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EQUILIBRIUM EARNINGS MANAGEMENT, INCENTIVE CONTRACTS, AND ACCOUNTING STANDARDS

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June 2003

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Abstract: In this paper, we model earnings management as a consequence of the interaction among self-interested economic agents, namely the managers, the shareholders, and the regulators. In our model, a manager controls a stochastic production technology and makes periodic accounting reports about his performance; an owner chooses a compensation contract to induce desirable managerial inputs and reporting choices by the manager; and a regulatory body selects and enforces accounting standards to achieve certain social objectives. We show various economic trade-offs give rise to endogenous earnings management. Specifically, the owner may reduce agency costs by designing a compensation contract that tolerates some earnings management because such a contract allocates the compensation risk more efficiently. The earnings management activity produces accounting reports that deviate from what is prescribed by accounting standards. Given such reports, the valuation of the firm may be nonlinear and S-shaped, recognizing the manager’s reporting incentives. We also explore policy implications, noting (i) the regulator may find enforcing a zero-tolerance policy – no earnings management is allowed – economically undesirable; and (ii) when selecting the optimal accounting standard, valuation concerns may conflict with stewardship concerns. We conclude earnings management is better understood in a strategic context involving various economic trade-offs.

Key Words: Earnings Management, Accounting Standards, and Agency Model.
Equilibrium Earnings Management, Incentive Contracts and Accounting Standards

1 Introduction

In this paper, we offer an equilibrium characterization of earnings management in a setting where the information content of accounting reports is determined by the interaction among self-interested economic agents. Specifically, we consider a two-period agency setting in which accounting reports facilitate managerial contracting as well as firm valuation. In this setting, an agent controls a stochastic production technology and makes periodic accounting reports about his performance; a principal chooses a compensation contract to induce desirable managerial inputs and reporting choices by the agent; and a regulatory body selects and enforces accounting standards to achieve certain social objectives.

We provide insights into the equilibrium nature of earnings management in balancing various economic trade-offs. First, given the productivity difference over time, it is economically beneficial to compensate the agent using different bonus coefficients in different periods. This creates an incentive for the agent to exert costly efforts to “move” accounting profits between periods for his personal benefit. Such incentives may be dampened by reducing the bonus coefficient differential. However, suppressing such incentives entirely may be too costly to the principal because it increases the total variability of the agent’s compensation. Allowing some earnings management reduces such risk and thus may lower agency costs. Second, as an alternative approach to suppress such incentives, the regulators may punish earnings management activities by adopting a zero-tolerance rule. We show this approach may not be socially optimal because it, too, may impose too high a compensation risk on the agent and cause agency costs to rise. Taken together, the two results point to earnings management as an equilibrium outcome from both the contracting and the policy perspectives. Finally, we show when selecting the optimal accounting standard, the regulator may face a conflict between the two objectives of reducing agency costs and increasing the valuation information content in the accounting report. In short, the equilibrium earnings management reflects various economic trade-offs.
In this type of inquiry, framing plays a subtle, yet important, role. The auditor independence is a good example. Antle [1999] distinguishes moral and economic framing of the auditor independence notion. In a moral framing, “auditors are professionals, with professional obligations to the public. They should not engage in any activity that appears to impair their effectiveness as professionals, ... Cost and benefits are not relevant in discussing moral issues. Right and wrong is what is relevant.” In contrast, “an economic framing stresses independence as an instrumental value. ... If auditors’ activities create independence problems, economics suggests a cost-benefit test: Do the benefits to society of the auditors’ activities outweigh the cost due to impairment of independence? If the benefit outweigh the cost, we are better off with these activities than without them.” (p. 9)
significantly. In particular, the Revelation Principle does not apply in designing optimal contracts, which implies insisting on unconditional truthful reporting may not be desirable in equilibrium. This is the basis for our key results.

Second, our model assumes the managerial compensation contracts must be linear in accounting profits. Although accounting-based bonus schemes are common in executive compensation, stock and stock-option based contracts (such as employee stock options or ESO programs) are also widespread, more visible and may be one of the contributing factors to the recent accounting scandals. In practice, both bonus- and option-based contracts may provide unintended incentives to manage earnings. With a bonus program, managers have an incentive to move earnings to the period where a (higher) bonus can be claimed. With a stock-option program, the incentive is to manipulate accounting earnings in order to influence the stock price and exercise options accordingly to maximize personal income. The main difference between the two is that the stock market plays an important role in determining executive compensation with the stock-option program. A number of economy/industry-wide and firm specific factors, in addition to accounting earnings, affect the firm’s share price at a particular point in time. Consequently, stock-option compensation may invite other managerial activities, not only earnings management, to indirectly manage the firm’s share price in order to favor executives when excising their options. The linear contract assumption in our model does capture the pay-for-performance essence of these compensation schemes. But it does not fully capture their richness and complexity. For example, typically, there is a bonus lower-bound in most compensation contracts (e.g., bonus is set to zero if earnings fail to hit a threshold). The strike-price is the explicit price lower-bound for stock options to be in the money. Failure to capture these specific features is a caveat of the paper.

The work here is related to recognizable areas of accounting research. First, costly information distortion has been studied in the literature. Dye [1988] posits an “internal demand” for earnings management based, in part, on a personal cost of reporting inaccurate accounting reports. Maggi and Rodriguez-Clare [1995] show that a costly distortion of information can be used to reduce information
rents by providing “counter-veiling” incentives. In Dutta and Gigler [2002], the manager can exert costly “window dressing” activities to affect the stochastic properties of the contracting signal, which is used to discipline an earlier voluntary disclosure. They show “window dressing” activities are desirable when it reduces the cost of eliciting a truthful report. Further, Dutta and Gigler [2002] show in some cases, the principal may choose not to prohibit “window dressing” if even it is costless to do so. The reason is inducing a selective “window dressing” – exerting “window dressing” effort only for some realization of the underlying output – can be utilized to better separate the agent’s type. While both results in Dutta and Gigler [2002] are similar to this paper, the key difference is that the economic benefit of tolerating earnings management is, in this paper, the efficient allocation of compensation risk across periods (as opposed to the better separation of the agent’s types in a one-period model in Dutta and Gigler [2002]).

Second, there is a sizeable accounting literature on earnings management that does not rely on the assumption that such activities are personally costly to managers. Instead, these studies emphasize other trade frictions such as limited communication (Dye [1988], Evans and Sridhar [1996] and Demski [1998]) and limited commitment (Arya, et al [1998], Demski and Frimor [1999], Christensen, Demski, and Frimor [2000]) as underlying earnings management practices. Demski [1998] shows when the agent’s ability to manage performance measures is linked to his other productive activities, earnings management may be used to motivate productive activities and therefore may appear as an equilibrium behavior. In Evans and Sridhar [1996], earnings management can occur in equilibrium because the agent’s message space is state-dependent and there is partial verifiability (i.e., the agent must report the truth with non-zero probability). In Arya, et al [1998], the lack of commitment (e.g., at-will contracts) enables earnings management to serve as a “device that effectively commits her [the principal] to making firing decisions that are better from an ex ante perspective.” (p.4)

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2 See review papers by Schipper [1989], Healy and Wahlen [1999] and Beneish [2001].
Third, our specification of accounting standards facilitates a cost-benefit calculus of selecting and enforcing accounting standards. A related study here is Dye [2000], who investigates standard-setting and transaction-classification manipulation in an investment decision framework. The dynamics of a Nash accounting standard is emphasized but conflicting objectives are not the focus.

Finally, the paper adds to our understanding of the nonlinear valuation response to accounting reports. In this paper, the underlying reporting incentives imply a nonlinear and S-shaped valuation response to accounting reports, in contrast to existing, non-strategic, explanations (e.g., Freeman and Tse [1992]) and Subramanyam [1996]).

The rest of the paper proceeds as follow. We first introduce elements of the agency model. Next, benchmark information regimes and the accounting setting are considered. We then analyze the equilibrium reporting in managerial contracting and in firm valuation, followed by an analysis of policy implications. The last section concludes.

2. Model and Benchmark Settings

2.1 Basic Elements of the Agency Model

A stochastic production technology is operated by a manager (the agent) who is hired by the owner (the principal). This agency lasts for two periods. The agent provides two unobservable inputs, \( a_0 \) \( A(t = 1, 2) \), at a personal cost of \( v(a_1) + v(a_2) \). Each input can be high or low: \( A = \{ H, L \} \) with \( v(H) > v(L) \) and \( v(L) = 0 \). At the end of period \( t \), an output is realized and privately observed by the agent, denoted \( x_t \). However, total output \( x (= x_1 + x_2) \) is publicly observed. At the end of the first period, the agent also privately observes a signal informative about the second period output. The monetary value of output \( x \) is given by \( q x \) with \( q > 0 \). The principal pays \( w_t \) to the agent at the end of period \( t \). Figure 1 (on page 23) summarizes the sequence of events.

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3 This type of normative analysis has been plagued by general impossibility results (Demski [1973]). Dye [2000] suggests “development of both narrower specifications of accounting standards and adoption of narrower efficiency criteria than Pareto-optimality.” (p. 4)
The specifications of the model constitute a variation of the LEN framework. Specifically, the periodic outputs are generated by the following random processes:

\[ x_1 = k_1 a_1 + \theta_1 \]
\[ x_2 = k_2 a_2 + \theta_2 \]

We normalize \( L = 0 \) and further assume the productivity parameters, \( k_1 \) and \( k_2 \), are known and \( k_1 \neq k_2 \). This time-varying production technology is a key assumption of the model and is the source of demand for varying compensation coefficients across the two periods.

