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The Agency Problems of Hedging and Earnings Management

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Abstract

This paper uses a principal-agent model to study the interaction between hedging and earnings management. Hedging makes earnings management more difficult and they appear to be strategic substitutes in this model, which is both consistent with existing empirical evidence and provides a new explanation for that evidence.

If hedging decision is contractible, hedging is efficient since it reduces both the risk premium and the equilibrium amount of earnings management. If hedging decision is not contractible, however, hedging does not always alleviate the agency problem. Surprisingly, sometimes a scenario of no hedging but allowing earnings management is efficient. The reason is that motivating hedging may require a more costly compensation scheme to mitigate the appeal of earnings management.

In addition, this paper shows that tolerating some earnings management is always efficient when there is no hedge option, since it is costly to eliminate
earnings management. Sometimes it is inefficient to take any action against earnings management. However, with the encouraged hedge option, the cost to eliminate earnings management can be reduced significantly and zero-tolerance of earnings management may be efficient.

1 Introduction

The hedging activities of firms have changed dramatically with the extraordinary development and expansion of financial derivatives since the early 1990s. According to the 2004 Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity, the global gross market value of over-the-counter (OTC) derivatives more than doubled, from $3.0 trillion at the end of June 2001 to $6.4 trillion at the end of June 2004.\(^1\) This has vastly increased the accessibility and diversity of hedging, and managers’ hedging decisions become much more important. Smith and Stulz (1985) suggest that managers’ compensation is a determinant of firms’ hedging decisions. However, the effect of hedging activities on firms incentive schemes has been far from fully explored in the literature. Among the few studies on this effect, Campbell and Kracaw (1987) focus on optimal insurance through hedging. They show that under certain incentive contracts, shareholders will be hurt by the manager’s hedging behavior since the manager will deviate from the

\(^1\)Gross market value is defined in the Central Bank Survey as the replacement value of all open contracts before counterparty or any other netting.
optimal managerial effort level with the acquisition of insurance. However, if the contract anticipates the hedging, shareholders can share the direct gain from hedging with the managers. DeMarzo and Duffie (1995) analyze hedging behavior from the perspective of information content and indicate that financial hedging improves the extent to which corporate earnings reflect managerial ability and project quality. However, unlike my study, the manager’s action in DeMarzo and Duffie’s model is given, so there is no need to motivate the manager’s hard working by contracting. Jorgensen (1999) uses Kyle’s model and a correlation between operating income and the settlement value of futures contracts, and shows that hedging provides strict benefits for contracting. Using a non-linear general agency model, Fischer (1999) illustrates that with a risk-neutral principal and a perfectly hedged earning, contracting on the hedged earnings is optimal and equivalent to contracting on both controllable and uncontrollable events.

The relationship between earnings management and hedging has been explored in some recent empirical studies, but the topic is underrepresented in the literature. Barton (2001) finds a significant negative association between the use of derivatives and the magnitude of discretionary accruals, which serves as a proxy for earnings management. Pincus and Rajgopal (2002), focusing on the oil and gas industry, find similar results. However, to my best knowledge, there is no theoretical study that addresses the ways in which hedging affects the firms’ incentive schemes in the presence of earnings management.
Using a principal-agent model and a mean preserving spread hedging structure with a normal distribution, this paper studies the interaction between hedging and earnings management. Hedging and earnings management appear to be strategic substitutes in this paper, which is both consistent with existing empirical evidence and provides a new explanation for that evidence.

If the decision to hedge is contractible, hedging alleviates the agency problem by reducing both the risk premium and the equilibrium amount of earnings management. If the decision to hedge is not contractible, however, hedging does not always alleviate the agency problem. Surprisingly, sometimes a scenario of no hedging but allowing earnings management is efficient. The reason is that motivating hedging may require a more consistent compensation scheme across periods to mitigate the appeal of earnings management, but the more consistent compensation scheme brings inefficiency.

In addition, this paper shows that when there is no hedge option the principal tolerates some earnings management, since it is costly to eliminate earnings management. Sometimes it is efficient not to take any action against earnings management. However, with the encouraged hedge option, the principal may achieve efficiency by stringently forbidding earnings management, since the cost to eliminate dead-weight loss from earnings management can be lowered significantly by hedging.

In the following section, I describe the basic setting of the model. Section 3 studies a setting in which the principal determines to eliminate earnings
management, while Section 4 looks at a setting in which earnings manage-
ment is allowed. In both Section 3 and 4, I examine how the introduction of
a hedge option affects the firm’s incentive scheme. Section 5 concludes this
paper and discusses the caveats.

2 Basic Setting

Suppose there is a risk-neutral principal and a risk-averse agent (manager)
in a two-period setting. The principal tries to minimize her expected pay-
ment to the manager while motivating the manager to choose high effort in
each period. The manager’s preference for total (net) compensation is char-
acterized by constant absolute risk aversion, implying a utility function of
\[ u(S - c) = -e^{-r(S - c)}, \]
where \( S \) is the payment to the agent after two periods, \( c \) is the cost of the manager’s efforts, and \( r \) is the Arrow-Pratt measure of
risk aversion. Without loss of generality, the manager’s reservation is set at
0. In other words, his reservation utility is \( -e^{-r(0)} \).

The performance signals (outputs) are stochastic, and their probabilities
are affected by two factors: the manager’s action and some exogenous factor.
The manager’s action is binary. In each period, the manager either supplies
low effort, \( L \), or high effort, \( H \), where \( H > L \). Without loss of generality,
\( L \) is normalized to zero. The manager’s personal cost for low effort is zero.
His personal cost for high effort is \( C > 0 \) in each period. The principal can-
not observe the manager’s efforts. An exogenous factor also affects realized
output. The effect of this exogenous factor can be hedged at least partially by using derivatives. Neither the principal nor the manager can foresee the realization of the exogenous factor. Here, "output" represents a noisy performance measurement of the manager’s effort levels (e.g., earnings); "output" does not narrowly refer to production and can be negative. I use $x_1$ to represent the output for the first period, and $x_2$ to represent the output for the second period.

Assume $x_1 = k_1 a_1 + \epsilon_1$ and $x_2 = k_2 a_2 + \epsilon_2$, with $a_i \in \{H, 0\}$, $i \in \{1, 2\}$. $a_i$ represents the action level for period $i$. $k_1, k_2$ are positive constants and represent the profitability in the first and second periods, respectively. Suppose $k_1 \neq k_2$, that is, that profitability varies through time. The uneven profitability follows a design of different productivity in Liang (2004). The different profitability induces different bonus rates through time and is important in creating the manager’s demand of earnings management. It is also a realistic assumption. A company’s profitability may change through time. For example, the profitability of a sunrise industry keeps increasing, while that of a sunset industry keeps declining. For my convenience, I assume $k_1 < k_2$, but the main conclusions of this paper still hold if $k_1 > k_2$.

The vector $[\epsilon_1, \epsilon_2]$ follows a joint normal distribution with a mean of $[0, 0]$. There is no carryover effect of actions across periods, and the outputs of each period are independent of one another.

If the outputs are not hedged, the covariance matrix of $[\epsilon_1, \epsilon_2]$ is $\Sigma =$
If the output in the second period is hedged, the matrix is

\[
\Sigma_d = \begin{bmatrix}
\sigma^2 & 0 \\
0 & \sigma^2_d
\end{bmatrix}, \quad \sigma^2_d < \sigma^2.
\]

The hedging process, with normal distributions, is stylized with a mean preserving spread structure: assuming the same effort level, the hedged production plan has a lower variance, \( \sigma^2_d \), than that of the unhedged one, \( \sigma^2 \), though they share the same mean. Thus the unhedged production plan is a mean preserving spread of its hedged counterpart. In this way, hedging lowers the variance of output due to the uncontrollable exogenous factor and reduces the noisy output risk. This structure captures the risk reduction theme of Rothschild and Stiglitz (1970), and also offers tractability.

