for Publicly Traded Firms

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Operations of publicly traded firms differ from privately owned firms because public firms' managers make decisions based on their own interests. In this paper, we study how stock market pressure may influence a manager's inventory and operational management. Our model is a straightforward extension of a two-period inventory management problem with correlated demand. The manager's compensation is partially based on the firm's stock price which is influenced by the reported sales revenues. With better information about the "real" demand, the manager may manipulate the stock price by shipping more than the real demand to downstream customers and claim higher than real sales revenues using a well known form of real earnings management called "channel stuffing" or "sales padding." As it does not correspond to real demand, the padded demand will return later to the firm after additional costs are incurred. Hence, channel stuffing destroys the firm's value. Based on a game between the manager and the stock market, we identify three factors that determine the manager's incentive to use channel stuffing: the *marginal incentive*, the *boundary* effect and the *carryover* effect. The marginal incentive is independent of the inventory problem. The boundary and carryover effects arise from the nature of the inventory management problem. When examining the initial inventory investment decision, we find that, compared to the optimal initial inventory level of an otherwise identical private firm, the manager who is aware of the costly consequence of padding may under-invest inventory due to the loss of margin as well as to limit the carryover effect, while he may also over-invest to limit the boundary effect on padding. Our theoretical analysis provides insights about how a public firm's inventory decision may be different from a private firm's inventory decision, which is the classical reference framework in operations management.

*Keywords:* Stock Market; Inventory Management; Channel Stuffing; Real Earnings Management

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## 1 Introduction

Operations Management research usually examines private firms whose owners/decision makers interests are perfectly aligned with the long-term interests of the firm. In private firms, the owners' and managers' interests are perfectly aligned. However, most major firms in each industry sector are public firms. A public

firm is owned by its shareholders but is run by its managers (CEOs and senior executives), and the managers may not perfectly represent the interest of the firm's long-term growth. In practice, a large portion of the managers' compensation is stock-based pay, such as stocks and stock options. According to Morgenson (1998), in 1997, the 200 largest US firms reserved more than 13% of their common stocks for compensation awards to managers. For instance, IBM's 2006 Annual Report shows that IBM recognized a \$541 million stock-based compensation expense in the year 2006, and at December 31, 2006, the unrecognized stock-based compensation cost related to non-vested awards was \$1,238 million. With their compensation directly linked to the firm's stock market performance, the managers may have an incentive to deviate from the "optimal" operational strategies in order to increase their stock-based compensation.

Information about the firm plays a key role in stock market pricing. In theory, the stock price represents the real value of a firm. Unfortunately, this will not always be true if investors do not have access to all relevant information. In practice, the most common way for outside investors to retrieve information about a firm's operation is through required quarterly and annual financial reports and ad hoc information release. However, it has been widely acknowledged that to achieve their compensation goals, managers have manipulated the earnings numbers in the reports through discretionary accruals or even creative accounting. The most famous examples include the cases of Enron and WorldCom, which directly led to the passage of the Sarbanes-Oxley Act (SOX) in 2002. Nevertheless, in addition to accrual-based earnings management, managers are likely to employ real operational activities to manipulate their financial reports as well, i.e., they use "real earnings management." As documented by Cohen et al. (2007), real earnings management activities increased significantly after the passage of SOX, in contrast to the decline of accrual-based earnings management. In practice, many operational factors, such as the actual demand, the future forecast, the expected return from R&D, etc., are only observable to managers. Sometimes it is even difficult to convey this kind of private information to investors. The financial reports are usually condensed with the aggregated values of a firm's revenues, costs, assets etc. without necessarily specifying every item in detail. This provides a broad space for managers to manipulate their operation. For example, to get a favorable stock market reaction and thus achieve higher stock-based compensation, managers may be tempted to sell excess units of inventory, known as channel stuffing or sales padding; they may also decrease discretionary spending on R&D, advertising, or maintenance to meet an earnings target (see various examples discussed in Lai (2008), Graham et al. (2005), as well as the well-known "hockey-stick phenomenon" widely mentioned in literature such as Lee et al. (2004), Kapuscinski et al. (2004), and Sohoni et al. (2005)). In contrast to accrual-based earnings management, such real earnings management activities will drive the operational decisions away from the best levels from the investor's perspective. With information asymmetry, managers may sacrifice the firm's long-term interest in exchange for a short-term boost in the stock market price. However, to evaluate

a firm, rational investors may anticipate the managers' incentive and strategies. They may infer a firm's real business environment and operational performance based on available information. Although accounting and finance literature has thoroughly documented accrual-based earnings management and rational valuation in practice, to the best of our knowledge, no theoretical study has examined how operational decisions deviate from the first best decisions and how investors incorporate operational information into their valuation of the firm.

In this paper, we analyze a two period inventory management problem with correlated demand, which is a corner-stone model in operations management literature. We study an environment for a public firm with the potential of real earnings management—channel stuffing. The demand in each period is stochastic and must be satisfied from inventory. The firm is managed by a self-interested manager. The manager's compensation partially depends upon the stock price of the firm and partially depends on the long term value of the firm. We consider a typical source of information asymmetry between the manager and the investors: the “real” demand is unobservable to investors, they only know the demand distribution, while the manager observes the real demand realization. The manager can thus influence the stock price through financial reporting: Because of the positive correlation with the future demand (and thus stock price), if there is any leftover inventory after the real demand has been satisfied, the manager may have an incentive to pad the sales by means of channel stuffing. The manager induces downstream parties to purchase more than the actual demand and recognizes the revenue immediately in its accounting books. The manager reports (possibly padded) sales revenue to the investors. All other information, like cost of goods sold, initial inventory and the managerial stock-based compensation are public knowledge.<sup>1</sup> Since padded sales do not correspond to any “real” demand, they will be returned to the firm and the revenue from this padded part will have to be written off.<sup>2</sup> Moreover, the firm incurs a channel stuffing cost (e.g., costs associated with handling and incentive compensation for the downstream parties). The investors infer the real demand from the financial report and assign a stock price equal to the expected fair value of the firm contingent upon the

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<sup>1</sup>In this paper we assume there is no explicit buyback agreement between the firm and its downstream parties. According to SFAS No. 49, if a company sells a product in one period and agrees to buy it back in the next period, no sales revenue should be recognized since the risks of ownership are retained by the seller. However, if there is no explicit buyback agreement, the seller can recognize revenue in the current period and recognize returns in “sales returns,” which is a contra-revenue account, in the next period when the inventory is returned. Even if the seller expects more than usual returns in the future, the seller's incentive is unobservable and the revenue is recognized in the current period without breaking the accounting rules.

Another accounting rule used to prevent overstatement of revenue is to ask the company to set up a separate account called sales returns and allowance, which is a reduction in sales revenue, to estimate the potential returns (SFAS No. 48). However, since this allowance is a subjective estimation by the manager, the manager can understate the potential returns. For simplicity, in this paper we do not consider the estimation of sales returns and allowance, since it only complicates the analysis by introducing an additional decision variable for the manager to manipulate.

<sup>2</sup>In practice, the downstream parties may or may not return all padded sales to the seller in the future. However, for simplicity we assume that all the sales beyond the realized first period demand will be returned in the second period.

information in the report.<sup>3</sup>

We characterize perfect Bayes market equilibria on channel stuffing and stock price. We find that in equilibrium, the manager's incentive to pad the sales is driven by three factors: the *marginal incentive*, the *boundary effect*, and the *carryover effect*. The boundary and carryover effect are intrinsically tied to the inventory problem that the manager deals with. The marginal incentive is independent of the inventory problem. Interestingly, when examining the optimal decision on the initial inventory investment within such an environment, we find that the inventory level can be either higher or lower, compared to the classical newsvendor solution for an otherwise identical private firm. While under-investment might be generally expected since channel stuffing deteriorates the margin of the business, it is especially interesting to see that over-investment may also occur. Our findings characterize the potential that managers might apply real earnings management not only to manipulate the financial status ex post but also to alleviate the stock market pressure ex ante.

The remaining sections are organized as follows: The next section reviews the relevant literature. Section 3 describes the model. In Section 4 and 5, we analyze the market equilibrium on channel stuffing and stock price both with and without inventory carryover. Section 6 investigates the incentives on initial inventory investment. We conclude in Section 7.

## 2 Related Literature

This paper connects two streams of research: the operation management literature on how operations interact with stock performance, and the accounting and finance literature on agency problems.

While the link between operation and stock performance is fundamental, it has been an under-explored area for a long time in operation management research. The recent empirical studies by Hendricks and Singhal (2001, 2005, 2006), Gaur et al. (2002), Raman (2006), Chen et al. (2005), Lai (2005, 2006, 2008), etc., are filling in this gap. Hendricks and Singhal (2001) investigate the relationship between the adoption of total quality management (TQM) and long-term stock performance. Their empirical finding asserts an average positive linkage between a public firm's stock performance and its quality control. In Hendricks

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<sup>3</sup>Although channel stuffing is illegal, it is a practice that is hard to catch, and widely exists as shown in Lai (2008). Kieso and Weygandt(1998) have the following comments on channel stuffing: "Some companies record revenues at date of delivery with neither buyback nor unlimited return provisions. Although they appear to be following acceptable point of sale revenue recognition, they are recognizing revenues and earnings prematurely... Trade loading and channel stuffing are management and marketing policy decisions and actions that hype sales, distort operating results, and window dress financial statements. End-of-period accounting adjustments are not made to reduce the impact of these types of sales on operating results. The practices of trade loading and channel stuffing need to be discouraged. Business managers need to be aware of the ethical dangers of misleading the financial community by engaging in such practices to improve their financial statements."

and Singhal (2005), they examine the impact of supply chain disruption on a firm's stock price. They demonstrate with stock price data that the negative effect from supply chain disruption can be profound. Gaur et al. (2002) set out the goal to understand the link between operational measurements and stock return. With the data from the sector of public retailers, they reveal that return on assets (ROA) and sales growth have a positive association, while standard deviation of ROA has a negative association, with long-term stock returns. They also examine the measurements of return on equity (ROE) and gross margin return on inventory investment (GMROI), from which they find the similar links. Raman (2006) explores the question echoed by numerous retail executives and investors of how stock market shall correctly reflect a retailer's inventory level and inventory management capabilities. Although there is no universal answer for whether the stock price will move up or down with an observation of high inventory level, Raman discusses that the link between inventory management and stock return is substantial. This is echoed by Chen et al. (2005) and Hendricks and Singhal (2006). They find empirical evidence that financial reports of excess inventory can drive the stock price down.

The above papers mainly document how stock price moves along with a firm's operational management. Differently, Lai (2005, 2006, 2008) tries to answer the question of how a public firm may "cooperate" with the stock market through intentionally managing the operation, which is closest to the purpose of our study. Lai (2005, 2006) investigates the stock market reaction to a firm's inventory level and studies how a firm may intentionally under-invest inventory. In particular, empirical evidence documented by Lai (2008) is consistent with our analytical results. Examining the financial data of public retailers, Lai (2008) demonstrates that the fiscal year end effect is widely observed among public retailers. Lai shows the examples of RadioShack as well as several public firms in Germany where the inventory levels are substantially low at the end of fiscal year that may not necessarily coincide with the annual year end. Lai documents strong evidence of consistent sales pushing.

Along with this ongoing trend of empirical study, our paper investigates the stock market link analytically. Moreover, our paper sets out the goal to close the loop. We address how the stock market may react to a firm's reported sales and then how this reaction in turn will impact the manager's incentive on selling and stocking. Lai et al. (2008) share the same motivation. That paper studies the forecasting and inventory signaling effect. The firm that has better forecast of the future demand may intentionally stock more inventory and report the forecast along with the inventory signal to public investors. But their paper does not address the sales effect or channel stuffing. Our paper is also related to several papers that address the agency problem in inventory management: Chen (2000, 2005) and Chen and Xiao (2005a,b). In these papers, the firm's sales effort that determines the demand is controlled by a self-interested manager, while the inventory decision is made by the owner. However, they focus on the contracting issue of how to motivate

the manager’s sales effort, while we examine the equilibrium between stock market reaction and the firm’s inventory decision with earnings management. Finally, Debo et al. (2008), study an agency problem in a queuing context analytically. In their model, the service manager observes the amount of work that is necessary for the customer, but the customer does not. This creates an incentive to pad the work, especially when the payments are time-based. In a different context, we consider an inventory manager that observes the “real” market demand and faces an incentive to pad the sales, especially when more sales make the stock price increase and the stock price is a component of the manager’s compensation.

There are numerous accounting and finance studies on earnings management. However, most of those works focus on accrual-based earnings management. This line of literature addresses the discretion of accounting accruals and how a manager may misreport the real earnings number, which has little to do with real operation. We therefore only discuss several studies that are closely related to our work.

Among the early works is Stein (1989), which studies a market equilibrium on earnings management between rational investors and the manager of a public firm. In Stein’s model, the manager consistently borrows from next period’s earning (for instance, to forsake good investment) to inflate current earnings, even though the stock market takes this into account when setting the stock price. The incentive from the current stock compensation drives the manager away from the first-best decision. Our paper shares the same interest in how the manager’s short-term interest impacts the operational decision, but we explore a more detailed operational problem—inventory management. More importantly, we show that although the incentive of earnings management may always exist, the manager may alter the initial inventory investment to reduce this incentive within a rational stock market.

Dye and Sridhar (2004) and Liang and Wen (2007) are the other two papers closely related to ours. Both of them address the impact of earnings management on public firms’ investment decisions. They show that with the chance of earnings manipulation, a firm may deviate from the first-best decision by over-investing or under-investing. Their papers are similar to Stein’s study because they also assume a stylized investment problem, but they focus more on the exploration of appropriate accounting rules.

### 3 The Model

**The Inventory Model.** We consider a firm that sells a product in two periods  $t \in \{1, 2\}$ . The “real” demand in period  $t$  is  $\xi_t$ . We assume that:  $\xi_1 = \eta_1$  and  $\xi_2 = a\xi_1 + \eta_2$ , where  $\eta_1$  and  $\eta_2$  are independent random variables. This demand structure is a two period autoregressive time series with order 1. We assume that  $\eta_t$  is distributed according to a continuous distribution function  $F_t(\cdot)$ , with density  $f_t(\cdot)$ , mean  $\mu_t$  and variance  $\sigma_t^2$ .  $a$  is a constant that determines the correlation of the demands (i.e.,  $\frac{a\sigma_1}{\sqrt{a^2\sigma_1^2 + \sigma_2^2}}$ ) in the two

periods. If  $a = 0$ , the demands are independent, while  $a \geq 0$  reflects a positive (negative) correlation. In particular, for the business motivations we are interested in, we focus on the case where  $a \geq 0$  (our model is general for  $a < 0$ ).<sup>4</sup>

In both periods, the selling price per unit is fixed at  $p$ . The production or purchasing cost per unit is  $c$  (and strictly less than  $p$ ). The firm needs to invest inventory  $q_t$  in each period  $t \in \{1, 2\}$ , before the real demand of that period,  $\xi_t$ , is realized. The unmet demand in each period is lost. This is appropriate in consumer settings where an out of stock demand leads to a substitution among competing products. For reasons of expositional clarity, we divide the analysis into two cases. We start in Section 4 with the case in which the leftover inventory at the end of the first period is not carried over to the second period. Leftover inventory in each period has zero salvage value. We extend in Section 5 the model by allowing carryover of leftover inventory at the end of the first period to the second period. At the end of the second period, leftover inventory has zero salvage value. The first case is more appropriate when there are significant design changes in between the two periods or when the products are seasonal. The second case is more appropriate when the product design is stable.

Notice that without any further considerations, we have described a classical two period inventory management problem with correlated demand whose objective is to determine the optimal inventory investment levels  $q_t$  for  $t \in \{1, 2\}$ . A manager-owner of a private firm would optimize the inventory management decision according to classical inventory management theory. Next, we describe how the firm structure modifies the objective function of a manager of a public firm.

**Firm structure.** We consider a public firm whose shares are traded in the stock market. It is run by a self-interested manager. We assume for the sake of clarity that the firm has initial liquid assets  $A_0$  at the beginning of the first period which is sufficient to cover any cost during the two periods.<sup>5</sup> In order to capture how the manager will be influenced by the stock market, we adopt, as in Stein (1989), the most simple model for the manager's payment structure: a fraction  $\beta$  of the manager's compensation is based on stock price performance and a fraction  $1 - \beta$  is based on the long-term value of the company. More complex managerial compensation mechanisms do exist in reality. However, the structure that we assume captures, in the simplest way, the key feature that we are interested in: It introduces stock-price considerations in operational decision making. Consistent with our two period model, we assume that at the end of the second period, the firm will be liquidated, i.e. the true value of the accumulated assets over two periods is revealed. The manager receives a fraction  $1 - \beta$  of this value. An alternative way to interpret the compensation

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<sup>4</sup>We allow  $a$  to be larger than 1. This can be viewed as if the second period is an aggregate of multiple periods in terms of the length of the first period.

<sup>5</sup> $A_0$  does not play a key role in the analysis, but avoids having to deal with the complexity of such as financing or bankruptcy which may arise otherwise.

structure, as also discussed in Stein (1989), is that the manager is granted the all firm's shares which are normalized to one unit and will sell a proportion  $\beta$  of his shares at the end of the first period and retain the other part  $1 - \beta$  to the end of the second period. The liquidation assumption is not crucial. It can also be seen as a way to capture that the term of the manager expires in the second period when the firm will be thoroughly audited. We make these assumptions for the convenience of analysis and exposition. Finally, for simplicity, the firm does not pay out any dividend in the first period and thus the profit made in this period will be accumulated to the second period. This assumption also simplifies our problem. Otherwise, there might be issues of refinancing to cover the inventory investment requirement in the second period.

**Channel stuffing.** Different from the classical newsvendor setting, in our model, the manager may pad the sales to provide an inflated signal of the business. Given that the stock market does not only care about the current sales realization but also, due to the correlation, regards the first period sales realization as a predictor of the future earnings, the manager is tempted to pad the “real demand” in order to obtain a more favorable short-term stock compensation. More precisely, we assume that the manager may “sell” an excess amount of  $x$  units to the downstream parties after the real first period demand,  $\xi_1$ , is privately observed. Obviously, the manager can choose not to pad the sales at all. Then,  $x = 0$ . On the other hand, the maximum possible padding amount is determined by the available inventory;  $(q_1 - \xi_1)^+$ . The constraint is due to the “real” earnings management character of our model. The firm recognizes revenue  $p$  and cost of goods sold  $c$  per unit for this padded sales, the same as the real demand, and then reports the aggregated sales to the financial market. The  $x$  units that are padded sales return to the firm at the beginning of the second period. Then, the customer needs to be reimbursed (i.e. the firm incurs a cost  $p$  per unit), the warehouse is credited by  $c$  per unit and, an additional cost  $\gamma$  per unit is incurred, representing the incurred extra cost of inducing the downstream parties to purchase more. For example, in the notorious case of Coca-Cola's gallon pushing between 1997 and 1999, Coca-Cola offered downstream distributors extra credit terms to induce them to purchase more than demanded, which resulted in extra costs to Coca-Cola.

In order to gradually build up the intuition, we analyze two models: In the first model (in Section 4) we examine the case where the inventory cannot be carried over from period 1 to period 2. Then, when the inventory from this padded sales is returned, it will be written off to zero value. In the second model (in Section 5), we examine the case where the inventory can be carried over from period 1 to period 2. Then, the inventory from the padded sales, when returned, can be used for the second period. However, the firm incurs a holding cost  $h$  per unit to hold the inventory to the next period. At the end of the second period, all remaining inventory is written off to zero value.

