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# Optimizing the Tactical Planning in the FMCG Industry Considering Shelf-Life Restrictions

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## Abstract

In this paper we address the optimization of the tactical planning for the Fast Moving Consumer Goods industry using an MILP model. Shelf-life restrictions are introduced into this model to prevent unnecessary waste and missed sales. Three methods for implementing shelf-life restriction are compared. In the direct method the age of each product is tracked. While this method can provide optimal solutions, it is computationally inefficient. In the indirect method, products are forced to leave inventory at the end of their shelf-life. For supply chains consisting of two or more storage echelons this method cannot guarantee optimality. Nevertheless, the solutions obtained with the indirect method were always within a few percent of optimality. Moreover, on average, the computational time was reduced by a factor 32 when using the indirect method instead of the direct method. Finally, the hybrid method models the product age directly in the first storage stage, while considering the shelf-life indirectly in the second stage. The hybrid method obtains near-optimal solutions and, on average, the computational time is reduced more than 5 times compared to the direct method. Cases of up to 25 SKUs were optimized using the direct method, up to 100 SKUs using the hybrid method, and up to 1000 SKUs using the indirect method.

## Keywords:

## 1. Introduction

Due to the increasingly competitive global market, companies with a global supply chain have to continuously optimize their supply chain operations. Optimizing the supply chain operations could, for example, allow a company to reduce the inventory while maintaining high customer satisfaction levels (Papageorgiou,2009). Grossmann (2005) and Varma et al. (2007) review the research on Enterprise-Wide Optimization (EWO), which focuses on optimizing the procurement, production and distribution operations.

In this paper, we consider these procurement, production and distribution operations over a one year horizon. In specific, we want to optimize these decisions on the tactical planning level for a Fast Moving Consumer Goods Company (FMCG). Examples of FMCG are yoghurt, ice cream and shampoo.

FMCG are products that are replaced/used up within a relatively short period, which depending on the product ranges from days to a year. They are usually quickly substituted when not available, and they are generally produced in large quantities. Because of these large quantities, they are profitable despite typically low profit margins. Therefore, optimizing the

tactical planning of a FMCG company is important to ensure that the products remain profitable, while ensuring that they are available in the right place at the right time.

For example, Kellogg greatly reduced its production, distribution, and inventory cost through the use of Linear Programming (LP) planning models (Brown et al., 2001). For an extensive review on quantitative optimization methods for the food supply chain, we refer to Akkerman et al. (2010). They mention that the perishability of the products is an important challenge in the optimization of the operations in a food supply chain.

Considering this perishability is important because product freshness is one of the primary concerns for consumers when buying food products. Consumers can judge the freshness of a product either by evaluating the sensory qualities of the product or by the Best-Before-Date (BBD) listed on the packaging. Since many products are fully packed, the consumer must often rely on calculating the remaining shelf-life based on this BBD. (Entrup, 2005)

Shelf-life is defined by the Institute of Food Science & Technology (1993) as “*the time during which the food product will remain safe, be certain to retain the sensory, chemical, physical and microbiological characteristics, and comply with any label declaration of nutritional data.*”

Because the product freshness is important for consumers, the retailers will require the products they receive to have a certain minimum remaining shelf-life. Therefore, only part of the shelf-life can be used in the supply chain up to the retailers. For the remainder of this paper, shelf-life refers to the part of the shelf-life that may be used in the supply chain before the retailers.

If the shelf-life is not considered in the tactical planning problem, part of the inventory could exceed its shelf-life. This would not only result in disposal costs, but the reduced inventory might not be sufficient to meet the demand, which would lead to missed sales. Therefore, considering shelf-life limitations in the tactical planning problem is crucial. Nevertheless, the implementation of shelf-life limitations in the tactical planning has only received limited attention in literature.

Much of the research regarding implementing shelf-life limitations focuses on adding shelf-life constraints to the Economic Lot Scheduling Problem (ELSP). An overview of the major contributions in this area is given in Soman et al. (2004) and Entrup et al. (2005). However, these models typically assume a constant demand rate. This is unrealistic for the food industry, which has many seasonal products and intense promotional activities. (Entrup et al., 2005).

Another part of the research in this area focusses on the quality degradation over time. Entrup (2005) integrates shelf-life in the advanced planning for fresh food industries. He relates the revenue of a product to its remaining shelf-life. The longer the remaining shelf-life, the more valuable the product. The shelf-life is modeled by tracking the production day and selling day of each product.

Farahani et al. (2011) propose an iterative scheme that integrates the production and distribution decisions for a perishable food company. They compare their integrated approach to a sequential planning approach. A penalty is added to the objective function for the quality decay of the products. They assume a linear decay for each day that a product remains in storage. Ahumada and Villalobos (2009) consider a similar linear decay penalty for the production and distribution of fresh produce. In addition, they limit the maximum shelf-life based on the harvest period and the sales period.

Rong et al. (2011) optimize a food supply chain, while managing the food quality. The quality degradation per period is linearly dependent on the temperature, which can be varied for each location. The shelf-life is then considered by imposing a minimum quality requirement.

Amorim et al. (2012) consider the shelf-life using two methods. In the first method, the maximum shelf-life is enforced directly through the production and sales dates of the products. In the second method, similar to Rong et al. (2011), they adjust the remaining shelf-life in each period according to the storage conditions. They use two objective functions. In the first one, the overall costs are minimized. In the second objective, the remaining shelf-life of the products sent to the distribution centers is maximized. Using these two objectives, they consider the trade-off between costs and the value of freshness.

Eksioglu and Jin (2006) optimize the tactical planning for perishable products in a two-stage supply chain, consisting of production facilities and retailers. They add a constraint to ensure that the inventory at a production facility in any period cannot exceed the amount that is sent to the retailers in the next  $X$  periods, where  $X$  is the shelf-life. However, their model formulation limits the retailers to receiving product from a single factory.

Gupta and Karimi (2003) consider the shelf-life of intermediate products in the short-term scheduling of a two-stage multiproduct process. They introduce a constraint that forces the second stage processing of a batch of product to start before the end of the first stage processing of a product lot plus the shelf-life of the product. Using a big-M formulation, they relax this constraint for second stage batches that are not produced from this first stage lot.

Finally, Susarla and Karimi (2012) optimize the tactical planning for pharmaceutical companies while considering the shelf-life. They directly model the age of each product, and set the maximum age equal to the shelf-life.

In summary, when shelf-life is considered in literature, it is typically considered directly: Either by tracking the age of products, by tracking the production and sales dates, or through the product quality. While directly tracking the shelf-life is accurate, it is relatively inefficient, as we will show in this paper. Therefore, it might not be a tractable method for larger, more realistically sized problems. In this paper, we propose two other, computationally more efficient, methods that also accurately track the shelf-life.

The remainder of this paper is organized as follows. The problem is defined in Section 2. In Section 3, the base tactical planning MILP model without shelf-life constraints is given. Section 4 introduces the three methods to include shelf-life restrictions into this tactical planning model. The results are discussed in Section 5, and the conclusions are drawn in Section 6.

## **2. Problem Definition**

Given is a set of Stock-Keeping Units (SKUs). These SKUs are products that may differ in composition and/or packaging. Given is also a supply chain consisting of suppliers, factories, warehouses, distribution centers and retailers. The operation of the supply chain is considered over a one year horizon that is divided into 52 weekly time periods to account for the seasonality of the demand.

The objective is to minimize the total costs of operating the supply chain. The costs include the procurement costs, set-up costs, transportation costs, inventory costs, safety stock violation costs, SKU waste disposal costs, and missed sales costs. For each week, the availability and cost of procuring the ingredients is known for each supplier. The initial ingredient inventory and the ingredient inventory capacity of all factories are given. Using known recipes, SKUs are produced from these ingredients at the factories. The production process is a two-stage make and

pack production process where either stage could be the bottleneck depending on the selected SKUs. The mixing and packing rates are known for all SKUs in all factories, and the available mixing and packing time is given for each factory as well. Each SKU that is produced in a factory in a week requires a given set-up time. In addition, all SKU families from which at least one SKU is produced require a given family set-up time.

There is no storage of SKUs at the factories and SKUs must, therefore, be transported to one of the warehouses in the same week they are produced. The initial inventory and inventory capacities of the warehouses and distribution centers are known. A desired safety stock level is given for each SKU in each location. The storage costs and the safety stock violation costs are also given.

The transportation costs between all facilities in the supply chain are known. An SKU can only be transported from one echelon of the supply chain to the next; it cannot skip an echelon. All SKUs must leave the supply chain before the end of their shelf-life. The shelf-life is known for each SKU. Any SKU that remains in the supply chain for longer than its shelf-life will become waste. The disposal cost of this SKU waste is given. Finally, a forecast for the weekly demand is given as well as costs for missed sales. It is not allowed to have a backlog of demand. Either the demand is met in the week that it occurs in, or missed sales costs are incurred.