The manager’s contract or compensation function is assumed to be "linear" in the reported outputs. Specifically, \( S = S(\hat{x}_1, \hat{x}_2) = W + \alpha \hat{x}_1 + \beta \hat{x}_2 \), where \( W \) is a fixed wage, and \( \alpha \) and \( \beta \) are the bonus rates assigned respectively to the first period reported output, \( \hat{x}_1 \), and the second period reported output, \( \hat{x}_2 \).\(^2\) The reported outputs have to satisfy the constraint that \( \hat{x}_1 + \hat{x}_2 = x_1 + x_2 \). The manager may misreport the output by moving \( \Delta \) between periods. (\( \Delta \) can be either positive or negative. I define a positive \( \Delta \) as an amount moved from the second to the first period, and a negative \( \Delta \) as the amount moved from the first to the second period.) The earnings

\(^2\)Assigning different bonus rates to \( \hat{x}_1 \) and \( \hat{x}_2 \) instead of compensating on the aggregate reported output is necessary in this model to ensue earnings management.
management has a personal cost to the manager of \( \frac{1}{2\sigma^2}\Delta^2 \) (If the output is hedged, then the cost is \( \frac{1}{2\sigma_d^2}\Delta^2 \)). The personal cost of manipulation is quadratic in the amount of manipulation, and decreases with the variance of the second period output. Intuitively, it is easy to manipulate a tiny amount but much harder to manipulate a big amount. Moreover, with a material discretion, a firm gets difficulty in convincing the auditor and is more likely to get penalized. In addition, it is harder to manipulate the earnings when the noise in earnings is reduced. For example, if a firm uses credit derivatives to hedge the credit default risk, then the firm has less reasons to estimate and recognize related bad debt expenses, since this risk is supposed to be taken away by hedging.

When the outputs were observed privately by the manager, the manager would have a natural incentive to move some output to the period with a higher bonus rate to get more compensation. With the personal cost of manipulation \( \frac{1}{2\sigma^2}\Delta^2 \), the manager’s optimal manipulation amount is \( \Delta^* = \sigma^2(\alpha - \beta) \).\(^3\) With hedging, the manager’s optimal manipulation amount is \( \Delta^* = \sigma_d^2(\alpha - \beta) \), which is smaller.

Barton (2001) and Pincus and Rajgopal (2002) show a negative association between hedging and earnings management. They suggest that the negative association may indicate that hedging and earnings management are substitutes to smooth earnings. In my paper a substitutional relation-

\(^3\)The manager’s optimal manipulation amount will occur when the marginal cost of manipulation equals the marginal benefit, that is, when \( \frac{\partial}{\partial\Delta}\left(\frac{1}{2\sigma^2}\Delta^2\right) = \frac{\partial\Delta^2}{\partial\Delta} \). This implies \( \Delta^* = \sigma^2(\alpha - \beta) \).
ship is achieved through hedging’s effect on the earnings management cost. The manager either to choose hedging and less earnings management, or to choose no hedging but cheaper earnings management. This substitutional relationship is both consistent with existing empirical evidence and may provide an alternative explanation for that evidence.

Assume that the second period hedging decision \( d \in \{0, 1\} \) is made at the beginning of the second period, after the first period output is privately observed.\(^4\) \( d = 1 \) represents the decision to hedge and \( d = 0 \) represents the decision not to hedge. The cost of hedging through derivatives in the model is zero.\(^5\) The time line is shown in Figure 1.

\[
\begin{array}{c|c|c}
\text{date 0} & \text{date 1} & \text{date 2} \\
\hline
\text{1st period} & \text{2nd period} & \\
\hline
\text{Manager chooses} & x_1 \text{realized. } d \in \{0, 1\}. & x_2 \text{ realized. Manager} \\
\text{ } & a_1 \in \{H, 0\}. & a_2 \in \{H, 0\}.
\end{array}
\]

Manager reports \( \hat{x}_1 \), \( \hat{x}_2 \), gets paid.

Figure 1: Time line

In the following sections we are going to examine two different settings

\(^4\)The results in this paper still hold if the second period hedging decision is made before the realization of \( x_1 \).

\(^5\)If hedge options are available in both period, the results about hedging in the second period still hold, and hedging in the first period is always efficient.

\(^5\)The financial carry cost of derivatives should be the arbitrage free price. In the economy of this model, the arbitrage free price (risk-free return) is zero since I do not consider the time value. Thus the cost of using derivatives is zero.
and four scenarios. We will first study the affect of a hedge option in a setting in which the principal eliminates earnings management by designing a suitable compensation contract. I call this setting "stringent setting." Later, we are going to examine a setting in which the principal allows but may restrain earnings management. I call it "tolerant setting." For our convenience, I display the four scenarios in the table below. I use A to denote the stringent setting in which hedge option is not available, B the stringent setting with encouraged hedging, C the tolerant setting without hedging, and D the tolerant setting with encouraged hedging.

<table>
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<th>No hedging</th>
<th>Encouraged hedging</th>
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Table 1

3 Stringent Setting

3.1 Stringent Setting With No Hedge Option — Scenario A

In the following two subsections, we first look at the stringent case in which there is no hedge option, and then introduce hedge into the stringent setting.

If there is no hedge option, the manager’s certainty equivalents at date 1, given $a_1 = H$ and $x_1$, are
\[ CE_1\{a_2 = H, d = 0, \hat{x}_1 = x_1\} = W + \alpha x_1 + \beta k_2 H - \frac{r}{2} \beta^2 \sigma^2 - 2C; \]

\[ CE_1\{a_2 = H, d = 0, \hat{x}_1 = x_1 + \Delta\} = W + \alpha x_1 + \alpha \sigma^2 (\alpha - \beta) + \beta [k_2 H - \sigma^2 (\alpha - \beta)] \]

\[ -\frac{r}{2} \beta^2 \sigma^2 - \frac{1}{2\gamma^2} [\sigma^2 (\alpha - \beta)]^2 - 2C = W + \alpha x_1 + \beta k_2 H + \frac{1}{2} \sigma^2 (\alpha - \beta)^2 - \frac{r}{2} \beta^2 \sigma^2 - 2C; \]

\[ CE_1\{a_2 = 0, d = 0, \hat{x}_1 = x_1\} = W + \alpha x_1 - \frac{r}{2} \beta^2 \sigma^2 - C; \]

\[ CE_1\{a_2 = 0, d = 0, \hat{x}_1 = x_1 + \Delta\} = W + \alpha x_1 + \frac{1}{2} \sigma^2 (\alpha - \beta)^2 - \frac{r}{2} \beta^2 \sigma^2 - C. \]

The manager’s certainty equivalents at date 0, given \( a_2 = H, d = 0, \) and \( \hat{x}_1 = x_1, \) are

\[ CE_0\{a_1 = H\} = W + \alpha k_1 H + \beta k_2 H - \frac{r}{2} \alpha^2 \sigma^2 - \frac{r}{2} \beta^2 \sigma^2 - 2C; \]

\[ CE_0\{a_1 = 0\} = W + \beta k_2 H - \frac{r}{2} \alpha^2 \sigma^2 - \frac{r}{2} \beta^2 \sigma^2 - C. \]

When the principal encourages truth-telling, the incentive constraint for the first period is \( CE_0\{a_1 = H\} \geq CE_0\{a_1 = 0\}, \) which implies \( \alpha \geq \frac{C}{k_1 H}. \)

The incentive constraints for the second period are \( CE_1\{a_2 = H, d = 0, \hat{x}_1 = x_1\} \geq CE_1\{a_2 = H, d = 0, \hat{x}_1 = x_1 + \Delta\}, \) \( CE_1\{a_2 = H, d = 0, \hat{x}_1 = x_1\} \geq CE_1\{a_2 = 0, d = 0, \hat{x}_1 = x_1\}, \) and \( CE_1\{a_2 = H, d = 0, \hat{x}_1 = x_1\} \geq CE_1\{a_2 = 0, d = 0, \hat{x}_1 = x_1 + \Delta\}. \) They imply \( \beta \geq \alpha, \) \( \beta \geq \frac{C}{k_2 H}, \) and 
\[ \beta \geq \frac{C + \frac{1}{2} \sigma^2 (\alpha - \beta)^2}{k_2 H}. \]

The principal’s design program to motivate truth-telling is

\[
\min_{\alpha, \beta} \frac{r}{2} \alpha^2 \sigma^2 + \frac{r}{2} \beta^2 \sigma^2 + 2C \quad \text{Program[1]}
\]

s. t. \( \alpha \geq \frac{C}{k_1 H} \)
\[ \beta \geq \frac{C + \frac{1}{2} \sigma^2 (\alpha - \beta)^2}{k_2 H} \]
\[ \beta \geq \alpha \]
Lemma 1 In Scenario A, the optimal contract satisfies $\alpha^* = \beta^* = \frac{C}{k_1H}$, and the principal’s expected cost is $PC_A = r(\frac{C}{k_1H})^2\sigma^2 + 2C$.