**Stock market.** We consider rational investors that infer the real demand from the publicly released information and assign a value to the firm (and its shares). The firm releases a financial report to the public



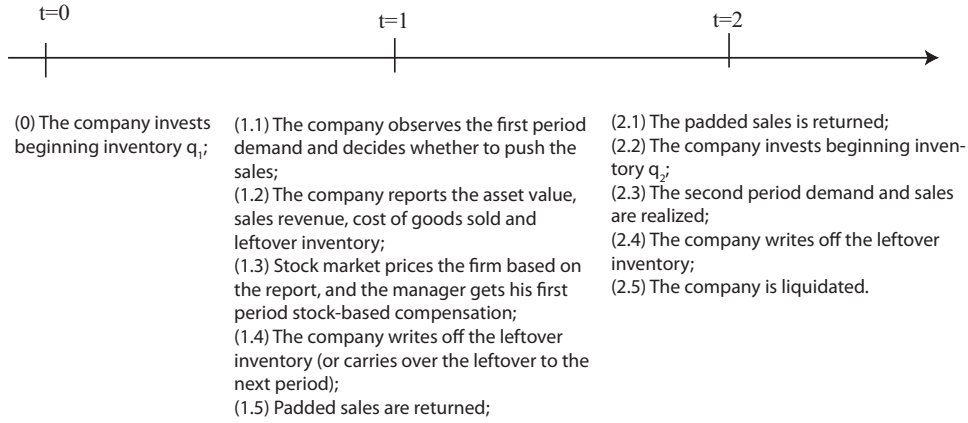


Figure 1: The timeline of the model.

at the end of the first period. This report includes the values of sales revenue  $pz$ , cost of goods sold  $cz$  and leftover inventory  $cI$ , where  $z$  is the sum of real demand  $\xi_1$  and padded sales  $x$ , and  $I$  is the unit of leftover inventory (i.e.,  $I = q_1 - z$ ). The information about the manager's compensation structure, i.e.,  $\beta$ , is also known to the investors.<sup>6</sup> Except for the real demand  $\xi_1$  and the amount of padded sales  $x$ , all other information is observable to the investors. In other words, provided by the sales revenue  $pz$ , the investors can infer the amount of sales  $z$  (but not  $\xi_1$  or  $x$ ). Similarly, provided by the leftover inventory  $cI$ , the investors will know the quantity  $I$ , and thus because of  $I$  and  $z$ , the investors will know the first period inventory investment,  $q_1$ , as well.

**Timeline.** Figure 1 details the timeline of the model. At the beginning of the first period, the firm invests inventory  $q_1$ . The manager then observes the actual demand  $\xi_1$  and decides how much sales  $x$  to pad beyond  $\xi_1$ . The firm releases the financial report. The stock market reacts to the information and assigns the stock price reflecting the firm's value based on the available information. Then, the manager obtains his first period stock-based compensation. At the beginning of the second period, the firm invests inventory  $q_2$ . The demand  $\xi_2$  is realized. The firm satisfies the demand up to the available inventory level. Finally, the firm is liquidated and the manager gets his remaining compensation. Notice that the stock price is defined at each point in time, however, only at  $t = 1$ , the manager has different information than the investors.

**The Equilibrium.** We introduce a stock-market equilibrium formally as follows: Since the investors cannot observe the actual demand, they have to infer it from the financial report. Suppose the investors hold the belief that the amount of sales the manager will pad is  $x^o(\xi_1, q_1) \geq 0$  for given  $\xi_1$  and  $q_1$ . Then, they believe

<sup>6</sup>For example, regarding stock option compensation, the companies should provide forecasts about when the stock option will be exercised by the employees, according to SFAS No. 123.

the total sales the firm reports satisfies:

$$z = \xi_1 + x^o(\xi_1, q_1). \quad (1)$$

To infer the real demand,  $\xi_1$ , from the reported demand,  $z$ , the investors face two situations: (1)  $\xi_1$  and  $x^o$  can be perfectly inferred, for instance, if  $z$  is a strictly increasing function of  $\xi_1$ ; and (2)  $\xi_1$  and  $x^o$  cannot be perfectly inferred, for instance, if there are multiple pairs of  $\xi_1$  and  $x^o$  that map to the same  $z$ , and then the investors have to assign probabilities for each pair. Let  $v_2(\xi_1)$  be the expected second period revenues when the first period real demand is  $\xi_1$  (in the case that no inventory is carried over) and, similarly,  $v_2(\xi_1, q_1)$  in the case that inventory is carried over (then,  $q_1 - \xi_1$  is the leftover inventory). Suppose that the manager pads the real demand by an amount  $x$ . With this information, the firm's expected value is:

$$\begin{aligned} v_1(x; \xi_1, q_1) &= A_0 + p \min(q_1, \xi_1) - cq_1 - \gamma x + v_2(\xi_1) \quad \text{and} \quad (\text{without carryover}) \\ v_1(x; \xi_1, q_1) &= A_0 + p \min(q_1, \xi_1) - cq_1 - \gamma x - h(q_1 - \xi_1)^+ + v_2(\xi_1, q_1) \quad (\text{with carryover}) \end{aligned}$$

for  $0 \leq x \leq (q_1 - \xi_1)^+$ , for each case respectively. Note that, as padding is expensive and does not increase the real value, it is obvious that if there is no stock market reaction, the manager will never pad the sales;  $\frac{\partial v_1}{\partial x} = -\gamma < 0$ . Let  $P(z, q_1)$  be the stock price, given  $q_1$  is the initial inventory investment level and  $z$  is the reported demand. The manager's padding decision is:

$$\max_{0 \leq x \leq (q_1 - \xi_1)^+} \pi(x; \xi_1, q_1) = \beta P(\xi_1 + x, q_1) + (1 - \beta) v_1(x; q_1, \xi_1). \quad (2)$$

As padding may increase the stock price, there is a possibility that the manager pads, i.e.  $x^* > 0$ . Let  $x^*(\xi_1, q_1) \in \arg \max_{0 \leq x \leq (q_1 - \xi_1)^+} \pi(x; \xi_1, q_1)$ . We define the following market equilibrium:

**Definition 1** *A perfect Bayes equilibrium  $(x^o(\cdot, \cdot), P^o(\cdot, \cdot))$  is reached if, for given inventory level  $q_1$  and first period demand  $\xi_1$ , the sales padding strategy  $x^o(\xi_1, q_1)$  and stock price  $P^o(z, q_1)$  satisfy the following two conditions:*

- (1) *Given  $P^o(z, q_1)$ ,  $x^o(\xi_1, q_1)$  maximizes the manager's payoff  $\pi(x; \xi_1, q_1)$ ;*
- (2)  *$P^o(z, q_1)$  is consistent with  $x^o(\xi_1, q_1)$  in the way that*

$$P^o(z, q_1) = \mathbb{E}_{\xi_1} [v_1(z - \xi_1; \xi_1, q_1) | z = \xi_1 + x^o(\xi_1, q_1)]$$

This definition of a perfect Bayes equilibrium outlines the following process. Anticipating the stock market reaction, the manager makes a padding decision that maximizes his utility over two periods. The investors, knowing the manager's incentive structure, value the firm and determine the stock price based on the available information and their conjecture about the manager's behavior. In equilibrium the manager's optimal padding decision solved from Equation (2) matches the investors' belief for any given set  $(\xi_1, q_1)$ , i.e.,  $x^*(\xi_1, q_1) = x^o(\xi_1, q_1)$ .

## 4 Market Equilibrium without Inventory Carryover

In this section, we assume that the leftover inventory cannot be carried over for future sales and has no salvage value. This does not imply that the two periods are separate. Recall that the demand in the first period is correlated to the demand in the second period. Hence, investors adjust the firm's value after obtaining information about the first period sales. To investigate the market equilibrium, we first analyze the second period decision and then examine the equilibrium on channel stuffing and stock price with given first period inventory  $q_1$ . We examine the manager's decision on  $q_1$  in Section 6, combined with the case where the leftover inventory can be carried over.

### 4.1 Second Period Decision

Since the firm will be liquidated after the second period and there is no stock selling within this period, the firm effectively behaves like a private firm in the second period. Therefore, the inventory investment is determined by:

$$\max_{q_2} \mathbb{E}_{\eta_2} [p \min(a\xi_1 + \eta_2, q_2)] - cq_2.$$

Following the newsvendor solution, the optimal order quantity and the corresponding expected profit are:

$$q_2^* = k_2 + a\xi_1 \text{ and } v_2(\xi_1) = (p - c)a\xi_1 + \mathbb{E}_{\eta_2} [p \min(\eta_2, k_2)] - ck_2,$$

where  $k_2 = F_2^{-1}\left(\frac{p-c}{p}\right)$ .  $v_2(\xi_1)$  provides the link between the first period (real) demand realization and the second period value of the firm. From

$$\frac{dv_2(\xi_1)}{d\xi_1} = a(p - c), \quad (3)$$

we can see that each extra unit of first period realized "real" demand increases the second period value with  $a(p - c)$ . As the second period value is reflected in the stock price, which influences the manager's short-term compensation, it is easy to see that the manager's utility increases in the first period's realized demand. This will determine the incentive for the manager to pad the first period sales.

### 4.2 Equilibrium Channel Stuffing and Stock Price

In the first period, the manager may pad the sales after observing the actual demand  $\xi_1$  to inflate the reported sales for his own interest. In the following, we characterize the market equilibrium. We define

$$\theta \equiv \frac{\beta(p + a(p - c)) - (1 - \beta)\gamma}{\gamma}. \quad (4)$$

$\theta$  indexes the marginal incentive on channel stuffing. The numerator captures the main intuition of  $\theta$ , which we interpret as follows. Assume that the investors believe that the sales report is truthful, then, we observe

that if  $\xi_1$  increases, the firm's value increases with  $p$  per unit from the *current* sales (note, the inventory cost has already been sunk) and  $a(p-c)$  per unit from the *second* period sales (see Equation (3)). Therefore, if the manager can increase the investors' perception of  $\xi_1$  by padding the sales,  $\beta(p+a(p-c))$  is the marginal benefit he can obtain from his short-term compensation. However, channel stuffing indeed does not benefit the firm but incurs the cost  $\gamma$  that will be subtracted from the long term compensation. Therefore,  $(1-\beta)\gamma$  represents the marginal cost of channel stuffing to the manager. Notice that  $\theta$  is independent of the inventory problem at hand,  $\theta$  only depends on the product price, the margin, the correlation coefficient, the channel stuffing cost and the manager's compensation structure.

**The boundary effect.** It would be natural to expect that if  $\theta < 0$ , the manager would not have any incentive to use channel stuffing. This holds only if the investors can perfectly infer the actual demand. However, the initial inventory level  $q_1$  acts as a bound on the system. A "sold out" sales report,  $z = q_1$ , will not convey perfect information to the investors, as any  $\xi_1 \geq q_1$  will lead to such a report. If the investors assigned a stock price equal to the average value of the firm for all  $\xi_1 \geq q_1$ , the manager might have strong incentive to pad the sales since such a stock price over the high demand region could be very attractive. The market equilibrium for  $\theta < 0$  is presented in Proposition 1.

**Proposition 1** *If  $\theta < 0$ , there is a unique threshold  $\xi_a \in [0, q_1]$  such that the market equilibrium follows*

$$x^o(\xi_1, q_1) = \begin{cases} 0 & \xi_1 < \xi_a \\ q_1 - \xi_1 & \xi_a \leq \xi_1 < q_1 \\ 0 & \xi_1 \geq q_1 \end{cases} \quad (5)$$

and

$$P^o(z, q_1) = \begin{cases} P_a(z, q_1) & 0 \leq z < q_1 \\ \bar{P}(\xi_a, q_1) & z = q_1 \end{cases} \quad (6)$$

where

$$P_a(z, q_1) = A_0 + pz - cq_1 + v_2(z) \quad (7)$$

and

$$\bar{P}(\xi_a, q_1) = A_0 + \mathbb{E}_{\xi_1} [p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + v_2(\xi_1) | \xi_1 \geq \xi_a] - cq_1 \quad (8)$$

$\xi_a$  satisfies

$$\beta \bar{P}(\xi_a, q_1) - (1-\beta)\gamma(q_1 - \xi_a) = \beta P_a(\xi_a, q_1) \quad (9)$$

or  $\xi_a = 0$  if Equation (9) does not hold for any  $\xi_a \in [0, q_1]$ .

**Corollary 1** *When  $\theta < 0$ ,*

$$\begin{cases} \xi_a = q_1 & a = 0 \\ \xi_a \in [0, q_1) & a > 0 \end{cases}$$

Proposition 1 and Corollary 1 assert that for  $\theta < 0$ , if the correlation coefficient  $a = 0$ , the manager will not pad the sales, but if  $a > 0$ , there is a positive region  $\xi_1 \in [\xi_a, q_1)$  where the manager will “clear the shelves,” i.e. pad all inventory left over from the real demand. We refer to the latter as the “boundary effect.” The equilibrium is separating in the region  $\xi_1 \in [0, \xi_a)$  and pooling in the region  $\xi_1 \in [\xi_a, \infty)$ . We explain the results in the following.

By Definition 1.2, the stock price contains two meaningful components: The profit that the firm made in the current period and the expected profit that the firm will make in the second period. If there is no correlation between the demand in both periods (i.e.,  $a = 0$ ), the second component is a constant. If there is correlation (i.e.,  $a > 0$ ), the stock price linearly increases in  $\xi_1$  with a slope  $a(p - c)$  (see Equation 3). Imagine that the investors trust the manager, always taking the reported sales as the actual demand for  $z < q_1$  and averaging the firm’s value over the region  $\xi_1 \in [q_1, \infty)$  for  $z = q_1$ , i.e., they set  $\xi_a = q_1$ . When  $a = 0$ , the stock price would linearly increase in  $z \in [0, q_1)$  and be continuous at  $q_1$  (i.e.  $\lim_{z \uparrow q_1} P_a(z, q_1) = \bar{P}(\xi_a, q_1)$  if  $a = 0$ ). Given that  $\theta < 0$ , the marginal gain from channel stuffing is no higher than zero and thus the manager will not choose to pad the sales, which matches the investors’ belief. However, when  $a > 0$ , the stock price will have a discontinuity at  $z = q_1$  with  $\lim_{z \uparrow q_1} P_a(z, q_1) < \bar{P}(\xi_a, q_1)$ . This discontinuity is determined by the inventory constraint: When the initial inventory is sold out, the investors only know that the “real” demand is higher than the initial inventory and average over all possible high demand realizations. When the inventory is not sold out (and the manager tells the truth), the investors perfectly know the real demand realization. Hence, an upward jump in the stock price would exist at  $z = q_1$ . Due to this discontinuity, the manager has an incentive to push the sales to  $q_1$  if the actual demand is close to  $q_1$  because then, the investors would think that the real first period demand is higher than  $q_1$  (though they would not know how much higher) and assign a high future value to the firm because of the positive correlation with the first period demand, averaged over all possible realizations of the first period demand that are more than  $q_1$ . It implies that the manager’s behavior would not match the investors’ belief when  $a > 0$  and hence it cannot be an equilibrium that the investors trust the manager.

Therefore, for the case with strictly positive correlation between the demand in the two periods,  $a > 0$ , the investors will expect the manager’s incentive to use channel stuffing when the demand is close to the inventory bound even when  $\theta < 0$ . They will adjust their belief of the level  $\xi_a$  from which the manager will push the sales to the bound. It is straightforward to see that the pooling stock price  $\bar{P}(\xi_a, q_1)$  decreases as  $\xi_a$  decreases since it averages the firm’s value over a larger region;  $[\xi_a, \infty)$ . As a result, when  $\xi_a$  in the investors’ belief decreases, to push the sales to the inventory bound becomes less attractive. An equilibrium will be achieved if there is a  $\xi_a$  such that the pooling stock price  $\bar{P}(\xi_a, q_1)$  for a report  $z = q_1$  makes the manager indifferent to push the sales to the bound or not to pad the sales at all when observing a demand

$\xi_1 = \xi_a$ . This is the intuition of Equation (9). The manager will push the sales to the inventory bound when an actual demand  $\xi_1 \geq \xi_a$  and does not pad the sales when  $\xi_1 < \xi_a$ . The stock price is consistent with the manager's incentive by taking the reported sales as the actual demand for  $z < \xi_a$  and averaging all the possibilities over  $[\xi_a, \infty)$  if  $z = q_1$ . If such a  $\xi_a$  cannot be found over  $[0, q_1]$ , it implies that the boundary incentive is so strong that in equilibrium the manager will always push the sales to the inventory bound even if the pooling stock price is over the whole horizon of the demand  $\xi_1 \in [0, \infty)$ .

It is remarkable and yet easily understood that, even though with complete inference of demand information padding would not be attractive (because a negative return of stock compensation  $\theta < 0$ ), padding occurs in equilibrium in our model because of the private information and the inventory constraint. As a result, the padding is driven by the "real" inventory constraint that "filters" the demand information and creates an upward discontinuous jump in the stock-price that the manager wants to cash in.

With this boundary effect, for the case of positive correlation  $a > 0$ , the sales report,  $z$ , contains a jump at  $\xi_1 = \xi_a$  from  $\xi_a$  to  $q_1$ . In other words, a financial report with  $z \in [\xi_a, q_1)$  will not appear in equilibrium. However, for the stock price, it is necessary to specify this off-equilibrium path. The stock price takes the reported sales as the actual demand if it is within the interval  $[\xi_a, q_1)$ . The equilibrium is sustained under this specification for the off-equilibrium path; moreover, the stock price matches the exact value of the firm if the manager did not pad the sales in this region. As a result, the stock price follows  $P_a(z, q_1)$  for a report  $z \in [0, q_1)$  and  $\bar{P}(\xi_a, q_1)$  for a report  $z = q_1$ . It is not hard to check that the equilibrium stock price contains an upward jump at  $z = q_1$ . Figure 2 demonstrates the equilibrium structure for  $\theta < 0$  and  $a > 0$ . The dashed part of the stock price in the right subplot specifies the off-equilibrium path, and there is a very slight upward jump at  $z = q_1$ . It can be proven (in Appendix A, Proposition 5) that the upward jump disappears when  $\theta \uparrow 0$ . Next, we turn to the case with  $\theta \geq 0$ .

**The marginal effect.** When  $\theta > 0$ , the incentive to pad the sales is strong even without the boundary effect. If the investors trusted the manager, always taking the reported sales as the actual demand, the manager would rather push the sales to the limit, given that the marginal gain from channel stuffing is positive. As a result, "no padding" cannot be an equilibrium when  $\theta > 0$ . In particular, when  $\theta = 0$ , the manager is indifferent to padding the sales with any amount or not at all. Without loss of generality, we combine this case with  $\theta > 0$  in discussion. Proposition 2 characterizes the market equilibrium for  $\theta \geq 0$ .

**Proposition 2** *If  $\theta \geq 0$ , there is a unique threshold demand  $\xi_b \in [0, q_1)$  such that the market equilibrium follows*

$$x^o(\xi_1, q_1) = \begin{cases} \theta \xi_1 & \xi_1 < \xi_b \\ q_1 - \xi_1 & \xi_b \leq \xi_1 < q_1 \\ 0 & \xi_1 \geq q_1 \end{cases} \quad (10)$$

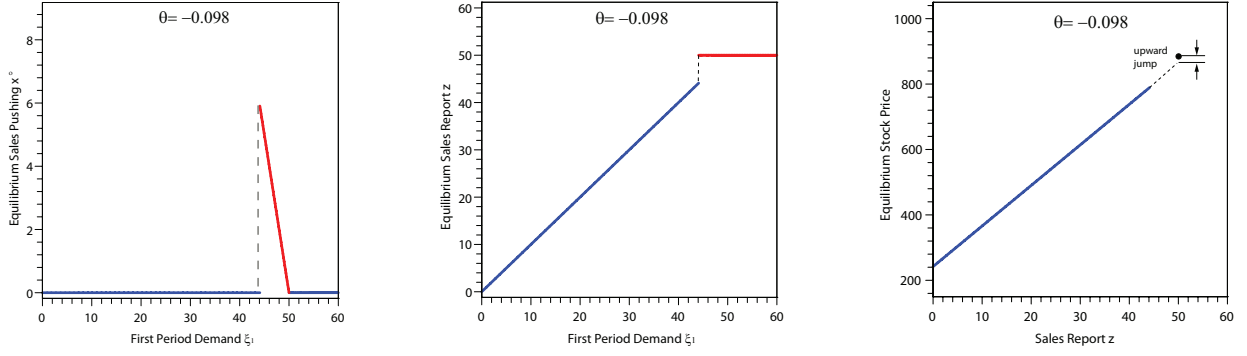


Figure 2: Market equilibrium with low pushing incentive. The parameters:  $A_0 = 400$ ,  $p = 10$ ,  $c = 6$ ,  $\gamma = 4$ ,  $a = 0.6$ ,  $\beta = 0.22$ ,  $q_1 = 50$ ,  $\eta_1, \eta_2 \sim N(50, 15^2)$  ( $N(\mu, \sigma^2)$  stands for normal distribution (mean and variance)).

and

$$P^o(z, q_1) = \begin{cases} P_b(z, q_1) & 0 \leq z < q_1 \\ \bar{P}(\xi_b, q_1) & z = q_1 \end{cases} \quad (11)$$

where

$$P_b(z, q_1) = A_0 + \frac{pz}{1+\theta} - \gamma \left(1 - \frac{1}{1+\theta}\right) z - cq_1 + v_2 \left(\frac{z}{1+\theta}\right) \quad (12)$$

and

$$\bar{P}(\xi_b, q_1) = A_0 + \mathbb{E}_{\xi_1} [p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + v_2(\xi_1) | \xi_1 \geq \xi_b] - cq_1 \quad (13)$$

$\xi_b$  satisfies

$$\beta \bar{P}(\xi_b, q_1) - (1 - \beta)\gamma(q_1 - \xi_b) = \beta P_b(\xi_b + \theta \xi_b, q_1) - (1 - \beta)\gamma \theta \xi_b \quad (14)$$

or  $\xi_b = 0$  if Equation (14) does not hold for any  $\xi_b \in [0, q_1]$ .

