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Quantifying Industrial Quality in Economic Models

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EDRC 24-91-92

Quantifying Industrial Quality in Economic Models

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Abstract

Due to several possible production modes, an industrial output may have different quality levels. We present a method to select the optimum production mode, based on cost and quality considerations. We introduce the idea of a Pareto efficient quality level that we relate with the concept of value loss. The traditional tradeoff cost versus quality is studied in a formalism that allows industrial quality to be treated as an economic variable. For validation and forecasting purposes, the effects of quality differences are studied in various contexts: perfect competitive market, monopoly, and oligopoly situation. In each case, consequences are drawn in terms of prices and profits. Depending on the economic situation considered, failure to produce at the Pareto efficient quality level is shown to have different detrimental effects.

Introduction

More than ever, industrial quality is one of the critical elements in the fierce competition between manufacturing companies. From an engineering standpoint as well as from a management perspective, quality is a challenge that cannot be ignored: a lack of competitiveness can often be attributed to quality problems. In these cases, the first logical approach is to question design and manufacturing practices. An abundant literature deals with systematic methods to improve quality at zero or low cost (*e.g.*, Freedman et al., 1948; Deming, 1950; Shewhart, 1986; Taguchi, 1989; Phadke, 1989). One of the main emphases in these works is to control the manufacturing process within pre-specified bounds. However, this paper addresses a more general problem that may arise and has received much less attention: several alternate production modes can often be employed to manufacture the same product, resulting in costs and quality levels that are significantly different. Then, the choice of a production mode, and thereby of a quality level, consists of trading off cost for quality. Here, instead of examining the control of a process within certain bounds, it is the values of those bounds that are determined. Consequently, an economic analysis is required to estimate the benefits in terms of market share or profit resulting from engineering decisions like choosing a tighter manufacturing process or screening a production. For that matter, we present a model *quantifying* quality at the engineering level in order to define an optimum quality level. We also model the impact of quality differences between firms in various contexts.

1. Modelization

The relative neglect of quality considerations in economic theory is probably due to the fact that quality is often thought of as a qualitative variable that cannot be incorporated in a mathematical model. The prevailing assumption is that competing products are standard commodities sold for different prices. A product with several quality grades is generally treated as several different products (Debreu, 1959). Unless it is possible to compare consumers' satisfaction for the different quality grades (*e.g.*, through demand curves or utility functions) this approach provides little insight as to the profitability of the various quality levels. To avoid this obstacle, we limit the scope of this paper to a mass produced industrial product for which a quality indicator can be developed. This requirement implies that the product has a function or a characteristic that is measurable. In some cases, the quality of a production will be assessed by the percentage of defective products shipped to the consumers. However, recent developments in the field of quality have shown that more often than not, the quality of a production is better estimated by its

consistency: the variability of any manufacturing operation induces variations in the characteristics of the products, and therefore possible performance degradations (Taguchi, 1986). Under this paradigm, the quality of a production can be evaluated by statistical means (*e.g.*, variance of the performance of the products). Various quality indicators can be developed that are appropriate for a particular product. For instance, in (Vasseur, Kurfess, Cagan, 1992), the quality of an assembly is measured by the ratio $\frac{\sigma}{\Delta}$, where Δ is a customer defined tolerance and σ the actual standard deviation of the nominal dimension of the assembly. The determination of a quality indicator will not be investigated in this paper; we will assume that the product studied can have various quality levels measured by the indicator q such that $0 \leq q \leq 1$. The highest possible quality is achieved when q is zero (*e.g.*, zero defect, or no significant deviation from expected performance). A value of one for q corresponds to the lowest quality level (*e.g.*, 100 % defective products).

We denote by C the cost of production of the product studied. Because different quality levels usually require different manufacturing processes, different inspection procedures, or different raw materials, C is a function of the quality level, q, of the production ($C = C(q)$). As mentioned in the introduction, when quality problems are detected, design and manufacturing practices are reviewed and sometimes can be improved at low cost. It must be understood that this is not contradictory with our models: any quality improvement that can be achieved at zero or low cost must be made. However, a manufacturing process has inherent capability limits; requiring performances beyond these limits involves changes in the production mode, that generally results in significantly higher costs. The function C(q) used in our models denotes the lowest possible production cost associated with an output with quality level, q. A good illustration of the aim of this paper is provided by statistical process control theory (SPC): we do *not* discuss on-line quality control procedures to provide feedback to a manufacturing process and ensure that the mean and variance of the output have the values that have been fixed. Rather, we discuss what the variance *should* be, based on an estimate of its cost, and the quality level it provides.