As long as there is a gap between the bonus rates in the two periods, the manager moves $\sigma^2(\alpha - \beta)$ between periods. To encourage truth-telling, the principal must increase the bonus rate of the second period to that of the first period.

3.2 Stringent Setting With Encouraged Hedging — Scenario B

Now I introduce a hedge option into the setting. If the hedging decision is contractible, or if the principal makes the hedging decision, then it is always efficient to hedge. The principal’s design program will be identical to Program [1] except the second period variance will be $\sigma^2_d$. Hedging improves efficiency since the principal’s expected cost is reduced to be $r(\frac{C}{k_1H})^2\sigma^2_d + 2C$.

In Scenario B I assume hedging decision is not contractible and the principal motivates hedging. With the presence of the hedge option at date 1, the manager’s certainty equivalents at date 1 if he chooses to hedge are:

$$CE_1\{a_2 = H, d = 1, \hat{x}_1 = x_1\} = W + \alpha x_1 + \beta k_2H - \frac{r}{2}\beta^2\sigma^2_d - 2C;$$
$$CE_1\{a_2 = H, d = 1, \hat{x}_1 = x_1 + \Delta\} = W + \alpha x_1 + \beta k_2H + \frac{1}{2}\sigma^2_d(\alpha - \beta)^2 - \frac{r}{2}\beta^2\sigma^2_d - 2C;$$
$$CE_1\{a_2 = 0, d = 1, \hat{x}_1 = x_1\} = W + \alpha x_1 - \frac{r}{2}\beta^2\sigma^2_d - C;$$

All proofs are in the appendix.
\[ CE_1 \{ a_2 = 0, d = 1, \hat{x}_1 = x_1 + \Delta \} = W + \alpha x_1 + \frac{1}{2} \sigma_d^2 (\alpha - \beta)^2 - \frac{r}{2} \beta^2 \sigma_d^2 - C. \]

If the manager reports the true \( x_1 \), then the strategy of hedging dominates that of no hedging, for hedging reduces the variance of the second period output, thus resulting in a lower risk to the manager’s compensation and a higher certainty equivalent. However, since a reduced variance implies a higher cost of manipulation, the manager may choose not to hedge but manipulate to obtain a higher payoff.

With the hedging option, if the principal encourages high efforts, truth-telling and hedging, the second period’s incentive constraints are

\[ CE_1 \{ a_2 = H, d = 1, \hat{x}_1 = x_1 \} \geq CE_1 \{ a_2 = H, d = 0, \hat{x}_1 = x_1 + \Delta \}, \]

which implies \( \alpha = \beta; \)

\[ CE_1 \{ a_2 = H, d = 1, \hat{x}_1 = x_1 \} \geq CE_1 \{ a_2 = H, d = 0, \hat{x}_1 = x_1 + \Delta \}, \]

which implies \( r \beta^2 (\sigma^2 - \sigma_d^2) \geq \sigma^2 (\alpha - \beta)^2; \)

\[ CE_1 \{ a_2 = H, d = 1, \hat{x}_1 = x_1 \} \geq CE_1 \{ a_2 = 0, d = 1, \hat{x}_1 = x_1 \}, \]

which implies \( \beta \geq \frac{C}{k_H}; \)

\[ CE_1 \{ a_2 = H, d = 1, \hat{x}_1 = x_1 \} \geq CE_1 \{ a_2 = 0, d = 1, \hat{x}_1 = x_1 + \Delta \}, \]

which implies \( \beta k_2 H - C \geq \frac{1}{2} \sigma_d^2 (\alpha - \beta)^2; \)

and \( CE_1 \{ a_2 = H, d = 1, \hat{x}_1 = x_1 \} \geq CE_1 \{ a_2 = 0, d = 0, \hat{x}_1 = x_1 + \Delta \}, \)

which implies \( \beta k_2 H - \frac{r}{2} \beta^2 \sigma_d^2 - C \geq \frac{1}{2} \sigma_d^2 (\alpha - \beta)^2 - \frac{r}{2} \beta^2 \sigma_d^2. \)

The last two constraints are redundant.

The incentive constraint for the first period, given \( a_2 = H, d = 1, \hat{x}_1 = x_1 \), is still \( \alpha \geq \frac{C}{k_H} \). The reason that the constraint on \( \alpha \) remains unchanged is that the amount of earnings management is independent of the realized first
To motivate hedging and truth-telling, the principal’s design program is

\[
\min_{\alpha, \beta} \frac{r}{2}(\alpha^2 \sigma^2 + \beta^2 \sigma_d^2) + 2C \quad \text{Program}[2]
\]

s. t. \(\alpha \geq \frac{C}{k_1H}\)
\[
\beta \geq \frac{C}{k_2H}
\]
\[
r \beta^2 (\sigma^2 - \sigma_d^2) \geq \alpha^2 (\alpha - \beta)^2
\]
\[
\alpha = \beta
\]

**Lemma 2** In Scenario B, the optimal contract satisfies \(\alpha^* = \beta^* = \frac{C}{k_1H}\), and the principal’s expected cost is

\[
PC_B = \frac{r}{2} \left(\frac{C}{k_1H}\right)^2 (\sigma^2 + \sigma_d^2) + 2C.
\]

The optimal bonus rates are still \(\alpha^* = \beta^* = \frac{C}{k_1H}\), same as in the no-hedge case, for the only way to encourage truth-telling is to set equal bonus rates to eliminate manipulation. However, with the presence of hedging, the principal’s expected cost is now

\[
\frac{r}{2} \left(\frac{C}{k_1H}\right)^2 (\sigma^2 + \sigma_d^2) + 2C,
\]

which is lower than that \(PC_A\) in the no-hedge setting.

In addition, hedging lowers the output variance but has no effect on the output mean, while the manager’s effort level affects the output mean but not the variance. Therefore, the effort choice is distinct from the hedging choice. Thus the optimal bonus rates are not affected by the introduction of a hedge option.

**Proposition 1** *Encouraging hedging is efficient in the stringent setting.*

Proposition 1 shows that Scenario A is never efficient, for it is always dominated by Scenario B in which the principal encourages hedging. Notice
that in the stringent case, hedging is preferred by both the manager and the principal. Hedging reduces the future output variance and therefore reduces the manager’s compensation risk. While the principal is herself risk neutral and unconcerned with risk, she pays a lower risk premium to compensate the manager’s risk by encouraging hedging.

Previous studies such as Jorgensen (1999) and Fisher (1999) suggest that hedging activities reduce agency cost. Proposition 1 is consistent with these previous works and further confirms that hedging improves efficiency even in the presence of earnings management, provided a stringent policy is enforced.

4 Tolerant Setting

4.1 Tolerant Setting With No Hedge Option — Scenario C

From this section, I consider a "tolerant" case in which the principal still encourages high efforts but allows earnings management. Again, I first exclude the hedge option, then introduce the hedge option into the setting to see the affect of hedging on the agency problem.

