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Industrial Quality:
A Missing Economic Variable

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Industrial Quality: A Missing Economic Variable

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Abstract

Industrial quality is presented as a direct result of manufacturing practices (e.g. tolerance allocation, process selection, inspection procedures). A quality indicator is developed that allows for the computation of a function representing the cost of an industrial product versus its quality level. This cost function is then incorporated in an economic model that estimates the consumer's demand for the product as a function of both its price and quality level. The profit maximizing values of the price and quality level are derived, and in turn indicate the optimum production mode.

Introduction

More than ever, industrial quality is one of the critical elements in the fierce competition between manufacturing companies. From an engineering standpoint as well as from a management standpoint, quality is a challenge that cannot be ignored. Yet, one striking feature of most economic models in firm competition theory is the absence of quality considerations. The prevailing assumption is that competing products are standard commodities for which the consumer's demand is a function only of the price charged by the manufacturer. A possible explanation is that quality is often thought of as a qualitative variable, and is therefore difficult to include in an economic analysis: the benefits in terms of market share or profit resulting from tightening a process or screening a production cannot be easily estimated. For that matter, our paper presents a model *quantifying* quality at the engineering level in order to provide an evaluation of its impact on the financial results of a firm. The methodology can be used for assessing manufacturing practices as well as for guiding company policy and resource allocation. The model is exemplified by the study of the production of a simple three component assembly.

1. The Cost of Quality

1.1 What Is Industrial Quality ?

Since the beginning of mass production, manufacturers have been struggling with the variability inherent in any manufacturing operation. Because no production process can produce any two items that are exactly identical, there is a need to spot and isolate "defective products". On one hand the basic requirement for mass production is that all

parts of a kind must be interchangeable and functionally equivalent, yet, on the other hand, it is not necessary or cost effective to try to make things exactly alike. It is therefore common to permit constrained variations in part characteristics by allocating tolerances and removing out-of-tolerance parts. The most widely used format for these specifications is

$$\text{Nominal Value} \pm \text{Tolerance.}$$

Traditionally, quality was defined as conformance to specifications where manufacturers measure the quality of their production by the percentage of defective items produced. This approach is reasonable when defective items have a significant probability of being shipped to customers. When the rate of defects is reduced significantly, or when the final products are inspected, quality cannot be assessed solely by the percentage of defective items produced; this measurement does not differentiate between items that possess characteristics near to the design nominal values and those that, despite being within specifications, deviate significantly from those target values. In many cases, the nominal value is that which provides the desired performance and, thus, the best product (Taguchi, 86). Tolerances exist only to limit the degradation of the performance. An item off target may involve later costs because is it more likely to break down than a product that has parameters closer to the target values. Therefore, it is important to determine if a product is within specifications, and if so, how far it is from the target value. Because we address mass production problems, it is necessary to be able to assess the quality of an entire population of products. It seems therefore natural to study a production with statistical means and estimate the average deviation of the characteristics of the products with respect to the target value. Considering the characteristic of a product as a random variable, the standard deviation of this variable provides a reasonable amount of information on the variable spread. Therefore, rather than the percentage of defects, we will use as an indicator of quality, the ratio

$$\text{Quality Indicator} = \frac{\sigma_{out}}{\Delta}, \tag{E1}$$

commonly referred to as process capability, where σ_{out} is the standard deviation of the population of products shipped to the customers and Δ the tolerance. It is clear that the smaller this ratio, the better the quality of the products.

1.2 The Cost of Producing at a Given Quality Level

As described in the previous section, process variability is a major cause of quality problems in manufacturing. The performance or characteristics of the assemblies produced

may vary and deteriorate because it is impossible to make all similar parts exactly alike. This degradation can be reduced if the part characteristics vary within constrained limits. Thus, it is believed that quality may be improved by improving the accuracy of the manufacturing processes, minimizing variations. Unfortunately, an accurate process is more likely to require more expensive machines, more complex setups, more maintenance, longer processing times, and better skilled workers; *the more precise a process, the higher the cost per part.* Several papers have presented the machining cost as a function of the tolerance held on a part (e.g. Peat 68, Spotts 73, Wilde and Prentice 75, Lee and Woo 90). Various representations of the cost of an individual item have been used. Generally, the cost is decomposed into the sum of a constant part (Fixed cost, cost of raw material, F_i) and a variable part (cost of holding the tolerance Δ_i). The functions most commonly used are:

$$C_i(\Delta_i) = F_i + \frac{\alpha_i}{\Delta_i^2}, \text{ or } C_i(\Delta_i) = F_i + \frac{\alpha_i}{(\beta_i - \Delta_i)}, \text{ or } C_i(\Delta_i) = F_i + \lambda_i e^{-(\Delta_i/\tau_i)},$$

where α_i, β_i, λ_i and τ_i are problem dependent parameters. We believe that this representation may camouflage part of the problem because the tolerance on a part is often an arbitrary number, chosen by the designer, that has nothing to do with the process itself. This model is based on the implicit, arbitrary relationship between the tolerance and the yield of the process (i.e. the famous relationship $\Delta = 3\sigma$) on which the theory of statistical process control is based. In fact, there is no reason why there should be a constant linear relationship between Δ and σ. Moreover, it is very likely that the coefficient 3 was originally chosen only because it is simple and "probably" not too far from the optimal.

A better model would view machining cost as a function of the process output spread, rather than the tolerance on the part. Since we are addressing industrial mass production, processes produce part populations that are approximately spread about a nominal target value: unavoidable sporadic trends are eventually detected, corrected and averaged to the nominal value. Because of the high quantities produced it is also reasonable to assume that the parts are normally distributed. The standard deviation of the part population can then be easily estimated, and constitutes an intrinsic representation of the process quality. Therefore, our model treats the cost of machining a part with a given process as a decreasing function of the *standard deviation* of the part population produced by the process. There is usually a best possible accuracy for which the cost is maximum, and the cost decreases to become almost constant. Tolerances and process accuracies are

thus decoupled independent variables. Once their values are determined, the resulting scrap rates can be calculated.

1.3 The Cost of Inspecting

Inspecting is the second means that can be employed to assure consistency in production quality. The idea is to allow constrained variations in part dimensions by allocating tolerances and removing out-of-tolerance parts. The allocation of these tolerances has a dramatic impact on the cost and quality of the final product. By the old quality standards (percentage of defective items produced) inspection improves the quality of a product since it removes defects. However, when quality is more accurately measured by the ratio $\frac{\sigma_{out}}{\Delta}$, inspecting a production does not really change its average quality level unless the rate of defect is important (Taguchi 86). This is particularly true when inspection is performed only on the final products, but as will be seen in the example, locating inspection at various production steps may significantly affect the distribution of the final products and thus their quality.

Inspection is a costly procedure involving two separate costs. The first is a fixed cost and corresponds to the wages and equipment of the people that actually inspect the parts. This cost is proportional to the number of inspections performed. The second type of cost is due to the loss incurred when a part, sub-assembly, or final assembly is found defective and has to be scrapped. This loss is equal to the value of the raw materials used to produce the defective item, plus the value that has been added to the material during its manufacture. Based on this, it would seem suitable to locate the inspection at the beginning of the fabrication process in order to avoid scrapping a part with substantial added value. Similarly, it would seem that assemblies should be made of reliable components in order to avoid scrapping an entire assembly because of a single defective component.

One approach, therefore, is to allocate tolerances on the components in such a way that an assembly made out of components within specifications is never defective. Stringent inspection procedures at the beginning of the fabrication could, therefore, assure essentially defect free assemblies. This strategy results in considerable costs because of the random nature of the distribution of the part populations. Indeed, in many cases, there are tolerance accumulation effects: a part too short on its tolerance has a significant chance of being assembled with another part that will make up for the defect of the first one. The probability for an assembly to actually be made out of parts, each at the limit of the tolerances, is very low. Thus, it may not be cost effective to allocate tolerances based on

the unlikely occurrence that all of the actual dimensions are at the limits of the specified tolerance zone; the interactions between geometric deviations which may negate each other must be considered. Therefore, a cost-effective tolerance allocation procedure may permit a percentage of defects in the final assemblies; a properly chosen manufacturing process will then yield a low defect percentage.