We interpret Proposition 2 as follows: As we have discussed above, if the investors took the reported sales as the actual demand, the manager would always push the sales to the inventory bound given that the marginal gain from channel stuffing is positive. Therefore, the investors will adjust their belief and the manager's channel stuffing strategy will change at the same time. An equilibrium will be reached only if there is such a stock price that correctly reflects the amount of sales the manager pads. In other words, an equilibrium requires that given this stock price, the manager will have no incentive to deviate from padding the amount of sales that is reflected in the stock price. This is possible only if the marginal gain from channel stuffing equals zero at such a point. It is exactly what  $P_b(z, q_1)$  in Equation (12) achieves. Under  $P_b(z, q_1)$ , the marginal benefit of channel stuffing from the stock price is  $(1 - \beta)\gamma$  (the derivative  $\frac{\partial P_b(z, q_1)}{\partial z} = \frac{(1-\beta)\gamma}{\beta}$  and thus  $\beta \frac{\partial P_b(z, q_1)}{\partial z} = (1 - \beta)\gamma$ ), which equals the manager's marginal loss due to channel stuffing  $(1 - \beta)\gamma$ . In particular, the payoff structure (the benefit and the loss) in this model is always linear in the padding

amount  $x$ . When the marginal gain reaches zero, it implies that the manager will be indifferent from padding any amount of sales<sup>7</sup>. However, in equilibrium, the amount of sales the manager pads needs to be consistent with the investors' belief. Therefore, the unique pure strategy equilibrium is that the manager will follow the linear padding strategy  $\theta\xi_1$  as provided in Proposition 2 that matches the investors' belief which is specified in the stock price  $P_b(z, q_1)$  in Equation (12).

Notice that we haven't yet discussed the role of the boundary effect. With the presence of the inventory bound, the linear channel stuffing strategy as well as the reported sales is capped by the available inventory. Therefore, the equilibrium further needs to specify the stock price at the inventory bound. We have discussed that under the linear channel stuffing strategy and the corresponding stock price  $P_b(z, q_1)$ , the manager's payoff function is constant and independent of the padding amount. Then, with the inventory bound, the equilibrium will still hold only if there is no jump of the manager's payoff at this bound, i.e., the manager needs to be always indifferent to follow the linear channel stuffing strategy or to push the sales to the bound. This leads to the boundary stock price in Equation (13) and the condition for  $\xi_b$  in Equation (14). In equilibrium, the manager's strategy needs to be consistent with the investors' belief. Since the manager is always indifferent about how much to pad, the equilibrium will hold in the way such that the manager follows the linear channel stuffing strategy for  $\xi_1 \in [0, \xi_b)$  and pushes the sales to the bound for  $\xi_1 \in [\xi_b, q_1)$ . If such a  $\xi_b$  cannot be found in the region  $[0, q_1)$ , it indicates that the boundary effect is so strong that the manager will always push the sales to the bound. Similarly, in this equilibrium, the reported sales  $z$  also contains a jump at  $\xi_1 = \xi_b$  from  $(1 + \theta)\xi_b$  to  $q_1$ . The stock price that specifies this off-equilibrium path follows  $P_b(z, q_1)$  in the region  $z \in ((1 + \theta)\xi_b, q_1)$  as if the manager would apply the linear channel stuffing strategy. Different from the case  $\theta < 0$  and  $a > 0$ , the stock price now does not have jump at the bound due to the above property of the pooling stock price. Figure 3 demonstrates the structure of the market equilibrium for  $\theta > 0$ . The manager follows a linear channel stuffing strategy when  $\xi_1 < \xi_b$ , and pushes the sales to the bound when  $\xi_1 \geq \xi_b$ .

### 4.3 Comparative Statics

In the following, we present the results common for both  $\theta < 0$  and  $\theta \geq 0$ . From Propositions 1 and 2, we observe that the equilibrium channel stuffing strategy for both  $\theta < 0$  and  $\theta \geq 0$  can be uniquely characterized by a threshold  $\xi_a$  or  $b$  and a slope  $(\theta)^+$ . When the actual demand is lower than the threshold, the manager follows a linear channel stuffing strategy with slope  $(\theta)^+$ ; when the demand exceeds the threshold, the manager pushes the sales to the inventory bound. If the sales revenue  $p$  or the correlation coefficient  $a$

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<sup>7</sup>Note that we made, for simplicity, the assumption that the costs of padding are linear in the padding amount. This assumption necessarily leads to the indifference result and allows us characterizing analytically the equilibrium stock price.



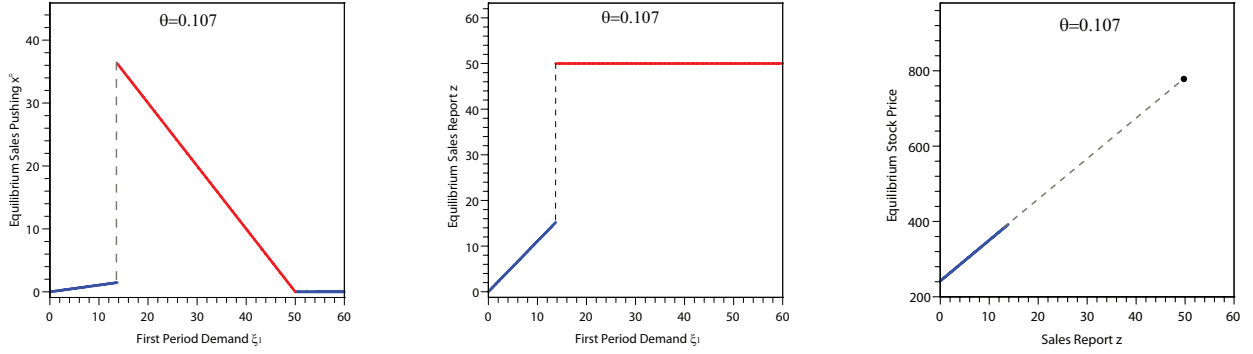


Figure 3: Market equilibrium with high pushing incentive. The parameters:  $A_0 = 400$ ,  $p = 10$ ,  $c = 6$ ,  $\gamma = 4$ ,  $a = 0.6$ ,  $\beta = 0.27$ ,  $q_1 = 50$ ,  $\eta_1, \eta_2 \sim N(50, 15^2)$

increases, the marginal benefit from an increase of the sales increases. This will provide the manager more incentive to pad the sales. Similarly, if the fraction  $\beta$  increases, the proportion of the manager's payoff from the short-term stock-based compensation increases, which stimulates the manager's incentive to inflate the sales. In contrast, if the inventory cost  $c$  increases, the margin of the sales decreases; if  $\gamma$  increases, the penalty cost of channel stuffing increases. Therefore, both of them negatively impact the manager's sales padding incentive. We now directly obtain Proposition 3.

**Proposition 3** *Given  $q_1$  and  $\xi_1$ , the equilibrium  $x^o(\xi_1, q_1)$  increases in  $p$ ,  $\beta$ ,  $a$ , but decreases in  $c$  and  $\gamma$ .*

## 5 Market Equilibrium with Inventory Carryover

So far we have assumed that the leftover inventory cannot be carried over for future use. In this section, we analyze the model with inventory carryover. We assume that the firm needs to pay a holding cost  $h < c$  per unit for carrying the leftover inventory from the first period to the second period. As a result, for channel stuffing, the firm incurs a cost of  $\gamma + h$  per unit. The inventory that is used to pad the sales, when returned, can be used to satisfy the demand in the second period. After the second period sales, any leftover inventory has zero salvage value.

### 5.1 Second Period Decision

Let  $I = (q_1 - \xi_1 - x)^+$  denote the leftover inventory from the first period. Then, the firm's second period inventory investment is determined by:

$$\max_{q_2} \mathbb{E}_{\eta_2} [p \min(a\xi_1 + \eta_2, I + x + q_2)] - cq_2.$$

Note that the padded amount,  $x$ , (which is less than  $(q_1 - \xi_1)^+$ ) does not matter for the second period optimal investment<sup>8</sup> as the padded demand is not real and will be returned at the beginning of the second period. Hence, only  $q_1 - \xi_1$  matters for the optimal second period inventory investment:  $q_2^* = \max\{k_2 + a\xi_1 - (q_1 - \xi_1)^+, 0\}$ . The corresponding profit in the second period is:

$$v_2(\xi_1, q_1) = \begin{cases} pa\xi_1 + \mathbb{E}_{\eta_2} [p \min(\eta_2, q_1 - (1+a)\xi_1)], & q_2^* = 0, \\ (p-c)a\xi_1 + c(q_1 - \xi_1)^+ + \mathbb{E}_{\eta_2} [p \min(\eta_2, k_2)] - ck_2, & q_2^* > 0, \end{cases} \quad (15)$$

where  $k_2 = F_2^{-1}\left(\frac{p-c}{p}\right)$ . Similar to the case without inventory carryover, we take the derivative of  $v_2(\xi_1, q_1)$  with respect to  $\xi_1$ ,

$$\frac{dv_2(\xi_1, q_1)}{d\xi_1} = \begin{cases} ap - (1+a)p\bar{F}_2(q_1 - (1+a)\xi_1), & 0 \leq \xi_1 < \bar{\xi}(q_1), \\ a(p-c) - c, & \bar{\xi}(q_1) \leq \xi_1 < q_1, \\ a(p-c), & \xi_1 \geq q_1, \end{cases} \quad (16)$$

where  $\bar{\xi}(q_1) \equiv \left(\frac{q_1 - k_2}{1+a}\right)^+$ .

Notice that the derivative depends on both the inventory level  $q_1$  and the actual demand  $\xi_1$  in the first period. If  $q_1$  is high and  $\xi_1$  is low, there will be a lot of inventory leftover at the end of the first period and the firm will not need to invest new inventory in the second period. It is clear from the first branch of Equation (15): An increase of  $\xi_1$  directly leads to a net margin  $ap$  in the second period due to the correlation. However, an increase of  $\xi_1$  leads to the decrease of inventory with a ratio  $(1+a)$  (“1” captures the first period consumption and “ $a$ ” captures the second period consumption). When the inventory decreases, the expected satisfaction for the part of the stochastic demand  $\eta_2$  decreases, which has an expected loss of the margin  $(1+a)p\bar{F}_2(q_1 - (1+a)\xi_1)$ . These two factors lead to the first branch of Equation (16). When  $\xi_1$  is higher than  $\bar{\xi}(q_1)$ , the firm needs to replenish inventory. In this case, an increase of  $\xi_1$  will lead to an increase of the expected profit by only  $a(p-c)$  per unit. Moreover, it also brings an additional marginal purchasing cost, which the firm otherwise does not need to incur in the second period. This leads to the second branch of Equation (16). Finally, if  $\xi_1$  is higher than  $q_1$ , then the firm uses up the inventory in the first period, which becomes identical to the case that the inventory cannot be carried over. Equation (16) will be useful to determine the manager’s incentive to pad the first period sales. Comparing with Equation (3), for the case of no inventory carryover, we see that each unit of the first period sales ( $\xi_1$ ) impacts the second period profits (and hence the stock price) in a different way, especially when the demand realization is low. In the next subsection, we describe how this impacts the incentive to pad the sales.

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<sup>8</sup> $I + x + q_2$  is independent of  $x$ .

## 5.2 Equilibrium Sales Padding and Stock Price

In this section, we characterize the market equilibrium. Let

$$\theta_c(\xi_1, q_1) = \begin{cases} \theta_c^I(\xi_1, q_1), & 0 \leq \xi_1 < \bar{\xi}(q_1), \\ \theta_c^{II}, & \bar{\xi}(q_1) \leq \xi_1 < q_1, \\ \theta, & \xi_1 \geq q_1, \end{cases} \quad (17)$$

with

$$\begin{aligned} \theta_c^I(\xi_1, q_1) &= \frac{\beta(p + ap - (1+a)p\bar{F}_2(q_1 - (1+a)\xi_1) + h) - (1-\beta)\gamma}{\gamma} \text{ and} \\ \theta_c^{II} &= \frac{\beta(p + a(p-c) + h - c) - (1-\beta)\gamma}{\gamma}. \end{aligned}$$

Similar to  $\theta$  (see Equation (4)) in the case without inventory carryover,  $\theta_c(\xi_1, q_1)$  indexes the manager's marginal incentive to pad the sales. However,  $\theta_c(\xi_1, q_1)$  is more complicated than the case in which no inventory is carried over. The difference comes from: (1) the change of the expected marginal profit in the second period, (2) the potential holding cost saving, and (3) the potential additional marginal inventory cost.

It is interesting to compare  $\theta_c(\xi_1, q_1)$  with  $\theta$ . From Equation (17), we observe that  $\theta_c(\xi_1, q_1)$  can be either higher or lower than  $\theta$  in the region  $0 \leq \xi_1 < \bar{\xi}(q_1)$ , which depends on  $q_1$ ,  $\xi_1$  and the distribution function  $F_2(\cdot)$ . It is easy to see that  $\theta_c^I(\xi_1, q_1)$  increases in  $q_1$  but decreases in  $\xi_1$ , since the term  $\bar{F}_2(q_1 - (1+a)\xi_1)$  decreases in  $q_1$  but increases in  $\xi_1$ . In the extreme case if  $q_1$  is sufficiently large and  $\xi_1$  is sufficiently small such that  $\bar{F}_2(q_1 - (1+a)\xi_1)$  is close to zero,  $\theta_c^I(\xi_1, q_1)$  goes to  $\frac{\beta(p+ap+h)-(1-\beta)\gamma}{\gamma}$  which is larger than  $\theta$ . Thus, the padding incentive increases in  $q_1$ . This observation will be useful later, when we discuss the incentives to invest in  $q_1$ . When  $\xi_1$  is close to  $\bar{\xi}(q_1)$ ,  $\theta_c(\xi_1, q_1)$  tends to  $\theta_c^{II}$  which is less than  $\theta$  given that  $h < c$ . In the region  $\bar{\xi}(q_1) \leq \xi_1 < q_1$ , it is direct that  $\theta_c(\xi_1, q_1)$  is always less than  $\theta$  as  $\theta_c^{II} < \theta$ . In the region  $\xi_1 \geq q_1$ ,  $\theta_c(\xi_1, q_1)$  coincides with  $\theta$ , however, the manager is not able to pad the sales as the inventory is used up. Figure 4 provides a graphic demonstration between  $\theta_c(\xi_1, q_1)$  and  $\theta$  for different initial inventory  $q_1$ . Therefore, with inventory carryover, the manager may have a stronger incentive to use channel stuffing if the first period inventory is high and the real demand is low, and weaker incentive if the first period real demand is relatively high.

Proposition 4 presents the structure of the market equilibrium. In this proposition, we combine different scenarios where  $\theta_c(\xi_1, q_1)$  might be positive or negative, and provide a general structure of the equilibrium.

**Proposition 4** *Given inventory  $q_1$ , there is a unique threshold  $\xi_c \in [0, q_1]$  such that the market equilibrium*

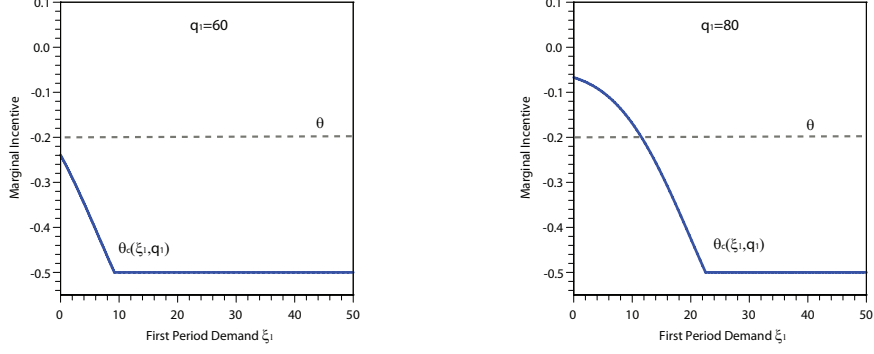


Figure 4: Comparison of the pushing indicators. The parameters:  $A_0 = 400$ ,  $p = 10$ ,  $c = 6$ ,  $\gamma = 4$ ,  $h = 0$ ,  $a = 0.6$ ,  $\beta = 0.2$ ,  $\theta = 0.8$ ,  $\eta_1, \eta_2 \sim N(50, 15^2)$ ,  $q_1 = 60$  ( $80$ ) in the left (right) subplot.

can be characterized by

$$x^o(\xi_1, q_1) = \begin{cases} \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+, & 0 \leq \xi_1 < \xi_c, \\ q_1 - \xi_1, & \xi_c \leq \xi_1 < q_1, \\ 0, & q_1 \leq \xi_1, \end{cases} \quad (18)$$

and

$$P^o(z, q_1) = \begin{cases} P_c(z, q_1), & 0 \leq z < q_1, \\ \bar{P}(\xi_c, q_1), & z = q_1, \end{cases} \quad (19)$$

where

$$P_c(z, q_1) = A_0 + p\xi_{1,c}^o(z, q_1) - cq_1 - \gamma x^o(z, q_1) - h(q_1 - \xi_{1,c}^o(z, q_1)) + v_2(\xi_{1,c}^o(z, q_1), q_1) \quad (20)$$

$$\xi_{1,c}^o(z, q_1) \text{ solves: } \xi_1 + \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+ = z$$

and

$$\bar{P}(\xi_c, q_1) = A_0 + \mathbb{E}_{\xi_1} [p \min(\xi_1, q_1) - (\gamma + h)(q_1 - \xi_1)^+ + v_2(\xi_1, q_1) | \xi_1 \geq \xi_c] - cq_1. \quad (21)$$

$\xi_c$  satisfies:

$$\beta \bar{P}(\xi_c, q_1) - (1 - \beta)\gamma(q_1 - \xi_c) = \beta P_c \left( \xi_c + \left( \int_0^{\xi_c} \theta_c(t, q_1) dt \right)^+, q_1 \right) - (1 - \beta)\gamma \left( \int_0^{\xi_c} \theta_c(t, q_1) dt \right)^+ \quad (22)$$

or  $\xi_c = 0$  if Equation (22) does not hold for any  $\xi_c \in [0, q_1]$ .