In economic theory, the production cost is often viewed as a function of the output of the firm. However, in our modelization, we currently neglect possible economies of scale over the cost variations due to quality differences. On one hand economies of scale sometimes disappear beyond a certain output level (Johnston, 1960), and on the other hand, a more accurate modelization describing the production cost as a function of both q and the firm output is not believed to affect the qualitative results presented.

A problem arises in modelization when attempting to define the impact of quality on a consumer. Obviously, a quality product is worth more to a consumer than an inferior item. Therefore, it is important to measure what value a given product represents to a consumer.

Marshall presented a way to measure this value by explaining that, when a consumer is willing to make a purchase, he believes that the satisfaction provided by the purchase is greater than the price paid. Therefore, the value of a product for a consumer must be greater than the price he pays for it. The price reaches the value of the product when the consumer would rather make do without the product, than pay the price. *"The excess price which he would be willing to pay rather than go without the thing, over that which he actually does pay, is the economic measure of [the] surplus satisfaction. It may be called consumer's surplus* " (Marshall, 1947). On a larger scale, for a given market, let us denote by V_{max} the first price for which the consumers' demand vanishes. This resistance price can be considered as the maximum value of the product (Cook and DeVor, 1991). More precisely, no matter what the quality of the product is, consumers will not buy it if it is priced higher than V_{max}. The determination of this value can be done by extrapolation of the demand curve if it is partially known, or by considering a possible product substitution that is likely to occur if the price of a product is higher than that of another product with a similar function (*e.g.*, if the price of a personal computer gets close to that of a workstation, people will probably choose the workstation).

The value attached by consumers to a product is a function of q. We denote this function by V(q). It is possible to approximate V(q) by a second degree Taylor expansion about q = 0 (maximum quality level, $V(0) = V_{max}$):

$$V(q) = V_{max} + V'(0)\, q + \frac{V''(0)}{2!}\, q^2.$$

Two relationships allow for the determination of the derivatives of V: because V is maximum for q = 0, V'(0) = 0; for q = 1, the product is defective therefore its value is 0. As a result, V can be approximated by

$$V(q) = V_{max}\, [\, 1 - q^2 \,].$$

Under the assumptions described above, it is clear that consumers' satisfaction can be measured by the surplus $S(p, q) = V(q) - p$, and the manufacturers' profit per item by $\pi(p, q) = p - C(q)$, where p is the actual price charged by the manufacturers. This formulation clearly enhances the contradictory interests of the two parties: raising prices benefits the manufacturers but decreases consumers' satisfaction, and increasing quality improves consumers' satisfaction but is detrimental for manufacturers' profits.

2. An efficient use of the resources

Let us consider a situation where a product is manufactured at a quality level, q, and sold to consumers for a price p. We model the consumers' behavior as a maximization of the surplus $S(p, q) = V(q) - p$, and manufacturers' behavior as a maximization of the profit $\pi(p, q) = p - C(q)$. As emphasized in the previous section, consumers and the manufacturers have contradictory interests. Both objectives are functions of the independent variables p and q. However, it is interesting to notice that the sum $S(p, q) + \pi(p, q)$ is only a function of q:

$$S(p, q) + \pi(p, q) = V(q) - C(q).$$

In other words, the sum of the two satisfaction measurements depends exclusively on the quality level. $V(q) - C(q)$ can be seen as the value created (denoted by $V_c(q)$) by the manufacture of a product. The benefits of this value creation is shared by the manufacturers and the consumers: q determines the amount to be shared, and p decides of the allotment of this value.

Let us suppose that C(q) is differentiable and convex; this situation is likely to occur when only one process is available for the manufacture of the product (Kurfess and Cagan, 1991). Because V(q) is differentiable and strictly concave, $V_c(q)$ is a differentiable and strictly concave function over the compact set [0; 1], therefore it has a unique maximum (see Figure 1).

