Inflation Determination with Taylor Rules:

Is New-Keynesian Analysis Critically Flawed?

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Cochrane (2007) has strongly questioned the basic economic logic of current monetary policy analysis, arguing that New Keynesian (NK) models imply rational expectations paths with explosive inflation that do not imply explosions in real variables relevant for transversality conditions. Consequently, the usual logic does not rule out solutions with explosive inflation. That result does not, however, justify negative conclusions about NK analysis, for a different criterion is logically satisfactory. It is that, to be plausible, a RE solution must satisfy the property of least-squares learnability. Adoption of this criterion serves to justify in principle the bulk of current mainstream analysis.

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1. Introduction

There are several significant reasons for concern regarding various aspects of current mainstream monetary policy analysis as widely practiced and described by Clarida, Gali, and Gertler (2000), King (2000), Svensson and Woodford (2005), Taylor (1999), Woodford (2003a), and other prominent writers.[1] Among these are the absence of any significant role for monetary aggregates, the failure to distinguish various interest rates important in the transmission mechanism, the non-observability of crucial policy variables such as the natural rate of output, and possible empirical weaknesses of key behavioral relationships including the "expectational IS function" and the usual Calvo-style price adjustment mechanism.[2]

There are also theoretical issues regarding the role of "determinacy"—defined in this literature as the existence of a single rational expectations (RE) solution that is dynamically stable—in the mainstream approach, issues that have been raised by, e.g., Bullard (2006), Bullard and Mitra (2002), and McCallum (2003). In this regard, John Cochrane (2007) has recently expressed a new concern regarding the mainstream approach, which includes as a central ingredient the assumption that monetary policy is conducted by means of a central-bank policy rule for period-by-period adjustment of a short-term nominal interest rate. To achieve determinacy, it is typically specified that this rule will call for adjustments of the policy interest rate by more than one-for-one in response to incipient movements in inflation—thereby satisfying a condition that is

[1] Some other notable items in the literature include Taylor (1993), Clarida, Gali, and Gertler (1999), Goodfriend and King (1997), King and Wolman (1996), Rotemberg and Woodford (1997). Goodfriend (2007) presents an overview that links formal analysis and actual central bank practice.
[2] Several of these issues have been studied in the July 2006 issue of the Journal of Monetary Economics.

widely referred to as the Taylor Principle.[3] Cochrane's contention is that the standard notion—i.e., that determinacy serves to guarantee that stable inflation behavior around target will be generated—is incorrect. His basic reasoning is expressed in the following quote:

> "I argue that the Taylor principle, in the context of new-Keynesian models, does not, in fact, determine inflation or the price level. Nothing in economics rules out explosive or "non-local" nominal paths. Transversality conditions can rule out real explosions, but not nominal ones" (Cochrane, 2007, p. 2).

Consideration of Cochrane's argument indicates that there is much merit in this point. Specifically, the equations of the mainstream's New Keynesian (NK)[4] models are typically consistent with the existence of RE solutions with explosive inflation rates, in addition to one or more stable paths. These imply "nominal explosions" that bring about paths along which real money balances tend to decrease in magnitude, in contrast to ones along which real balances grow without limit. More generally, explosive paths for inflation rates—or nominal interest rates—do not normally imply explosions in real variables relevant for transversality conditions, which are crucial for individual agent optimization. Consequently, the usual logic, in the usual models, does not imply the absence of explosive inflation.

It is argued below that this point of Cochrane's is itself correct. The point as just stated does not, however, justify negative conclusions about NK analysis.[5] For there is a

[3] The principle is often extended to cases in which policy responds also to incipient movements in the the output gap. See, e.g., Woodford (2003a, pp. 90-94, 252-261), Taylor (1999).
[4] New-Keynesian (NK) models are basically the same as those referred to by Goodfriend and King (1997) as "New Neoclassical Synthesis" (NNS) models. Some would consider the latter label to be more apt, from a historical perspective. Nevertheless, in what follows I will use the more standard label NK to refer to the models of current mainstream analysis.
[5] It should be said that the term "critically flawed" that appears in the present paper's title is my own. I

different criterion—in many (but not all) cases implied by determinacy and not generally implying determinacy—that is logically satisfactory for the purpose at hand. This is the requirement that, to be plausible, a RE solution should satisfy the property of learnability, of the type made prominent in the work of Evans and Honkapohja (E&H) (1999, 2001).[6] Adoption of this criterion, which should arguably be attractive to analysts concerned with actual monetary policy issues, serves to justify the bulk of current mainstream analysis in principle. Indeed, as argued in McCallum (2003, 2007), it eliminates some other problematic aspects of the NK analysis.

