Accounting for Banks, Capital Regulation and Risk-Taking

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Accounting for Banks, Capital Regulation and Risk-Taking∗

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Abstract

This paper examines risk-taking incentives in banks under different accounting regimes with capital regulation. In the model the bank’s decisions of capital issuance and investment policy are jointly determined. Given exogenous minimum capital requirement, the bank is more likely to issue equity capital in excess of the minimum required level and implement less risky investment policy under either lower-of-cost-or-market accounting or fair value accounting than under historical cost accounting. But fair value accounting may induce more risk-taking compared to lower-of-cost-or-market accounting due to short term interest in the part of the bank. However, the disciplining role of lower-of-cost-or-market accounting may discourage bank’s incentive to exert project discovery effort ex-ante if the ex-ante effort plays an important role. From the regulator’s perspective, the optimal accounting choice will be governed by a tradeoff between the social cost of capital regulation and the efficiency of the bank’s project discovery efforts. When the former effect dominates, the regulator prefers lower-of-cost-or-market accounting; when the later effect dominates, the regulator may prefer other regimes.

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1 Introduction

The current banking crisis has raised much criticism of fair value accounting due to the mandatory adoption of SFAS 157 (Fair Value Measurement) in 2007 which resulted in large amounts of write-downs and recognition of credit losses in banks and financial institutions. This criticism has mainly focused on the unreliable value estimation for assets with illiquid markets and the systematic risk induced by excessive volatility under fair value accounting (Andrea et al., 2004; Landsman, 2005), and it has intensified during the current credit crunch.¹ Many financial institutions blame fair value accounting for aggravating the financial crisis at a time where markets are extremely illiquid and proper valuation models are unavailable; some even call on FASB to reassess the new fair value standard.² Advocates for fair value accounting, on the other hand, emphasize the benefits in terms of improved transparency and disclosure, promoting market discipline and providing relevant information for decision makers.³

Given the ongoing debate amid the financial crisis, it is crucial to have a better understanding of the desirability of different accounting regimes for banks so as to provide guidance for policymakers and regulators in the post-crisis regulatory reform. To that end, this paper examines whether different financial reporting standards for banks provide relevant information for the prudential regulation and discipline of banks. Specifically, in a theoretical model I examine how accounting regimes affect the effectiveness of capital regulation in restricting banks’ risk-taking behaviors.

Banks have incentives to engage in excessive risk-taking as a result of high leverage, as shown by Jensen and Meckling (1976). The incentives for risk-taking are greater when banks’ investment decisions are not observable or verifiable to outsiders. Due to the nature of deposit financing, depositors are typically dispersed and uninformed small investors with deposits insured by the government, therefore they lack both the capability and incentives

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¹Research on limitations and potential problems of market value accounting dates back to the early 1990s, for example, Berger et al., 1991; Shaffer, 1994; Robert, 1992; etc.
²See for example “Fair-value Accounting’s Atmosphere of Fear” (CFO.com, May.19, 2008), “Bankers cry foul over fair value accounting rules” (FT.com, March.13, 2008), etc.
³See for example, Barth, 1994; Bernard et al., 1995, Bies, 2004; Landsman, 2005; and most recently, Ryan, 2008b.
to monitor banks’ investment decisions.\textsuperscript{4} While debtholders in other industries may protect themselves through various instruments such as covenants and close monitoring, banks are subject to prudential regulation where the regulator serves as the representative of small investors (Dewatripont and Tirole, 1994). An important aspect of the current regulatory system is the explicit minimum capital requirement, which was introduced in the Basel Accords as part of the bank regulatory reform in the late 1980s in response to the Savings and Loans (S&L) crisis. By forcing banks to hold more capital, it is expected that risk-taking incentives can be reduced.\textsuperscript{5} The move toward market-value based accounting in banks and financial institutions has also been triggered by the S&L crisis, which in part was attributed to a lack of transparency under historical-cost based accounting (Benson et al., 1986; Kaufman, 1996). Consistent with the proposal’s recommendation, the use of current valuations among banks and financial institutions has increased over the past 20 years, with FASB’s issuance of a number of accounting standards related to fair value accounting.\textsuperscript{6} FASB is also advocating moving toward the comprehensive or full fair value accounting, in which all financial assets and liabilities are recorded at fair value on the balance sheet and changes in fair value recorded in earnings.

Whether or not capital requirement can effectively restrict the risk-taking depends crucially on the extent to which the measure of capital is accurate and informative. Therefore capital regulation confers an important role to accounting methods that largely determine how the net worth (capital) is measured. Three accounting regimes are analyzed in this paper: historical cost accounting (HC), lower-of-cost-or-market accounting (LCM), and fair

\textsuperscript{4} The deposit insurance is assumed as an inherent feature of the banking sector in this paper. Diamond and Dybvig (1983) model the bank’s function as a liquidity provider in the economy; thus rationalize the deposit insurance as an instrument to prevent bank runs. But as John, et al. (1991) point out, even though banks’ deposits are insured, the root of banks’ risk-taking incentives is not in the deposit insurance (whether or not the insurance premium is risk based); but rather attributable to the convexity of levered equity payoff resulting from limited liability.

\textsuperscript{5} The role of capital requirement to reduce risk-taking in banks is modeled in Keeley and Furlong, 1989 and 1990; Rochet, 1991; John et al., 1991. However, other papers such as Kim and Santomero (1988) and Koehn and Santomero (1980) argue that capital requirement can increase banks’ riskiness within a simple portfolio model in an incomplete market setting.

\textsuperscript{6} These standards include SFAS 107 (Disclosures about fair values of financial instruments), SFAS 114 (Accounting by creditors for impairment of a loan), SFAS 115 (Accounting for certain investments in debt and equity securities), SFAS 119 (Disclosures about derivatives), SFAS 133 (Accounting for derivative instruments and hedging activities), SFAS 140 (Accounting for transfers and servicing of financial assets and extinguishment of liabilities), SFAS 141 (Accounting and reporting for business combinations), SFAS 157 (Fair value measurements) and SFAS 159 (The fair value option for financial assets and financial liabilities).
value accounting (FV). I assume in the model that LCM and FV are equivalent when economic losses are realized; the only difference between these two arises when economic gains are realized.

The basic model in the paper follows John et al. (1991), capturing the key feature of banks’ risk-taking incentives in a simple framework. The bank chooses between a safe investment and a risky investment, where the risky investment opportunity only appears after the bank exerts certain effort ex-ante and the information about project risk is privately observable only to the bank. The bank also simultaneously decides the amount of equity capital to be issued along with the investment policy, and raises the rest of investment through deposits. I assume that the deposits are fully insured by government insurance agencies such as Federal Deposit Insurance Corporation (FDIC). The bank is somewhat myopic in that it maximizes a weighted average of the short term earnings recognized and the final expected payoff to shareholders, subject to the cost of capital regulation. Different accounting regimes determine the expected earnings to be recognized and the expected regulatory cost when the interim capital falls below the regulatory requirement.

I first consider the bank’s problem when the risky investment is always available. Under HC, no accounting information is revealed in the interim period and thus there is no risk of violating the minimum capital requirement ex post. Therefore the bank will not issue more equity than the minimum capital required and the investment policy will be more risky than the first best investment policy, the well known risk-shifting problem due to debt financing. Under LCM, the bank may incur a regulatory cost in face of loss realizations and hence is likely to issue equity capital in excess of the minimum requirement. The optimal investment policy is less risky under LCM than under HC. FV also helps restrict the bank’s risk-taking behavior; however, the interest in short term earnings makes the bank more risk-taking under

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7The practice of lower-of-cost-or-market accounting conforms to the conservatism principle; in the current accounting framework, it is not the same as ‘one-side’ fair value accounting, since for some assets the book value is written down only when the impairment can be proved to be “other than temporary”. SFAS 157 only provides more comprehensive guidance on fair value measurements to estimate fair value more rigorously without changing current framework in banks, which still has mixed features with some assets like bank loans recorded at historical based accounting subject to impairments and other assets such as held-for-sale securities recorded at lower-of-cost-or-market or full fair value.

8See Nissim and Penman (2008) for a detailed reference of the applications of fair value accounting in banks.
FV than under LCM as the upside gain is also recognized.

When the bank needs to exert effort in order to discover a risky investment opportunity, ITS ex-ante incentive to do so depends on the benefit from the risk-shifting under the accounting regime in place. LCM, which is most effective in controlling excessive risk-taking, also most severely discourages the ex-ante effort incentives.

From the regulator's perspective, he can always adjust the capital requirement to alter the bank's capital and investment policy decisions under different accounting regimes. Therefore the preference over different accounting regimes will depend on the social cost of such capital requirement (e.g., restricting the liquidity provision function of banks (Diamond and Rajan, 2000; Gorton and Worton, 1995)) and the trade-off between the ex-ante effort and ex-post risk taking incentives induced by adjusting the capital requirement. I find that when the bank's ex-ante effort is costless but the capital requirement bears non-negligible social cost, LCM is the most favorable regime while HC is the least favorable. However, if the cost of ex-ante effort is non-negligible but capital requirement bears no social cost, this ranking may change. In particular, when the bank is highly short term oriented, LCM may make its investment choice too conservative and thereby depresses ex-ante effort incentives. In this scenario, HC is preferred if the bank's cost of violating capital regulation is very high; otherwise FV may be the preferred regime. These results shed light on the recent debate about suitable accounting regimes for banks.

Another feature of the results in the model is that banks raise more capital and implements less risky investment policy when the accounting system is more fair value based (either lower-of-cost-or-market or full fair value accounting). This is consistent with the empirical evidence that banks started to hold more excess capital in 1990s during which the accounting regime moves toward a more market-value based system (Flannery and Rangan, 2008). In addition, previous studies on banks typically examine only one aspect of these two decisions, yet the result in this paper demonstrates that the regulator can influence

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9 Flannery and Rangan (2008) suggests one reason for the build-up of bank capital is the market discipline force, which is also related to the market-based accounting measurement.

