12-2008

Limited Managerial Attention and Endogenous Precision of Performance Measures

Lin Nan
Carnegie Mellon University, linnan@andrew.cmu.edu

Pierre Jinghong Liang
Carnegie Mellon University, liangj@andrew.cmu.edu

Limited Managerial Attention and Endogenous Precision of Performance Measures*

Pierre Jinghong Liang
Tepper School of Business
Carnegie Mellon University
5000 Forbes Ave
Pittsburgh, PA 15213
liangj@andrew.cmu.edu

Lin Nan
Tepper School of Business
Carnegie Mellon University
5000 Forbes Ave
Pittsburgh, PA 15213
linnan@andrew.cmu.edu

December 2008
(First Draft: November 2008)

Abstract

In this paper, we model two drivers which underlie the economic trade-off shareholders face in designing incentives for optimal effort allocation by managers. The first driver is limited managerial attention, by which we mean that performing one task may have an adverse effect on the cost-efficiency of performing another. The second is the presence of a performance reporting task, by which we mean the manager's ability to exert personally costly effort to improve the precision (or quality) of his/her performance measures. We show that the subtle interactions of the two drivers may alter the characteristics of incentive provision. First, we show the interaction may lead to a positive relation between the strength of the incentive and the endogenous variance of the performance measures. Second, the interaction may render an otherwise informative performance signal unused in equilibrium incentive contracts. We show two cases in which an informative signal is unused, for two distinct reasons. In one case the principal does not use the signal whose precision can be improved by the manager, in order to discourages the manager from diverting attention to the performance reporting task (which makes the productive effort more costly). In another case with asymmetric information about the nature of the performance measurement system, the principal may discard the signal which cannot be influenced by the manager in order to encourage a truthful self-report by the manager.

*We wish to thank Jon Glover for initial encouraging discussions and Mark Bagnoli, Jeremy Bertomeu, John O'Brien, Susan Watts and other seminar participants at Carnegie Mellon University and Purdue University for their comments.

Limited Managerial Attention and Endogenous Precision of Performance Measures

1 Introduction

In a modern firm, a well-motivated management team has become a vital source of organizational success. One important component of designing managerial incentives is to properly induce an optimal allocation of managerial effort over multiple tasks (see Roberts 2004, p. 140-153). Among crucial tasks inviting the limited managerial attention, the performance reporting task (both internal and external) stands out as of significant interests by the press, policy makers, and the academic accounting profession. While many wide-publicized cases have been negative (e.g., Enron and Worldcom), most managerial efforts on reporting firm performance to outsiders are legitimate, such as improving reporting information systems and various investor relation activities in order to improve the quality of information shareholders receive about firm performance. An example of management's performance reporting effort is to maintain and improve internal control over financial reporting (ICOFR). Recent SEC guidence on ICOFR (SEC 2007) imposes significant demand of attention on the management of public companies.[1] In this light, managers face a trade-off between productive efforts such as identifying real investment opportunities and "non-productive" effort such as performance reporting tasks.[2] More broadly, limited attention is a pervasive issue in the management of large organizations. In a classic work, Herb Simon points out

> "... the scarce resource is not information; it is processing capacity to attend to information. Attention is the chief bottleneck in organizational activity, and the bottleneck becomes narrower and narrower as we move to the tops of organizations, where parallel processing capacity become less easy ..." (Simon 1973, page 270.)

In this paper, we formally model the two drivers underlying the economic trade-off in managerial effort allocation. The first driver is limited managerial attention, by which we mean that performing one task may have an averse effect on the cost-efficiency of performing another. The second is the presence of a performance reporting task, by which we mean the manager's ability to exert personally costly effort to improve the precision (or quality) of his/her performance measures. We show that the subtle interactions of the two drivers may alter the characteristics of incentive provision. Such alterations shed light on our understanding of some recognizable practices. First, we show that the interaction may lead to a positive relation between the strength of incentive and the endogenous variance of the performance measures. This is consistent with many empirically mixed findings of the relation (see Prendergast 2002 and Lafontaine

[1] In particular, "management should evaluate the design of the controls to determine whether they adequately address the risk that a material misstatement in the financial statements would not be prevented or detected in a timely manner. ... that the evaluation of evidence about the operation of controls should be based on assessments of the controls' associated risk." (KPMG 2007)

[2] The following quote in London Stock Exchange's *A Practical Guide to Listing* (as quoted in Peng and Roell 2008), speaks to the significance of this trade-off. "Both the floatation process itself and the continuing obligation–particularly the vital investor relations activities ... –use up significant amounts of management time which might otherwise be directed to running the business. ... It is vital that you maintain your company's profile, and stimulate interest in its shares on a continuing basis. ... you cannot leave press or investor relations to your advisors. Top executives will commonly devote at least a couple of days a month to developing and nurturing such contacts." (page 11 and pages 47-48)

and Bhattacharyya 1995 among others). Second, we show two cases in which the interaction may render an otherwise informative performance signals unused in equilibrium contracts. This second result provides an incentive reason that certain contractible, informative signals are not used in incentive contracts.

Specifically, we employed an agency model similar to the single-period multi-task model of Feltham and Xie (1994), which is further examined by Christensen, Sabac and Tian (2008). We choose a LEN (linear-exponential-normal) model to exploit its clarity in conveying economic intuitions. We show, in the appendix, that the main intuition and the first result hold in more general settings (e.g., optimal contracts with concave utility functions). The main distinguishing features of our model are (1) that the agent may exert personally costly performance-reporting effort to improve the accuracy of the measured performance, which is a noisy signal of the productive effort, and (2) that the two efforts compete for limited managerial attention such that the exertion of higher effort in one leads to a higher marginal cost of exerting effort in another. In other words, two tasks in our setting (labeled productive task and performance reporting task) are hinged together in two respects: They affect the same performance measure, with one affecting the mean and the other the precision, and the performance reporting effort (which reduces the variance) affects the agent's marginal cost of the productive effort. The family of performance signals in such a setting are most likely those generated by a sophisticated information system such as an accounting information system (for internal as well as external use) which requires active managerial attention in order to maintain its accuracy and precision.

When designing the optimal incentive contract in such an environment, the principal must consider subtle interactions induced by the two drivers. Any pay-for-performance scheme using the performance measure will induce the agent to exert performance reporting as well as productive effort, since a risk-averse agent would enjoy a reduced variance in his compensation. The principal also enjoys the reduced variance, as compensation costs (those due to the risk-premium) are lower. Thus, the induced response from the agent is desirable from the principal's perspective. However, such a response may also complicate the problem if the performance reporting effort has a "spillover" effect on the moral hazard problem involving the productive effort. This would take place if exerting performance reporting effort increases the marginal cost of the agent's productive effort, which indeed makes the moral hazard problem more severe. This is an undesirable aspect of the induced response from the agent. When facing such a problem, the principal must balance the benefits and costs from the desirable as well as the undesirable aspects of multi-tasking.

We use the model to explore several aspects of incentive provision practice. Generally, our analysis points to the subtleties in extending standard agency results to settings in which the agent has an influence over the precision of his own performance measures. First, we investigate the properties of incentive provision when there exists the "spillover" effect between the agent's two effort choices. We show that, in contrast to standard results, a positive relation between incentive strength and endogenous performance variance is possible. That is, high-powered incentives may be associated with high performance variances. The economic intuition is driven by subtle interactions between the two drivers. In some cases, the principal would like to redirect the agent's attention from the performance reporting task to the productive task by employing higher powered incentive. Coupled with a positive increase in variance caused by a lower performance reporting effort, the shift in managerial attention leads to a positive relation between incentive strength and endogenous performance variance. Even though we present this result in the LEN setting, we show, in Appendix II, that this result extends to setting with optimal contracts and concave utility functions, not necessarily in the exponential functional form. This finding offers a multi-task-based rationale to explain the mixed empirical results on the relation between variance of performance measures and equilibrium incentive strength.

Second, we slightly enrich the setting by adding a second performance signal whose precision is not affected by the agent. Within the LEN framework, we derive conditions under which it is efficient for the principal to discard the signal with an endogenous variance, even if the signal is informative. This is because when the second signal is sufficiently informative, placing any incentive weight on the first signal may be too costly due to the "spillover" effect of increasing the agent's marginal cost of productive effort. As a result, a contractible, informative signal is left unused due to the induced drain of the agent's limited attention. In another case, when the manager has private information about the precision of the performance measurement system, we find that sometimes the principal, somewhat surprisingly, discards the signal that cannot be manipulated by the manager but keeps the signal with an endogenous variance. This is because in doing so, the principal lowers the cost of inducing a truthful self-report of the manager's private information. Our finding here offers a noval explanation why informative signals are left unused, complementing other competing reasons (such as incompele contracts and subjective performance measures).

Previous agency studies of multi-tasking, such as Holmstrom and Milgrom (1991), Feltham and Xie (1994), Zhang (2003) and Christensen, Sabac, and Tian (2008), usually focus on the productive efforts and assume exogenous variance (and covariance) of performance measures. Holmstrom and Milgrom (1991) examine a multi-task setting in which the agent allocates his effort to more than one productive activities. They show that the incentive in one activity should decrease with the difficulty of measuring performance in other activities, so that the agent will not be induced to input his effort only in the activity that is easy to measure. In Holmstrom and Milgrom's model, the principal seeks a more balanced allocation of the agent's effort among productive activities, while in our study the principal restrains the agent's effort from performance reporting activity to avoid a high marginal cost of productive effort. Feltham and Xie (1994) examine a similar model to Holmstrom and Milgrom's setting, but focus more on the congruence of a performance measure with the principal's interest. They show that any informative additional signal can reduce risk and non-congruity (see extentions by Christensen, Sabac and Tian 2008), while in our model it may be efficient to exclude an informative signal from the contract so that the marginal cost of productive effort is reduced, or the cost to motivate truth-telling is lowered. Zhang (2003) studies the multiple tasks that are complements; in our paper the two tasks have a substitutional relationship in the sense that a high variance-reduction effort leads to a higher marginal cost of productive effort. We extend the multi-task literature by enriching the tasks to include those focusing on increasing the precision of performance measures.

Standard moral hazard models usually predict a negative association between risk and incentives. However, empirical studies show the existence of a positive association between risk and incentives. Recently there have been several theoretical studies that explore this positive association. Using a discrete model which shares the same basic property as the standard model, Hemmer (2006) demonstrates that changes in incentives that affect the optimal effort level also affect the variance of the outcome distribution, thus resulting in a negative or positive relation between risk and incentives. In our model, the positive relation is the result of limited managerial attention and the manager's ability to reduce the variance of his performance measure. Our paper is similar to a study by Dutta (2008) in the sense that both examine the endogeneity of variances in a LEN model. Dutta (2008) introduces an additional information risk from the uncertainty about the manager's expertise, while our paper focuses on the endogenous variance affected by the agent's effort. Also related is a recent paper by Liang, Rajan, and Ray (2008) where variance of performance measurement is endogenous, not because of a performance reporting task but because of the endogeneity of the worker team size. Hughes (1982), Danielsson, Jorgensen and Vries (2002), and Bertomeu (2008) also consider the agent's ability to change the risk profile of the firm output (thus the agent's performance measure). In all these papers, limited managerial attention is not a key research issue.

Value of additional signals has also been a foucs of agency work since its early years. Holmstrom (1979) pioneered this inquiry and established the early standard result called the *Informativeness Critieron*. In accounting, this work is followed by Antle and Demski (1988), Demski (1994), Feltham and Xie (1994), Arya, Glover, and Radhakrishnan (2005), and Christensen, Sabac, and Tian (2008), among others. The focus has been on the conditional nature of the informativeness critieron, or on the different informational roles in valuation versus control settings, or on signal aggregation over time. We extend this literature by bringing into focus the role of limited managerial attention on the value of additional information.

Limited managerial attention has been examined in the prior literature, but from different perspectives. Geanakoplos and Milgrom (1991) focus on how to allocate different tasks among managers with different ability and the optimal organization structure of a firm, while our paper looks at how a manager allocates his effort between different tasks. Darrough and Melumad (1995) examine a setting in which a principal motivates a manager with unknown ability to allocate his effort between his own division and other division, and illustrate that sometimes it is optimal to motivate the manager to concentrate on his own division. Unlike the setting in our paper in which the cost of efforts spillover into each other, in their study the manager's effort is costless. Peng and Roell (2008) study earnings manipulation within a setting where productive and manipulation tasks compete for the limited managerial attention.[3]

Finally, we utilize the so-called LEN framework to analyze and present our model and results. But we verified that the main intuition and the first result do carry over to settings with optimal contract and general concave utility functions. In this light, our work complements recent work by Christensen, Sabac, and Tian (2008), which also extend results they discovered in LEN settings to optimal contracting settings.

The rest of the paper is organized as follows: Section 2 lays out the basic model and analyzes the key economic tension caused by the introduction of the two drivers. Section 3 analyzes the relation between incentive strength and endogenous performance variance and shows forces that cause a positive relation.

[3]Other studies include an experimental study in which Bruggen and Moers (2007) examine a setting in which the agent makes an effort-level choice and effort-allocation choice. The agent's effort is allocated between two tasks, A and B. However, only Task A has an observable and verifiable performance measure and thus the agent has incentive to input effort on Task A only. Introducing social incentives congruent with the principal's interest helps mitigate the distortion in the agent's effort allocation, but may lead to lower total effort. Their paper, though also look at effort allocation problem, has a different focus. In their setting the agent chooses the *total* effort level, and the proportion of total effort allocated to Task B. Therefore the focus is the trade-off between higher total effort and more congruent effort allocation. In our paper, however, the agent makes two effort level decisions on two tasks, and the focus is the interaction between these two efforts.

Sections 4 and 5 analyze two variations of the basic model in which an additional signal is introduced, and illustrates two reasons that an informative signal should be ignored. Section 6 concludes the paper.

2 Basic model

We consider a single-period two-task LEN agency setting. We show, in Appendix II, that the main result holds in optimal contract settings. Here we exploit the clarity in LEN to focus on the economic intuition (see Christensen, Sabac, and Tian 2008 for a similar approach). Consider a risk-neutral principal contracting with a risk-averse agent. The agent provides two-dimensional effort $\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$, where $e_i \in \mathbb{R}^+$, at a personal cost $C(e_1, e_2)$. The output, denoted x, is an increasing function of e_1, which we call the agent's productive effort. We assume a constant return to scale $x'(e_1) = q > 0$.[4] Also assume that the output x is realized too late for contracting, but there is a contractible signal y which is a noisy signal of the agent's productive effort e_1:

$$y = e_1 + \varepsilon_y,$$

where ε_y is a zero-mean normally distributed random variable with variance $V(e_2, \sigma^2)$. That is,

$$\varepsilon_y \sim N\left(0, V(e_2, \sigma^2)\right).$$

We regard e_2 as the agent's performance reporting effort to reduce the error in his performance measures.[5] We assume function $V(.)$ is such that a higher e_2 leads to a more accurate performance measure (i.e., $V_1 \equiv \frac{\partial}{\partial e_2} V(e_2, \sigma^2) < 0$), but there is a decreasing return on performance reporting effort (i.e., $V_{11} \equiv \frac{\partial^2}{\partial (e_2)^2} V(e_2, \sigma^2) > 0$). In addition, it also satisfies typical regularity conditions: $V_1|_{e_2=0} = -\infty$ and $V_1|_{e_2=+\infty} = 0$. Except in Section 5, parameter σ^2 is a known constant and can be regarded as the exogenous component of the variance of the performance measure. Let $V_2 \equiv \frac{\partial}{\partial \sigma^2} V(e_2, \sigma^2) > 0$. We further assume $\frac{\partial}{\partial e_2 \partial \sigma^2} V(e_2, \sigma^2) < 0$; that is, that the marginal effect of performance reporting effort on V is stronger when σ^2 is higher.

As usual, we assume $C(e_1, e_2)$ is increasing and convex in both e_1 and e_2. Further, we assume $C_{12}(e_1, e_2) \equiv \frac{\partial}{\partial e_1 \partial e_2} C(e_1, e_2) \geq 0$ to highlight the interaction between the two actions.[6] In particular, a positive cross-

[4] All results remain if x is modeled as the expected output instead.

[5] Generally these activities may include any choices or decisions of managers to make their performance measure more accurate on their managerial abilities/efforts. In practice, these may include "real" operating decisions such as hedging. The financial derivatives market has been developing rapidly since the 1990s and offers managers greater availability and accessibility of hedging instruments. Managers are now able to modify the variances of the reported outcomes.

Dye and Sridhar (2007) and Stocken and Verrecchia (2004) also look at the case in which the precision of a disclosed estimate or that of a firm's accounting reporting system is a choice variable. In Dye and Sridhar's study, a risk-averse initial owner discloses an estimate of future cash flow mean to risk-neutral investors. The study shows that whether the initial owner's precision choice is private or public and whether his disclosure is voluntary or mandatary lead to different equilibria of allocating risk between the owner and the investors. Their paper focuses on the allocational effects while our paper focuses on the interaction between the agent's productive effort and precision choice. Stocken and Verrecchia's study examines the interaction between the manager's choice of the precision of a firm's accounting reporting system and his disclosure management decision. It shows that the manager may not choose the most precise reporting system when he has the option to manipulate the financial report. Again, their study does not consider the precision choice's effect on productive effort.