2. Cochrane's Critique

Let us begin by outlining the central aspects of the argument developed by Cochrane (2007) in the context of the basic NK model that is used for his main presentation. It is common, in the NK literature, to utilize a three-equation structure that includes an expectational IS function,[7] a Calvo-type price adjustment relation, and a Taylor-style policy rule in a system that could be written as

$$y_t = b_0 + b_1(R_t - E_t\pi_{t+1}) + E_t y_{t+1} + v_t \qquad b_1 < 0 \qquad (1)$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa(y_t - \bar{y}_t) \qquad \kappa > 0 \qquad (2)$$

$$R_t = \mu_0 + (1 + \mu_1)\pi_t + \mu_2(y_t - \bar{y}_t) + e_t \qquad (3)$$

where y_t represents output/consumption, $\bar{y}_t$ is its "natural rate" flexible-price value, π_t is

would think, however, that most exponents of NK analysis would regard a model/policy-rule combination, that does not rule out explosive inflation, to be critically flawed.

[6] More precisely, the requirement is least-squares (LS) learnability, as a necessary condition for a RE solution to be plausible and therefore of potential economic relevance. A similar learning process is featured in an influential early contribution by Marcet and Sargent (1989).

[7] This function represents a consumption Euler equation together with the economy's overall resource constraint.

inflation, and R_t is the one-period (nominal) rate of interest.[8] Here v_t and e_t are preference and policy shocks while $\bar{y}_t$ is exogenously generated by a (possibly) autocorrelated technology process. In his initial exposition, Cochrane simplifies by assuming full price flexibility so that $y_t = \bar{y}_t$ in each period, which eliminates the Calvo equation (2) and the output-gap term in (3).[9] If we also let $\bar{y}_t$ be a constant, then the IS relation (1) becomes

$$0 = b_0 + b_1(R_t - E_t\pi_{t+1}) + v_t. \tag{4}$$

Then if the shock term is neglected and $r = -b_0/b_1$ is recognized as a constant real rate of interest, we have the relation that is often termed a "Fisher equation." From the perspective of monetary policy, I think it better to describe it as is done here, but there is no substantive difference.[10] Thus the system at hand reduces to (3) and (4), identical to Cochrane's (1) and (2) if we delete the shock v_t from ours while also setting $\mu_0 = r$ in the policy rule, as a sensible central bank would do.

To solve this simple system we combine (3) and (4) to obtain

$$0 = b_0 + b_1[\mu_0 + (1+\mu_1)\pi_t + e_t - E_t\pi_{t+1}] + v_t \tag{5}$$

and conjecture that with $v_t = 0$ and $e_t = \rho e_{t-1}$ + white noise $(|\rho| < 1)$, there will be a solution of the form

$$\pi_t = \phi_0 + \phi_1 e_t \tag{6}$$

with expectations therefore obeying $E_t\pi_{t+1} = \phi_0 + \phi_1\rho e_t$. Substitution in (5) then implies

[8] The rule is written assuming a zero target rate of inflation. If it were non-zero, then a non-zero constant term would appear in the solution equation (7) below.

[9] Later in the paper, Cochrane (2007, pp. 23-27) extends his argument to a more standard NK model with Calvo-style price adjustments.

[10] My preference is to think of the "Fisher equation" as an identity, one which defines the one-period real rate of interest as a nominal rate minus expected inflation. The construct under discussion here is a model of individual saving behavior together with a market-clearing condition that consumption equals output in the aggregate plus the assumption that output is constant.