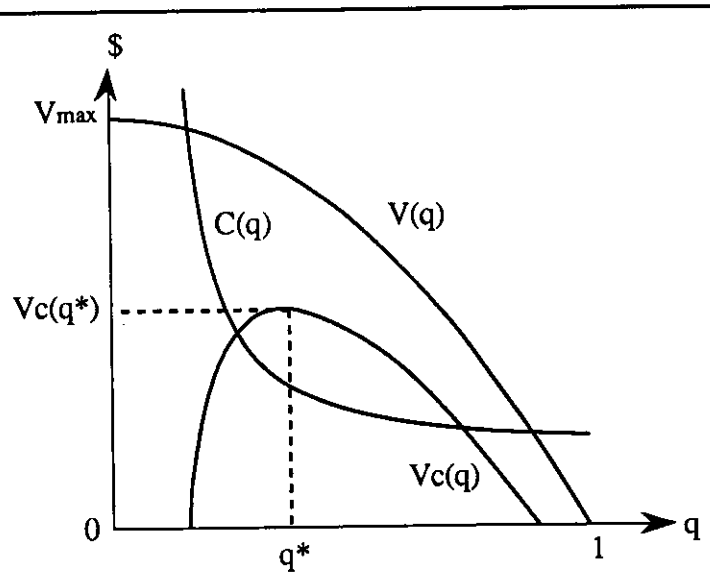

Figure 1: Quality level maximizing the value created.

<u>Proposition</u>: Let us denote by q* the (unique) quality level that maximizes $S(p, q) + \pi(p, q)$. q* is the only Pareto efficient quality level, *i.e.* given a price p and a quality level q, unless q = q*, it is possible to find p* such that, by manufacturing the product with a quality level q* and

selling it at the price p*, either the consumers' surplus or the manufacturers' profit is increased, and none of them is decreased.

Proof: In a situation where the quality level is q and the price is p, the consumers' surplus is V(q) - p, and the manufacturers' profit is p - C(q). If the quality level is too high ($q < q^*$), then let us choose $p^* = V(q^*) - V(q) + p$. The consumers' surplus is unchanged, and the manufacturers' profit becomes $p^* - C(q^*)$ *i.e.*, $V(q^*) - V(q) + p - C(q^*)$. Because q* is defined as the value that maximizes $S(p, q) + \pi(p, q)$, necessarily, $V(q^*) - C(q^*) \geq V(q) - C(q)$. Substituting $V(q^*) - C(q^*)$ by $V(q) - C(q)$ in the expression of the profit reveals that the profit is greater than $V(q) - C(q) - V(q) + p = p - C(q)$, which is the original profit. In other words, the manufacturers are better off and the consumers are not harmed. The case where the original quality level is lower than q* can be handled in a similar way; in the resulting situation the consumers are better off and the manufacturers are not harmed. A similar reasoning also proves that q* is a Pareto efficient quality level.

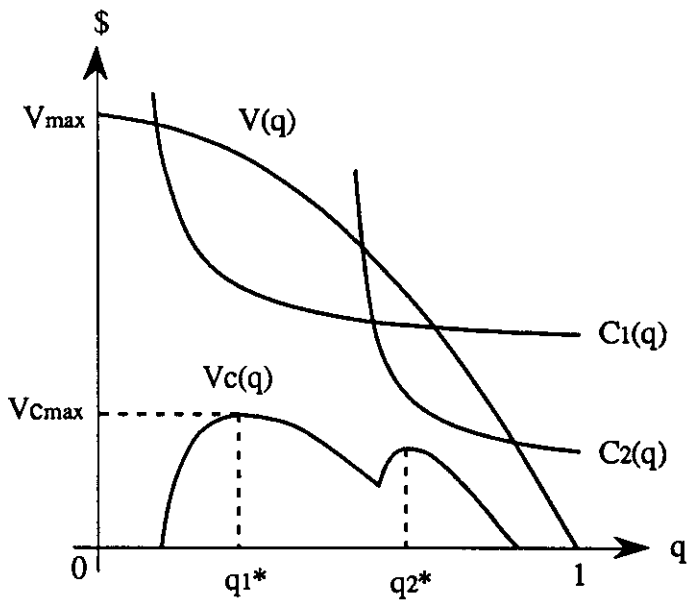

Figure 2: Several market segments may have to be considered.

So far, we have assumed that C(q) was a differentiable convex function. This assumption assured the existence and uniqueness of a local maximum for Vc(q). However, C(q) may not be convex, especially when several processes can be used (Cagan and Kurfess, 1992). In this case, several local maxima may appear. This situation suggests a partitioning of the market into several segments, each local maximum corresponding to a Pareto solution over a limited quality (and probably also price) range. Figure 2 illustrates a situation where two processes can be used. Each

process is best suited over different quality ranges. q^{1*} is the global Pareto optimum solution (maximizing $V_C(q)$ over [0, 1]); q^{1*} is situated in the high quality / price range. Therefore, it may be profitable to offer the quality level q^{2*} (*e.g.*, for low budget customers), even though it is a sub-optimum solution. q^{1*} may be what is desirable from a social viewpoint, but q^{2*} could be what a large portion of the customers can actually afford. This explains the "portfolio" of products that companies often offer. In the rest of the paper, we will consider a given market segment, for which a unique Pareto efficient quality level exists.