### 3. Tactical Planning Model

This problem can be represented by an MILP model. For this model we start with the MILP model by van Elzakker et al. (2013), which describes this problem without considering the shelf life restrictions. An overview of this model will be given below and afterwards the possible methods for introducing shelf-life constraints into this model will be discussed.

$$\sum_f TransIng_{h,f,s,t} \leq MaxSupply_{h,s,t} \quad \forall h,s,t \quad (1)$$

$$\sum_h INVIng_{h,f,t} \leq INVIngCap_f \quad \forall f,t \quad (2)$$

$$INVIng_{h,f,t} = INVIng_{h,f,t-1} + \sum_s TransIng_{h,f,s,t} - \sum_i (Recipe_{h,i} \cdot Prod_{i,f,t}) \quad \forall h,f,t \quad (3)$$

$$\sum_{(i \in IM_{mfam})} \frac{Prod_{i,f,t}}{MixRate_{i,f}} \leq MixTime_{mfam,f} \quad \forall mfam,f,t \quad (4)$$

$$\begin{aligned} & \sum_{i \in IP_{pfam}} \left( \frac{Prod_{i,f,t}}{PackRate_{i,f}} + SUT_i \cdot WSU_{i,f,t} \right) \\ & + \sum_{fam \in FAM_{pfam}} [FamSUT_{fam} \cdot YFamSU_{fam,f,t}] \\ & \leq PackTime_{pfam,f} \quad \forall pfam,f,t \quad (5) \end{aligned}$$

$$\frac{Prod_{i,f,t}}{PackRate_{i,f}} \leq PackTime_{pfam,f} \cdot WSU_{i,f,t} \quad \forall i \in IP_{pfam}, pfam, f, t \quad (6)$$

$$YFamsU_{fam,f,t} \geq WSU_{i,f,t} \quad \forall i \in IF_{fam}, fam, f, t \quad (7)$$

$$\sum_w TransFW_{i,f,w,t} = Prod_{i,f,t} \quad \forall i, f, t \quad (8)$$

$$\sum_i INVWH_{i,w,t} \leq WHCap_w \quad \forall w, t \quad (9)$$

$$\sum_i INVDC_{i,dc,t} \leq DCCap_{dc} \quad \forall dc, t \quad (10)$$

$$INVWH_{i,w,t} = INVWH_{i,w,t-1} + \sum_f TransFW_{i,f,w,t} - \sum_{dc} TransWDC_{i,w,dc,t} \quad \forall i, w, t \quad (11)$$

$$INVDC_{i,dc,t} = INVDC_{i,dc,t-1} + \sum_w TransWDC_{i,w,dc,t} - \sum_r TransDCR_{i,dc,r,t} \quad \forall i, dc, t \quad (12)$$

$$SSVioWH_{i,w,t} \geq SSWH_{i,w,t} - INVWH_{i,w,t} \quad \forall i, w, t \quad (13)$$

$$SSVioDC_{i,dc,t} \geq SSDC_{i,dc,t} - INVDC_{i,dc,t} \quad \forall i, dc, t \quad (14)$$

$$\sum_{dc} TransDCR_{i,dc,r,t} \leq D_{i,r,t} \quad \forall i, r, t \quad (15)$$

$$MissedSales_{i,r,t} \geq D_{i,r,t} - \sum_{dc} TransDCR_{i,dc,r,t} \quad \forall i, r, t \quad (16)$$

$$\begin{aligned} \text{Min TotalCosts} = & \sum_{h,f,s,t} TransIng_{h,f,s,t} \cdot (CostIng_{h,s,t} + TCSF_{f,s}) \\ & + \sum_{h,f,t} INVIng_{h,f,t} \cdot SCIng_{h,f} + \sum_{i,w,t} INVWH_{i,w,t} \cdot SCWH_{i,w} + \sum_{i,dc,t} INVDC_{i,dc,t} \cdot SCDC_{i,dc} \\ & + \sum_{i,f,w,t} TransFW_{i,f,w,t} \cdot TCFW_{f,w} + \sum_{i,w,dc,t} TransWDC_{i,w,dc,t} \cdot TCWH_{w,dc} \\ & + \sum_{i,dc,r,t} TransDCR_{i,dc,r,t} \cdot TCDCR_{dc,r} \\ & + \sum_{i,w,t} SSpencerCost \cdot SSVioWH_{i,w,t} + \sum_{i,dc,t} SSpencerCost \cdot SSVioDC_{i,dc,t} \\ & + \sum_{i,f,t} SUCost_i \cdot WSU_{i,f,t} + \sum_{fam,f,t} FAMSUCost_{fam} \cdot YFAMSU_{fam,f,t} \\ & + \sum_{i,r,t} MSpencerCost_{i,r,t} \cdot MissedSales_{i,r,t} \end{aligned} \quad (17)$$

Constraint (1) limits the procurement by the availability of supply. Constraint (2) limits the storage of ingredient by the storage capacity and constraint (3) is the inventory balance.

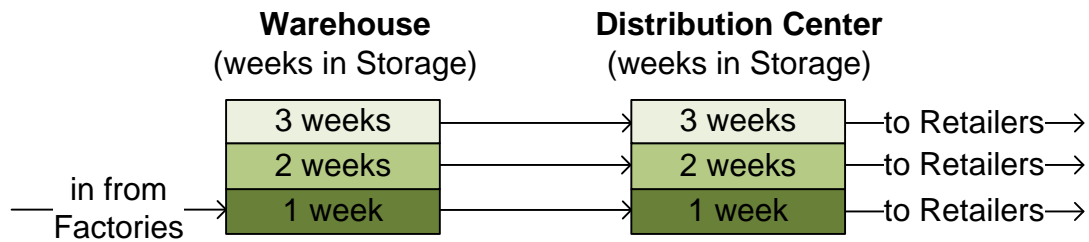
Constraints (4) and (5) ensure that the available mixing and packing time is not exceeded. Constraint (6) dictates that an SKU can only be produced if there is a set-up for this SKU, while constraint (7) enforces an SKU family set-up if there is a set-up to at least one of the SKUs of this family. The total weekly transport to the warehouses from a factory is set equal to the production in that factory through constraint (8). The storage in warehouses and distribution centers is limited by the storage capacity in constraints (9) and (10). The inventory balances of the warehouses and distribution centers are given in constraints (11) and (12), and the safety stock violations are calculated through constraints (13) and (14). Finally, the total amount of each SKU sent to each retailer is limited by the demand in constraint (15), and the missed sales is defined as the retailer demand minus the amount sent to the retailer by constraint (16). The objective of the model is to minimize the total cost which, as shown in constraint (17), is the sum of the procurement, inventory, transport, safety stock violation, set-up, and missed sales costs. For a more detailed description of this model we refer to van Elzakker et al. (2013).

#### 4. Shelf-Life

We consider three methods of implementing the shelf-life restrictions into the tactical planning model.

##### 4.1. Direct Shelf-Life Implementation

In the direct shelf-life implementation, the shelf-life is considered directly. An additional index  $a$ , the age of an SKU, is introduced for all inventory and transportation variables. This index represents the number of weeks since an SKU has been produced. As shown in Figure 1, this method keeps track of the age of each SKU. When the shelf-life is considered in literature, it is typically considered using this direct shelf-life implementation. For example, Susarla and Karimi (2012) directly model the age of the products in their supply chain to enforce shelf-life restrictions.