10 For example, Keeley and Furlong (1989), Rochet (1991), and John et al. (1991) focus on the bank's risk choice taking the capital requirement as exogenous binding constraint; other studies such as Peura and Keppo (2006) focus on the optimal capital decision with capital regulation in the model with costly recapitalization but no risk-shifting incentive.
both the level of excess capital and investment risk by adjusting the capital requirement when accounting regime is not historical-based.

In a closely related paper, Strausz and Burkhart (2009) study the risk-shifting problem in banks but arrive at a different conclusion from this paper. They find that fair value accounting intensifies the risk-shifting problem in banks due to higher liquidity of banks’ assets as a result of less asymmetric information about the value of assets under fair value accounting. In addition, their setting does not distinguish between fair value accounting and lower-of-cost-or-market accounting, while my setting allows me to examine different risk-taking incentives under these two accounting regimes.

There are also several other studies on the implication of fair value accounting in financial institutions with more emphasis on the criticisms of fair value accounting. O’Hara (1993) examines the effect of market value accounting on the loan maturity in that market value accounting introduces a bias into the valuation of long-term illiquid assets and hence increases interest rates for long-term loans and induces a shift to short term loans. More recently, Allen and Carletti (2008) show that mark-to-market accounting can lead to contagion between a banking sector and an insurance sector when the bank’s illiquid assets are carried at the market value, while no contagion would occur under historical cost accounting. Plantin et al. (2008) focus on the problem of fire sales induced by the artificial volatility due to mark-to-market accounting. Another study by Bleck and Liu (2008) adds to the debate by showing that greater opacity in financial markets leads to more frequent and more severe crashes in asset prices under historical cost accounting. These studies have adopted mostly an ex-post approach in examining the impact of accounting system. However, Ryan (2008b) argues that although criticisms about fair value accounting are correct in some aspects, ‘the subprime crisis that gave rise to the credit crunch was primarily caused by firms, investors, and households making bad operating, investing, and financing decisions, and managing risks poorly’. This paper hence provides analytical support for Ryan (2008b)’s argument by focusing on the ex-ante role of accounting in banks’ decision making process, yet the results also show that moving to full fair value accounting may not be optimal for disciplining banks.

More broadly, this paper is also related to prior studies on alternative accounting regimes in other settings. Bachar et al. (1997) compare different accounting valuation approaches
in communicating information to investors in a setting with transaction costs and auditing costs. Kirschenheiter (1997) compares the historical cost and market value methods in the valuation of assets. Other papers compare accounting regimes in a contracting setting (Kirschenheiter, 1999) or in a hedge-accounting setting (Melumad et al., 1999 and Gigler et al., 2007). This paper contributes to the literature by comparing accounting regimes in banks and financial institutions and, in the course of doing so, supporting the role of accounting conservatism in financial reporting under certain conditions.

The rest of the paper proceeds as follows. Section 2 describes the basic model and assumptions. Section 3 analyzes the bank’s problem of choosing the investment policy and equity capital under different accounting regimes. Section 4 analyzes the regulator’s problem of choosing the level of capital requirement and the welfare effects of different accounting regimes. Finally Section 5 concludes the paper.

2 The model

The basic model is built on the risk shifting model developed in John et al. (1991) and John et al. (2000), which captures the key feature of the bank’s moral hazard problem in a simple framework. I first lay out the analytical framework in a general setting and then analyze and compare the bank’s behavior under different accounting regimes. While in John, et al. (1991) the bank’s capitalization decision is exogenous, the current model endogenizes both the equity issuance and risk choice of the bank.

2.1 The basic model setup

Consider a three-period model. In period \( t = 0 \), the bank may exert some effort \( a \) to discover a risky investment opportunity. Assume that \( a \in [0, 1] \) and the cost of effort to the bank is \( g(a) = \frac{1}{m} a^2 \). With probability \( a \), a risky investment opportunity appears at the beginning of period \( t = 1 \). A safe investment opportunity which generates zero NPV is always available to the bank at the beginning of period \( t = 1 \). This assumption is reasonable for the banking industry as banks can either purely function as an intermediary to provide liquidity and payment services to the depositors, or can be more active in making commercial loans and
other types of investments that can generate positive values to the economy as a whole. This latter type of investment require more expertise and effort from banks in screening and monitoring the borrowers.

The amount of investment required by either the safe or the risky investment is $I$. Following John et al. (1991), I assume that the risky investment generates either high or low cash flow (represented as $H$ or $L$) at the end of period $t = 2$, with $H > I > L$. The menu of risky investment opportunities is represented by $\bar{q}$, which is the probability of generating the high cash flow $H$. $\bar{q}$ is privately observed by the bank when the risky investment opportunity appears. However, ex-ante all parties know that $\bar{q}$ is uniformly distributed over the interval $[0,1]$ and the bank’s effort does not affect the distribution of $\bar{q}$.

At the beginning of $t = 1$, the bank will seek financing by issuing equity $K$ and collecting deposits of $D$ to a total amount of $I$. For simplicity, all deposits are assumed to be insured by the government in the case of default. Thus, the pricing of deposits does not incorporate the default risk of the bank. We can normalize the interest rate of deposits to zero, and the bank promises to pay $D$ at $t = 2$.

After the equity and deposits are raised, the bank may choose between the safe and risky projects if the risky investment opportunity appears; otherwise the bank can only invest in the safe project. At the end of $t = 2$, the terminal cash flow is realized, i.e, $I$ if the safe investment is chosen and $H$ or $L$ if the risky investment is chosen. The final realized cash flow is observable and verifiable. The bank will pay the full promised payment $D$ to depositors if realized cash flows are higher than $D$ and a partial payment of $L$ if realized cash flows are lower than $D$ (i.e., the bank defaults). In the case of default, the government insurance agency will pay depositors the remaining amount of $D - L$.

In the following discussion of this section and next section, we will ignore the bank’s effort incentive and focus on the risk-taking incentives by the bank. That is, assume for now that the risky investment opportunity always appears at the beginning of $t = 1$.

**Definition 1** An investment policy indexed by $q$ is defined as follows: for a given cutoff value of $q$, the bank will choose the risky investment for $\bar{q} \geq q$ and the safe investment for $\bar{q} < q$. 

7
Given that $\tilde{q}$ is uniformly distributed over $[0,1]$, an investment policy $q$ produces the following terminal cash flow distribution: $H$ with a probability $\frac{1}{2}(1-q^2)$, $I$ with a probability $q$, and $L$ with a probability $\frac{1}{2}(1-q)^2$. The total expected value of terminal cash flows for an investment policy $q$ is thus given by:

$$V(q) = qI + \frac{(1-q)^2}{2}L + \frac{1-q^2}{2}H$$

(1)

**First-best investment policy:** The first best investment policy ($q^{fb}$), which maximizes $V(q)$ above, is:

$$q^{fb} = \frac{I-L}{H-L}$$

(2)

The first-best investment policy ($q^{fb}$) can be implemented if the bank is financed entirely by equity so that the bank maximizes the firm value, or if the information about $\tilde{q}$ is perfectly observed by all parties. Thus $q^{fb}$ is a benchmark for comparing the investment incentives with deposit financing under the alternative accounting regimes.

If the bank finances the investment by issuing both equity and deposits, then deposit financing will induce excessive risk-taking. Suppose now the bank simultaneously chooses the level of riskiness of its investment policy ($q$) and the equity issued ($K$), and raises the remaining amount of the investment by insured deposits ($D$). Then the expected future payoff to shareholders is represented by:

$$\pi(q,K) = q(I-D) + \frac{1-q^2}{2}(H-D) - K$$

(3)

The bank’s joint choice of equity issuance and investment policy so as to maximize the shareholders’ payoff is summarized in Lemma 1:

**Lemma 1** **Risk-shifting incentives:** In the absence of capital regulation, the bank will raise the full investment by deposits and implement the most risky investment policy, i.e., $K^d = 0$ and $q^d = 0$.

In the absence of any regulatory constraint, the bank will always invest in the risky project and not issue any equity capital to finance the investment. It should be noted that even if, in contrast to my model’s assumptions, deposit insurance were fairly priced, the
risk shifting problem could not be reduced (See Appendix A for the analysis with the fairly priced deposit insurance premium). The reason is that the insurance premium could only reflect the anticipated riskiness of the investment, as the actual realization of $\tilde{q}$ is privately observed by the bank. The insurance premium only adds a lump sum to the payoff once the equity issued ($K$) is chosen. Excessive risk-taking by banks increases the default probability and hence the likelihood of bank failures, which may result in an industry-wide crisis when most banks choose risky investments for individual profit-maximization objectives.

2.2 Information and accounting regimes

Assume that the bank’s private information about $\tilde{q}$ is not verifiable or contractible. However, the bank has an information system in place that generates signals about the terminal cash flows for the risky investment at the end of the period $t = 1$. The signal can be either good ($G$) or bad ($B$). When the safe investment is chosen, no signal is generated by the information system. The following conditional probabilities represent the properties of the information system:

$$
P(G \mid H) = \alpha
$$

$$
P(B \mid L) = \beta
$$

$\alpha \in [\frac{1}{2}, 1]$ and $\beta \in [\frac{1}{2}, 1]$

When $\alpha = 1$ and $\beta = 1$, the information system generates perfect signals about the terminal cash flows. Given the investment policy of $q$, the probabilities of generating good and bad signals can be derived as:

$$
P(G) = \alpha \frac{1 - q^2}{2} + (1 - \beta) \frac{(1 - q)^2}{2}
$$

$$
P(B) = (1 - \alpha) \frac{1 - q^2}{2} + \beta \frac{(1 - q)^2}{2}
$$

Let $E[V(q) \mid G]$ and $E[V(q) \mid B]$ denote the expected future cash flows conditional on the signals:
\begin{align*}
E[V(q) \mid G] &= \frac{\alpha(1-q^2)H + (1-\beta)(1-q)^2L}{\alpha(1-q^2) + (1-\beta)(1-q)^2} \\
E[V(q) \mid B] &= \frac{(1-\alpha)(1-q^2)H + \beta(1-q)^2L}{(1-q^2)(1-\alpha) + \beta(1-q)^2} 
\end{align*}

I also assume that the properties of the information system satisfy the following condition:

\[
\frac{1-\alpha}{\beta} < \frac{I-L}{H+I-2L} \tag{7}
\]

Overall when the information quality (as measured by \(\alpha\) and \(\beta\)) increases, this condition is easier to be satisfied. Given the condition in (7), it can be shown that if \(q \leq q^{fb}\), the following conditions always hold:

\[
E[V(q) \mid G] > I \\
E[V(q) \mid B] < I
\]

Hence if the bank takes excessive risk, the expected future payoff to the investment given a bad (good) signal represents a loss (gain).