3. Notion of Value Loss

The existence of a quality level desirable from a society point of view can also be explained by the notion of value loss. This loss is incurred by society when a product does not have the maximum possible quality level. The concept was introduced by Taguchi (Taguchi, 1986), who used it to define his quality loss function. Yet, we propose to define society loss in terms of value to account for people's needs and desires, rather than pure cost which only relates to manufacturing issues. We define our value loss function as the potential value L(q) which is lost, when a product does not have the maximum possible quality level. When producing at the quality level q, L(q) is defined as

$$L(q) = V(0) - V(q).$$

Society as a whole is affected by this loss: besides the immediate loss to the consumers, imperfect products also hurt manufacturers' sales. The total cost of a product to society is the sum of its actual manufacturing cost and the value loss it generates. Therefore, society is better off when the expression

$$C(q) + L(q)$$

is minimized. Since

$$V_C(q) = V(0) - [L(q) + C(q)],$$

maximizing the value created is equivalent to minimizing C(q) + L(q). Therefore q*, as defined in section 2, is also the solution of this minimization problem.

The concept of society loss is by nature difficult to perceive in terms of individual or personal deprivation; the concept is somewhat fuzzy, let alone the assignment of the loss between manufacturers and consumers. Moreover, although real and significant, the value loss does not appear in any accounting procedure and, therefore, it has little appeal to corporate decision makers.

For that matter, this section emphasizes its importance but more tangible measures will be developed in the next sections. Further assumptions will be made to provide a more elaborate modelization. The consistency of the two approaches will be verified.

4. Perfect Competitive Model

The first situation considered is that of a perfect competitive market, as described for instance in (Cohen and Cyert, 1965). A large number of independent manufacturers sell a product for which consumers have perfect information about the prices being charged, and perfect knowledge of the quality levels provided by each producer. In these conditions, different quality grades between producers have to be fairly compensated by price differences, resulting in a uniform consumers' surplus offered by all the producers. Indeed, any firm attempting to offer a lower surplus than the prevailing one would lose all its customers, and a firm offering a higher surplus than the prevailing one would make all its competitors match its offer. Therefore, we can assume without loss of generality, that a price, p_o, is established for the best possible quality, and that any item with a different quality level is discounted in such a way that the consumers' surplus remains constant. Therefore, the price of an item with quality level q is

$$p = p_o - [V(0) - V(q)].$$

The manufacturer's profit per item of quality level q is

$$\pi = p - C(q) = p_o - C(q) - [V(0) - V(q)] = p_o - C(q) - L(q).$$

In this situation, the quality loss incurred by society is a direct cut in the manufacturers' profits. The society loss is entirely incurred by the manufacturers, constituting a strong incentive to produce at the q* quality level.

This model obviously views consumers as more responsive to prices than they actually are, and more knowledgeable about product quality than they realistically can be. Therefore, we can expect that consumers are also partially affected in a situation where the quality level is different from q*: they probably incur a small portion of the value loss. Yet, despite its unrealistic aspects, the model suggests that in a near perfect competitive market, the major impact of imperfect production is to burden manufacturers' profits.

5. Case of the Monopolistic Firm

5.1 Modelization

We now consider a situation completely different from that of the previous section: we consider a market dominated by a single enterprise, manufacturing and selling a single industrial product. Let us determine the profit maximizing values of the price p and the average quality level q of the product. If N is the consumers' demand and C the cost of production of a single product (function of q), the total profit π_T of the firm can be expressed as

$$\pi_T(p, q) = N\,[p - C(q)].$$

It is assumed that the firm can make accurate predictions as to the consumers' demand and can generate a supply to meet that demand. We assume the demand to be an increasing function of the consumers' surplus S ($S = V(q) - p$) (Cook, 1991). As a first approximation, we assume the demand to be linear. We assume there is no demand for zero surplus, and denote by α the slope of the demand function (see Figure 3). This representation allows for a rough comparison between the demand curves at various quality levels (see Figure 4). Note that in reference to the product portfolio discussed in Figure 2, each locally optimal q^{i*} would be represented by a different demand curve in Figure 4.