[6] See Peng and Roell (2008) for a recent example of limited managerial attention in a simple agency model.

partial derivative implies a limited managerial attention where higher level of one effort increases the marginal cost of performing the other effort. When $C_{12}(e_1, e_2) > 0$, a "spillover" is present between the cost of two actions; and when $C_{12}(e_1, e_2) = 0$, we call this reference point the "separable cost" condition.[7]

The principal offers a linear contract on y, with a fixed wage α and a bonus rate β on the performance measure y.

$$w = \alpha + \beta y$$

The time line of the events is:

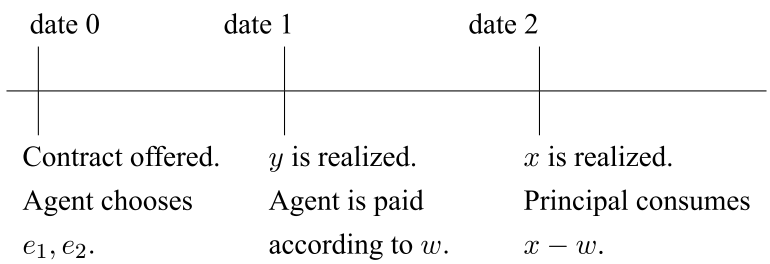

Figure 1: Time line

The agent's preference is represented by a negative exponential utility function with Arrow-Pratt measure r. This allows the standard transformation of the agent's problem into

$$\max_{e_1,\, e_2} \alpha + \beta E[y] - \frac{r}{2}\beta^2 V(e_2, \sigma^2) - C(e_1, e_2),$$

which yields a standard incentive constraint on the equilibrium choice of e_1 (in equilibrium this constraint always binds):

$$C_1(e_1, e_2) = \beta. \tag{1}$$

In addition, it yields an additional incentive constraint on the equilibrium choice of e_2, which we define as a function F of agent's choices e_1, e_2, the principal's choice β, and model parameter σ^2.

$$F(e_1, e_2, \beta, \sigma^2) \equiv -\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - C_2(e_1, e_2) = 0 \tag{2}$$

Notice from (2), if $\beta > 0$, then $e_2^* > 0$. Intuitively, when performance measure y is used in contract, the agent always has an incentive to exert performance effort (e_2) to reduce the variance of that measure.[8] Conditions (1) and (2) implicitly define the agent's best response (e_1 and e_2) to a given choice β by the

[7] Formally, we assume $C(e_1, e_2)$ is continuous and differentiable over $\left(\mathbb{R}^+\right)^2$, where $C_1(), C_{11}, C_{111} > 0$ and $C_2(), C_{22}, C_{222} > 0$, and $C_{12}(), C_{112}, C_{221} \geq 0$. Further, $C_1|_{e_1=0,e_2=0} = 0 = C_2|_{e_1=0,e_2=0} = 0$ and $C_1|_{e_1=+\infty} = +\infty$ for all e_2 and $C_2|_{e_2=+\infty} = +\infty$ for all e_1. In some examples, we may consider specific cost function to illustrate economic intuition using closed-form solutions. In these example, we consider $C(e_1, e_2) = \frac{1}{2}a(e_2)e_1^2 + b(e_2)$, where $a(e_2) > 0$ and $b(e_2) \geq 0$. One can think of $b(e_2)$ as the direct cost of e_2 (so $b'(e_2) \geq 0$) and $a(e_2)$ reflects e_2's indirect effect. Also in this case, condition $a'(e_2)e_1 \geq 0$ reflects the limited managerial attention.

[8] Formally, for any given positive bonus weight β, a manager choosing $e_2 = 0$ is not optimal because at $e_2 = 0$, the marginal benefit is proportional to $-V_1(e_2, \sigma^2) = +\infty$ and the marginal cost is $C_2(e_1, e_2) < +\infty$. By continuity, the manager can always find an $e_2 > 0$ to equate the marginal benefit and the marginal costs.

principal. Without loss of generality, the reservation wage for the agent is set at zero. The principal will set the fixed wage α so that the agent's individual rationality constraint binds.[9] The principal's problem, labeled (PP), is

$$\max_{\beta} x\left(e_1\right) - \frac{r}{2}\beta^2 V(e_2, \sigma^2) - C(e_1, e_2) \tag{PP}$$

yielding a first-order condition for optimal choice of incentive β:

$$\left[q - C_1(e_1, e_2)\right]\frac{de_1}{d\beta} - r\beta V(e_2, \sigma^2) + \left[-\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - C_2(e_1, e_2)\right]\frac{de_2}{d\beta} = 0 \tag{3}$$

From (1), we deduce that $\frac{de_1}{d\beta} = \frac{1}{C_{11}(e_1, e_2)}$. If (2) is satisfied in equilibrium (thus eliminating the third term in equation 3) and substituting in $C_1(e_1, e_2)$ using equation (1), (3) can be simplified into

$$\beta = B\left(e_1, e_2, \sigma^2\right) \equiv \frac{q}{1 + rV(e_2, \sigma^2)C_{11}(e_1, e_2)}. \tag{4}$$

We assume an interior solution to (PP) exists and define $\langle \alpha^*, \beta^*, e_1^*, e_2^* \rangle$ to be the optimal solution. This model differs from traditional multi-task models in several ways. First, the performance reporting task e_2 is endogenous to the moral hazard problem of the productive task e_1. Notice it is easy to see that the first best action combination is $\langle e_1^{FB} > 0, e_2^{FB} = 0 \rangle$ while in the second-best, $\langle e_1^{FB} > e_1^{SB} > 0, e_2^{SB} > 0 \rangle$. In other words, without the moral hazard problem with respect to e_1 (e.g., if the principal could contract directly on e_1), the principal would not demand any agent's effort to reducing the error in his performance metric.

Second, managerial attention (e.g., $C_{12}(e_1, e_2)$) is a key factor in determining the optimal choice of performance reporting effort. If no such indirect effect is in place (i.e., $C_{12}(e_1, e_2) = 0$), it is easily verified that the optimal e_2 supplied by the agent at the solution to (PP) is identical to the solution of a slightly modified problem (PP') where e_2 is supplied by the principal (at the same cost). In other words, without "spillover" costs, there is no conflict of interest with respect to the provision of e_2.

However, if marginal cost of the productive effort, $C_1(e_1, e_2)$, is an increasing function of e_2, the issue becomes more complicated. In particular, inducing the agent to provide performance reporting effort leads to an interaction (or a "spillover") effect on the agent's choice of productive effort. From the agent's perspective, one obvious effect is that inducing a higher e_2-choice makes the agent lower his e_1-choice for a given bonus rate (such that equation 1 holds). From the principal's perspective, inducing a higher e_2-choice makes e_1 marginally more costly (i.e., a higher $C_1(e_1, e_2)$). Normally this amounts to a "more severe" moral hazard problem and would cause the principal to lower the bonus coefficient as a response. Indeed, from equation (4), we see that a higher e_2 leads to higher $C_{11}(e_1, e_2)$, which would press the principal to lower the optimal bonus rate β. However, there is a countervailing effect. Notice that from the same equation (4), a higher e_2 also leads to a more precise performance measure (i.e., a lower $V(e_2, \sigma^2)$), which amounts to a "less severe" moral hazard problem and would encourage the principal to increase the bonus rate β. This two-way interaction is a result due to the combination of (i) induced demand for the performance reporting task and (ii) limited managerial attention.

[9] This is because the principal can always adjust the fixed wage α, without affecting any incentive constraints, to make sure the agent takes the contract by setting $\alpha = -\beta E[y] + \frac{r}{2}\beta^2 V(e_2, \sigma^2) + C(e_1, e_2)$.

Now we use this two-task model to address two questions on management control, and we show that there are subtleties in extending standard results to settings in which the agent has influence over the variance of his own performance measures. In Section 3, we investigate how the "spillover" affects the characteristics of the optimal incentive provision. We show that unlike the setting in which performance variance is exogenous, it is possible the relation between incentive strength (β) and endogenous performance variance ($V(e_2, \sigma^2)$) is positive. In Sections 4 and 5, we examine two cases with an alternative performance signal whose variance is not affected by e_2. In the first case we derive conditions under which it is efficient for the principal to discard the signal with endogenous variance, even if the signal is informative. In the second case we introduce an uncertainty about the ex ante internal control information (i.e., σ^2 parameter is private information) and study the setting in which the manager may mis-report the exogenous variance. We find a surprising result that sometimes the principal keeps the signal with endogenous variance but discards the signal that cannot be influenced by the manager. This is because this way the principal has a lower cost to motivate truth-telling.

3 Incentive-Variance Relation

In standard LEN moral hazard models, the variance of the performance measures is typically unaffected by the agent's effort. In these settings, a typical prediction is that risk and incentive are negatively related. That is, the principal offers a lower bonus rate when the agent's performance is measured with high variance (risk). Our model nests such a special case and such a prediction. Consider the case where e_2 is a known constant denoted by E (and thus not a choice of the agent). Then, the principal's trade-off is captured by the following modification of equation (4):

$$\beta \ = B\left(e_1, E, \sigma^2\right) = \frac{q}{1 + rV(E, \sigma^2)C_{11}(e_1, E)}. \tag{5}$$

The negative relation between incentive and signal variance is intuitive: principal lowers incentive rates in response to a higher variance in the performance measure imposed on a risk-averse agent. Indeed, from equation (5), an increase in σ^2 leads to a decrease in β. The key is that such an increase in σ^2 does not generate a response in the agent's choice of e_2, which would have affected β indirectly.

Outside this special case, an increase in σ^2 would induce a response from the agent's performance reporting effort (e_2). This is because the σ^2-parameter affects the agent's trade-off in choosing e_2. (Notice that σ^2 changes the marginal benefit of e_2, which is represented mathematically by the first term in equation (2)). Anticipating this change in the agent's effort calculus, the principal would react by adjusting the incentive provision (i.e., bonus rate β). In the principal's calculus, an increase in σ^2 induces two effects on β, as shown in equation (4). First, similar to the special case above, it leads to an incentive to lower β as performance measure is more noisy. However, because of the effect of σ^2 on the agent's choice of e_2, the principal's choice of β is also influenced by how e_2 changes. As discussed earlier, the presence of e_2 has a subtle, two-way interaction effect on the incentive rate, and the nature of such an effect depends on whether there is a "spillover" effect of e_2 on the marginal cost of productive effort e_1. The overall impact of an exogenous change in σ^2 on incentive rate β is far more complicated than it is in the standard setting.

We examine precisely how the principal would react to an exogenous change in the variance parameter, taking into account the fact that the agent's performance reporting effort changes accordingly. Further, we wish to understand how the direction of the relation between incentive strength and performance variance is affected by the endogenous nature of the performance variance (due to the agent's performance reporting effort) and by the possible "spillover" effect (i.e., the agent's performance reporting effort may change the marginal cost of productive effort).

We propose two measures of the incentive-variance relation:

- Incentive-to-exogenous-variance relation measured by $\Gamma_{ex} \equiv \frac{d}{d\sigma^2}\beta^*\left(\sigma^2\right)$. The idea here is to compare the incentive rate change in response to an exogenous change in the noise of the performance measure. The noise of the performance measure does not take into account the change in manager's effort in reducing the noise.
- Incentive-to-endogenous-variance relation measured by $\Gamma_{en} \equiv \frac{\frac{d}{d\sigma^2}\beta^*\left(\sigma^2\right)}{\frac{d}{d\sigma^2}V(e_2^*,\sigma^2)}$. The idea here is to compare the change in the incentive rate as a response to an exogenous change in the noise of the performance measure to the change in the endogenous change in variance actually born by the manager. The noise of the performance measure does take into account the change in manager's effort in reducing the noise. This relation reflects the endogenous nature of performance variance and is also closer to empirical measures.

Proposition 1 characterizes the determinants of incentive-variance relation and its underlying economic trade-offs.

Proposition 1 *Suppose there is a solution $\langle\alpha^*, \beta^*, e_1^*, e_2^*\rangle$ to the principal's problem. Then, at the solution,*

$$\frac{d}{d\sigma^2}\beta^*\left(\sigma^2\right) = \frac{B_3 + \left[B_2 - B_1\frac{C_{12}}{C_{11}}\right]\frac{de_2}{d\sigma^2}}{1 - B_1/C_{11}} \tag{6}$$

$$\frac{d}{d\sigma^2}V(e_2^*,\sigma^2) = V_2 + V_1\frac{de_2}{d\sigma^2} \tag{7}$$

where $B_1 \equiv \frac{\partial}{\partial e_1}B\left(e_1, e_2, \sigma^2\right) \leq 0$, $B_2 \equiv \frac{\partial}{\partial e_2}B\left(e_1, e_2, \sigma^2\right)$, and $B_3 \equiv \frac{\partial}{\partial \sigma^2}B\left(e_1, e_2, \sigma^2\right) < 0$.

Proof. all proofs appear in the appendix. ■

Notice that by equation (4), $B_3 < 0$, that is, the direct effect of σ^2 on bonus rate is negative. By assumption $V_2 > 0$. Therefore,

$$\text{If } V(.) \text{ is exogenous } \left(\text{thus } \frac{de_2}{d\sigma^2} = 0\right), \Gamma_{en} < 0 \text{ and } \Gamma_{ex} < 0$$

or, the incentive and compensation variance is always negatively related, as measured by either Γ_{en} or Γ_{ex}, as predicted by standard agency models (recall that the term $1 - B_1/C_{11}$ is always positive). This confirms the economic intuition we have obtained by casually observing equation (5) above.

However, when the variance is endogenous and can be reduced by the agent's e_2-choice, there is an indirect effect caused by changes in the σ^2-environment, which works through the effect of σ^2 on manager's performance reporting effort (e_2). For measure Γ_{ex}, the extra-term $\left[B_2 - B_1 \frac{C_{12}}{C_{11}}\right] \frac{de_2}{d\sigma^2}$ captures the indirect effect of the environment (σ^2) on the principal's incentive choice β through the agent's induced choice on e_2. When the exogenous measurement noise increases, it may induce the agent to increase e_2 to reduce its negative effect on compensation risk (in this case, $\frac{de_2}{d\sigma^2} > 0$). This, in turn, may make the marginal cost of e_1 higher, which leads to lowering the principal's choice of incentive coefficient β. It may also lead to a higher β, since the variance is reduced by e_2 and the performance measure becomes a better signal of e_1. As a result, the incentive-to-exogenous-variance relation, as measured by Γ_{ex}, may remain negative or turn positive. Bringing in the endogenous variance into the measure, the extra-term in (7) captures the indirect effect of the environment (σ^2) on the endogenous compensation variance through the agent's induced choice on e_2. Notice here we assume V_1 to be unambiguously negative ($V_1 < 0$). With the indirect effects, the sign of Γ_{en} may remain negative or, interestingly, may turn positive. Therefore,

$$\text{If } V(.) \text{ is endogenous } \left(\text{thus } \frac{de_2}{d\sigma^2} \neq 0\right), \Gamma_{en} \lesseqqgtr 0 \text{ and } \Gamma_{ex} \lesseqqgtr 0$$

This conclusion is in contrast to the economic intuition in the standard agency model. In Appendix II, we show, in detail, that the ambiguity result extends to settings with optimal contracts and concave utility functions.

One such case highlights the importance of looking at the endogenous nature of incentive-to-risk relation. Using an example, we show that it is possible to have a negative relation using the incentive-to-exogenous-variance measure ($\Gamma_{ex} < 0$) and, at the same time, a positive incentive-to-endogenous-variance measure ($\Gamma_{en} > 0$). Suppose $\frac{de_2}{d\sigma^2} > 0$. Therefore, $\Gamma_{en} > 0$ if $V_2 + V_1 \frac{de_2}{d\sigma^2}$ turns negative while the Γ_{ex} remains negative. Intuitively, the indirect effect of e_2 on V (i.e., $V_1 \frac{de_2}{d\sigma^2} < 0$) dominates the direct effect of σ^2 on V (i.e., V_2) and the indirect effect of σ^2 on β is dominated by the direct effect ($B_3 + \left[B_2 - B_1 \frac{C_{12}}{C_{11}}\right] \frac{de_2}{d\sigma^2} < 0$). Combined, they result in a higher powered incentive alongside a lower exogenous compensation variance but a higher endogenous compensation variance. In other words, the principal finds it optimal to respond to an increase in σ^2 by motivating a substitution between the two efforts in favor of e_2. As a result, the relation between the incentive and endogenous compensation variance is positive.[10]

Corollary 1 shows a specific example of this positive-relation case.

Corollary 1 *Suppose* $V = \frac{\sigma^2}{e_2}$, *and* $C(e_1, e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$, *The optimal contract satisfies* $\beta^* = \frac{q}{1+\sigma\sqrt{kr}}$, *thus,* $\Gamma_{ex} < 0$. *In addition,* $\Gamma_{en} > 0$ *if* $1 - 2\sigma\sqrt{rk} < 0 < 1 - \sigma\sqrt{rk}$. *Further the agent's optimal efforts are* $e_1^* = \frac{q(1-\sigma\sqrt{rk})}{c_1(1+\sigma\sqrt{kr})}$ *and* $e_2^* = \frac{c_1\sigma}{1-\sigma\sqrt{rk}}\sqrt{\frac{r}{k}}$.

[10] In another case, suppose $\frac{de_2}{d\sigma^2} < 0$ instead. $\Gamma_{ex} < 0$ because the term $B_3 + \left[B_2 - B_1 \frac{C_{12}}{C_{11}}\right] \frac{de_2}{d\sigma^2}$ remains negative. Intuitively, the indirect effect of σ^2 on β reenforces the direct effect of σ^2 on β (i.e., B_3). This reenforcing indirect effect, coupled with a definite positive $V_2 + V_1 \frac{de_2}{d\sigma^2}$ (because $V_2 > 0$, and $V_1 < 0$) leads to $\Gamma_{en} < 0$ as well. That is, the principal always decreases incentives when the compensation variance increases even in the presence of the "spillover" effect. In this case, the principal finds it optimal to respond to an increase in σ^2 by motivating less e_2.