With earnings management, the manager’s certainty equivalents at date 1 are always higher than those of truth-telling, regardless of his effort choices. That is, $CE_1\{a_2, d = 0, \hat{x}_1 = x_1 + \Delta \} \geq CE_1\{a_2, d = 0, \hat{x}_1 = x_1 \}$. Therefore the manager will always overreport the first period output and underreport
the second period output by \( \sigma^2(\alpha - \beta) \), as long as the principal does not set equal bonus rates. To motivate high effort in the second period, there are two incentive constraints at date 1, \( CE_1\{a_2 = H, d = 0, \hat{x}_1 = x_1 + \Delta\} \geq CE_1\{a_2 = 0, d = 0, \hat{x}_1 = x_1 + \Delta\} \) and \( CE_1\{a_2 = H, d = 0, \hat{x}_1 = x_1 + \Delta\} \geq CE_1\{a_2 = 0, d = 0, \hat{x}_1 = x_1\} \). They imply \( \beta \geq \frac{C}{k_2H} \).

Given the manager’s date 1 decision to choose \( a_2 = H \) and manipulate, his incentive constraint at date 0 is \( \alpha \geq \frac{C}{k_1H} \). The principal’s expected compensation cost, taking earnings management into consideration, is increased by \( (\alpha - \beta)\Delta = \sigma^2(\alpha - \beta)^2 \), for the principal pays a higher bonus for the manipulated amount of output. Her design program is

\[
\min_{\alpha, \beta} \frac{r}{2} \alpha^2 \sigma^2 + \frac{r}{2} \beta^2 \sigma^2 + \sigma^2(\alpha - \beta)^2 + 2C \quad \text{Program[3]}
\]

s. t. \( \alpha \geq \frac{C}{k_1H} \)
\[ \beta \geq \frac{C}{k_2H} \]

**Proposition 2** In Scenario C,

if \( \frac{2}{r+2}k_2 < k_1 < k_2 \), the optimal contract satisfies \( \alpha^* = \frac{C}{k_1H} \), \( \beta^* = \frac{C}{k_2H} \), and the principal’s expected cost is \( PC_c = \frac{r}{2} \left( \frac{C}{k_1H} \right)^2 \sigma^2 + \frac{r}{2} \left( \frac{C}{k_2H} \right)^2 \sigma^2 + \sigma^2(\frac{C}{k_1H} - \frac{C}{k_2H})^2 + 2C \);

if \( k_1 \leq \frac{2}{r+2}k_2 \), the optimal contract satisfies \( \alpha^* = \frac{C}{k_1H} \), \( \beta^* = \frac{C}{k_1H} \), and the principal’s expected cost is \( PC_c' = \frac{r}{2} \left( \frac{C}{k_1H} \right)^2 \sigma^2 + \frac{r}{2} \left( \frac{C}{k_1H} \right)^2 \sigma^2 + \sigma^2(\frac{C}{k_1H})^2(\frac{r}{r+2})^2 + 2C \).

\( \frac{C}{k_1H} \) and \( \frac{C}{k_2H} \) are the lower bounds for \( \alpha \) and \( \beta \) to motivate the manager’s high efforts. When \( k_1 \) and \( k_2 \) are close, \( \frac{C}{k_1H} \) and \( \frac{C}{k_2H} \) are close and keeping
the bonus rates at these lower bounds is efficient, since the extra payment to the manipulated amount is immaterial. When \( k_1 << k_2 \), the principal raises the second period bonus rate to restrain but not to eliminate the earnings management.

**Corollary 1** Without hedging, the tolerant policy is always efficient.

Again, Corollary 1 indicates that Scenario A is never efficient, since Scenario C also dominates A. Liang (2004) shows that zero tolerance of earnings management is not efficient, and that the optimal contracts always show different bonus rates. Corollary 1 is consistent with Liang’s study. The manipulation option merely garbles the information and increases agency cost, but tolerating some earnings management is always better than eliminating it by setting \( \alpha = \beta \). Moreover, Proposition 2 shows that occasionally the principal maintains the bonus rates at \( \frac{C}{k_1 H} \) and \( \frac{C}{k_2 H} \) respectively, as if there were no earnings management problem.

Since the induced earnings management is \( \Delta^* = \sigma^2(\alpha - \beta) \), the dead-weight loss of earnings management can be reduced by lowering \( \alpha - \beta \). To lower \( \alpha - \beta \), the principal either lowers \( \alpha \) or raises \( \beta \). However, \( \alpha \) has a binding lower bound at \( \frac{C}{k_1 H} \), and the principal cannot reduce \( \alpha \) below that bound. Thus the optimal \( \alpha \) remains at its bound. By raising \( \beta \), the principal reduces the dead-weight loss of misreporting. However, it simultaneously increases the riskiness of the unmanaged compensation scheme \( \left( \frac{r}{2} \alpha^2 \sigma^2 + \frac{r}{2} \beta^2 \sigma^2 \right) \) goes up). Due to this trade off, sometimes it is better not to raise \( \beta \) and keep \( \beta \)
at its lower bound, $\frac{C}{k_2 H}$. Furthermore, when the second period profitability $k_2$ is relatively high, the lower bound for the second period bonus is low, and the principal is more willing to raise the second period bonus above the lower bound to reduce the earnings management dead-weight loss.

The fight against earnings management has increased in urgency after the Enron and World.Com scandals. Former SEC Chairman Arthur Levitt commented in a speech in 1998 that earnings management is "[a] game that, if not addressed soon, will have adverse consequences for America’s financial reporting system. A game that runs counter to the very principles behind our market’s strength and success." However, according to the results above, sometimes it may be better to live with earnings management instead of taking action against it. Even when eliminating or restraining earnings management is feasible, it is not always efficient. This conclusion does not imply that earnings management is harmless; instead, it indicates that we need to consider the costs and benefits of the fight against earnings management.

4.2 Tolerant Setting With Encouraged Hedging — Scenario D

Before examining Scenario D, notice that if the hedging decision is contractible, then it is always efficient to hedge. The principal’s design program will be identical to Program [3] except that the second period variance is lower. With the lower variance $\sigma_d^2$, the principal’s expected cost is reduced,
since hedging reduces both the risk premium and the dead-weight loss from earnings management, $\sigma^2_d(\alpha - \beta)$.

In Scenario D I assume the manager’s hedging decision is not contractible and the principal motivates hedging. The interaction between hedging and earnings management is now more subtle. In addition, it has been shown that the hedging option alleviates the agency problem in the stringent setting. We now examine whether hedging is still efficient in Scenario D and whether the introduction of the hedging option still alleviates the agency problem in the tolerant setting.

Given that the manager chooses hedging, his certainty equivalents at date 1 when manipulating are always higher than those of truth-telling, regardless of his effort choices. That is, $CE_1\{a_2, d = 1, \hat{x}_1 = x_1 + \Delta'\} \geq CE_1\{a_2, d = 1, \hat{x}_1 = x_1\}, \Delta' = \sigma^2_d(\alpha - \beta)$. In addition, we see from the previous section that the hedging strategy dominates the no-hedge strategy when the manager reports the true outputs. That is, $CE_1\{a_2, d = 1, \hat{x}_1 = x_1\} \geq CE_1\{a_2, d = 0, \hat{x}_1 = x_1\}$.

When the principal encourages hedging and allows earnings management, the incentive constraints at date 1, given $a_1 = H$, are

$$CE_1\{a_2 = H, d = 1, \hat{x}_1 = x_1 + \Delta'\} \geq CE_1\{a_2 = H, d = 0, \hat{x}_1 = x_1 + \Delta\},$$

which implies $r\beta^2 \geq (\alpha - \beta)^2$;

$$CE_1\{a_2 = H, d = 1, \hat{x}_1 = x_1 + \Delta'\} \geq CE_1\{a_2 = 0, d = 1, \hat{x}_1 = x_1 + \Delta'\},$$

which implies $\beta \geq \frac{C}{\sigma^2_d}$;

$$CE_1\{a_2 = H, d = 1, \hat{x}_1 = x_1 + \Delta'\} \geq CE_1\{a_2 = 0, d = 1, \hat{x}_1 = x_1\},$$

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which is redundant given \( \beta \geq \frac{C}{k_2 H} \) and \( CE_1 \{ a_2 = H, d = 1, \hat{x}_1 = x_1 + \Delta' \} \geq CE_1 \{ a_2 = H, d = 1, \hat{x}_1 = x_1 \} \);

\[
CE_1 \{ a_2 = H, d = 1, \hat{x}_1 = x_1 + \Delta' \} \geq CE_1 \{ a_2 = 0, d = 0, \hat{x}_1 = x_1 + \Delta \},
\]

which implies \( \frac{1}{2} \sigma_d^2 (\alpha - \beta)^2 - \frac{r}{2} \beta^2 \sigma_d^2 + \beta k_2 H - C \geq \frac{1}{2} \sigma^2 (\alpha - \beta)^2 - \frac{r}{2} \beta^2 \sigma_d^2 \), and is redundant given \( r \beta^2 \geq (\alpha - \beta)^2 \) and \( \beta \geq \frac{C}{k_2 H} \).