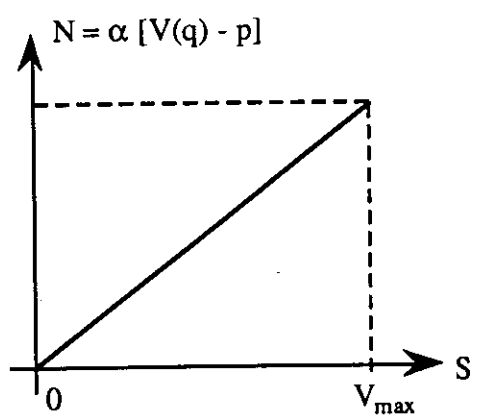

Figure 3: Consumers' demand is a linear function of consumers' surplus.

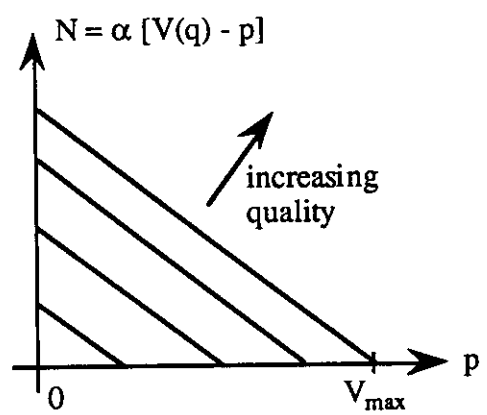

Figure 4: Demand curves for various quality levels.

The modelization is not valid over the whole range of possible values for S; nevertheless, it allows us to approximately locate the optimum surplus. Once a first value is determined, it can be further refined by the same method as more precise information is gathered about the shape of the

demand function in the neighborhood of the initial value. Then, as a local approximation, the linear assumption is substantially more legitimate.

Under the assumption of differentiability for C, the profit maximizing values have to satisfy:

$$\frac{\pi_T}{\partial p} = \alpha\,[V(q) + C(q) - 2p] = 0,$$

which sets the optimal price at

$$p = \frac{1}{2}[V(q) + C(q)]\ , \tag{1}$$

and

$$\frac{\partial \pi_T}{\partial q} = \frac{\alpha}{2}[V(q) - C(q)]\,[V'(q) - C'(q)] = 0,$$

which sets the quality level at the value of q satisfying

$$[V'(q) - C'(q)] = 0 \tag{2}$$

(Note that V - C = 0 is obviously not a profit maximizing solution).

5.2 Analysis

As expected, the profit maximizing quality level given by (2) is the Pareto optimal solution: the value created per product is maximized. It is equally shared by the manufacturer and the consumers: the consumers' surplus is

$$V(q) - p = V(q) - \frac{1}{2}[\,V(q) + C(q)] = \frac{1}{2}[\,V(q) - C(q)] = \frac{1}{2}\,Vc(q),$$

and the manufacturer's profit per item is

$$p - C(q) = \frac{1}{2}[V(q) + C(q)] - C(q) = \frac{1}{2}\,[\,V(q) - C(q)] = \frac{1}{2}\,Vc(q).$$

The total profit is formulated as proportional to the consumers' surplus times the profit per product. As seen in section 2, the sum of these two components is the value created per product. It is maximum when the quality level is q*. Our problem consists of maximizing the multiplication of two variables (S and π), with an upper bound on their sum. The objective is maximum when each variable is equal to half the upper bound. The profit maximizing solution is reached when the

profit per product and the consumers' surplus are both equal to half of the maximum possible value created.

It is dubious that the quality level of a production can be adjusted as easily as its price. In most cases, major changes in the production processes are required, and retooling may be necessary. Therefore, the model provides an indication as to the quality level towards which it is suitable to go, but except for predictions in the case when new factory lines are being installed, q should be seen more as a parameter specified by production capabilities rather than a variable. It can be noticed that no matter what the quality level is, the model recommends that the value created per product be equally shared by the manufacturer and the consumers. The consumers' surplus and the manufacturer 's profit per item can be expressed as

$$\frac{1}{2} Vc(q) = \frac{1}{2} [\ V(0) - C(q) - L(q)],$$

revealing that the value loss affects both the manufacturer and the consumers.