It must be understood that inspection has a double purpose: it is used to generate feedback on the behavior of a process and to filter out bad parts. The first type of inspection requires only sampling procedures providing estimations used to check whether a process is under control or not, and possibly take a corrective action. The use of this inspection type is always recommended; the study of optimal sampling procedures is out of the scope of this paper (some important results can be found in Taguchi 86, 89). This paper deals with the second type of inspection, used when the defect rate of a process is too high to be acceptable.

Two remarks on inspection are in order. First, because of economic constraints it is not always recommended that inspection occur after all of the stages of the manufacturing process. Rather, inspection procedures should be employed at strategic points in the production process. The example presented below considers four possible scenarios for inspection of assembled products: *i.)* no inspection at all, *ii.)* inspection of the components *only*, *iii.)* inspection of the assemblies *only*, *iv.)* inspection of both components and assemblies. To determine the optimal inspection strategy, mathematical models of the assembly fabrication are required. Once these models are defined, optimal use of inspection systems may be determined by cost considerations. Second, tolerance allocation and inspection procedures are intimately related and must be determined concurrently for an optimal strategy. This also enhances the need for a mathematical model of the fabrication.

1.4 The Key Cost Function

Any attempt to incorporate quality in an economic model requires a cost analysis. For the quality indicator we defined (E1), it is necessary to have an estimation of the production cost for all feasible values of the ratio $\frac{\sigma_{out}}{\Delta}$. Except for products made of a single component, product performance Y is usually influenced by several variables x_1, x_2, ...x_n, corresponding to the characteristics of different components of the product. A mathematical model is necessary to define the influence of each component on product performance:

$$Y = F(x_1, x_2, \ldots x_n).$$

Since the x_i's are random variables, Y is also random. Each x_i has a probability density function which depends on the manufacturing process and possibly the inspection strategy used to produce component i. Once the production mode of each component is determined, the probability density function of Y is theoretically known as well as its standard deviation σ. In fact, except for special cases (e.g. F is a linear function), statistical difficulties arise when computing σ and it may be necessary to linearize F or use other methods like Monte Carlo simulations (Grossman 76). The computation of the cost function

$$\text{Cost} = C\left(\frac{\sigma_{out}}{\Delta}\right) \tag{E2}$$

is therefore not a simple task but this cost function is the most important one. It is the only tool to assess the benefits of tightening a process or inspecting a production. Depending on production opportunities, C may be continuous or discrete.

2. The Model

We investigate the case where customer expectations or requirements for a given industrial product can be measured in terms of performance. The performance can be the geometry of the product, or any other measurable characteristic (e.g. speed, torque, power, composition, hardness, noise, weight). It must be clear that this paper addresses quality in the manufacturing domain. We do not address the problem of quality of design. The design of the product is assumed given and we want to determine the best possible way to manufacture it. Extensions to concurrent design issues are considered for future work. As stated in a previous section, the assumption of mass production is needed to provide meaningful average process accuracies.

2.1 Benefits of Quality

The ratio $\frac{\sigma_{out}}{\Delta}$ is useful to compare different quality levels but still does not allow for a monetary quantification of the quality benefits. This ratio can be used for a cost comparison between quality levels using the cost function developed above (E2) but this information is not sufficient to determine the optimal quality level from an economic standpoint. In order to do so, it is necessary to relate the quality and price of a product to the customer's demand for this product. A simple observation of the market for most industrial products reveals that different manufacturing firms selling roughly the same products have different market shares, depending on the price and quality of their products.

Given different qualities, we believe that advertising campaigns can induce short term fluctuations but cannot account for differences that persist in the long run. Therefore a firm has to take account of demand behavior when determining its pricing *and* production policies.

Let us denote by N the demand for a given product, in a given market. N denotes the number of products the consumers are willing to purchase, given the price charged by the firm *and* of the average quality level of the products. The price paid by a customer cannot exceed the highest price he would be willing to pay rather than make do without the product. *"The excess price which he would be willing to pay rather than go without the thing, over that which he actually does pay, is the economic measure of [the] surplus satisfaction. It may be called consumer's surplus"* (Marshall 47). Let us denote by V the first price for which the consumers surplus is 0. For this price called the resistance price, the consumer's demand vanishes: a transaction at this price does not make any consumer better off. This resistance price can be considered as the value of the product (Cook 91). V, as a measure of the value given by customers to a product is a function of the average quality level of the product. Let us compute an approximation of the function V for an individual product. If the product performance Y deviates from the target value T, a second order Taylor expansion of V about T provides the following approximation:

$$V(Y) \approx V(T) + V'(T)(Y-T) + \frac{V''(T)}{2!}(Y-T)^2.$$

Two relationships allow for the determination of the derivatives of V: because V is maximum for $Y = T$, $V'(T) = 0$. For $Y = T + \Delta$, the product is defective therefore its value is 0, hence the approximation for V:

$$V(Y) = V(T) - V(T)\left[\frac{(Y-T)^2}{\Delta^2}\right].$$

Because we are addressing mass production issues, we will consider the expected value of V:

$$E[V(Y)] = V_0\left[1 - \frac{\sigma_{out}^2}{\Delta^2}\right],$$

where Vo is the maximum possible product value. The average product value can be seen as a function of the quality indicator $\frac{\sigma_{out}}{\Delta}$:

$$V(\frac{\sigma_{out}}{\Delta}) = V_o\left[1 - \frac{\sigma_{out}^2}{\Delta^2}\right].$$

If P is the price of a product, the consumer's surplus becomes

$$V - P = V_o\left[1 - \frac{\sigma_{out}^2}{\Delta^2}\right] - P.$$

This approach is similar to that taken by Taguchi (Taguchi 86) to model his loss function. When quality is not modelled, consumer's choices are based on prices alone since maximizing the surplus means picking the lowest price. Surplus maximizing when quality is odelled, involves a maximization of V - P, and therefore does not necessarily favor the lowest price.

2.2 Case of the Monopolistic Firm

Let us consider a market dominated by a single enterprise manufacturing and selling a single product. Let us determine the profit maximizing values of the price P and the average quality level $\frac{\sigma_{out}}{\Delta}$ of the product. If Fc is the fixed cost of production, N the demand and C the variable cost of production of a single product, the profit π of the firm can be expressed as

$$\pi(P, \frac{\sigma_{out}}{\Delta}) = N\left[P - C(\frac{\sigma_{out}}{\Delta})\right] - Fc. \tag{E3}$$

To keep the model simple, we describe the demand as a linear function of the consumer's surplus:

$$N = \alpha\left[V_o\left[1 - \frac{\sigma_{out}^2}{\Delta^2}\right] - P\right].$$

Under the assumption of differentiability for C, the profit maximizing values have to satisfy:

$$\frac{\partial\pi}{\partial P} = \alpha\left[V(\frac{\sigma_{out}}{\Delta}) + C(\frac{\sigma_{out}}{\Delta}) - 2P\right] = 0,$$

which sets the optimal price at

$$P = \frac{1}{2}\left[V(\frac{\sigma_{out}}{\Delta}) + C(\frac{\sigma_{out}}{\Delta})\right], \tag{E4}$$

and

$$\frac{\partial \pi}{\partial \frac{\sigma_{out}}{\Delta}} = \frac{\alpha}{2}\left[V(\frac{\sigma_{out}}{\Delta}) - C(\frac{\sigma_{out}}{\Delta})\right]\left[V'(\frac{\sigma_{out}}{\Delta}) - C'(\frac{\sigma_{out}}{\Delta})\right] = 0,$$

which sets the quality level at the value of $\frac{\sigma_{out}}{\Delta}$ for which the expression

$$\left[V'(\frac{\sigma_{out}}{\Delta}) - C'(\frac{\sigma_{out}}{\Delta})\right] \tag{E5}$$

is zero ($V - C = 0$ is obviously not a profit maximizing solution). The optimal quality level maximizes the value created per product, which for the value of the price P defined above is nothing but the consumer's surplus.

3. Example Application

The example consists of the study of the industrial production of a set of three friction wheels. The wheels have different diameters and, therefore, different production costs. Each wheel, from largest to smallest, has a given nominal diameter of: 4, 3, and 1 inches. The assembly specifications require that its total length must be the sum of the three nominal wheel diameters, with a given overall tolerance, Δ, of 0.2 inches. The quality of the product depends on the deviation of the average length of an assembly from the nominal value. Each wheel is manufactured with an accuracy that has to be determined (see Figure 1), and that influences the distribution of the assembly lengths. We use the model developed in the previous section to determine i) the optimal quality level of this product and ii) the cheapest way to reach this quality level.