Proposition 4 provides us with a complete overview of all sales padding incentives in our model. When the real demand realization is close to the available inventory ( $\xi_c \leq \xi_1 < q_1$ ), the equilibrium padding strategy  $x^o$ , with inventory carryover also exhibits the boundary effect that we discussed for the model without carryover (see Section 4.2): The remaining inventory is padded to the sales and the manager reports that all inventory is sold. When the real demand realization is less than  $\xi_c$ , the equilibrium padding strategy

is determined by  $\left(\int_0^{\xi_1} \theta_c(t, q_1) dt\right)^+$ . From Equation (17), we see that the padding strategy will be composed of a linear part (when the real demand realization is neither high nor low,  $\bar{\xi}(q_1) \leq \xi_1 < q_1$ ). This is the marginal effect that we discussed for the model without carryover (see Section 4.2). When the real demand realization is low, we identify a new incentive for padding.

**The carryover effect.** With inventory carryover and low demand realizations,  $0 \leq \xi_1 < \bar{\xi}(q_1)$ , no replenishment will be necessary in the second period. Then, each left-over unit contributes more revenues in the second period than was the case without inventory carryover; no purchasing cost is incurred as the second period demand can be satisfied from the leftover first period inventory. The expected increase in profits for each unit of realized real first period demand is  $ap$  instead of  $a(p - c)$  without carryover. We refer to this increased attractiveness as the “carryover” effect. It is the third effect that determines the padding incentives. Like the boundary effect, it is directly linked to the inventory problem that we study. However, this time it is within the low demand region. A high demand realization will reduce the first period final inventory that otherwise could be used to serve the second period demand. Therefore, there is a region where the manager’s incentive increases if the demand realization is lower. This carryover effect occurs for *low* realizations of the real demand, which is interesting because the boundary effect only occurs at *high* realizations of the real demand.

Similarly to  $\xi_a$  and  $\xi_b$  (defined in Equations (8) and (13)), the threshold  $\xi_c$  represents the actual demand at which the manager is indifferent to push the sales to the inventory bound or not to.  $\xi_c$  can easily be computed numerically, although it is difficult to analyze in closed form due to the complexity of  $x^o(\xi_1, q_1)$ . Figure 5 demonstrates the market equilibrium with inventory carryover.<sup>9</sup> We plot three scenarios in this figure (we omit the case where the manager always chooses not to pad sales, i.e., when  $\theta_c(\xi_1, q_1) < 0$  and  $a = 0$ ). In the first scenario (the top three subplots),  $\theta_c(\xi_1, q_1)$  is always less than zero but  $a > 0$ . As a result, the manager does not have an incentive to pad except when the actual demand is close to the inventory bound due to the boundary effect. This scenario is similar to the case without inventory carryover with  $\theta < 0$  and  $a > 0$ . In the second scenario (the middle three subplots),  $\theta_c(\xi_1, q_1)$  is positive when the actual demand  $\xi_1$  is low but becomes negative as  $\xi_1$  increases. The amount of channel stuffing first increases and then decreases to zero before the boundary effect appears. This scenario illustrates in the best way the two main padding factors in real earnings management that we identify in this paper and are due to the nature of the inventory problem: the boundary and carryover effect. In the third scenario,  $\theta_c(\xi_1, q_1)$  is always positive. As a result, the manager always pads the sales. The manager continuously increases the padding amount as  $\xi_1$  increases, and pushes the sales to the inventory bound once  $\xi_1$  reaches  $\xi_c$ .

<sup>9</sup>Note that, because  $\theta_c(\xi_1, q_1)$  depends on both  $\xi_1$  and  $q_1$ , we cannot use  $\theta_c(\xi_1, q_1)$  to index the plots and thus we alternatively use  $\beta$  to index the plots. This is similar for all the figures later on with inventory carryover.

So far, we have explained in the models with and without inventory carryover why operations managers with short-term stock-based compensation have incentives to pad the sales, even though this does not create any value for the firm. In the next section, we study how these effects are impacted by the first period inventory investment.

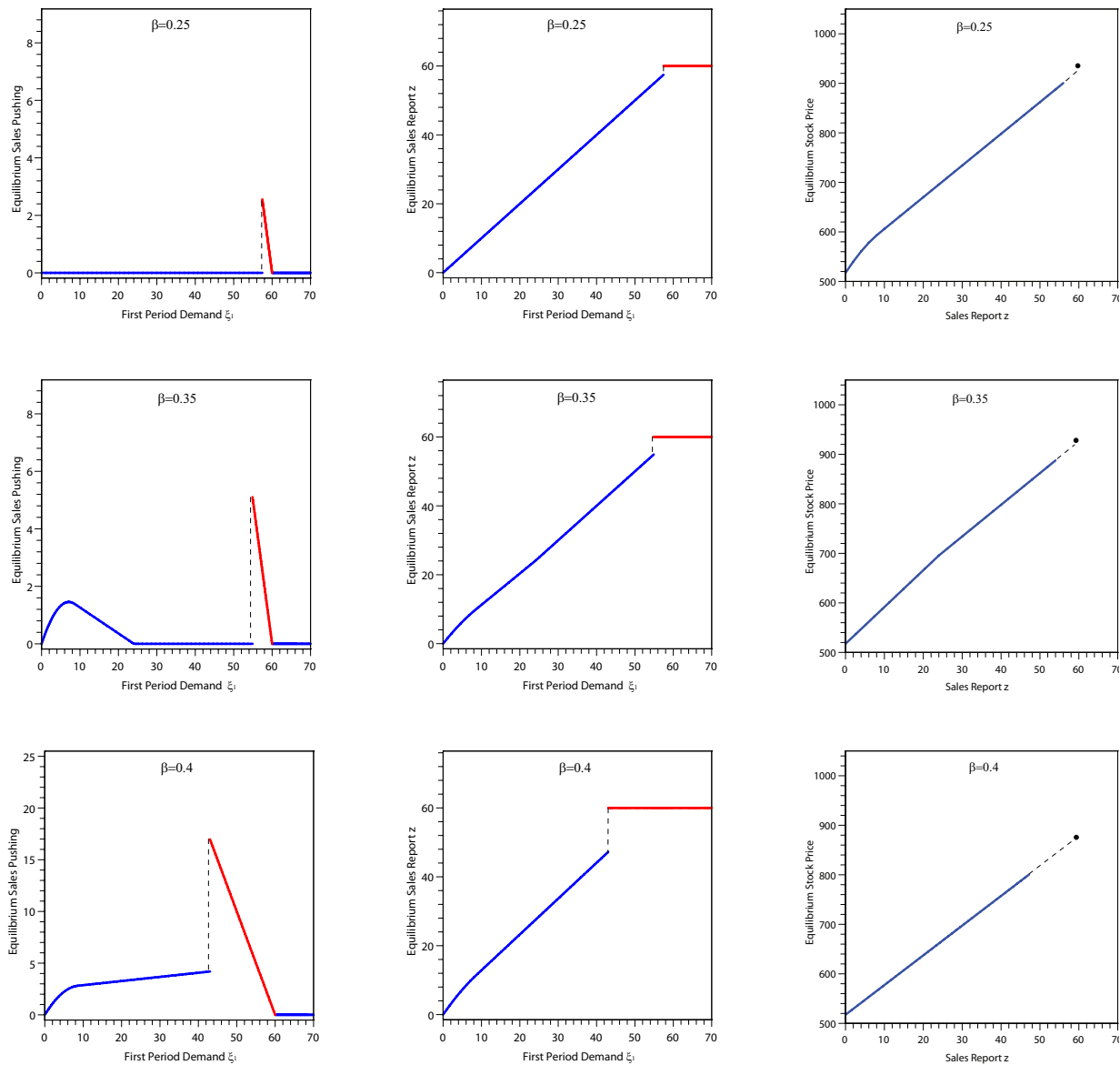


Figure 5: Market equilibrium with inventory carryover. The parameters:  $A_0 = 400$ ,  $p = 10$ ,  $c = 6$ ,  $\gamma = 4$ ,  $a = 0.6$ ,  $q_1 = 60$ ,  $\eta_1, \eta_2 \sim N(50, 15^2)$ . From top to bottom,  $\beta = 0.25, 0.35$  and  $0.4$ .

## 6 Inventory Incentives

In previous sections, we regarded the first period inventory level,  $q_1$ , as given. In this section, we discuss the manager's incentive on initial inventory investment decisions. Our main question is whether the sales padding leads to over or under investment, compared to the initial inventory investment level of a private firm. We present a general formulation of the manager's optimization program with respect to the first period inventory decision. Then, we discuss the cases without and with inventory carryover and reveal the key insights on inventory incentives.

Incorporating the market equilibrium, we obtain the manager's optimization program with respect to the first period inventory decision  $q_1$  for both cases with/without inventory carryover as (see Appendix B for the derivation):

$$\text{Without carryover: } \max_{q_1} \underbrace{A_0 - cq_1 + \mathbb{E}_{\xi_1} [p \min(\xi_1, q_1) + v_2(\xi_1)]}_{\text{Classical Newsvendor}} - \underbrace{\gamma \mathbb{E}_{\xi_1 \in [0, q_1]} [x^o(\xi_1, q_1)]}_{\text{Stock Market Pressure}} \quad (23)$$

and

$$\text{Carryover: } \max_{q_1} \underbrace{A_0 - cq_1 + \mathbb{E}_{\xi_1} [p \min(\xi_1, q_1) - h(q_1 - \xi_1)^+ + v_2(\xi_1, q_1)]}_{\text{Classical Newsvendor}} - \underbrace{\gamma \mathbb{E}_{\xi_1 \in [0, q_1]} [x^o(\xi_1, q_1)]}_{\text{Stock Market Pressure}} \quad (24)$$

Notice that this optimization problem follows a combination of the classical newsvendor terms and an extra negative term that reflects the friction from potential channel stuffing. The negative term represents the stock market pressure. The self-interested manager has an incentive to pad the sales to inflate the sales report; even though his strategy is anticipated by the investors in a rational stock market. Recall that padding does not create any value for the company and only induces costs ( $\gamma > 0$ ). Ex ante, the presence of stock market pressure and the chance of channel stuffing always reduce the firm's value as well as the manager's expected payoff, although ex post the manager may benefit from padding. Nevertheless, this effect is observed in public firms. This effect of stock market pressure has been documented earlier in Stein (1989), "...the excessive capital market pressure may have adverse effects on firm performance..." Although firms should improve their operations in order to enhance stock performance, in the cases where such fundamental changes cannot be achieved in a short term, it has been well documented that managers use various earnings management means despite of the long run harm. In our model, the (rational) operations manager is aware of this perverse, but, inescapable incentive. He can partially control for the negative impact by deliberately selecting the first period inventory investment level taking the downstream padding incentives into account. In the following, we examine the inventory investment. Due to the complexity of the analysis, we show the key insights by numerical examples.<sup>10</sup> (In Appendix B, we derive the first order conditions with respect

<sup>10</sup>In the numerical examples, we use line search to find the  $q_1$  that provides close-maximal payoff.

to  $q_1$  and show the intuition of our numerical findings.) We compare how this investment under market equilibrium differs from an otherwise identical private firm (i.e., the classical newsvendor investment of a private firm), which we denote as the *benchmark*.

In Figure 6, left panel, we characterize the optimal inventory level  $q_1^*$  for the case where inventory cannot be carried over, according to the fraction of short-term stock compensation  $\beta$ . We vary  $\beta$  with the other parameters fixed. We observe that the optimal inventory level is slightly higher than the benchmark when  $\beta$  is low (note that a low  $\beta$  implies a low  $\theta$  (or  $\theta_c(\xi_1, q_1)$  for the case with inventory carryover) and thus the marginal incentive on padding is low). This over-investment first increases in  $\beta$ , reaches a maximum, then drops, and the manager starts to under-invest inventory. When  $\beta$  increases over some threshold, the inventory level stays at a fixed level below the benchmark. The intuition is the following: When  $\beta$  is small (such that  $\theta$  is negative), the incentive of channel stuffing is exclusively determined by the boundary effect. To increase the inventory level, i.e., to move the bound upwards, decreases the probability that the realized demand will fall into the boundary effect zone, i.e., the region  $\xi_1 \in [\xi_a \text{ or } b, \infty)$  (In Corollary 2 in Appendix A, we show that both  $\xi_a$  and  $\xi_b$  increase in  $q_1$ ). Moreover, with a higher bound, the cost to pad all remaining inventory is also higher and thus the incentive to do so will be lower. With a low  $\beta$  (and thus a low  $\theta$ ), the expected loss from channel stuffing is not that high. As a result, the expected gain from limiting the boundary effect outweighs the expected loss of inventory over-investment. However, when  $\beta$  increases (such that  $\theta$  becomes positive), the marginal incentive starts to play its role and the loss due to channel stuffing increases relatively high. In that case, the margin from the potential sales reduces, and from the newsvendor property we know inventory investment decreases. Finally, when  $\beta$  is too large, the manager always pads all leftover inventory, for any realized demand. As a result, the equilibrium result does not change any longer and the inventory level stays flat. Note that in this set of experiments we only vary  $\beta$ —the fraction of stock compensation, which does not change the properties of the business (i.e.,  $p$ ,  $c$ ,  $F_t(\cdot)$ , etc.).

The middle and right panels characterize the results with inventory carryover. Recall, in Section 5, we have discussed that in this case in the region  $0 \leq \xi_1 < \bar{\xi}(q_1)$ , the manager's padding incentive increases in  $q_1$ ; i.e., the carryover effect. As a result, the manager may want to invest lower inventory to limit the incentive in this region. However, the manager still needs to consider the boundary effect, where the channel stuffing incentive increases as the inventory bound decreases. Hence, the boundary and carryover effects will drive the first period inventory investment in different directions.

Since  $\bar{\xi}(q_1)$  depends on  $q_1$  (the first period inventory level) and  $k_2$  (the second period optimal inventory level) (see the definition of  $\bar{\xi}(q_1)$  in Equation (16)), we discuss two situations. In the first situation, we have  $q_1^*$  always less than  $k_2$ . As a result, the firm needs to replenish inventory in the second period and we have  $\bar{\xi}(q_1^*) < 0$ . In such a situation, effectively, the carryover effect does not exist. The middle panel of



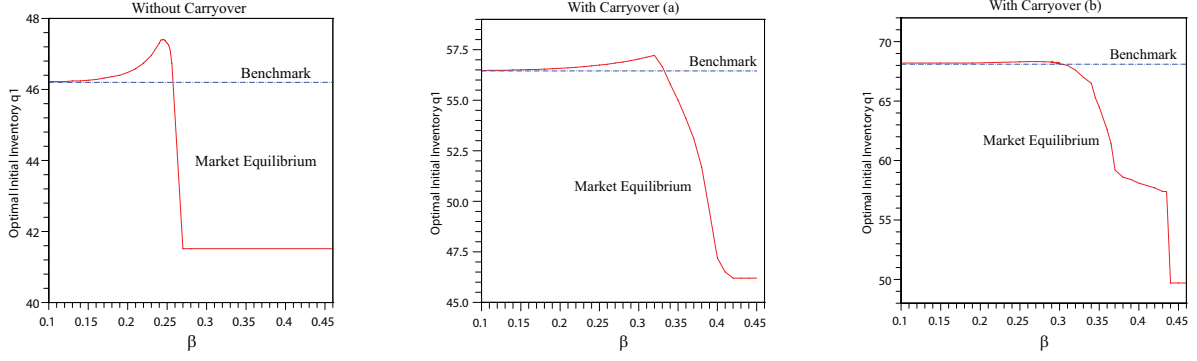


Figure 6: Comparison of the first-period inventory levels between market equilibrium and benchmark. Left panel: with no inventory carryover; middle and right panels: with inventory carryover. Common parameters:  $A_0 = 400$ ,  $p = 10$ ,  $c = 6$ ,  $\gamma = 4$ ,  $a = 0.6$ ,  $\eta_1 \sim N(50, 15^2)$ ,  $\beta \in [0.1, 0.46]$ . Different parameters: left panel:  $\eta_2 \sim N(50, 15^2)$ ; middle panel:  $\eta_2 \sim N(80, 30^2)$ ,  $h = 2$ ,  $\bar{\xi}(q_1^*) < 0$ ; right panel:  $\eta_2 \sim N(50, 15^2)$ ,  $h = 0$ ,  $\bar{\xi}(q_1^*) > 0$ . Note: in the middle (right) panel, there effectively does not (does) exist carryover effect given that  $\bar{\xi}(q_1^*) < (>)0$ .

Figure 6 illustrates that the dependency of  $q_1^*$  on  $\beta$  is similar to the case without inventory carryover. This is reasonable because this case is almost the same as the one without inventory carryover except for the additional term  $h - c$  in the marginal incentive  $\theta_c(\xi_1, q_1)$  (see Equation(17)). There is clear over-investment when  $\beta$  is small and under-investment when  $\beta$  is large.

In contrast, in the second situation, we have the optimal first period inventory level  $q_1^*$  more than  $k_2$  and  $\bar{\xi}(q_1^*) > 0$ . The carryover effect appears. Figure 6, right panel illustrates this situation. From the panel, we see that the dependency of  $q_1^*$  on  $\beta$  is more complicated. First, over-investment is very slight although it exists. This is due to the carryover effect; the manager's incentive to use channel stuffing increases in  $q_1$  when the realized demand  $\xi_1$  falls in the region  $[0, \bar{\xi}(q_1)]$ . This acts as a force limiting high inventory investment. Second, as  $\beta$  increases, there are two sharp drops of  $q_1^*$  but in between there is a narrow region which is slowly shrinking. The reason is the following: When  $\beta$  increases, the amount of padding besides the boundary effect first increases slowly (see the left middle subplot in Figure (5) where the padding amount increases and then decreases to zero before approaching the boundary effect). However, when  $\beta$  increases to some level, the amount of padding increases sharply (see the left bottom subplot in Figure (5) where the padding amount continuously increases and never decreases to zero), i.e. the marginal incentive effect kicks in. This leads to the first sharp drop of  $q_1^*$  since it is beneficial to decrease the inventory level to limit the marginal incentive on channel stuffing. When  $\beta$  continues to increase, there is a point from which the manager will always push the sales to the inventory bound. This will lead to a big decrease of the margin. Therefore, there is the second sharp drop of  $q_1^*$ .  $q_1^*$  does not change after that, since the equilibrium will

always be the same that the manager pads all leftover inventory for any realized demand  $\xi_1$ .

These observations indicate an interesting insight that the optimal inventory investment under the stock market pressure can be either lower or higher than the classical newsvendor solution, which is formally stated in the following Observation 1.

**Observation 1** *Under the market equilibrium, the manager may over- or under- invest the first period inventory  $q_1^*$ , both possible, compared to the classical newsvendor solution.*

Under-investment of inventory may generally be expected because the expected loss from channel stuffing, which is an agency cost, reduces the margin of the potential sales. To put it another way, the existence of channel stuffing reduces the underage cost. This will lead to a low inventory investment based on the newsvendor property. More interestingly, the existence of over-investment (as well as the incentive of under-investment to limit the carryover effect) reveals the intuition worth pondering. Anticipating the “myopia” of playing a costly game with rational investors, a rational manager may use the operational lever, investment in inventory, to limit this potential. As discussed in Abegglen and Stalk (1985), many U.S. executives are clearly aware of the cost of this gaming to push the short-term stock price high. Therefore, the above results not only explore how a public firm’s inventory decision may differ from the classical theory for private firms, but also characterize the potential that managers might apply real earnings management to alleviate the stock market pressure ex ante, beyond the incentive to manipulate the financial status ex post.

## 7 Conclusion

In this paper, we study the impact of stock market pressure on public firms’ selling and inventory decisions. We focus on a public firm controlled by a self-interested manager whose compensation depends partially on the stock price of the firm. When the market demand is correlated across periods, the manager may pad the reported sales in the current period in order to influence the stock price. As with the increased attention after the Enron and WorldCom scandals and the Sarbanes-Oxley Act, accrual-based earnings management becomes more difficult and real earnings management becomes more attractive for managers that are subject to stock market pressure. With real earnings management, the manager faces “real” constraints, like bounds on physical inventory. We capture this feature in our model of channel stuffing. We identify three factors that drive the manager’s incentive on channel stuffing: the marginal incentive, the boundary effect and the carryover effect. Our analysis reveals that with a rational stock market, although, ex post, to apply such real earnings management means may benefit the manager from the short-term stock market mis-pricing, ex ante, the existence of stock market pressure and the chance of channel stuffing hurt the firm as well as the

manager himself. Furthermore, when examining the up front inventory decision, we show that the manager may not only under-invest the inventory but over-invest the inventory compared to an otherwise identical private firm. With the friction loss due to channel stuffing, the margin of the business reduces, which acts as the incentive to under-invest inventory. It is surprising to see that over-investment also exists. We find that the incentive of over-investment comes from the desire of the manager to mitigate the boundary effect. With a higher inventory level, the chance that the realized demand will fall into the boundary effect zone will be lower and thus less negative boundary effect. These findings show how the existence of compensations based on the stock market price may influence operations management of a publicly traded firm.