that this solution is

$$\pi_t = 0 - \frac{1}{1+\mu_1-\rho} e_t . \tag{7}$$

The latter is often referred to as the "fundamentals" or "MSV" solution.[11] With $\mu_1 > 0$ and $|\rho| < 1$, it implies that π_t is negatively related to e_t and that larger values of μ_1 serve to reduce the variability of π_t around its target. Now suppose, however, that instead of (6) one looks for a solution of the form

$$\pi_t = \phi_0 + \phi_1 e_t + \phi_2 \pi_{t-1}. \tag{8}$$

Then $E_t\pi_{t+1} = \phi_0 + \phi_1\rho e_t + \phi_2(\phi_1 e_t + \phi_2\pi_{t-1})$ and a second solution, in addition to (7), is

$$\pi_t = (1/\rho)e_t + (1+\mu_1)\pi_{t-1}. \tag{9}$$

Clearly, with $\mu_1 > 0$, as specified by the Taylor Principle, this expression (9) implies an explosive process for the inflation rate.[12] That is the type of solution referred to above and it seems clear that in the model at hand there is no transversality condition that would rule out this explosive solution for the inflation rate. Further investigation, pertaining to models in which the medium-of-exchange properties of money are explicitly recognized, will be considered below in Section 4. Pending that discussion, it would appear appropriate to accept, on a preliminary basis, the validity of Cochrane's critique of mainstream monetary policy analysis, with NK models and Taylor-style policy rules.

3. LS Learnability

Several analysts have argued, however, that for any RE solution to be considered

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[11] The identifying characteristic of this solution is that it does not include state variables in addition to those required by the structural model.

[12] It appears from (9) that this solution will not be defined in the measure-zero case with $\rho = 0$. But in that case one can add e_{t-1} as an additional state variable in (8) and obtain an infinity of explosive solutions that could be indexed by the start-up value of π_{t-1}.

plausible, and thereby relevant for policy analysis, it should be learnable.[13] Since there are some analysts who dispute that presumption,[14] I will proceed by arguing in effect that LS learnability (as defined by, e.g., E&H (2001, pp. 200, 232-233), should be regarded as a part of the requirement for a RE equilibrium, a part that amounts to a requirement of feasibility in terms of information. The motivation for this additional requirement is simple. It is that in any RE model intended to represent behavior in an actual market economy, the individual agents should be able to learn quantitative details concerning the behavior of variables—which they must forecast for decision-making purposes—from data generated by the economy itself.[15]

The first task at hand is to describe, in a reasonably precise way, the learning process that is being discussed. Rather than beginning with the highly special model given by equation (5), I will at first proceed in terms of a rather general linear framework that will give a better idea of the nature and scope of the learning process without requiring any significant complication in the presentation. Then a short presentation in terms of the special model (5) will also be included, at the end of this section. In the discussion, I assume that we are concerned with an economy in which the agents are alike, but are not aware of that fact, and behave entirely independently. Suppose that the

[13] These writers include Bullard (2006), Evans and Honkapohja (2001, p. 12-13), McCallum (2003, 2007), and (arguably) Woodford (2003a, pp. 261-276; 2003b, p. 1183).

[14] For example, Cochrane (2007, p. 44) states that "… a wide variety of almost philosophical principles have been advocated to prune equilibria. For example, Evans and Honkapohja (2001) advocate criteria based on least-squares (LS) learnability, and McCallum (2003) advocates a 'minimum state variable criterion,' which he relates to learnability. These refinements go beyond the standard definitions of economic equilibria. One may argue that when a model gives multiple equilibria, we need additional selection criteria. I argue instead that we need a different model."

[15] For RE to obtain, the implied forecasting relationships must be quantitatively accurate. The statement in this sentence is part of a rationale for my position concerning learnability; it is not intended to serve as a definition of learnability. In addition it should be noted that the present paper is concerned throughout with RE equilibria that are conventional except with respect to the informational aspect just mentioned. In particular, the paper does not address game-theoretic issues such as those implied by monetary policy actions that depart systematically from the specified policy rule.

behavior of per-capita values is given by

$$x_t = AE_t x_{t+1} + Cx_{t-1} + Dz_t \tag{10}$$

where x_t is a n×1 vector of endogenous variables, while exogenous variables z_t are

$$z_t = Rz_{t-1} + \varepsilon_t \tag{11}$$

with ε_t white noise and R is a stable matrix. Considering fundamental solutions of form

$$x_t = \Omega x_{t-1} + \Gamma z_t, \tag{12}$$

standard undetermined-coefficient reasoning establishes that, with RE specified in (10), Ω and Γ must satisfy

$$A\Omega^2 - \Omega + C = 0 \tag{13a}$$

$$\Gamma = A\Omega\Gamma + A\Gamma R + D. \tag{13b}$$

Given Ω, Γ is determined uniquely but there are many matrices Ω that satisfy (13a) and if more than one has all its eigenvalues smaller than 1.0 in modulus there are multiple stable solutions, i.e., a situation of "indeterminacy."