The preceding analysis has implications on empirical analysis in managerial accounting research. Cross-sectionally, empirical measures of compensation variance are subject to varying degrees of influence by managers. Our model indicates that the relation between the precision of the performance metrics and the strength of managerial incentives depends on the extend to which the managers are induced to improve the precision of the performance signals. When the signals are not subjected to such e_2-like efforts, the relation is likely to be negative. Otherwise, the relation can be positive or negative depending on the nature of the "spillover" effect identified by our model.

4 Additional Signal

This far, we have limited the way with which the principal can address the incentive problem caused by limited attention. That is, the only way to promote more attention to production is for the principal to offer a higher incentive β. In this section, we consider an alternative method of redirecting managerial attention, which is to employ an additional performance signal whose variance is unaffected by the agent's performance reporting effort. Compared with signals generated by a sophisticated accounting information system, the precisions of certain other signals (such as hours worked, output quantities, cash flows, or stock price) are affected by managers' performance reporting task to a lesser degree. Here we abstract away from the richness in the different sensitivities to managerial reporting efforts and explore the extreme case of signals with precision unaffected by the management. This exploration allows us to compare, qualitatively, the optimal use of two different signals with such a distinctive difference and offers new insights into the value of an additional signal, a vital theoretical interest in agency theory since Holmstrom (1979). In particular, it may be efficient to exclude a signal with endogenous variance from contracting in the presence of a signal with exogenous variance.

To begin, we modify the model to include an additional performance measure z. Both z and y are noisy measures of the agent's productive effort e_1.[11]

$$\begin{aligned} y &= e_1 + \varepsilon_y \\ z &= e_1 + \varepsilon_z \end{aligned}$$

However, unlike y, the additional signal z's variance σ_z^2 cannot be reduced through the agent's effort. That is,

$$\begin{bmatrix} \varepsilon_y \\ \varepsilon_z \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} V(e_2, \sigma^2) & 0 \\ 0 & \sigma_z^2 \end{bmatrix} \right).$$

The principal offers a linear contract on y and z. As in previous setting, α is a fixed wage and β is the bonus rate on y. In addition, the contract also assigns a bonus rate δ on z.

$$w = \alpha + \beta y + \delta z$$

[11] If the additional signal (z) is informative about e_2 (e.g., $z = e_2 + \varepsilon_z$), we show that z is not used in optimal contract if in equilibrium e_2 is interior.

We first examine the optimal use of the two signals in two benchmarks. In the first benchmark, both variances of performance measure y and z are exogenous, as in most standard agency models. In the second benchmark, the variance of y can be reduced by e_2, and the costs of e_1 and e_2 are separable. In these two benchmarks, we find that both measures are useful (that is, the principal is better off by including both measures into the contract) as long as their variances are non-degenerate. Then we consider a setting in which the variance of y is endogenous and e_2 has a "spillover" effect on the marginal cost of e_1. In this setting, we show it may be efficient to exclude measure y from the contract even if the variance of y is non-degenerate. The reason is, again, that using y would, via e_2, induce a higher marginal cost of productive effort e_1 and y's incentive benefit cannot offset this cost increase in the presence of another performance signal.

4.1 Benchmark settings

Consider the following two settings:

- In the first benchmark, we return to a simpler setting in which the agent's effort does not affect the variance of performance measures. This setting is consistent with standard agency studies such as Holmstrom (1979) and Feltham and Xie (1994). Without loss of generality, we parameterize this benchmark by setting $V(e_2, \sigma^2) = \sigma^2$. We label this setting *exogenous variance.*
- In the second benchmark, the agent is able to exert e_2 to reduce variance of the performance measure y. However, the personal cost of the agent's effort is separable in e_1 and e_2. Without loss of generality, we parameterize this benchmark by setting $C(e_1, e_2) = L(e_1) + K(e_2)$. We label this setting *separable costs.*

Lemma 1 summarizes the optimal use of the two performance measures in these two benchmark settings.

Lemma 1 *Under either exogenous variance setting $V(e_2, \sigma^2) = \sigma^2$ or the separable cost setting $C(e_1, e_2) = L(e_1) + K(e_2)$,*

$$\beta^*,\ \delta^* > 0 \Longleftrightarrow V(e_2, \sigma^2),\ \sigma_z^2 < +\infty \textit{ for all } e_2 \tag{8}$$

In Lemma 1, the result of the first benchmark is a reproduction of the standard agency conclusion from Holmstrom (1979), Banker and Datar (1989), and Feltham and Xie (1994). The standard agency models with exogenous variances show that any informative signal about the agent's productive effort, no matter how imperfect, can be used in contracting to improve the principal's welfare. The key argument is that the principal will always use a signal as long as its variance is finite, because the principal can always place a sufficiently small weight on the signal to balance the marginal cost from a higher risk premium against the marginal benefit from a higher productive effort.

The result of the second benchmark shows that the standard agency conclusion still holds with an endogenous variance, as long as the cost of performance reporting effort is separable from the cost of productive effort (no "spillover"). Again, the principal can always choose a proper weight on the signal to balance the marginal cost and benefit. However, there are two marginal costs in the second benchmark: one from the increased risk premium, and the other from the cost of performance reporting effort. When the bonus weight is close to zero, the marginal benefit is positive and the two marginal costs, due to their quadratic nature, are both zero; thus the principal can always benefit from slightly increasing the bonus weight from zero.

4.2 Endogenous Variance Setting with Additional Signal

Now we return to a setting with the "spillover" effect. That is, where exerting performance reporting effort e_2 may affect the marginal cost of productive effort e_1.

The agent's problem with the additional signal is

$$\max_{e_1,\, e_2} \alpha + \beta E[y] + \delta E[z] - \frac{r}{2}\beta^2 V(e_2, \sigma^2) - \frac{r}{2}\delta^2\sigma_z^2 - C(e_1, e_2)$$

The first order condition with respect to $e_1$shows that the optimal e_1 satisfies

$$\beta + \delta = C_1(e_1, e_2). \tag{9}$$

In addition, it yields an additional incentive constraint on the equilibrium choice of e_2, which we define as a function G of agent's choices e_1, e_2, the principal's choice β, and model parameter σ^2.

$$G(e_1, e_2, \beta, \sigma^2) \equiv -\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - C_2(e_1, e_2) = 0 \tag{10}$$

The principal's problem, labeled (PP2), is

$$\max_{\beta,\delta} E[x\,(e_1)] - \frac{r}{2}\beta^2 V(e_2, \sigma^2) - \frac{r}{2}\delta^2\sigma_z^2 - C(e_1, e_2) \tag{PP2}$$

yielding a first-order condition for optimal choice of incentive δ, which after substituting (9), the incentive constraint for e_1, can be written as

$$q\frac{de_1}{d\delta} - r\delta\sigma_z^2 - (\beta + \delta)\,\frac{de_1}{d\delta} = 0, \tag{11}$$

Substituting $\frac{de_1}{d\delta} = \frac{1}{C_{11}(e_1,e_2)}$(derived from equation 9), leading to

$$\delta = \Delta(e_2, \beta, \sigma_z^2) \equiv \frac{q - \beta}{r\sigma_z^2 C_{11}(e_1, e_2) + 1}.$$

Additionally, the first order condition with respect to β, which after substituting (9) and $\frac{de_1}{d\delta}$, can be written as

$$\frac{q}{C_{11}(e_1, e_2)} - \frac{\beta + \delta}{C_{11}(e_1, e_2)} - r\beta V(e_2, \sigma^2) + \left[-\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - C_2(e_1, e_2)\right]\frac{\partial e_2}{\partial \beta} = 0 \tag{12}$$

where the $\frac{q}{C_{11}(e_1,e_2)}$ term is the marginal benefit of increasing β. The rest of terms are the marginal costs.

If in equilibrium, (10) is satisfied (that is, if $e_2^* > 0$), (12) is reduced to

$$\beta = \frac{q - \delta}{rV(e_2, \sigma^2)C_{11}(e_1, e_2) + 1}. \tag{13}$$

In this case, it is easy to see both signals are used in contracts, similar to the benchmark cases. However, what if in equilibrium, (10) is not satisfied (that is, if $e_2^* = 0$, a corner solution)? Is it possible that $\beta = 0$ and $\delta > 0$ can be true in equilibrium? Unlike the benchmark cases, such a scenario cannot be ruled out in the endogenous variance setting because of the "spillover" effect. To see this, consider what would sustain such an equilibrium. Suppose the solution of $\beta = 0$ and $\delta > 0$ is contemplated. From (11), we learn $\delta = \Delta(e_2, \beta = 0, \sigma_z^2) \equiv \frac{q}{r\sigma_z^2 C_{11}(e_1,e_2)+1} < q$. From (12), we learn the marginal net benefit (i.e., marginal benefit minus all marginal costs) is

$$\frac{q}{C_{11}(e_1, e_2)} - \frac{\delta}{C_{11}(e_1, e_2)} - r\beta V(e_2, \sigma^2) - \left[\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) + C_2(e_1, e_2)\right] \frac{\partial e_2}{\partial \beta}$$

Let us consider each of the three marginal costs evaluated at the contemplated solution $\beta = 0$ and $\delta > 0$,

- the $\frac{\delta}{C_{11}(e_1,e_2)}$ term: this first marginal cost is always less than the marginal benefit (the $\frac{q}{C_{11}(e_1,e_2)}$ term);
- (the $r\beta V(e_2, \sigma^2)$ term: this second marginal cost is zero at the contemplated $\beta = 0$.
- The third marginal cost is the key factor.
  - If e_2 is interior, then by (10), this third marginal cost term is zero.
  - At the contemplated $\beta = 0$, e_2 is a corner solution so the marginal cost term is not zero. In particular, notice $\frac{\partial e_2}{\partial \beta} > 0$ and $\left[\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) + C_2(e_1, e_2)\right]|_{\beta=0,\delta>0} = C_2(e_1, e_2)|_{\beta=0,\delta>0}$ > 0 if "spillover" or limited attention is present.

If this third marginal cost is high enough, at $\beta = 0$ is indeed optimal because the total marginal cost (first and third) is greater than the marginal benefit. As a result, $\beta = 0$ and $\delta > 0$ can be sustained as an equilibrium. The key, again, is the "spillover" effect between the two efforts. This result is summarized by Proposition (2).

Proposition 2 *In the case of endogenous variance with additional signal, it is efficient to ignore signal y when* $\frac{qr\sigma_z^2}{r\sigma_z^2 C_{11}(e_1,e_2)+1} \leq [C_2(e_1, e_2)] \frac{\partial e_2}{\partial \beta} \;|_{\beta=0,\delta=\frac{q}{r\sigma_z^2 C_{11}(e_1,e_2)+1}}$.

Intuitively, the principal would always use z. No matter whether signal y is used or not ($\beta > 0$ or $\beta = 0$), the marginal benefit of increasing δ is always greater than the marginal cost. However, at $\beta = 0$ the marginal benefit of increasing β may be less than the marginal cost because of the "spillover" effect. That is, if the principal uses y in the contract ever so slightly, the marginal cost from risk-sharing is zero but the marginal cost from limited attention is positive, which may overweigh the positive marginal benefit. Therefore, sometimes it is efficient for the principal to ignore signal y ($\beta^* = 0$), though signal y is informative.

4.3 An example where y is useless

Now we show a specific example in which the signal y can be useless. We follow the previous example of Corollary 1, except that now there is an additional signal z. For this modified specific example, we define $\widehat{\beta}$, $\widehat{\delta}$ as the optimal incentives on y and z, and $\widehat{e_1}$, $\widehat{e_2}$ as the optimal effort by the agent. Corollary 2 also compares the optimal solutions in the modified example with those of the previous one without the additional signal (i.e., the optimal solutions shown in Corollary 1).

Corollary 2 *Suppose* $V(e_2, \sigma^2) = \frac{\sigma^2}{e_2}$, *and* $C(e_1, e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$, *then the optimal solution has*

$$\widehat{\delta} = \frac{q(1 - \sigma\sqrt{rk})\sigma\sqrt{rk}}{(1 - \sigma^2 rk)(1 + c_1 r\sigma_z^2) - 1} > 0$$

$$\widehat{e_1} = \frac{q(1-\sigma\sqrt{rk}) + \sigma\sqrt{rk}\widehat{\delta}}{c_1\left(1+\sigma\sqrt{kr}\right)} > e_1^*, \qquad \widehat{e_2} = \frac{c_1\sigma\beta^*}{\beta'^*\left(1-\sigma\sqrt{rk}\right)+\widehat{\delta}}\sqrt{\frac{r}{k}} < e_2^*.$$

In addition,

$$\begin{aligned} \widehat{\beta} &= \frac{q[(1 - \sigma\sqrt{rk})(1 + c_1 r\sigma_z^2) - 1]}{(1 - \sigma^2 rk)(1 + c_1 r\sigma_z^2) - 1} \quad if \quad c_1 r\sigma_z^2 > \frac{\sigma\sqrt{rk}}{1 - \sigma\sqrt{rk}} \\ \widehat{\beta} &= 0 \quad if \;\; c_1 r\sigma_z^2 < \frac{\sigma\sqrt{rk}}{1 - \sigma\sqrt{rk}} \end{aligned}$$

Corollary 2 indicates that by introducing an additional performance measure z, the principal is able to redirect the agent's attention from performance reporting to production ($\widehat{e_1} > e_1^*$ and $\widehat{e_2} < e_2^*$). An additional performance measure that cannot be modified by the agent may help the principal alleviate the tension in managerial attention. Further, we see sometimes it is efficient for the principal to exclude the performance measure y from contracting ($\widehat{\beta} = 0$). When the performance reporting effort has the "spillover" effect on the cost of productive effort, including y in contracting draws the agent's attention to performance reporting, thus making the productive tasks more costly. Therefore, when performance measure z is sufficiently precise (σ_z^2 is sufficiently small), the principal would opt to use z exclusively so that the agent concentrates his attention on production. [12]

5 Additional Signal with a Privately Informed Manager

In previous sections the exogenous component of the performance measurement variance, σ^2, is common knowledge. However, in practice, σ^2 may be observed only by the manager. For example, the manager may have better information on the internal control and reporting information system than the principal does. We now consider a setting in which the manager privately observes the exogenous component and has an option to mis-report σ^2. For tractability we assume $C(e_1, e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$ for the rest of the paper.

[12] This result is in contrast to Lemma 1 where any signal with a bounded variance will be used in contracting. With the "spillover" effect, the standard agency conclusion applies to the bonus rate on z (i.e., the optimal δ is always positive) but may not necessarily hold for the bonus rate on y (i.e., β may become zero).

5.1 No Additional Signal Benchmark

We first look at a setting in which there is no additional signal. Same as in the basic model, we assume the output x is an increasing function of e_1, with a constant return to scale $x'(e_1) = q > 0$; performance measurement y is a noisy signal of the manager's productive effort $e_1 : y = e_1 + \varepsilon_y$, where ε_y is a zero-mean normally distributed random variable with variance $V(e_2, \varphi) = \frac{\varphi}{e_2}$, where φ can be either $(1+\tau)\sigma^2$ or $(1-\tau)\sigma^2$. That is,

$$\varepsilon_y \sim N\left(0, V(e_2, \varphi)\right), \varphi \in \{(1+\tau)\sigma^2, (1-\tau)\sigma^2\}.$$

φ is unknown to the principal and is observed privately by the manager. The principal only knows that the probability of a high variance $(1+\tau)\sigma^2$ is ρ. The principal offers contracts to the manager depending on the manager's report of the variance status $\widehat{\varphi}$. The contract for reported high variance is $w_h = \alpha_h + \beta_h y$, and that for reported low variance is $w_l = \alpha_l + \beta_l y$. The subscription h refers to the status that $\varphi = (1+\tau)\sigma^2$, and l refers to the status that $\varphi = (1-\tau)\sigma^2$. For our convenience, we use $\varphi = h$ to represent $\varphi = (1+\tau)\sigma^2$, and $\varphi = l$ to represent $\varphi = (1-\tau)\sigma^2$. We further use $\widehat{\varphi}$ to denote the reported φ.

Figure 2 shows the time line.

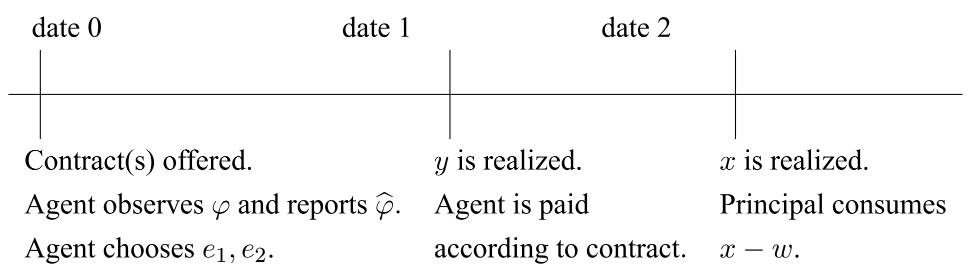

Figure 2

The manager's optimal effort levels when $\varphi = h$ are

$$\begin{aligned} e_{1h}^* &= \frac{\beta_h(1-\sqrt{kr(1+\tau)\sigma^2})}{c_1}, e_{2h}^* = \frac{c_1\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1+\tau)\sigma^2}}; \\ e_{1h}'^* &= \frac{\beta_l(1-\sqrt{kr(1+\tau)\sigma^2})}{c_1}, e_{2h}'^* = \frac{c_1\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1+\tau)\sigma^2}} = e_{2h}^*, \end{aligned}$$

where e_{1h}, e_{2h} are the manager's effort levels when reporting φ truthfully, and e_{1h}', e_{2h}' are those when mis-reporting.