Our paper contributes to both the operations management literature and the earnings management literature. The traditional operations management literature usually examines private firms whose owners make operational decisions. However, in practice, most major firms are public. Stock market interaction has been widely documented which has substantial impact on firms' operational decisions. Similarly, the firm's operations also influence the move of stock prices. The ongoing pioneer research in operation management by Hendricks and Singhal (2001, 2005, 2006), Gaur et al. (2002), Raman (2006), Chen et al. (2005), Lai (2005, 2006, 2008) try to answer such questions how operation and stock market performance are exactly connected. Our study contributes to this stream of literature by studying the problem formally in a two period inventory management context with correlated demand in presence of a rational stock market as a game between the operations manager of a firm and rational investors. The information asymmetry that we introduce, the manager observes the demand realization while the investors only know the distribution of the demand, is a logical relaxation of the usual assumption that all information is public knowledge. Our theoretical analysis is, to the best of our knowledge, new and provides meaningful insights such as how a public firm's inventory decisions are different from a private firm's inventory decisions, which is the classical reference framework in operations management. We also believe that our way of introducing the stock-market, adapted from the finance and accounting literature, is novel and relevant in an operational context. Finally, our paper also contributes to the earnings management literature. The existing studies in this stream include empirical research to document the phenomena of earnings management (e.g., Roychowdhury (2006), Cohen et al. (2007)) and analytical studies on related agency problems (e.g., Dechow et al. (1995), Dye (2002), Stein (1989), Dye and Sridhar (2004), Liang and Wen (2007)). These studies usually focus on accrual-based earnings management or stylized investment. We enrich this literature by addressing a detailed operational problem and relating it with widely reported real earnings management activities that are known as channel stuffing, truck loading, demand borrowing, etc. More importantly, our paper identifies two new factors that may drive a manager to use such means in real operation and are determined by the physical constraints that inventories impose on real earnings management. Rather surprisingly, we find that real earnings management

may also be used ex ante, to limit the incentives on ex post real earnings management activities.

We conclude by discussing the assumptions in our model and indicating directions for future research. Generally, when the ownership and the management of a firm are separated, agency problems may arise. The information advantage of the manager cultivates the incentive to game. First, in this paper, we investigate the information asymmetry about the realized actual demand. It is natural to assume that investors do not know the real demand realization, while the manager has much better idea, hence, we focus on this type of information asymmetry and assume that all other parameters are common knowledge. However, the games between inside managers and outside investors can have many more applications, for example, if the efficiency (e.g., the internal management cost, the creativity, the forecasting capability, etc.) of the firm is private knowledge instead of the actual demand. Furthermore, our paper addresses only one instrument of operational manipulation. As discussed in Roychowdhury (2006), managers in practice however may have many alternative choices to manipulate their operation. For instance, managers can manipulate the sales alternatively through intensive discounts; managers can also intentionally reduce expenditures such as cost on R&D activities; managers can sometimes overproduce to report lower cost of goods sold.

Second, in this paper, we have assumed that initial inventory is perfectly observable and hence, investors know exactly how much can be satisfied from inventory. In practice, there will be noise on the observation of the initial inventory (multiple products, pipeline inventory, ...), hence, the boundary effect will not be so sharp that reporting the initial inventory will lead to a discrete jump in stock price (because of the pooling effect), but, the stock price may be rather steeply increasing around the expected initial inventory. Differently, the carryover effect is somewhat less sensitive to the initial inventory, as it occurs for low realizations of the demand. Irrespective of what the initial inventory is, no replenishment is expected in the next period. The latter drives the steeper increase in stock price as a function of the realized sales.

Finally, our paper assumes that the financial report only contains information of the past operation. However, in practice, a financial report may also contain direct information of the ongoing business and future forecast. For example, firms may report information of the newly installed facility or inventory along with the future forecast. In such situations, managers may have incentive to manipulate the initial investment, to signal the potential of the business to investors. We address this forecasting and signaling effect in our parallel work, Lai et al. (2008). We believe real earnings management is a rich area for further research.

## References

Abegglen, J., G. Stalk. 1985. *Kaisha*, The Japanese corporation. New York: Basic Books.

- Babich, V., M. J. Sobel. 2004. Pre-IPO operational and financial decisions. *Mgmt. Sci.* 50(7): 935-948.
- Buzacott, J.A., R.Q. Zhang. 2004. Inventory management with asset-based financing. *Mgmt. Sci.* 50(9): 1274-1292.
- Chen, F. 2000. Sales-force incentives and inventory management. *M&SOM*. 2(2): 186-202.
- Chen, F. 2005. Salesforce incentives, market information, and production/inventory planning. *Mgmt. Sci.* 51(1): 60-75.
- Chen, F., W. Xiao. 2005(a). Salesforce incentives, information acquisition, and production planning. Working paper, Columbia Business School.
- Chen, F., W. Xiao. 2005(b). Salesforce incentives and information screening: individualistic vs. competitive schemes. Working paper, Columbia Business School.
- Chen, H., M. Z. Frank, O. Q. Wu. 2005. What actually happened to the inventories of American companies between 1981 and 2000? *Mgmt. Sci.* 51(7): 1015-1031.
- Cohen, D. A., A. Dey, T. Z. Lys. 2007. Real and accrual-based earnings management in the pre- and post-Sarbanes Oxley periods. Working paper, Northwestern University.
- Debo, L.G., L.B. Toktay, L.N. Van Wassenhove. 2008. Queuing for expert services. *Mgmt. Sci.* Forthcoming.
- Dechow, P. M., R. G. Sloan, A. P. Sweeney. 1995. Detecting earnings management. *Accounting Rev.* 70(2): 193-225.
- Dye, R. A. 2002. Classifications manipulation and Nash accounting standards. *J. Accounting Res.* 40(4):1125-1162.
- Dye, R. A., S. S. Sridhar. 2004. Reliability-relevance trade-off and the efficiency of aggregation. *J. Accounting Res.* 42(1): 51-88.
- Gaur, V., M. Fisher, A. Raman. 2002. Linking finance and operations in retailing. Working paper, New York University.
- Gaur, V., M. Fisher, A. Raman. 2005. An econometric analysis of inventory turnover performance in retail services. *Mgmt. Sci.* 51(2): 181-194.
- Graham, J. R., C. R. Harvey, S. Rajgopal. 2005. The economic implications of corporate financial reporting. *J. Accounting Econ.* 40(1-3): 3-73.
- Hendricks, K. B., V. R. Singhal. 2001. The long-run stock price performance of firms with effective TQM programs as proxied by quality award winners. *Mgmt. Sci.* 47(3): 359-368.
- Hendricks, K. B., V. R. Singhal. 2005. An empirical analysis of the effect of supply chain disruptions on

- long-run stock price performance and risk of the firm. *Production & Oper. Mgmt.* 14(1): 35-52.
- Hendricks, K. B., V. R. Singhal. 2006. Demand-supply mismatch and stock market reaction: Evidence from excess inventory announcements. Working paper, Georgia Institute of Technology.
- Kapuscinski, R., R.Q. Zhang, P. Carbonneau, R. Moore, B. Reeves. 2004. Inventory Decisions in Dell's Supply Chain. *Interfaces.* 34(3): 191-205.
- Kieso, D. E., J. J. Weygandt. 1998. Intermediate Accounting. Wiley, 9th Edition.
- Lai, R. 2005. Inventory and the stock market. Working paper, Harvard Business School.
- Lai, R. 2006. Inventory signals. Working paper, Harvard Business School.
- Lai, R. 2008. Is inventory's fiscal year end effect caused by sales timing? A test using a natural experiment from Germany. Working paper, Harvard Business School.
- Lai, G., L.G. Debo, L. Nan. 2008. Market information and financial reporting with inventory signal for publicly traded firms. Working paper, Carnegie Mellon University.
- Lee, H., V. Padmanabhan, S. Whang. 2004. Information distortion in a supply chain: The bullwhip effect. *Mgmt. Sci.* 50(12): 1875-1886.
- Liang, P.J., X. Wen. Accounting measurement basis, market mispricing, and firm investment efficiency. *J. Accounting Res.* 45(1): 155-197.
- Libby, R., P. A. Libby, D. G. Short. 2005. Financial accounting. McGraw-Hill Irwin. 5th Edition.
- Morgenson, G. 1998. Stock options are not a free lunch. *Forbes.* May 18.
- Raman, A. 2006. The impact of inventory management capabilities on retailers' stock-market valuation. Working paper, Harvard Business School.
- Roychowdhury, S. 2006. Earnings management through real activities manipulation. *J. Accounting Econ.* 42(3): 335-370.
- SFAS No. 48. 1981. Revenue recognition when right of return exists. Financial Accounting Standard Board, June.
- SFAS No. 49. 1981. Accounting for product financing arrangements. Financial Accounting Standard Board, June.
- SFAS No. 123. 2004. Share-based payment. Financial Accounting Standard Board, December.
- Sohoni, M., S. Chopra, U. Mohan, M.N. Sendil. 2005. Impact of stair-step incentives and dealer structures on manufacturers' sales variance. Working paper, Northwestern University.

Stein, J. 1989. Efficient capital markets, inefficient firms: A model of myopic corporate behavior. *Quart. J. Econ.* 104(4): 655-670.

Xu, X., J. R. Birge. 2005. Operational decisions, capital structure, and managerial compensation: A newsvendor perspective. Working paper, University of Chicago.

## Appendix A

In this Section, we provide the proofs for the Propositions and Corollaries.

To prove Propositions 1 and 2, we first show the following Lemma.

**Lemma 1** Recall  $\theta = \frac{\beta(p+a(p-c))-(1-\beta)\gamma}{\gamma}$ . Suppose there is no inventory bound (we temporarily take the inventory level  $q_1$  being infinite such that  $q_1$  will not be hit no matter how large the demand  $\xi_1$  is and how much sales  $x$  the manager pads). Then, there is a unique pure-strategy separating equilibrium on channel stuffing and stock price, which follows

$$x^o(\xi_1, q_1) = \begin{cases} 0 & \theta < 0 \\ \theta \xi_1 & \theta \geq 0 \end{cases} \quad (25)$$

and

$$P^o(z, q_1) = A_0 + \frac{pz}{1+\theta} - \gamma \left(1 - \frac{1}{1+\theta}\right) z - cq_1 + v_2 \left(\frac{z}{1+\theta}\right) \quad (26)$$

**Proof of Lemma 1:** We show this Lemma in two steps: (i) We verify that the padding strategy and the stock price provided in Equations (25) and (26) satisfy the equilibrium conditions in Definition 1. (ii) We show that this is the unique separating equilibrium.

(i) First, based on the manager's padding strategy in Equation (25), for a given report with sales  $z$ , the real demand follows  $\frac{z}{1+(\theta)^+}$  (as  $z = \xi_1 + x^o(\xi_1, q_1)$ ). Therefore, the stock price in Equation (26) correctly values the firm by taking the demand as  $\frac{z}{1+(\theta)^+}$  for given report  $z$ .

Second, given the stock price in Equation (26), the manager's payoff function follows (see Equation (2))

$$\begin{aligned}
\pi(x; \xi_1, q_1) &= \beta P^o(x + \xi_1, q_1) + (1 - \beta)v_1(x; \xi_1, q_1) \\
&= \beta \left[ A_0 + \frac{p(x + \xi_1)}{1 + (\theta)^+} - \gamma \left( 1 - \frac{1}{1 + (\theta)^+} \right) (x + \xi_1) - cq_1 + v_2 \left( \frac{x + \xi_1}{1 + (\theta)^+} \right) \right] \\
&\quad + (1 - \beta) [A_0 + p\xi_1 - cq_1 - \gamma x + v_2(\xi_1)] \\
&= A_0 + p\xi_1 - cq_1 + v_2(\xi_1) + \beta \left[ \frac{px}{1 + (\theta)^+} - \frac{\gamma(\theta)^+ x}{1 + (\theta)^+} + \frac{(p - c)ax}{1 + (\theta)^+} \right] - (1 - \beta)\gamma x \\
&\quad - \beta \left[ p \frac{(\theta)^+}{1 + (\theta)^+} \xi_1 + \gamma \frac{(\theta)^+}{1 + (\theta)^+} \xi_1 + (p - c)a \frac{(\theta)^+}{1 + (\theta)^+} \xi_1 \right] \\
&= A_0 + p\xi_1 - cq_1 + v_2(\xi_1) + \left[ \beta(p + (p - c)a) - (1 - \beta)\gamma - \gamma(\theta)^+ \right] \frac{x}{1 + (\theta)^+} \\
&\quad - \beta [p + (p - c)a + \gamma] \frac{(\theta)^+}{1 + (\theta)^+} \xi_1 \\
&= A_0 + p\xi_1 - cq_1 + v_2(\xi_1) - \beta [p + (p - c)a + \gamma] \frac{(\theta)^+}{1 + (\theta)^+} \xi_1
\end{aligned}$$

We observe that the manager's payoff function under this stock price is independent of the padding amount  $x$ . As a result, any amount of sales padding provides the same payoff to the manager and the one specified in Equation (25) is one of those in the sense weakly "maximizing" the manager's payoff under this stock price. (Note that the reason that we do not have a padding strategy that strictly maximizes the manager's payoff is the linearity of the payoff structure. Both of the benefit and the cost are linear in this problem.)

Therefore, the equilibrium conditions in Definition 1 are satisfied with the padding strategy and the stock price provided in Equations (25) and (26).

(ii) To show the uniqueness of the separating equilibrium, we first suppose that a separating equilibrium exists without specifying the detail and then we find that the unique padding strategy and stock price satisfying the equilibrium conditions are those provided in Equations (25) and (26).

Based on the property of separating equilibrium, we know that for any realized demand  $\xi_1$ , there are unique reported sales  $z$  corresponding to it; vice versa. In other words, there is a one-to-one mapping between  $\xi_1$  and  $z$ . Now, assume the investors hold the belief that if the reported sales are  $z$ , then the actual demand will be  $\xi_1^o(z, q_1)$  and thus the padded sales will be  $x^o(z, q_1) = z - \xi_1^o(z, q_1)$ . Consequently, the stock price satisfying Definition 1 will follow

$$\begin{aligned}
P^o(z, q_1) &= \mathbb{E}_{\xi_1} [v_1(z - \xi_1; \xi_1, q_1) | z = \xi_1 + x^o(\xi_1, q_1)] \\
&= A_0 + p \min(q_1, \xi_1) - cq_1 - \gamma x + v_2(\xi_1) \\
&= A_0 + p\xi_1^o(z, q_1) - cq_1 - \gamma(z - \xi_1^o(z, q_1)) + v_2(\xi_1^o(z, q_1))
\end{aligned} \tag{27}$$

The second identity of the above equation is due to the property of separating equilibrium. The third identity is based on the investors' belief and the assumption that  $q_1$  is sufficiently large.



As a result, the first derivative of the manager's payoff function  $\pi(x; \xi_1, q_1)$  with respect to ("w.r.t" thereafter)  $x$  follows:

$$\begin{aligned} \frac{d\pi(x; \xi_1, q_1)}{dx} &= \beta \frac{dP^o(z, q_1)}{dz} \frac{dz}{dx} + (1 - \beta) \frac{dv_1(x; \xi_1, q_1)}{dx} \\ &= \beta \left( \left( p + \gamma + \frac{dv_2(\xi_1)}{d\xi_1} \Big|_{\xi_1 = \xi_1^o(z, q_1)} \right) \frac{d\xi_1^o(z, q_1)}{dz} - \gamma \right) \frac{dz}{dx} - (1 - \beta) \gamma \\ &= \beta \left( (p + \gamma + a(p - c)) \frac{d\xi_1^o(z, q_1)}{dz} - \gamma \right) \frac{dz}{dx} - (1 - \beta) \gamma \end{aligned}$$

Since  $z = \xi_1 + x$ , we have  $\frac{dz}{dx} = 1$ , which leads to

$$\frac{d\pi(x; \xi_1, q_1)}{dx} = \beta (p + a(p - c) + \gamma) \frac{d\xi_1^o(z, q_1)}{dz} - \gamma. \quad (28)$$

Based on Definition 1, in equilibrium, the manager will choose  $x$  to maximize his payoff; i.e., from the first order condition  $\frac{d\pi(x; \xi_1, q_1)}{dx} = 0$ . To satisfy this first order condition, we shall have

$$\frac{d\xi_1^o(z, q_1)}{dz} = \frac{\gamma}{\beta(p + a(p - c) + \gamma)}. \quad (29)$$

By the property of separating equilibrium, there is a one-to-one mapping between  $\xi_1$  and  $z$ . In other words, under a separating equilibrium, the investors can correctly infer the real demand from the reported sales  $z$ ; i.e., the investors' belief shall be  $\xi_1^o(z, q_1) = \xi_1$ . Therefore, we obtain from Equation (29)

$$\frac{d\xi_1}{dz} = \frac{\gamma}{\beta(p + a(p - c) + \gamma)} \quad (30)$$

This differential equation has a general solution

$$\xi_1 = \frac{\gamma}{\beta(p + a(p - c) + \gamma)} z + l$$

where  $l$  is a constant. However, the boundary condition of a separating equilibrium indicates that the manager will not pad any sales at the lower bound  $\xi_1 = 0$  (which we will explain in more detail later on).

Therefore,  $l = 0$  and we obtain a unique solution:

$$\begin{aligned} \xi_1 &= \frac{\gamma}{\beta(p + a(p - c) + \gamma)} (x + \xi_1) \\ \Rightarrow x &= \frac{\beta(p + a(p - c)) - (1 - \beta)\gamma}{\gamma} \xi_1 \end{aligned}$$

Since the model does not allow the manager to pad negative sales,  $x$  must be no less than zero. We end up with the padding strategy as provided in Equation (25)

$$x^o(\xi_1, q_1) = \begin{cases} 0 & \theta < 0 \\ \theta \xi_1 & \theta \geq 0 \end{cases}$$

Given this padding strategy, the stock price is straightforward. The investors can derive the real demand from the reported sales  $z$  by  $\xi_1 = \frac{z}{1+\theta}$  and then the stock price shall follow

$$P^o(z, q_1) = A_0 + \frac{pz}{1+\theta} - \gamma \left(1 - \frac{1}{1+\theta}\right) z - cq_1 + v_2 \left(\frac{z}{1+\theta}\right)$$

This equilibrium is unique because  $x^o(\xi_1, q_1)$  in Equation (25) is the only solution to the first order condition of the manager's payoff function for a separating equilibrium.

We discuss the different situations of this separating equilibrium, when (a)  $\theta > 0$ , (b)  $\theta < 0$  and (c)  $\theta = 0$  in the following.

(a) Since in a separating equilibrium the investors can perfectly infer the actual demand, channel stuffing does not benefit the manager but rather reduces his payoff. Therefore, the manager should not pad any sales if he could. However, when  $\theta > 0$ , the manager is not trustworthy. Imagine that if the investors trust the manager and take the reported sales as the actual demand, the manager would pad as much sales as he could since the marginal gain from the short-term stock selling (i.e.,  $\beta(p + a(p - c))$ ) is higher than the marginal loss (i.e.,  $(1 - \beta)\gamma$ ). Thus, no channel stuffing cannot be an equilibrium for  $\theta > 0$ . In equilibrium, the manager has to pad sales, which effectively acts as a costly signaling tool. The only exception is when  $\xi_1 = 0$ . At this bound, if the manager reports sales  $z = 0$ , the investors will trust him since no one with a positive demand will report zero sales. For  $\xi_1 > 0$ , the manager pads positive sales, which linearly increases in the actual demand  $\xi_1$ .

