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Abstract

The valuation of the real option to store liquefied natural gas (LNG) at the downstream terminal of an LNG value chain is an important problem in practice. As the exact valuation of this real option is computationally intractable, we develop a novel and tractable heuristic model for its strategic valuation that integrates models of LNG shipping, natural gas price evolution, and inventory control and sale into the wholesale natural gas market. We incorporate real and estimated data to quantify the value of this real option and its dependence on the throughput of an LNG chain, the type of price variability, the type of inventory control policy employed, and the level of stochastic variability in both the shipping model and the natural gas price model used. In addition, we develop an imperfect information dual upper bound to assess the effectiveness of our heuristic, and find that our method is highly accurate. Our approach also has potential relevance to value the real option to store other commodities in facilities located downstream from a commodity production or transportation stage, such as petroleum and agricultural products, chemicals, and metals, or the real option to store the input used in the production of a commodity, such as electricity.

1 Introduction

Liquefied natural gas (LNG) is natural gas cooled to liquid state at -260F; liquefaction reduces the volume of natural gas by a factor of more than 600, making storage and shipping practical (Greenwald [26], Tusiani and Shearer [51]). Special ocean going vessels load LNG at liquefaction facilities (for example in Trinidad and Tobago, Australia or Qatar), transport it (for days or weeks), and unload it at terminals (for example in the U.S., Europe or Japan). At these terminals LNG is pumped into storage tanks, regasified, and then distributed via pipelines or, sometimes, by trucks.

The Energy Information Administration (EIA) has projected that local production of natural gas will soon be unable to meet its increasing demand in several industrialized countries, and expects LNG imports to play an important role in bridging this gap (EIA [20, 21]). This long term projected increase in the world's natural gas demand is primarily due to natural gas being a relatively environmentally clean and abundant fuel, which has helped to make it the fuel of choice for many new power generation projects. Many of these long term forecasts predate the current economic recession, however, as of March 2009 EIA (EIA [22]) forecasts that in 2018 U.S. LNG imports will peak at 4.9 times their 2008 levels before declining to 2.7 times these levels in 2030.

We have started to see the unfolding of some of these predicted increases, which Jensen [31] refers to as the "LNG revolution." Several liquefaction capacity expansion and greenfield projects have been announced, a number of new terminals have been proposed in North America, and some of these have been recently completed. Obtaining access to these terminals is necessary to bring LNG into the natural gas distribution system, and requires leasing storage space and regasification capacity from the terminal's operating companies. Hence, industry players face the challenge of assessing the value of downstream terminal leasing contracts.

The value of such a contract consists of the delivery value and the storage value. In this paper, we focus on determining the storage value, which requires valuing the real option to store LNG at a regasification terminal. This topic has not yet been studied in the literature and may not be well understood in practice. For instance, Holcomb [29] attributes little storage value to LNG regasification terminals, but Cheniere Energy (www.cheniere.com) attributes strategic importance to LNG storage at such terminals, having embarked on the construction of a network of LNG terminals along the U.S. Gulf Coast that, once completed, will feature the largest availability of LNG storage and regasification capacity in the U.S.

Our interest in this paper is the valuation of the real option to store LNG at regasification facilities in the presence of a wholesale market for natural gas, which is the case in the U.S., U.K., and some parts of Europe (Tusiani and Shearer [51, p. 26]). Exact valuation of this real option is computationally intractable, especially using an operational model (because of its fine time scale). Thus, we take a strategic approach and develop a novel and tractable model for the heuristic valuation of this real option. Our approach integrates models of shipping, commodity price evolution, inventory control, and storage valuation based on closed queueing networks (CQNs), lattice approximations of Ito processes, a Markov decision process (MDP), and Monte Carlo simulation, respectively. Specifically, we extend the CQN model of LNG shipping of Koenigsberg and Lam [33] and integrate it with (1) lattice approximations of the commodity price models of Jaillet et al. [30] and Schwartz and Smith [44], modeling seasonality as in Jaillet et al. [30], and (2) an inventory control MDP, whose policy is evaluated by simulation. We also develop an imperfect information dual upper bound (Brown et al. [11]) to assess the effectiveness of our practical heuristic model.