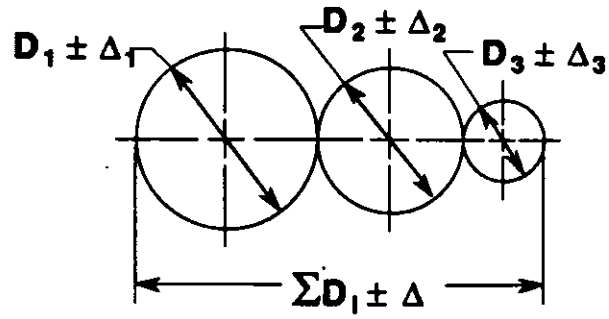

Figure 1: System of friction wheels.

The wheels can be manufactured by three different processes: sawing, turning on a lathe, or grinding. For each of those processes, there is a best possible accuracy involving the highest cost. Decreasing the processing times decreases the accuracy as well as the cost per part. Each machine is best suited for a given range of accuracy over which the manufacturing cost per part can be represented as a hyperbolic function of the standard deviation of the population of parts produced (see Figure 2) (Cagan and Kurfess, 91). The global curve representing the wheel manufacturing cost as a function of process accuracy is found by taking the minimum of the three possible costs for any accuracy. Each wheel has its own cost function due to the different sizes of the wheels: the larger the diameter, the more expensive the part. We are addressing a two step production process of manufacturing components and assembling them; therefore, inspection may or may not be utilized. If employed, it can take place after manufacturing the components, after assembling the wheels, or after both. For each strategy, we determine the cost, C, of an assembly within specifications by summing the costs of machining the individual wheels, $C_i(\sigma_i)$, with a process of accuracy σ_i. When inspection is performed, additional costs are taken into account: cost of scrapping and cost of inspecting. The cost of inspecting an individual wheel is almost negligible ($ 0.05) since it can be done with a go-no go type gauge. Inspecting an assembly is more time consuming and therefore more expensive ($0.20). We denote the tolerance on the ith wheel by Δ_i, the variance of the population of the final assemblies σ^2 and that of the final assemblies shipped by σ^2_{out} (these two variances may be different due to inspection). The populations of parts are assumed to be normally distributed about their nominal or target diameters.

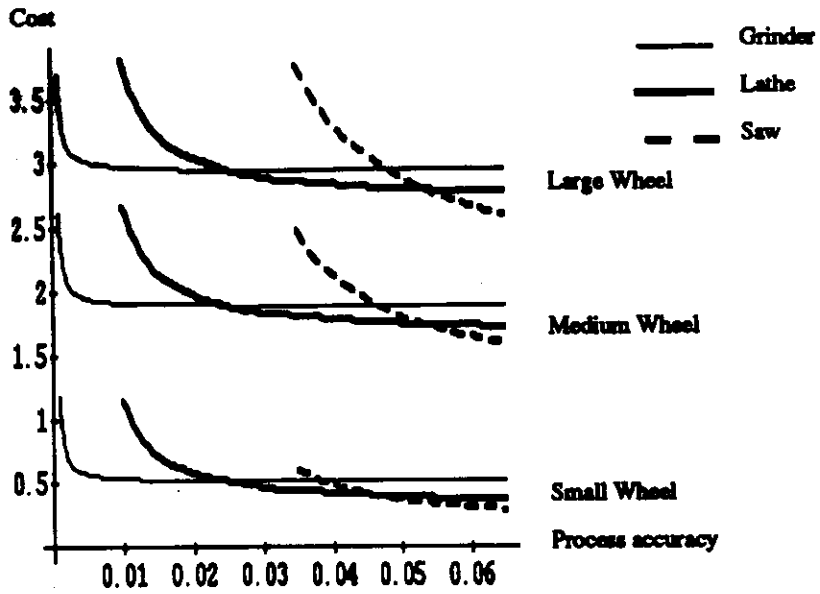

Figure 2: Cost functions of the components for different processes

We need to develop a cost function representing the minimum possible cost of production as a function of the quality level $\frac{\sigma_{out}}{\Delta}$. Each point on this curve can be found by minimizing the cost of production under a constraint fixing the value of $\frac{\sigma_{out}}{\Delta}$. The assembly can be produced using different inspection strategies for which the cost of production has different expressions. Therefore, we use a two step minimization method: the first step minimizes the cost of production for a given inspection strategy, and the second step consists of choosing the lowest cost over all the inspection strategies. Because of the complexity or non-differentiability of certain functions, the stochastic optimization technique of simulated annealing (Kirkpatrick 83) is selected to perform the optimization.