6. Case of the Oligopoly

6.1 Modelization

The case of an oligopoly, where a limited number of firms are competing on a given market to mass produce and sell a given product, can be modeled similarly to that of the monopolistic case. We assume that the products sold by the different firms are similar by their functions, but have different prices and different quality levels (imperfect competition). The cost of production is assumed to be identical for all firms. The same simplifying assumptions are made as in the monopolistic case concerning the consumers' demand: it is treated as a linear function of the consumers' surplus. If p_i and q_i are the price and quality level of the products sold by firm i, the demand for firm i can be written as:

$$N_i(p_i, q_i) = \alpha_i\ [\ V(q_i) - p_i].$$

It does not seem reasonable to use the same coefficients for the demand function of all the firms: the sales of a given firm depend on what it offers to the consumers, but also on what the competitors provide. Firm i's demand function is proportional to the consumers' surplus it offers; this component measures consumers' satisfaction on an absolute scale. To model the effect of the presence of the competitors, α_i must reflect the position of the firm with respect to the others. An immediate solution consists of taking α_i proportional to the ratio of the surplus offered by firm i to the sum of the surplus offered by all the firms:

$$\alpha_i = K \frac{V(q_i) - p_i}{\sum_{k=1}^{n} [V(q_k) - p_k]} .$$

The proportionality coefficient can be determined by considering the case where the n firms manufacture and sell products with the same quality levels and prices. Then, we can assume that the total demand will equal that of the monopolistic case, therefore

$$\sum_{k=1}^{n} \alpha_i = \alpha,$$

where α is the demand at 0 price, as defined in section 5. Since in this case there is no reason to assume different coefficients (*i.e.*, the market shares of all the firms should be identical), each α_i equals $\frac{\alpha}{n}$. Therefore, $K = \alpha$, and in the general case, the coefficients α_i are

$$\alpha_i = \alpha \frac{V(q_i) - p_i}{\sum_{k=1}^{n} [V(q_k) - p_k]} .$$

This modelization of consumers' demand takes into account two different effects: by increasing the surplus it offers, a firm increases its sales volume by *i)* depriving the other firms of a part of their customers, and *ii)* by increasing the total sales of the product considered.

Under these assumptions, the profit of firm i is given by:

$$\pi_{Ti}(p_i, q_i) = \alpha \frac{[V(q_i) - p_i]}{\sum_{k=1}^{n} [V(q_k) - p_k]} [V(q_i) - p_i][p_i - C(q_i)],$$

or, in terms of consumers' surplus and value created,

$$\pi_{Ti}(p_i, q_i) = \alpha \frac{S_i^2}{\sum_{k=1}^{n} S_k} [V_C(q_i) - S_i] .$$

This formulation introduces a coupling between the variables related to a firm and that of the other firms. It models the impact of competition on the profit of an individual firm. In fact, the modelization takes into account the sum of the surplus offered by the competitors. The profit of an individual firm is affected similarly, whether the competitors are numerous or whether they offer a large surplus.

The unrealistic assumptions of perfect information and knowledge of the consumers made in the perfect competitive model are relaxed. A firm may offer a lower surplus than its competitors and still remain in business: we only assume that it sales are lowered.

6.2 Profit maximizing decisions

We will not consider the case where a collusion between the firms artificially maintains high prices, since this case is not very different from a monopoly situation. We study the case where each firm defines its optimum quality level and price, taking its competitors' quality levels and prices as given and fixed (no expected reaction from the other firms). This setting is similar to that of the Cournot model (Cournot, 1897), where the variables are prices and outputs. Setting the partial derivatives of π_{Ti} to zero yields the following equations:

$$\left[S_i - 2\sum_{k=1}^{n} S_k\right][p_i - C(q_i)] = -S_i\sum_{k=1}^{n} S_k,$$

and

$$\left[S_i - 2\sum_{k=1}^{n} S_k\right][p_i - C(q_i)] = \frac{-S_i\, C'(q_i)}{V'(q_i)}\sum_{k=1}^{n} S_k.$$

Combining these two equations reveals that q_i must satisfy:

$$C'(q_i) - V'(q_i) = 0.$$

The optimum value of q_i is once again the Pareto optimal solution.