(b) In contrast, for the case  $\theta < 0$ , if we assume that the investors always take the reported sales as the actual demand, we will have  $\frac{d\pi(x; \xi_1, q_1)}{dx} < 0$  for any  $\xi_1$  and  $x$ . This implies that the manager has no incentive to pad the sales even if the investors always take the reported sales as the actual demand. As a result, when  $\theta < 0$ , the manager pads zero sales for any  $\xi_1$  and the investors trust the manager.

(c) It is special for the case  $\theta = 0$ . In this situation, if we assume that the investors take the reported sales as the actual demand, the first derivative of the manager's payoff function satisfies  $\frac{d\pi(x; \xi_1, q_1)}{dx} = 0$  for any  $\xi_1$  and  $x$ . As a result, the manager indeed is indifferent to pad any amount of sales in such a situation. However, only if the manager's strategy matches the investors' belief, the equilibrium condition in Definition 1 will be satisfied. Therefore, for  $\theta = 0$ , a separating equilibrium can be sustained only at the point where the investors trust the manager and the manager indeed pads zero sales. Without loss of generality, we combine this case with  $\theta > 0$ . ■

**Proof of Proposition 1:** Lemma 1 has shown that if there is no inventory bound, then there is a unique separating equilibrium. Moreover, for the case  $\theta < 0$ , the manager does not pad any sales in the separating equilibrium. However, if the inventory bound is finite, the reported sales  $z$  will be bounded by the inventory level. As a result, the separating equilibrium in Lemma 1 cannot always be sustained. Rather, a hybrid

structure with both separating and pooling equilibria may arise. In the following, we prove this Proposition in two steps: (i) We verify that the padding strategy and stock price provided in Proposition 1 satisfy the equilibrium conditions in Definition 1. (ii) We discuss the problem of the off-equilibrium path.

(i) First, the padding strategy in Equation (5) specifies that the manager does not pad the sales if the real demand  $\xi_1 < \xi_a$  but pushes the sales to the inventory bound for  $\xi_1 \leq \xi_1 < q_1$ . As a result, the manager reports sales  $z = \xi_1$  for  $\xi_1 < \xi_a$  and reports  $z = q_1$  for all  $\xi_1 \geq \xi_a$ . It is easy to see that given this padding strategy, the stock price specified in Equations (6), (7) and (8) satisfies condition (2) in Definition 1; i.e., it provides the exact value of the firm for  $\xi_1 < \xi_a$  and the expected value of the firm for  $\xi_1 \geq \xi_a$  given the report  $z = q_1$ .

Second, we verify that given the stock price in Equations (6), (7) and (8) with the definition of  $\xi_a$ , the padding strategy in Equation (5) satisfies condition (1) in Definition 1. (a) It is directly true for  $\xi_1 \geq q_1$  because all the inventory is used up and the manager can do nothing but report  $z = q_1$ .

(b) For  $0 \leq \xi_1 < q_1$ , the manager has the choices of padding  $x$  from 0 to  $q_1 - \xi_1$ . In particular, if the manager does not push the sales to the inventory bound (i.e.,  $z < q_1$ ), the stock price  $P_a(z, q_1)$  in Equation (7) is the same as the one provided in Lemma 1 (see Equation (26)). Lemma 1 has shown that the manager shall pad zero sales if  $\theta < 0$ . However, if the manager pushes the sales to the inventory bound (i.e.,  $z = q_1$ ), the manager achieves the pooling stock price  $\bar{P}(\xi_a, q_1)$  in Equation (8) which gives the manager a different payoff. Therefore, there are only two rational choices for the manager: to pad zero sales or to push the sales to the inventory bound.

Here, we assume that there is a unique  $\xi_a$  as defined in this Proposition and compare the payoffs corresponding to these two choices for any demand  $0 \leq \xi_1 < q_1$ . We later verify that there does exist such a unique  $\xi_a$  in the following point (c).

Let  $\pi(0; \xi_1, q_1)$  ( $\pi(q_1 - \xi_1; \xi_1, q_1)$ ) indicate to pad zero sales (to push the sales to the inventory bound) (note that if the manager pads zero sales, the stock price is equal to the exact value of the firm).

$$\begin{aligned}
& \pi(0; \xi_1, q_1) - \pi(q_1 - \xi_1; \xi_1, q_1) \tag{31} \\
&= A_0 - cq_1 + p\xi_1 + v_2(\xi_1) - \beta\bar{P}(\xi_a, q_1) - (1 - \beta)(A_0 + p\xi_1 - \gamma(q_1 - \xi_1) + v_2(\xi_1) - cq_1) \\
&= \beta(A_0 + p\xi_1 + v_2(\xi_1) - cq_1) + (1 - \beta)\gamma(q_1 - \xi_1) \\
&\quad - \beta(A_0 + p\xi_a + v_2(\xi_a) - cq_1) - (1 - \beta)\gamma(q_1 - \xi_a) \\
&= (\xi_1 - \xi_a)(\beta(p + a(p - c)) - (1 - \beta)\gamma) \\
&= (\xi_1 - \xi_a)\gamma\theta
\end{aligned}$$

The second identity is due to Equation (9); i.e., the definition of the threshold  $\xi_a$ .

Since  $\theta < 0$ , the manager shall choose not to pad the sales for  $\xi_1 < \xi_a$  and choose to push to the inventory bound for  $\xi_a < \xi_1 < q_1$  (when  $\xi_1 = \xi_a$  the manager is indifferent). Therefore, the padding strategy in Equation (5) satisfies condition (1) in Definition 1, given the above stock price.

(c) We verify that there is a unique  $\xi_a$  as defined in this Proposition. To show this, we first assume that such a  $\xi_a$  exists and then verify its existence and uniqueness. When  $\xi_1 = \xi_a$ , the manager is indifferent between padding zero sales and pushing to the inventory bound. We therefore have  $\pi(0; \xi_a, q_1) = \pi(q_1 - \xi_a; \xi_a, q_1)$ , where

$$\begin{aligned}\pi(0; \xi_a, q_1) &= \beta P_a(z, q_1) + (1 - \beta) v_1(0; \xi_a, q_1) \\ &= A_0 - cq_1 + p\xi_a + v_2(\xi_a)\end{aligned}$$

and

$$\begin{aligned}\pi(q_1 - \xi_a; \xi_a, q_1) &= \beta \bar{P}(\xi_a, q_1) + (1 - \beta) v_1(q_1 - \xi_a; \xi_a, q_1) \\ &= \beta \left( A_0 + \int_{\xi_a}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + v_2(\xi_1)) \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1 - cq_1 \right) \\ &\quad + (1 - \beta) (A_0 + p\xi_a - \gamma(q_1 - \xi_a) + v_2(\xi_a) - cq_1) \\ &= A_0 - cq_1 + \beta \int_{\xi_a}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + v_2(\xi_1)) \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1 \\ &\quad + (1 - \beta) (p\xi_a - \gamma(q_1 - \xi_a) + v_2(\xi_a))\end{aligned}$$

Equating these two equations, we obtain

$$\begin{aligned}(\beta p - (1 - \beta) \gamma) \xi_a + \beta v_2(\xi_a) + (1 - \beta) \gamma q_1 \\ = \beta \int_{\xi_a}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + v_2(\xi_1)) \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1\end{aligned}$$

We substitute  $v_2(\xi_a)$  into the above equation and obtain

$$\begin{aligned}(\beta(p + a(p - c) + \gamma) - \gamma) \xi_a + \beta (\mathbb{E}_{\eta_2} [p \min(\eta_2, k_2)] - ck_2) + (1 - \beta) \gamma q_1 \\ = \beta \int_{\xi_a}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + v_2(\xi_1)) \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1\end{aligned}$$

Organizing the terms, we have

$$\begin{aligned}(\beta(p + a(p - c)) - (1 - \beta)\gamma) \xi_a + (1 - \beta) \gamma q_1 \\ = \beta \int_{\xi_a}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p - c) \xi_1) \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1\end{aligned} \tag{32}$$

Notice that LHS of Equation (32) is a linear and decreasing function of  $\xi_a$  ( $\beta(p + a(p - c)) - (1 - \beta)\gamma < 0$  as  $\theta < 0$ ), while RHS is an increasing function of  $\xi_a$ . To show this, we derive the first derivative of RHS

w.r.t  $\xi_a$  as

$$\begin{aligned}\frac{dRHS}{d\xi_a} &= -\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p-c)\xi_a) \frac{f_1(\xi_a)}{F_1(\xi_a)} + \frac{f_1(\xi_a)}{F_1(\xi_a)} RHS \\ &= \frac{f_1(\xi_a)}{F_1(\xi_a)} [-\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p-c)\xi_a) + RHS]\end{aligned}$$

Because RHS of Equation (32) is the average value of  $p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p-c)\xi_1$  over the interval  $\xi_1 \in [\xi_a, \infty)$  it must be larger than  $\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p-c)\xi_a)$ . This implies that  $\frac{dRHS}{d\xi_a} > 0$ . Therefore, if there exists such a threshold point  $\xi_a$  which makes Equation (32) hold, it is unique. If there is not such a  $\xi_a$  for the whole interval  $[0, q_1]$ , it implies that LHS of Equation (32) is always less than RHS. This is because RHS is always no less than LHS when  $\xi_a = q_1$ , which we show by replacing  $\xi_a$  by  $q_1$  in both LHS and RHS of Equation (32) and organizing them to

$$LHS = \beta(p + a(p-c))q_1, \text{ and } RHS = \beta \int_{q_1}^{\infty} (pq_1 + a(p-c)\xi_1) \frac{f_1(\xi_1)}{F_1(q_1)} d\xi_1$$

It is easy to see that  $RHS \geq LHS$  with the equality only if  $a = 0$ . Therefore, in such a situation, the manager will push the sales up to  $q_1$  for any  $\xi_1 < q_1$ , and thus we set  $\xi_a = 0$ . This completes the proof.

(ii) In the above, we identified the equilibrium structure. From this structure, we see that the reported sales  $z$  in equilibrium have a jump at  $\xi_1 = \xi_a$  from  $\xi_a$  to  $q_1$ . A report with  $z \in [\xi_a, q_1)$  would not appear in the equilibrium. However, the stock price needs to specify this off-equilibrium path. In particular, there are infinite possible stock prices for this off-equilibrium path that can support the characterized equilibrium. For example, if we set  $P^o(z, q_1) = -\infty$  for any  $z \in [\xi_a, q_1)$ , the characterized equilibrium still holds. We take the one that follows the function of the separating equilibrium. The equilibrium holds under such a stock price for the off-equilibrium path; moreover, this stock price would award the exact value of the firm if the manager chose not to pad the sales. ■

**Proof of Corollary 1:** First, suppose  $a = 0$ . Equation (32) is reduced to

$$(\beta p - (1-\beta)\gamma)\xi_a + (1-\beta)\gamma q_1 = \beta \int_{\xi_a}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+) \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1$$

We observe that this equation can hold only at  $\xi_a = q_1$ . Therefore, when  $a = 0$ ,  $\xi_a = q_1$ .

Second, suppose  $a > 0$ . In the proof of Proposition 1 we have shown that when  $a > 0$ , if there is a solution that makes Equation (32) hold, it must be less than  $q_1$ . This completes the proof. ■

**Proof of Proposition 2:** We prove this Proposition in the same way as we did for Proposition 1. (i) We verify that the given padding strategy and stock price satisfy the equilibrium condition. (ii) We discuss the off-equilibrium path.

(i) First, it is easy to see that given the padding strategy in Equation (10), the stock price specified in Equations (11), (12) and (13) satisfies condition (2) of Definition 1. When  $\xi_1 < \xi_b$ , the manager takes the

linear padding strategy. The stock price correctly anticipates the padding amount and assigns the exact value of the firm. When  $\xi_1 \geq \xi_b$ , the manager always pushes the sales to the inventory bound as long as there is inventory. Given a report with  $z = q_1$ , the stock price then assigns the expected value of the firm for  $\xi_1 \geq \xi_b$ .

Second, we verify that given the stock price, the padding strategy in Equation (10) satisfies condition (1) of Definition 1. (a) It is true for  $\xi_1 \geq q_1$  since the manager can do nothing but report  $z = q_1$  with all the inventory used up.

(b) For  $0 \leq \xi_1 < q_1$ , the manager has the choices of padding  $x$  from 0 to  $q_1 - \xi_1$ . In particular, if the manager does not push the sales to the inventory bound (i.e.,  $z < q_1$ ), the stock price  $P_b(z, q_1)$  in Equation (12) is the same as the one provided in Lemma 1 (see Equation (26)). Lemma 1 has shown that the manager shall pad  $\theta\xi_1$  if  $\theta \geq 0$ . Differently, if the manager pushes the sales to the inventory bound (i.e.,  $z = q_1$ ), the manager achieves the pooling stock price  $\bar{P}(\xi_b, q_1)$  in Equation (13) which gives the manager a different payoff. We assume that there is a unique  $\xi_b$  as defined in this Proposition and compare the payoffs corresponding to these two choices for any demand  $0 \leq \xi_1 < q_1$  as follows, where  $\pi(\theta\xi_1; \xi_1, q_1)$  ( $\pi(q_1 - \xi_1; \xi_1, q_1)$ ) indicates to pad  $\theta\xi_1$  sales (to push the sales to the inventory bound).

$$\begin{aligned}
& \pi(\theta\xi_1; \xi_1, q_1) - \pi(q_1 - \xi_1; \xi_1, q_1) & (33) \\
= & A_0 - cq_1 + p\xi_1 + v_2(\xi_1) - \gamma\theta\xi_1 - \beta\bar{P}(\xi_b, q_1) - (1 - \beta)(A_0 + p\xi_1 - \gamma(q_1 - \xi_1) + v_2(\xi_1) - cq_1) \\
= & \beta(A_0 + p\xi_1 + v_2(\xi_1) - cq_1) - \gamma\theta\xi_1 + (1 - \beta)\gamma(q_1 - \xi_1) \\
& - \beta(A_0 + p\xi_b + v_2(\xi_b) - cq_1) - (1 - \beta)\gamma(q_1 - \xi_b) + \gamma\theta\xi_b \\
= & (\xi_1 - \xi_b)(\beta(p + a(p - c)) - (1 - \beta)\gamma - \gamma\theta) \\
= & 0
\end{aligned}$$

The second identity is due to Equation (14); i.e., the definition of the threshold  $\xi_b$ . Therefore, given the stock price, the padding strategy in Equation (10) satisfies condition (1) of Definition 1 in the sense of “weakly maximizing” the manager’s payoff (as the manager is indifferent).

(c) We verify that there is a unique  $\xi_b$ . To show this, we first assume that such a  $\xi_b$  exists and then verify its existence and uniqueness. When  $\xi_1 = \xi_b$ , the manager is indifferent between padding  $\theta\xi_b$  sales and pushing to the inventory bound. We therefore have  $\pi(\theta\xi_b; \xi_b, q_1) = \pi(q_1 - \xi_b, \xi_b, q_1)$ , where

$$\begin{aligned}
\pi(\theta\xi_b; \xi_b, q_1) &= \beta P_b(z, q_1) + (1 - \beta)v_1(\theta\xi_b; \xi_b, q_1) & (34) \\
&= A_0 - cq_1 + p\xi_b - \gamma\theta\xi_b + v_2(\xi_b)
\end{aligned}$$

and

$$\begin{aligned}
\pi(q_1 - \xi_b, \xi_b, q_1) &= \beta \bar{P}(\xi_b, q_1) + (1 - \beta) v_1(q_1 - \xi_b; \xi_b, q_1) \\
&= A_0 - cq_1 + \beta \int_{\xi_b}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + v_2(\xi_1)) \frac{f_1(\xi_1)}{F_1(\xi_b)} d\xi_1 \\
&\quad + (1 - \beta) (p\xi_b - \gamma(q_1 - \xi_b) + v_2(\xi_b))
\end{aligned} \tag{35}$$

Equating these two payoff functions

$$\begin{aligned}
&(\beta p - (1 - \beta)\gamma - \gamma\theta)\xi_b + \beta v_2(\xi_b) + (1 - \beta)\gamma q_1 \\
&= \beta \int_{\xi_b}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + v_2(\xi_1)) \frac{f_1(\xi_1)}{F_1(\xi_b)} d\xi_1
\end{aligned} \tag{36}$$

and rearranging the terms, we obtain the condition function for  $\xi_b$  as

$$(1 - \beta)\gamma q_1 = \beta \int_{\xi_b}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p - c)\xi_1) \frac{f_1(\xi_1)}{F_1(\xi_b)} d\xi_1 \tag{37}$$

Notice that LHS of Equation (37) is independent of  $\xi_b$  and LHS is increasing in  $\xi_b$ . As a result, if there is a  $\xi_b$  such that Equation (37) holds, it is unique. If such a  $\xi_b$  does not exist, it implies that the manager will push the sales up to  $q_1$  for any  $\xi_1 < q_1$  and we have  $\xi_b = 0$ . This completes the proof.

(ii) Similar to the case with  $\theta < 0$ , the reported sales  $z$  in equilibrium also have a jump at  $\xi_1 = \xi_b$  from  $\xi_b$  to  $q_1$ . A report with  $z \in [\xi_b, q_1)$  would not appear in the equilibrium. However, the stock price needs to specify this off-equilibrium path. Similarly, there are infinite possible stock prices for this off-equilibrium path that can support the characterized equilibrium. We take the one that follows the price function of the separating equilibrium. The equilibrium holds under such a stock price for the off-equilibrium path; moreover, this stock price is consistent with the separating equilibrium. ■

**Corollary 2**  $\xi_a$  and  $\xi_b$  both increase in  $q_1$ .

**Proof of Corollary 2:** We show  $\frac{\partial \xi_a}{\partial q_1} > 0$  and  $\frac{\partial \xi_b}{\partial q_1} > 0$  in (i) and (ii), respectively.

(i) Using Implicit Theorem, we take the first derivative of  $\xi_a$  w.r.t.  $q_1$  based on Equation (32):

$$\frac{\partial \xi_a}{\partial q_1} = - \frac{\frac{1}{F(\xi_a)} \beta [-\gamma(F_1(q_1) - F_1(\xi_a)) + p\bar{F}_1(q_1)] - (1 - \beta)\gamma}{\frac{f_1(\xi_a)}{F_1(\xi_a)} [-\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p - c)\xi_a) + G(\xi_a, q_1)] - (\beta(p + a(p - c) + \gamma) - \gamma)} \tag{38}$$

where

$$G(\xi_a, q_1) = \beta \int_{\xi_a}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p - c)\xi_1) \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1.$$

It is easy to see that  $G(\xi_a, q_1) > \beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p - c)\xi_a)$  because  $p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p - c)\xi_1$  increases in  $\xi_1$  and  $G(\xi_a, q_1)$  is the average value of  $p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p - c)\xi_1$  over  $\xi_1 \in [\xi_a, \infty)$ .

Organizing the terms of Equation (38), we obtain

$$\frac{\partial \xi_a}{\partial q_1} = \frac{\frac{1}{\bar{F}_1(\xi_a)} [\gamma \bar{F}_1(\xi_a) - \beta(p + \gamma) \bar{F}_1(q_1)]}{\frac{f_1(\xi_a)}{\bar{F}_1(\xi_a)} [-\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p - c)\xi_a) + G(\xi_a, q_1)] - (\beta(p + a(p - c) + \gamma) - \gamma)} \quad (39)$$

Given that  $\theta < 0$  for this case,  $\beta(p + \gamma) < \beta(p + a(p - c) + \gamma) < \gamma$ . Furthermore, we have shown in Proposition 1 that  $\xi_a < q_1$  and thus  $\bar{F}_1(\xi_a) > \bar{F}_1(q_1)$ . Therefore, the numerator and denominator of Equation (39) are both positive. Consequently,  $\frac{\partial \xi_a}{\partial q_1} > 0$ .