Inspection of the Final Assemblies Only

This strategy requires inspection of the final assemblies only. Components are manufactured, assembled and the assemblies are tested. A defective assembly is entirely scrapped, no matter what causes the defect. This model consists of compounding independent normal distributions. The resulting distribution is truncated to simulate the inspection. Since there is no inspection of the components, there is no need for assigning tolerances on the components. Therefore, the only variables are the σ_i's (accuracies of the processes with which the parts are manufactured). The cost of an assembly (good or bad) is the cost of production of the components and the inspection cost. The proportion of

good assemblies produced, A, is a function of Δ and σ. If Ic denotes the fixed inspection cost, the cost of an assembly within specifications is

$$C = \frac{1}{A(\sigma, \Delta)}\left[\sum_{i=1}^{3} C_i(\sigma_i) + Ic\right].$$

The inspection also changes the population variance of the final assemblies. The population variance of the assemblies produced is

$$\sigma^2 = \sum_{i=1}^{3} \sigma_i^2 ,$$

but after inspection, it can be proven that the new variance is

$$\sigma_{out}^2 = \sigma^2 - \frac{2\Delta\sigma}{A(\sigma,\Delta)\sqrt{2\pi}} e^{-(\Delta^2/2\sigma^2)} .$$

Therefore, the optimization problem consists of finding the values of the σ_i's that minimize C, under the constraint that σ_{out} is given. Once the values of the σ_i's are determined, the processes to be used are easily found.

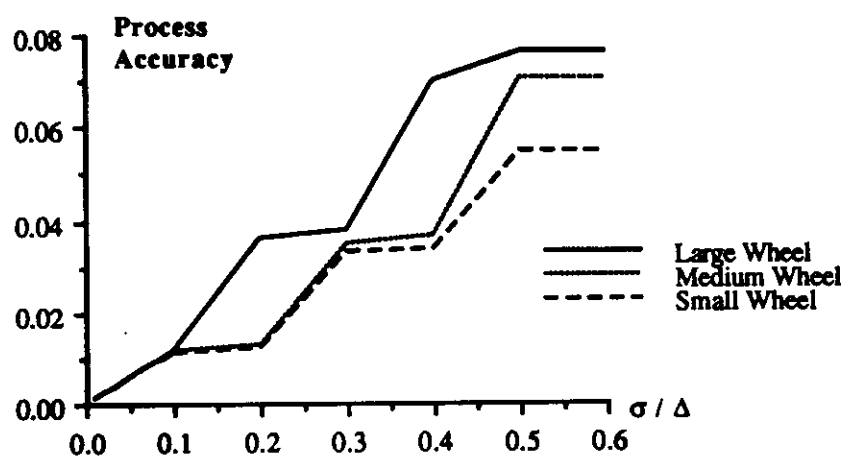

Figure 3: Optimal process accuracies versus the quality level of the assembly

The results of the optimization for this problem are presented in Figures 3, 4 and 5. Figure 3 shows some trends common to all the inspection strategies. The components with the highest manufacturing costs (the largest wheels) are manufactured with the cheapest processes (the less accurate), and the components with the lowest manufacturing costs are

required to be accurately manufactured. This is due to the fact that it is cheaper to hold tighter tolerances on the smaller components. Note that the abrupt changes in the process accuracies in Figure 3 relate to production process changes (from grinding to turning to saw cutting with increasing overall tolerance).

As shown in Figure 5, the cost of producing high quality items (small ratio $\frac{\sigma_{out}}{\Delta}$) decreases as $\frac{\sigma_{out}}{\Delta}$ increases, up to a point where the manufacturing cost becomes constant (Figure 2), whereas the scrap rate increases sharply (Figure 4), which causes the production cost to go up again. The use of normal distributions (with infinite tails) probably leads to an overestimation of the scrap rate. Still, it is interesting to notice that the optimal values for the process accuracies sometimes involve a significant scrap rate (see Figure 4).