Now, for RE to prevail, agents need to have their expectations based on accurate quantitative knowledge of Ω and Γ; what the agents need to learn about is the system's law of motion. Such knowledge cannot, in reality and therefore in a satisfactory model, be obtained by introspection or divine revelation, but must be determined from data generated by the economy itself. The LS learning process for acquiring such knowledge is as follows. In period t, agents obtain <u>estimates</u> Ω_t and Γ_t by ordinary least squares using past data: they estimate the relationship $x_\tau = \Omega_t x_{\tau-1} + \Gamma_t z_\tau$ using data from periods dated $\tau = t-1$ and earlier. Using these estimates, the agents forecast x_{t+1} as

$x^e_{t+1} = \Omega_t x_t + \Gamma_t R z_t$.[16] [17] Then from substitution in (10), but using x^e_{t+1} in place of $E_t x_{t+1}$, the outcome x_t is generated—as a consequence of the supply and demand behavior summarized in (10)—as

$$x_t = A[\Omega_t x_t + \Gamma_t R z_t] + C x_{t-1} + D z_t. \tag{14}$$

Next, in period t+1 agents add the newly generated observation to their data, estimate Ω_{t+1} and Γ_{t+1}, form expectations x^e_{t+2}, make supply-demand choices via (10), and observe x_{t+1}. In periods t+2, t+3, ... the process continues.[18]

We visualize this process as going on indefinitely. Then, for a particular value of Ω (with associated Γ) we can ask if the process is "locally stable" in the sense that the estimates Ω_t and Γ_t approach the (hypothetically) true values Ω and Γ as $t \to \infty$, provided that they begin in the first period with estimates that are close to these true values. If the answer is "yes," then that particular RE solution is "stable under LS learning" and the model economy can be viewed as tending to behave as implied by that RE solution if it (the economy) has been operating for a large number of periods. If, on the other hand, a particular RE equilibrium, relating to a particular solution to (13a) for Ω, is not stable under LS learning, the implication/prediction of the analysis is that the model economy—and thus the actual economy represented by the model—will not be found in an

[16] The discussion proceeds for simplicity as if R were known. E&H (2001, p. 181) mention, however, that this assumption is not needed for the relevant results (since exogenous variables can be treated as endogenous). It might be mentioned incidentally that in the discussion of E&H (2001) the least squares estimation calculations are often described as being conducted by recursive least squares.

[17] It is being assumed that x_t and z_t are observed by agents in period t. It is possible, however, that the model specifies that expectations (and other determinants of actions in t) relevant in t are based on data only from earlier time periods.

[18] Evans and Honkapohja (2001) also consider a second information assumption, namely, that in period t an observation on x_t is not available in the learning process. In this case the term x_t on the right-hand side of (14) would be replaced with x^e_t. In the monetary model under discussion in Section 2, this would result in no change in the learnability findings—although it will in cases in which $C \neq O$ in (10).

equilibrium corresponding to this particular RE solution.[19]

Under what conditions will a particular RE solution, itself dynamically stable, be stable under LS learning? For the model (10)-(11), the basic analytical result of Evans and Honkapohja is, in my notation, that the LS learning process will be locally stable if all of the eigenvalues of the following matrices have real parts less than 1.0:

$$F = (I - A\Omega)^{-1} A \tag{15a}$$

$$\Omega' \otimes F \tag{15b}$$

$$R' \otimes F. \tag{15c}$$

The proof of this result is presented by E&H (1998) on pp. 26-32, the last two pages being an application to the linear framework of equations (10)-(12) of a more general analysis [by Benveniste, Métivier, and Priouret (1990)] that is summarized on E&H's pp. 26-30. Alternatively, the result is presented in the treatise of E&H (2001), with the summary statement of pp. 236-238 utilizing analysis developed on pp. 229-235 and 121-134. If, on the other hand, conditions (15) are not satisfied, E&H indicate that "… one can show convergence [obtains] with probability zero" (1998, p. 32).

At this point a critic might object on the grounds that there are many possible learning processes, and one cannot know that the one described above is realistic. My response to that observation emphasizes that LS learnability is being used, in the present argument concerning monetary policy, primarily as a <u>necessary</u> condition for plausibility of a RE equilibrium. In that context I will begin by quoting myself:

[19] It is the case that in some of their work E&H use a "constant gain" version of the LS learning process as a model of expectation formulation, rather than as a justification of a particular RE solution. See e.g., E&H (2001, pp. 333-359). Analysts following this approach include Orphanides and Williams (2005), among others.