**Accounting regimes** Three accounting regimes are considered in this paper: historical cost accounting, lower-of-cost-or-market accounting, and fair value accounting. Accounting earnings, indicated by \(e_j\), \(j \in h,l,f\), to be recognized at the end of \(t = 1\) under different accounting regimes are as follows:

**Historical cost accounting.** No accounting earnings are recognized, i.e:

\[
e_h = 0
\]

**Lower-of-cost-or-market accounting.** LCM is a form of conservative accounting. The common practice is to write down the book value of the asset to its current market value when the market value falls below the historical book value. In this model, I assume that the current market value equals to the expected future cash flows at the end of \(t = 1\) conditional on the signals observed. The book value is initially carried at \(I\) at the beginning of \(t = 1\).
When the bad signal is generated, the market value $E[V(q) \mid B]$ is lower than the book value $I$ and the bank needs to recognize negative earnings. When the good signal is generated, or no signal is observed, the bank recognizes no earnings. Therefore, accounting earnings under LCM are:

$$
e_t = \begin{cases} 
0 & \text{if no (or good) signal is generated} \\
e^B = E[V(q) \mid B] - I < 0 & \text{if bad signal is generated}
\end{cases}
$$

*Fair value accounting.* Under FV, the bank has to recognize both the accounting gain (for a good signal) and the accounting loss (for a bad signal):

$$
e_f = \begin{cases} 
0 & \text{if no signal is generated} \\
e^G = E[V(q) \mid G] - I > 0 & \text{if good signal is generated} \\
e^B = E[V(q) \mid B] - I < 0 & \text{if bad signal is generated}
\end{cases}
$$

### 2.3 Bank capital and regulation

Besides the investment choice, the bank also endogenously chooses the level of equity capital. I assume that the bank can only issue equity at the beginning of the investment period and the bank’s equity balance is subject to changes due to accounting earnings recognized under different accounting regimes. It is reasonable to assume that the new equity issuance is allowed only at the beginning of the investment period in this setting, as the bank faces the investment opportunity only at the beginning of $t = 1$.

**Capital regulation:** From the preceding discussion, it is apparent that without capital requirements the bank will prefer to hold no capital at all. An important element of the current capital regulation is the minimum capital adequacy ratio that the bank needs to meet continually. I model the role of this regulatory constraint as follows:

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11 However, it is possible that with capital regulation, the bank may need to raise additional equity from the market when its capital falls below the regulatory requirement at the interim stage. This possibility is not directly modeled here. But since the equity issuance in time of breaching capital requirement is often more costly, thereby the equity issuance can be viewed as part of the regulatory cost that the bank may need to incur in breach of regulatory requirements, as modeled in the regulatory cost function.

12 The 1988 Basel Accord (Basel I) requires two levels of minimum capital requirements for banks: minimum Tier 1 capital is set at 4% of risk-weighted assets and minimum Tier 2 capital is set at 8% of risk weighted assets. Banks with at least 5% Tier 1 and 10% Tier 2 capital are considered to be ‘well-capitalized’. Basel I was replaced by Basel II in 2004. Basel II better aligns the regulatory capital requirements with ‘economic
1. At $t = 0$, the initial equity issued has to strictly satisfy the capital requirement, which is to hold a minimum capital of $k$ per unit of deposits.

2. At $t = 1$, the bank violates the minimum capital requirement if the new equity balance after recognizing accounting earnings falls below the requirement. The expected regulatory cost of violation is given by the function $C(u_j(k))$, where $u_j(k), \ j \in \{h, f, l\}$, denotes the amount of inadequate capital under respective accounting regimes. If the bank issues equity of $K$ at $t = 0$, then the capital balance at $t = 1$ will be $K + e_j$. The total amount of inadequate capital $u_j$ at $t = 1$ can be represented as:

$$u_j(k) = \text{Max}\{0, kD - K - e_j\}, \ j \in \{h, l, f\} \quad (8)$$

I assume that the cost function is convex, i.e., $C' \geq 0$ and $C'' > 0$, with $C'(0) = 0$. Specifically, this cost can be viewed as the penalty levied by the regulator when the bank violates the capital requirement. The cost function is assumed to be exogenous, and the regulator can only choose the level of capital requirement ($k$) to affect the regulatory cost incurred by the bank. Empirical studies have documented that many banks hold capital above the minimum regulatory requirement, which is consistent with the model in this paper where the level of buffer capital (i.e., the amount of capital in excess of the regulatory requirement) will be endogenously determined.\(^\text{13}\)

To summarize the model setup, Figure 1 illustrates the timeline of events.

3 The bank’s problem without the effort incentive

Now I consider the bank’s problem at the beginning of $t = 1$ when the risky investment opportunity is always available, given the exogenous capital requirement. Assume that the capital demanded by investors, which allows the use of internal ratings based (IRB) approach of choosing regulatory capital.\(^\text{12}\) The capital regulation modeled in this paper is consistent with ex-ante regulation approach in bank capital regulation. Basel I and Pillar I of Basel II are examples of ex ante capital constraint, which imposes a fixed ratio of the minimum capital requirement. However, Pillar II of Basel II introduces some elements of ex-post regulation, in which the bank has the freedom to choose capital and portfolio risk. This paper does not attempt to model the feature under the new Basel Accord, but it is a possible future direction for research. See Giammarino et al. (1993) and Kupiec and O'Brien (1997) for more details about the ex post regulation approach.
bank is interested not only in the long run expected payoff to shareholders, but also in the short term earnings reported under the prevailing accounting system. This assumption is in line with the managerial myopia literature which typically assumes that firms give some weight to short term earnings in addition to the long-term fundamental value. I assume that the bank will assign some weight, \( \gamma > 0 \), on the earnings reported in the interim period. The expected regulatory cost at the end of \( t = 1 \) is fully internalized by the bank. Accounting earnings play a dual role in the model: first, it determines the ex-post cost of violating capital regulation; second, it directly affects the bank’s incentive due to its short term orientation.

The bank chooses the equity issuance and investment policy to maximize the following objective function under each regime, subject to the regulatory constraint of equity capital at the beginning of \( t = 1 \):

\[
\max_{q,K} \Pi_j(q,K) = \gamma E[e_j] + (1-\gamma)\pi(q,K) - E[C(u_j)], \quad j \in \{h,l,f\} \\
\text{s.t.} \\
D + K = I \\
K \geq \hat{K} 
\]

Here \( \hat{K} = \frac{k}{k+1}I \) represents the minimum equity capital that satisfies the regulatory re-

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14 See for example Stein, 1989; Narayanan, 1985.
quirement at the beginning of $t = 1$.

For any given $k$, denote the optimal solution to the bank’s problem above as $(q^*_j(k), K^*_j(k))$, with $j \in \{h, l, f\}$ indicating different accounting regimes.

### 3.1 Historical cost accounting

Consider first HC regime. Under HC, no accounting earnings are recognized in the interim period. Therefore the expected regulatory cost is $E[C(u_h)] = 0$ if the initial equity satisfies the minimum capital requirement constraint. Solving the bank’s problem under HC gives the following proposition:

**Proposition 1** Under HC, the bank’s optimal investment policy $(q^*_h(k))$ and equity issue $(K^*_h(k))$ are given by:

$$
q^*_h(k) = \frac{K^*_h(k)}{H - I + K^*_h(k)}
$$

$$
K^*_h(k) = K^*_0(k)
$$

As long as the minimum capital requirement is greater than zero, the bank’s investment policy under HC is less risky than in Lemma 1 as the bank now invests partially in the safe project as a result of the minimum capital requirement. As the minimum capital requirement $(k)$ increases, the bank is forced to raise more capital and will therefore also reduces the riskiness of its investment policy. Another implication of Proposition 1 is that, with HC, the bank will issue no more equity than the minimum required level and finance the rest of investment by deposits. In sum, the result is consistent with the prior literature that prudential regulation of banks through minimum capital requirements can reduce the excessive risk-taking by forcing the bank to share the investment’s riskiness to some extent. However, HC fails to capture any new information about the investment’s future cash flows and hence accounting information does not affect the effectiveness of capital requirement in controlling the risk-taking by the bank.
3.2 Lower-of-cost-or-market accounting

LCM constitutes a move toward forward-looking, market based accounting. It requires the write-down of assets when the asset value is impaired or when its market value falls below the book value. This is consistent with the general conservatism principle in GAAP and other accounting standards. Overall, LCM provides more information about the bank’s economic activities, especially when the expected future economic conditions deteriorate.

In the model, the accounting system reports a loss of $e^B$ when the bad signal is generated and zero when either the good signal is generated or no signal is generated at all. Therefore the expected earnings to be recognized at $t = 1$ are:

$$E[e_i] = P(B)e^B$$

And the expected regulatory cost is given by:

$$E[C(u_t)] = P(B)C(u_t)$$

Ignoring for now the constraint for the minimum equity capital requirement at the beginning of $t = 1$, the bank’s optimal choices of equity capital and the investment policy are determined by the first order conditions to the objective function in (9).