The production shocks (\( \theta_1 \) and \( \theta_2 \)) are mutually independent and normally distributed with mean \( \mu_1 \) and standard deviation \( \sigma_\theta \), or \( \theta_i \sim \mathcal{N}(\mu_\theta, \sigma_\theta^2) \). Both \( \sigma_\theta^2 \) are known constants, and so is \( \mu_1 \), which is normalized to zero (\( \mu_1 = 0 \)). However, \( \mu_2 \) is random at the outset. For simplicity, we assume \( \mu_2 \) is binary, \( \mu_2 \in \{0, M\} \) with \( M > 0 \), with common prior belief of equal (50/50) probability. We also assume the agent’s signal after period 1 reveals \( \mu_2 \). Further \( \mu_2 \) is independent of \( \theta_1 \) and does not depend upon the agent’s actions.

LEN refers to linear contract, exponential utility function and normal distribution technology. There are two reasons for the modeling choice of the LEN model. First, the tractability of LEN model offers an opportunity to gain insights into the accounting phenomenon. The research question here is not about contract shapes, but on the reporting behavior induced by imperfect contracting. Linear contract, as a form of imperfect contracting, offers an avenue to pursue these questions. (See Lambert [2001] review paper for a discussion on the use of LEN framework.) Second, to allow for earnings management to play a non-trivial role, perfect contracting conditions must be relaxed (see Arya, et al [1998] for a discussion in the context of earnings management literature). The linear contract restriction is a relaxation of the perfect contracting assumption and has been studied extensively in the accounting literature in a variety of contexts (see Feltham and Xie [1994], Bushman and Indjejikian [1993], Dutta and Reichelstein [1999], etc.).

The underlying idea is that there are externalities in production over time. Another specification would be to assume earlier actions have long term effects: \( x_2 \) may be a function of \( a_1 \). If the long-term effect continues to be linear (i.e., \( x_2 = k_{12} a_1 + k_2 a_2 + \theta_2 \)), the main results of the paper do not change.

One can think of \( \mu_2 \) as a binary shock to the second period cash flow. Christensen and Demski [2003] study a similar two-period setting where a normally distributed component of income may properly “belong” to first (second) when it is correlated with the first (second) period income.
We assume \( q \) is large enough that the principal always prefers high effort in both periods regardless of what information might become available to either party.\(^7\) The principal is risk-neutral with utility \( q x_1 \! w_1 \! w_2 \). The agent is risk-averse with utility \( U(w_1, w_2; a_1, a_2) = \exp(-r(w_1 + w_2 \! v(a_1) \! v(a_2))) \). The utility function exhibits constant absolute risk aversion (CARA) with the Arrow-Pratt measure \( r > 0 \) and is multiplicatively separable over time. If the agent chooses not to participate in the agency, his opportunity utility is \( U \).

Given the preference structure, the principal can collapse the two periodic payments into a single payment \( w(.) \) at the end of the game. We restrict the contract form to be linear: \( w = \delta_0 + \delta \Gamma \). The total compensation to the agent consists of a fixed salary \( \delta_0 \) and a bonus scheme \( \delta \Gamma \) where \( \delta \) is an \( N \times 1 \) vector representing the bonus coefficients and \( \Gamma \) is an \( N \times 1 \) vector representing potential performance measures.\(^8\)

### 2.2 Benchmark Information Regimes

In this section, we study the principal’s contract design problem by considering two benchmark information regimes: disaggregate output observation and aggregate output observation. These two benchmark settings are essential for understanding the underlying economic forces.

#### 2.2.1 Disaggregate Output Observation

This is a benchmark case where both output measures \((x_1 \text{ and } x_2)\) are publicly observed. In this case, \( \delta \Gamma = [\delta_1, \delta_2] \) and \( \Gamma = [x_1, x_2] \). The agent’s second period policy \( \alpha \) is a mapping \( \alpha: E \times \{-M, +M\} \rightarrow \Gamma \).

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\(^7\) In general, the optimal managerial input is endogenous to the principal’s problem and accounting may be used for such production decisions. In this paper, we neutralize the production decision in order to focus on the incentive use of accounting information. The same assumption has been made in accounting studies such as Demski [1998] and Gigler and Hemmer [1999].

\(^8\) While we do not claim a linear contract is optimal among all possible contracts, we do believe the restriction is justified because empirically contracts are typically simple, in sharp contrast to the unusually complex contracts predicted by optimal contract models. For the similar reason, the same linearity restriction has been used in a large and growing literature in accounting literature on various topics. See e.g., Baiman and Verrecchia [1995], Boylan and Villadsen [1996], Bushman and Indjejikian [1993], Chaney and Lewis [1995], Demski and Dye [1999], Feltham and Xie [1994], and Holmström and Tirole [1993]. Healy [1985], in an empirical study, considers step-linear contracts.
A. Let \( a^H \) (resp. \( a^L \)) denote the particular \( a_2 \)-policy where the agent provides high (resp. low) effort regardless of the realizations of \( x_1 \) and \( \mu_2 \). The principal’s problem can be represented by the following optimization program.

\[
\text{C}_{\text{full}} / \min \ E[w(x_1, x_2)|H, a^H] \\
\delta_0, \delta_1, \delta_2 \\
\text{Subject to } E[U(w(.); @H, a^H)] \leq U \\
E[U(w(.); @H, a^H)] \leq E[U(w(.); @a_1, a)] \forall a_1, a
\]

Lemma 1: The optimal linear contract in the disaggregate output observation case exhibits:

\[
\delta_1^* = \frac{\delta}{k_1}, \text{ and } \delta_2^* = \frac{\delta}{k_2}, \text{ where } \delta = \nu(H)/H.
\]

(All proofs appear in the appendix)

In this benchmark case, two IC constraints, those associated with the \( \{L, a^H\} \) and \( \{H, a^L\} \) policies, are binding and the resulting two equalities determine the two bonus coefficients (\( \delta_1 \) and \( \delta_2 \)). Given these two coefficients, \( \delta_0 \) is chosen so the IR constraint (expression 2) binds. In the appendix, we provide a running numerical example to illustrate this lemma as well as subsequent lemmas and propositions.

Two observations stand out. First, when \( k_1 < k_2 \), the first period input (\( a_1 \)) is less productive than the second period input (\( a_2 \)). However, observe \( \delta_1^* > \delta_2^* \); that is, the bonus coefficient on the first period output (\( x_1 \)) is higher than the second period output (\( x_2 \)). This is because a less productive but unobservable act – \( a_1 \) in this case – is “harder for the principal to infer from the output.” Therefore, the principal must place a steeper incentive on \( x_1 \) to induce the high \( a_1 \). Intuitively, temporal differences in managerial productivity induce temporal differences in the (degrees of) moral hazard problems, which, in

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9 The underlying technology and the preference structure allow us to deal with IR and IC constraints independently. This is a variant of the argument in Holmström and Milgrom [1987]. See the proof of Lemma 1 for details. The basic idea is the following. After the agent observes \( \mu_2 \), the remaining uncertainty is described by a normally distributed random variable (\( \theta \) with mean \( \mu_2 \) and variance \( \sigma^2 \)) so the same intuition in Holmström and Milgrom [1987] applies. However, ex ante, \( \mu_2 \) has a binomial distribution. So at the start of the game, \( \theta \) is normal with a known mean and a known variance; but \( \theta \) is normal with a binomial mean (\( \mu_2 \)) and a known variance. Note \( \mu_2 \) is independent of \( \theta \) and the variance of \( \theta \), and is not affected by the agent’s actions. So the effect of \( \mu_2 \) on agent’s expected utility appears as a multiplicative term that does not depend on the agent’s action choices. So it drops out of the IC constraints. However, to the extent the agent is asked to bear the risk of \( \mu_2 \) ex ante, a risk premium must be paid to the agent as a result. Lemma 1 need not hold if the agent’s actions also affect the variance of future cash flows.
10 Since this feature is a driving force of the main results in the following sections, we provide some explanations for such temporal differences. The difference in productivity may, in the real world, be driven by technological nature of a staged multi-task production process (e.g., signing a customer may be more critical than servicing the customer). It may also be driven by learning-by-doing in repeated production settings.

11 This interaction among nested IC constraints is also present in Demski [1994]. In his single-period-multiple-task setting, certain IC constraint may “free ride” other IC constraints. The interaction “between the problem of motivating input for the task per se and the problem of coordinating supply of inputs across the array of tasks” may result in “good” performance measures driving out “bad” measures or “bad” driving out “good” measures (p. 577). In our multi-period setting, a similar force is in play.

12 As an aside, it is not obvious the principal, in general, would prefer disaggregate to aggregate performance measures. It depends on what the agent knows as well. If the aggregation prevents the agent from observing the detailed information, it may provide a benefit to the principal because there are fewer IC constraints to deal with. See Arya, Glover, and Liang [2002] for more on this point.
equilibrium contract induces “managed” accounting reports. Finally, we explore valuation response to the equilibrium report.

3.1 Accounting Standard

To begin, we specify an accounting standard which prescribes the “desirable reporting” of the entity for the two periods, a series denoted \((y_1^*, y_2^*)\). A simple standard may call for reporting the output series itself (i.e., \(y_t^* = x_t\) for all \(t\)). We shall call this the Realization Standard. Another standard may call for realization plus some recognition about future events (e.g., \(y_t^* = x_t + \text{expected gains} \leftarrow \text{expected losses}\)) if information about future events are available. Yet another standard may call for some type of conservative reporting (e.g., \(y_t^* = x_t \leftarrow \text{expected losses}\)). In most of our analysis, we use the simple Realization Standard except in section 4.2 when we focus on selecting the optimal standard.\(^{13}\)

3.2 Accounting Report

Now we move to the actual reporting. The agent makes two accounting reports, denoted by the series \((y_1, y_2)\). We interpret any deviation from \(y_t^*\) (i.e., \(y_t - y_t^*\)) as earnings management. In the model, we focus on three determinants of earnings management. The manager makes the actual accounting reports \((y_t)\) to maximize his expected utility. The board of directors, representing shareholders, designs compensation contracts to induce the desirable (operating and reporting) behavior from the manager. A regulatory body selects and enforces the accounting standard to maximize social welfare. The interaction among the three parties determines earnings management activities in equilibrium, if any.