We apply our model in a numerical study to estimate and analyze the storage value. We consider a realistic LNG chain consisting of liquefaction in North Africa, shipping to Lake Charles, Louisiana, and regasification and sale into the Louisiana natural gas wholesale market using the spot price at Henry Hub, the delivery location of the New York Mercantile Exchange (NYMEX) natural gas futures contract. We calibrate the price evolution models to prices of traded NYMEX natural gas futures and options on futures. This application provides the following findings:

(1) The estimates of the value of the real option to store during 12 years range from $89M to $726M, when the available storage space and LNG chain nominal throughput (ignoring congestion) vary from 3 billion cubic feet (BCF) and 0.7536 million tons per annum (MTPA) to 24BCF and 7.5362MTPA, respectively, and the regasification capacity is 2BCF/Day (Table 4 in Online Appendix A reports relevant units of measurements and conversion factors); this takes between 5 and 53 Cpu minutes, depending on the LNG chain configuration.

(2) Comparisons with our upper bound estimates indicate that our model's valuations are very accurate, being on average more than 99% of the upper bound estimates.

(3) The value of the real option to store can be nonmonotonic in the throughput of the LNG chain, which with homogeneous ships is the number of LNG cargos delivered per unit time. This implies that discretionary regasification capacity is necessary for LNG storage to be most valuable, and suggests that the discussion in Holcomb [29] is mainly relevant to situations when the regasification capacity is comparable to throughput.

(4) Depending on the system configuration and the price model used, approximately between 50% and 62% of the value of storage can be attributed to natural gas price volatility (stochastic variability), with the remaining part attributable to price seasonality (deterministic variability).

(5) Analysis of our results shows that usage of the forward looking optimal policy of our MDP is important when the LNG throughput is low relative to the storage space.

(6) The storage value is fairly insensitive to how one models stochastic variability in the shipping process, due to the low level of congestion in the system configurations that we consider. Thus, an essentially deterministic model of throughput is often sufficient for valuation, in which case the relevant Cpu times decrease to 1.4-1.7 minutes. In contrast, the storage value decreases substantially when we model the price evolution using one factor, rather than two factors. Hence, how one models price uncertainty has a much stronger impact on the value of storage than how one models stochastic variability in the processing times in an LNG chain.

Our model and computational results have managerial relevance: Our model is very accurate and can be used by LNG players to support the negotiation of contracts for access to LNG regasification terminals, and our computational results significantly enhance the understanding of the value of LNG storage at such a terminal. Our model also has potential applicability, with suitable modifications, in other settings in which a storage facility is located downstream from the production or shipping stages of a given commodity, or upstream of such processes when it is used to store their input. Examples include petroleum and agricultural products, chemicals, metals, and the production of electricity from nonrenewable and renewable sources, such as coal, natural gas, water, sunlight, and wind (see, e.g., Bannister and Kaye [4], Geman [25], and Baxter [6]).

We proceed by reviewing the relevant literature in §2. We introduce our operational and strategic valuation models in §3 and §4, respectively. We present our heuristic valuation and upper bound models in §5 and §6, respectively, and quantity the value of storage in §7. We conclude in §8.

2 Literature Review

Our work is unique with respect to the LNG literature: Kaplan et al. [32] and Koenigsberg and Lam [33] address the modeling of the shipping stage of an LNG chain, but do not investigate the quantification of the storage value of an LNG regasification terminal, as we do here. Özelkana et al. [38] present a deterministic optimization model to analyze the design of LNG terminals. In contrast, our model captures both price and shipping uncertainty. Grønhaug and Christiansen [27] propose a deterministic optimization model for the tactical management of LNG inventory and ship routing, whereas we investigate the strategic valuation of downstream LNG storage accounting for both price and shipping uncertainty.