Denote the solution to this relaxed maximization problem under LCM as $(\hat{q}_l, \hat{K}_l)$, which is given by the solution to the maximization problem in (9) without the constraint in (11). The following system of equations represents the solution:

$$\left\{ \begin{array}{l}
\frac{\partial}{\partial q}\Pi_t(q,K)|_{\hat{q}_l,\hat{K}_l} = 0 \\
\frac{\partial}{\partial K}\Pi_t(q,K)|_{\hat{q}_l,\hat{K}_l} = 0
\end{array} \right.$$  

However, the relaxed optimal equity capital $(\hat{K}_l)$ is not always feasible as it may be lower than the minimum capital requirement. Before presenting the complete solution to

\footnote{I show in the proof of Proposition 2 in the Appendix that the second order condition is met given the assumption of sufficiently large $C''$.}
the bank’s problem under LCM, I also define the minimum capital investment policy as the bank’s optimal risk choice conditional on the initial equity level being equal to the minimum capital required:

**Definition 2** A minimum capital investment policy \( (\hat{q}_j) \) is defined as below:

\[
\hat{q}_j \in \max_q \Pi_j(q, \hat{K}), \text{ where } \hat{K} = \frac{k}{k+1} I, \ j \in \{h,l,f\}
\]

One can easily see that the optimal investment policy under HC coincides with the minimum capital investment policy, \( q^*_h = \hat{q}_h = \frac{K}{H-I+K} \). Under LCM, the minimum capital investment policy \( (\hat{q}_l) \) can be derived from the first-order condition of the maximization problem of \( \Pi_l(q, \hat{K}) \), shown as follows:

\[
\hat{q}_l = \frac{(1-\gamma)\hat{K} - \frac{\partial P(B)}{\partial q} C(-e^B) + P(B)C'(-e^B) \frac{\partial e^B}{\partial q} + \gamma \beta (I-L)}{H-I + (1-\gamma)\hat{K} - \gamma \alpha (H-I) + \gamma \beta (I-L)}
\]  

The following lemma compares the minimum capital investment policies under HC and lower-of-cost-or-market accounting.

**Lemma 2** The minimum capital investment policy under historical cost accounting is more risky than under LCM, i.e., \( \hat{q}_h < \hat{q}_l \).

**Proof.** See Appendix. ■

Lemma 2 suggests LCM may alleviate the excessive risk-taking problem. Yet this result is preliminary because it exogenously imposes identical capital structures under different accounting regimes. The bank’s risk-taking incentive is potentially mitigated by the fact that increasing risk also increases the bank’s expected regulatory cost, since the future equity capital may be reduced through the loss recognition under LCM. However, implementing the minimum capital investment policy may not necessarily be optimal for the bank, given that the bank also has the option to raise more equity ex-ante to reduce the expected regulatory cost. The marginal impact of raising additional equity at the minimum capital level can be represented by the following equation:
\[
\frac{\partial}{\partial K} \Pi_l(q, K)\bigg|_{\hat{q}_l, \hat{K}} = -(1 - \gamma) \frac{(1 - \hat{q}_l)^2}{2} + P(B)C'(-e^B(\hat{q}_l)) \tag{16}
\]

The bank trades off the marginal benefit and cost of increasing equity. On the one hand, increasing equity capital reduces the expected future regulatory cost of violating the capital requirement; on the other hand, it reduces the bank’s benefit from risk shifting. Therefore one of two scenarios may obtain:

- **Case I:** \[C'(-e^B(\hat{q}_l)) \leq (1 - \gamma) \frac{(1 - \hat{q}_l)^2}{2P(B)}\]

- **Case II:** \[C'(-e^B(\hat{q}_l)) > (1 - \gamma) \frac{(1 - \hat{q}_l)^2}{2P(B)}\]

When the bank’s marginal regulatory cost \((C')\) is high enough, Case II is more likely to obtain. Given these two different cases, the bank’s optimal decisions under LCM are characterized in the following proposition:

**Proposition 2** Under LCM, the bank’s optimal investment choice \((q^*_l)\) and equity issue \((K^*_l)\) are given by:

- **Case I:** \(K^*_l = \hat{K}\) and \(q^*_l = \hat{q}_l\)

- **Case II:** \(K^*_l = \hat{K}_l\) and \(q^*_l = \hat{q}_l\)

**Proof.** See Appendix. ■

To better understand the intuition behind Proposition 2, note that the optimal investment policy \((q^*_l)\) always satisfies the first order condition \(\frac{\partial}{\partial q_l} \Pi_l(q^*_l, K) = 0\) for any level of equity capital \(K\). However, the optimal equity issuance decision at \(t = 0\) involves a tradeoff between the marginal benefit and cost of increasing equity. Only in Case II, when the expected marginal regulatory cost is larger than the benefit, the bank will have an incentive to increase its equity capital to the relaxed optimal level (above the minimum capital requirement). Hence one would expect to find banks holding excess capital under LCM, consistent with the empirical evidence that banks started to hold more excess capital in 1990s when the accounting regime is moving toward a more market-value based system (Flannery and Rangan, 2008).
Another observation is that it is never optimal for the bank to issue more capital than 
\( kD - e^B \), which is the capital level that fully insures the bank against incurring any regulatory 
cost. This can be shown following the fact that \( \frac{\partial}{\partial K} \Pi_l(q, kD - e^B) < 0 \). Given the assumption 
that \( C'(0) = 0 \), lowering \( K \) slightly, starting from \( K = kD - e^B \), only comes at a second-order 
loss in terms of expected regulatory costs, while yielding a first-order gain in terms of risk 
shifting benefits. Thus, instead of holding the capital too safe, the bank will always prefer 
being exposed to some degree of future regulatory cost.

In terms of the investment policy, Corollary 1 gives the complete comparison of the 
investment policies under these two accounting regimes:

**Corollary 1** The bank’s investment policy is always less risky under LCM than under HC, 
i.e., \( q^*_l > q^*_h \)

**Proof.** See Appendix. ■

Aside from the minimum capital requirement, the short term interest in earnings also 
plays a discipline role in reducing the risk-taking incentive of the bank under LCM. When 
the bank puts more attention on the interim earnings reported (i.e., \( \gamma \) increases), the optimal 
investment policy under LCM will be less-risky, the bank will be more likely to issue equity 
in excess of the minimum requirement, and the level of buffer capital will increase further.

### 3.3 Fair value accounting

FV is a forward looking accounting regime that requires the recognized asset value to incor-
porate current information about future cash flows in a fully symmetric fashion. It requires 
the recognition of both unrealized gains and losses consistently.\(^{16}\) In the context of this 
model, FV is identical to LCM when there is bad news about future expected cash flows. 
The only difference between these two regimes arises when there is good news about future 
cash flows.

\(^{16}\)SFAS 157 provides an extensive practical guidance regarding how to measure fair values, however, it does 
not require fair value accounting for any position (Ryan, 2008a). SFAS 159 offers the fair value option to 
measure certain financial assets and liabilities at fair value, with changes in fair value recognized in current 
earnings.
In this model, due to the binary nature, the bank has the same expected regulatory cost under both LCM and FV, given in (13). However, the expected earnings recognized under FV will be higher:

\[
E[e_f] = P(B)e^B + P(G)e^G = \frac{1-q^2}{2}(H-I) + \frac{(1-\hat{q})^2}{2}(L-I) \tag{17}
\]

This transparency property is the main advantage of fair value accounting. Note in particular that the properties of the accounting system do not affect the expected earnings to be recognized under FV. Since FV provides full recognition of both gains and losses symmetrically, there is no distortion in the recognized earnings with respect to the expected future cash flows.

Given (13) and (17), the bank’s problem can be solved under FV. Following a similar procedure as under LCM, I first characterize the relaxed solution to the objective function, ignoring the constraint in (11). The following system of equations summarizes the solution:

\[
\begin{cases}
\frac{\partial}{\partial q}\Pi_f(q, \hat{K}_f) = 0 \\
\frac{\partial}{\partial \hat{K}}\Pi_f(q, \hat{K}_f) = 0
\end{cases} \tag{18}
\]

Again, the relaxed optimal equity capital ($\hat{K}_f$) is not always feasible as it may be lower than the minimum capital required. Therefore we also need to consider the minimum capital requirement policy under FV, $\hat{q}_f$, which is derived from the first order condition of the maximization problem of $\Pi_f(q, \hat{K})$, shown as follows:

\[
\hat{q}_f = (1-\gamma)\hat{K} - \frac{\partial P(B)}{\partial q}C(-e^B) + P(B)C'(-e^B)\frac{\partial e^B}{\partial q} + \gamma(I-L) \frac{H-I + (1-\gamma)\hat{K} + \gamma(I-L)}{H-I + (1-\gamma)\hat{K} + \gamma(I-L)} \tag{19}
\]

The following lemma compares the minimum capital investment policy under FV with those under the other two accounting regimes.

**Lemma 3** The bank’s minimum capital investment policy under FV is less risky than under
HC, but more risky than under LCM, i.e., \( q_h < \hat{q}_f < \hat{q}_l \).

**Proof.** See Appendix. ■

Lemma 3 suggests that although the bank’s minimum capital investment policy is less risky under FV than under HC, it is more risky than under LCM. Now we can finalize the optimal decisions of the bank \((q^*_f, K^*_f)\) given the above relaxed optimal solution and the minimum investment policy under FV. The bank’s choice of optimal equity capital also depends on two different scenarios under FV, which are parallel to Case I and Case II under LCM:

- **Case I´:** \( C'(-e^B(\hat{q}_f)) \leq (1 - \gamma)(1 - \hat{q}_f)^2 \frac{2P(B)}{2P(B)} \)
- **Case II´:** \( C'(-e^B(\hat{q}_f)) > (1 - \gamma)(1 - \hat{q}_f)^2 \frac{2P(B)}{2P(B)} \)

The following Proposition 3 characterizes the bank’s optimal decisions under FV:

**Proposition 3** Under FV, the bank’s optimal investment policy \((q^*_f)\) and equity issuance \((K^*_f)\) are given by:

- **Case I´:** \( K^*_f = \hat{K} \) and \( q^*_f = \hat{q}_f \)
- **Case II´:** \( K^*_f = \bar{K}_f \) and \( q^*_f = \bar{q}_f \)

**Proof.** Similar to the proof of Proposition 2 and hence omitted. ■

Under FV the bank is also likely to hold capital in excess of the minimum requirement level when the expected marginal regulatory cost is high enough. Moreover, we can also derive a conclusion similar to Corollary 1 that under FV the bank’s investment policy is no more risky than its minimum capital investment policy, i.e., \( q^*_f \geq q_h^* \). Therefore FV always induces less risky investment than HC, i.e.:

\[
q^*_f \geq q^*_h
\]  

(20)

Now we focus on a comparison of the optimal decisions under FV and LCM as shown in Corollary 2.
Corollary 2  Under FV the bank’s investment policy is more risky than under LCM, i.e., $q^*_f < q^*_l$; Moreover, the bank is less likely to issue capital in excess of the minimum requirement and the level of capital issued is also lower, i.e., $K^*_f \leq K^*_l$

Proof. See Appendix.