> "... Of course any particular learning scheme might be incorrect in its depiction of actual learning behavior. But in this regard it is important to note that the LS learning process in question assumes that (i) agents are collecting an ever-increasing number of observations on all relevant variables while (ii) the structure is remaining unchanged. Furthermore, (iii) the agents are estimating the relevant unknown parameters (iv) with an appropriate estimator in (v) a properly specified model. Thus if a proposed RE solution is not learnable by the process in question—the one to which the E&H results pertain—then it would seem highly implausible that it could prevail in practice" (McCallum, 2007, p. 1378).

In other words, the process is distinctly "biased" toward a finding of learnability, which is suitable for a necessary condition.

It might be countered, as in footnote 14, that the notion being advanced—that individual agents should have some way of obtaining the information necessary to form expectations—represents an additional requirement not included in standard definitions of economic equilibria. That is so, but in standard (i.e., RE) analysis, for the determination of each period's endogenous variables, one specifies information sets that include quantitative features of the system plus, at a maximum, current and past values of relevant variables.[20] In stochastic models, therefore, knowledge of future variables is excluded as infeasible. In that manner, standard analysis does typically include a form of information feasibility as a requirement for equilibrium. What LS learnability does, in a sense, is to extend the requirement of information feasibility so as to cover quantitative knowledge of the economy's structure. More specifically, for agents to be able to form

[20] For some purposes perfect-foresight analysis is useful, of course, but one would not use that assumption in an analysis that is concerned with (e.g.) the variability of asset prices or macroeconomic variables.

expectations rationally—i.e., without systematic expectational errors—they must be able to develop quantitative knowledge of the economy's law of motion on the basis of observations from its past. Then LS learnability posits a particular process of information acquisition that is highly optimistic with respect to the possibility of the requisite information being acquired. In this sense, the absence of LS learnability may be regarded as representing a type of informational infeasibility.

With regard to monetary policy, the point of the present analysis is, of course, that application of the LS learnability criterion does, in the monetary policy model of Section 2, support solution (7) while eliminating the explosive solution of equation (9). That conclusion is implied by the analyses of Bullard and Mitra (2002) and Honkapohja and Mitra (2004), and is mentioned by McCallum (2003, p. 1161), but let us verify it here by reference to the setup of equations (1)-(6) and conditions (15).

To begin, it will be noted that, if there are no lagged endogenous variables in the system, then C = 0 implying that $\Omega = 0$ and F = A. Then the first two of conditions (15) amount to the requirement that the eigenvalues of A all have real parts less than 1. In the basic system summarized in (5) above, the fundamentals solution has $\Omega = 0$ and A = $1/(1 + \mu_1)$. Thus it is clear that the LS learnability requirements (15a-c) are satisfied. By contrast, the non-fundamentals solution (9) yields $\pi_t = (1/\rho)e_t + (1+\mu_1)\pi_{t-1}$, implying that $\Omega = 1 + \mu_1$ and thus that $F = (I - A\Omega)^{-1}A = [1-((1+\mu_1)/(1+\mu_1))]^{-1}(1/(1+\mu_1)) =$ $[1-1]^{-1}(1/(1+\mu_1)) = (1/(1+\mu_1))/0$. Thus in this case $F > 1 + \mu_1$ (an understatement) and at least two of the three conditions (15) are violated. Accordingly, the explosive solution is not learnable—convergence to (9) occurs with probability zero. Although (9) satisfies the orthogonality conditions for a RE solution, it is implausible (according to the present argument) that an economic system matching the specification would generate outcomes of the type that it describes.

It may be useful, for expository purposes, to describe the LS learning process again, but now for the special model of equation (5), which we now write as $\pi_t = aE_t\pi_{t+1} + u_t$, where $a = 1/(1+\mu_1)$ and $u_t = -ae_t$. The RE solution corresponding to (7) is then $\pi_t = \psi_1 u_t$ with $\psi_1 = \frac{1}{1-a\rho}$. For the LS learning process we assume that agents do not know the values of a or ψ_1 and accordingly use in place of $E_t\pi_{t+1}$ the value π^e_{t+1} to be defined momentarily. Thus in each period the agents estimate the relationship $\pi_t = \psi_{1t}u_t$, where the estimate ψ_{1t} is obtained by least squares regression with data from all previous periods: $\psi_{1t} = [\sum_{\tau=1}^{t-1} u_\tau^2]^{-1}\sum_{\tau=1}^{t-1} u_\tau\pi_\tau$.[21] Then expectations are given by $\pi^e_{t+1} = \psi_{1t}\rho u_t$ and π_t is generated as