The incentive constraint at date 0 is still \( \alpha \geq \frac{C}{k_1 H} \).

The principal’s design program is

\[
\begin{align*}
\min_{\alpha, \beta} & \quad \frac{1}{2} \alpha^2 \sigma_d^2 + \frac{r}{2} \beta^2 \sigma_d^2 + \sigma_d^2 (\alpha - \beta)^2 + 2C \\
\text{s. t.} & \quad \alpha \geq \frac{C}{k_1 H} \\
& \quad \beta \geq \frac{C}{k_2 H} \\
& \quad r \beta^2 \geq (\alpha - \beta)^2
\end{align*}
\]

Program [4]

**Proposition 3** In Scenario D,

if \( \frac{1}{1+\sqrt{r}} k_2 < k_1 < k_2 \), the optimal contract satisfies \( \alpha^* = \frac{C}{k_1 H}, \beta^* = \frac{C}{k_2 H} \), and the principal’s expected cost is

\( PC_D = \frac{r}{2} \left( \frac{C}{k_1 H} \right)^2 \sigma_d^2 + \frac{r}{2} \left( \frac{C}{k_2 H} \right)^2 \sigma_d^2 + \sigma_d^2 \left( \frac{C}{k_1 H} - \frac{C}{k_2 H} \right)^2 + 2C \);

if \( k_1 \leq \frac{1}{1+\sqrt{r}} k_2 \), the optimal contract satisfies \( \alpha^* = \frac{C}{k_1 H}, \beta^* = \frac{C}{k_1 H} \left( 1 + \sqrt{r} \right) \), and the principal’s expected cost is

\( PC_D' = \frac{r}{2} \left( \frac{C}{k_1 H} \right)^2 \sigma_d^2 + \frac{r}{2} \left( \frac{C}{k_1 H} \left( 1 + \sqrt{r} \right) \right)^2 \sigma_d^2 + \sigma_d^2 \left( \frac{C}{k_1 H} \right)^2 \frac{r}{(1+\sqrt{r})^2} + 2C \).

As in Scenario C, when \( k_1 \) and \( k_2 \) are close, the principal keeps the bonus rates at these lower bounds. The manager’s high efforts are easily motivated, and there is only an immaterial dead-weight loss from earnings management. However, when the profitability increases greatly through time, the principal
will raise the second period bonus rate to restrain earnings management while tolerating a small manipulated amount.

**Proposition 4** With encouraged hedging,

When \( r < (\sqrt{3} - 1)^2 \) and \( k_1 \leq \frac{2-r}{2+r^2} k_2 \), the stringent policy is efficient; otherwise, the tolerant policy is efficient.

Corollary 1 in the setting with no hedge option shows that it is always better to tolerate some earnings management, for it is costly to eliminate the manipulation. However, with the hedge option encouraged, the principal may find the stringent policy (Scenario B) efficient. The efficiency of the stringent policy has two reasons. First, when the principal raises the second period bonus to \( \frac{C}{h_1 H} \) to eliminate the dead-weight loss from earnings management, the risk premium increases simultaneously. However, with a lower variance in the second period, the risk premium is now only \( \frac{r}{2} \alpha^2 \sigma^2 + \frac{r}{2} \beta^2 \sigma^2_d \); thus the cost of stringent policy is reduced. In addition, this risk premium is even lower with a small risk aversion \( r \). Secondly, since hedging reduces the risk in the manager’s compensation and increases the cost of earnings management from \( \frac{1}{2} \sigma^2 \Delta^2 \) to \( \frac{1}{2} \sigma^2_d \Delta^2 \), it is easier for the principal to discourage earnings management.

Proposition 1 shows that encouraging hedging always alleviates the agency problem in the stringent setting. Will the introduction of the hedge option into the tolerant setting improve efficiency too?

**Proposition 5** In the tolerant setting,
if $\max\{\frac{1}{1+\sqrt{r}}k_2, \frac{2}{2+r}k_2\} < k_1 < k_2$, then the encouraged hedge option alleviates the agency problem;

if $\frac{1}{1+\sqrt{r}}k_2 \leq k_1 \leq \frac{2}{2+r}k_2$, then the encouraged hedge option aggravates the agency problem when $\sigma^2 < \frac{|rk_2^2 + (k_2 - k_1)^2|(r+2)}{rk_2^2}$, \(7\)

if $\frac{2}{2+r}k_2 \leq k_1 \leq \frac{1}{1+\sqrt{r}}k_2$, then the encouraged hedge option aggravates the agency problem when $\sigma^2 < \frac{3r k_2^2}{rk_2^2 + 2(k_2 - k_1)^2}$, \(8\)

if $k_1 < \min\{\frac{1}{1+\sqrt{r}}k_2, \frac{2}{2+r}k_2\}$, then the encouraged hedge option aggravates the agency problem when $\sigma^2 < \frac{3(r+2)}{2(1+\sqrt{r})^2} \sigma_d^2$. \(9\)

In the tolerant setting, the encouraged hedge option alleviates the agency problem when $k_1$ and $k_2$ are close ($\max\{\frac{1}{1+\sqrt{r}}k_2, \frac{2}{2+r}k_2\} < k_1 < k_2$). If $k_1$ and $k_2$ are not sufficiently close, then hedging option alleviates the agency problem when hedging sufficiently reduces the output variance. Without hedging, the principal’s expected cost is $\frac{r}{2} \alpha^2 \sigma^2 + \frac{r}{3} \beta^2 \sigma^2 + \sigma^2 (\alpha - \beta)^2 + 2C$. With an encouraged hedge option, the expected cost is $\frac{r}{2} \alpha^2 \sigma^2 + \frac{r}{3} \beta^2 \sigma_d^2 + \sigma_d^2 (\alpha - \beta)^2 + 2C$, and there is an extra constraint, $r \beta^2 \geq (\alpha - \beta)^2$, to motivate hedging. When $k_1$ and $k_2$ are close, the bonus rates tend to be similar and the dead-weight loss due to earnings management is trivial. The introduction of a hedge option merely reduces the risk premium and thus the principal’s

---

\(7\) $\frac{|rk_2^2 + (k_2 - k_1)^2|(r+2)}{rk_2^2} > 1$, for $\frac{2r^3 k_2^2 + r^2 (k_2 - k_1)^2 + 2r^2 k_2^2 + r(k_2 - k_1)^2 + 5rk_2^2}{r k_2^2} + 4(k_2 - k_1)^2 > 0$, thus $r^3 k_2^2 + r^2 k_2^2 - 2rk_1 k_2 + 3r^2 k_2^2 + rk_2^2 - 4rk_1 k_2$

$+ 6r k_1^2 + 4k_2^2 - 8k_1 k_2 + 4k_1^2 > 0$, which implies $|rk_2^2 + (k_2 - k_1)^2|(r+2) > rk_2^2$.

\(8\) $\frac{3r k_2^2}{rk_2^2 + 2(k_2 - k_1)^2} > 1$, for $\frac{2}{2+r} < \frac{1}{1+\sqrt{r}}$ implies $r > 4$, and $r k_2^2 + 2r k_2^2 > rk_2^2 + 2(k_2 - k_1)^2$.