Because there are only two periods and accounting articulation prevails (i.e., \(\Sigma y_t = \Sigma y_t^* = \Sigma x_t\)), once \(y_1\) is declared by the agent, he does not have any discretion over the \(y_2\) report. The agent observes the realizations of \(x_1\) and \(\mu_2\) before issuing \(y_1\). Therefore, in general the agent’s reporting strategy can be any mapping \(\psi: \hat{\mathbb{E}} \times \{! M, +M\} \rightarrow \hat{\mathbb{E}}\). Specifically we model the agent’s report as:

\(^{13}\) Notice after period 1 the standard asks for a scalar report \(y_1^*\) when both \(x_1\) and \(\mu_2\) are informative about the firm. This assumption is appropriate to the extent it reflects an important feature of the accounting standard – the inherent aggregation function. See similar assumption in Dye [2000]. For a broader perspective on the role of aggregation in accounting, see Ijiri [1975] and Sunder [1997]
\[ y_1 = y_1^* + \beta = x_1 + \beta \quad (4) \]

Notice the Realization Standard (i.e., \( y_1^* = x_1 \)) is in effect and \( \beta \) represents the amount of earnings management. We assume there are personal costs to the agent for reporting \( y_1 \) to be different from \( y_1^* \).\(^{14}\)

Specifically, we assume the personal cost is quadratic in \( \beta \): \( \frac{1}{2} \beta^2 c(.) \). So the agent’s utility becomes

\[
U(w; \beta, a_1, a_2) = \exp(-r(w - \frac{1}{2} \beta^2 c(.) - v(a_1) - v(a_2))).
\]

In addition, we assume the positive cost parameter \( c(.) \) depends upon the agent’s private information \((\mu_2)\) as well as \( \beta \). Specifically, we assume \( c(.) \) is lower (resp. higher) if the deviation in reported output is consistent (resp. inconsistent) with the expected future output.\(^{15}\) That is \( c(\mu_2, \beta) = c_L \), when \( \beta > 0 \) and \( \mu_2 > 0 \) or when \( \beta < 0 \) and \( \mu_2 < 0 \). Presumably, if future cash flow is higher in expectation, the agent may have an easy time (or incur less personal cost) convincing the auditor that reporting a higher current accounting performance is warranted. If \( \beta \) and \( \mu_2 \) do not have the same sign, \( c(\mu_2, \beta) = c_H \) (\( \$c_L \)). The varying personal cost parameter is a key feature of the model as it allows earning management to imperfectly convey the underlying information (see proposition 2).

In the accounting regime, the linear contract takes the form: \( \delta N = [\delta_1, \delta_2] \) and \( \Gamma N = [y_1, y_2] \). The principal’s problem can be written as the following optimization program:\(^{16}\)

\(^{14}\) See a similar assumption in Dye [1988], Maggi and Rodriguez-Clare [1995] and Dutta and Gigler [2002]. We recognize the limited nature of such a personal cost representation. Ideally, a comprehensive model would endogenize these costs as the result of the interplay of potential economic, political, and legal forces.

\(^{15}\) In general, \( c(.) \) may also depends all private information \((x_1, \mu_2)\) and actions \((a_1, a_2)\). Due to tractability concern, we focus on the simpler cases where \( c(.) \) depend on \( \mu_2 \) and \( \beta \). There are some empirical evidence in the earnings management literature that the sign discretionary accruals (analogous to \( \beta \) here) tends to be consistent with the expected future profitability. See DeFond and Park [1997].

\(^{16}\) The Revelation Principle does not apply in the principal’s mechanism design problem. This is because (i) the communication between the agent and the principal \((y_1, y_2)\) is costly and (ii) contracts are restricted to be linear. Therefore, we write the principal’s problem as choosing a linear contract that minimizes the expected compensation to the agent subject to the IR and IC constraints which incorporate any subsequent earnings management by the agent (his choice of \( \beta \)). See Dye [1988] and Arya, et al [1998] for extensive discussions on the significance of the Revelation Principle in earnings management studies.
This characterization of reporting incentive is consistent with the earnings management literature, say the Bonus Hypothesis in Healy [1985]. When the performance is out of bonus range (or close to maximum), the manager tends to move income out of current period if possible. In our model, this can be thought of as the setting with \( \delta_1 < \delta_2 \). On the other hand, when the performance is within bonus range, the manager tends to move income into current period if possible. In our model, this can be thought of as the setting with \( \delta_1 > \delta_2 \).

\[
C^* = \min_{\delta_0, \delta_1, \delta_2} E[w(y_1, y_2)|H, \alpha^{H}, \beta^*] \\
\text{Subject to } E[U(w(.); \alpha^{H}, \beta^*)] \geq U \text{ argmax } E[U(w(.); \alpha^{H}, \alpha, \beta)]
\]

3.3 Analysis of the Equilibrium Accounting Report

3.3.1 Agent’s Optimal First Period Report (\( y_1 \))

After \( x_1 \) and \( \mu_2 \) have realized, the agent’s reporting and effort choice are made to maximize his conditional expected utility, given contract parameters \( \delta_0, \delta_1 \) and \( \delta_2 \). Specifically,

\[
\text{argmax } E[U(w(.); a_2, \beta; a_1, x_1, \mu_2)]
\]

Since the agent’s expected utility has a certainty equivalent representation, we can rewrite (8) into:

\[
\text{argmax } \delta_0 + \delta_1 (k_1 a_1 + \theta_1 + \beta) + \delta_2 (k_2 a_2 + \mu_2 \beta) + v(a_1) + v(a_2) + c(.)|\beta|^2/2 + (1/2) \rho \delta_2 \sigma_2^2
\]

From the first-order condition, we derive the optimal \( \beta^* \)

\[
\beta^* = \begin{cases} 
\frac{\delta_1 - \delta_2}{c_l} & \text{if } \text{sign}(\delta_1 - \delta_2) = \text{sign}(\mu_2) \\
\frac{\delta_1 - \delta_2}{c_H} & \text{if } \text{sign}(\delta_1 - \delta_2) \neq \text{sign}(\mu_2)
\end{cases}
\]

In the symmetric cost case (\( c_L = c_H = c \)), \( \beta^* = (\delta_1 - \delta_2)/c \).

Three properties of \( \beta^* \) emerge. First, the sign of \( \beta \) is determined by the sign of the bonus coefficient differential across the two periods (\( \delta_1 \) ! \( \delta_2 \)). If \( \delta_1 > \delta_2 \), the first period performance commands the higher bonus and the agent would choose a positive \( \beta \), effectively “borrowing” some second period performance to the first period. Similarly, if \( \delta_1 < \delta_2 \), the agent would choose a negative \( \beta \), “lending” some current performance to the second period.\(^{17}\) Second, the size of \( \beta^* \) depends on the cost parameter as well as the magnitude of the bonus coefficients differential. For example, when \( \delta_1 > \delta_2 \), the

\(^{17}\) This characterization of reporting incentive is consistent with the earnings management literature, say the Bonus Hypothesis in Healy [1985]. When the performance is out of bonus range (or close to maximum), the manager tends to move income out of current period if possible. In our model, this can be thought of as the setting with \( \delta_1 < \delta_2 \). On the other hand, when the performance is within bonus range, the manager tends to move income into current period if possible. In our model, this can be thought of as the setting with \( \delta_1 > \delta_2 \).
agent wants to “borrow,” he would “borrow” more if $\mu_2 > 0$ ($\beta = (\delta_1 ! \delta_2)/c_1$) than if $\mu_2 < 0$ ($\beta = (\delta_1 ! \delta_2)/c_2$). Third, given any bounded cost structure, the principal may deter any earnings management by setting $\delta_1 = \delta_2$, corresponding to the aggregate output observation case. Alternatively, a regulator may deter any earnings management by setting both $c_L$ and $c_H$ to be $+4$ (e.g., enforcing a zero-tolerance policy). This corresponds to the disaggregate output observation case. Therefore, aggregate and disaggregate observations are simply special cases of the accounting regime.

### 3.3.2 Endogenous “Managed” Accounting Report ($\beta* \neq 0$)

A natural question at this point is whether it is optimal for the principal to design a contract to induce unmanaged accounting reports in equilibrium (i.e., setting $\delta_1 = \delta_2$ to get $y_t = x_t$). We return to the principal’s problem specified by (5) to (7) and have:

**Proposition 1:** In the accounting regime, if $k_1 \neq k_2$, and $c_L, c_H < +4$, the optimal linear contract assigns uneven bonus coefficients $\delta_1* \neq \delta_2*$ and so $\beta* \neq 0$.

Inducing earnings management utilizes the natural incentive to the principal’s advantage. Specifically, when $k_1 < k_2$, the agent has a natural incentive to “borrow” output from the second period, and the principal may be better off encouraging some “borrowing”($\beta > 0$) as opposed to truthful reporting ($\beta = 0$). Similarly with lending when $k_1 > k_2$. By encouraging such “borrowing/lending,” the principal can reduce the total variability of the compensation and therefore reduce the risk premium paid to the agent. However, $\beta \neq 0$ also implies more bonuses paid to the agent (by the amount $|(\delta_1 - \delta_2) \beta|$) than inducing $\beta = 0$. Since the marginal costs of earnings management is zero around truthful reporting, the principal can always deviate from $\delta_1 = \delta_2$ just enough to make sure benefits outweigh the costs.\(^{19}\)

---

\(^{18}\) Given the assumption that acts are binary and the principal wishes to induce high effort all the time, reducing compensation risk becomes the key issue in designing optimal contracts.