$$\pi_t = a(\psi_{1t}\rho u_t) + u_t\,. \tag{16}$$

In this simple setting, it is easy to see that if the process is such that $\psi_{1t} \to \psi_1 = \frac{1}{1-a\rho}$, then in the limit we have

$$\pi_t = a\left[\left(\frac{1}{1-a\rho}\right)\rho u_t\right] + u_t = \left[\frac{a\rho}{1-a\rho} + 1\right]u_t = \left[\frac{1}{1-a\rho}\right]u_t\,, \tag{17}$$

that is, the RE solution (7). Does the process converge? Clearly the learnability conditions analogous to (15a)-(15c) are that the 1×1 matrices a, 0, and $a\rho$ all have eigenvalues with real parts less than 1, that is, that $a < 1$ and $a\rho < 1$. But $a < 1$ is implied

[21] In principle, a constant term should also be included. It is omitted here for expositional simplicity.

by the Taylor Principle condition $\mu_1 > 0$ whereas $|\rho| < 1$ by assumption. Consequently, we see that learnability prevails for the MSV solution (7'). In addition, the non-learnability result for the alternative solution (9) can be obtained in an analogous fashion.

Before moving on, brief mention should be made of the concept of "expectational stability," or "E-stability," which occurs frequently in the writings of Evans and Honkapohja. E-stability is a concept that is closely related to LS learnability, but is easier to establish technically, so is often used in learnability analysis. It pertains to a process that takes place not in actual (model economy) time with data being augmented from period to period, but to iterations in conceptual meta-time in the reasoning process of agents—with no reference to data collection. In practice, use of the E-stability concept, which concerns properties of a mapping from perceived to actual laws of motion, can be of analytical convenience but introduction of the concept is in principle unnecessary. A non-technical description of the E-stability process is provided in McCallum (2003, pp. 1157-1159) and extensive discussion appears in E&H (2001, e.g., pp. 39-43).

4. Medium of Exchange Money

To this point the discussion has included no explicit recognition of money, i.e., an asset that serves as a medium of exchange and thus facilitates transactions in the economy under consideration. It might be thought that such recognition could itself overturn the contention that has been under discussion, i.e., that NK models admit RE solutions with explosive inflation, since recognition of a monetary asset would give rise to a transversality condition pertaining to real money holdings—and transversality conditions tend to rule out certain explosive paths as equilibria because they violate conditions necessary for individual optimality. Such a conclusion would render the analysis above irrelevant. The literature on this topic does not support, however, the possibility of paths in which the discounted increment to utility provided by future real money balances fails to approach zero as the horizon considered increases indefinitely. The presumption in the literature is that an explosive inflation rate would tend to reduce real money holdings and, with the $T-1$ power of the discount factor approaching zero, thus to induce the relevant product to approach zero. A well known discussion is that of Obstfeld and Rogoff (1983), in which a model with medium-of-exchange money tends to rule out paths along which the price level approaches zero but not paths along which the price level explodes.

It is possible, however, that critics could object to the model used by Obstfeld and Rogoff on the grounds that its money-in-the-utility-function specification does not represent money's transaction-facilitating properties as adequately as would a shopping-time or transaction-cost model in which time or physical resources used up by transactions are specified to depend on real money holdings and real quantities of transactions. Additional analysis in that direction has been conducted by Gray (1984). Her paper does not consider every specification that one might desire, but it goes some substantial way toward justifying the conclusion of Obstfeld and Rogoff (1983, p. 686) that under a regime of pure fiat money "… explosive price-level paths—speculative hyperinflations can be ruled out only when severe restrictions are placed on individual preferences." [22]

Gray's analysis includes model specifications in which the medium-of-exchange

[22] (My emphasis.) Also, Obstfeld and Rogoff say "preferences" because their way of incorporating transaction costs is to include real money balances in agents' utility function. In Gray's models both preference and transaction-cost specifications are relevant.