The optimal effort levels in the case of $\varphi = l$ are

$$\begin{aligned} e_{1l}^* &= \frac{\beta_l(1-\sqrt{kr(1-\tau)\sigma^2})}{c_1}, e_{2l}^* = \frac{c_1\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1-\tau)\sigma^2}}; \\ e_{1l}'^* &= \frac{\beta_h(1-\sqrt{kr(1-\tau)\sigma^2})}{c_1}, e_{2l}'^* = \frac{c_1\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1-\tau)\sigma^2}} = e_{2l}^*, \end{aligned}$$

where e_{1l}, e_{2l} are the manager's effort levels when reporting φ truthfully, and e'_{1l}, e'_{2l} are those when mis-reporting. Notice that the optimal productive efforts depend on the reporting strategy, while the performance reporting efforts do not.[13]

To motivate truth-telling, the principal's design program becomes

$$\max_{\alpha_h,\beta_h,\alpha_l,\beta_l} q[e^*_{1h}\rho + e^*_{1l}(1-\rho)] - \rho[\alpha_h + \beta_h e^*_{1h}] - (1-\rho)[\alpha_l + \beta_l e^*_{1l}],$$

$s.t.$

$$\alpha_h + \beta_h e^*_{1h} - \frac{r}{2}\beta_h^2 \frac{(1+\tau)\sigma^2}{e^*_{2h}} - \frac{1}{2}(c_1 + ke^*_{2h})e^{*2}_{1h} \geq 0,$$

$$\alpha_l + \beta_l e^*_{1l} - \frac{r}{2}\beta_l^2 \frac{(1-\tau)\sigma^2}{e^*_{2l}} - \frac{1}{2}(c_1 + ke^*_{2l})e^{*2}_{1l} \geq 0,$$

$$\alpha_h + \beta_h e^*_{1h} - \frac{r}{2}\beta_h^2 \frac{(1+\tau)\sigma^2}{e^*_{2h}} - \frac{1}{2}(c_1 + ke^*_{2h})e^{*2}_{1h} \geq \alpha_l + \beta_l e'^*_{1h} - \frac{r}{2}\beta_l^2 \frac{(1-\tau)\sigma^2}{e^*_{2l}} - \frac{1}{2}(c_1 + ke^*_{2l})e'^{*2}_{1h},$$

$$\alpha_l + \beta_l e^*_{1l} - \frac{r}{2}\beta_l^2 \frac{(1-\tau)\sigma^2}{e^*_{2l}} - \frac{1}{2}(c_1 + ke^*_{2l})e^{*2}_{1l} \geq \alpha_h + \beta_h e'^*_{1l} - \frac{r}{2}\beta_h^2 \frac{(1-\tau)\sigma^2}{e^*_{2l}} - \frac{1}{2}(c_1 + ke^*_{2l})e'^{*2}_{1l}.$$

Proposition 3 *With an option to mis-report φ, the optimal contract is such*

$$\begin{aligned}
\alpha^*_h &= \frac{[(A_h+2)^2 - 7]}{2c_1}\beta^{*2}_h, \\
\beta^*_h &= \frac{q\rho(1-A_h)}{(A_h+2)^2 - 5\rho - 2\rho A_h - (1-\rho)(A_l+2)^2}, \\
\alpha^*_l &= \alpha^*_h + \frac{[(A_l+2)^2 - 7]}{2c_1}(\beta^{*2}_l - \beta^{*2}_h), \textit{ and} \\
\beta^*_l &= \frac{q(1-A_l)}{(1+A_l)^2 - 2}
\end{aligned}$$

where $A_h \equiv \sqrt{kr(1+\tau)\sigma^2}$ and $A_l \equiv \sqrt{kr(1-\tau)\sigma^2}$.

Corollary 3 *The optimal contract shows the following properties:*

(i). The contract offered to the manager who reports $\widehat{\varphi} = h$ provides reservation utility;

(ii). The contract offered to the manager who reports $\widehat{\varphi} = l$ provides non-zero information rent;

(iii). The manager who observes $\widehat{\varphi} = h$ strictly prefers to report high variance;

(iv). The manager who observes $\widehat{\varphi} = l$ is indifferent between reporting high variance or low variance.

The optimal contract in the case where there is a "spillover" effect between two types of effort still shows the standard property of an adverse selection problem. Notice a pooling reporting strategy is strictly dominated.

In a previous section we have shown that a positive incentive-variance relation emerges in a setting without mis-reporting option. We also examine the relation between incentive and variance in this setting

[13] This is because the bonus rate (β_l or β_h) appears in both the marginal cost and marginal benefit of the first-order-condition for e_2 and exactly offset.

with a mis-reporting option and again we find that a positive incentive-variance relation is possible. Detailed analysis is available in Appendix II at the end of this paper.

5.2 Additional Signal in the Setting with a Privately Informed Manager

In a previous section we have shown that the principal would drop an informative signal to discourage performance reporting effort when the "spillover" effect is strong. Now we return to this question in a setting with the mis-reporting option.

We include an additional performance measure z. Both z and y are noisy measures of the agent's productive effort e_1.

$$\begin{aligned} y &= e_1 + \varepsilon_y \\ z &= e_1 + \varepsilon_z \end{aligned}$$

As before, the additional signal z's variance σ_z^2 cannot be reduced through the agent's effort. That is,

$$\begin{bmatrix} \varepsilon_y \\ \varepsilon_z \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} V(e_2,\varphi) & 0 \\ 0 & \sigma_z^2 \end{bmatrix} \right),$$

where $\varphi \in \{(1+\tau)\sigma^2, (1-\tau)\sigma^2\}$.

The principal offers contracts to the manager depending on the manager's report of the variance status $\widehat{\varphi}$. The contract for reported high variance is $w_h = \alpha_h + \beta_h y + \delta_h z$, and that for reported low variance is $w_l = \alpha_l + \beta_l y + \delta_l z$. δ_h is the incentive coefficient assigned to z when the agent reports $\widehat{\varphi} = (1+\tau)\sigma^2$, and δ_l is the incentive coefficient to z when the agent reports $\widehat{\varphi} = (1-\tau)\sigma^2$. As in previous sections, we assume $V(e_2,\varphi) = \frac{\varphi}{e_2}$ and $C(e_1,e_2) = \frac{1}{2}(c_1 + ke_2)e_1^2$.

The optimal effort levels in the case of $\varphi = h$ are

$$\begin{aligned} e_{1h}^* &= \frac{\beta_h(1-\sqrt{kr(1+\tau)\sigma^2}) + \delta_h}{c_1}, e_{2h}^* = \frac{c_1\beta_h\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k}(\beta_h+\delta_h) - k\beta_h\sqrt{r(1+\tau)\sigma^2}}; \\ e_{1h}'^* &= \frac{\beta_l(1-\sqrt{kr(1+\tau)\sigma^2}) + \delta_l}{c_1}, e_{2h}'^* = \frac{c_1\beta_l\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k}(\beta_l+\delta_l) - k\beta_l\sqrt{r(1+\tau)\sigma^2}}. \end{aligned}$$

Following a similar analysis, we get the optimal effort levels when the manager observes $\varphi = l$,

$$\begin{aligned} e_{1l}^* &= \frac{\beta_l(1-\sqrt{kr(1-\tau)\sigma^2}) + \delta_l}{c_1}, e_{2l}^* = \frac{c_1\beta_l\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k}(\beta_l+\delta_l) - k\beta_l\sqrt{r(1-\tau)\sigma^2}}; \\ e_{1l}'^* &= \frac{\beta_h(1-\sqrt{kr(1-\tau)\sigma^2}) + \delta_h}{c_1}, e_{2l}'^* = \frac{c_1\beta_h\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k}(\beta_h+\delta_h) - k\beta_h\sqrt{r(1-\tau)\sigma^2}}. \end{aligned}$$

Unlike the benchmark case without additional signal in last subsection, now both the optimal productive efforts and the optimal performance reporting efforts depend on the reporting strategy. Again, for our convenience we denote $A_h \equiv \sqrt{kr(1+\tau)\sigma^2}$ and $A_l \equiv \sqrt{kr(1-\tau)\sigma^2}$. The principal's expected payoff, which is denoted by PP, becomes $q\frac{\rho[\beta_h(1-A_h)+\delta_h]+(1-\rho)[\beta_l(1-A_l)+\delta_l]}{c_1} - \rho[\alpha_h + \frac{(\beta_h+\delta_h)[\beta_h(1-A_h)+\delta_h]}{c_1}] - (1-\rho)[\alpha_l + \frac{(\beta_l+\delta_l)[\beta_l(1-A_l)+\delta_l]}{c_1}]$.

The principal's design program to motivate truth telling is

$$\max_{\alpha_h,\beta_h,\delta_h,\alpha_l,\beta_l,\delta_l} PP,$$

$s.t.$

$$\alpha_h + \frac{[\beta_h(1-A_h)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 \geq 0, \tag{14}$$

$$\alpha_l + \frac{[\beta_l(1-A_l)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 \geq 0, \tag{15}$$

$$\alpha_h + \frac{[\beta_h(1-A_h)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 \geq \alpha_l + \frac{[\beta_l(1-A_h)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2, \tag{16}$$

$$\alpha_l + \frac{[\beta_l(1-A_l)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 \geq \alpha_h + \frac{[\beta_h(1-A_l)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2. \tag{17}$$

The first two constraints are IR constraints to ensure that the manager gets at least his reservation in either $\varphi = h$ case or $\varphi = l$ case. The last two constraints are IC constraints to motivate truth telling.

Proposition 4 *In the setting with a mis-reporting option,*

(i) when the manager reports $\widehat{\varphi} = l$, it is efficient to ignore signal y when $(1-A_l)(1+r\sigma_z^2c_1) < 1$, but signal z is always useful;

(ii) when the manager reports $\widehat{\varphi} = h$, it is efficient to ignore signal y when $\rho(1-A_h)(1+r\sigma_z^2c_1) < (1-\rho)(A_h - A_l)$, and it is efficient to ignore signal z when $\rho(1+A_h) < (1-\rho)(A_h - A_l)$.

Corollary 4 *If $\rho(1+A_h) < (1-\rho)(A_h - A_l) < \rho(1-A_h)(1+r\sigma_z^2c_1)$, the principal uses both signals y and z when $\widehat{\varphi} = l$ but uses only signal y when $\widehat{\varphi} = h$.*

Surprisingly, now the principal sometimes drops signal z. Moreover, in the case shown in Corollary 4, the principal uses both signals when the manager reports low variance, but ignores z and uses only y when the manager reports high variance. This is different from both the classical result and the result we had in the previous setting with endogenous variance but no mis-reporting option, that the additional signal z is always useful and helps improve efficiency no matter how noisy it is.

If the manager reports high variance, the principal now may ignore signal z when the probability of low variance $(1-\rho)$ is high, and when $A_h - A_l$ is large. When the probability of low variance is high and/or when there is a large gap between high variance and low variance, the principal worries more about low-variance manager reporting high variance to get additional compensation for "higher risk." Dropping signal z when $\widehat{\varphi} = h$ makes it easier for the principal to prevent this mimicking strategy. If a low-variance-type manager reports high variance, his certainty equivalent is $\alpha_h + \frac{[\beta_h(1-A_l)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2$. Dropping z (in other words, $\delta_h = 0$) lowers the gain from mimicking, as long as $r\sigma_z^2$ is sufficiently small. With a lowered gain from mimicking, the mimicking strategy is less attractive to the manager and the principal lowers her compensation cost by dropping z.

Notice that the principal discarding the signal which cannot be manipulated by the manager only occurs with the "spillover" effect between the two types of efforts. If there is no "spillover" effect (that is , if $k = 0$), according to Proposition 4 the principal will always use both signals no matter whether the manager reports $\widehat{\varphi} = h$ or $\widehat{\varphi} = l$, which comes back to the classical result that all informative signals are useful.

A couple of other studies also illustrate cases in which it may be efficient to ignore an informative signal, but the reasons that the informative signal is ignored are different from that in our paper. Kanodia, Singh, and Spero (2005) show a case that an imprecise but informative signal of a firm's investment is ignored in equilibrium by the market when pricing the firm. They study a setting in which the investment has both short-term return and long-term return which is reflected as the firm's price in the capital market, and the market has perfect information about the profitability of the investment. The market thus prices the firm based on its anticipated investment rather than the firm's actual investment, and attributes the difference between the investment signal and the anticipated level to the imprecision of the signal. Under this situation, the only sustainable equilibrium is that the market anticipates the manager to invest to maximize only the short-term return and the manager has to invest myopically. There is no agency conflict in their study either. In our paper, however, the ignorance of an informative signal is driven by directing the managerial attention or reducing the agency cost from mis-reporting. Similar to our result in Corollary 4, Demski (1997) shows a case that bad measures might drive out good measures of a manager's input. In that setting, there is a perfect monitor of the manager's effort level in one task, but the manager's effort in the second task can only be reflected by the total output from both tasks. Demski illustrates that the information for the manager's second task effort is so bad that it is better not to have the monitor of the first task effort, while in our paper the signal that cannot be manipulated might be dropped so that the principal prevents the manager from mimicking.

6 Conclusion

The paper focuses on the trade-off between two competing demands on managerial attention. One is the productive effort which increases the expected output of the firm and the other is the performance reporting effort which increases the quality of the manager's own performance measure. This research identifies a complication in the manager's effort allocation. The analysis shows that the characteristics of the incentive contract show a mixed risk-incentive relation. Further, we show that with the "spillover" effect sometimes an informative signal is discarded to avoid high marginal cost of productive effort, or to prevent a mimicking strategy.

The main analysis is carried out in a tractable LEN framework, and we verified that the incentive-variance result also carries over to settings with optimal contract and general concave utility functions. It would be interesting to see if the result regarding additional signals holds in generalized non-linear contracts. In addition, a multiple-period version of this model that allows for an intertemporal performance reporting effort may elicit additional features. These are potential extensions of the current setting.

References

[1] Antle, R., and J. Demski. 1988. "The controllability principle in responsibility accounting." *The Accounting Review* 63: 700-718.

[2] Arya, Anil, Jonathan Glover, Suresh Radhakrishnan. "The Controllability Principle in Responsibility Accounting: Another Look." In *Essays on Accounting Theory in Honour of Joel S. Demski*, Rick Antle, Froystein Gjesdal, and Pierre Jinghong Liang, editors, (New York, NY: Springer, 2005).

[3] Banker, R. D., and S. M. Datar. 1989. "Sensitivity, precision, and linear aggregation of signals for performance evaluation." *Journal of Accounting Research* 27, No. 1, Spring, 21-39.

[4] Bertomeu, Jeremy, 2008. "Risk Management, Executive Compensation and the Cross-Section of Corporate Earnings." Northwestern University working paper. Available at SSRN: http://ssrn.com/abstract=1276412.

[5] Bruggen, A., and F. Moers. 2007. "The role of financial incentives and social incentives in multi-task settings." *Journal of Management Accounting Research*, Vol. 19, 25-50.

[6] Christensen, P., Florin Sabac, and Jie Tian. 2008 "Licence to LEN." University of Alberta Working paper.

[7] Danielssen, J., B. Jorgensen, and C. de Vries. 2002. "Incentives for Effective Risk Management." Journal of Banking and Finance, Vol. 26, page 1407-1425.

[8] Darrough, M. N., and N. D. Melumad. 1995. "Divisional versus company-wide focus: The trade-off between allocation of managerial attention and screening of talent." *Journal of Accounting Research*, Vol. 33 Supplement, 65-94.

[9] Demski, J. S. 1997. *Managerial Uses of Accounting Information*, Chapter 22, Kluwer Academic Publishers, USA. ISBN: 0792398475

[10] Dutta, S. 2008. "Managerial expertise, private information, and pay-performance sensitivity." *Management Science* 54, No. 3, March, 429-442.

[11] Dye, R., and S. Sridhar. 2007. "The allocational effects of the precision of accounting estimates." *Journal of Accounting Research*, Vol. 45, No. 4, September, 731-769

[12] Geanakoplos, J., and P. Milgrom. 1991. "A theory of hierarchies based on limited managerial attention." *Journal of the Japanese and International Economics*, 5, 205-225.

[13] Hughes, J. 1982. "Agency Theory and Stochastic Dominance." *Journal of Financial and Quantitative Analysis*, Vol. 17, No. 3, page 341-361.

[14] Kanodia, C., R. Singh, and A. Spero. 2005. "Imprecision in accounting measurement: Can it be value enhancing?" *Journal of Accounting Research*, Vol. 43, Number 3, June, 487-519.

[15] KPMG LLP. 2007. Defining the Issues. No. 07-20 (June 2007). Available at http:\\www.us.kpmg.com/definingissues.

[16] Liang, Pierre Jinghong, Madhav Rajan, and Korok Ray. 2008. "Optimal Team Size and Monitoring in Organizations." *The Accounting Review*, vol. 83, No. 3 (May), page 789-822.

[17] London Stock Exchange. 2002. *A Practical Guide to Listing*. Downloadable from http://www.londonstockexchange.com/en-gb/products/companyservices/.