\(^{19}\) If the agent’s acts are continuous, as opposed to binary, the temporal difference in productivity would cause the principal to design different action choices for each period. This would, in general, result in different bonus coefficients for each period. So the basic tension remains even though the result in proposition 1 may be strengthened or weakened depending on the model specifics.
Intuitively, the principal must cope with the temporal difference in moral hazards and the problem of earnings management activities. By Lemma 1, the temporal difference favors contracts with an uneven bonus structure due to the smaller overall variability in the agent’s compensation (therefore a smaller risk premium). However, these contracts invite earnings management so in balancing these two effects, it is not surprising that an uneven bonus structure becomes optimal when the benefit outweighs the cost. Therefore, earnings management is a consequence of balancing contracting trade-offs.

Note Proposition 1 does not depend on the assumption that $\mu_2$ is random. If $\mu_2$ is known for certain, the result still holds because the key is the temporal difference in moral hazard problems caused by the underlying temporal difference in managerial productivities. In addition, the result does not depend on the absence of a self-report on $\mu_2$. Suppose the agent is allowed to self-report $\mu_2$ and is offered a menu of linear contracts indexed by the self-report, denoted $\mathcal{M} \{! M, +M\}$. Consider the class of contracts in the form: $w(y_1, y_2, m) = \delta_0 + \delta_1 y_1 + \delta_2 y_2$. We show, in the appendix, that Proposition 1 still holds in this setting. However, the result does depend on the assumption that it is personally costly to manage reported earnings. If it is costless to management accounting reports, the only feasible solution to the problem is to assign equal bonus coefficients (otherwise the problem degenerates because the manager would choose an unbounded $\beta$).20

3.3.3 Valuation in the Presence of “Managed” Accounting Report

In this section, we explore the valuation consequence of the earnings management. To this end, we simply calculate the expected value of the firm’s net cash flows, fully anticipating the ongoing

---

20 Linear contracts also contribute to the result. Mirrlees [1974] shows that with normal density production function, risk neutral principal and risk averse agent with separable (additive or multiplicative) utility representation, first-best solutions can be approximated arbitrarily closely by using a non-linear two-tiered contract. Such a contract punishes the agent by a large sum for extreme low outcomes and pays the agent a constant wage otherwise. With normal distribution, extreme low outcomes occur with very small probability, an appropriately chosen cut-off point would satisfy both IC and IR constraints, and it generates a principal’s expected utility as close to the first-best outcome as possible. See Holmström and Milgrom [1987]
earnings management activities. Even though the accounting report is “managed,” one may still be able to infer some information about the underlying cash flow series to form a consistent valuation of the firm.

After the contract is in place but before the first period accounting report is released, the expectation of the total output is:

$$E[x_1 + x_2] = k_1H + E[\theta_1] + k_2H + E[\theta_2] = k_1H + k_2H$$  \hfill (10)

Notice under the optimal linear contract, the agent will exert high efforts in both periods and the unconditional expectations of all random variables are zero.

The first period accounting report $y_1$ provides information about these random variables. By equation (4), a high reported $y_1$ may be due to a high $x_1$ (i.e., a high realization of $\theta_1$) or due to a high positive $\beta^*$. If $\delta_1 > \delta_2$, equation (9) suggests that a high positive $\beta^*$ implies a high $\mu_2$ while a low positive $\beta^*$ implies a low $\mu_2$. Notice if $c_L = c_H$, $\beta$ is a constant and $y_1$ does not convey any information about $\mu_2$.

To formalize the inference process, we calculate the revised expectation of total output below:

$$E[x_1 + x_2|y_1] = E[y_1 \beta^* + k_2H + \theta_2|y_1] = y_1 E[\beta^*|y_1] + E[\theta_2|y_1] + k_2H$$  \hfill (11)

Anticipating the reporting incentives, a rational valuer would not equate $y_1$ to $y_1^* (= x_1)$. Rather, $y_1$ is used to update the expectation of future cash flows. First, the valuation subtracts the expected shifting (the term $E[\beta^*|y_1]$) from $y_1$, undoing the expected earnings management. Second, it takes advantage of the information about the mean of $\theta_2$ (the term $E[\theta_2|y_1]$). If $y_1$ reveals $\mu_2$ perfectly, $E[\beta^*|y_1] = \beta^*$ and $E[\theta_2|y_1] = \mu_2$, and the effect of earnings management can be completely undone.

However, such perfect updating is not the case in our model. To further explore the shape of the valuation response to $y_1$, suppose $\delta_1 > \delta_2$; substituting $\beta^*(\mu_2 = +M) = (\delta_1 ! \delta_2)/c_L$ and $\beta^*(\mu_2 = -M) = (\delta_1 ! \delta_2)/c_H$, we have:

$$E[x_1 + x_2|y_1] = y_1 ! \left[ \text{Prob}(\mu_2 = +M|y_1) (\delta_1 ! \delta_2)(1/c_L ! 1/c_H) + (\delta_1 ! \delta_2)/c_H \right] + (2 \text{Prob}(\mu_2 = +M|y_1) ! 1) M + k_2H$$  \hfill (12)

From here it is a short step to calculate the expectation of the net cash flow to the firm (after deducting the expected compensation to the agent): $E[q(x_1 + x_2) - (\delta_0 + \delta_1y_1 + \delta_2y_2)|y_1]$. Given $y_1$ is mixture of a
normal random variable \((x_i)\) with a binary random variable \((\beta'(\mu_2))\), the conditional probability \(\text{Prob}(\mu_2 = +M|y_1)\) has a closed-form solution (see appendix for derivation):

\[
\text{Prob}(\mu_2 = +M|y_1) = \frac{1}{1 + \exp \left[ \frac{\delta_2 - \delta}{c_L - 1/c_H}(2k_H + (\delta_1 - \delta_2)(1/c_L + 1/c_H) - 2y_1) \right]}
\]  

(13)

Figure 2 (on page 24) plots the expected net cash flow to the firm against the realization of \(y_1\).

The valuation response to the first period report is nonlinear and S-shaped. The intuition for the S-shaped response is the following. As the first period accounting report exceeds its expected value (i.e., the top half of Figure 2), there are three valuation factors in (12). First, the first period cash flow \((\theta_1)\) may be higher than expected, the valuation response is one-for-one (the first term in equation 12). Second, earnings management \((\beta)\) is positive, which drives the valuation lower (the second term in equation 12). Third, second period cash flow \((\mu_2)\) may be higher, which drives the valuation higher (the third term in equation 12). Given expression (13), as \(y_1\) gets higher, it is more likely to be driven by earnings management, so the valuation response gets smaller. As a result, the response is concave when \(y_1\) exceeds its expected value. For a similar reason, the response is convex when \(y_1\) misses its expected value (the bottom half of Figure 2).

There exists a literature on nonlinear valuation response to accounting numbers. Some consider the trade-off of permanent vs. transitory components of the reported earnings (e.g., Freeman and Tse [1992] and Das and Lev [1994]), which, in essence, look to the properties of the earnings-generation process for explanations. Along a similar line, Subramanyam [1996] considers the uncertainty about the precision of accounting numbers as the underlying reason for the non-linearity. Some look at legal constraints (e.g., limited liability rule) as the explanation for the empirical regularity (Fischer and Verrecchia [1997]). Antle, Demski, and Ryan [1994] focus on the fact that accounting reflects only a part of all available information (i.e., accounting is on an information “diet”). In contrast, nonlinear valuation shows up in this paper as a response to the reporting incentive of the firm.
4 Analysis of Regulator’s Preference

In this section, we focus on the regulator’s choices. In particular, we consider the trade-off in his decision to enforce existing standards (by setting cost parameters $c_{HL}$ and $c_L$) and in his decision to select an optimal accounting standard ($y^*_t$).

4.1 Preference Over Cost Parameters

We have shown it may be sub-optimal for the principal to design contracts that deter earnings management, given bounded personal cost parameters. Now we ask if it is socially optimal to set cost parameters to induce $\beta = 0$ by making personal cost parameters $c_{HL}$ and $c_L$ positive infinity. In other words, is it optimal to have a zero-tolerance policy?\textsuperscript{21}

Proposition 2: In the accounting regime, if $k_1 > k_2$ and $M$ is sufficiently large, the principal is strictly worse off with $c_L = c_{HL} = +4$ than with $c_L < c_{HL} < +4$.

With $c_L = c_{HL} = +4$, disaggregate output observation prevails. If $k_1 > k_2$, the variability of $\mu_2$ is completely absorbed by the second period performance measure with a higher bonus coefficient ($\delta_2 = \delta/k_2$) and, therefore, imposes more compensation risk on the agent. On the other hand, setting $c_L < c_{HL} < +4$ encourages moving some accounting performance across the two periods, which “spreads” the variability of $\mu_2$ across periods. In turn, it lowers the risk premium associated with the ex ante uncertainty on $\mu_2$. If the benefit of the lower risk premium is not completely outweighed by the cost due to earnings management, the principal prefers earnings management to the disaggregate output observation case. As a result, it is not universally the case principal would prefer a regulator who sets $c_L = c_{HL} = +4$.\textsuperscript{22}

\textsuperscript{21} An obvious concern here is the social cost of such a zero-tolerance rule. Since it is costly for the society to design and execute social institutions (like courts and enforcement agencies) to carry out any such regulation. The private nature of underlying information may dictate such a zero-tolerance rule may be prohibitively costly to enforce. Healy and Wahlen [1999] also make this point.