role of money is represented by an explicit transaction-cost function. There is a remaining weakness in her argument, however, in that this function specifies that costs depend only upon real money holdings; one might instead specify that the costs depend also on the quantity of transactions conducted (per unit of time, of course). To represent that type of extension, consider the following setup, which is based in part on McCallum (2001).[23]

The economy consists of a large number of similar but independently-acting households, each of which has an intertemporal utility function

$$u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + ... \tag{18}$$

where c_t is consumption in period t. Each household supplies one unit of labor inelastically each period and uses n_t units of labor (the excess, if any, coming from the labor market) to produce $f(n_t, k)$ units of output. Its budget constraint for period t is

$$f(n_t, k) - tx_t - w_t(n_t - 1) = c_t + m_t - (1+\pi_t)^{-1}m_{t-1} + (1+r_t)^{-1}b_{t+1} - b_t + \psi(c_t,m_t). \tag{19}$$

Here tx_t is lump-sum taxes, w_t is the real wage, m_t is end-of-period real money balances, $1 + \pi_t = P_t/P_{t-1}$ where P_t is the price level in t, b_{t+1} is real one-period private bonds purchased in t, and r_t is the real rate of interest on these bonds. Finally, transaction costs are given by

$\psi(c_t, m_t)$, where $\psi_1 > 0$, $\psi_{11} < 0$, $\psi_2 < 0$, and $\psi_{22} > 0$ Both f and ψ have the Inada properties, as does u.

In this setting, a household's first-order optimality conditions include

$$u'(c_t) - \lambda_t[1 + \psi_1(c_t, m_t)] = 0 \tag{20a}$$

$$-\lambda_t w_t + \lambda_t f_1(n_t, k) = 0 \tag{20b}$$

[23] In McCallum (2001) there are a few "typos" including the following: In its equations (26) and (27), $\psi_1(c,m)$ should be $\psi_2(c,m)$.

$$-\lambda_t[1 + \psi_2(c_t, m_t)] + \beta E_t\lambda_{t+1}(1 + \pi_{t+1})^{-1} = 0 \tag{20c}$$

$$-\lambda_t(1 + r_t)^{-1} + \beta E_t\lambda_{t+1} = 0 \tag{20d}$$

where λ_t is the Lagrangian multiplier attached to (19). There is a transversality condition (TC) for bonds,

$$\lim_{T\to\infty} b_{T+1}\beta^{T-1}\lambda_T = 0 \tag{21}$$

and another for money:

$$\lim_{T\to\infty} m_T\beta^{T-1}\lambda_T = 0\ . \tag{22}$$

The latter two will be taken as necessary for individual-household optimality.[24]

For general equilibrium, we must also have

$$n_t = 1 \tag{23}$$

$$b_{t+1} = 0 \tag{24}$$

$$m_t = M_t/P_t \tag{25}$$

and the government budget constraint[25]

$$tx_t + m_t - (1+ \pi_t)^{-1}m_{t-1} = 0. \tag{26}$$

In the latter we take government consumption to equal zero, for simplicity. Thus money is injected via lump-sum transfers (negative taxes). With the government exogenously setting values of M_t, equilibrium paths for the nine endogenous variables c_t, m_t, λ_t, n_t, w_t, P_t, π_t, r_t, and b_t are given by equations (19), (20a), (20b), (20c), (20d), (23), (24), (25), and the definition $1 + \pi_t = P_{t+1}/P_t$.

[24] The TC (22) can be thought of as arising, from a T-period version of the optimization problem, as a limiting version as $T \to\infty$ of the second part of the Kuhn-Tucker condition $\partial L/\partial m_T \leq 0$ plus $m_T\partial L/\partial m_T = 0$, since $\partial L/\partial m_T = -\beta^{T-1}\lambda_T$. (Here L refers to the problem's Lagrangian expression.)
[25] By excluding bonds from (26) I am implicitly assuming that bonds are private loans from one household to another.

In this economy, the demand for money relation is, from equations (20c) and (20d),

$$1+\psi_2(c_t, m_t) = \frac{1}{1+R_t} \tag{27}$$

where the nominal interest rate R_t satisfies the Fisher identity

$$1+R_t = (1+r_t)(1+\pi_{t+1}) . \tag{28}$$

(I ask the reader to please include expectation operators where appropriate.) With flexible prices, r_t will be a constant and so as π_{t+1} explodes, $1+\psi_2(c_t, m_t)$ will approach zero. That is, $-\psi_2(c_t, m_t)$, the marginal benefit from holding money, will approach 1 from below as m_t/c_t declines with the increasing cost of holding money.