Our work is related to the real option literature (Trigeorgis [49]) dealing with applications in commodity and energy industries (see, e.g., Smith and McCardle [47], Clewlow and Strickland [16], Eydeland and Wolyniec [23], and Geman [25]). To the best of our knowledge, this literature has not yet studied the valuation of LNG storage: Geman [25, pp. 246-249] briefly describes the LNG storage setting and Abadie and Chamorro [1] use Monte Carlo simulation to value natural gas investments, including an LNG plant, but not LNG storage.

Several authors have studied the related real option valuation of natural gas storage, including Carmona and Ludkovski [12], Chen and Forsyth [14], Boogert and de Jong [10], Lai et al. [34], Secomandi [45], and Thompson et al. [48], among others. The main difference between natural gas and LNG storage is that the inflow of commodity into the storage facility is controllable in the natural gas case, but not in the LNG case. In addition, our work differs from that of Lai et al. [34] in the natural gas price model, the dynamic programming approximation, and the upper bound that we use, as well as in some of the managerial insights that we obtain.

Our model is also related to models of hydropower production and sale, especially those with price uncertainty (see, e.g., the review by Wallace and Fleten [52]). Some of these models are based on stochastic (mathematical) programming, whereas our approach uses an MDP within a heuristic decomposition scheme with an imperfect information dual upper bound (Brown et al. [11]), which differ from MDP type models available in this literature, such as that of Näsäkkälä and Keppo [36] (see also related work by Harrison and Taksar [28], Drouin et al. [17], and Lamond et al. [35]).

3 Operational Model

In this section we present an operational model for the management of an LNG chain. This model is computationally intractable; we use it to motivate the strategic valuation model in §4.

Shipping and Terminal Operations. Faithful representation of the interplay between the LNG shipping, storage, and regasification activities would require modeling them using a continuous time framework. For ease of exposition, we use a discrete time approach where a given finite time interval of length T, which we denote by set $\mathcal{T} := [0, T]$, is subdivided into I small time intervals, each of length Δt, in set $\mathcal{I} := \{1, \ldots, I\}$ (the length Δt is discussed further below). We index this set by i and denote time by t and the time when time period i starts by t_i ($t_1 := 0$ and $t_{I+1} := T$).

A fleet of N identical LNG ships, each with cargo size C, perform the following activities: loading at the upstream port, loaded transit to the downstream port, entering the downstream port, unloading at this port, leaving this port, and ballast transit to the upstream port. Ships are dedicated and loop between the liquefaction and regasification facilities, by far the most typical setting in the LNG industry (Greenwald [26], Tusiani and Shearer [51]). Congestion may occur at the upstream and downstream ports, that is, ships may queue up at these facilities.

We abstract from the details of natural gas liquefaction assuming ample supply. The fact that LNG liquefaction facilities are designed to run at full capacity, being served by an appropriate number of ships to satisfy this capacity (Flower [24, p. 96]), supports this. Loading aggregates the following activities: entering the port (traversing the entry channel), loading the ship, and leaving the port (traversing the exit channel or the entry channel again if there is only one channel at the given port). In contrast, as will become apparent soon, it is useful to separately model these activities, with loading replaced by unloading, for the downstream port.

The state of the shipping system at time t_i is $\underline{s}_i$; we simplify t_i to i when used as a subscript. (Quantities that are underlined will be simplified in later sections.) Each $\underline{s}_i$ is an N dimensional vector of triples that describe the activity performed by each ship, the elapsed time since the start of this activity, and the position of each ship in any relevant queue. The set of all possible shipping states at time t_i is $\underline{\mathcal{S}}_i$. There is usually uncertainty associated with shipping operations (Kaplan et al. [32], Ronen [40]). In this section we do not postulate a specific model of this uncertainty, but we suppose that Δt is chosen such that it is reasonable to assume that at most one activity can complete with positive probability during a time interval of length Δt. This assumption would give rise to daily or even smaller time intervals in applications.