\textsuperscript{22} Several technical notes are worth mentioning. First, if $c_L$ and $c_{HL}$ are bounded and equal, the amount of output shifted would be fixed and the result in Proposition 2 disappears because the benefit of allowing earnings management comes from spreading the ex ante variability of $\mu_2$ across the two periods. For similar reasons, if there is no uncertainty about $\mu_2$, the result in Proposition 2 disappears since there is certainly no benefit to earnings management. Second, if $k_1 < k_2$, the result in Proposition 2 also disappears because there is no need to shift some of the variability of $\mu_2$ to the first period. This is because the fact the bonus coefficient in first period is set to be
Intuitively, the assigned bonus coefficients on accounting performances ($y_1$ and $y_2$) place weights on the three primitive random components in the agent’s compensation: $\theta_1$ in the first period cash flow, and $\mu_2$ and $\theta_2|\mu_2$ in the second period cash flows. When costs are infinity, $\mu_2$ and $\theta_2|\mu_2$ are bundled together and receive the same weights ($\delta_2$). When costs are different and bounded, earnings management de-bundles $\mu_2$ and $\theta_2|\mu_2$ such that $\mu_2$ may be assigned a lower weight than $\theta_2|\mu_2$. This is the source of the benefit provided by different and bounded costs.

Suppose a regulator is responsible for setting the cost parameters (e.g., the SEC sets and enforces rules that limit reporting flexibility). Further suppose the regulator’s goal is to maximize social welfare that place positive weights on the expected utilities of the principal and the agent. Because the agent’s ex ante expected utility is held constant (=U), the regulator’s preference over cost parameters is the same as that of the principal.

### 4.2 Preference Over Accounting Standards

So far in our analysis, the Realization Standard has been the maintained accounting standard. In this section, we allow a standard-setting role of the regulator, who selects an accounting standard from a nontrivial set of possible standards. Suppose there is a continuum of standards which call for reporting $y_{1^*}$ as $x_1$ plus some recognition of information about $\mu_2$. We operationalize the regulator’s choice as

<table>
<thead>
<tr>
<th>Presumably, the regulator sets cost parameter before the principal chooses the optimal contract, which is followed by the agent’s managerial input and performance reporting. Schipper [1989] points out this timing implies a temporal rigidity in the policy and the contracting environment. A fully dynamic model (where policy and contract-design change over time) is beyond the scope of this study.</th>
</tr>
</thead>
</table>
selecting $\gamma \in [0, 1]$ such that $y_1^* = x_1 + \gamma \mu_2$. In this formulation, the Realization Principle is a special case: $\gamma = 0$. Notice $\gamma = 1$ calls for full recognition of $\mu_2$. 24

The choice of the accounting standard affects both managerial contracting and firm valuation.

We study the firm valuation first. It is intuitive that an investor may prefer a higher $\gamma$ as it reveals “more” about future prospects of the firm. To confirm this intuition, we sketch a valuation demand for information as follows: suppose there exists a set of “small” shareholders who, unlike those represented by the board of directors, are risk-averse and have consumption smooth demand. 25 Their investment-consumption decisions depend upon their beliefs about the total output a period ahead. The less the remaining uncertainty, the better-off they are. As a gauge, we calculate the conditional variance of $x_1 + x_2$ given $y_1$ as a function of $\gamma$: 26

$$\text{Var}[x_1 + x_2 \mid y_1] = \sigma_x^2 + 4(1 - \gamma)^2 M^2 \left( \text{Prob}(\mu_2 = +M) \cdot (1 - \text{Prob}(\mu_2 = -M)) \right)$$

Notice the conditional variance achieves its minimum when $\gamma = 1$.

Now consider a contracting point of view. For any given standards, the agent’s personal cost of deviating from the accounting standard is given by $\frac{1}{2} \beta^2 c(.)$ with $\beta = y_1^* - y_1$. Like the analysis before, 27

24 A more elaborate representation of the standard choices may involve a pair of parameters, say $\{\gamma_1, \gamma_2\}$, which may call for reporting conditional on the realization of the signal $\mu_2$ and set $y_1^* = x_1 + \gamma_1 \mu_2$ given $\mu_2 = +M$ and $y_1^* = x_1 + \gamma_2 \mu_2$ given $\mu_2 = -M$. This formulation allows for a conservative standard which calls for early recognition of $\mu_2 = +M$ (i.e., $\gamma_1 = 1$) and late recognition of $\mu_2 = +M$ (i.e., $\gamma_2 = 0$).

25 We acknowledge a small investor may hold a well-balanced portfolio to diversify the idiosyncratic part of the risk away. However, regulators, such as the SEC, has been pushing firms to disclose both firm-specific as well as industry-wide information and has long championed itself as the defender of “small” investors, who are considered vulnerable and whose welfare may not be fully represented by the board. In this capacity, it is reasonable to assume the SEC cares for disclosing more valuation information in its policy making.

26 Suppose the preference of this set of “small” investors can be described by utility function: $u = \frac{1}{2} \beta^2 c(.)$ with the budget constraint $c_1 + c_2 = \pi q(x_1 + x_2)$ where $\pi$ is the “small” investors’ share of the firm. Given a realization of $y_1$, the optimal consumption choice calls for $c_1 = \frac{1}{2} \pi E[x_1 + x_2 \mid y_1]$. It can be shown the ex ante expected utility for a given $\gamma$ is decreasing in $\text{Var}[x_1 + x_2 \mid y_1]$, the conditional variance of the total output. We also assume the inherent risk involved in owning the shares of the firm cannot be diversified away.

27 Notice $\beta$ is the difference between the actual report $y_1$ and the standard $y_1^*$, not necessarily $x_i$. This is important as standards are only effective to the extent the economic agents respond to them. If $y_1^*$ does not figure into the agency personal cost calculation, standards will have no economic substance because the agent would not respond at all. See Dye [2000] for discussion of a same point.
we are interested in whether earnings management remains in equilibrium. In addition, we are interested in whether the choice of $\gamma$ affects agency costs. To the extent the agent responds to accounting standards, the choice of $\gamma$ affects the variance of reported output ($y_i$), which, in turn, affects the trade-off between incentive provision and risk-sharing.

**Proposition 3**: Assume $c_L = c_H$. If $k_1 \neq k_2$, then $\beta^* \neq 0$. Further, (1) if $k_1 > k_2$, the principal is better off with higher $\gamma$; and (2) if $k_1 < k_2$, the principal is better off with lower $\gamma$.

The $\beta \neq 0$ result is not surprising since the choice of $\gamma$ does not change the fact principal must balance the temporal difference in moral hazards and the problem of earnings management activities. The economic intuition of the latter results is that the choice of $\gamma$ moves the risk associated with $\mu_2$ across the two periods. As a result, it moves the required risk premiums across periods. If $k_1 > k_2$, the second period incentive problem is more severe, the total risk premium is reduced by setting a higher $\gamma$ because with a higher $\gamma$, the increase in the first period risk premium is less than the decrease in the second period risk premium.

Proposition 3, combined with the valuation analysis, implies that there may exist a conflict in the standard-setting from the two perspectives. From the valuation perspective, early recognition standards (i.e., $\gamma=1$) may benefit “small” shareholders by making the accounting report more valuation informative. However, it may increase the agency cost in the labor market which hurts all investors. This conflict in standard-setting reflects the fundamental distinction between the stewardship and the valuation use of information (as in Gjesdal [1981] and see Dye [1988] for a formulation of an internal and an external demand for earnings management, which exhibits a similar conflict).

If Proposition 1 is seen as endogenizing a demand for earnings management, Proposition 2 and 3, which focus on optimal policy choices of $c_L$, $c_{li}$, and $\gamma$, may be interpreted as steps toward endogenizing the supply side of earnings management (because the personal cost of earning management is a function of $c_L$, $c_{li}$, and $\gamma$). However, we acknowledge this characterization is not at all comprehensive so the
results reported here should be interpreted with the specific model in mind and should not be taken as literal policy recommendations. In reality, implementing such policies would require a concerted effort from a number of parties such as auditors, the board, lawmakers, and judges.

5. Conclusions and Limitations

In this paper we formulate earnings management as endogenously determined by the interaction among various economic agents: the manager, the owner, and the regulator. With the business headlines these days, it seems tempting to eliminate all earnings management incentives, either through designing compensation contracts (setting $\delta_1 = \delta_2$) or through enforcing accounting standard (setting $c_L = c_H = +4$). Given the various economic trade-offs considered in the model, this paper shows neither approach is unconditionally and universally preferred. In addition, when selecting an optimal accounting standard, valuation concerns may conflict with stewardship concerns. This is critical in understanding accounting attributes as informational and economic phenomena.

Two caveats deserve special emphases in interpreting the results of the paper. First, the contracting form is restricted to a fixed salary and a linear bonus based on accounting numbers. This precludes other, more complex, compensation contracts we see in practice, such as stock options. The consideration of these contrasts is beyond the scope of this paper and shall be an interesting line of research. Second, the valuation analysis in Section 3.3.3 presumes the investors know the precise distribution of firm underlying cash flows and the managerial incentive structure. In practice these details are far from transparent. In addition, non-linear valuation response may be driven, in practice, by a combination of various factors including the incentive-based factor presented in this paper as well as non-incentive factors studied by the existing literature.