To get a feel for the situation, let's consider the specific transaction-cost function used in McCallum (2001), viz.,

$$\psi(c, m) = a_1 c(c/m)^{a_2}, \qquad a_1 > 0,\ a_2 > 0 \tag{29}$$

in which the average transaction cost has a constant-elasticity relationship to c/m. Note that this function is one in which the average transaction cost grows without limit as real money balances approach zero. But as inflation explodes, $1+\psi_2(c_t, m_t) = 1 - a_2 a_1 (c/m)^{1+a_2}$ approaches zero and the limiting condition has $c/m = (1/a_2 a_1)^{1/(1+a_2)}$ so m does not approach zero; it approaches a limiting value of (say) m* > 0. The calibration adopted in McCallum (2001) has $a_1 = 0.00102$ and $a_2 = 4$, in which case c/m* equals 3.96. This calibration is designed to pertain to the quarterly frequency, suggesting that real money holdings in this limiting high-inflation situation would be about 1/4 of quarterly spending. In the recent U.S. data, by contrast, the value of m/c (M1 concept) is of the order of magnitude of 1, about four times this limiting value.

The relevance of all this for the issue at hand is that the transversality condition for money holdings does not fail in the model at hand as inflation explodes. The value of λ_t in our model is, from (20a),

$$\lambda_t = \frac{u'(c_t)}{1+\psi_1(c_t, m_t)} = \frac{u'(c_t)}{1+(1+a_2)a_1(c_t/m_t)^{a_2}}, \tag{30}$$

But from our calculations in above, we see that the latter does not grow without bound as inflation explodes; the right-hand side of (30) approaches a positive limit. Then with λ_T remaining finite, violation of the TC (22) will not occur, since both m_T and λ_T remain finite while β^{T-1} approaches zero as the horizon T extends indefinitely.

A heuristic but more general argument can be made without reference to any specific transaction technology, as follows. Transversality conditions for money holdings are in general limiting values (as $T \to \infty$) of the product of three terms: β^{T-1}, m_T, and λ_T, where the latter is a Lagrange multiplier on the budget constraint for period T. If this limit is zero, the TC is satisfied; if for some path it is not satisfied, that path is not optimal for the agent. Now clearly β^{T-1} approaches zero as T grows, so for the TC to fail, either m_T or λ_T must explode. Exploding inflation tends, however, to drive the former toward small values. So the only way that the TC can fail is for $\lambda_T \to \infty$ as $T \to \infty$.

But since λ_T is a multiplier on the budget constraint, with each term in the latter expressed in real terms, λ_T represents the increase in utility made possible by a one-unit decrease in end-of-period-T real money holdings, at the margin. This magnitude will tend to increase as money holdings decrease, but it cannot plausibly increase without bound. Marginal utility of consumption may $\to \infty$ as $c_t \to 0$, but a model specification that drives consumption to zero (as real money holdings decrease) implies that a barter economy would necessarily feature zero consumption. That should be regarded as an inadmissible assumption, however, even by an enthusiast for the MOE role of money. This, I suggest, is what lies behind the argument to the effect that exploding inflation does not imply the failure of any TC.

5. Conclusions

It is clearly important that the logical foundations of the dynamic models used in current mainstream monetary policy analysis be clearly understood. In that regard, the recent paper by Cochrane (2007) makes a substantial contribution by pointing out that determinacy, in the sense of a unique solution that is dynamically stable, does not provide an adequate foundation for analyses of the standard NK type. It is the case, however, that a more satisfactory criterion, that of least-squares learnability, is available. Basic informational requirements suggest that learnability should be considered necessary for the plausibility of any rational expectations equilibrium,[26] and acceptance of this view rules out explosive solutions of the type discussed above.[27] A useful by-product of such acceptance may be that "indeterminacy" ceases to be a problem in other circumstances, such as strenuous inflation-forecast targeting.[28]

[26] Indeed, perhaps, for the definition of an equilibrium.
[27] In this paper I have focused on the argument of Cochrane (2007) and accordingly have not considered more esoteric classes of sunspot solutions.
[28] This position is argued by McCallum (2003). Woodford's (2003b) comment on the latter stresses objections to McCallum's "minimal state variable" solution concept, but features little disagreement with that paper's substantive positions (which do not rely upon that concept).

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