*Figure 1. Overview of the direct shelf-life method for an SKU with a shelf-life of 3 weeks*

For the direct shelf-life implementation, the following constraints are introduced. The total inventory of all SKUs  $i$  of any age  $a$  cannot be greater than the inventory capacity in any location at any time.

$$\sum_{i,a} INVWH_{i,w,t,a} \leq WHCap_w \quad \forall w,t \quad (18)$$

$$\sum_{i,a} INVDC_{i,dc,t,a} \leq DCCap_{dc} \quad \forall dc,t \quad (19)$$

The inventory of an SKU  $i$  with an age of one week in a warehouse  $w$  is equal to the incoming amount from the factories minus the amount of SKU  $i$  that is one week old that is sent on to the distribution centers.

$$INVWH_{i,w,t,a} = \sum_f TransFW_{i,f,w,t} - \sum_{dc} TransWDC_{i,w,dc,t,a} \quad \forall i, w, t, a = 1 \quad (20)$$

The inventory of an SKU  $i$  with an age  $a$  in a warehouse  $w$  is equal to the inventory that was  $a-1$  weeks old in the previous week, minus the amount of SKU  $i$  that is  $a$  weeks old that is sent to the distribution centers, minus the amount that becomes waste. This waste variable is only defined for SKUs with an age  $a$  equal to their shelf life since it is assumed that no SKUs will be disposed unless they have reached the limit of their shelf-life.

$$INVWH_{i,w,t,a} = INVWH_{i,w,t-1,a-1} - \sum_{dc} TransWDC_{i,w,dc,t,a} - WasteWH_{i,w,t,a} \quad \forall i, w, t, 1 < a \leq SL_i \quad (21)$$

The inventory of an SKU  $i$  with an age of  $a$  weeks in a distribution center  $dc$  is equal to inventory that was  $a-1$  weeks old in the previous week, plus the incoming amount from the warehouses that is  $a$  weeks old, minus the amount that that is sent to the retailers that is  $a$  weeks old, minus the amount that becomes waste. Similarly to the warehouses, the waste variable is only defined for SKUs that have reached the end of their shelf life.

$$INVDC_{i,dc,t,a} = INVDC_{i,dc,t-1,a-1} + \sum_w TransWDC_{i,w,dc,t,a} - \sum_r TransDCR_{i,dc,r,t,a} - WasteDC_{i,dc,t,a} \quad (22)$$

$\forall i, dc, t, a$

The safety stock violation in a location is larger than or equal to the safety stock minus the total inventory level of an SKU in that location.

$$SSvioWH_{i,w,t} \geq SSWH_{i,w,t} - \sum_a INVWH_{i,w,t,a} \quad \forall i, w, t \quad (23)$$

$$SSvioDC_{i,dc,t} \geq SSDC_{i,dc,t} - \sum_a INVDC_{i,dc,t,a} \quad \forall i, dc, t \quad (24)$$

The total amount of SKU  $i$  of all ages sent to a retailer is limited by the demand at the retailer.

$$\sum_{dc,a} TransDCR_{i,dc,r,t,a} \leq D_{i,r,t} \quad \forall i, r, t \quad (25)$$

Finally, the missed sales are equal to the retailer demand of SKU  $i$  minus the total amount of all ages of this SKU sent to this retailer.

$$MissedSales_{i,r,t} \geq D_{i,r,t} - \sum_{dc,a} TransDCR_{i,dc,r,t,a} \quad \forall i, r, t \quad (26)$$

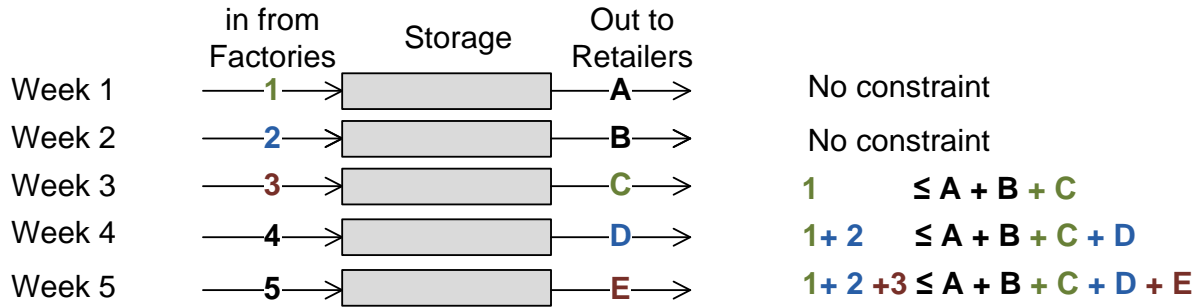
These constraints replace constraints (9)-(16) of the base tactical planning model. In addition, a cost term for disposing waste is added to the objective function. While the direct shelf-life implementation allows the tactical planning to be optimized considering the exact



shelf-life limitations, it also greatly increases the model size. Therefore, we also consider two other options for modeling the shelf-life.

#### 4.2. Indirect Shelf-Life Implementation

Instead of tracking the age of all SKUs directly, we can also introduce constraints that force an SKU to leave the supply chain at the end of its shelf life. For a supply chain with a single storage echelon such constraints are relatively straight forward as shown in Figure 2. For simplicity the initial inventory and the waste streams are assumed to be zero in this example.

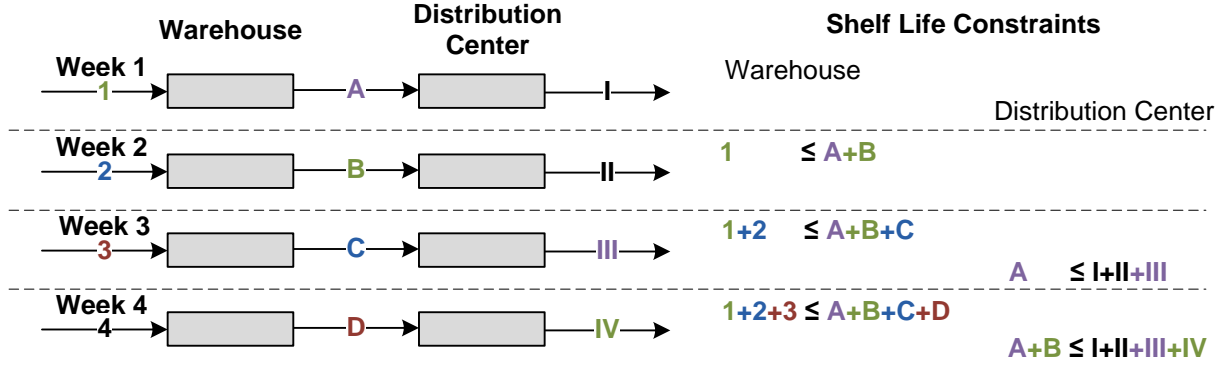


*Figure 2. Example of the indirect shelf-life constraints for a supply chain with a single storage echelon and an SKU with a 3-week shelf life*

The incoming SKUs from the factories in week 1 have an age of 1 week at the end of week 1. At the end of week 3, these SKUs have reached their maximum shelf life of 3 weeks. Therefore, the sum of the amount sent to the retailers in weeks 1-3 ( $A+B+C$ ) must be at least as large as the amount that was received in week 1. It could be larger, since part of the SKU that was received in weeks 2 and 3 could already be sent on to the retailers. At the end of week 4, the SKUs that arrived in week 2 have reached the end of their shelf-life and we, therefore, know that the total amount sent to the retailers in weeks 1-4 must be at least sufficient to cover the incoming SKUs in weeks 1-2. Similarly, the outgoing flow in weeks 1-5 can be coupled with the incoming flow in weeks 1-3. This concept is similar to the concept behind the shelf-life constraint introduced by Eksioglu and Jin (2006). They limit the inventory to the amount of product that leaves the storage in the next  $X$  weeks, with  $X$  being the shelf-life of the product.

However, in a supply chain with two storage echelons, we would not know the age of the SKUs arriving in the second storage stage and we could, therefore, not apply these constraints. Nevertheless an indirect shelf-life implementation seems attractive since the model would be considerably smaller than a direct shelf-life model. Therefore, we propose to manually divide the shelf-life over the storage echelons.

For example, if the total shelf-life of an SKU is 4 weeks, we could dedicate 2 weeks to the warehouses and 2 weeks to the distribution centers. If an SKU arrives in a warehouse in week 1, it can thus remain in this warehouse for at most two weeks. Therefore, the SKU that is produced in week 1, must be sent to a distribution center by the end of week 2. The SKU that arrives in the distribution center in week 2 is at most 2 weeks old by the end of week 2. Therefore, it must be sent to the retailers before the end of week 4, when the SKU is at most 4 weeks old. An overview of this example is given in Figure 3.