In this paper, as with most scholarly work, certain aspects of economic forces have assumed passive (or even silent) roles. While institutional constraints are admitted into the model through the personal cost of earning management, the model stops short of explicitly introducing the standard setting
process or enforcement policy as responsive forces in the reporting game. While valuation responses
have been considered, the endogenous demand for valuation consideration is outside the model because
productive decisions are exogenously given (i.e., high efforts are wanted for both period) and because
consumption decision does not entice mid-game information acquisition (i.e., both the principal and the
only agent care the end-of-the-game cash flows). Lastly, welfare analysis contained in the last section is
rather limited. The regulator is only interested in the welfare of the principal, the agent, and a “small”
investor (who cares about the noted valuation issue). The welfare of other parties, such as consumers,
suppliers, competitors, or even the regulator himself, are outside the model.
Figure 1: Sequence of Events

\[
\begin{align*}
t &= 1 & t &= 2 \\
\hline
\text{agent's inputs} & a_1 \in \{H, L\} & a_2 \in \{H, L\} \\
\text{agent’s private information} & \mu_2 \in \{M, +M\} \\
\text{output} & x_1 = k_1 a_1 + \theta_1 & x_2 = k_2 a_2 + \theta_2 & x = x_1 + x_2 \\
& \theta_1 \sim N(0, \sigma_1^2) & \theta_2 \sim N(\mu_2, \sigma_2^2) \\
\text{agent’s reporting choice} & y_1 = x_1 + \beta y_2 = x \\
\text{agent’s personal costs} & v(a_1) & v(a_2) \text{ efforts} \quad \text{earnings management} \\
& c = c_L \text{ if } \text{sign}(\beta) = \text{sign}(\mu_2) & c = c_H \text{ if } \text{sign}(\beta) \neq \text{sign}(\mu_2) \\
\text{linear contract forms} & \text{full output observation} & w = \delta_1 x_1 + \delta_2 x_2 \\
& \text{aggregate output observation} & w = \delta x \\
& \text{accounting regime} & w = \delta_1 y_1 + \delta_2 y_2 \\
& \text{accounting regime with self-report } m(\mu_2) & w = \delta_{0m} + \delta_{1m} y_1 + \delta_{2m} y_2 \\
\text{agent’s net compensation} & \text{w ! v(a_1) ! } \frac{1}{2} \beta^2 c(.) ! v(a_2) \\
\text{principal’s net cash flow} & q x ! w
\end{align*}
\]
This graph plots the valuation response to first period accounting ($y_1$) report:

$$E[q(x_1 + x_2) - (\delta_0 + \delta_1 y_1 + \delta_2 y_2) |y_1]$$

where

$$E[x_1 + x_2 | y_1] = y_1 ! [\text{Prob}(\mu_2 = +M | y_1) (\delta_1 ! \delta_2)(1/c_L ! 1/c_H) + (\delta_1 ! \delta_2)/c_H ]$$

$$+ (2 \text{Prob}(\mu_2 = +M | y_1) ! 1) M + k_2 H$$

and

$$\text{Prob}(\mu_2 = +M | y_1) = \frac{1}{1 + \exp \left( \frac{(\delta_1 - \delta_2)(1/c_L - 1/c_H)(2k_H + (\delta_1 - \delta_2)(1/c_L + 1/c_H) - 2y_1)}{2\sigma_1^2} \right)}$$

The numerical specifications are the following: $q = 30$, $H = 100$, $L = 0$, $M = 200$, $\sigma_1 = \sigma_2 = 100$, $k_1 = 1$, $k_2 = 2$, $\delta_1 = 20$, $\delta_2 = 10$, $c_L = 1$, and $c_H = 5$. So the ex ante expectation of total cash flow is $E[q(x_1 + x_2)] = q(k_1H + k_2H) = 9,000$, and expectation of the agent’s first period report is expected output plus expected shifting: $E[y_1] = k_1H + E[\beta'] = 100 + .5 (20! 10) (1/1 + 1/5) =106$. If the actual report turns out $y_1 = 106$, the expected total cash flow remains at 9,000. The expected compensation would be $20 x (100 + 6) + 10 x (200 - 6) = 4,060$. So when $y_1 = 106$, the expected net cash flow is 4,940 (9,000 − 4,060). The fixed component of the compensation ($\delta_0$) is normalized to zero since it does not change the shape of the plot.
APPENDIX

A Running Numerical Example

Let $H = 100$, $L = 0$, $v(H) = 2,000$, $v(L) = 0$, $M = 200$, $\sigma_1 = \sigma_2 = 100$, $r=.0001$, $U = \exp(\frac{1}{r} 5,000)$, $k_1 = 1$, $k_2 = 2$. Given disaggregate output observation, the optimal contract is $w^*(x_1, x_2) = 5,448 + 20 x_1 + 10 x_2$, and the expected compensation $E[w^*(x_1, x_2)] = 9,448$. On the other hand, if $k_2 < k_1$ (say $k_1 = 4$) the solution yields $w^*(x_1, x_2) = 5,261 + 5 x_1 + 10 x_2$, and the expected compensation is $E[w^*(x_1, x_2)] = 9,261$.

For aggregate output observation, when $k_1 = 1$, the optimal contract under aggregate observation is $w^*(x) = 4,180 + 20x$ and the expected compensation is $E[w^*(x)] = 10,180$. When $k_1 = 4$, the optimal contract is $w^*(x) = 3,299 + 10 x$ and the expected compensation is $E[w^*(x)] = 9,299$.

For proposition 1, assume symmetric cost structure, $c_L = c_H = 1$ for simplicity. In the case of $k_1 = 1$, the optimal linear contract is $w^*(y_1, y_2) = 5,399 + 20 y_1 + 10 y_2$, and the expected compensation is $E[w^*(y_1, y_2)] = 9,599$. In equilibrium, $\beta^* = 10 (= (20! 10)/1)$. Inducing $\beta = 0$ requires writing a contract that is identical to the aggregate output observation case. Such a contract, $w^*(y_1, y_2) = 4,180 + 20 (y_1 + y_2)$, yields a more costly expected compensation of 10,180. On the other hand, if $k_1 = 4$, the optimal contract is $w^*(y_1, y_2) = 4,274 + 7.5 y_1 + 10 y_2$, and the expected compensation is $E[w^*(y_1, y_2)] = 9,286$. In equilibrium, $\beta^* = 2.5 (= (7.5! 10)/1)$. Inducing $\beta = 0$ requires writing a contract $w^*(y_1, y_2) = 3,299 + 10 (y_1 + y_2)$, with a more costly expected compensation of $E[w^*(y_1, y_2)] = 9,299$.

Proof of Lemma 1:

Under disaggregate output observation, the principal’s problem is the following mechanism design program:

$$
\min_{C^{full}} \text{subject to } E[U(I(.); \alpha^{li})] \geq U \\
E[U(I(.); \alpha^{H}, \alpha^{li})] \geq E[U(I(.); \alpha^{li})] \forall \alpha^{li}, \alpha
$$

-25-
Consider the following restricted version of the above program:

\[
\text{minimum } E[I(x_1, x_2) \mid H, \alpha^H] \\
\delta_0, \delta_1, \delta_2
\]

Subject to \( E[U(I(.); \alpha^H)] \geq U \)

\[
E[U(I(.); \alpha^H)] \geq E[U(I(.); \alpha^L)]
\]

\[
E[U(I(.); \alpha^H)] \geq E[U(I(.); \alpha^L)]
\]

given assumptions on the preference and technologies, we can write the agent’s expected utility of adopting strategy \( \alpha^H \), as the following:

\[
E[U(I(x_1, x_2); \alpha^H)] = \sum_{\mu_2} \text{Prob}(\mu_2) \int \exp[r(\delta_0 + \delta_1(k_1H + \theta_1) + \delta_2(k_2H + \theta_2) - 2v(H))] f(\mu_2, \sigma_2^2) d\theta_2 f(0, \sigma_1^2) d\theta_1
\]

where \( f(\mu, \sigma^2) \) is the density function of a normal distribution with mean \( \mu \) and variance \( \sigma^2 \). Because all three random variables \( (\mu_2, \theta_1, \theta_2) \mid \mu_2 \) are independent, we can rewrite the expression as:

\[
E[U(I(x_1, x_2); \alpha^H)] = \sum_{\mu_2} \text{Prob}(\mu_2) \{ \exp[r(\delta_0 + \delta_1(k_1H + \theta_1) + \delta_2(k_2H + \mu_2) - 2v(H)) r/2 (\delta_1^2 \sigma_1^2 + \delta_2^2 \sigma_2^2)] \}
\]

Since \( \text{Prob}(\mu_2) = .5 \) for \( \mu_2 \in \{! M, +M \} \), we have:

\[
E[U(I(x_1, x_2); \alpha^H)] = \eta(\delta_2, \delta_2) V(CE(H, H))
\]

where \( \eta(i, j) = .5 \exp(! r i M) + \exp(! r j (! M))] V(CE(a_i, a_j)) / \exp(! r CE(a_i, a_j)); \) and

\[
CE(a_1, a_2) / \delta_0 + \delta_1 k_1 a_1 + \delta_2 k_2 a_2 + v(a_1) + v(a_2) / r/2 (\delta_1^2 \sigma_1^2 + \delta_2^2 \sigma_2^2)
\]

Similarly, we can write:

\[
E[U(I(x_1, x_2); \alpha^L)] = \eta(\delta_2, \delta_2) V(CE(H, L))
\]

\[
E[U(I(x_1, x_2); \alpha^H)] = \eta(\delta_2, \delta_2) V(CE(L, H))
\]

The objective function is

\[
E[I(x_1, x_2) \mid H, \alpha^H] = \delta_0 + \delta_1 k_1 H + E[\theta_1] + \delta_2 (k_2 H + E[\theta_2])
\]