Therefore, overall FV is less effective in controlling the bank’s risk-taking behavior than LCM. The bank’s concern about the regulatory cost is identical under these two accounting regimes, but the short term interest in earnings makes the bank more aggressive under FV as the upside gain recognized adds to take on more risk. In terms of the equity issuance, the result in Corollary 2 means that when the bank’s marginal regulatory cost is high enough so that the optimal decision is to issue equity in excess of the minimum requirement under LCM, the same level of marginal regulatory cost will also drive the issuance of equity in excess of the minimum requirement under FV. The opposite, however, does not hold. Therefore the likelihood of observing excess capital is larger under LCM.

4 The ex-ante incentive of effort

The previous section assumes that the bank always faces a risky investment opportunity and then chooses optimal equity and investment decisions at the beginning of period $t = 1$ under different accounting regimes. However, access to the risky investment opportunity is not guaranteed, but depends on the effort, $a$, exerted by the bank in period $t = 0$. In this section, I consider the bank’s problem of choosing ex-ante effort to discover a risky investment opportunity. The risky investment opportunity itself is desirable, as the bank can only generate positive NPV through investing in the risky project. But under different accounting regimes, the bank’s effort incentive will depend on their anticipated benefit from risk-shifting. Therefore the regulator needs to balance the effectiveness of controlling the bank risk-taking behavior and the incentive to motivate the bank to exert effort ex-ante when comparing across different accounting regimes.

Consider the bank’s problem of effort choice under a given accounting regime. The bank will choose between the safe and risky investments as before, if a risky investment is available at $t = 0$; otherwise, the bank can only invest in the safe investment.
When the bank’s effort and investment choices are not contractible, he chooses the effort level \((a_j^*)\) in the period \(t = 0\) and equity issuance and investment decisions \((q_j^*(k), K_j^*(k))\) at the beginning of period \(t = 1\) to maximize his own utility under each accounting regime.

Denote the bank’s problem given in (9)-(11) in the preceding section as \(P_j^{t1}(q, K)\). Still assuming the minimum capital requirement is exogenously set by the regulator, the bank’s problem at \(t = 0\) now becomes:

\[
\max_a \ a \cdot \Pi_j(q_j^*(k), K_j^*(k), k) - g(a) \\
\text{s.t.} \quad q_j^*(k), K_j^*(k) \in \arg \max_{q, K} P_j^{t1}(q, K|k) \tag{21}
\]

Where \(g(a) = \frac{1}{m} a^2\).

The bank’s period-1 subproblem under each accounting regime is the same as in Section 3. Hence the effort level is determined as follows:

\[
a_j^*(k) = \frac{m}{2} \Pi_j(q_j^*(k), K_j^*(k)) \tag{22}
\]

The bank’s effort increases with his expected payoff from the risk project, taking as given his capital and investment policy decisions. These decisions are made assuming the same level of exogenous capital requirement under different accounting regimes. LCM, which is most effective in disciplining the bank’s risk-taking incentive, also discourages the most the bank’s incentive to exert effort ex-ante in discovering a risky project.

Before discussing the regulator’s problem in detail in the next section, first consider the following benchmark case where the bank’s marginal cost of effort goes to zero, i.e., \(g'(a) \to 0, \forall a \in [0, 1]\). In addition, the regulator can adjust the capital requirement without any cost.

Then the bank will always exert the maximum effort \(a = 1\) at the beginning of period \(t = 0\). The following corollary then holds:

**Corollary 3** When the capital regulation and the bank’s effort are both costless, there exists
an optimal minimum capital requirement under each accounting regime, $\bar{k}_j$:

$$\bar{k}_l < \bar{k}_f < \bar{k}_h = \frac{I - L}{L}$$

such that the first best investment policy is always chosen by the bank:

$$V(q^*_l(\bar{k}_l)) = V(q^*_f(\bar{k}_f)) = V(q^*_h(\bar{k}_h)) = V(q^{fb})$$

**Proof.** The proof follows by setting the optimal investment policy $q^*_j(k)$ under each regime equal to $q^{fb}$. ■

In this scenario the regulator finds himself indifferent choosing among three accounting regimes. Under HC, when the capital requirement is $\bar{k}_h$, the bank issues only safe deposits as $D = L$.

Under the other two accounting regimes, the requirement for issuing safe deposits cannot induce the first best investment policy, as the bank is also subjected to the regulatory cost. The setting in Corollary 3 is clearly unrealistic but serves as a useful benchmark for the following analysis.

## 5 The regulator’s problem

In the previous analysis the capital requirement was assumed exogenous and the bank’s optimal decisions under different accounting regimes were compared for some given minimum capital requirement. In this section I allow the regulator to adjust the capital requirement optimally under each accounting regime to maximize its own objective function, which is to maximize social welfare.

The regulator now needs to take into consideration both the ex-ante effort and ex-post risk-taking incentives of banks. Notice that the cost to the insurance agency in default is offset by the benefit to shareholders regardless of the insurance premium scheme, and the regulatory cost is a wealth transfer between the regulator and the bank’s shareholders.

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17 In fact for any capital requirement above this level, the bank’s investment policy will also achieve the first best level under HC as the bank internalizes the default risk when only safe deposits are issued.

18 Note that if we assume the regulatory cost comes from the costly new equity issuance, which represents a deadweight loss, the objective function of the regulator will change accordingly. In this case, the assumption...
Therefore the regulator maximizes the expected investment return when choosing the optimal capital regulation under different accounting regimes. While in the benchmark case of Corollary 3 I have assumed that such capital requirement adjustment is costless, in general there will be an associated deadweight cost, e.g., due to the restriction of liquidity creation provided by the bank to investors through deposits. \footnote{Banks’ function as liquidity provider has been extensively studied in the literature following Diamond and Dybvig (1983). For example, Diamond and Rajan (2000) study the consequences of regulatory capital requirements in trading off credit and liquidity creation functions with the possibility of financial distress. Gorton and Winton (1995) also show in a general equilibrium framework to that bank capital is costly because of the restriction on the liquidity provision. Other types of costs associated with capital regulation involve the supervision and compliance costs in general. In a recent study, Van den Heuvel (2008) quantifies the social welfare cost of capital requirements as the percentage of consumption by comparing the benefit of limiting the moral hazard problem and the cost of reducing liquidity creation.}

Rather than adopting a full-fledged the general equilibrium framework, I will employ a reduced form of the regulator’s objective function to capture both the cost and benefit of imposing capital requirements in a parsimonious fashion. Assume that the economic benefit from liquidity provision can be expressed by some function $L(k)$ with $L'(k) < 0$, $L''(k) < 0$ and $L(0) = 0$. The regulator’s objective THEN is to maximize the following social welfare function under each accounting regime:

\[
\max_k W_j(k) = a_j^*(k)V(q_j^*(k)) - g(a_j^*(k)) + L(k)
\]
\[
\text{s.t. } a_j^*(k) \in \arg \max_a a\Pi_j(q_j^*(k), K_j^*(k), k) - g(a)
\]
\[
q_j^*(k), K_j^*(k) \in \arg \max_{q,K} \Pi_j(q, K|k)
\]  \hspace{1cm} (23)

Denote the solution to the above problem under accounting regime $j$ as $k_j^*$. In general the optimal capital requirement to the regulator’s problem will be lower than the level that induces the first best investment policy as in Corollary 3 respectively under each regime. This is because slightly lowering the capital requirement at this level has a positive first order effect on both the ex-ante effort and the liquidity provision benefit while the marginal effect on the investment policy is of second order. A comparison of the overall effect on the total welfare is intractable when the regulator chooses the optimal capital requirement under each regime to maximize the objective function as in (23). Therefore in the following
discussion I focus on two specific cases which allow us to compare the regulator’s preference over different accounting regimes with endogenous capital regulation.

First consider the case when the marginal cost of effort goes to zero, \( g'(a) \to 0 \), but the capital requirement itself is costly, \( L'(k) < 0 \). In this case the bank always exerts the maximum effort level ex-ante to discover the risky investment opportunity. The regulator is only concerned with the cost of capital regulation that limits the bank’s ability to accept more deposits and provide liquidity. The following proposition compares social welfare under different accounting regimes when the regulator optimally chooses the capital requirement accordingly:

**Proposition 4** Social welfare at the optimal capital requirement level is the highest under LCM, and the lowest under HC:

\[
W_l(k_l^*) > W_f(k_f^*) > W_h(k_h^*)
\]

**Proof.** See Appendix. ■

Social welfare at the respective optimal capital requirement levels will be the highest under LCM, and the lowest under HC. The conservative bias under LCM reduces the risk-taking incentives by banks, thereby allowing the regulator to set more lenient capital requirements. This in turn improves social welfare in the presence of opportunity costs of imposing capital requirements.

The next case I consider is when the marginal cost of effort is non-negligible, i.e. \( g'(a) > 0 \), but the capital requirement is not socially costly, \( L'(k) \to 0 \) for any \( k \). The regulator now takes into consideration both the bank’s ex-ante and ex-post incentives under each accounting regime. Given the same capital requirement, the bank’s effort level under LCM is lower than the other two. However, the regulator may further lower the capital requirement under LCM to induce higher effort and thereby the comparison of welfare is not clear in this case. In the first case the regulator always prefer LCM when the capital regulation is costly, therefore it is more interesting to look for a scenario when the regulator may prefer other accounting regimes. Proposition 5 below characterizes such a special case:

**Proposition 5** When the bank’s short term interest is high (\( \gamma \to 1 \)), HC achieves the highest
welfare if the bank’s cost of violating capital regulation is very high; otherwise FV achieves the highest welfare.

Proof. See Appendix.