**Figure 3.** Example of the indirect shelf-life constraints for a supply chain with two storage echelons and an SKU with a 4-week shelf-life which is divided into a 2 week warehouse and a 2 week distribution center shelf-life

The following two constraints ensure that the SKUs will not exceed their shelf-life. The part of the initial inventory that reaches the end of its warehouse shelf life before or at the end of the current period, plus the amount received from the factories that reaches the end of its warehouse shelf life before or at the end of the current period is less than or equal to the amount that is transported to the distribution centers until the end of the current period, plus the amount that is disposed of before the end of the current period. This constraint ensures that an SKU will be disposed of if it is not transported to the distribution centers before the end of its warehouse shelf-life.

$$\begin{aligned}
 & \sum_{a \geq WHSL_t + 1 - t} INVWHini_{i,w,a} + \sum_{f,t' \leq t+1-WHSL_t} TransportFW_{i,f,w,t'} \\
 & \leq \sum_{dc,t' \leq t} TransportWDC_{i,w,dc,t'} + \sum_{t' \leq t} WasteWH_{i,w,t'} \quad \forall i, w, t
 \end{aligned} \tag{27}$$

Similarly, a constraint is introduced that ensures that an SKU will be disposed of if it is not transported to the retailers before the end of its distribution center shelf-life.

$$\begin{aligned}
 & \sum_{a \geq ShelfLife_t + 1 - t} INVDCini_{i,dc,a} + \sum_{w,t' \leq t-DCSL_t} TransportWDC_{i,w,dc,t'} \\
 & \leq \sum_{r,t' \leq t} TransportDCR_{i,dc,r,t'} + \sum_{t' \leq t} WasteDC_{i,dc,t'} \quad \forall i, dc, t
 \end{aligned} \tag{28}$$

In addition, a waste term is added to the inventory balances:

$$INVWH_{i,w,t} = INVWH_{i,w,t-1} + \sum_f TransFW_{i,f,w,t} - \sum_{dc} TransWDC_{i,w,dc,t} - WasteWH_{i,w,t} \quad \forall i, w, t \tag{29}$$

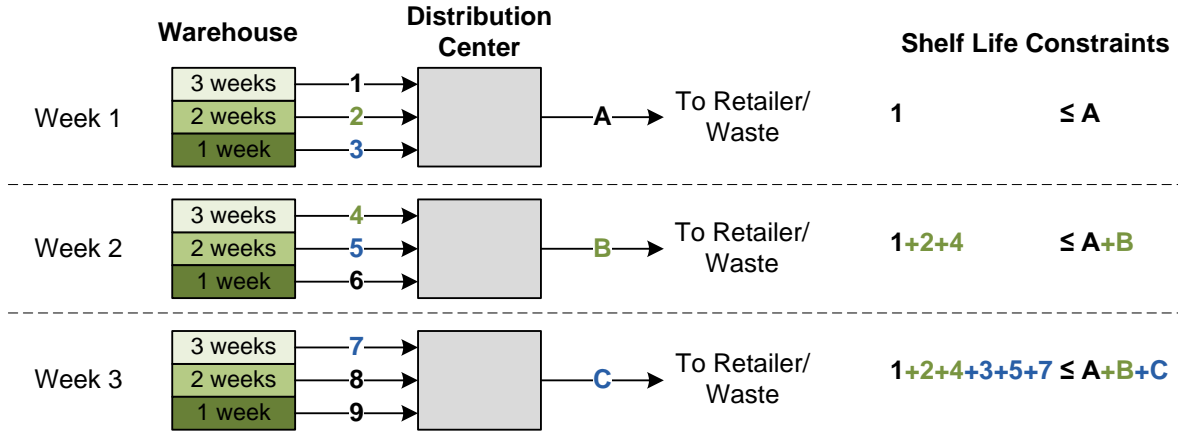
$$INVDC_{i,dc,t} = INVDC_{i,dc,t-1} + \sum_w TransWDC_{i,w,dc,t} - \sum_r TransDCR_{i,dc,r,t} - WasteDC_{i,dc,t} \quad \forall i, dc, t \tag{30}$$

The indirect shelf-life model consists of constraints (1)-(10), (14)-(17), and (27)-(30). A term for the cost of disposing waste is added to the objective function. The advantage of this indirect method is that the resulting models are considerably smaller than those of the direct method. In addition, it still ensures that each SKU leaves the supply chain before the end of its shelf-life. While we can obtain optimal solutions using the indirect method for a supply chain

with a single storage echelon, we cannot guarantee that we will obtain the optimal solution for a supply chain with two storage echelons. It might be beneficial for some SKUs to stay in one of the storage echelons longer than the maximum we have allocated.

### 4.3. Hybrid Shelf-Life Implementation

Therefore, we consider a third method of implementing the shelf-life restrictions. This method combines the direct and indirect methods. The age of all SKUs is tracked directly in the first stage, but in the second storage stage the shelf-life restrictions are enforced indirectly. The number of weeks an SKU may remain in the second storage stage can be calculated from the shelf-life minus the age of the SKU when it was sent to the second storage stage. An overview of the hybrid shelf-life method is given in Figure 4.



**Figure 4.** Example of the hybrid shelf-life method for an SKU with a 3-week shelf life

Similar to the indirect and direct methods, a cost term for disposing of SKUs is added to the objective function. The constraints for the hybrid shelf-life model are constraints (1)-(8), (10), and (14)-(17) of the base tactical planning model, constraints (18), (20), (21), and (23) of the direct shelf-life model, and the following constraints.

The distribution center inventory of SKU  $i$  in the current period is equal to the distribution center inventory in the previous period, plus the total amount of this SKU of any age received from the warehouses, minus the total amount sent to the retailers, minus the amount that is disposed of.

$$INVDC_{i,dc,t} = INVDC_{i,dc,t-1} + \sum_{w,a} TransWDC_{i,w,dc,t,a} - \sum_r TransDCR_{i,dc,r,t} - WasteDC_{i,dc,t} \quad (31)$$

$\forall i, dc, t$

The part of the initial inventory that reaches the end of its shelf life before or at the end of the current period, plus the amount received from the warehouses that reaches the end of its shelf life before or at the end of the current period is less than or equal to the total amount that is transported to the retailers until the end of the current period, plus the amount that is disposed of before the end of the current period. This constraint ensures that an SKU that reaches the end of its shelf-life is sent to a retailer or is disposed of.

$$\begin{aligned}
& \sum_{a \geq \text{Shelflife}_i + 1 - t} \text{INVDCini}_{i,dc,a} + \sum_{w, t' \leq t, a \geq \text{Shelflife}_i - (t-t')} \text{TransWDC}_{i,w,dc,t',a} \\
& \leq \sum_{r, t' \leq t} \text{TransDCR}_{i,dc,r,t'} + \sum_{t' \leq t} \text{WasteDC}_{i,dc,t'} \quad \forall i, dc, t
\end{aligned} \tag{32}$$

In some cases this constraint might not be sufficient. This constraint enforces that the total amount of SKU leaving a distribution center is at least equal to the total amount of SKU that was sent to this distribution center that would reach its maximum shelf-life at the end of the current period. However, it relies on the assumption that we can always send the SKU with the shortest remaining shelf-life to the retailers. However, as will be demonstrated in the following example, the SKU with the shortest remaining shelf-life might still be in one of the warehouses.

In week  $t$ , a batch of SKU  $i$  with 6 weeks remaining shelf-life is sent to a distribution center with no inventory leading up to week  $t$ . This batch of SKU is used to meet the retailer demand in weeks 1-3. In week 4, a second batch of SKU  $i$  is sent to this distribution center. This second batch is smaller and already at its maximum shelf-life in week 4. This batch should, therefore, immediately leave the distribution center in week 4.

However, constraint (32) is already met because the first batch is larger, is already sent on, and would still have some remaining shelf-life. Therefore, based on constraint (32), the inventory at the end of week 4 could be used to meet demand in weeks 5 and 6 as well. However, when applying that solution we would discover that we have to dispose of the second batch at the end of week 4, and we would thus incur missed sales in weeks 5 and 6.

The solution obtained with the hybrid method can be corrected to account for this problem using the following correction procedure. First, the SKU waste that is not accounted for by the hybrid model is identified using a small LP model. This LP model considers only the distribution centers and the retailers, and is comprised of the following constraints.

The amount of SKU  $i$  in distribution center  $dc$  in week  $t$  with an age of  $a$  weeks is equal to the amount of this SKU that was in the distribution center last week with an age of  $a-1$ , plus the amount received from the warehouses, minus the amount sent to the retailers, minus the amount that becomes waste.  $\text{TransWDC}$  is input for this LP model and is thus a parameter set at the value obtained from the hybrid model.

$$\begin{aligned}
\text{INVDC}_{i,dc,t,a} &= \text{INVDC}_{i,dc,t-1,a-1} + \sum_w \text{TransWDC}_{i,w,dc,t,a} \\
&- \sum_r \text{TransDCRC}_{i,dc,r,t,a} - \text{WasteDCC}_{i,dc,t,a} \quad \forall i, dc, t, a < SL_i
\end{aligned} \tag{33}$$

The total amount of each SKU sent from each distribution center to each retailer in each week must be equal to the amount sent in the hybrid model solution. For the correction model,  $\text{TransDCR}$  is an input parameter obtained from the solution of the hybrid model.

$$\sum_a \text{TransDCRC}_{i,dc,r,t,a} = \text{TransDCR}_{i,dc,r,t} \quad \forall i, dc, r, t \tag{34}$$

The total amount of each SKU disposed from each distribution center in each week must be equal to the amount disposed of in the hybrid model solution. For the correction model,  $\text{WasteDC}$  is an input parameter obtained from the solution of the hybrid model.

$$\sum_a \text{WasteDCC}_{i,dc,t,a} = \text{WasteDC}_{i,dc,t} \quad \forall i, dc, t \tag{35}$$

If an SKU remains in inventory at the end of its shelf-life it indicates an infeasibility.