Since \( E[\theta_1] = E[\theta_2] = 0 \), we have:

\[
E[I(x_1, x_2) \mid H, \alpha^H] = \delta_0 + \delta_1 k_1 H + \delta_2 k_2 H
\]
The IC constraint (A-3) can be rewritten as:

$$\eta(\delta_2, \delta_2) \ V(CE(H, H)) \geq \eta(\delta_2, \delta_2) \ V(CE(H, L))$$

which is the same as CE (H, \(\alpha^H\)) \$CE (H, \(\alpha^L\)) because \(V(.)\) is increasing monotonically, it can be rewritten as \(\delta_2^k H \ v(H) \geq \delta_2^k L \ v(L)\), which implies:

$$\delta_2 \geq \frac{\delta_2}{k_2}$$

Similarly, IC constraint \(E[U(I(.); \alpha^H)] \geq \ E[U(I(.); \alpha^L)]\) can be rewritten as CE(H, H) \$CE(L, H). We further reduce it to \(\delta_1^k H \ v(H) \geq \delta_1^k L \ v(L)\), which implies (given \(L=0\) and \(v(L)=0\)):

$$\delta_1 = \frac{\delta_1}{k_1}$$

The IR constraint can be rewritten as: \(\eta(\delta_2, \delta_2) \ V(CE(H, H)) \geq U\)

This constraint must bind for similar reasons. So it must be the case that \(\delta_0^*\) is such that:

$$\eta(\delta_2, \delta_2) \ V(CE(H, H)) = U$$

so \(\delta_0^*\) must be such \(U = \eta(\delta_2, \delta_2) \{ ! \exp[! r(\delta_0^*+\delta_1^k a_1+\delta_2^k a_2) ! 2v(H) ! r/2 (\delta_1^2 \sigma_1^2+\delta_2^2 \sigma_2^2))]\}

\(! \exp[! r(\delta_0^*+\delta_1^k a_1+\delta_2^k a_2) ! 2v(H) ! r/2 (\delta_1^2 \sigma_1^2+\delta_2^2 \sigma_2^2)]) = \frac{U}{\eta(\delta_2, \delta_2)}\)

\(! r(\delta_0^*+\delta_1^k a_1+\delta_2^k a_2) ! 2v(H) ! r/2 (\delta_1^2 \sigma_1^2+\delta_2^2 \sigma_2^2)) = \log(! U) ! \log (\eta(\delta_2, \delta_2))\)

\(\delta_0^* = ! \log(! U)/r +(1/r) \log (\eta(\delta_2, \delta_2)) \) ! \(\delta_1^k a_1+\delta_2^k a_2) ! 2v(H) ! r/2 (\delta_1^2 \sigma_1^2+\delta_2^2 \sigma_2^2))\)

Substitute \(\delta_0^*\) into \(E[I(x_1, x_2)|H, \alpha^H] = \delta_0 + (\delta_1^k k_1+\delta_2^k k_2)H\), we have:

\(E[I(x_1, x_2)|H, \alpha^H] = \delta_0 + (\delta_1^k k_1+\delta_2^k k_2)H\)

So the risk-premium portion of the expected payment is \(RP = \frac{1}{2} (\delta_1^2 \sigma_1^2+\delta_2^2 \sigma_2^2) + (1/r) \log (\eta(\delta_2, \delta_2))\)

Therefore, we can simplify the program using a risk-premium formulation:

\[
\begin{align*}
\text{minimize} & \quad \text{RP} = \frac{1}{2} (\delta_1^2 \sigma_1^2+\delta_2^2 \sigma_2^2) + (1/r) \log (\eta(\delta_2, \delta_2)) \\
\text{Subject to} & \quad \delta_2 \geq \frac{\delta_2}{k_2} \quad \text{(A-5)} \\
& \quad \delta_1 \geq \frac{\delta_1}{k_1} \quad \text{(A-6)} \\
\end{align*}
\]

Let \(\lambda_1, \lambda_2\) be the Lagrange multipliers and we obtain the following first order conditions:

$$\delta_1 r \sigma_1^2 \neq 0 \text{ or } \lambda_2 = \delta_1 r \sigma_1^2 \quad \text{(FOC-1)}$$
\[
\delta_2 r \sigma_2^2 + \left(\frac{1}{r}\right) M \log(\eta(\delta_2, \delta_2)) = 0 \quad \text{(FOC-2)}
\]

If \(\lambda_2 = 0, \delta_1 = 0\), violating (A-6) so \(\lambda_2 > 0\). Given (FOC-2) and \(\delta_2 > 0\) (by A-5), we have \(\lambda_1 = \frac{(\delta_2! k_1(\delta_2) + \delta_2) \sigma^2 + (1/r) \log(\eta(\delta_2, \delta_2))/M_2 > 0}\). Both IC constraints are binding, or \(\delta^*_2 = \delta/k_2\) and \(\delta^*_1 = \delta/k_1\).

Notice at the solution to the restricted version of the program, the expected utility of continuation given any \(u_1, x_1, u_2\), history is such that the agent will prefer to provide high effort. So all other input combinations (e.g., \(\alpha(.) = H\) some of the time and \(\alpha(.) = L\) otherwise) are inferior to \(H, \alpha_H\), at the solution. There is no loss of generality to consider the restricted version of the program.

Proof of Lemma 2:

Under aggregate output observation, the principal’s problem is equivalent to the disaggregate observation case except there is an additional constraint of \(\delta_1=\delta_2=\delta\). The mechanism design program becomes:

\[
\begin{align*}
\text{minimum} & \quad \text{RP} = 2 v(H) + \delta^2 \sigma^2 + \left(\frac{1}{r}\right) \log(\eta(\delta, \delta)) \\
\text{Subject to} & \quad \delta \leq \delta/k_2 \quad \text{(A-7)} \\
& \quad \delta \leq \delta/k_1 \quad \text{(A-8)}
\end{align*}
\]

and since one IC constraint must bind here, we have \(\delta^* = \max \{\delta/k_1, \delta/k_2\}\).

Proof of Proposition 1

If the principal decides to induce the “reliable” reporting (\(\beta = 0\)), he must deter both “borrow” and “lend” incentives, and the only way to do that is to equate \(\delta_1\) to \(\delta_2\) (assuming the exogenous cost of shifting, \(c\), bounded), which essentially leads to the optimal linear contract under aggregate observation. Specifically, suppose, \(k_1 > k_2\) substituting optimal linear contract in Lemma 2, \(\delta_1^* = \delta_2^* = \delta^* = \delta/k_2\) with appropriate \(\delta_0\) such that IR binds, we can write the expected compensation as the following as \(E[w] = \frac{1}{\rho} \log(\frac{1}{\rho}) + 2 v(H) + \frac{1}{2} r (\delta/k_2)^2 (\sigma_1^2 + \sigma_2^2) + g(\delta/k_2)\), where \(g(\delta/k_2) = (1/r) \log \frac{1}{2} [\exp(\frac{1}{r} \log M) + \exp(\frac{1}{r} \log M)]\). However, consider another feasible contract which allows for shifting. Specifically, suppose \(c_L = c_H = c\) and let \(\delta_2 = \delta/k_2\) and \(\delta_1 = \delta_2! \epsilon\) (with \(\delta_0\) appropriately adjusted such that
IR binds), we can write the expected compensation as the following as 

\[ E[w] = \log(\frac{U}{r}) + 2v(H) + \varepsilon/2c + \frac{1}{2} r \delta_1^2 \sigma_1^2 + \frac{1}{2} r \delta_2^2 \sigma_2^2 + g(\delta_2). \]

The difference in expected payment between the two feasible contracts is:

\[ E[w] - E[w]' = \varepsilon/2c + \frac{1}{2} r \sigma_1^2 [\delta_2^2 - \delta_1^2] \]

Examining this difference reveals that if \( \sigma_1 \) is positive, \( E[w] - E[w]' > 0 \) for a small enough \( \varepsilon \). In other words, the \( \beta=0 \) contract is strictly dominated by a contract with \( \beta \neq 0 \). Therefore the optimal contract cannot be such that \( \beta=0 \), thus in equilibrium, \( \beta^* \neq 0 \). A similar argument can be made for the case with \( c_L \neq c_H \) and cases with \( k_1 > k_2 \).

~

**Proof of Proposition 1 in the presence of self-report \( \mu_2 \):**

One of the assumptions in the earlier sections is that the agent is not able to send a separate, self-report on \( \mu_2 \) after he observes it. In the case of symmetric cost (\( c_L = c_H \)), information about \( \mu_2 \) is blocked. (Notice with symmetric costs, \( \beta \), thus \( y_j \), does not depend on \( \mu_2 \).) In the case of asymmetric costs, \( \mu_2 \) is partially communicated through \( y_1 \) (with asymmetric costs, \( \beta \) depends on \( \mu_2 \)). Now we allow for a separate self-report on \( \mu_2 \) along with the accounting report. Specifically, the self-report, denoted by \( m \), is a mapping from \{ \( ! M, +M \) \} to \{ \( ! M, +M \) \}.

To stay in the linear contract framework and to keep the tractability of the model, we choose to consider the following set of contract forms:

\[ w(y, m) = \delta_0^m + \delta_1^m y_1 + \delta_2^m y_2 \]  

(A-10)

The agent is offered a menu of linear contracts indexed by his self-report \( m(z) \). Given the binary structure of \( \mu_2 \), we consider two \( m \) mappings: truthful (\( m = \mu_2 \)) and pooling (\( m(\mu_2 = M) = m(\mu_2 = ! M) \)). The principal’s problem of inducing the desirable managerial input and a particular \( m \) mapping can be written as:

\[ C'(m) / \min \{ E[w(y_1, y_2)|H, \alpha^H, \beta^*, m] \} \]

subject to \( E[U(w(.);@H, \alpha^H, \beta^*, m)] \geq U \)

(A-11)

(A-12)
Proposition: In the accounting regime with self-report, within the class of linear contracts in (A-10), if $\sigma_1$ and $\sigma_2$ positive and $c_{11}, c_{12} < +4$, $\beta^* \not\equiv 0$.