[18] Feltham, G. A., and J. Xie. 1994. "Performance measure congruity and diversity in multi-task principal/agent relations." *The Accounting Review*, Vol. 69, No. 3, July, 429-453.

[19] Hemmer, T. 2006. "On the subtleties of the principal-agent model." in Antle, Rick, Froystein Gjesdal, and Pierre Jinghong Liang (eds.) *Essays in Accounting Theory in honour of Joel S. Demski*, Nowell, MA: Springer.

[20] Holmstrom, B. 1979. "Moral hazard and observability." *The Bell Journal of Economics*, Vol. 10, No. 1, 74-91.

[21] Holmstrom, B., and P. Milgrom. 1991. "Multitask principal-agent analysis: Incentive contracts, assets ownership, and job design." *Journal of Law, Economics, & Organization,* Vol. 7, September, 24-52.

[22] Lafontaine F., and S. Bhattacharyya. 1995. "The role of risk in franchising." *Journal of Corporate Finance*, Vol. 2, Issues 1-2 , October, 39-74.

[23] Peng, Lin, and Ailsa Roell. 2008. "Manipulation and Equity-Based Compensation." *American Economic Review: Papers and Proceedings*, Vol 98:2, p. 285-290.

[24] Prendergast, C. 2002. "The tenuous trade-off between risk and incentives." *Journal of Political Economy*, Vol. 110, No. 5, 1071-1102.

[25] Roberts, John. 2004. *The Modern Firm: Organizational Design for Performance and Growth,* Oxford University Press, USA. ISBN: 019829376.

[26] Simon, Herbert A. 1973. "Applying Information Technology to Organizational Design." *Public Administration Review*, Vol. 33, No. 3, May/June 1973, pp. 268-278.

[27] Stocken, P. C. and R. E. Verrecchia. 2004. "Financial reporting system choice and disclosure management." *The Accounting Review*, Vol. 79, No. 4, 1181-1203.

[28] SEC. 2007. Commission Guidance Regarding Management's Report on Internal Control Over Financial Reporting Under Section 13(a) or 15(d) of the Securities Exchange Act of 1934. SEC Releases No. 33-8810, June 20. Available at www.sec.gov.

[29] SEC. 2007. Amendments to Rules Regarding Management's Report on Internal Control Over Financial Reporting, SEC Release No. 33-8809, June 20. Available at www.sec.gov.

[30] Zhang, L. 2003. "Complementarity, task assignment, and incentives." *Journal of Management Accounting Research*, Vol. 15, 225-246.

Appendix I

Proof of Proposition 1

Totally differentiate (1) and (4) with respect to β, e_1, e_2 and σ^2, we have:

$$\begin{aligned} 0 &= C_{11}(e_1, e_2)de_1 + C_{12}(e_1, e_2)de_2 - d\beta \\ d\beta &= B_1 de_1 + B_2 de_2 + B_3 d\sigma^2 \end{aligned}$$

substituting out de_1, we have

$$d\beta = (d\beta - C_{12}de_2)B_1/C_{11} + B_2 de_2 + B_3 d\sigma^2$$

solving for $d\beta/d\sigma^2$, we have

$$\frac{d\beta}{d\sigma^2} = \frac{B_3 + [B_2 - C_{12}B_1/C_{11}]\frac{de_2}{d\sigma^2}}{1 - B_1/C_{11}}$$

combined with $\frac{d}{d\sigma^2}V(e_2, \sigma^2) = V_2 + V_1\frac{de_2}{d\sigma^2}$, and collecting terms, we have

$$\Gamma = \frac{B_3 + [B_2 - C_{12}B_1/C_{11}]\frac{de_2}{d\sigma^2}}{V_2 + V_1\frac{de_2}{d\sigma^2}} \frac{1}{1 - B_1/C_{11}}.$$

Here we provide the derivation of $\frac{de_2}{d\sigma^2}$. The rest of the conclusion is evident from the text.

Totally differentiate (2), we have:

$$0 = F_1 de_1 + F_2 de_2 + F_3 d\beta + F_4 d\sigma^2$$

substituting out de_1, we have

$$\begin{aligned} 0 &= F_1\left(d\beta/C_{11} - de_2 C_{12}/C_{11}\right) + F_2 de_2 + F_3 d\beta + F_4 d\sigma^2 \\ &= \left(F_1/C_{11} + F_3\right)d\beta + \left(F_2 - F_1 C_{12}/C_{11}\right)de_2 + F_4 d\sigma^2 \end{aligned}$$

substituting out $d\beta$, we have

$$0 = \left(F_1/C_{11} + F_3\right)\frac{B_3 d\sigma^2 + [B_2 - C_{12}B_1/C_{11}]\,de_2}{1 - B_1/C_{11}} + \left(F_2 - F_1 C_{12}/C_{11}\right)de_2 + F_4 d\sigma^2$$

solving for $de_2/d\sigma^2$ leads to

$$\frac{de_2}{d\sigma^2} = -\frac{\left(F_1/C_{11} + F_3\right)\frac{B_3}{1 - B_1/C_{11}} + F_4}{\left(F_1/C_{11} + F_3\right)\frac{[B_2 - C_{12}B_1/C_{11}]}{1 - B_1/C_{11}} + \left(F_2 - F_1 C_{12}/C_{11}\right)}$$

The sign of $\frac{de_2}{d\sigma^2}$ is ambiguous in general. Notice by assumption, we have $C_{11}, F_3 > 0, C_{12} \geq 0, B_1 \leq 0$, and $B_3, F_1, F_4 < 0$. However, either B_2 or F_2 may be positive or negative.

Proof of Corollary 1

The agent's first order conditions, in closed form, are:

$$e_1^* = \frac{\beta(1-\sigma\sqrt{rk})}{c_1}$$
$$e_2^* = \frac{c_1\sigma}{1-\sigma\sqrt{rk}}\sqrt{\frac{r}{k}}$$

To ensure we have feasible solutions, we assume $1-\sigma\sqrt{rk} > 0$.

The principal's problem becomes:

$$\max_\beta \; qe_1^* - \frac{1}{2}\left(c_1 + ke_2^*\right)e_1^{*2} - \frac{r}{2}\beta^2\frac{\sigma^2}{e_2^*}$$
$$\max_\beta \; q\frac{\beta(1-\sigma\sqrt{rk})}{c_1} - \frac{(1-\sigma\sqrt{rk})\beta^2}{2c_1} - \beta^2\frac{(1-\sigma\sqrt{rk})\sigma}{2c_1}\sqrt{rk}$$

From the principal's first-order condition, we get $\beta^* = \frac{q}{1+\sigma\sqrt{kr}}$.Thus the predicted relation is that incentive is negatively related to risk: $\frac{\partial}{\partial\sigma}\beta^*(q,k,\sigma,r) < 0$.

In equilibrium:

$$e_1^* = \frac{\beta(1-\sigma\sqrt{rk})}{c_1} = \frac{q(1-\sigma\sqrt{rk})}{c_1\left(1+\sigma\sqrt{kr}\right)}$$
$$e_2^* = \frac{c_1\sigma}{1-\sigma\sqrt{rk}}\sqrt{\frac{r}{k}}$$

In addition, $V(e_2^*) = \frac{\sigma^2}{e_2^*} = \frac{\left(1-\sigma\sqrt{rk}\right)\sigma}{c_1\sqrt{\frac{r}{k}}} = \frac{\left(1-\sigma\sqrt{rk}\right)\sigma\sqrt{rk}}{c_1 r}$ in equilibrium.

Thus, $\frac{\partial}{\partial\sigma}V(e_2^*) = \frac{\sqrt{rk}(1-2\sigma\sqrt{rk})}{c_1 r}$. Therefore,
$0 < 1-2\sigma\sqrt{rk} < 1-\sigma\sqrt{rk} \Rightarrow \frac{\partial}{\partial\sigma}V(e_2^*) > 0$
$1-2\sigma\sqrt{rk} < 0 < 1-\sigma\sqrt{rk} \Rightarrow \frac{\partial}{\partial\sigma}V(e_2^*) < 0$
If $1-2\sigma\sqrt{rk} < 0 < 1-\sigma\sqrt{rk}$, $\Gamma \equiv \frac{\frac{\partial}{\partial\sigma}\beta^*}{\frac{\partial}{\partial\sigma}V(e_2^*)} > 0$.

Proof of Lemma 1

(1) Benchmark 1: Exogenous Variance ($V(e_2,\sigma^2) = \sigma^2$)

Define $V(e_2^0,\sigma^2) = \sigma^2$. In Benchmark 1, the agent can only choose e_2^0 so that the variance is not modified.

The agent chooses his productive effort e_1 to maximize his payoff $\alpha+\beta E[y]+\delta E[z]-\frac{r}{2}\beta^2 V-\frac{r}{2}\delta^2\sigma_z^2-C(e_1,e_2^0)$. From the first order condition with respect to e_1, we have $C_1(e_1,e_2) = \beta+\delta$, $\frac{de_1}{d\beta} = \frac{1}{C_{11}(e_1,e_2)}$, and $\frac{de_1}{d\delta} = \frac{1}{C_{11}(e_1,e_2)}$.

The principal's problem is:

$$\max_{\beta,\delta} qe_1^* - \frac{r}{2}\beta^2\sigma^2 - \frac{r}{2}\delta^2\sigma_z^2 - C(e_1^*,e_2^0)$$

The principal's first order conditions show that:

$$\frac{q}{C_{11}(e_1,e_2^0)} - r\beta\sigma^2 - \frac{\beta+\delta}{C_{11}(e_1,e_2^0)} = 0;$$
$$\frac{q}{C_{11}(e_1,e_2^0)} - r\delta\sigma_z^2 - \frac{\beta+\delta}{C_{11}(e_1,e_2^0)} = 0.$$

From the principal's first order conditions, we have:

$$\beta^* = \frac{q\sigma_z^2}{C_{11}(e_1,e_2^0)r\sigma^2\sigma_z^2+\sigma_z^2+\sigma^2}$$
$$\delta^* = \frac{q\sigma^2}{C_{11}(e_1,e_2^0)r\sigma^2\sigma_z^2+\sigma_z^2+\sigma^2}$$

Therefore, in Benchmark 1, β^* and δ^* are both positive, as long as $\sigma_z^2, \sigma^2 < +\infty$. In addition, β^* and δ^* have a relation that $\beta^*\sigma^2 = \delta^*\sigma_z^2$.

(2) Benchmark 2: Separable Cost ($C(e_1, e_2) = L(e_1) + K(e_2)$ and following convention, the $'$ symbol indicates partial derivatives such as $L'(e_1)$ and $K'(e_2)$)

The agent's problem with the additional signal is

$$\max_{e_1,\, e_2} \alpha + \beta E[y] + \delta E[z] - \frac{r}{2}\beta^2 V(e_2, \sigma^2) - \frac{r}{2}\delta^2\sigma_z^2 - C(e_1, e_2)$$

The first order condition with respect to e_1 shows that $L'(e_1) = \beta + \delta$, $\frac{de_1}{d\beta} = \frac{1}{L'(e_1)}$, and $\frac{de_1}{d\delta} = \frac{1}{L''(e_1)}$. In addition, it yields an additional incentive constraint on the equilibrium choice of e_2, which we define as a function J of agent's choices e_1, e_2, the principal's choices β and δ, and model parameter σ^2.

$$J(e_2, \beta, \sigma^2) \equiv -\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - K'(e_2) = 0 \tag{18}$$

(18) implies that if $\beta > 0$, then $e_2^* > 0$ must be true since $V_1 = -\infty$ at $\beta = 0$.

The principal's problem is

$$\max_{\beta,\delta} E[x\,(e_1)] - \frac{r}{2}\beta^2 V(e_2, \sigma^2) - \frac{r}{2}\delta^2\sigma_z^2 - L(e_1) - K(e_2)$$

The problem yields a first-order condition for optimal choice of incentive β:

$$\frac{q}{L''} - \frac{\beta+\delta}{L''} - r\beta V(e_2, \sigma^2) + \left[-\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - K'(e_2)\right]\frac{de_2}{d\beta} = 0 \tag{19}$$

$\frac{q}{L''}$ is the marginal benefit of increasing β, and $\frac{\beta+\delta}{L''} - r\beta V(e_2, \sigma^2) + \left[-\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - K'(e_2)\right]\frac{de_2}{d\beta}$ is the marginal cost. If marginal benefit is lower than marginal cost, then the optimal β will be zero, which is a corner solution.

If (18) is satisfied (that is, if $e_2^* > 0$), according to (18), $-\frac{r}{2}\beta^2 V_1(e_2, \sigma^2) - K'(e_2) = 0$, we have

$$\beta \;= \beta\left(e_2, \delta, \sigma^2\right) \equiv \frac{q-\delta}{rV(e_2, \sigma^2)L'' + 1}. \tag{20}$$

The first order condition with respect to δ yields

$$\frac{q}{L''} - \frac{\beta+\delta}{L''} - r\delta\sigma_z^2 = 0. \tag{21}$$

$$\delta = \Delta(e_2, \beta, \sigma_z^2) \equiv \frac{q - \beta}{r\sigma_z^2 L'' + 1}$$

There are four possible cases to consider:

1. $\beta^* = 0, \delta^* = 0$, leading to $e_1^* = 0, e_2^* = 0$. This cannot be the optimal solution, since (21) shows a marginal benefit of $\frac{q}{L''} > 0$ and zero marginal cost. It can be improved by increase δ slightly.

2. $\beta^* > 0, \delta^* = 0$, Because (18) holds, and $\beta = \frac{q}{rV(e_2,\sigma^2)L''+1} < q$ according to (20). However, according to (21), the marginal cost $\frac{\beta}{L''}$ is less than the marginal benefit $\frac{q}{L''}$ if $\beta < q$. Therefore $\delta = 0$ cannot be optimal since the principal can slightly increase δ to get better off.

3. $\beta^* = 0, \delta^* > 0$, leading to $e_2^* = 0$. From (21), when $\beta^* = 0$ we have $\delta^* = \frac{q}{r\sigma_z^2 L''+1} > 0$. Substitute δ^* into (19), we have $\frac{q}{L''}$ as the marginal benefit of β, and $\frac{q}{r\sigma_z^2(L'')^2+L''} + K'(e_2)\frac{de_2}{d\beta} \mid_{\beta=0}$ as the marginal cost of β. Evaluated at $e_2^* = 0$, $\beta^* = 0$ holds when

$$\frac{q}{L''} \leq \frac{q}{r\sigma_z^2(L'')^2 + L''} + K'(e_2)\frac{de_2}{d\beta} \mid_{\beta=0} \tag{22}$$

   Because $K'(e_2)|_{\beta=0} = 0$ when costs are separable, inequality (22) doesn't hold, and it is impossible to have $\beta^* = 0, \delta^* > 0$.

4. $\beta^* > 0, \delta^* > 0$ leading to $e_1^* > 0, e_2^* \geq 0$. When (18) holds ($e_2^* > 0$), the optimal β^* and δ^* are:

$$\beta^* = \frac{q\sigma_z^2}{rV(e_2,\sigma^2)\sigma_z^2 L'' + \sigma_z^2 + V(e_2,\sigma^2)} \tag{23}$$

$$\delta^* = \frac{qV(e_2,\sigma^2)}{rV(e_2,\sigma^2)\sigma_z^2 L'' + \sigma_z^2 + V(e_2,\sigma^2)} \tag{24}$$

When (18) shows a greater marginal cost of e_2 than its marginal benefit ($e_2^* = 0$), we have the optimal β^* and δ^* decided by (23) and (24).

Proof of Proposition 2

Similar to the second part of the proof of Lemma 1, there are four possible cases. Cases 1, 2 and 4 are follows nearly identical arguments. We only provide case-3 with details.

If $\beta^* = 0, \delta^* > 0$, then $e_2^* = 0$. From (11), when $\beta^* = 0$ we have $\delta^* = \frac{q}{r\sigma_z^2 C_{11}(e_1,e_2)+1} > 0$. Substitute δ^* into (12), we have $\frac{q}{C_{11}(e_1,e_2)}$ as the marginal benefit of β, and

$$\frac{q}{r\sigma_z^2[C_{11}(e_1,e_2)]^2 + C_{11}(e_2)} + \left[-\frac{r}{2}\beta^2 V_1(e_2,\sigma^2) + C_2(e_1,e_2)\right]\frac{de_2}{d\beta} \mid_{\beta=0}$$

as the marginal cost of β. Evaluated at $e_2^* = 0$, $\beta^* = 0$ holds when

$$\frac{qr\sigma_z^2}{r\sigma_z^2 C_{11}(e_1,e_2)+1} \leq [C_2(e_1,e_2)]\,\frac{\partial e_2}{\partial \beta}\;|_{\beta=0,\delta=\frac{q}{r\sigma_z^2 C_{11}(e_1,e_2)+1}} \tag{25}$$

Notice that this condition requires a non-negative $\frac{de_2}{d\beta}$ at $\beta = 0$. In (10), we see that a slight increase of β from $\beta = 0$ will increase the marginal benefit of e_2 tremendously (from zero to positive infinity). Therefore we have $\frac{\partial e_2}{\partial \beta}\;|_{\beta=0}> 0$.