\(9\) $\frac{3(r+2)}{2(1+\sqrt{r})^2} > 1$, for $(\sqrt{r} - 2)^2 > 0$, thus $r - 4\sqrt{r} + 4 > 0$, which implies $3r + 6 > 2 + 4\sqrt{r} + 2r$ or $3(r + 2) > 2(1 + \sqrt{r})^2$. 

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expected compensation cost. When $k_1$ and $k_2$ are quite different, the gap between the two bonus rates tends to be large. Although hedging reduces the second period output variance thus reducing the risk premium, the principal may need to raise $\beta$ further to satisfy the incentive constraint on motivating hedging. Raising $\beta$ has two effects on the principal’s expected cost. The dead-weight loss $\sigma^2_d(\alpha - \beta)^2$ is reduced, but part of the risk premium, $\frac{1}{2} \beta^2 \sigma^2_d$, goes up. Thus the introduction of a hedge option has an ambiguous effect on the agency problem. As shown in Proposition 5, if $k_1$ and $k_2$ are quite different, then only when hedging sufficiently reduces the output variance would the introduction of such a hedge option improves efficiency.

Propositions 4 and 5 imply that none of Scenarios B, C, and D strictly dominates others. Which scenario is efficient depends on the balance of several forces. In the following table, I use $X \to Y$, where $X, Y \in \{A, B, C, D\}$, to represent that Scenario X provides the principal with a lower expected cost than Scenario Y does.

<table>
<thead>
<tr>
<th></th>
<th>no hedging</th>
<th>encouraged hedging</th>
</tr>
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<tbody>
<tr>
<td>stringent</td>
<td>Scenario A</td>
<td>← Scenario B</td>
</tr>
<tr>
<td></td>
<td>↑</td>
<td>/\</td>
</tr>
<tr>
<td>tolerant</td>
<td>Scenario C</td>
<td>⇐ Scenario D</td>
</tr>
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</table>

Table 2

To illustrate the above results more clearly, I list in the following corollaries some cases in which Scenario D, B, or C is efficient.
Corollary 2 If \( r > (\sqrt{3} - 1)^2 \), \( \frac{2}{3} k_2 \leq k_1 \leq \frac{1}{1+\sqrt{r}} k_2 \), and \( \sigma^2 > \frac{3r^2 k_2^2}{r k_2^2 + 2(k_2 - k_1)^2} \sigma_d^2 \), Scenario D is efficient.

Corollary 3 If \( r < (\sqrt{3} - 1)^2 \), \( \frac{1}{1+\sqrt{r}} k_2 \leq k_1 \leq \frac{2-r}{2+r} k_2 \), and \( \sigma^2 > \frac{3(r+2)}{2(1+\sqrt{r})^2} \sigma_d^2 \), Scenario B is efficient.

Corollary 4 If \( r > (\sqrt{3} - 1)^2 \), \( k_1 < \min\{\frac{1}{1+\sqrt{r}} k_2, \frac{2-r}{2+r} k_2\} \), and \( \sigma^2 < \frac{3(r+2)}{2(1+\sqrt{r})^2} \sigma_d^2 \), Scenario C is efficient.

In Corollary 2, since \( r \) is high, it is efficient to motivate hedging to reduce the risk premium \( \frac{r}{2} \beta^2 \sigma_d^2 \) in the compensation cost, and hedging also reduces the earnings management dead-weight loss \( \sigma_d^2 (\alpha - \beta)^2 \). In addition, since \( \sigma_d^2 \) is small and the dead-weight loss \( \sigma_d^2 (\alpha - \beta)^2 \) is low, the earnings management problem is better tolerated than eliminated.

In Corollary 3, when the manager does not care much about risk and hedging reduces the variance significantly, the second period risk premium \( \frac{r}{2} \beta^2 \sigma_d^2 \) is very low with hedging, even if \( \beta \) is raised to eliminate earnings management. It is also easier and efficient to eliminate earnings management dead-weight loss since the manager’s personal cost of manipulation soars with hedging.

In Corollary 4, When \( k_1 << k_2 \) and the manager is highly risk averse, it is not efficient to eliminate earnings management by raising \( \beta \) because this will bring a high risk premium \( \frac{r}{2} \beta^2 \sigma_d^2 \). Hedging does not help in this case, since it can reduce the variance only slightly and will not sufficiently increase the manager personal cost of manipulation to discourage earnings management.
The introduction of hedging can lower the risk premium by reducing the output variance. In addition, hedging makes earnings management costly, therefore may discourage manipulation and reduce the dead-weight loss of earnings management. However, it is surprising that hedging may aggravate the agency problem, and sometimes the scenario of no hedge but earnings management is efficient.

Some empirical evidence may support our result that sometimes it is inefficient to motivate hedging. Guay and Kothari (2002) find that most firms hold only derivatives positions that are small in magnitude relative to the entity-level risk exposures. The inefficiency to motivate hedging may be one of the explanations to this empirical phenomenon.

5 Conclusions and Caveats

This paper explores the interaction between hedging activities and earnings management. I employ a mean preserving spread structure to model the risk-reducing hedging activities, and assume that hedging is linked with earnings management through the variance of the output to achieve a substitutional relationship.

When hedging decision is contractible, hedging always improves efficiency, since it reduces the risk premium, the equilibrium amount of earnings management and the dead-weight loss from earnings management. When the decision to hedge is not contractible, however, hedging may aggravate the
agency problem. It is shown that sometimes a scenario of no hedging but allowing earnings management is efficient. Since hedging makes earnings management more difficult, to motivate hedging the principal may make earnings management less appealing by a more consistent contract across periods. However, the cost of setting similar bonus rates may outweigh the benefit from hedging.

Another result of this paper shows that tolerating earnings management is always efficient when there is no hedge option, since it is costly to eliminate earnings management. However, with the encouraged hedge option, the cost to eliminate earnings management can be significantly reduced, and the principal may achieve efficiency by stringently forbidding earnings management.

In the paper I use a LEN framework (LEN refers to linear contract, negative exponential utility function, and normal distribution). LEN is a helpful technology for research in agency and has been employed in analytical studies such as Feltham and Xie (1994), Indjejikian and Nanda (1999), Autrey, Dikolli and Newman (2003), Dutta and Reichelstein (1999), Liang (2004) and Christensen and Demski (2003). Among the three assumptions of the LEN framework, exponential utility and normal distribution have been widely used and accepted, while the linear contract assumption is more controversial. Linear contracts are usually not the optimal contracts, but they provide great tractability and help researchers explore some agency questions that were hard to analyze in the conventional agency models. However, as Lambert (2001) points out, we achieve this tractability with a cost of restricting the
generality.

The mean preserving spread structure in this paper provides us with a straightforward instrument to model risk reduction through hedging. However, the simplification also limits the analysis. The direct cost of hedging due to basis risk is not addressed, and hedging is assumed to be always effective in this paper. However, the ineffectiveness of hedging and the basis risk are important factors in the firms’ hedging decision. For example, Haushalter (2000) shows evidence that there is a negative association between the extent to which a firm should hedge to reduce risks and the basis risk the firm is facing. In addition, the hedge effectiveness tests required by SFAS No. 133 may change firms’ hedging behavior since it may result in the volatility of earnings due to the recognition of ineffective hedges. I believe future studies on the ways in which basis risk and the ineffectiveness of hedging affect firms’ incentive schemes will further enhance our understanding of hedging activities.

Reference

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**Appendix**

**Proof of Lemma 1:**

**Proof.** If

\[ \frac{C + \frac{1}{2} \sigma^2 (\alpha - \beta)^2}{k_2 H} \geq \frac{C}{k_1 H}, \]

then

\[ \alpha = \frac{C}{k_1 H}, \beta = \frac{C}{k_1 H} + \frac{k_2 H + \sqrt{2\sigma^2 C (k_2 / k_1 - 1)}}{\sigma^2}, \]

Thus,

\[ \frac{C}{k_1 H}. \]
If \( \frac{C + \frac{1}{2} \sigma^2 (\alpha - \beta)^2}{k_2 H} \geq \frac{C}{k_1 H} \), then \( \alpha = \beta = \frac{C}{k_1 H} \). With \( \alpha = \beta \), \( \frac{C + \frac{1}{2} \sigma^2 (\alpha - \beta)^2}{k_2 H} \geq \frac{C}{k_1 H} \) always holds, for \( k_1 < k_2 \).