Proposition 5 shows that the regulator may prefer HC or FV when the capital regulation is costless but costly project discovery effort on the part of banks is an issue. This is in contrast to the previous case where the regulator always prefers the most conservative accounting regime when ex-ante effort incentive is not important but capital regulation cost is non-negligible. When the ex-ante effort plays an important role in discovering the bank’s investment opportunity, conservative accounting will discourage such effort and therefore reduce the overall efficiency.

6 Conclusion

This paper examines banks’ risk-taking incentives in the presence of minimum capital regulation under three different accounting regimes: HC, LCM and FV. LCM, which requires banks to recognize economic losses earlier when information becomes known to the market, is shown to be more effective than the other two regimes in controlling risk-taking behaviors by banks. Moreover, banks are more likely to hold buffer capital to avoid future costly violation of capital regulation when the accounting system incorporates more market-based information. Compared to LCM, FV may be less effective in controlling the risk-taking, because recognizing positive news gives banks additional incentives to be more aggressive ex-ante in risk-taking when banks also care about short term earnings recognized in addition to the expected final payoff to shareholders.

The results taken together provide policy implications for bank regulators and accounting standard setting bodies. In terms of safe and sound banking, LCM provides better risk control than other accounting regimes. Banks will be more cautious in making investment decisions being aware of potential costs of violating capital regulation and negative market responses to earnings in the future. While the results support for incorporating market information into the accounting system, they also suggest that moving toward a full FV should be carefully considered by policymakers.
When the regulator may adjust the minimum capital requirement optimally under each accounting regime, the social welfare is the highest under LCM and the lowest under HC if increasing the capital requirement also increases the social cost. On the other hand, when the role of ex-ante effort by the bank in discovering the investment opportunity is more important, I show that the above preference order may reverse if the bank is sufficiently short term oriented.

Another relevant concern for standard setters is the recognition versus disclosure of fair value. The model in this paper can provide indirect implications about this concern from the following two aspects. First, for the effective capital regulation, recognition of economic losses are essential to get an accurate measure of the capital; disclosure of fair value itself can not bring into regulator’s attention about the declining economic value of banks’ capital. Second, the short term interest in earnings likely depends on the market’s reaction to accounting information. Given that the degree of market reaction is larger for recognized earnings than for disclosed numbers, the recognition of upside gains may induce more risk-taking by banks than pure disclosure; however the recognition of downside losses can better discipline the risk-taking as shown in the model. Therefore, this paper suggests that LCM with disclosure of full fair value is a better combination for the accounting framework in banks.
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Appendix A: Fairly priced deposit insurance

In the model, I assume that deposits are fully insured by the insurance agency and the insurance agency may demand an insurance premium from the bank for each dollar of deposit raised. Will the fairly priced (risk-sensitive) deposit insurance premium solve the problem of risk taking? In the analysis of the main body, the bank’s payment for the insurance premium is not included in the objective function. The following analysis explains why the bank’s optimal decisions are not altered by the existence of a fairly priced insurance system, even if the bank incorporates the insurance premium cost in the objective function.

In this appendix, I analyze the bank’s problem in Lemma 1 considering fairly priced deposit insurance. Suppose that the insurance agency now prices the insurance of deposits $D$ based on the expected default cost when the bank chooses its investment policy of $q$. A fairly priced insurance premium is specified as follows:

$$p(D, q) = \frac{(1 - q)^2}{2} (D - L)$$ (24)

Ideally, if the bank internalizes the insurance cost in the objective function, the bank faces the problem as stated below:

$$\max_q \pi_h(q) = q(I - D) + \frac{1 - q^2}{2} (H - D) - p(D, q) - K$$

The investment policy that solves the above problem is $I - L / H - L$, which equals the first best investment choice $q^{fb}$. However, since the bank’s investment riskiness is not observable to the regulator, the regulator can not enforce or monitor the bank’s investment decision once deposits are raised. If the bank issues deposits with the insurance premium priced as $p(D, q^{fb})$, it will always have the incentive to deviate from $q^{fb}$ so as to maximize the expected payoff in the following equation:

$$\max_q \pi_h'(q) = q(I - D) + \frac{1 - q^2}{2} (H - D) - p(D, q^{fb}) - K$$

Then the optimal solution to the above problem is given by $q^* = \frac{I - D}{H - D}$, which yields the same investment policy as in Lemma 1. Essentially the risk-shifting problem of the bank in my model is driven by the incomplete contractable investment choice, which can not be solved through the fairly pricing of insurance premium.

The insurance agency can, nonetheless, still set a fairly priced insurance premium based
on the predicted bank’s optimal decisions under different accounting regimes. As specified below, the insurance premium depends on the capital structure and the anticipated investment policy of the bank:

$$\pi(D, q_j) = \frac{(1 - q_j^*)^2}{2} (D - L), \text{ where } j \in \{h, l, f\}$$  \hspace{1cm} (25)

With the fairly priced insurance premium, the bank’s shareholders actually pay the cost of the sub-optimal investment choice induced by the deposit financing. The bank’s investment riskiness can only be controlled through the effective capital regulation or other mechanisms not examined in this paper.

**Appendix B: Proof**

**Proof. Lemma 2** Compare the minimum capital investment policy under HC and LCM:

$$\hat{q}_h = \frac{\hat{K}}{H - I + \hat{K}}$$
$$\hat{q}_l = \frac{(1 - \gamma)\hat{K} + \gamma\beta(I - L) - \frac{\partial P(B)}{\partial q} C(-e^B) + P(B)C'(e^B) \frac{\partial e^B}{\partial q}}{H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)}$$

Let

$$a = \frac{(1 - \gamma)\hat{K} + \gamma\beta(I - L)}{H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)} \text{ and } b = \frac{\hat{K}}{H - I + \hat{K}}$$

Then using assumption in (7), it can be shown that:

$$a - b = \frac{\gamma(H - I)[\beta(I - L) - (1 - \alpha)\hat{K}]}{[H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)] \cdot [H - I + \hat{K}]}$$

$$> \frac{\gamma(H - I)\beta(I - L) [1 - \frac{\hat{K}}{H + I - 2L}]}{[H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)] \cdot [H - I + \hat{K}]}$$

$$= \frac{\gamma(H - I)\beta(I - L) \frac{H - L + D - L}{H + I - 2L}}{[H - I + (1 - \gamma)\hat{K} - \gamma\alpha(H - I) + \gamma\beta(I - L)] \cdot [H - I + \hat{K}]}$$

$$> 0 \text{ (risky debt is issued, } D > L)$$  \hspace{1cm} (26)
In addition, for any $q$, the following conditions hold:

\[
\frac{\partial P(B)}{\partial q} = -\left[q(1 - \alpha) + (1 - q)\beta\right] < 0
\]

\[
\frac{\partial e^B}{\partial q} = \frac{2\beta(1 - \alpha)(H - L)}{[1 + q(1 - \alpha) + (1 - q)\alpha]^2} > 0
\]  

(27)

Hence combine (22) and (23), we have $\hat{q}_h < \hat{q}_l$

**Proof. Proposition 2**

To solve the objective function in (9) subject to the capital requirement constraint, first look at the case when $u = 0$, i.e, $K \geq kD - e^B$. Since $e^B < 0$, $K \geq kD$ is automatically satisfied. Now the objective function becomes:

\[
\Pi_l(q, K) = \gamma P(B)e^B + (1 - \gamma)(qK + \frac{1 - q^2}{2}(H - I + K) - K)
\]

Take the first order derivative with respect to $K$, we get:

\[
\frac{\partial}{\partial K}\Pi_l(q, K) = -(1 - \gamma)(1 - q)\frac{(1 - q)^2}{2} < 0
\]

Hence it is never optimal to issue equity more than $kD - e^B$.

Now we examine the case when the equity issuance level is less than $kD - e^B$:

If $K < kD - e^B$, $u = kD - K - e^B > 0$, the objective function becomes:

\[
\Pi_l(q, K) = \gamma P(B)e^B + (1 - \gamma)(qK + \frac{1 - q^2}{2}(H - I + K) - K) - P(B)C(kD - K - e^B)
\]

Take the first order derivative of the above function with respect to $K$ and $q$, we get:

\[
\frac{\partial}{\partial K}\Pi_l(q, K) = -(1 - \gamma)(1 - q)\frac{(1 - q)^2}{2} + P(B)C'(u)
\]

\[
\frac{\partial}{\partial q}\Pi_l(q, K) = \gamma\left[\beta(1 - q)(I - L) - q(1 - \alpha)(H - I)\right] + (1 - \gamma)[K - q(H - I + K)]
\]

\[
-\frac{\partial P(B)}{\partial q}C(u) + P(B)C'(u)\frac{\partial e^B}{\partial q}
\]

The second order derivatives and cross partial derivative with respect to $K$ and $q$ are given by:
\[
\frac{\partial^2}{\partial K^2} \Pi_l(q, K) = -P(B)C''(kD - K - e^B) < 0
\]
\[
\frac{\partial^2}{\partial q^2} \Pi_l(q, K) = \gamma[(1 - \alpha)(H - I) - \beta(I - L)] - (1 - \gamma)(H - I + K) < 0
\]

By assumption (7)
\[
-\frac{\partial^2 P(B)}{\partial q^2} C(u) + 2 \frac{\partial P(B)}{\partial q} C'(u) \frac{\partial e^B}{\partial q} - P(B)C''(u)(\frac{\partial e^B}{\partial q})^2 + P(B)C'(u) \frac{\partial^2 e^B}{\partial q^2} < 0
\]

\Rightarrow \frac{\partial^2}{\partial q^2} \Pi_l(q, K) < 0

Check whether the Hessian Matrix is positive definite, i.e.,
\[
\frac{\partial^2 \Pi_l}{\partial K^2} \frac{\partial^2 \Pi_l}{\partial q^2} - (\frac{\partial^2 \Pi_l}{\partial K \partial q})^2 > 0
\]

It turns out that as long as the \(C''\) is sufficiently large, the above condition always holds. Therefore the second order condition for maximizing \(\Pi_l(q, K)\) without the second constrained is satisfied.