Proof of Corollary 2

The agent's problem:

$$\max_{e_1,\,e_2} \alpha + \beta E[y] + \delta E[z] - \tfrac{r}{2}\beta^2 V(e_2) - \tfrac{r}{2}\delta^2\sigma_z^2 - \tfrac{1}{2}(c_1 + ke_2)e_1^2$$

The agent's first order conditions, in closed form, are:

$$\widehat{e_1} = \tfrac{\beta(1-\sigma\sqrt{rk})+\delta}{c_1}$$
$$\widehat{e_2} = \tfrac{c_1\sigma\beta}{\beta\left(1-\sigma\sqrt{rk}\right)+\delta}\sqrt{\tfrac{r}{k}}$$

The principal's Problem is:

$$\max_{\beta,\delta}\; q\widehat{e_1} - \tfrac{1}{2}\left(c_1 + k\widehat{e_2}\right)\widehat{e_1}^{\,2} - \tfrac{r}{2}\beta^2\tfrac{\sigma^2}{\widehat{e_2}} - \tfrac{r}{2}\delta^2\sigma_z^2$$
$$\max_{\beta,\delta}\; q\tfrac{\beta(1-\sigma\sqrt{rk})+\delta}{c_1} - \tfrac{\left[\beta(1-\sigma\sqrt{rk})+\delta\right]^2}{2c_1} - \tfrac{\beta\sigma\sqrt{rk}\left[\beta(1-\sigma\sqrt{rk})+\delta\right]}{c_1} - \tfrac{r}{2}\delta^2\sigma_z^2$$

It leads to a first-order condition and an interior solution (if binding) of β

$$\tfrac{q(1-\sigma\sqrt{rk})}{c_1} - \tfrac{\beta(1-\sigma\sqrt{rk})+\delta}{c_1}\left[1-\sigma\sqrt{kr}\right] - \tfrac{2\beta(1-\sigma\sqrt{rk})+\delta}{c_1}\sigma\sqrt{kr} = 0,$$
$$\widehat{\beta} = \tfrac{q(1-\sigma\sqrt{kr})-\widehat{\delta}}{1-\sigma^2 kr},$$

and a first-order condition with respect to δ,

$$\widehat{\delta} = \tfrac{q-\widehat{\beta}}{1+c_1 r\sigma_z^2}.$$

From the principal's first order conditions, we have:

$$\widehat{\beta} = \tfrac{q[(1-\sigma\sqrt{rk})(1+c_1 r\sigma_z^2)-1]}{(1-\sigma^2 rk)(1+c_1 r\sigma_z^2)-1}$$
$$\widehat{\delta} = \tfrac{q(1-\sigma\sqrt{rk})\sigma\sqrt{rk}}{(1-\sigma^2 rk)(1+c_1 r\sigma_z^2)-1}$$

Notice that $\widehat{\delta}$ is always positive. In addition, since $\widehat{\beta} = \frac{q(1-\sigma\sqrt{kr})-\widehat{\delta}}{1-\sigma^2 kr}$,we have $\widehat{\beta} > \frac{q(1-\sigma\sqrt{kr})}{1-\sigma^2 kr} = \beta^*$. We also have:

$$\widehat{e_1} = \tfrac{q(1-\sigma\sqrt{rk})+\sigma\sqrt{rk}\widehat{\delta}}{c_1\left(1+\sigma\sqrt{kr}\right)} > \tfrac{q(1-\sigma\sqrt{rk})}{c_1\left(1+\sigma\sqrt{kr}\right)} = e_1^*$$
$$\widehat{e_2} = \tfrac{c_1\sigma\widehat{\beta}}{\widehat{\beta}\left(1-\sigma\sqrt{rk}\right)+\widehat{\delta}}\sqrt{\tfrac{r}{k}} < \tfrac{c_1\sigma}{1-\sigma\sqrt{rk}}\sqrt{\tfrac{r}{k}} = e_2^*$$

If $\widehat{\delta} = 0$, then we have $\widehat{\beta} = \frac{q(1-\sigma\sqrt{kr})-\widehat{\delta}}{1-\sigma^2 kr} = \frac{q}{1+\sigma\sqrt{kr}} < q$. Equation $\widehat{\delta} = \frac{q-\widehat{\beta}}{1+c_1 r\sigma_z^2}$ implies $\widehat{\delta} \neq 0$, which is a contradiction.

However, $\widehat{\beta}$ is no longer guaranteed to be positive with the non-separable costs of e_1 and e_2. Suppose $\widehat{\beta} = 0$, then $\widehat{\delta} = \frac{q-\widehat{\beta}}{1+c_1 r\sigma_z^2}$ gives $\widehat{\delta} = \frac{q}{1+c_1 r\sigma_z^2}$. Substituting into the first order condition with respect to β, we have the marginal benefit net of marginal cost, evaluated at the proposed solution, equal to:

$$\begin{aligned}&\tfrac{q(1-\sigma\sqrt{rk})}{c_1} - \tfrac{0+\widehat{\delta}}{c_1}[1-\sigma\sqrt{rk}] - \tfrac{0+\widehat{\delta}}{c_1}\sigma\sqrt{rk} \\ &= \tfrac{q}{c_1(1+c_1 r\sigma_z^2)}[(1+c_1 r\sigma_z^2)(1-\sigma\sqrt{rk})-1]\end{aligned}$$

Thus, if $(1+c_1 r\sigma_z^2)(1-\sigma\sqrt{rk})-1<0$,or equivalently, $c_1 r\sigma_z^2 < \frac{\sigma\sqrt{rk}}{1-\sigma\sqrt{rk}}$, then marginal benefit is less than marginal cost, and $\widehat{\beta} = 0$ is indeed optimal. Finally, notice that regularity mandates $(1-\sigma^2 rk)(1+c_1 r\sigma_z^2)-1>0$.

Proof of Proposition 3

When the manager observes $\varphi = h$, (that is, $\varphi = (1+\tau)\sigma^2$), his certainty equivalents are

$$CE(\widehat{\varphi} = h; \varphi = h) = \alpha_h + \beta_h e_{1h} - \frac{r}{2}\beta_h^2 \frac{(1+\tau)\sigma^2}{e_{2h}} - \frac{1}{2}(c_1 + k e_{2h})e_{1h}^2$$

and

$$CE(\widehat{\varphi} = l; \varphi = h) = \alpha_l + \beta_l e'_{1h} - \frac{r}{2}\beta_l^2 \frac{(1+\tau)\sigma^2}{e'_{2h}} - \frac{1}{2}(c_1 + k e'_{2h})e'^2_{1h},$$

The first order conditions with respect to the effort levels show that

$$e^*_{1h} = \frac{\beta_h}{c_1 + k e^*_{2h}}, e^*_{2h} = \frac{c_1\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1+\tau)\sigma^2}}; e'^*_{1h} = \frac{\beta_l}{c_1 + k e'^*_{2h}}, e'^*_{2h} = \frac{c_1\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1+\tau)\sigma^2}}.$$

Solving for the optimal effort levels, we have

$$\begin{aligned} e^*_{1h} &= \frac{\beta_h(1-\sqrt{kr(1+\tau)\sigma^2})}{c_1}, e^*_{2h} = \frac{c_1\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1+\tau)\sigma^2}}; \\ e'^*_{1h} &= \frac{\beta_l(1-\sqrt{kr(1+\tau)\sigma^2})}{c_1}, e'^*_{2h} = \frac{c_1\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1+\tau)\sigma^2}} = e^*_{2h}. \end{aligned}$$

When $\varphi = l$, (that is, $\varphi = (1-\tau)\sigma^2$), we follow a similar analysis and get

$$e^*_{1l} = \frac{\beta_l}{c_1 + k e^*_{2l}}, e^*_{2l} = \frac{c_1\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1-\tau)\sigma^2}}; e'^*_{1l} = \frac{\beta_h}{c_1 + k e'^*_{2l}}, e'^*_{2l} = \frac{c_1\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k} - k\sqrt{r(1-\tau)\sigma^2}},$$

Therefore the optimal effort levels when $\varphi = l$ are

$$\begin{aligned}
e_{1l}^* &= \frac{\beta_l(1-\sqrt{kr(1-\tau)\sigma^2})}{c_1}, e_{2l}^* = \frac{c_1\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k}-k\sqrt{r(1-\tau)\sigma^2}};\\
e_{1l}'^* &= \frac{\beta_h(1-\sqrt{kr(1-\tau)\sigma^2})}{c_1}, e_{2l}'^* = \frac{c_1\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k}-k\sqrt{r(1-\tau)\sigma^2}} = e_{2l}^*.
\end{aligned}$$

The principal's program becomes

$$\max_{\alpha_h,\beta_h,\alpha_l,\beta_l} q[e_{1h}^*\rho + e_{1l}^*(1-\rho)] - \rho[\alpha_h + \beta_h e_{1h}^*] - (1-\rho)[\alpha_l + \beta_l e_{1l}^*],$$

$s.t.$

$$\alpha_h + \beta_h e_{1h}^* - \frac{r}{2}\beta_h^2\frac{(1+\tau)\sigma^2}{e_{2h}^*} - \frac{1}{2}(c_1 + ke_{2h}^*)e_{1h}^{*2} \geq 0,$$

$$\alpha_l + \beta_l e_{1l}^* - \frac{r}{2}\beta_l^2\frac{(1-\tau)\sigma^2}{e_{2l}^*} - \frac{1}{2}(c_1 + ke_{2l}^*)e_{1l}^{*2} \geq 0,$$

$$\alpha_h + \beta_h e_{1h}^* - \frac{r}{2}\beta_h^2\frac{(1+\tau)\sigma^2}{e_{2h}^*} - \frac{1}{2}(c_1 + ke_{2h}^*)e_{1h}^{*2} \geq \alpha_l + \beta_l e_{1h}'^* - \frac{r}{2}\beta_l^2\frac{(1-\tau)\sigma^2}{e_{2l}^*} - \frac{1}{2}(c_1 + ke_{2l}^*)e_{1h}'^{*2},$$

$$\alpha_l + \beta_l e_{1l}^* - \frac{r}{2}\beta_l^2\frac{(1-\tau)\sigma^2}{e_{2l}^*} - \frac{1}{2}(c_1 + ke_{2l}^*)e_{1l}^{*2} \geq \alpha_h + \beta_h e_{1l}'^* - \frac{r}{2}\beta_h^2\frac{(1-\tau)\sigma^2}{e_{2l}^*} - \frac{1}{2}(c_1 + ke_{2l}^*)e_{1l}'^{*2}.$$

Rewrite the principal's program, we have

$$\begin{aligned}
&\max_{\alpha_h,\beta_h,\alpha_l,\beta_l} q[\rho\frac{\beta_h(1-\sqrt{kr(1+\tau)\sigma^2})}{c_1} + (1-\rho)\frac{\beta_l(1-\sqrt{kr(1-\tau)\sigma^2})}{c_1}] - \rho[\alpha_h + \frac{\beta_h^2(1-\sqrt{kr(1+\tau)\sigma^2})}{c_1}]\\
&-(1-\rho)[\alpha_l + \frac{\beta_l^2(1-\sqrt{kr(1-\tau)\sigma^2})}{c_1}],
\end{aligned}$$

$s.t.$

$$\begin{aligned}
0 &\leq \alpha_h + \frac{\beta_h^2}{c_1}[2-3\sqrt{kr(1+\tau)\sigma^2} - \frac{1}{2}(1-\sqrt{kr(1+\tau)\sigma^2})^2]\\
0 &\leq \alpha_l + \frac{\beta_l^2}{c_1}[2-3\sqrt{kr(1-\tau)\sigma^2} - \frac{1}{2}(1-\sqrt{kr(1-\tau)\sigma^2})^2]\\
0 &\leq \left(\alpha_h + \frac{\beta_h^2}{c_1}[2-3\sqrt{kr(1+\tau)\sigma^2} - \frac{1}{2}(1-\sqrt{kr(1+\tau)\sigma^2})^2]\right)\\
&\quad - \left(\alpha_l + \frac{\beta_l^2}{c_1}[2-3\sqrt{kr(1+\tau)\sigma^2} - \frac{1}{2}(1-\sqrt{kr(1+\tau)\sigma^2})^2]\right)\\
0 &\leq \left(\alpha_l + \frac{\beta_l^2}{c_1}[2-3\sqrt{kr(1-\tau)\sigma^2} - \frac{1}{2}(1-\sqrt{kr(1-\tau)\sigma^2})^2]\right)\\
&\quad - \left(\alpha_h + \frac{\beta_h^2}{c_1}[2-3\sqrt{kr(1-\tau)\sigma^2} - \frac{1}{2}(1-\sqrt{kr(1-\tau)\sigma^2})^2]\right).
\end{aligned}$$

For convenience, we define $A_h \equiv \sqrt{kr(1+\tau)\sigma^2}$ and $A_l \equiv \sqrt{kr(1-\tau)\sigma^2}$. Rewrite the constraints, we have

$$\alpha_h + \frac{\beta_h^2}{2c_1}[7-(A_h+2)^2] \geq 0, \tag{26}$$

$$\alpha_l + \frac{\beta_l^2}{2c_1}[7-(A_l+2)^2] \geq 0, \tag{27}$$

$$\alpha_h + \frac{\beta_h^2}{2c_1}[7-(A_h+2)^2] \geq \alpha_l + \frac{\beta_l^2}{2c_1}[7-(A_h+2)^2], \tag{28}$$

$$\alpha_l + \frac{\beta_l^2}{c_1}[7-(A_l+2)^2] \geq \alpha_h + \frac{\beta_h^2}{c_1}[7-(A_l+2)^2]. \tag{29}$$

Define $\mu_{1,}$ $\mu_{2,}$ μ_3 and μ_4 as the Lagrangian multipliers for (26), (27), (28) and (29). First order conditions with respect to $\alpha_h, \beta_h, \alpha_l$ and β_l give us

$$-\rho + \mu_1 + \mu_3 - \mu_4 = 0 \tag{30}$$

$$-(1-\rho) + \mu_2 - \mu_3 + \mu_4 = 0 \tag{31}$$

$$q\rho\frac{1}{c_1}(1-A_h) - \rho\frac{2}{c_1}(1-A_l)\beta_h + (\mu_1+\mu_3)\frac{1}{c_1}[7-(A_h+2)^2]\beta_h - \mu_4\frac{1}{c_1}[7-(A_l+2)^2]\beta_h = 0 \tag{32}$$

$$\frac{q(1-\rho)(1-A_l)}{c_1} - \frac{2(1-\rho)(1-A_l)}{c_1}\beta_l + \frac{\mu_2+\mu_4}{c_1}[7-(A_l+2)^2]\beta_l - \frac{\mu_3}{c_1}[7-(A_h+2)^2]\beta_l = 0 \tag{33}$$

From (30) and (31), we see μ_1, μ_3 cannot both be zero, μ_1, μ_2 cannot both be zero, and μ_2, μ_4 cannot both be zero. We therefore start from the following potential combinations: (1) $\mu_1 > 0$, $\mu_2 = 0$, $\mu_3 = 0$ and $\mu_4 > 0$; (2) $\mu_1 = 0$, $\mu_2 > 0$, $\mu_3 > 0$ and $\mu_4 = 0$; (3) $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 = 0$ and $\mu_4 = 0$.

(2) implies $\alpha_h + \frac{\beta_h^2}{2c_1}[7-(A_h+2)^2] > 0$ and $\alpha_l + \frac{\beta_l^2}{2c_1}[7-(A_l+2)^2] = 0$.Since $\alpha_l + \frac{\beta_l^2}{2c_1}[7-(A_h+2)^2] < \alpha_l + \frac{\beta_l^2}{2c_1}[7-(A_l+2)^2]$, it cannot be true that $\mu_3 > 0$ (in other words, it cannot be true that $\alpha_h + \frac{\beta_h^2}{2c_1}[7-(A_h+2)^2] = \alpha_l + \frac{\beta_l^2}{2c_1}[7-(A_h+2)^2]$). Therefore (2) is not a feasible solution.

(3) implies $\alpha_h + \frac{\beta_h^2}{2c_1}[7-(A_h+2)^2] = 0$ and $\alpha_l + \frac{\beta_l^2}{2c_1}[7-(A_l+2)^2] = 0$.Since $\alpha_h + \frac{\beta_h^2}{2c_1}[7-(A_l+2)^2] > \alpha_h + \frac{\beta_h^2}{2c_1}[7-(A_h+2)^2]$, it cannot be true that $\mu_4 = 0$ (in other words, it cannot be true that $\alpha_l + \frac{\beta_l^2}{c_1}[7-(A_l+2)^2] > \alpha_h + \frac{\beta_h^2}{c_1}[7-(A_l+2)^2]$). Therefore (3) is not feasible, and (1) is the only feasible optimal solution.

(1) $\mu_1 > 0$, $\mu_2 = 0$, $\mu_3 = 0$ and $\mu_4 > 0$:

From (30) and (31), we have $\mu_1 = 1$ and $\mu_4 = 1-\rho$. Then from (32) we have $q\rho\frac{1}{c_1}(1-A_h) - \rho\frac{2}{c_1}(1-A_h)\beta_h + \frac{2}{c_1}[\frac{7}{2} - \frac{1}{2}(A_h+2)^2]\beta_h - (1-\rho)\frac{2}{c_1}[\frac{7}{2} - \frac{1}{2}(A_l+2)^2]\beta_h = 0$, which implies

$$\beta_h^* = \frac{q\rho(1-A_h)}{(A_h+2)^2 - 5\rho - 2\rho A_h - (1-\rho)(A_l+2)^2}.$$

From (33), we have $q(1-\rho)\frac{1}{c_1}(1-A_l) - (1-\rho)\frac{2}{c_1}(1-A_l)\beta_l + (1-\rho)\frac{2}{c_1}[\frac{7}{2} - \frac{1}{2}(A_l+2)^2]\beta_l = 0.$

Therefore

$$\beta_l^* = \frac{q(1-A_l)}{(1+A_l)^2 - 2}.$$

From (26) and (29), we have

$$\alpha_h^* = \frac{[(A_h+2)^2 - 7]}{2c_1}\beta_h^{*2},$$

and

$$\alpha_l^* = \alpha_h^* + \frac{[(A_l+2)^2 - 7]}{2c_1}(\beta_l^{*2} - \beta_h^{*2}).$$

Proof of Corollary 3

Directly from Proposition 3.