It is obvious that the principal’s expected cost is lower with \( \alpha = \beta = \frac{C}{k_1 H} \). Thus \( \alpha^* = \beta^* = \frac{C}{k_1 H} \).

**Proof of Lemma 2:**

Skipped

**Proof of Proposition 1:**

Skipped

**Proof of Proposition 2:**

**Proof.** Define \( \mu_1 \) as the Lagrangian multiplier for \( \alpha \geq \frac{C}{k_1 H} \) and \( \mu_2 \) for \( \beta \geq \frac{C}{k_2 H} \). The first order conditions for Program [3] are:

\[
\begin{align*}
-r \alpha \sigma^2 - 2(\alpha - \beta)\sigma^2 + \mu_1 &= 0 \quad \text{(FOC1)} \\
-r \beta \sigma^2 + 2(\alpha - \beta)\sigma^2 + \mu_2 &= 0 \quad \text{(FOC2)}
\end{align*}
\]

FOC1 implies \( \mu_1 > 0 \), thus \( \alpha^* = \frac{C}{k_1 H} \).

If \( \mu_2 = 0 \), then \( \beta^* = \frac{2\beta}{r + 2} \frac{C}{k_1 H} \) and \( \frac{2\beta}{r + 2} \frac{C}{k_1 H} \geq \frac{C}{k_2 H} \). That is, \( \beta^* = \frac{2\beta}{r + 2} \frac{C}{k_1 H} \) when \( k_1 \leq \frac{2}{r + 2} k_2 \).

If \( \mu_2 > 0 \), then \( \beta^* = \frac{C}{k_2 H} \) and \( \mu_2 = r \beta \sigma^2 - 2(\alpha - \beta)\sigma^2 = \frac{k_1 (r+2) - 2k_2}{k_1 k_2} \sigma^2 C > 0 \). This requires \( \frac{2}{r + 2} k_2 < k_1 < k_2 \).

**Proof of Corollary 1:**

**Proof.** The principal’s expected compensation cost in Program [1] is \( PC_1 = r \sigma^2 (\frac{C}{H})^2 \frac{1}{k_1^2} + 2C \). The principal’s expected compensation cost in Program[3] is

\[
PC_2 = \frac{r}{2} (\frac{C}{H})^2 \frac{1}{k_1} \sigma^2 + (\frac{2}{r + 2})^2 \sigma^2 + \sigma^2 (\frac{r}{r + 2})^2 (\frac{C}{k_1 H})^2 + 2C \text{ when } k_1 \leq \frac{2}{r + 2} k_2, \text{ and}
\]

is \( PC_3 = \frac{r}{2} (\frac{C}{H})^2 \frac{1}{k_1} \sigma^2 + \frac{1}{k_2^2} \sigma^2 + \sigma^2 (\frac{C}{H})^2 (\frac{1}{k_1} - \frac{1}{k_2})^2 + 2C \text{ when } \frac{2}{r + 2} k_2 < k_1 < k_2. \)
expected cost is
Program [4] when the principal allows earnings management, the principal’s expected cost is

Proof. In Program [2] when the principal encourages truth-telling and hedging, the principal’s expected cost is $PC_4 = \frac{r}{2}(\frac{C}{H})^2[\frac{1}{k^2_1} \sigma^2[1 - (\frac{2}{r+2})^2] - \sigma^2(\frac{r}{r+2})^2(\frac{C}{k_1H})^2] = (\frac{C}{H})^2 \frac{1}{k^2_1} \sigma^2[\frac{r}{2} - \frac{2r+r^2}{(r+2)^2}] \]

\[= (\frac{C}{H})^2 \frac{1}{k^2_1} \sigma^2 \frac{r^2}{2(r+2)} > 0\]

$PC_1 - PC_2 = \frac{r}{2}(\frac{C}{H})^2(\frac{1}{k_2^2} - \frac{1}{k_2^2}) - \sigma^2(\frac{C}{k_1H})^2\]

$PC_1 - PC_3 = \frac{r}{2}(\frac{C}{H})^2\sigma^2(\frac{1}{k_2^2} - \frac{1}{k_2^2}) - \sigma^2(\frac{C}{k_1H})^2\]

Proof of Proposition 3:

Proof. Define $\mu_1$ as the Lagrangian multiplier for $\alpha \geq \frac{C}{k_1H}$, $\mu_2$ for $\beta \geq \frac{C}{k_2H}$, and $\mu_3$ for $r \beta^2 \geq (\alpha - \beta)^2$. The first order conditions for Program [4] are:

\[-r \alpha \sigma^2 - 2(\alpha - \beta)\sigma^2 + \mu_1 - 2(\alpha - \beta)\mu_3 = 0 \quad \text{(FOC1)}\]

\[-r \beta \sigma^2 + 2(\alpha - \beta)\sigma^2 + \mu_2 + 2r \beta \mu_3 + 2(\alpha - \beta)\mu_3 = 0 \quad \text{(FOC2)}\]

(FOC1) implies $\mu_1 > 0$, thus $\alpha^* = \frac{C}{k_1H}$. If $\mu_2 > 0$, $\mu_3 = 0$, then $\beta = \frac{C}{k_2H}$ and $r(\frac{C}{k_2H})^2 > \frac{C}{k_1H} - \frac{C}{k_2H}$. In other words, $\alpha^* = \frac{C}{k_1H}$, $\beta^* = \frac{C}{k_2H}$, if $\frac{r}{1+\sqrt{r}}k_2 < k_1 < k_2$.

If $\mu_2 = 0$, $\mu_3 > 0$, then $r \beta^2 = (\frac{C}{k_1H} - \beta)^2$ and $\beta \geq \frac{C}{k_2H}$. This implies $\alpha^* = \frac{C}{k_1H}$, $\beta^* = \frac{C}{k_1H} \frac{1}{1+\sqrt{r}}$, if $k_1 \leq \frac{1}{1+\sqrt{r}}k_2$. ■

Proof of Proposition 4:

Proof. In Program [2] when the principal encourages truth-telling and hedging, the principal’s expected cost is $PC_4 = \frac{r}{2}(\frac{C}{H})^2[\frac{1}{k^2_1} \sigma^2 + \frac{1}{k^2_1} \sigma^2] + 2C$. In Program [4] when the principal allows earnings management, the principal’s expected cost is $PC_5 = \frac{r}{2}(\frac{C}{H})^2[\frac{1}{k^2_1} \sigma^2 + \frac{1}{k^2_1}([\frac{1}{1+\sqrt{r}}\sigma^2] + \sigma^2(\frac{1}{k_1H})^2) + 2C\]

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implies \( r < \frac{1}{1+\sqrt{r}} k_2 \), and \( PC_6 = \frac{r}{2} (\frac{C}{H})^2 [\frac{1}{k_2} \sigma^2 + \frac{1}{k_2} \sigma_d^2] + \sigma_d^2 (\frac{C}{H})^2 (\frac{1}{k_1} - \frac{1}{k_2})^2 + 2C \) if \( \frac{1}{1+\sqrt{r}} k_2 < k_1 < k_2 \).

\[
PC_4 - PC_5 = (\frac{C}{k_1 H})^2 \sigma_d^2 \left[ \frac{r}{2} - \frac{1}{2} \left( \frac{1}{1+\sqrt{r}} \right)^2 - \frac{r}{(1+\sqrt{r})^2} \right]
= (\frac{C}{k_1 H})^2 \sigma_d^2 \left( \frac{1+\sqrt{r}}{2} \right)^2 - \frac{2\sqrt{r} + r - 2}{2(1+\sqrt{r})}.
\]