Zero Defect Output

Having a final output without any defect is the recent trend in industry. In our case, it can be achieved if and only if any combination of components meant for assembly provides an assembly within specifications. Therefore, any component that may lead to a defective assembly must be removed before the assembly stage. This requires a total inspection of the components. The variables that can be controlled are the accuracy of the manufacturing processes used and the tolerance assigned to the components. The cost of the ith wheel is:

$$\frac{C_i(\sigma_i) + (Ic_i)}{A(\sigma_i, \Delta_i)},$$

where $A(\sigma_i, \Delta_i)$ is the proportion of wheels that pass the inspection, and Ic_i the fixed cost of inspection. From a statistical point of view, this model consists of compounding truncated normal distributions. It can be proven that the variance of the final assembly population is

$$\sigma_{out}^2 = \sum_{i=1}^{3} \left[\sigma_i^2 - \frac{2\Delta_i \sigma_i}{A(\sigma_i, \Delta_i)\sqrt{2\pi}} e^{-(\Delta_i^2/2\sigma_i^2)} \right].$$

The optimization problem consists of finding the values of the σ_i's and Δ_i's that minimize

$$C = \sum_{i=1}^{3} \frac{C_i(\sigma_i) + (I_{ci})}{A(\sigma_i, \Delta_i)},$$

under the constraint that σ_{out} is given. The zero defect requirement induces a constraint stipulating that the sum of the tolerances on the components has to be less than the overall tolerance. The minimization of C shows that the zero defect requirement generates large costs. It is an expensive strategy because it requires accurate processes while resulting in substantial scrap rates. It is a worst case analysis: the tolerance allocation is based on the unlikely assumption that all the components of an assembly can be at the limit of the allowed variations.

Inspection of the Final Assemblies and Components

In this scenario, the components are inspected as well as the final assemblies. The control variables are the accuracies of the manufacturing processes used to make the components (σ_i's), and the tolerances allocated to each component (Δ_i's). The model compounds truncated normal distributions into a resulting distribution that also must be truncated. The resulting distribution cannot be simply expressed and must be numerically computed.

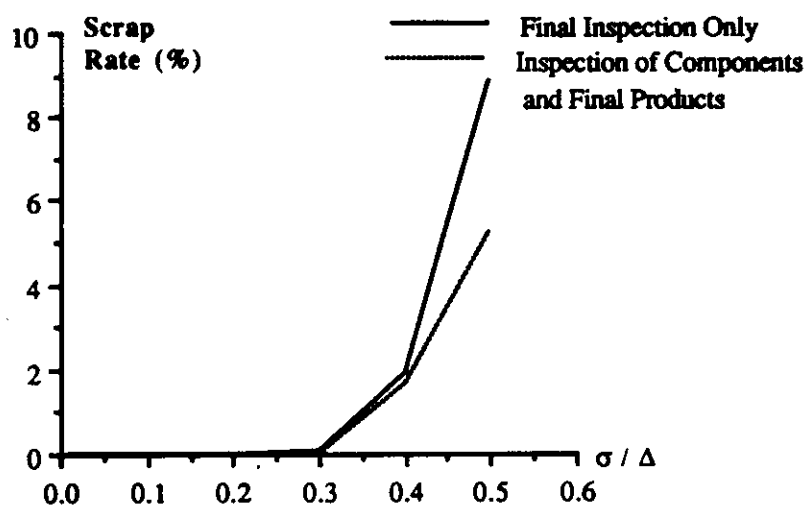

Figure 4: Percentage of defective items for final assembly inspection

The cost minimization gives interesting results. This inspection strategy provides the lowest manufacturing cost of all the strategies modeled when viewed without fixed inspection cost. However, burdened by the fixed inspection costs, this strategy is efficient only for low quality items (see Figure 5). The optimal values for the process accuracies are similar to that of the previous strategies. The difference is, therefore, due to the inspection.

The scrap rate of assemblies in the case of double inspection is much lower than in the other case (see Figure 4). This is due to the fact that the first inspection removes the components that deviate from target significantly and that are likely to result in a defective assembly. Concerning the components, the more expensive the parts, the lower the scrap rate.