Now define the solution that satisfies the first order condition as \(\hat{K}_{lc}^e\) and \(\hat{q}_{lc}^e\), which are given as follows:
\[
\left\{ \begin{array}{l}
\frac{\partial}{\partial q} \Pi_l(\hat{q}_l, \hat{K}_l) = 0 \\
\frac{\partial}{\partial K} \Pi_l(\hat{q}_l, \hat{K}_l) = 0
\end{array} \right.
\]

Considering the capital requirement constraint that \(K \geq kD\), let \(\hat{K} = kD\). The following scenarios are considered:

- If \(\frac{\partial}{\partial K} \Pi_l(\hat{q}_l, \hat{K}_l) \leq 0\), i.e., \(P(B)C'(-e^B) \leq (1 - \gamma)^2 (1 - \hat{q}_l)^2\), then the bank would want to further decrease the equity capital to the relaxed optimal \(K\) which is below the capital requirement level, but couldn’t do so because of the capital requirement. Hence, given the capital requirement constraint the bank’s optimal equity level is \(K^*_l = \hat{K}\).

- If \(\frac{\partial}{\partial K} \Pi_l(\hat{q}_l, \hat{K}_l) > 0\), i.e., \(P(B)C'(-e^B) > (1 - \gamma)^2 (1 - \hat{q}_l)^2\), then the bank could further increase the equity capital to the relaxed optimal level, which is \(K^*_l = \hat{K}_l\).

Given the optimal level of the equity capital \(K^*_l\), the optimal investment policy \(q^*_l\) is always determined by the first order condition \(\frac{\partial}{\partial q} \Pi_l(q, K^*_l) = 0\). Therefore, when \(K^*_l = \hat{K}\),

\[
34
\]
it is the minimum capital investment policy \( q^*_l = \hat{q}_l \); when \( K^*_l = \hat{K}_l \), it is the relaxed optimal investment policy \( \hat{q}^*_l = \hat{q}_l \).

**Proof. Corollary 1**

First we need to show that the investment policy under LCM is always no more risky than the minimum capital investment policy, i.e, \( q^*_l \leq \hat{q}_l \).

Define the function \( \Gamma \) as:

\[
\Gamma = \frac{\partial}{\partial q} \Pi(\hat{q}_l, \hat{K}_l) = \gamma [\beta (1 - \hat{q}_l)(I - L) - \hat{q}_l (1 - \alpha)(H - I)] + (1 - \gamma) [\hat{K}_l - \hat{q}_l(H - I + \hat{K}_l)]
- \frac{\partial P(B)}{\partial \hat{q}_l} C(u) + P(B) C'(u) \frac{\partial e^B}{\partial \hat{q}_l} \tag{28}
\]

Since \( \frac{\partial}{\partial K} \Pi(\hat{q}_l, \hat{K}_l) = 0 \) also holds at FOC, i.e,

\[-(1 - \gamma) \frac{(1 - \hat{q}_l)^2}{2} + P(B) C'(u(\hat{K}_l)) = 0 \tag{29}\]

Substitute \( P(B) C'(u) \) from (29) into the function of \( \Gamma \) in (28), we have:

\[
\Gamma = \gamma [\beta (1 - \hat{q}_l)(I - L) - \hat{q}_l (1 - \alpha)(H - I)] + (1 - \gamma) [\hat{K}_l - \hat{q}_l(H - I + \hat{K}_l)]
- \frac{\partial P(B)}{\partial \hat{q}_l} C(u) + (1 - \gamma) \frac{(1 - \hat{q}_l)^2}{2} \frac{\partial e^B}{\partial \hat{q}_l} \tag{30}\]

Now take the partial derivative of \( \Gamma \) with respect to \( \hat{K}_l \), we have:

\[
\frac{\partial \Gamma}{\partial \hat{K}_l} = (1 - \gamma)(1 - \hat{q}_l) + \frac{\partial P(B)}{\partial \hat{q}_l} C'(u) \tag{31}\]

Then substitute the function of \( C'(u) \) from (29) into (31), and also substitute \( P(B) \) and \( \frac{\partial P(B)}{\partial \hat{q}_l} \) into (31), we have the following result:

\[
\frac{\partial \Gamma}{\partial \hat{K}_l} = (1 - \gamma)(1 - \hat{q}_l) + \frac{\partial P(B)}{\partial \hat{q}_l} \frac{(1 - \hat{q}_l)^2}{2P(B)}
= (1 - \gamma)(1 - \hat{q}_l)[1 - \frac{\hat{q}_l(1 - \alpha) + (1 - \hat{q}_l)\beta}{(1 + \hat{q}_l)(1 - \alpha) + (1 - \hat{q}_l)\beta}] > 0 \tag{32}\]
Then take the total derivative of the function $\Gamma$ with respect to $\hat{K}_l$, we have:

\[
\frac{\partial \Gamma}{\partial \hat{K}_l} + \frac{\partial \Gamma}{\partial \hat{q}_l} \frac{\partial \hat{q}_l}{\partial \hat{K}_l} = 0
\]

Given the second order condition in the proof of Proposition 2, we have $\frac{\partial \Gamma}{\partial \hat{q}_l} < 0$; and $\frac{\partial \Gamma}{\partial \hat{K}_l} > 0$ from (32), therefore we have:

\[
\frac{\partial \hat{q}_l}{\partial \hat{K}_l} > 0 \quad (33)
\]

Hence at the relaxed optimal solution, the higher equity capital always induces the less risky investment. Since $\hat{K}$ and $\hat{q}_l$ is also a set of solution that satisfies the FOC, thereby it is easy to see that $q^*_l \geq \hat{q}_l$ given $K^*_l > \hat{K}$.

Then combined with Lemma 2, which suggests that $\hat{q}_l > \hat{q}_h = q^*_h$, we can show that:

\[
q^*_l > q^*_h
\]

**Proof. Lemma 3**

Following a similar proof of Lemma 2, we can easily show that $\hat{q}_f > \hat{q}_h$. Now we need to compare $\hat{q}_f$ with $\hat{q}_l$. Compare the partial derivative of the objective function with respect to $q$ at $\hat{K}$:

\[
\frac{\partial}{\partial q} \Pi_l(q, \hat{K}) = \gamma[\beta(1-q)(I-L)-q(1-\alpha)(H-I)] + (1-\gamma)[\hat{K}-q(H-I+\hat{K})] - \frac{\partial P(B)}{\partial q} C(u) + P(B)C'(u) \frac{\partial e^B}{\partial q}
\]

\[
\frac{\partial}{\partial q} \Pi_f(q, \hat{K}) = \gamma[(1-q)(I-L)-q(H-I)] + (1-\gamma)[\hat{K}-q(H-I+\hat{K})] - \frac{\partial P(B)}{\partial q} C(u) + P(B)C'(u) \frac{\partial e^B}{\partial q}
\]

The only difference in the partial derivative functions is the underlined part. Compare
these two underlined parts:

\[
[(1-q)(I-L) - q(H-I)] - [\beta(1-q)(I-L) - q(1-\alpha)(H-I)]
= (I-L)(1-q)(1-\beta) - \alpha q(H-I)
< \frac{H-I}{H-I+\hat{K}} [(I-L)(1-\beta) - \alpha \hat{K}] \quad \text{(since } \hat{q}_{l,f} > \frac{\hat{K}}{H-I+\hat{K}} \text{ holds)}
< 0 \quad \text{(by the assumption in (7))}
\]

\[
\frac{\partial \Pi_f}{\partial \hat{q}_f} < \frac{\partial \Pi_l}{\partial \hat{q}_l}, \forall q
\]

Therefore the optimal solution must satisfy \( \hat{q}_f < \hat{q}_l \)

**Proof. Corollary 2**

Following the proof of Lemma 3, we can also show that for any given equity capital \( K \), the optimal investment policy under fair value accounting is always more risky than under LCM, i.e, \( q^*_f(K) > q^*_l(K) \)

If under both regimes, the bank issues the minimum capital capital, then \( q^*_f < q^*_l \) holds.

If under both regimes, the bank issues the capital in excess of the minimum requirement, we need to compare \( \hat{K}_l \) and \( \hat{K}_f \) and the corresponding optimal investment policies.

Now since \( \hat{q}_f \) and \( \hat{K}_f \) satisfy the FOC condition for \( K \), we have:

\[
\frac{\partial}{\partial \hat{K}_f} \Pi_f(\hat{q}_f, \hat{K}_f) = -(1-\gamma)(1-\hat{q}_f^2) + P(B)C'(u) = 0
\]

Under LCM, we also have the same form of FOC condition for \( K \):

\[
\frac{\partial}{\partial \hat{K}_l} \Pi_l(\hat{q}_l, \hat{K}_l) = -(1-\gamma)(1-\hat{q}_l^2) + P(B)C'(u) = 0
\]

From the proof in Corollary 1, we have the following condition for the FOC solution under FV:

\[
\frac{\partial^2}{\partial \hat{K}_f \partial \hat{q}_f} \Pi_f(\hat{q}_f, \hat{K}_f) > 0
\]

In addition \( q^*_l(\hat{K}_f) > \hat{q}_f(\hat{K}_f) \) by (29), therefore \( \frac{\partial}{\partial \hat{K}_f} \Pi_f(q^*_l(\hat{K}_f), \hat{K}_f) > 0 \)
Given that FOC functions under FV and LCM have the same form:

$$\frac{\partial}{\partial K_{l}} \Pi_{l}(q_{l}^{*}(K_{f}), K_{f}) > 0 \Rightarrow \dot{K}_{l} > \dot{K}_{f}, \dot{q}_{l} > \dot{q}_{f}$$

The only question remains about the likelihood of issuing equity capital in excess of the minimum requirement under two regimes. Suppose under fair accounting, the minimum capital $\dot{K}$ also satisfies the FOC, i.e,

$$\Lambda_{f}(\dot{K}, \dot{q}_{f}) = -(1 - \gamma) \frac{(1 - \dot{q}_{f})^{2}}{2} + P(B)C'(e^{B}(\dot{q}_{f})) = 0$$

From the proof of Proposition 2, we know that at the optimal solution, the cross partial derivative $\frac{\partial^{2} \Pi_{f}}{\partial q \partial K} > 0$, therefore we can get:

$$\Lambda_{l}(\dot{K}, \dot{q}_{l}) > 0, \text{ as } \dot{q}_{l} > \dot{q}_{f}$$

This means under LCM, the optimal solution for the bank is to issue equity in excess of the minimum requirement. Therefore, the bank is more likely to issue buffer capital under LCM than under FV.