If \( 2\sqrt{r} + r - 2 < 0 \) and \( k_1 \leq \frac{1}{1+\sqrt{r}} k_2 \), then \( PC_4 - PC_5 < 0 \). \( 2\sqrt{r} + r - 2 < 0 \) implies \( r < (\sqrt{3} - 1)^2 \).

\[
PC_4 - PC_6 = \frac{r}{2} (\frac{C}{H})^2 [\frac{1}{k_1} \sigma^2 - \frac{1}{k_2} \sigma_d^2] - \sigma_d^2 (\frac{C}{H})^2 (\frac{1}{k_1} - \frac{1}{k_2})^2
= \sigma_d^2 (\frac{C}{H})^2 [\left( \frac{r}{2} + \frac{1}{k_1} \right) - (\frac{1}{k_1} - \frac{1}{k_2})^2]
= \sigma_d^2 (\frac{C}{H})^2 (\frac{1}{k_1} - \frac{1}{k_2})[\frac{r}{2} - 1] (\frac{1}{k_1} + \frac{1}{k_2})
\]

If \( \left( \frac{r}{2} - 1 \right) \frac{1}{k_1} + \left( \frac{r}{2} + 1 \right) \frac{1}{k_2} < 0 \) and \( \frac{1}{1+\sqrt{r}} k_2 < k_1 < k_2 \), then \( PC_4 - PC_6 < 0 \). \( \left( \frac{r}{2} - 1 \right) \frac{1}{k_1} + \left( \frac{r}{2} + 1 \right) \frac{1}{k_2} < 0 \) implies \( k_1 < \frac{2-r}{2+r} k_2 \). Thus \( PC_4 - PC_6 < 0 \) if \( \frac{1}{1+\sqrt{r}} k_2 < k_1 < \frac{2-r}{2+r} k_2 \). This requires \( \frac{1}{1+\sqrt{r}} < \frac{2-r}{2+r} \), which turns out to be \( r < (\sqrt{3} - 1)^2 \).

To summarize, if \( r \geq (\sqrt{3} - 1)^2 \), the tolerant policy is always better than the stringent policy (\( PC_4 - PC_6 > 0 \)); if \( r < (\sqrt{3} - 1)^2 \) and \( k_1 < \frac{2-r}{2+r} k_2 \), the stringent policy is better than the tolerant policy (\( PC_4 - PC_5 < 0 \), or \( PC_4 - PC_6 < 0 \)). □

**Proof of Proposition 5:**

**Proof.** (i) If \( \max\{\frac{1}{1+\sqrt{r}} k_2, \frac{2-r}{2+r} k_2\} < k_1 < k_2 \), the principal’s expected compensation cost in Program [3] is \( PC_3 = \frac{r}{2} (\frac{C}{H})^2 [\frac{1}{k_2} \sigma^2 + \frac{1}{k_2} \sigma_d^2] + \sigma_d^2 (\frac{C}{H})^2 (\frac{1}{k_1} - \frac{1}{k_2})^2 + 2C \) and the principal’s expected cost in Program [4] is \( PC_6 = \frac{r}{2} (\frac{C}{H})^2 [\frac{1}{k_1} \sigma^2 + \frac{1}{k_2} \sigma_d^2] + \sigma_d^2 (\frac{C}{H})^2 (\frac{1}{k_1} - \frac{1}{k_2})^2 + 2C \).
(ii) If $\frac{1}{1+\sqrt{r}} k_2 \leq k_1 \leq \frac{2}{2+r} k_2$, the principal’s expected compensation cost in Program [3] is $PC_3 = \frac{r}{2}(\frac{C}{k_1})^2[\sigma^2 + (\frac{2}{r+2})^2\sigma_2^2] + \sigma^2(\frac{r}{r+2})^2(\frac{C}{k_1H})^2 + 2C$, and the principal’s expected cost in Program [4] is $PC_6 = \frac{r}{2}(\frac{C}{k_1})^2\left[\frac{1}{k_1^2}\sigma^2 + \frac{1}{k_2^2}\sigma_2^2\right] + \frac{r^2}{2}\left(\frac{C}{k_1H}\right)^2\left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2$.

\[
PC_2 = \frac{r}{2}(\frac{C}{k_1})^2\left[\frac{1}{k_1^2}\sigma^2 + \frac{1}{k_2^2}\sigma_2^2\right] + \frac{r^2}{2}\left(\frac{C}{k_1H}\right)^2\left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2 - \sigma^2(\frac{r}{r+2})^2(\frac{C}{k_1H})^2 \left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2
\]

and $PC_2 - PC_6 = \frac{r}{2}(\frac{C}{k_1})^2\left[\frac{1}{k_1^2}\sigma^2 + \frac{1}{k_2^2}\sigma_2^2\right] + \frac{r^2}{2}\left(\frac{C}{k_1H}\right)^2\left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2 - \sigma^2(\frac{r}{r+2})^2(\frac{C}{k_1H})^2 \left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2$

If $\frac{1}{k_1^2} - \frac{1}{k_2^2} > 0$, then $PC_2 - PC_6 < 0$, and the introduction of encouraged hedging increases the principal’s expected cost.

(iii) If $\frac{2}{2+r} k_2 \leq k_1 \leq \frac{1}{1+\sqrt{r}} k_2$, the principal’s expected compensation cost in Program [3] is $PC_3$, and the principal’s expected cost in Program [4] is

\[
PC_5 = \frac{r}{2}(\frac{C}{k_1})^2\left[\frac{1}{k_1^2}\sigma^2 + \frac{1}{k_2^2}\left(\frac{1}{1+\sqrt{r}}\right)^2\sigma_2^2\right] + \frac{r^2}{2}\left(\frac{C}{k_1H}\right)^2\left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2 - \sigma^2(\frac{r}{r+2})^2(\frac{C}{k_1H})^2 \left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2
\]

and

\[
PC_3 - PC_5 = \frac{r}{2}(\frac{C}{k_1})^2\left[\frac{1}{k_1^2}\sigma^2 + \frac{1}{k_2^2}\left(\frac{1}{1+\sqrt{r}}\right)^2\sigma_2^2\right] + \frac{r^2}{2}\left(\frac{C}{k_1H}\right)^2\left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2 - \sigma^2(\frac{r}{r+2})^2(\frac{C}{k_1H})^2 \left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2
\]

If $\left(\frac{1}{k_1^2} - \frac{1}{k_2^2}\right) < 0$, that is, if $\sigma^2 < \frac{3(\sqrt{r}k_1^2 + (k_2 - k_1)^2)(r+2)^2}{\sqrt{r}k_1^2 + (k_2 - k_1)^2}$, then $PC_3 - PC_5 < 0$ and the introduction of encouraged hedging increases the principal’s expected cost.

(iv) If $k_1 < \frac{1}{1+\sqrt{r}} k_2$, the principal’s expected compensation cost in Program [3] is $PC_2$, and the principal’s expected cost in Program [4] is $PC_6$. $PC_2 - PC_6 = \frac{r}{2}(\frac{C}{k_1H})^2\left[\frac{2}{r+2}\sigma^2\right] - \frac{r^2}{2}\left(\frac{C}{k_1H}\right)^2\left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2 - \sigma^2(\frac{r}{r+2})^2(\frac{C}{k_1H})^2 \left(\frac{1}{k_1} - \frac{1}{k_2}\right)^2$

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\[
\left( \frac{C}{k_1H} \right)^2 \left[ 1 + \frac{r}{r+2} \frac{3r}{2(1+\sqrt{r})^2} \sigma_d^2 \right].
\]
When \( \frac{r}{r+2} \frac{3r}{2(1+\sqrt{r})^2} \sigma_d^2 < 0 \), that is, \( \sigma^2 < \frac{3(r+2)}{2(1+\sqrt{r})^2} \sigma_d^2 \), then \( PC_2 - PC_5 < 0 \) and the introduction of encouraged hedging increases the principal’s expected cost. ■

**Proof of Corollaries 2, 3, and 4:**

**Proof.** Directly from Propositions 4 and 5. ■