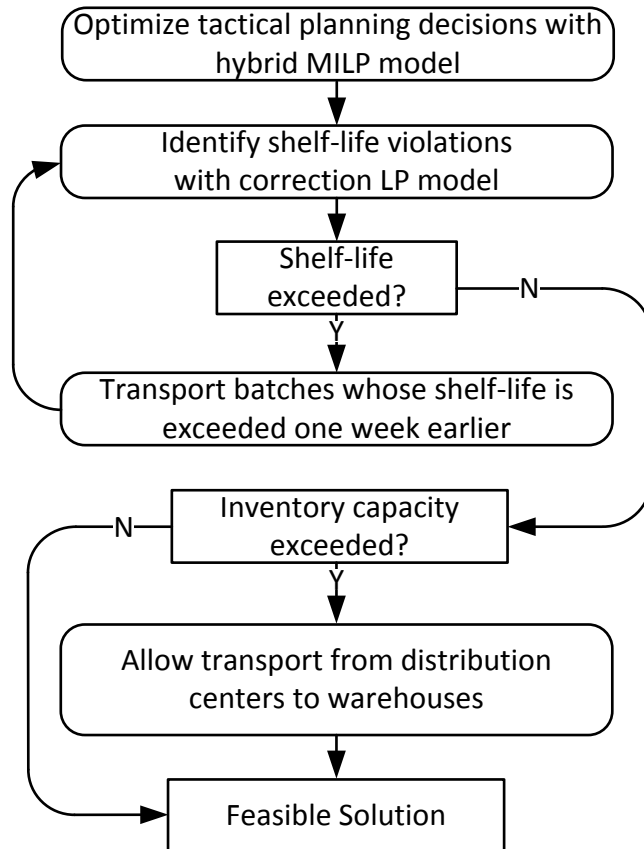
$$Infeasibility_{i,dc,t} = INVDC_{i,dc,t-1,a=SL} \quad \forall i,dc,t \quad (36)$$

The objective of the LP model is to minimize these infeasibilities. This identifies which SKUs exceed their shelf-life in each distribution center in each week. If a batch exceeds its shelf-life, it should have been sent from the warehouse to the distribution center earlier so that it can be used to meet earlier demand.

Therefore, in step 2 of the correction procedure, the batches that exceed the shelf-life are transported one week earlier from warehouses to distribution centers. Afterwards, the LP model is optimized again to identify any remaining infeasibilities. If no infeasibilities remain, the decisions of the hybrid model are updated. Otherwise, the batches that exceed their shelf-life are transported another week earlier. This procedure is repeated until no infeasibilities remain.

It should be noted that the age of SKUs sent to retailers is not limited by their shelf-life in this correction model. However, if an SKU that is sent to the retailers has exceeded its shelf-life, the inventory of that SKU must have reached the end of its shelf-life at some point. The total amount of SKUs that reach the end of their shelf-life while still in inventory is minimized in this correction procedure. Therefore, an SKU past its expiration date will only be used to meet the demand if there is no other option. At the end of the correction procedure, no SKUs will exceed their expiration date in inventory, and thus no SKUs past their shelf-life are used to meet demand.

The corrections might lead to an inventory capacity violation at one of the distribution centers. However, this can easily be corrected by sending SKU with a relatively long remaining shelf-life to a warehouse with available capacity. Therefore, we allow SKUs to be transported back from distribution centers to warehouses in this step. It should be noted that this is rarely necessary. Even in those cases where it is required, the amounts sent back from distribution center to warehouse are typically very small. An overview of the correction procedure is given in Figure 5.



*Figure 5. Overview of the correction procedure for the hybrid shelf life model*

It should be noted that the hybrid shelf life model may not obtain the global optimal solution if the correction procedure is required. Nevertheless, the corrections are typically minor and have a very limited impact on the total cost.

## 5. Results

First, these three shelf-life implementation methods have been applied to several relatively small case studies. The time horizon in these case studies consists of 52 weekly periods, and the supply chain consists of 5 suppliers, 2 factories, 2 warehouses, 4 distribution centers and 8 retailers. Each of these case studies contained 10 ingredients and 5 SKUs. The SKUs belonged to 2 different mixing families, 4 packing families, and 5 SKU families. Afterwards, case studies with a larger supply chain and up to 1000 SKUs are considered.

Both due to confidentiality, and due to the extremely large amount of data that is required, hypothetical data is used in this paper. For example, the location of each facility is randomly generated on a grid, and the transportation costs are then calculated based on the distance between the locations. Most data is generated from uniform distributions. However, some additional limitations are applied. For example, each supplier only has a 33% chance of supplying a certain ingredient. But each ingredient must be supplied by at least one supplier. The production and storage capacities are chosen such that in the base case they are just sufficiently large to meet the demand. The demand is highly seasonal as 80% of the demand occurs in weeks 40 to 49.

### 5.1. 5 SKU case study results

All optimizations in this paper have been performed using CPLEX 12.4 in AIMMS 3.12 on a computer with an Intel(R) Core(TM) i7-3770 CPU @ 3.40 Ghz and with 16 GB. All optimizations have been performed with a one percent MIP optimality tolerance unless specified otherwise.

In this section, four models are compared with each other: The tactical planning model without shelf-life considerations (No SL), with indirect shelf-life constraints (ISL), with hybrid shelf-life constraints (HSL), and with direct shelf-life (DSL) constraints. For the model without shelf-life considerations, the total costs are adjusted based on SKUs reaching the end of their shelf-life. These SKUs incur disposal cost. Moreover, this typically leads to missed sales as the remaining inventory is reduced. For the indirect shelf-life constraints, three shelf-life ratios are used. One where 25% of the shelf-life is allocated to the warehouses and 75% to the distribution centers, one where 50% of the shelf-life is allocated to both warehouses and distribution centers, and one where 75% is allocated to the warehouses and 25% to the distribution centers.

For the base case, the shelf-life of all SKUs was set to 13 weeks. For the indirect methods, the allocation of the shelf life was 3 weeks to the warehouse and 10 to the distribution centers, 7 to the warehouses and 6 to the distribution centers, or 10 to the warehouses and 3 to the distribution centers. The total storage capacity of the warehouses is equal to the total storage capacity of the distribution centers. The results for the base case are given in Table 1. The cost increase is the increase in cost of a certain method compared to the best solution obtained with any of the methods.

*Table 1. Results of the various shelf-life methods for the base 5-SKU case*

Shelf-life method	Constraints	Variables (Binary)	Required CPU time[s]	Cost increase [%]
No shelf-life	11,493	23,141 (520)	4	32.12%
Indirect 3-10			20	13.50%
Indirect 7-6	13,001	24,649 (520)	16	0.25%
Indirect 10-3			13	2.84%
Hybrid	19,241	55,849 (520)	75+18	0.02%
Direct	48,881	180,649 (520)	524	0.00%

First of all, it is clear that the direct shelf-life implementation indeed leads to a significantly larger model. The number of constraints is more than 3.5 times larger than in the indirect method and more than 2.5 times larger than in the hybrid method. Moreover, the number of variables is increased by a factor 7 compared to the indirect method and by a factor 3 compared to the hybrid method. As a result, the direct method requires considerably more CPU time than the other two methods. While all models could be optimized within a reasonable time for this base case, it should be noted that the required times will increase drastically for more realistically sized cases.

Secondly, the costs when the shelf-life is not considered are 32.13% higher than the cost when the shelf-life is considered directly. Therefore, we conclude that it is extremely important that the shelf-life is considered in the tactical planning model. We also see that the corrections required by the hybrid method only lead to a cost increase of 0.02%. For the indirect method the results vary. If most of the shelf-life is allocated to the distribution centers, a poor solution with a

cost increase of 13.50% is obtained. However, if the shelf-life is distributed evenly between the warehouses and distribution centers, the costs only increase by 0.25%.

The main differences between the solutions of the various methods are in the inventory profiles. Figure 6 and 7 show that the inventory profiles obtained with the direct and hybrid shelf-life models are very similar. The only difference is that in the solution obtained with the hybrid shelf-life model, the inventory buildup in the distribution centers starts a few weeks earlier. This is caused by the correction procedure, which forces SKUs to be sent earlier from warehouses to distribution centers.

All models that consider shelf-life start increasing the inventory around week 25. Since the peak demand starts in week 40 and the shelf-life is 13 weeks, the first couple of weeks of inventory buildup are used to meet the demand until the peak, and the majority is used to meet the peak demand. On the other hand, the model that does not consider shelf-life starts building up inventory from the first week. Therefore, part of the production in the first 25 weeks exceeds the shelf-life and must thus be disposed of. As a result, the total inventory buildup is less than with the other models, and a considerably part of the peak demand cannot be met. In fact, 9.6% of the total demand cannot be met.