**Proof. Proposition 4**

When $g' \rightarrow 0$, the bank manager will always exert effort of $a_{j} = 1$, such that the problem of the regulator becomes:

$$\max_{k} W_{j}(k) = V(q^{*}_{j}(k)) + L(k)$$

s.t. $q^{*}_{j}(k), K^{*}_{j}(k) \in \arg \max_{q, K} \Pi_{j}(q, K | k)$

a. First show that under all regimes the optimal investment policy increases with $k$, i.e, $\frac{\partial q^{*}_{j}(k)}{\partial k} > 0$. This is obvious under HC. Under LCM and FV, we need to consider both the original and relaxed solutions. Now I only show the proof under LCM, the case for FV is similar.

(1) when $q^{*}_{l}(k) = \hat{q}_{l}$, the FOC function at $\dot{K}$ is given by:
\[\Gamma(\hat{K}) = (1 - \gamma)[\hat{K} - \hat{q}(H - I + \hat{K})] + \gamma((1 - \hat{q})\beta(I - L) - \hat{q}(1 - \alpha)(H - I)] - \frac{\partial P(B)}{\partial \hat{q}} C(-e^B) + P(B)C'(e^B) \frac{\partial e^B}{\partial \hat{q}} = 0\]

\[\Rightarrow (1 - \hat{q_l}) \frac{\partial \hat{K}}{\partial k} + \frac{\partial \Gamma}{\partial \hat{q_l}} \frac{\partial \hat{q_l}}{\partial k} = 0\]

\[\Rightarrow \frac{\partial \hat{q_l}}{\partial k} > 0 \text{ (Since } \frac{\partial \Gamma}{\partial \hat{q_l}} < 0 \text{ from SOC) (34)}\]

(2) when \(q^*_l(k) = \hat{q}_l\), the optimal solution is interior solution, the FOC functions are defined as:

\[\Gamma = \frac{\partial}{\partial q_t} \Pi_t(\hat{q}_l, \hat{K}_l), \quad \Lambda = \frac{\partial}{\partial K_t} \Pi_t(\hat{q}_l, \hat{K}_l)\]

By Cramer’s rule, we can get:

\[
\frac{\partial \hat{q}_l}{\partial k} = \begin{vmatrix}
-\frac{\partial \Gamma}{\partial k} & -\frac{\partial \Gamma}{\partial K_l} \\
-\frac{\partial \Lambda}{\partial k} & -\frac{\partial \Lambda}{\partial K_l} \\
\frac{\partial \Gamma}{\partial \hat{q}_l} & \frac{\partial \Gamma}{\partial \hat{K}_l} \\
\frac{\partial \Lambda}{\partial \hat{q}_l} & \frac{\partial \Lambda}{\partial \hat{K}_l}
\end{vmatrix}
\]

The denominator of the above equation is exactly the determinant of the Hessian Matrix, which by assumption is positive when \(C''\) is sufficiently large. Now we look at the numerator, we have the following conditions:

\[
\begin{align*}
\frac{\partial \Lambda}{\partial \hat{K}_l} &= P(B)C''(u)D > 0 \\
\frac{\partial \Lambda}{\partial K_l} &= -\frac{\partial P(B)}{\partial \hat{q}} C'(u)D + P(B)C''(u) \frac{\partial e^B}{\partial \hat{q}} D > 0 \\
\frac{\partial \Gamma}{\partial \hat{K}_l} &= 0 \text{ (see the proof in Corollary 1)} \\
\frac{\partial \Lambda}{\partial \hat{K}_l} &< 0 \text{ (SOC for } K) \\
\frac{\partial \Gamma}{\partial k} &< 0 \text{ (SOC for } K_l) \\
\frac{\partial \Gamma}{\partial k} &> 0 \text{ (SOC for } K_l) \\
\end{align*}
\]

\[\Rightarrow \frac{\partial \Lambda}{\partial \hat{K}_l} \frac{\partial \Gamma}{\partial k} - \frac{\partial \Gamma}{\partial k} \frac{\partial \Lambda}{\partial \hat{K}_l} > 0\]

Hence we can prove that:

\[\frac{\partial \hat{q}_l}{\partial k} > 0 \quad \text{(35)}\]
Therefore combine (35) and (36), we have \( \frac{\partial q^*_l}{\partial k} > 0 \).

b. Next since the optimal \( k \) is determined by the FOC condition:

\[
W'_j(k) = \frac{\partial L(k)}{\partial k} + \frac{\partial V(q^*_j(k))}{\partial q} \frac{\partial q^*_j(k)}{\partial k} = 0
\]

As shown above \( \frac{\partial q^*_j(k)}{\partial k} > 0, \) and \( \frac{\partial L(k)}{\partial k} < 0, \) we must have \( \frac{\partial V(q^*_j(k))}{\partial q^*_j(k)} > 0 \) at the optimal \( k^*_j \). In addition,

\[
\frac{\partial^2 V(q^*_j(k))}{\partial q^2} = L - H < 0 \tag{36}
\]

Therefore at the optimal capital requirement, we must have \( q^*_j(k^*_j) < q^{fb} \)

b. Given that \( \forall k, \ q^*_f(k) > q^*_f(k) > q^*_h(k) \) and \( \frac{\partial V(q^*_j(k))}{\partial k} > 0 \) (from \( q^*_j(k^*_j) < q^{fb} \)), the following condition holds:

\[
V(q^*_f(k^*_j)) \geq V(q^*_h(k^*_j)) \Rightarrow W_f(k^*_h) \geq W_h(k^*_h)
\]

\[
V(q^*_f(k^*_j)) \geq V(q^*_f(k^*_j)) \Rightarrow W_f(k^*_f) \geq W_f(k^*_f)
\]

Therefore we must have \( W_{i_j}(k^*_j) \geq W_f(k^*_j) \geq W_h(k^*_j) \)

**Proof. Proposition 5**

When \( L'(k) \rightarrow 0, \) the problem in (23) becomes:

\[
\max_k a^*_j(k)V(q^*_j(k)) - g(a^*_j(k))
\]

s.t.

\[
a^*_j(k) \in \arg\max_a a\Pi_j(q^*_j(k), K^*_j(k), k) - g(a)
\]

\[
q^*_j(k), K^*_j(k) \in \arg\max_{q,K} \Pi_j(q, K | k)
\]

Substitute the bank’s solution to his own problem in (22) to get the optimal social welfare under each accounting regime for any exogenous \( k \):

\[
W_j(k) = \frac{m}{2} \Pi_j(q^*_j(k), K^*_j(k), k)[V(q^*_j(k)) - \frac{1}{2} \Pi_j(q^*_j(k), K^*_j(k), k)] \tag{37}
\]

The maximum welfare under each accounting regime is given by \( W_j(k^*_j) \), where \( k^*_j \) is the optimal capital requirement for the regulator.

The maximum welfare under each accounting regime is given by \( W_j(k^*_j) \), where \( k^*_j \) is the optimal capital requirement for the regulator. The first step is to prove that \( W_i(k^*_i) < W_h(k^*_h) \)
and $W_l(k^*_f) < W_f(k^*_f)$ when $\gamma \to 1$. For any given $k$, we will show that $W_l(k) < W_h(k)$. Then it is easy to conclude that $W_l(k^*_f) < W_h(k^*_f) \leq W_h(k^*_h)$.

Under HC, the welfare is:

$$W_h(k) = (1 - \gamma)\pi(q^*_h(k), K^*_h(k))[V(q^*_h(k)) - \frac{1 - \gamma}{2}\pi(q^*_h(k), K^*_h(k))]$$  \hspace{1cm} (38)

Under LCM, define $\Delta_l = E[e_l] - E[C(u_l)]$, then the welfare function becomes:

$$W_l(k) = (1 - \gamma)\pi(q^*_l(k), K^*_l(k))[V(q^*_l(k)) - \frac{1 - \gamma}{2}\pi(q^*_l(k), K^*_l(k))] - \Delta_l[(1 - \gamma)\pi(q^*_l(k), K^*_l(k)) - V(q^*_l(k))]$$ \hspace{1cm} (39)

Define $U(q, K) = \pi(q, K)[V(q) - \frac{1}{2}\gamma\pi(q, K)]$, and take derivative with respect to $q$ and $K$. It can be shown that for any $q^*_h \leq q < q^*_f$, the following holds when $\gamma \to 1$:

$$\frac{\partial U(q, K)}{\partial K} \leq 0, \quad \frac{\partial U(q, K)}{\partial q} \leq 0$$

Therefore since $q^*_l(k) > q^*_h(k)$ and $K^*_l(k) \geq K^*_h(k)$, we have

$$U(q^*_l(k), K^*_l(k)) < U(q^*_h(k), K^*_h(k))$$  \hspace{1cm} (40)

In addition, under lower of cost or market accounting, we have $\Delta_l < 0$. And for any $q$, we can show that $\pi(q, K) < V(q)$. Therefore in (40), we have $\Delta_l[(1 - \gamma)\pi(q^*_l(k), K^*_l(k)) - V(q^*_l(k))] > 0$. Combined with the result in (41), we have for any given $k$,

$$W_l(k) < W_h(k)$$  \hspace{1cm} (41)

To compare the welfare under FV and HC, define $\Delta_f = E[e_f] - E[C(u_f)]$, then the result depends on the sign of $\Delta_f$. When the cost and marginal cost of violating regulatory constraint is sufficiently high, $\Delta_f < 0$; otherwise, $\Delta_f > 0$. In the former case, comparing FV to HC is similar to the proof shown above for LCM, i.e., $W_f(k^*_f) < W_h(k^*_h)$. In the later case, the result will be opposite, we can show that $W_f(k) > W_h(k)$ following the same process. Therefore under these conditions, we have $W_h(k^*_h) < W_f(k^*_f)$.