Proof of Proposition 4

When $\varphi = (1+\tau)\sigma^2$ (that is, $\varphi = h$), the manager's certainty equivalent if he reports $\widehat{\varphi} = h$ is

$$CE(\widehat{\varphi} = h; \varphi = h) = \alpha_h + (\beta_h + \delta_h)e_{1h} - \frac{r}{2}\beta_h^2\frac{(1+\tau)\sigma^2}{e_{2h}} - \frac{r}{2}\delta_h^2\sigma_z^2 - \frac{1}{2}(c_1 + ke_{2h})e_{1h}^2,$$

and his certainty equivalent if reporting $\widehat{\varphi} = l$ is

$$CE(\widehat{\varphi} = l; \varphi = h) = \alpha_l + (\beta_l + \delta_l)e'_{1h} - \frac{r}{2}\beta_l^2\frac{(1+\tau)\sigma^2}{e'_{2h}} - \frac{r}{2}\delta_l^2\sigma_z^2 - \frac{1}{2}(c_1 + ke'_{2h})e'^2_{1h}.$$

From the first order conditions with respect to the effort levels, we have

$$\begin{aligned} e_{1h}^* &= \frac{\beta_h + \delta_h}{c_1 + ke_{2h}^*}, e'^*_{1h} = \frac{\beta_l + \delta_l}{c_1 + ke'^*_{2h}}, \\ e_{1l}^* &= \frac{\beta_l + \delta_l}{c_1 + ke_{2l}^*}, e'^*_{1l} = \frac{\beta_h + \delta_h}{c_1 + ke'^*_{2l}}. \end{aligned}$$

Solving for the optimal effort levels, we have

$$\begin{aligned} e_{1h}^* &= \frac{\beta_h(1 - \sqrt{kr(1+\tau)\sigma^2}) + \delta_h}{c_1}, e_{2h}^* = \frac{c_1\beta_h\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k}(\beta_h + \delta_h) - k\beta_h\sqrt{r(1+\tau)\sigma^2}}; \\ e'^*_{1h} &= \frac{\beta_l(1 - \sqrt{kr(1+\tau)\sigma^2}) + \delta_l}{c_1}, e'^*_{2h} = \frac{c_1\beta_l\sqrt{r(1+\tau)\sigma^2}}{\sqrt{k}(\beta_l + \delta_l) - k\beta_l\sqrt{r(1+\tau)\sigma^2}}. \end{aligned}$$

Following a similar analysis, we get the optimal effort levels when the manager observes $\varphi = l$, (that is, $\varphi = (1-\tau)\sigma^2$) :

$$
\begin{aligned}
e_{1l}^* &= \frac{\beta_l(1-\sqrt{kr(1-\tau)\sigma^2})+\delta_l}{c_1}, e_{2l}^* = \frac{c_1\beta_l\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k}(\beta_l+\delta_l)-k\beta_l\sqrt{r(1-\tau)\sigma^2}};\\
e_{1l}'^* &= \frac{\beta_h(1-\sqrt{kr(1-\tau)\sigma^2})+\delta_h}{c_1}, e_{2l}'^* = \frac{c_1\beta_h\sqrt{r(1-\tau)\sigma^2}}{\sqrt{k}(\beta_h+\delta_h)-k\beta_h\sqrt{r(1-\tau)\sigma^2}}.
\end{aligned}
$$

Again, for our convenience we denote $A_h \equiv \sqrt{kr(1+\tau)\sigma^2}$ and $A_l \equiv \sqrt{kr(1-\tau)\sigma^2}$. Then the principal's expected payoff, which is denoted by PP, becomes $q\frac{\rho[\beta_h(1-A_h)+\delta_h]+(1-\rho)[\beta_l(1-A_l)+\delta_l]}{c_1} - \rho[\alpha_h + \frac{(\beta_h+\delta_h)[\beta_h(1-A_h)+\delta_h]}{c_1}] - (1-\rho)[\alpha_l + \frac{(\beta_l+\delta_l)[\beta_l(1-A_l)+\delta_l]}{c_1}]$.

The principal's design program to motivate truth telling is

$$\max_{\alpha_h,\beta_h,\delta_h,\alpha_l,\beta_l,\delta_l} PP,$$

$s.t.$

$$\alpha_h + \frac{[\beta_h(1-A_h)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 \geq 0,$$

$$\alpha_l + \frac{[\beta_l(1-A_l)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 \geq 0, \tag{34}$$

$$\alpha_h + \frac{[\beta_h(1-A_h)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 \geq \alpha_l + \frac{[\beta_l(1-A_h)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2, \tag{35}$$

$$\alpha_l + \frac{[\beta_l(1-A_l)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 \geq \alpha_h + \frac{[\beta_h(1-A_l)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2. \tag{36}$$

Define $\mu_{1,}$ $\mu_{2,}$ μ_3 and μ_4 as the Lagrangian multipliers for (14), (15), (16) and (17). First order conditions with respect to $\alpha_h, \alpha_l, \beta_h, \beta_l, \delta_h$ and δ_l give us

$$0 = -\rho + \mu_1 + \mu_3 - \mu_4 \tag{37}$$

$$0 = -(1-\rho) + \mu_2 - \mu_3 + \mu_4 \tag{38}$$

$$
\begin{aligned}
0 &= \frac{q\rho}{c_1}(1-A_h) - \frac{\rho}{c_1}(2\beta_h + 2\delta_h - 2\beta_h A_h - \delta_h A_h) \\
&\quad + \frac{\mu_1+\mu_3}{c_1}(\beta_h+\delta_h-\beta_h A_h)(1-A_h) - \frac{\mu_4}{c_1}(\beta_h+\delta_h-\beta_h A_l)(1-A_l)
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
0 &= \frac{q(1-\rho)}{c_1}(1-A_l) - \frac{(1-\rho)}{c_1}(2\beta_l + 2\delta_l - 2\beta_l A_l - \delta_l A_l) \\
&\quad + \frac{\mu_2+\mu_4}{c_1}(\beta_l+\delta_l-\beta_l A_l)(1-A_l) - \frac{\mu_3}{c_1}(\beta_l+\delta_l-\beta_l A_h)(1-A_h)
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
0 &= \frac{q\rho}{c_1} - \frac{\rho}{c_1}(2\beta_h + 2\delta_h - \beta_h A_h) + \frac{\mu_1+\mu_3}{c_1}(\beta_h + \delta_h - \beta_h A_h) \\
&\quad -(\mu_1+\mu_3-\mu_4) r\sigma_z^2 \delta_h - \frac{\mu_4}{c_1}(\beta_h + \delta_h - \beta_h A_l)
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
0 &= \frac{q(1-\rho)}{c_1} - \frac{(1-\rho)}{c_1}(2\beta_l + 2\delta_l - \beta_l A_l) \\
&\quad + \frac{\mu_2+\mu_4}{c_1}(\beta_h + \delta_h - \beta_h A_h) - (\mu_2+\mu_4-\mu_3) r\sigma_z^2 \delta_h - \frac{\mu_3}{c_1}(\beta_l + \delta_l - \beta_l A_h)
\end{aligned} \tag{42}
$$

(37) and (38) imply that μ_1 and μ_3 cannot both be zero, μ_2 and μ_4 cannot both be zero, and μ_1 and μ_2 cannot both be zero. Therefore, we only need to examine the following candidates: (1). $\mu_1 > 0, \mu_2 = 0, \mu_3 = 0, \mu_4 > 0$;(2). $\mu_1 > 0, \mu_2 > 0, \mu_3 = 0, \mu_4 = 0$;(3). $\mu_1 = 0, \mu_2 > 0, \mu_3 > 0, \mu_4 = 0$.

If (2) is the solution, then we have $\alpha_h + \frac{[\beta_h(1-A_h)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 = 0$, $\alpha_l + \frac{[\beta_l(1-A_l)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 = 0$,and $\alpha_h + \frac{[\beta_h(1-A_h)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 > \alpha_l + \frac{[\beta_l(1-A_h)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2$. However, since $\alpha_l + \frac{[\beta_l(1-A_h)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 < \alpha_l + \frac{[\beta_l(1-A_l)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 = 0$, we cannot have $\alpha_h + \frac{[\beta_h(1-A_h)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 > \alpha_l + \frac{[\beta_l(1-A_h)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2$. Therefore (2) is infeasible.

If (3) is the solution, then we have $\alpha_h + \frac{[\beta_h(1-A_h)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 > 0$, $\alpha_l + \frac{[\beta_l(1-A_l)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 = 0$, and $\alpha_l + \frac{[\beta_l(1-A_l)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 = \alpha_h + \frac{[\beta_h(1-A_l)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2$. However, since $\alpha_h + \frac{[\beta_h(1-A_l)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 > \alpha_h + \frac{[\beta_h(1-A_h)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2 > 0$, the equation $\alpha_l + \frac{[\beta_l(1-A_l)+\delta_l]^2}{2c_1} - \frac{r}{2}\delta_l^2\sigma_z^2 = \alpha_h + \frac{[\beta_h(1-A_l)+\delta_h]^2}{2c_1} - \frac{r}{2}\delta_h^2\sigma_z^2$ doesn't hold. Therefore (3) is infeasible too.

Then we only have (1) $\mu_1 > 0, \mu_2 = 0, \mu_3 = 0, \mu_4 > 0$.

From (37) and (38) we get $\mu_1 = 1$ and $\mu_4 = 1-\rho$. Substitute them into the first order conditions with respect to $\beta_h, \beta_l, \delta_h$ and δ_l, we get

$$
\begin{aligned}
\beta_h &= \frac{q\rho(1-A_h) - [(1-\rho)A_h + \rho(1-A_l)]\delta_h}{\rho(1-A_h^2)}, \\
\delta_h &= \frac{q\rho - [\rho + (1-\rho)(A_h - A_l)]\beta_h}{\rho(1+r\sigma_z^2 c_1)}, \\
\beta_l &= \frac{q(1-A_l) - \delta_l}{1-A_l^2}, \\
\delta_l &= \frac{q-\beta_l}{1+r\sigma_z^2 c_1}.
\end{aligned}
$$

If $\delta_l^* = 0$, then $\beta_l = \frac{q(1-A_l)-0}{1-A_l^2} = \frac{q}{1+A_l}$. Substitute it to (42), the first order condition with respect to δ_l, we have

$$
q - \frac{q}{1+A_l} > 0 \neq 0,
$$

therefore there is a contradiction and $\delta_l^* \neq 0$.

If $\beta_l^* = 0$, then $\delta_l = \frac{q}{1+r\sigma_z^2 c_1}$. Substitute it to (40), the first order condition with respect to β_l, we have

$$q(1 - A_l) - \frac{q}{1 + r\sigma_z^2 c_1} = 0.$$

As long as the marginal benefit of increasing β_l is less than the marginal cost (in other words, $q(1 - A_l) < \frac{q}{1+r\sigma_z^2 c_1}$, or $(1 - A_l)(1 + r\sigma_z^2 c_1) < 1$), we have $\beta_l^* = 0$.

If $\delta_h^* = 0$, then $\beta_h = \frac{q}{1+A_h}$. Substitute it to (41), the first order condition with respect to δ_h, we have

$$\rho - \frac{(1-\rho)(A_h - A_l)}{1 + A_h} = 0.$$

As long as the marginal benefit of increasing δ_h is less than the marginal cost (in other words, $\rho(1 + A_h) < (1 - \rho)(A_h - A_l)$), we have $\delta_h^* = 0$.

If $\beta_h^* = 0$, then $\delta_h = \frac{q}{1+r\sigma_z^2 c_1}$. Substitute it to (39), the first order condition with respect to β_h, we have

$$\rho(1 - A_h) - \frac{(1-\rho)(A_h - A_l)}{1 + r\sigma_z^2 c_1} = 0.$$

As long as the marginal benefit of increasing β_h is less than the marginal cost (in other words, $\rho(1 - A_h)(1 + r\sigma_z^2 c_1) < (1 - \rho)(A_h - A_l)$), we have $\beta_h^* = 0$.

Proof of Corollary 4

For our convenience we use $y_i, z_i, i \in \{h, l\}$ to represent signals y and z when $\widehat{\varphi} = h$ and when $\widehat{\varphi} = l$. From the proof of Proposition 4, δ_l^* can never be zero. That is, the principal always uses z_l. The possible combination of signal uses are $\{y_h, z_h, y_l, z_l\}$, $\{y_h, z_h, z_l\}$, $\{z_h, y_l, z_l\}$, $\{z_h, z_l\}$, $\{y_h, y_l, z_l\}$, and $\{y_h, z_l\}$.Since we are interested in the cases that the principal ignores signal z, we focus on the last two combinations, $\{y_h, y_l, z_l\}$, and $\{y_h, z_l\}$.

$\{y_h, z_l\}$ requires $\rho(1+A_h) < (1-\rho)(A_h - A_l) < \rho(1-A_h)(1+r\sigma_z^2 c_1)$ and $(1-A_l)(1+r\sigma_z^2 c_1) < 1$. Substitute$1 + r\sigma_z^2 c_1 < \frac{1}{1-A_l}$ into $\rho(1 + A_h) < (1 - \rho)(A_h - A_l) < \rho(1 - A_h)(1 + r\sigma_z^2 c_1)$, we have $\rho(1 + A_h) < \rho\frac{1-A_h}{1-A_l}$. Since $\frac{1-A_h}{1-A_l} < 1$, we get a confliction. Therefore $\{y_h, z_l\}$ is not feasible.

However, $\{y_h, y_l, z_l\}$ is possible as long as $\rho(1 + A_h) < (1 - \rho)(A_h - A_l) < \rho(1 - A_h)(1 + r\sigma_z^2 c_1)$, which is feasible. A numerical example is that $\rho = 0.2, A_h = 0.5, A_l = 0.1, r\sigma_z^2 c_1 = 3$.

Appendix II Incentive-Variance Relation in other setting

In this appendix, we consider the incentive-variance relation in settings not covered in the main text and show the same results do appear (i) when contracts are not restricted to linear and agent's utility function is not restricted to exponential and (ii) when the agent has a option to misreport his private knowledge about the nature of the performance measurement system.

General Settings (beyond LEN)

Here we drop the linear contract assumption and the negative exponential utility assumption. We show the positive incentive-variance relation may still hold in a generalized setting with the assumption that outcome follows a normal distribution, a concave utility function, and a sufficiently convex effort cost function.

The principal chooses optimal contract, $w(y)$, subject to IR and IC constraints, to maximize $E[x(e_1) - w(y)]$.The agent chooses his effort levels, e_1 and e_2, to maximize his expected utility $\int_{-\infty}^{+\infty}[u(w(y)) - C(e_1, e_2)]f(y \mid e_1, e_2)dy$, where $u(.)$ is a concave function.

By first order approach,[14] we have

$$\frac{1}{u'(w(y))} = \mu + \lambda_1 \frac{f_1(y \mid e_1, e_2)}{f(y \mid e_1, e_2)} + \lambda_2 \frac{f_2(y \mid e_1, e_2)}{f(y \mid e_1, e_2)}, \tag{43}$$

where μ is the Lagrangian multiplier of the IR constraint and λ_1, λ_2 are the Lagrangian multipliers of the IC constraints. $f_1(y \mid e_1, e_2) \equiv \frac{\partial f(y|e_1,e_2)}{\partial e_1}$,and $f_2(y \mid e_1, e_2) \equiv \frac{\partial f(y|e_1,e_2)}{\partial e_2}$.

With normal distribution assumption, we have

$f(y \mid e_1, e_2) = \frac{1}{\sqrt{2\pi V(e_2)}} e^{-\frac{(y-e_1)^2}{2V(e_2)}}$;

$f_1(y \mid e_1, e_2) = \frac{1}{\sqrt{2\pi V(e_2)}} e^{-\frac{(y-e_1)^2}{2V(e_2)}} [-\frac{2(y-e_1)}{2V(e_2)}](-1) = \frac{1}{\sqrt{2\pi V(e_2)}} e^{-\frac{(y-e_1)^2}{2V(e_2)}} \frac{(y-e_1)}{V(e_2)}$;

$f_2(y \mid e_1, e_2) = -\frac{1}{2}[2\pi V(e_2)]^{-\frac{3}{2}} 2\pi V'(e_2) e^{-\frac{(y-e_1)^2}{2V(e_2)}} + \frac{1}{\sqrt{2\pi V(e_2)}} e^{-\frac{(y-e_1)^2}{2V(e_2)}} \frac{(y-e_1)^2}{2} \frac{V'(e_2)}{V(e_2)^2}$

$= -\frac{1}{2} \frac{1}{\sqrt{2\pi V(e_2)}} \frac{V'(e_2)}{V(e_2)} e^{-\frac{(y-e_1)^2}{2V(e_2)}} + \frac{1}{\sqrt{2\pi V(e_2)}} e^{-\frac{(y-e_1)^2}{2V(e_2)}} \frac{(y-e_1)^2}{2V(e_2)^2} V'(e_2)$.