Taking the quality level as a given (e.g., imposed by existing manufacturing equipment), the total profit is now only a function of the price or equivalently of the surplus. The profit maximizing surplus for firm i is now expressed by:

$$S_i = V(q_i) - p_i = \frac{1}{4}\left[V_C(q_i) - 3\sigma_i + \sqrt{\left[V_C(q_i) - 3\,\sigma_i\right]^2 + 16\, V_C(q_i)\,\sigma_i}\,\right], \tag{3}$$

where σ_i is the sum of the surplus offered by the other firms. It can be seen that when σ_i is zero, the profit maximizing price is equal to that of the monopolistic case. Furthermore, S_i is an increasing and p_i a decreasing function of the sum of the surplus offered by the competitors. Figure 5 represents the variations of S_i as a function of σ_i. In a competitive situation, a firm has not only to entice as many consumers as possible to buy the product but also it has to prevent them from buying from the competitors. This is done by offering a larger surplus than in the monopolistic case. It is interesting to notice that the introduction of a competitor imposes a sharp

increase in the profit maximizing surplus; still, no matter what is offered by the competition, the surplus offered reaches a limit (prices do not drop below a given value): the consumers can receive as much as two thirds of the value created in the case of strong competition (versus only one half in the monopolistic case). The well known benefits of economic competition for the consumers appear in this model as in previous models based on price and output.

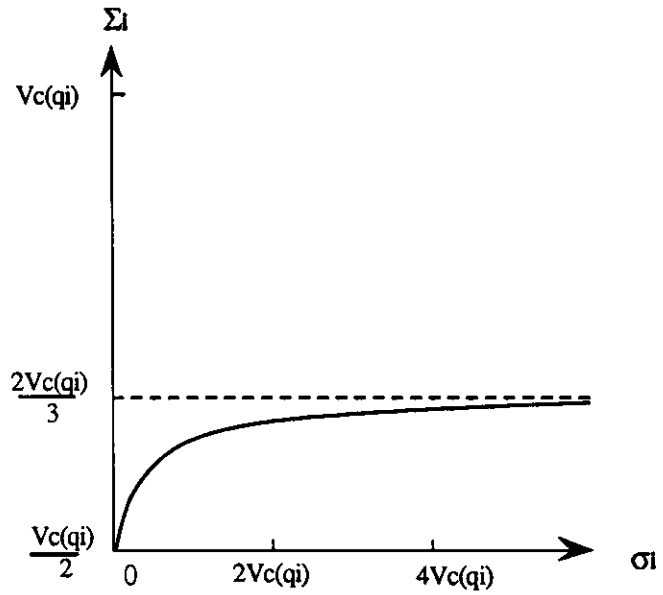

Figure 5: Firm i profit maximizing surplus as a reaction to the surplus offered by its competitors

6.3 Equilibrium

The profit maximizing scenario studied in section 6.2 implicitly defines a dynamic system: It was supposed that each firm decides on the surplus it offers to its customers assuming no reaction of the other firms. In fact, it is very likely that a reaction occurs, calling for an adjustment of the surplus initially chosen, thus inducing a new reaction and so on. The profit maximizing surplus of the n firms over a certain number of periods is the solution of a system of difference equations based on equation (3): if S_i^k denotes the surplus offered by firm i during period k, for each firm, we have

$$\begin{cases} S_i^{k+1} = \frac{1}{4}\left[Vc(q_i) - 3\sigma_i^k + \sqrt{(Vc(q_i) - 3\sigma_i^k)^2 + 16\, Vc(q_i)\, \sigma_i^k}\,\right] \\ \sigma_i^k = \sum_{j=1\ j\neq i}^{j=n} S_j^k. \end{cases}$$

The question of the convergence of the solutions towards an equilibrium state naturally arises.

Based on the results of section 6.2, the solutions of the system are necessarily bounded (see Figure 5); assuming that the surplus chosen in the first period is $\frac{V_c(q_i)}{2}$, the solutions can be recursively proved monotonically increasing. Therefore, the solutions are convergent. If σ_e denotes the sum of the surplus offered by all the firms at the equilibrium state, equation (3) squared can be rewritten as

$$- S_i^2 + [V_c(q_i) + 3\,\sigma_e\,]S_i - 2\,V_c(q_i)\sigma_e = 0, \tag{4}$$

and defines the limit of the solutions of the difference equations.

If all the competitors produce at the same quality level q, equation (4) is the same for all the firms, and since it only has one feasible solution (the other solution is larger than $V_c(q_i)$), the surplus at the equilibrium is the same for all firms. The value S_e of the common surplus offered by the n firms at the equilibrium, as given by equation (4) ($\sigma_e = nS_e$), is

$$S_e = \frac{2n-1}{3n-1} V_c(q).$$

The equilibrium surplus is a decreasing function of the number of competitors, and converges rapidly toward $\frac{2}{3} V_c(q)$. There again, consumers and manufacturers are better off if $V_c(q)$ is maximized.