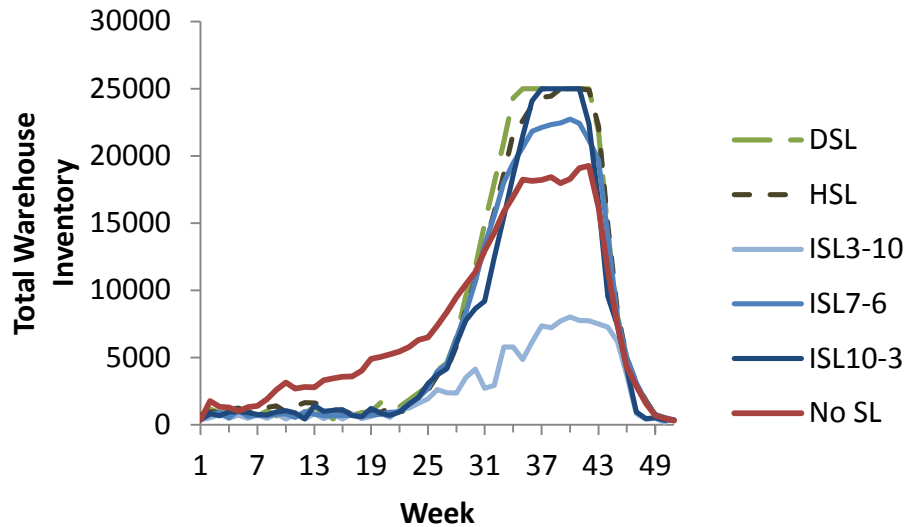


Figure 6. The total inventory of all SKUs in the warehouse when using the various models



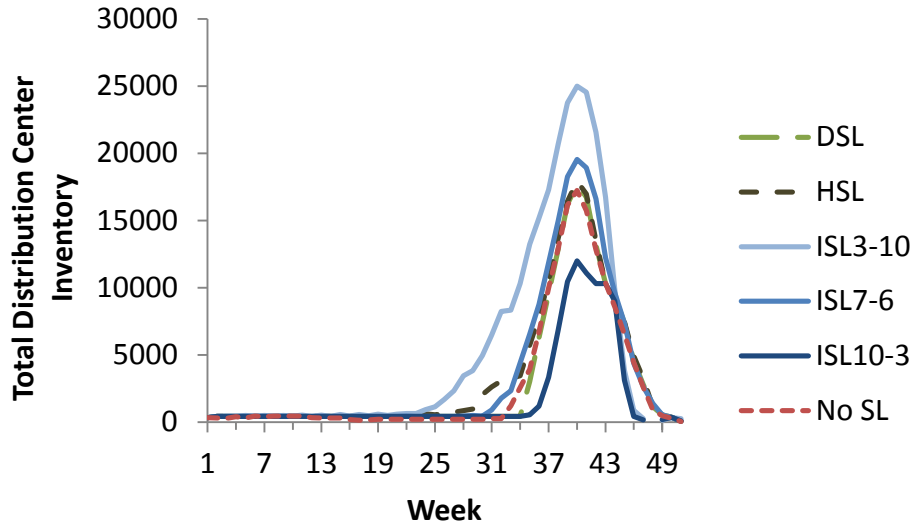


Figure 7. The total inventory of all SKUs in the distribution centers when using the various models

No SKUs have to be disposed of in the solutions obtained with the indirect, hybrid and direct shelf-life models. The hybrid and direct shelf-life models do not incur any missed sales costs. However, the indirect shelf-life model incurs 4.6% and 0.3% missed sales when the shelf-life is allocated in a 3-10 and 10-3 ratio respectively. The reason is that these ratios severely limit the flexibility in inventory storage.

As can be seen in Figure 6, the total inventory that is stored in the warehouses is considerably lower when the warehouse shelf-life is set to 3 weeks. This is because three weeks of production is considerably less than the total warehouse storage capacity. Therefore, the available storage capacity in the warehouses is reduced significantly. While the distribution center inventory can be increased slightly, the distribution center capacity is not sufficient to account for the difference. As a result, the inventory buildup is insufficient to meet the demand, and missed sales are incurred. Similarly, missed sales are incurred with the 10-3 ratio because the distribution center capacity is restricted too much.

### 5.1.1. Storage capacity ratios

Therefore, the right shelf-life ratio for the indirect method seems to be the storage capacity ratio. In other words, the fraction of the shelf-life allocated to the warehouses should be equal to the total warehouse capacity divided by the total storage capacity. To investigate this further, we have also optimized the base case with a warehouse:distribution center capacity ratio (WH:DC ratio) of 1:3 and a WH:DC ratio of 3:1. A 1:3 ratio indicates that 25% of the total storage capacity is in the warehouses and 75% in the distribution centers. The computational results are given in Table 2.

Table 2. Computational results for varying WH:DC capacity ratios.

WH:DC storage capacity ratio	Required CPU time[s]		Cost increase [%]	
	3:1	1:3	3:1	1:3
No shelf-life	4s	6s	32.16%	28.30%
Indirect 3-10	18s	18s	44.05%	1.75%

Indirect 7-6	16s	15s	5.37%	4.72%
Indirect 10-3	13s	9s	0.62%	32.39%
Hybrid	44s+19s	75s+18s	0.07%	0.04%
Direct	258s	351s	0.00%	0.00%

From Table 2 it is clear that the best shelf-life allocation ratio is indeed equal to the WH:DC storage capacity ratio. When using this ratio, solutions within 2% of the minimum cost are obtained with the indirect shelf-life model. On the other hand, the cost increase can be as much as 44% when using alternative ratios. These solutions are even worse than for the base case because choosing the shelf-life ratio opposite to the WH:DC ratio leads to an extremely limited available storage capacity. Similarly to the base case, the costs for not considering the shelf-life are approximately 30%. With respect to the required CPU time, the indirect shelf-life model is again more efficient than the hybrid shelf-life model which in turn is more efficient than the direct shelf-life model.

### 5.1.2. Varying Demand

Another aspect that might influence the quality of the solution obtained with the different methods is the demand. In the base case, all capacities are sufficient to meet the demand, but the overcapacity is limited. We will also compare the methods for case with 30% higher and 30% lower demand. For the high demand case, the capacity is insufficient to meet all demand, while for the low demand case the overcapacity is substantial. In addition, we compare the methods for a case with non-seasonal demand. The computational results for these cases are given in Table 3.

*Table 3. Computational results for cases with different demand*

Demand:	Required CPU time[s]			Cost increase [%]		
	Low	High	Non-seasonal	Low	High	Non-seasonal
No shelf-life	6s	4s	3s	20.57%	18.38%	8.27%
Indirect 3-10	19s	18s	22s	0.99%	21.35%	0.37%
Indirect 7-6	19s	14s	16s	0.15%	0.26%	0.30%
Indirect 10-3	12s	11s	15s	0.58%	13.40%	0.87%
Hybrid	38s+18s	70s+19s	61s+15s	0.00%	0.05%	0.10%
Direct	514s	336s	422s	0.06%	0.00%	0.00%

First of all, the best solution for the low demand case was obtained with the hybrid model. This was because the remaining MIP optimality gap was 0.43% for the hybrid model and 0.51% for the direct model. Both the hybrid and the direct model again obtained a close to optimal solution. Due to the significant overcapacity in the low demand case, the reduction in effective storage capacity by choosing a shelf-life ratio that is not equal to the WH:DC ratio does not lead to missed sales. Nevertheless, the indirect model still obtains the best results when the shelf-life ratio is set equal to the WH:DC ratio.

The costs of the solutions obtained with the various models for the non-seasonal demand case are very similar. Even when the shelf-life is not considered at all, the costs only increase by 8.27%. This is mainly because no large buildup of inventory is required when the demand is non-

seasonal. Therefore, even when the shelf-life is not considered, SKUs are rarely stored longer than their shelf-life.

### 5.1.3. Varying Shelf-life

Finally, the length of the shelf-life might also influence the quality of the solution obtained. Therefore, we have compared the models for cases with a longer, a shorter or a mixed shelf-life. The longer shelf-life is 26 weeks, the shorter shelf-life is 6 weeks, and the mixed shelf-life is 26,13,13,6, and 6 weeks for SKUs 1-5 respectively. The results are given in Table 4.