(ii) Similarly, to show  $\frac{\partial \xi_b}{\partial q_1} > 0$ , we can derive from Equation (37) by Implicit Theorem

$$\frac{\partial \xi_b}{\partial q_1} = -\frac{\frac{1}{\bar{F}_1(\xi_b)} \beta [-\gamma(F_1(q_1) - F_1(\xi_b)) + p\bar{F}_1(q_1)] - (1 - \beta)\gamma}{\frac{f_1(\xi_b)}{\bar{F}_1(\xi_b)} [-\beta(p\xi_b - \gamma(q_1 - \xi_b) + a(p - c)\xi_b) + G(\xi_b, q_1)]} \quad (40)$$

Because from Equation (37) we have

$$(1 - \beta)\gamma q_1 = \beta \int_{\xi_b}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p - c)\xi_1) \frac{f_1(\xi_1)}{\bar{F}_1(\xi_b)} d\xi_1$$

we can reorganize the terms of Equation (40) as

$$\begin{aligned} \frac{\partial \xi_b}{\partial q_1} &= -\frac{\frac{\beta}{\bar{F}_1(\xi_b)q_1} \left[ -\gamma q_1 (F_1(q_1) - F_1(\xi_b)) + p q_1 \bar{F}_1(q_1) - \int_{\xi_b}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p - c)\xi_1) \frac{f_1(\xi_1)}{\bar{F}_1(\xi_b)} d\xi_1 \right]}{\frac{f_1(\xi_b)}{\bar{F}_1(\xi_b)} [-\beta(p\xi_b - \gamma(q_1 - \xi_b) + a(p - c)\xi_b) + G(\xi_b, q_1)]} \\ &= \frac{\frac{\beta}{\bar{F}_1(\xi_b)q_1} \left[ \int_{\xi_b}^{q_1} (p\xi_1 + \gamma\xi_1) f_1(\xi_1) d\xi_1 + \int_{\xi_b}^{\infty} a(p - c)\xi_1 f_1(\xi_1) d\xi_1 \right]}{\frac{f_1(\xi_b)}{\bar{F}_1(\xi_b)} [-\beta(p\xi_b - \gamma(q_1 - \xi_b) + a(p - c)\xi_b) + G(\xi_b, q_1)]} \end{aligned} \quad (41)$$

For the same reason, we have  $G(\xi_b, q_1) > \beta(p\xi_b - \gamma(q_1 - \xi_b) + a(p - c)\xi_b)$  and thus the denominator of Equation (41) is positive. It is obvious that the numerator is also positive. Therefore,  $\frac{\partial \xi_b}{\partial q_1} > 0$ . ■

**Corollary 3** *Given inventory  $q_1$ , the slope  $(\theta)^+$  of the first part of  $x^o(\xi_1, q_1)$  increases in  $p$ ,  $\beta$  and  $a$ , but decrease in  $c$  and  $\gamma$ ; the thresholds  $(\xi_a$  and  $\xi_b)$  decrease in  $p$ ,  $\beta$  and  $a$ , but increase in  $c$  and  $\gamma$ .*

**Proof of Corollary 3:** In the following, we first show the properties of the slope  $(\theta)^+$  w.r.t  $p$ ,  $\beta, a$ ,  $c$  and  $\gamma$  in (i); and then show the properties of the thresholds  $(\xi_a$  and  $\xi_b)$  in (ii).

(i) Given that  $\theta = \frac{\beta(p+a(p-c))-(1-\beta)\gamma}{\gamma}$ , it is easy to see  $\theta$  increases in  $p$ ,  $\beta$  and  $a$ , but decreases in  $c$  and  $\gamma$ . Therefore, those properties in Corollary 3 w.r.t  $\theta$  directly hold.

(ii) To show those properties of the thresholds  $(\xi_a$  and  $\xi_b)$ , we take the partial derivatives of  $\xi_a$  w.r.t.  $p, \beta, a, c$  and  $\gamma$  in sequence in (ii.a) and those of  $\xi_b$  in (ii.b).

(ii.a) Using Implicit Theorem, we obtain from Equation (32) that

$$\frac{\partial \xi_a}{\partial p} = -\frac{\beta \int_{\xi_a}^{\infty} (\min(\xi_1, q_1) + a\xi_1) \frac{f_1(\xi_1)}{\bar{F}_1(\xi_a)} d\xi_1 - \beta(1 + a)\xi_a}{\frac{f_1(\xi_a)}{\bar{F}_1(\xi_a)} [-\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p - c)\xi_a) + G(\xi_a, q_1)] - (\beta(p + a(p - c) + \gamma) - \gamma)} \quad (42)$$



Since we have shown  $\xi_a < q_1$  in Proposition 1, it is easy to see that  $\int_{\xi_a}^{\infty} (\min(\xi_1, q_1) + a\xi_1) \frac{f(\xi_1)}{F(\xi_a)} d\xi_1 > (1+a)\xi_a$ . Therefore, the numerator of Equation (42) is positive. Furthermore, the denominator is also positive as we have shown in Corollary 2. Therefore,  $\frac{\partial \xi_a}{\partial p} < 0$ .

Similarly, we can obtain

$$\begin{aligned} \frac{\partial \xi_a}{\partial \beta} &= -\frac{\int_{\xi_a}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p-c)\xi_1) \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1 - (p\xi_a - \gamma(q_1 - \xi_a) + a(p-c)\xi_a)}{\frac{f_1(\xi_a)}{F_1(\xi_a)} [-\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p-c)\xi_a) + G(\xi_a, q_1)] - (\beta(p+a(p-c)+\gamma) - \gamma)} < 0 \\ \frac{\partial \xi_a}{\partial a} &= -\frac{\beta \int_{\xi_a}^{\infty} (p-c)\xi_1 \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1 - \beta(p-c)\xi_a}{\frac{f_1(\xi_a)}{F_1(\xi_a)} [-\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p-c)\xi_a) + G(\xi_a, q_1)] - (\beta(p+a(p-c)+\gamma) - \gamma)} < 0 \\ \frac{\partial \xi_a}{\partial c} &= -\frac{-\beta a \int_{\xi_a}^{\infty} \xi_1 \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1 + \beta a \xi_a}{\frac{f_1(\xi_a)}{F_1(\xi_a)} [-\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p-c)\xi_a) + G(\xi_a, q_1)] - (\beta(p+a(p-c)+\gamma) - \gamma)} > 0 \\ \frac{\partial \xi_a}{\partial \gamma} &= -\frac{-\beta \int_{\xi_a}^{\infty} (q_1 - \xi_1)^+ \frac{f_1(\xi_1)}{F_1(\xi_a)} d\xi_1 - (1-\beta)(q_1 - \xi_a)}{\frac{f_1(\xi_a)}{F_1(\xi_a)} [-\beta(p\xi_a - \gamma(q_1 - \xi_a) + a(p-c)\xi_a) + G(\xi_a, q_1)] - (\beta(p+a(p-c)+\gamma) - \gamma)} > 0 \end{aligned}$$

(ii.b) We obtain the partial derivatives of  $\xi_b$  w.r.t.  $p, \beta, a, c$  and  $\gamma$  in sequence and for the same reasons we can obtain their signs as follows:

$$\begin{aligned} \frac{\partial \xi_b}{\partial p} &= -\frac{\beta \int_{\xi_b}^{\infty} (\min(\xi_1, q_1) + a\xi_1) \frac{f_1(\xi_1)}{F_1(\xi_b)} d\xi_1}{\frac{f_1(\xi_b)}{F_1(\xi_b)} [-\beta(p\xi_b - \gamma(q_1 - \xi_b) + a(p-c)\xi_b) + G(\xi_b, q_1)]} < 0 \\ \frac{\partial \xi_b}{\partial \beta} &= -\frac{\int_{\xi_b}^{\infty} (p \min(\xi_1, q_1) - \gamma(q_1 - \xi_1)^+ + a(p-c)\xi_1) \frac{f_1(\xi_1)}{F_1(\xi_b)} d\xi_1 + \gamma q_1}{\frac{f_1(\xi_b)}{F_1(\xi_b)} [-\beta(p\xi_b - \gamma(q_1 - \xi_b) + a(p-c)\xi_b) + G(\xi_b, q_1)]} < 0 \\ \frac{\partial \xi_b}{\partial a} &= -\frac{\beta \int_{\xi_b}^{\infty} (p-c)\xi_1 \frac{f_1(\xi_1)}{F_1(\xi_b)} d\xi_1}{\frac{f_1(\xi_b)}{F_1(\xi_b)} [-\beta(p\xi_b - \gamma(q_1 - \xi_b) + a(p-c)\xi_b) + G(\xi_b, q_1)]} < 0 \\ \frac{\partial \xi_b}{\partial c} &= -\frac{-\beta a \int_{\xi_b}^{\infty} \xi_1 \frac{f_1(\xi_1)}{F_1(\xi_b)} d\xi_1}{\frac{f_1(\xi_b)}{F_1(\xi_b)} [-\beta(p\xi_b - \gamma(q_1 - \xi_b) + a(p-c)\xi_b) + G(\xi_b, q_1)]} > 0 \\ \frac{\partial \xi_b}{\partial \gamma} &= -\frac{-\beta \int_{\xi_b}^{\infty} (q_1 - \xi_1)^+ \frac{f_1(\xi_1)}{F_1(\xi_b)} d\xi_1 - (1-\beta)q_1}{\frac{f_1(\xi_b)}{F_1(\xi_b)} [-\beta(p\xi_b - \gamma(q_1 - \xi_b) + a(p-c)\xi_b) + G(\xi_b, q_1)]} > 0 \end{aligned}$$

■

**Proof of Proposition 3:** This proposition directly follows Corollary 3. ■

**Proposition 5** *When the marginal incentive reduces to zero, there is no upward jump in the stock price;  $\lim_{\theta \uparrow 0} P_a(q_1, q_1) = \bar{P}(\xi_a, q_1)$ .*

**Proof of Proposition 5:** This Proposition can be shown based on the proofs of Propositions 1 and 2.

(i) Recall the proof of Proposition 1. To find the pooling equilibrium, we equate the manager's two payoff functions with no sales padding and with pushing to the inventory bound. We see from Equation (31) that

for the case  $\theta < 0$ , in equilibrium, the boundary stock price at  $z = q_1$  must be strictly larger than those without sales padding and thus there is an upward jump at  $z = q_1$ . From Equation (31), we can see that this upward jump decreases in  $\theta$  and it becomes zero when  $\theta = 0$ .

(ii) Recall the proof of Proposition 2. For the case  $\theta \geq 0$ , the stock price at the bound is always smooth (see Equation (33)). This completes the proof. ■

**Proof of Proposition 4:** We show this Proposition in two steps: (i) We verify that the padding strategy and stock price provided in this Proposition satisfy the equilibrium conditions of Definition 1. (ii) We show the uniqueness of the equilibrium.

(i) First, given the padding strategy in Equation (18), as long as the reported sales  $z$  is monotonically increasing in  $\xi_1$  for  $0 \leq \xi_1 < \xi_c$  (i.e., there is a unique solution  $\xi_{1,c}^o(z, q_1)$  solving:  $\xi_1 + \left(\int_0^{\xi_1} \theta_c(t, q_1) dt\right)^+ = z$  and thus a one-to-one mapping between  $\xi_1$  and  $z$ ), the stock price provided in Equations (19), (20) and (21) satisfies condition (2) of Definition 1. This stock price provides the exact value of the firm for  $0 \leq \xi_1 < \xi_c$ , and the expected value of the firm for  $\xi_1 \geq \xi_c$ . Therefore, we only need to verify that  $z$  is monotone in  $\xi_1$  for  $0 \leq \xi_1 < \xi_c$ . To show this, we take the first derivative of  $z$  w.r.t  $\xi_1$

$$\frac{dz}{d\xi_1} = 1 + \theta_c(\xi_1, q_1)$$

Recall

$$\theta_c(\xi_1, q_1) = \begin{cases} \theta_c^I(\xi_1, q_1), & 0 \leq \xi_1 < \bar{\xi}(q_1), \\ \theta_c^{II}, & \bar{\xi}(q_1) \leq \xi_1 < q_1, \\ \theta, & \xi_1 \geq q_1, \end{cases}$$

with

$$\begin{aligned} \theta_c^I(\xi_1, q_1) &= \frac{\beta(p + ap - (1+a)p\bar{F}_2(q_1 - (1+a)\xi_1) + h) - (1-\beta)\gamma}{\gamma} \text{ and} \\ \theta_c^{II} &= \frac{\beta(p + a(p-c) + h - c) - (1-\beta)\gamma}{\gamma} \end{aligned}$$

Therefore, we have

$$\frac{dz}{d\xi_1} = \begin{cases} \frac{\beta(p + ap - (1+a)p\bar{F}_2(q_1 - (1+a)\xi_1) + h) + \beta\gamma}{\gamma}, & 0 \leq \xi_1 < \min(\xi_c, \bar{\xi}(q_1)), \\ \frac{\beta(p + a(p-c) + h - c) + \beta\gamma}{\gamma}, & \min(\xi_c, \bar{\xi}(q_1)) \leq \xi_1 < \xi_c. \end{cases}$$

It is easy to see that  $\frac{dz}{d\xi_1} > 0$ .

Second, given the stock price, we verify that the padding strategy satisfies condition (1) of Definition 1. We first assume that there is a unique  $\xi_c$  satisfying the conditions provided in the Proposition, based on which we show that the padding strategy satisfies the equilibrium condition given the stock price (in the following (a,b,c)). Then, we prove that a unique  $\xi_c$  does exist (in the following (d)).

(a) It is obvious for  $\xi_1 \geq q_1$  since the manager can do nothing but report  $z = q_1$ .

(b) For  $0 \leq \xi_1 < \xi_c$ , the manager has the choices of padding  $x$  from 0 to  $q_1 - \xi_1$ . Given the stock price  $P_c(z, q_1)$  in Equation (20), the manager's payoff function follows

$$\pi(x; \xi_1, q_1) = \beta P_c(x + \xi_1, q_1) + (1 - \beta)v_1(x; \xi_1, q_1)$$

Recall

$$v_1(x; \xi_1, q_1) = A_0 + p\xi_1 - cq_1 - \gamma x - h(q_1 - \xi_1) + v_2(\xi_1, q_1)$$

for  $0 \leq x \leq (q_1 - \xi_1)^+$ . Recall

$$v_2(\xi_1, q_1) = \begin{cases} pa\xi_1 + \mathbb{E}_{\eta_2} [p \min(\eta_2, q_1 - (1+a)\xi_1)] & q_2^* = 0 \\ (p-c)a\xi_1 + c(q_1 - \xi_1)^+ + \mathbb{E}_{\eta_2} [p \min(\eta_2, k_2)] - ck_2 & q_2^* > 0 \end{cases}$$

where  $q_2^* = \max\{k_2 + a\xi_1 - (q_1 - \xi_1)^+, 0\}$  is the optimal second-period inventory investment and  $k_2 = F_2^{-1}\left(\frac{p-c}{p}\right)$ , and

$$\frac{dv_2(\xi_1, q_1)}{d\xi_1} = \begin{cases} ap - (1+a)p\bar{F}_2(q_1 - (1+a)\xi_1) & 0 \leq \xi_1 < \bar{\xi}(q_1), \\ a(p-c) - c & \bar{\xi}(q_1) \leq \xi_1 < q_1, \end{cases}$$

(see Equations (15) and (16)).

We take the first derivative of  $\pi(x; \xi_1, q_1)$  w.r.t  $x$  as (remind  $\frac{dz}{dx} = 1$  since  $z = x + \xi_1$ )

$$\begin{aligned} \frac{d\pi(x; \xi_1, q_1)}{dx} &= \beta \frac{dP_c(z, q_1)}{dz} + (1 - \beta) \frac{dv_1(x; q_1, \xi_1)}{dx} \\ &= \beta \left[ \frac{p}{1 + \theta_c(\xi_1, q_1)} - \gamma \left( 1 - \frac{1}{1 + \theta_c(\xi_1, q_1)} \right) + \frac{h}{1 + \theta_c(\xi_1, q_1)} + \frac{\frac{dv_2(\xi_1, q_1)}{d\xi_1}}{1 + \theta_c(\xi_1, q_1)} \right] \\ &\quad - (1 - \beta)\gamma \\ &= \beta \left[ \frac{p - \gamma\theta_c(\xi_1, q_1) + h + \frac{dv_2(\xi_1, q_1)}{d\xi_1}}{1 + \theta_c(\xi_1, q_1)} \right] - (1 - \beta)\gamma \end{aligned}$$

Plugging  $\theta_c(\xi_1, q_1)$  and  $\frac{dv_2(\xi_1, q_1)}{d\xi_1}$  into the above equation, we obtain

$$\begin{aligned} &\frac{d\pi(x; \xi_1, q_1)}{dx} \\ &= \begin{cases} \beta \frac{p - \gamma \left[ \frac{\beta(p+a p - (1+a)p\bar{F}_2(q_1 - (1+a)\xi_1) + h) - (1-\beta)\gamma}{\gamma} \right] + h + a p - (1+a)p\bar{F}_2(q_1 - (1+a)\xi_1)}{1 + \left[ \frac{\beta(p+a p - (1+a)p\bar{F}_2(q_1 - (1+a)\xi_1) + h) - (1-\beta)\gamma}{\gamma} \right]} - (1 - \beta)\gamma & 0 \leq \xi_1 < \bar{\xi}(q_1), \\ \beta \left[ \frac{p - \gamma \left[ \frac{\beta(p+a(p-c) + h - c) - (1-\beta)\gamma}{\gamma} \right] + h + a(p-c) - c}{1 + \frac{\beta(p+a(p-c) + h - c) - (1-\beta)\gamma}{\gamma}} \right] - (1 - \beta)\gamma & \bar{\xi}(q_1) \leq \xi_1 < q_1, \end{cases} \\ &= 0 \end{aligned}$$

In other words, given this stock price, the manager's payoff function is independent of  $x$ . Therefore, the given padding strategy  $\left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+$  satisfies condition (1) of Definition 1 for  $0 \leq \xi_1 < \xi_c$ .

(c) For  $\xi_c \leq \xi_1 < q_1$ , the manager can choose to follow the padding strategy  $\left(\int_0^{\xi_1} \theta_c(t, q_1) dt\right)^+$  or to push the sales to the inventory bound. We compare the two payoff functions as

$$\begin{aligned}
& \pi \left( \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+ ; \xi_1, q_1 \right) - \pi(q_1 - \xi_1; \xi_1, q_1) \\
= & A_0 - cq_1 + p\xi_1 + v_2(\xi_1, q_1) - \gamma \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+ - h(q_1 - \xi_1) \\
& - \beta \bar{P}(\xi_c, q_1) - (1 - \beta)(A_0 - cq_1 + p\xi_1 + v_2(\xi_1, q_1) - \gamma(q_1 - \xi_1) - h(q_1 - \xi_1)) \\
= & \beta(p\xi_1 + v_2(\xi_1, q_1) - h(q_1 - \xi_1)) - \gamma \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+ \\
& - \beta \left( p\xi_c + v_2(\xi_c, q_1) - \gamma \left( \int_0^{\xi_c} \theta_c(t, q_1) dt \right)^+ - h(q_1 - \xi_c) \right) + (1 - \beta)\gamma(q_1 - \xi_1) \\
& - (1 - \beta)\gamma(q_1 - \xi_c) + (1 - \beta)\gamma \left( \int_0^{\xi_c} \theta_c(t, q_1) dt \right)^+ \\
= & \beta(p(\xi_1 - \xi_c) + v_2(\xi_1, q_1) - v_2(\xi_c, q_1) + h(\xi_1 - \xi_c)) + \gamma \left( \int_0^{\xi_c} \theta_c(t, q_1) dt \right)^+ - \gamma \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+ \\
& - (1 - \beta)\gamma(\xi_1 - \xi_c)
\end{aligned}$$

The second identity is due to Equation (22); i.e., the definition of the threshold  $\xi_c$ . We take the derivative of the above difference w.r.t  $\xi_1$

$$\begin{aligned}
& \frac{d \left[ \pi \left( \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+ ; \xi_1, q_1 \right) - \pi(q_1 - \xi_1; \xi_1, q_1) \right]}{d\xi_1} \\
= & \beta \left( p + h + \frac{dv_2(\xi_1, q_1)}{d\xi_1} \right) - \gamma \frac{d \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+}{d\xi_1} - (1 - \beta)\gamma \\
= & \begin{cases} \theta_c(\xi_1, q_1)\gamma \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+ = 0, \\ 0 \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+ \geq 0. \end{cases}
\end{aligned}$$

The second identity is obtained by substituting  $\frac{dv_2(\xi_1, q_1)}{d\xi_1}$  and  $\theta_c(\xi_1, q_1)$  into the equation. Notice that if  $\left(\int_0^{\xi_1} \theta_c(t, q_1) dt\right)^+ = 0$ , it implies that  $\theta_c(\xi_1, q_1) < 0$ ; otherwise,  $\left(\int_0^{\xi_1} \theta_c(t, q_1) dt\right)^+$  cannot be zero since  $\theta_c(\xi_1, q_1)$  is a decreasing function in  $\xi_1$ . Therefore, we see that under this stock price: if  $\xi_c$  satisfies  $\left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+ = 0$ , then the manager strictly prefers to push the sales to the inventory bound for  $\xi_1 > \xi_c$  and follow the padding strategy  $\left(\int_0^{\xi_1} \theta_c(t, q_1) dt\right)^+$  for  $0 \leq \xi_1 < \xi_c$ ; if  $\xi_c$  satisfies  $\left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+ \geq 0$ , then the manager is indifferent to following the padding strategy  $\left(\int_0^{\xi_1} \theta_c(t, q_1) dt\right)^+$  or pushing to the inventory bound. As a result, given this stock price, the padding strategy specified in Equation (18) satisfies condition (1) of Definition 1.