The global cost versus quality curve

The global cost versus quality curve is simply obtained by taking the minimum production cost over the three strategies, for any given value of $\frac{\sigma_{out}}{\Delta}$. For high quality assemblies ($\frac{\sigma_{out}}{\Delta} < 0.2$), no inspection is required since the values of the scrap rates are 0. Inspection in this case can be performed as a control procedure to make sure that the processes behave as they are supposed to, but inspection is not part of the actual production process. The differences between the three curves are due only to the fixed inspection costs. For quality levels between 0.2 and 0.275, inspecting the components only is the best procedure. Then, inspecting the final products is the optimal strategy until the quality level reaches 0.55 where a double inspection is necessary. It is interesting to notice that the higher the quality, the less inspection is necessary.

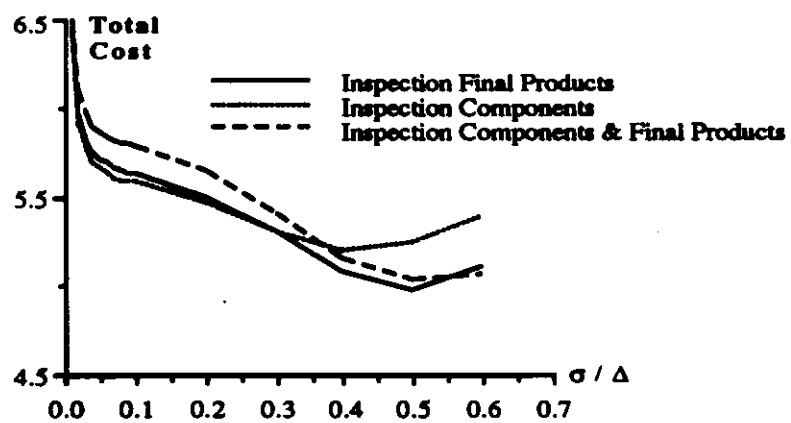

Figure 5: Cost versus quality for various inspection policies

Determination of the optimal quality level

The determination of the optimal quality level requires the value of the resistance price of the assembly (first price for which the demand is zero). The friction wheels

assembly is used as a component of high precision machines, to avoid the backlash produced by regular gears. The same desired effect can be obtained with high precision gears but at a higher cost. It can therefore be assumed that if the price of the friction wheels reached that of high precision gears, the demand would fall dramatically. Let us assume that the price of high precision gears is $20.00 which can then be taken as the resistance price for the friction wheels. Because of the non differentiability of the cost function $C(\frac{\sigma_{out}}{\Delta})$, the profit is directly maximized using (E3). For each value of the ratio $\frac{\sigma_{out}}{\Delta}$, the profit is a function only of the price and is maximum for

$$P = \frac{1}{2}\left[V(\frac{\sigma_{out}}{\Delta}) + C(\frac{\sigma_{out}}{\Delta}) \right],$$

as found in (E4). Since the function $C(\frac{\sigma_{out}}{\Delta})$ had to be discretized for computation, the maximum profit is computed for each value of the ratio $\frac{\sigma_{out}}{\Delta}$. The highest profit is still reached for the quality level that maximizes V - C, as given in (E5) for a differentiable function. The optimal quality level found is around $\frac{\sigma_{out}}{\Delta} = 0.1$ (very high quality level). Therefore, the assembly should be manufactured with a grinder with this high precision. Inspecting the parts and/or the final products is not necessary. The optimal price to charge is $12.6.

Concluding Remarks

We developed a model that allows for quantification of industrial quality by considering the statistical distribution of a production. The potential impact of inspection, tolerance allocation and process selection on the quality of a product was illustrated in the friction wheels example. Moreover, the consumer's behavior is modeled as a maximization of the surplus, function of price and quality. The producer's decisions for producing and pricing a product can be easily related to the consumer's behavior. This model is therefore a useful extension of conventional price theory for which industrial products are standard commodities with a constant quality level.

The scope of the model is limited to products with precise specifications. The format studied in this paper was the classical

Nominal Value ± Tolerance,

but other formats can be investigated (e.g. one-sided tolerances, tolerances with different boundaries), using the formalism of the Taguchi methods. A similar approach can also be developed for products with several specifications.

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