Notice at the time the agent chooses the self-report, he wishes to maximize the expected utility given $a_1, x_1, \mu_2$. Specifically,

$$\beta^*, H, m \arg\max E[U(w(.); a_2, \beta, m; a_1, x_1, \mu_2)]$$ \hspace{1cm} (A-14)

For a given contract, the agent’s expected utility has a certain equivalent representation so we can rewrite (A-14) into:

$$\beta^*, a_2 = H, m^* \arg\max \delta_0^m + \delta_1^m (k_1a_1 + \theta_1 + \beta) + \delta_2^m (k_2a_2 + \mu_2 - \beta)$$

$$\nu(a_1) \nu(a_2) c^2/2 (\frac{1}{2}) \sigma_2^2$$ \hspace{1cm} (A-15)

Suppose the principal insists on truthful self-report ($m(\mu_2) = \mu_2$) and no shifting ($\beta=0$), he must set $\delta_1^{m=M} = \delta_2^{m=M}$ (denoted $\delta^M$) and $\delta_1^{m=M} = \delta_2^{m=M}$ (denoted $\delta_1^{M}$), which makes sure $\beta=0$ regardless of the self-report. To ensure $m(\mu_2)=\mu_2$, the following conditions must also be satisfied (CE means certain equivalent):

CE$$[U(w(.), .)|m(z)=z, a_1=H, a_2=H; \theta_1, \mu_2=M]$$

$$= \delta_0^M + \delta^M (k_1H + \theta_1) + \delta^M (k_2H + M) \nu(H) (\frac{1}{2}) r \sigma_2^2$$

$\$ CE$$[U(w(.), .)|m(z)=z', a_1=H, a_2=H; \theta_1, \mu_2=M]$$

$$= \delta_0^M + \delta^M (k_1H + \theta_1) + \delta^M (k_2H + M) \nu(H) (\frac{1}{2}) r \sigma_2^2$$

for all $\theta_1$

and

CE$$[U(w(.), .)|m(z)=z, a_1=H, a_2=H; \theta_1, \mu_2=! M]$$

$$= \delta_0^M + \delta^M (k_1H + \theta_1) + \delta^M (k_2H ! M) \nu(H) (\frac{1}{2}) r \sigma_2^2$$

$\$ CE$$[U(w(.), .)|m(z)=z', a_1=H, a_2=H; \theta_1, \mu_2= ! M]$$

$$= \delta_0^M + \delta^M (k_1H + \theta_1) + \delta^M (k_2H ! M) \nu(H) (\frac{1}{2}) r \sigma_2^2$$

for all $\theta_1$
Since \( \theta_1 \)'s support is the real line, the only way to satisfy this constraint is the set \( \delta^M = \delta^{1M} \), which implies \( \delta_1 \) in \( \delta^M \) is \( \delta_1 \) in \( \delta^{1M} \). In other words, insisting on both truthful self-report and no shifting confines the principal to writing contracts as if the setting is the aggregate output observation case. Given Proposition 1, we can improve on aggregate observation cases by inducing some shifting (\( \beta \neq 0 \)).

This proposition confirms that, in this model, the endogenous earnings management is driven by the assumption on limited contractual forms, not the assumption on the agent’s inability to self-report \( \mu_2 \).

**Proof of Proposition 2:**

Consider the cases with no self-report on \( \mu_2 \). If \( c_L = c_H = +4 \), and suppose \( k_1 > k_2 \), substituting optimal linear contract in Lemma 1, \( \delta_1^* = \delta/k_1 \) and \( \delta_2^* = \delta/k_2 \) with appropriate \( \delta_0 \) such that IR binds, we can write the expected compensation as the following as \( E[w] = \! \log(! U)/r + 2 v(H) + \frac{1}{2} r (\delta/k_1)^2 \sigma_1^2 + \frac{1}{2} r (\delta/k_2)^2 \sigma_2^2 + g(\delta/k_2) \). Notice in the setting where \( c_L < c_H < +4 \), the same contract will induce shifting (because \( \delta_1^* \neq \delta_2^* \)). However, consider a slightly altered contract of \( \delta_1^* = \delta/k_1 \) and \( \delta_2^* = \delta/k_2 \) with \( \delta_0 \) such that IR binds (notice IR now is different from the setting \( c_L = c_H = +4 \)). We can write the expected compensation as \( E[w]^* = \! \log(! U)/r + 2 v(H) + \frac{1}{2} r (\delta/k_1)^2 \sigma_1^2 + \frac{1}{2} r (\delta/k_2)^2 \sigma_2^2 + g^*(\delta/k_2, c_L, c_H) \), where \( g^*(\delta/k_2, c_L, c_H) = \frac{1}{2} \log{1/2} [\exp(r(\delta/k_2 M)) + \exp(-r(\delta/k_2 (1 \! - M)))] \).

The difference in expended payment between the two feasible contracts is:

\[
E[w] - E[w]^* = \frac{1}{2} \delta^2(1/k_1 + 1/k_2)^2/c_L + [g(\delta/k_2) - g^*(\delta/k_2, c_L, c_H)]
\]  

(A-16)

Examining this difference reveals that if the first term in the RHS of (A-16) is negative but not a function of \( M \) and the second term of the RHS of (A-16) is positive and increasing function in \( M \), so if \( M \) “sufficiently large,” \( E[w] - E[w]^* > 0 \), or the \( \beta = 0 \) contract is strictly dominated by a contract with \( \beta \neq 0 \). Therefore the optimal contract cannot be such that \( \beta = 0 \), thus in equilibrium, \( \beta^* \neq 0 \). If \( k_1 < k_2 \), a similar difference like (A-16) will always be negative.
Derivation of Prob(z=+M|y_1) in equation (13)

Given $y_1$ is a mixture of binary random variable and the normal variable, we can write the unconditional density function of $y_1$ as:

$$f(y_1) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(y_1-k_1H-(\delta_1-\delta_2)/c_1)^2}{2\sigma^2}} + \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(y_1-k_1H-(\delta_1-\delta_2)/c_2H)^2}{2\sigma^2}}$$

The conditional density of $y_1$ given $\mu_2=+M$ is:

$$f(y_1|\mu_2 = +M) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(y_1-k_1H-(\delta_1-\delta_2)/c_1)^2}{2\sigma^2}}$$

By Bayes’s rule, we have equation (17):

$$f(\mu_2 = +M|y_1) = \frac{1}{1 + e^{-\frac{(\delta_1-\delta_2)(k_1H-c_1)k_2H+2(\delta_1H-c_2H)(k_1H+c_2H)-2y_1)^2}{2\sigma^2}}}$$

Proof of Proposition 3:

Given $c_H = c_L (= c)$, the optimal reporting by the agent is $\beta^* = (\delta_1 - \delta_2)/c$. In designing the optimal linear contract, the IR constrain binds as before, we have:

$$U = \exp[\! r (\delta_0 + \delta_1 k_1H + \delta_2 k_2 H - 2v(H) - \frac{1}{2} r(\delta_1^2 \sigma_1^2 + \delta_2^2 \sigma_2^2)) ]$$

$$\times \{.5 \exp[\! r (\delta_1(\gamma M + \beta) + \delta_2((1 - \gamma)M - \beta)) - \frac{1}{2} \beta^2 c)]$$

$$+ .5 \exp[\! r (\delta_1(\gamma M + \beta) + \delta_2((1 - \gamma) M + \beta)) - \frac{1}{2} \beta^2 c)]$$

Solve for $\delta_0$ and substituting $\beta^* = (\delta_1 - \delta_2)/c$, we have:

$$\delta_0 = \log(! U)/r! \delta_1 k_1H + \delta_2 k_2 H + 2v(H) + \frac{1}{2} r(\delta_1^2 \sigma_1^2 + \delta_2^2 \sigma_2^2)) + (\delta_1 - \delta_2)^2/(2c)$$

$$+ \log \frac{1}{2} \{\exp[\! r M(\delta_1(\gamma + \delta_2(1 - \gamma))M) + \exp[\! r (! M) (\delta_1(\gamma + \delta_2(1 - \gamma))]$$

Substituting $\delta_0$ into the expected payment, we have:

$$E[w(.)|H, a^H, \beta^*] = \log(! U)/r! + 2v(H) + \frac{1}{2} r(\delta_1^2 \sigma_1^2 + \delta_2^2 \sigma_2^2)$$

$$+ \log \frac{1}{2} \{\exp[\! r M(\delta_1(\gamma + \delta_2(1 - \gamma))M) + \exp[\! r (! M) (\delta_1(\gamma + \delta_2(1 - \gamma))]$$

(A-17)
For any given standard \((\gamma)\), similar to proposition 1, we can prove that optimal contract will involve \(\beta^* \neq 0\). To see this assume \(k_1 < k_2\), suppose the contrary, we must have \(\delta_1 = \delta_2 = \delta/k_1\). Using the same proof strategy as in proposition 1, we can prove \(\delta_1^* \neq \delta_2^*\) or \(\beta^* \neq 0\).

Knowing \(\delta_1^* \neq \delta_2^*\) or \(\beta^* \neq 0\). For any given pair \(\{\delta_1^*, \delta_2^*\}\), we see (from equation (A-17)), lower \(\gamma\) lowers the expected payment for the case of \(k_1 < k_2\). Similarly, higher \(\gamma\) lowers the expected payment for the case of \(k_1 > k_2\). To see the first claim, notice when \(k_1 < k_2\), \(\delta_1^* > \delta_2^*\), so \(\log \frac{1}{2} \{\exp[ r M(\delta_1 \gamma + \delta_2 (1! \gamma) M] + \exp[ r (1! M) (\delta_1 \gamma + \delta_2 (1! \gamma)]\}\) is increasing in \(\gamma\) by Jensen’s inequality. Similar argument holds for the second claim.
References


-35-