(d) We verify that there is a unique  $\xi_c$  satisfying the condition provided in this Proposition. Assume

such a  $\xi_c$  exists. Then, it shall satisfy the condition  $\pi\left(\left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+; \xi_c, q_1\right) = \pi(q_1 - \xi_c; \xi_c, q_1)$ ; i.e., Equation (22):

$$\beta\bar{P}(\xi_c, q_1) - (1 - \beta)\gamma(q_1 - \xi_c) = \beta P_c\left(\xi_c + \left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+, q_1\right) - (1 - \beta)\gamma\left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+$$

Organizing the terms, we have

$$\beta P_c\left(\xi_c + \left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+, q_1\right) - (1 - \beta)\gamma\left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+ + (1 - \beta)\gamma(q_1 - \xi_c) = \beta\bar{P}(\xi_c, q_1)$$

Note that for the part of the separating equilibrium, the stock price  $P_c\left(\xi_c + \left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+, q_1\right)$  equals the exact value of the firm.

We take the first derivative of both LHS and RHS of the above equation w.r.t  $\xi_c$ :

$$\begin{aligned} \frac{dLHS}{d\xi_c} &= \beta \left[ p - (\gamma - h) \frac{d\left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+}{d\xi_c} + \frac{dv_2(\xi_c, q_1)}{d\xi_c} \right] - (1 - \beta)\gamma \frac{d\left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+}{d\xi_c} - (1 - \beta)\gamma \\ &= \begin{cases} \theta_c(\xi_c, q_1)\gamma \left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+ = 0, \\ 0 \left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+ \geq 0. \end{cases} \end{aligned}$$

and

$$\frac{dRHS}{d\xi_c} = \frac{f_1(\xi_c)}{F_1(\xi_c)} \left[ \begin{aligned} & -\beta(p\xi_c - (\gamma + h)(q_1 - \xi_c) + v_2(\xi_c, q_1)) \\ & + \beta \int_{\xi_c}^{\infty} (p \min(\xi_1, q_1) - (\gamma + h)(q_1 - \xi_1)^+ + v_2(\xi_1, q_1)) \frac{f_1(\xi_1)}{F_1(\xi_c)} d\xi_1 \end{aligned} \right]$$

As discussed in the above, we have that  $\frac{dLHS}{d\xi_c} < 0$  if  $\left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+ = 0$  and  $\frac{dLHS}{d\xi_c} = 0$  if  $\left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+ \geq 0$ . In contrast, we always have  $\frac{dRHS}{d\xi_c} > 0$ . This indicates that the derivative of RHS always dominates that of LHS; moreover, both of them are monotone. Therefore, if there is a  $\xi_c$  under which the above equation holds, such a  $\xi_c$  is unique; if such a  $\xi_c$  does not exist, we set  $\xi_c = 0$ .

Consequently, the padding strategy and stock price provided in the Proposition satisfy the equilibrium conditions in Definition 1.

(ii) To show the uniqueness of the equilibrium, we only need to show that the part of the separating equilibrium is unique because the part of the pooling equilibrium is constructed based on the separating equilibrium. To show this, suppose a separating equilibrium exists and the investors hold the belief that: if the reported inventory level is  $q_1$  and the reported sales are  $z$ , then the real demand is  $\xi_1^o(z, q_1)$ . Then, the stock price follows

$$P^o(z, q_1) = A_0 + p\xi_1^o(z, q_1) - cq_1 - \gamma(z - \xi_1^o(z, q_1)) - h(q_1 - \xi_1^o(z, q_1)) + v_2(\xi_1^o(z, q_1), q_1).$$

We take the first derivative of the manager's payoff function w.r.t  $x$  (remind  $\frac{dz}{dx} = 1$  since  $z = x + \xi_1$ )

$$\begin{aligned}\frac{d\pi(x; \xi_1, q_1)}{dx} &= \beta \frac{dP^o(z, q_1)}{dz} \frac{dz}{dx} + (1 - \beta) \frac{dv_1(x; q_1, \xi_1)}{dx} \\ &= \beta \frac{dP^o(z, q_1)}{dz} + (1 - \beta) \frac{dv_1(x; q_1, \xi_1)}{dx}\end{aligned}$$

For the stock price, we have

$$\begin{aligned}\frac{dP^o(z, q_1)}{dz} &= \begin{cases} (p + \gamma + h) \frac{d\xi_1^o(z, q_1)}{dz} - \gamma + pa \frac{d\xi_1^o(z, q_1)}{dz} - p(1 + a)\bar{F}_2(q_1 - (1 + a)\xi_1^o(z, q_1)) \frac{d\xi_1^o(z, q_1)}{dz} & q_2^o = 0 \\ (p + \gamma + h) \frac{d\xi_1^o(z, q_1)}{dz} - \gamma + (p - c)a \frac{d\xi_1^o(z, q_1)}{dz} - c \frac{d\xi_1^o(z, q_1)}{dz} & q_2^o > 0 \end{cases} \\ &= \begin{cases} -\gamma + [p + \gamma + h + pa - p(1 + a)\bar{F}_2(q_1 - (1 + a)\xi_1^o(z, q_1))] \frac{d\xi_1^o(z, q_1)}{dz} & q_2^o = 0 \\ -\gamma + [p + \gamma + h + (p - c)a - c] \frac{d\xi_1^o(z, q_1)}{dz} & q_2^o > 0 \end{cases}\end{aligned}$$

where  $q_2^o$  denotes the optimal second-period inventory decision under the investors' belief. For the real firm value, we have

$$\frac{dv_1(x; q_1, \xi_1)}{dx} = -\gamma.$$

Therefore,

$$\frac{d\pi(x; \xi_1, q_1)}{dx} = \begin{cases} -\gamma + \beta [p + \gamma + h + pa - p(1 + a)\bar{F}_2(q_1 - (1 + a)\xi_1^o(z, q_1))] \frac{d\xi_1^o(z, q_1)}{dz} & q_2^o = 0 \\ -\gamma + \beta [p + \gamma + h + (p - c)a - c] \frac{d\xi_1^o(z, q_1)}{dz} & q_2^o > 0 \end{cases}$$

By the property of separating equilibrium, we have  $\xi_1^o(z, q_1) = \xi_1$ , which leads to

$$\frac{d\pi(x; \xi_1, q_1)}{dx} = \begin{cases} -\gamma + \beta [p + \gamma + h + pa - p(1 + a)\bar{F}_2(q_1 - (1 + a)\xi_1)] \frac{d\xi_1}{dz} & q_2^* = 0 \\ -\gamma + \beta [p + \gamma + h + (p - c)a - c] \frac{d\xi_1}{dz} & q_2^* > 0 \end{cases}$$

Recall the optimal inventory investment  $q_2^* = \max(k_2 + a\xi_1 - (q_1 - \xi_1)^+, 0)$  where  $k_2 = F_2^{-1}\left(\frac{p-c}{p}\right)$ . Consequently, we obtain the first order condition as

$$\begin{aligned}\frac{d\xi_1}{dz} &= \begin{cases} \frac{\gamma}{\beta[p + \gamma + h + pa - p(1 + a)\bar{F}_2(q_1 - (1 + a)\xi_1)]} & \xi_1 \leq \bar{\xi}(q_1) \\ \frac{\gamma}{\beta[p + a(p - c) + \gamma + h - c]} & \xi_1 > \bar{\xi}(q_1) \end{cases} \\ \Rightarrow \frac{dz}{d\xi_1} &= \begin{cases} \frac{\beta[p(1 + a)\bar{F}_2(q_1 - (1 + a)\xi_1) + \gamma + h]}{\gamma} & \xi_1 \leq \bar{\xi}(q_1) \\ \frac{\beta[p + a(p - c) + \gamma + h - c]}{\gamma} & \xi_1 > \bar{\xi}(q_1) \end{cases} \\ \Rightarrow \frac{dx}{d\xi_1} &= \begin{cases} \frac{\beta(p + ap - (1 + a)p\bar{F}_2(q_1 - (1 + a)\xi_1) + h) - (1 - \beta)\gamma}{\gamma} & \xi_1 \leq \bar{\xi}(q_1) \\ \frac{\beta(p + a(p - c) + h - c) - (1 - \beta)\gamma}{\gamma} & \xi_1 > \bar{\xi}(q_1) \end{cases}\end{aligned}$$

where  $\bar{\xi}(q_1) = \frac{q_1 - k_2}{1 + a}$ .

Based on the boundary condition of the separating equilibrium at  $\xi_1 = 0$ , the above ODE uniquely determines the separating equilibrium. Furthermore,  $x^o$  is always nonnegative by our assumption. As a result, the padding strategy in the separating equilibrium satisfies

$$x^o(\xi_1, q_1) = \left( \int_0^{\xi_1} \theta_c(t, q_1) dt \right)^+$$

It matches Equation (18). This completes the proof.

Note that in this Proposition there is also a jump of the reported sales  $z$  at  $\xi_1 = \xi_c$  from  $\xi_c + \left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+$  to  $q_1$ . A report with  $z \in \left[\xi_c + \left(\int_0^{\xi_c} \theta_c(t, q_1) dt\right)^+, q_1\right)$  will not appear in equilibrium. The stock price however needs to specify this off-equilibrium. We take the stock price following the price function in the separating equilibrium for this off-equilibrium path. ■

## Appendix B

In this section, we provide the derivation of the manager's expected payoff function w.r.t the first period inventory decision  $q_1$  and the first derivative.

**Derivation of Equation (23) and (24):** We show the derivation for the case where inventory can be carried over as an example. The procedure is similar for the case with no inventory carryover.

When the manager makes the first period inventory decision  $q_1$ , the manager takes the subgame into consideration. For the part of the separating equilibrium (i.e.,  $\xi_1 < \xi_c$ ), the investors can perfectly infer the real demand, and thus the stock price equals the real value of the firm. For the part of pooling equilibrium (i.e.,  $\xi_1 \geq \xi_c$ ), the investors provide the stock price equal to the expected value of the firm over the region  $\xi_1 \geq \xi_c$ . Therefore, the expected payoff w.r.t  $q_1$  follows

$$\begin{aligned}
\mathbb{E}_{\xi_1} [\pi(x; \xi_1, q_1)] &= \mathbb{E}_{\xi_1} [\beta P^o(\xi_1 + x, q_1) + (1 - \beta) v_1(x; q_1, \xi_1)] \\
&= A_0 - cq_1 + \int_0^{\xi_c} [p \min(\xi_1, q_1) - h(q_1 - \xi_1)^+ + v_2(\xi_1, q_1) - \gamma x^o(\xi_1, q_1)] f_1(\xi_1) d\xi_1 \\
&\quad + (1 - \beta) \int_{\xi_c}^{q_1} [p \min(\xi_1, q_1) - h(q_1 - \xi_1)^+ + v_2(\xi_1, q_1) - \gamma x^o(\xi_1, q_1)] f_1(\xi_1) d\xi_1 \\
&\quad + (1 - \beta) \int_{q_1}^{\infty} [p \min(\xi_1, q_1) - h(q_1 - \xi_1)^+ + v_2(\xi_1, q_1)] f_1(\xi_1) d\xi_1 \\
&\quad + \beta \int_{\xi_c}^{\infty} \int_{\xi_c}^{\infty} [p \min(\xi_1, q_1) - h(q_1 - \xi_1)^+ + v_2(\xi_1, q_1) - \gamma x^o(\xi_1, q_1)] \frac{f_1(\xi_1)}{F_1(\xi_c)} d\xi_1 f_1(\xi_1) d\xi_1 \\
&= A_0 - cq_1 + \mathbb{E}_{\xi_1} [p \min(\xi_1, q_1) - h(q_1 - \xi_1)^+ + v_2(\xi_1, q_1)] - \gamma \mathbb{E}_{\xi_1 \in [0, q_1]} [x^o(\xi_1, q_1)]
\end{aligned}$$

This leads to Equation (24).

Based on Proposition 1 and 2, we can determine the first derivative of Equation (23) with respect to  $q_1$  for the case without inventory carryover: If  $\theta < 0$ ,

$$\underbrace{-c + p\bar{F}_1(q_1)}_{\text{Classical Newsvendor}} + \gamma \underbrace{\left[ (q_1 - \xi_a) f_1(\xi_a) \frac{d\xi_a}{dq_1} - (F_1(q_1) - F_1(\xi_a)) \right]}_{\text{Stock Market Pressure}} = 0 \tag{43}$$

and if  $\theta \geq 0$ ,

$$\underbrace{-c + p\bar{F}_1(q_1)}_{\text{Classical Newsvendor}} + \underbrace{\gamma \left[ (q_1 - (1 + \theta)\xi_b) f_1(\xi_b) \frac{d\xi_b}{dq_1} - (F_1(q_1) - F_1(\xi_b)) \right]}_{\text{Stock Market Pressure}} = 0 \quad (44)$$

**Derivation of Equations (43) and (44):** (i) Without inventory carryover, we have the manager's decision model on inventory  $q_1$  for  $\theta < 0$  as

$$\mathbb{E}_{\xi_1} [\pi(x; \xi_1, q_1)] = A_0 - cq_1 + \int_0^\infty p \min(\xi_1, q_1) f_1(\xi_1) d\xi_1 - \gamma \int_{\xi_a}^{q_1} (q_1 - \xi_1) f_1(\xi_1) d\xi_1$$

Taking the first derivative w.r.t  $q_1$ , we obtain

$$-c + p\bar{F}_1(q_1) + \gamma \left[ (q_1 - \xi_a) f_1(\xi_a) \frac{d\xi_a}{dq_1} - (F_1(q_1) - F_1(\xi_a)) \right] = 0$$

(ii) For  $\theta \geq 0$ , we have

$$\mathbb{E}_{\xi_1} [\pi(x; \xi_1, q_1)] = A_0 - cq_1 + \int_0^\infty p \min(\xi_1, q_1) f_1(\xi_1) d\xi_1 - \gamma \int_0^{\xi_b} \theta \xi_1 f_1(\xi_1) d\xi_1 - \gamma \int_{\xi_b}^{q_1} (q_1 - \xi_1) f_1(\xi_1) d\xi_1$$

Taking the first derivative w.r.t  $q_1$ , we obtain

$$-c + p\bar{F}_1(q_1) + \gamma \left[ (q_1 - (1 + \theta)\xi_b) f_1(\xi_b) \frac{d\xi_b}{dq_1} - (F_1(q_1) - F_1(\xi_b)) \right] = 0$$

Equations (43) and (44) directly explain the inventory incentive for the case without inventory carryover. We have shown that both  $\xi_a$  and  $\xi_b$  increase in  $q_1$ . The derivatives  $d\xi_a/dq_1$  and  $d\xi_b/dq_1$  are positive which play the role on over-investment compared to the classical newsvendor solution.

The inventory decision is more complicated when the leftover inventory can be carried over to the next period. When determining the first period inventory level, the manager needs to consider also the marginal value of the inventory for the second period use. As a result, the problem must be divided into two cases: (a)  $q_1^* - k_2 \leq 0$  and (b)  $q_1^* - k_2 > 0$ . In case (a), the firm always needs to replenish inventory in the second period. As a result, there is only one marginal incentive to pad the sales; i.e., only  $\theta_c^I(\xi_1, q_1)$  will appear in the whole region  $\xi_1 \in [0, q_1)$  (see Equation (22)). In case (b), both marginal incentives in Equation (22) in the region  $\xi_1 \in [0, q_1)$  will appear; i.e.,  $\theta_c^{II}(\xi_1, q_1)$  will also appear if  $\xi_1$  falls into the region  $[0, \bar{\xi}(q_1))$ . In that case, the carryover sales padding factor plays a role. We write the optimization program according to these two cases as follows and take the first order derivatives.

**Case (a)  $q_1^* \leq k_2$ :** Then, the first order derivative is:

$$\underbrace{-c + p\bar{F}_1(q_1) + \frac{d}{dq_1} \mathbb{E}_{\xi_1} [-h(q_1 - \xi_1)^+ + v_2(\xi_1, q_1)]}_{\text{Classical Newsvendor}} - \underbrace{\gamma \frac{d}{dq_1} \mathbb{E}_{\xi_1} [x_a^o(\xi_1, q_1)]}_{\text{Stock Market Pressure}} = 0$$

with

$$x_a^o(\xi_1, q_1) = \begin{cases} \left( \theta_c^{II} \right)^+ \xi_1, & 0 \leq \xi_1 < \xi_c, \\ q_1 - \xi_1, & \xi_c \leq \xi_1. \end{cases}$$



**Case (b)**  $q_1^* > k_2$ : Then, the first order derivative is:

$$\underbrace{-c + p\bar{F}_1(q_1) + \frac{d}{dq_1}\mathbb{E}_{\xi_1}[-h(q_1 - \xi_1)^+ + v_2(\xi_1, q_1)]}_{\text{Classical Newsvendor}} - \underbrace{\gamma \frac{d}{dq_1}\mathbb{E}_{\xi_1}[x_b^o(\xi_1, q_1)]}_{\text{Stock Market Pressure}} = 0$$

with

$$x_b^o(\xi_1, q_1) = \begin{cases} \left(\int_0^{\xi_1} \theta_c^I(t, q_1) dt\right)^+, & 0 \leq \xi_1 < \min(\bar{\xi}(q_1), \xi_c), \\ \left(\int_0^{\bar{\xi}(q_1)} \theta_c^I(t, q_1) dt + (\xi_1 - \bar{\xi}(q_1)) \theta_c^{II}\right)^+, & \min(\bar{\xi}(q_1), \xi_c) \leq \xi_1 < \xi_c, \\ q_1 - \xi_1, & \xi_c \leq \xi_1 < q_1. \end{cases}$$

To solve  $q_1^*$ , we need to try both of these two cases and find the one which leads to the larger value. The optimization program in case (b) is complicated as the manager faces more trade-offs. We have discussed in Section 5 that in the region  $0 \leq \xi_1 < \bar{\xi}(q_1)$ , the manager's padding incentive increases in  $q_1$ . As a result, the manager may want to invest lower inventory to limit the incentive in this region. However, the manager still needs to consider the boundary effect, where the channel stuffing incentive increases as the inventory bound decreases. Hence, the boundary and carryover effects will drive the first period inventory investment in different directions. Due to the complexity of the analysis, we show the key insights by numerical examples in which we use line search to find the  $q_1$  that provides close-maximal payoff.