*Table 4. Computational results for cases with varying shelf-life*

Shelf-life:	Required CPU time[s]			Cost increase [%]		
	Short	Long	Mixed	Short	Long	Mixed
No shelf-life	3s	4s	4s	38.23%	2.06%	35.63%
Indirect 3-10	17s	14s	16s	2.63%	0.82%	3.04%
Indirect 7-6	22s	12s	15s	0.70%	0.07%	0.99%
Indirect 10-3	16s	13s	15s	1.11%	0.16%	1.00%
Hybrid	34s+3s	48s+58s	52s+24s	0.00%	0.00%	0.07%
Direct	60s	1504s	311s	0.30%	0.00%	0.00%

Even when the shelf-life is not considered, the 26 week shelf-life is sufficiently large that it is almost never violated. As a result, the costs obtained with the various models are very similar for the 26 week shelf-life case. On the other hand, not considering the shelf-life leads to a very large increase in cost when the shelf-life is short. Similar to the previous cases, the best solution for the indirect method is obtained when the shelf-life ratio is equal to the WH:DC ratio.

It is clear from Table 4 that the hybrid and, especially, the direct shelf-life models are less efficient when considering SKUs with a longer shelf-life. This is mainly because the model sizes increase with the length of the shelf-life. On the other hand, the required CPU time of the indirect shelf-life model is independent of the shelf-life length.

## 5.2. 10-1000 SKUs

For the various 5 SKU cases, all models could be optimized within a reasonable time. However, a more realistic case would contain a larger supply chain and up to 1000 SKUs. For these larger cases, these models quickly become intractable. In fact, even without considering the shelf-life, van Elzakker et al. (2013) showed that the tactical planning model becomes intractable for cases of 25 SKUs or more and a supply chain consisting of 10 suppliers, 4 factories, 5 warehouses, 10 distribution centers, and 20 retailers.

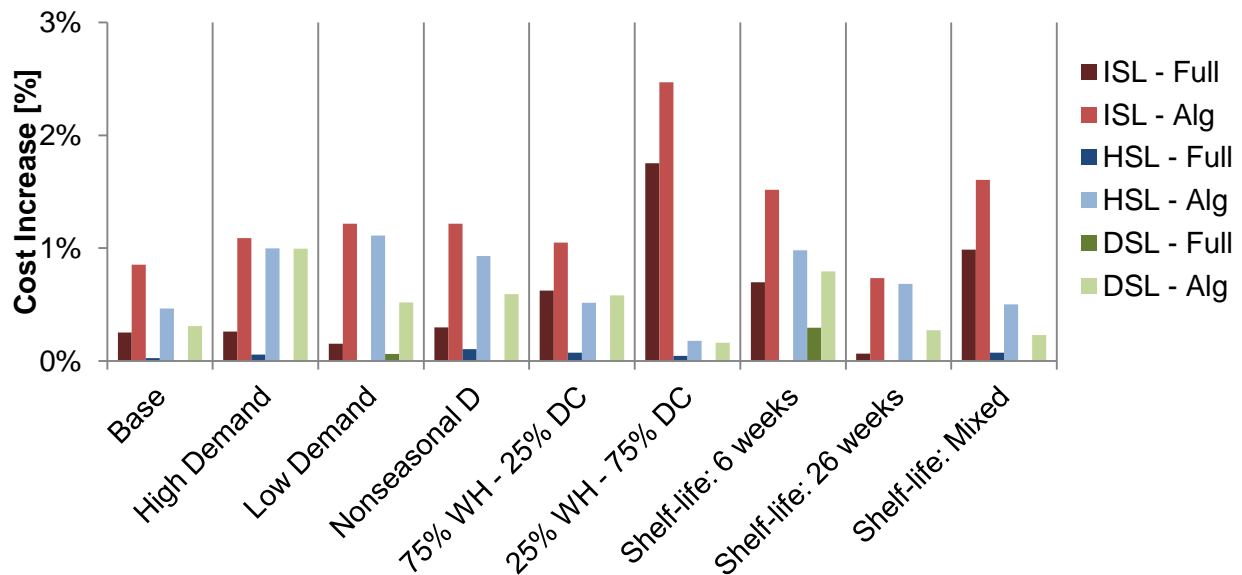
They proposed an SKU decomposition algorithm to solve cases of up to 1000 SKUs. This decomposition is based on limiting the domain of all variables and constraints to a single SKU. An initial solution for the complete problem is obtained by optimizing these single SKU models for all SKUs individually. In these submodels, the capacity used by the other SKUs is included as parameters, and a slack variable is added to all the capacity constraints. The capacity constraints are constraints (1), (2), (4), (5), (9), and (10). The slack variables are also added to the objective function with a penalty cost. Therefore, these slack variables allow the capacities to be violated at a certain cost.

Initially, this cost is set to zero, and therefore, the initial solution will most likely be infeasible as the capacities will be exceeded. In the second part of the algorithm, the penalty

costs are set to a small initial penalty, all SKUs are re-optimized consecutively, and the penalty cost are increased at the end of each iteration. The algorithm will continue to iterate until the penalty cost are sufficiently high such that a feasible solution is obtained. A more detailed description of the algorithm is given in van Elzaker et al. (2013). While there is no guarantee that the global optimal solution will be obtained with the SKU decomposition algorithm, they showed that the obtained solutions are typically within a few percent of the optimal solution. Moreover, while the full model became intractable for more than 25 SKUs, they optimized cases of up to 1000 SKUs using the SKU decomposition algorithm.

Therefore, we have selected this SKU decomposition algorithm to optimize cases containing between 10 and 1000 SKUs. In these cases we also consider a larger supply chain consisting of 10 suppliers, 4 factories, 5 warehouses, 10 distribution centers, and 20 retailers.

However, we have first tested the algorithm on the 5-SKU cases discussed in section 5.1. For the indirect shelf-life, we set the shelf-life ratio equal to the storage capacity ratio. An overview of the results is given in Figure 8. For all cases, the algorithm obtained a solution within 1.5% of the solution obtained with the corresponding full model. Therefore, we conclude that the algorithm can still obtain solutions within a few percent of optimality after introducing the shelf-life constraints. Because these cases are still relatively small, the required computational time of the algorithm was similar to that of the full model.



**Figure 8.** Overview of the cost increase compared to the best obtained solution for the 5-SKU cases when using the indirect, hybrid, or direct shelf-life method with or without the SKU decomposition algorithm.

However, without the algorithm, all three shelf-life models require more than 8 hours to obtain a solution within 10% of optimality for the case containing 10 SKUs and the larger supply chain. With the algorithm, and a MIP optimality tolerance of 2%, this 10-SKU case could be optimized by all three shelf-life models. The 2% MIP optimality tolerance was chosen to ensure that each submodel could still be solved relatively quickly. Smaller optimality tolerances greatly increased the required CPU time to solve some of the submodels, which greatly increases the total required CPU time.

As can be seen in Table 5, the computational differences between the models are more significant for the larger cases. For example, for the 10-SKU case, a solution can be obtained in 23 minutes when using the indirect shelf-life model, while the direct shelf-life model requires more than 8 hours. For this particular case, the best solution was obtained with the hybrid shelf-life model. This is again caused by smaller MIP optimality gaps for the hybrid model.

*Table 5. Computational results for the larger cases*

Case	Shelf-life Method	Required CPU time	Cost increase [%]
10-SKU	Indirect	0:23 hr	2.06%
	Hybrid	1:32 hr	0.00%
	Direct	8:36 hr	0.93%
25-SKU	Indirect	0:33 hr	2.51%
	Hybrid	3:36 hr	1.52%
	Direct	18:08 hr	0.00%
100-SKU	Indirect	3:14 hr	1.50%
	Hybrid	8:53 hr	0.00%
	Direct	> 72 hr	-
1000-SKU	Indirect	50:29 hr	-
	Hybrid	> 72 hr	-
	Direct	> 72 hr	-

For the 10- and 25-SKU cases, all models obtained solutions for which the costs are again within a few percent of each other. For the 100-SKU case, the direct shelf-life model is intractable as it requires more than 72 hours. For the 1000-SKU case, both the direct and the hybrid shelf-life models are intractable. Nevertheless, a feasible solution for this extremely large case can still be obtained with the indirect method.

## 6. Conclusions

Three different methods for introducing shelf-life restrictions into a tactical planning MILP for a FMCG company were compared. The direct method, which keeps track of the age of all SKUs, provides optimal solutions but is computationally inefficient. Therefore, it is the most suited method for small problems. For larger problems, the hybrid method is more suitable. It tracks the age of SKUs in the first storage stage directly, while indirectly enforcing the maximum shelf-life in the second storage stage. The hybrid method can be used to obtain near-optimal solutions in, on average, less than 20% of the required computational time of the direct method. For extremely large problems, even the hybrid method becomes intractable. For these cases, the indirect method can be used. This method models the shelf-life indirectly on both storage stages by manually dividing the shelf-life over the two stages. Using the indirect method instead of the hybrid method reduces the computational time by, on average, another factor 5. The solutions obtained with the indirect method are within a few percent of optimality. By combining this indirect method with a previously developed SKU-decomposition algorithm, cases of up to 1000 SKUs could be optimized.