The equilibrium point when the quality levels are different has a much more complicated expression. The unique feasible solution of equation (4) is

$$S_i = \frac{1}{2}\left[V_c(q_i) + 3\sigma_e - \sqrt{\left[V_c(q_i) - \sigma_e\right]^2 + 8\sigma_e^2}\,\right].$$

This expression does not provide the value of S_i since σ_e is undetermined. However, it reveals that the larger the value created, the larger the consumers' surplus. Figure 6 represents the reaction curves for a 2 competitor situation, with the equilibrium point. Firm 2 has a higher "value created" than Firm 1; at the equilibrium state, its surplus is larger than that of Firm 1. This indicates that a firm producing a lower value than its competitor is less profitable, and offers a lower surplus to its customers. In other words, even in a competition situation, the consumers are affected by the inability of Firm 1 to create as much value as Firm 2: indeed, the profit maximizing strategy for Firm 1 is to make a lower profit per item than Firm 2, while offering a lower surplus to its customers than Firm 2. Both Firm 1 and its customers incur the value loss. Moreover the customers of Firm 2 are also affected since Firm 2 takes advantage of the situation to lower the surplus it offers (compared to the situation with identical quality levels for all firms).

When the competition is very strong *i.e.*, when σ_e is large compared to $V(q_i)$, S_i gets close to $\frac{2}{3}V(q_i)$, which is the same limit as in the case where all the quality levels are equal. Quality differences are not completely compensated, and the impact of the value loss on the manufacturers is less than in the monopolistic case.

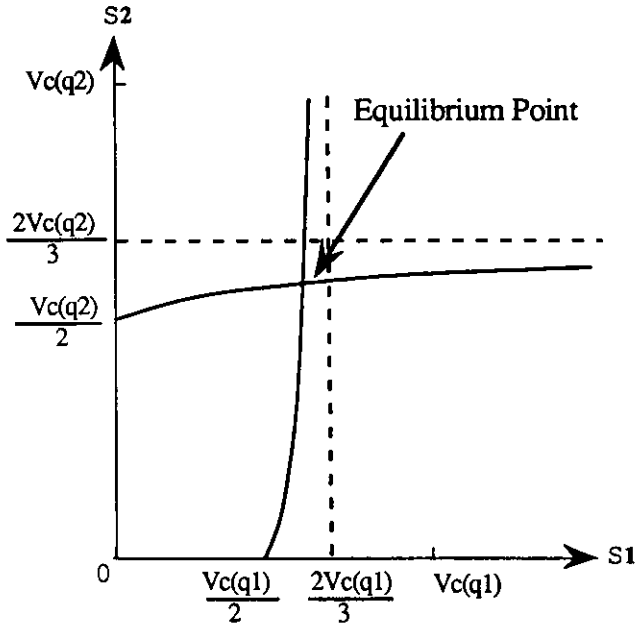

Figure 6: Reaction curves in a two competitor situation

Conclusion

In any of the situations studied, there is an incentive for a manufacturer to produce at the Pareto efficient quality level: q* maximizes the profit in any situation. However, a simple observation of the economic reality shows that there are quality differences between manufacturers, therefore some productions do not have the q* quality level. Two major reasons can be found; on one hand, a manufacturer may not be willing or able to offer the right quality level because of the investments and the technology required. On the other hand, the strength of the incentive to reach the q* quality level varies according to the type of market considered and the behavior of the consumers. The incentive is the strongest in the perfect competitive market: consumers' information, knowledge, responsiveness and possibilities of choice enforce fair compensations for quality differences. The value loss is entirely incurred by the manufacturer.

When the assumptions of perfect knowledge, information, and responsiveness are relaxed and the possibilities of choice reduced, the model suggests that quality differences are not fairly compensated. In the case of a monopoly, the manufacturer and the consumers equally share the value created. The incentive for a manufacturer to produce at the Pareto efficient quality level is reduced. Introducing some competition results in an increased share of the value created for the consumers, but quality differences are compensated only partially. However, another aspect of competition is the increased pace of innovation that changes the cost function over time. Since a cost reduction is more likely to occur in the high quality range, we can assume that the Pareto efficient quality level increases progressively. Consequently, making quality improvements becomes necessary at some point for all manufacturers in order to keep up with an innovative leader.

Acknowledgements

The authors are grateful to Richard M. Cyert, Kenneth Kotovsky, and Herbert A. Simon for their comments on this manuscript.

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