Therefore, we get

$$\begin{aligned}
\frac{f_1(y \mid e_1, e_2)}{f(y \mid e_1, e_2)} &= \frac{y - e_1}{V(e_2)} \\
\frac{f_2(y \mid e_1, e_2)}{f(y \mid e_1, e_2)} &= -\frac{V'(e_2)}{2V(e_2)}(1 - \frac{(y - e_1)^2}{V(e_2)}).
\end{aligned}$$

Substitute $\frac{f_1(y|e_1,e_2)}{f(y|e_1,e_2)}$ and $\frac{f_2(y|e_1,e_2)}{f(y|e_1,e_2)}$ into (43), we have

$$\frac{1}{u'(w(y))} = \mu + \lambda_1 \frac{y - e_1}{V(e_2)} - \lambda_2 \frac{V'(e_2)}{2V(e_2)}[1 - \frac{(y - e_1)^2}{V(e_2)}] \tag{44}$$

For our convenience, we call $\lambda_1 \frac{y-e_1}{V(e_2)}$ in (44) term 1, and $-\lambda_2 \frac{V'(e_2)}{2V(e_2)}[1 - \frac{(y-e_1)^2}{V(e_2)}]$ term 2. We discuss the incentive-variance relation in the following cases.

- Case 0: No e_2.

[14] A sufficient condition for first order approach to be valid is that both MLRP and CDFC hold. In this setting, the agent's expected utility function is $\int_{-\infty}^{+\infty}[u(w(y)) - C(e_1, e_2)]f(y \mid e_1, e_2)dy$. With the outcome y follows a normal distribution, CDFC is not satisfied. However, as long as $C(e_1, e_2)$ is sufficiently convex, the agent's expected utility will be concave and first order approach is valid.

Without e_2, (44) becomes $\frac{1}{u'(w(y))} = \mu + \lambda_1 \frac{y-e_1}{V}$. When σ^2 increases, V gets larger. Therefore the principal would increase $u'(w(y))$ to make (44) balanced. Since $u(w(y))$ is concave, to achieve a higher $u'(w(y))$ implies a decrease in $w'(y)$.

We use Figure 3 to help illustrate. We start from w_0 which is the compensation for y, and w_1 which is the compensation for y', $y, y' > e_1$.[15] Assume $y' = y + \varepsilon$, then $w'(y)$ at y can be represented by $w_1(y') - w_0(y)$. As σ^2 increases, the right hand side of (44) decreases, and the principal would choose $w(y)$ such that $u'(w(y))$ is higher (i.e., to raise the LHS) in order to maintain (44). That implies the principal would decrease the compensation for y' from $w_1(y')$ to $w_2(y')$ in order to achieve a steeper slope of $u(w(y))$.Therefore, $w'(y)$ decreases in σ^2, since difference in payment is now $w_2(y') - w_0(y)$ which is smaller than $w_1(y') - w_0(y)$. Therefore we observe a negative incentive-variance relation, which is the result of standard moral hazard models.

- Case 1: With e_2 and $\frac{de_2^*}{d\sigma^2} > 0$, and $\frac{dV^*(e_2)}{d\sigma^2} < 0$. That is, an increase in σ^2 makes the agent work harder to reduce the noise in his performance measure and that leads to a decrease in $V(e_2)$.

  Again, for our convenience, we use Figure 3 to illustrate. An increase in σ^2 results in an increase in term 1. In term 2, $V'(e_2)$ becomes more negative since $V'(e_2) < 0$ and $\frac{\partial V'(e_2)}{\partial \sigma^2} < 0$; $V(e_2)$ decreases since $\frac{dV^*(e_2)}{d\sigma^2} < 0$. If we look at the area where y is close to its expected value e_1, that is, when we focus on first order effects and ignore second order effect from $\frac{(y-e_1)^2}{V(e_2)}$, we see that term 2 becomes larger.[16] Therefore, both term 1 and term 2 increase, and the principal would decrease $u'(w(y))$ to maintain (44). That implies the principal would increase the compensation for y' to $w_3(y')$ in order to achieve a flatter slope of $u(w(y))$. As a result, $w'(y)$ increases in σ^2, since it is now $w_3(y') - w_0(y)$ which is larger than $w_1(y') - w_0(y)$. Therefore we observe a positive incentive-variance relation.

- Case 2: With e_2 and $\frac{de_2^*}{d\sigma^2} > 0$, and $\frac{dV^*(e_2)}{d\sigma^2} > 0$. That is, an increase in σ^2 makes the agent work harder to reduce the noise in his performance measure, but e_2^*'s effect on $V(e_2)$ is dominated by the effect from σ^2. Therefore $V^*(e_2)$ increases in σ^2.

  We again use Figure 3 to help us illustrate. In this case, as σ^2 increases, term 1 decreases since $V(e_2)$ becomes larger. To cope with the decrease on the right hand side of (44), the principal would increase $u'(w(y))$ to maintain (44).

  However, as σ^2 increases, term 2's change is ambiguous, since $V'(e_2)$ in the numerator becomes more negative while $V(e_2)$ in the denominator becomes larger.[17] There are two possible situations:

  (i) If the denominator effect dominates, then term 2 decreases. Since both term 1 and term 2 decrease, the principal would increase $u'(w(y))$ to maintain (44). As illustrated in Figure 3, the principal would decrease the compensation for y' in order to achieve a steeper slope of $u(w(y))$. In other words, $w'(y)$ decreases with σ^2 and there is a negative incentive-variance relation.

[15] The analysis is similar when $y' < e_1$.

[16] When y is far away from its expected value e_1, term 2 may decrease since $1 - \frac{(y-e_1)^2}{V(e_2)}$ decreases. That may lead to a negative incentive-variance relation when the decrease in term 2 dominates the increase in term 1.

[17] Again, we focus on first order effects and ignore the second order effect from $\frac{(y-e_1)^2}{V(e_2)}$. When y is far from its expected value e_1, the analysis may be different but it won't alter the fact that the change of the right hand side of (44) is ambiguous.

(ii) If the numerator effect dominates, then term 2 increases. When the decrease of term 1 dominates the increase of term 2, the principal would increase $u'(w(y))$ by decreasing the compensation for y'. Therefore we have a negative incentive-variance relation. However, when term 2's increase dominates the decrease in term 1, the principal would increase the compensation for y' in order to achieve a lower $u'(w(y))$.In other words, $w'(y)$ increases with σ^2. Therefore we observe a positive incentive-variance relation.

- Case 3: With e_2 and $\frac{de_2^*}{d\sigma^2} < 0$, and $\frac{dV^*(e_2)}{d\sigma^2} > 0$. That is, the agent's effort on performance reporting decreases when σ^2 is higher. Therefore $V^*(e_2)$ increases in σ^2. The analysis for this case is the same as that for Case 2. The incentive-variance relation is ambiguous.

Option to Mis-report

First, when there is no performance reporting effort (e_2) to reduce the risk of a project or the noise in the performance measure, we have $\beta_h^* = \frac{q\rho}{ar(2\tau-\rho-\tau\rho)\sigma^2+\rho}$ and $\beta_l^* = \frac{q}{ar(1-\tau)\sigma^2+1}$. It is easy to verify that the incentive-variance relation is negative in both cases of high risk ($\varphi = (1+\tau)\sigma^2$) and low risk ($\varphi = (1-\tau)\sigma^2$).

When the manager can influence the variance through e_2, again we find that a positive incentive-variance relation is possible. We use the previous measure Γ to analyze this relation. Specifically, we define

$$\Gamma_h \equiv \frac{\frac{\partial}{\partial(1+\tau)\sigma^2}\beta_h^*}{\frac{\partial V(e_{2h}^*,(1+\tau)\sigma^2)}{\partial(1+\tau)\sigma^2}}$$

to be the measure for the high risk case when $\varphi = (1+\tau)\sigma^2$, and define

$$\Gamma_l \equiv \frac{\frac{\partial}{\partial(1-\tau)\sigma^2}\beta_l^*}{\frac{\partial V(e_{2l}^*,(1-\tau)\sigma^2)}{\partial(1-\tau)\sigma^2}}$$

for the low risk case when $\varphi = (1-\tau)\sigma^2$. To simplify our analysis, we set $\rho = \frac{1}{2}$. Further, since we focus on interior solutions, we assume $\frac{(\sqrt{2}-1)^2}{1+\tau} < kr\sigma^2 < \frac{1}{1+\tau}$ so that both optimal incentive coefficients, β_h^* and β_l^*, are positive.

(i). Define $\varphi_h = (1+\tau)\sigma^2$. With $\rho = \frac{1}{2}$, β_h^* becomes $\frac{q(1-\sqrt{kr\varphi_h})}{2kr\varphi_h-1+6\sqrt{kr\varphi_h}-\frac{1-\tau}{1+\tau}kr\varphi_h-4\sqrt{\frac{1-\tau}{1+\tau}kr\varphi_h}}$.

We calculate $\frac{\partial\beta_h^*}{\partial\varphi_h}$ first:

$$\frac{\partial\beta_h^*}{\partial\varphi_h} = \frac{A}{B},$$

where $A \equiv -\frac{q\sqrt{kr}}{2\sqrt{\varphi_h}}[2kr\varphi_h - 1 + 6\sqrt{kr\varphi_h} - \frac{1-\tau}{1+\tau}kr\varphi_h - 4\sqrt{\frac{1-\tau}{1+\tau}kr\varphi_h}] - q(1-\sqrt{kr\varphi_h})[(2-\frac{1-\tau}{1+\tau})kr + (3-2\sqrt{\frac{1-\tau}{1+\tau}})\frac{\sqrt{kr}}{\sqrt{\varphi_h}}]$, and $B \equiv [2kr\varphi_h - 1 + 6\sqrt{kr\varphi_h} - \frac{1-\tau}{1+\tau}kr\varphi_h - 4\sqrt{\frac{1-\tau}{1+\tau}kr\varphi_h}]^2$.

Since $\frac{(\sqrt{2}-1)^2}{1+\tau} < kr\sigma^2 < \frac{1}{1+\tau}$, we have $2kr\varphi_h - 1 + 6\sqrt{kr\varphi_h} - \frac{1-\tau}{1+\tau}kr\varphi_h - 4\sqrt{\frac{1-\tau}{1+\tau}kr\varphi_h} > 0$ and

$1-\sqrt{kr\varphi_h}>0$. In addition, we see $(2-\frac{1-\tau}{1+\tau})kr+(3-2\sqrt{\frac{1-\tau}{1+\tau}})\frac{\sqrt{kr}}{\sqrt{\varphi_h}}>0$ since $2>\frac{1-\tau}{1+\tau}$ and $3>2\sqrt{\frac{1-\tau}{1+\tau}}$. Therefore

$$\frac{\partial \beta_h^*}{\partial \varphi_h}>0.$$

For $\frac{\partial V(e_{2h}^*,\varphi_h)}{\partial \varphi_h}$, we have

$$\begin{aligned}
\frac{\partial V(e_{2h}^*,\varphi_h)}{\partial \varphi_h} &= \frac{\partial \frac{\varphi_h\sqrt{k}(1-\sqrt{kr\varphi_h})}{c_1\sqrt{r\varphi_h}}}{\partial \varphi_h} \\
&= \frac{\frac{1}{2}c_1\sqrt{kr\varphi_h}-\frac{1}{2}c_1 kr(3-\varphi_h)}{c_1^2 r\varphi_h} \\
&= \frac{\frac{1}{2}c_1\sqrt{kr}\{\sqrt{\varphi_h}-\sqrt{kr}(3-\varphi_h)\}}{c_1^2 r\varphi_h}.
\end{aligned}$$

When $\sqrt{\varphi_h}<\sqrt{kr}(3-\varphi_h)$, (that is, $\sqrt{(1+\tau)\sigma^2}<\sqrt{kr}[3-(1+\tau)\sigma^2]$), $\frac{\partial V(e_{2h}^*,\varphi_h)}{\partial \varphi_h}<0$, and $\Gamma_h\equiv\frac{\frac{\partial}{\partial(1+\tau)\sigma^2}\beta_h^*}{\frac{\partial V(e_{2h}^*,(1+\tau)\sigma^2)}{\partial(1+\tau)\sigma^2}}>0$.

(ii). Define $\varphi_l=(1-\tau)\sigma^2$. We have $\beta_l^*=\frac{q(1-\sqrt{kr\varphi_l})}{(1+\sqrt{kr\varphi_l})^2-2}$. (Notice that β_l^* doesn't depend on ρ.)

We now calculate $\frac{\partial \beta_l^*}{\partial \varphi_l}$:

$$\frac{\partial \beta_l^*}{\partial \varphi_l}=\frac{-\frac{q\sqrt{kr}}{2\sqrt{\varphi_l}}[(1+\sqrt{kr\varphi_l})^2-2]-q(1-\sqrt{kr\varphi_l})(1+\sqrt{kr\varphi_l})\frac{\sqrt{kr}}{\sqrt{\varphi_l}}}{[(1+\sqrt{kr\varphi_l})^2-2]^2}.$$

Since $\frac{(\sqrt{2}-1)^2}{1+\tau}<kr\sigma^2<\frac{1}{1+\tau}$, we have $1-\sqrt{kr\varphi_l}>0$ and $(1+\sqrt{kr\varphi_l})^2-2>0$. Therefore

$$\frac{\partial \beta_l^*}{\partial \varphi_l}>0.$$

For $\frac{\partial V(e_{2l}^*,\varphi_l)}{\partial \varphi_l}$, we have

$$\begin{aligned}
\frac{\partial V(e_{2l}^*,\varphi_l)}{\partial \varphi_l} &= \frac{\partial \frac{\varphi_l\sqrt{k}(1-\sqrt{kr\varphi_l})}{c_1\sqrt{r\varphi_l}}}{\partial \varphi_l} \\
&= \frac{\frac{1}{2}c_1\sqrt{kr\varphi_l}-\frac{1}{2}c_1 kr(3-\varphi_l)}{c_1^2 r\varphi_l} \\
&= \frac{\frac{1}{2}c_1\sqrt{kr}\{\sqrt{\varphi l}-\sqrt{kr}(3-\varphi_l)\}}{c_1^2 r\varphi_l}.
\end{aligned}$$

When $\sqrt{\varphi_l}<\sqrt{kr}(3-\varphi_l)$, (that is, $\sqrt{(1-\tau)\sigma^2}<\sqrt{kr}[3-(1-\tau)\sigma^2]$), $\frac{\partial V(e_{2l}^*,\varphi_l)}{\partial \varphi_l}<0$, and $\Gamma_l\equiv$

$$\frac{\frac{\partial}{\partial(1-\tau)\sigma^2}\beta_l^*}{\frac{\partial V(e_{2l}^*,(1-\tau)\sigma^2)}{\partial(1-\tau)\sigma^2}} > 0.$$

Therefore, when the manager is able to influence the variance of the performance measure and has an option to mis-report the status of the exogenous variance,

$$\Gamma_h > 0 \text{ if } \sqrt{(1+\tau)\sigma^2} < \sqrt{kr}[3-(1+\tau)\sigma^2],$$

and

$$\Gamma_l > 0 \text{ if } \sqrt{(1-\tau)\sigma^2} < \sqrt{kr}[3-(1-\tau)\sigma^2].$$

.

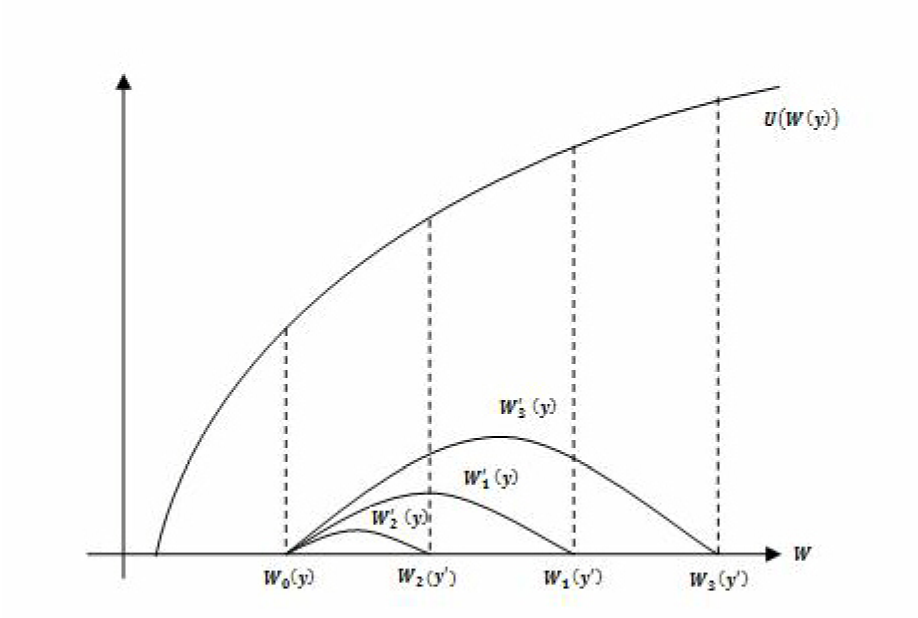

Figure 3: Incentive-Variance Relation for the general setting

- U(w(y)): Agent's utility function
- W_0(y): payment for the signal realization y for an initial level of σ^2
- W_1(y'): payment for the signal realization y' for an initial level of σ^2
- $W'_1(y) = W_1(y') - W_0(y)$
- W_2(y'): payment for the signal realization y' for a level of σ^2 slightly higher than the initial level of σ^2
- $W'_2(y) = W_2(y') - W_0(y)$
- W_3(y'): payment for the signal realization y' for a level of σ^2 slightly higher than the initial level of σ^2
- $W'_3(y) = W_3(y') - W_0(y)$