## Acknowledgements

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## Indices

$a$	Age of an SKU in weeks.
$dc$	Distribution centers
$f$	Factories
$fam$	SKU families
$h$	Ingredients
$i$	SKUs
$mfam$	Mixing families
$pfam$	Packing Families
$r$	Retailers
$SKU$	Current SKU
$t, t'$	Weeks
$w$	Warehouses

## Subsets

$FAM_{pfam}$	SKU families belonging to packing family $pfam$
$IM_{mfam}$	SKUs belonging to mixing family $mfam$
$IP_{pfam}$	SKUs belonging to packing family $pfam$

## Variables

$Infeasibility_{i,dc,t}$	Amount of SKU $i$ in distribution center $dc$ in week $t$ that exceeds its shelf-life
$INVDC_{i,dc,t}$	Amount of SKU $i$ stored in distribution center $dc$ in week $t$
$INVDC_{i,dc,t,a < SL}$	Amount of SKU $i$ stored in distribution center $dc$ in week $t$ with an age of $t'$ weeks. Since the inventory is the inventory at the end of the week, the age of all SKUs must be less than their shelf-life. Otherwise they would need to be disposed of.
$INVIng_{h,f,t}$	Inventory of ingredient $h$ at factory $f$ in week $t$
$INVWH_{i,w,t}$	Amount of SKU $i$ stored in warehouse $w$ in week $t$
$INVWH_{i,w,t,a < SL}$	Amount of SKU $i$ stored in warehouse $w$ in week $t$ with an age of $t'$ weeks. Since the inventory is the inventory at the end of the week, the age of all SKUs must be less than their shelf-life. Otherwise they would need to be disposed of.
$INVWHini_{i,w,a}$	The initial inventory of SKU $i$ in warehouse $w$ that has been in storage for $a$ weeks.
$MissedSales_{i,r,t}$	Shortage of SKU $i$ at retailer $r$ in week $t$
$Prod_{i,f,t}$	Amount of SKU $i$ produced in factory $f$ in week $t$
$SSVioDC_{i,dc,t}$	Amount of SKU $i$ short of the safety stock in distribution center $dc$ in week $t$
$SSVioWH_{i,w,t}$	Amount of SKU $i$ short of the safety stock in warehouse $w$ in week $t$
$TransDCR_{i,dc,r,t}$	Amount of SKU $i$ transported from distribution center $dc$ to retailer $r$ in week $t$

$TransDCR_{i,dc,r,t,a}$	Amount of SKU $i$ with age $a$ transported from distribution center $dc$ to retailer $r$ in week $t$
$TransDCRC_{i,dc,r,t,a}$	Amount of SKU $i$ with age $a$ transported from distribution center $dc$ to retailer $r$ in week $t$ (This variable is only used in the correction model of the hybrid model)
$TransFW_{i,f,w,t}$	Amount of SKU $i$ transported from factory $f$ to warehouse $w$ in week $t$
$TransFW_{i,f,w,t,a}$	Amount of SKU $i$ with age $a$ transported from factory $f$ to warehouse $w$ in week $t$
$TransIng_{h,f,s,t}$	Amount of ingredient $h$ procured from supplier $s$ to factory $f$ in week $t$
$TransWDC_{i,w,dc,t}$	Amount of SKU $i$ transported from warehouse $w$ to distribution center $dc$ in week $t$
$TransWDC_{i,w,dc,t,a}$	Amount of SKU $i$ with age $a$ transported from warehouse $w$ to distribution center $dc$ in week $t$
$WasteDC_{i,dc,t}$	Amount of SKU $i$ that is disposed of at the end of week $t$ in distribution center $dc$
$WasteDC_{i,dc,t,a=SL_i}$	Amount of SKU $i$ that is disposed of at the end of week $t$ in distribution center $dc$ . This variable is only defined for SKUs that have reached the end of their shelf life.
$WasteDCC_{i,dc,t,a=SL_i}$	Amount of SKU $i$ that is disposed of at the end of week $t$ in distribution center $dc$ . This variable is only defined for SKUs that have reached the end of their shelf life and is only used in the correction model of the hybrid model
$WasteWH_{i,w,t,a=SL_i}$	Amount of SKU $i$ that is disposed of at the end of week $t$ in warehouse $w$ . This variable is only defined for SKUs that have reached the end of their shelf life.
$WSU_{i,f,t}$	Binary variable, indicates a set-up of SKU $i$ in factory $f$ in week $t$
$YFAMSU_{fam,f,t}$	0-1 continuous variable, indicates if there is a set-up of SKU family $fam$ in factory $f$ in week $t$

## Parameters

$CostIng_{h,s,t}$	Unit cost of ingredient $h$ at supplier $s$ in week $t$
$D_{i,r,t}$	Demand of SKU $i$ at retailer $r$ in week $t$
$DCCap_{dc}$	Available storage capacity in distribution center $dc$
$DCSL_i$	Part of the shelf-life of SKU $i$ that is dedicated to the distribution centers.
$FAMSUCost_{fam}$	Average set up cost for SKU family $fam$
$FAMSUT_{fam}$	Average set up time for SKU family $fam$
$INVDCP_{i,dc,t}$	Amount of SKU $i$ stored in distribution center $dc$ in week $t$ . This parameter is used when the decisions for SKU $i$ are frozen in the current optimization.
$INVIngCAP_f$	Available storage capacity for ingredients at factory $f$
$INVIngP_{h,f,t}$	Inventory of ingredient $h$ at factory $f$ in week $t$ . This parameter is used when the decisions for ingredient $h$ are frozen in the current optimization.
$INVWHP_{i,w,t}$	Amount of SKU $i$ stored in warehouse $w$ in week $t$ . This parameter is used when the decisions for SKU $i$ are frozen in the current optimization.
$MaxSupply_{h,s,t}$	Available supply of ingredient $h$ at supplier $s$ in week $t$
$MixTime_{mfam,f}$	Available mixing time at factory $f$ for SKUs that are part of mixing family $mfam$
$MixRate_{i,f}$	Mixing rate of SKU $i$ in factory $f$

$MSPen_{i,r,t}$	Penalty costs per unit of missed sales of SKU $i$ at retailer $r$ in week $t$
$PackRate_{i,f}$	Packing rate of SKU $i$ in factory $f$
$PackTime_{pfam,f}$	Available packing time at factory $f$ for SKUs that are part of packing family $pfam$
$ProdP_{i,f,t}$	Amount of SKU $i$ produced in factory $f$ in week $t$ . This parameter is used when the decisions for SKU $i$ are frozen in the current optimization.
$Recipe_{h,i}$	Amount of ingredient $h$ consumed per unit produced of SKU $i$
$SCIng_{h,f}$	Storage costs of ingredient $h$ at factory $f$
$SCDC_{i,dc}$	Storage costs of SKU $i$ at distribution center $dc$
$SCWH_{i,w}$	Storage costs of SKU $i$ at warehouse $w$
$SL_i$	Maximum shelf-life of SKU $i$
$SSDC_{i,dc,t}$	Minimum safety stock of SKU $i$ in distribution center $dc$ in week $t$
$SSWH_{i,w,t}$	Minimum safety stock of SKU $i$ in warehouse $w$ in week $t$
$SSpenCost$	Safety stock violation penalty cost
$SUCost_i$	Average set-up cost for SKU $i$
$SUT_i$	Average set-up time for SKU $i$
$TCDCR_{dc,r}$	Transportation cost between distribution center $dc$ and retailer $r$
$TCFW_{f,w}$	Transportation cost between factory $f$ and warehouse $w$
$TCSF_{f,s}$	Transportation cost between supplier $s$ and factory $f$
$TCWDC_{w,dc}$	Transportation cost between warehouse $w$ and distribution center $dc$
$WHCap_w$	Available storage capacity in warehouse $w$
$WHSL_i$	Part of the shelf-life of SKU $i$ that is dedicated to the warehouses
$WSUP_{i,f,t}$	Binary parameter, indicates a set-up of SKU $i$ in factory $f$ in week $t$ . This parameter is used when the decisions for SKU $i$ are frozen in the current optimization.
$YFAMSUP_{fam,f,t}$	Binary parameter, indicates if there is a set-up of SKU family $fam$ in factory $f$ in week $t$ . This parameter is used to indicate a required set up for one of the SKUs of SKU family $fam$ that are frozen in the current optimization.

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