We denote by x_i the inventory available at the downstream facility at time t_i. Let 0 and X, respectively, denote the minimum and maximum levels of inventory that can be held in storage at this facility (a positive minimum inventory level can be easily accommodated at the expense of additional notation). Hence, the quantity x_i is constrained to be in set $\mathcal{X} := [0, X]$.

We assume that at most one ship can unload its cargo at any one time. The LNG unloading and regasification rates are deterministic. We let $z_i \in \mathcal{Z} := [0, C]$ denote the LNG inventory remaining onboard the ship that is unloading at time t_i. Consistent with the objective of running the liquefaction facility at full capacity, we restrict attention to control policies that do not intentionally slow down the shipping system. Thus, the amount of LNG unloaded during time period i is not discretionary and is a deterministic function of x_i and z_i that we denote by $\underline{u}_i(x_i, z_i)$. We denote by $\underline{q}_i$ the amount of LNG one chooses to regasify during time period i. The set of feasible values that this quantity can take on depends on x_i, z_i, and $\underline{u}_i(x_i, z_i)$. We denote this set by $\underline{\mathcal{Q}}(x_i, z_i)$, since $\underline{u}_i(x_i, z_i)$ is a function of x_i and z_i. We let $\tilde{z}_i$ be the random variable that describes the inventory onboard the ship that unloads during time period i, which depends on the evolution of the shipping process (we denote random entities as $\tilde{\cdot}$); z_i is equal to C if a ship starts unloading its cargo at time t_i, $z_{i-1} - \underline{u}_{i-1}(x_{i-1}, z_{i-1})$ otherwise (here $i > 1$; z_1 is determined by $\underline{s}_1$).

Revenue and Cost Structures. We use the Markovian vector-valued stochastic process $\{\tilde{p}_t, t \in \mathcal{T}\}$ to describe the continuous time evolution of a price state random vector $\tilde{p}_t$, with realization $p_t \in \Re^P$ ($P \geqslant 1$ is an integer). The natural gas spot price at the downstream facility at time t is the known function $g_t(p_t) : \Re^P \to \Re_+$. We assume an arbitrage free and complete market for natural gas futures at this location and denote by $\mathbb{E}[\cdot \mid p_t]$ risk neutral conditional expectation given p_t (see, e.g., Duffie [18]). We assume that $\mathbb{E}[g(\tilde{p}_{t'}) \mid p_t] < \infty$, $\forall t,\ t' \in \mathcal{T}$ and $t \leqslant t'$. This ensures that the value functions of the MDPs discussed below and in §§4-6 are finite.

This representation allows us to capture single ($P = 1$) and multiple ($P > 1$) factor models of the evolution of the natural gas spot price, such as the one- and two-factor models of Jaillet et al. [30] and Schwartz and Smith [44], the latter modified to use deterministic monthly seasonality factors as in the one factor model. In the two factor model, the natural gas spot price at time t is $g_t(\tilde{p}_t) := f_{m(t)} \exp(\tilde{p}_t^1 + \tilde{p}_t^2)$, with $f_{m(t)}$ the time t seasonality factor and $m(t)$ the month corresponding to time t. Using the same notation and terminology of Schwartz and Smith [44], we let $\chi_t := p_t^1$ and $\xi_t := p_t^2$, and refer to χ_t and ξ_t as the time t values of the short term deviation factor and long term (equilibrium) factor, respectively. The risk neutral dynamics of these factors are (Schwartz and Smith [44])

$$d\chi_t = (-\kappa_\chi \chi_t - \lambda_\chi)dt + \sigma_\chi dz_\chi^*, \quad (1)$$

$$d\xi_t = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dz_\xi^*, \quad (2)$$

$$d\chi_t d\xi_t = \rho_{\chi\xi} dt. \quad (3)$$

Here κ_χ and σ_χ are the speed of mean reversion and the volatility of χ_t; μ_ξ and σ_ξ are the drift and the volatility of ξ_t; λ_χ and λ_ξ are the risk premia associated with the two factors; and dz_χ^* and dz_ξ^* are increments to standard Brownian motions with instantaneous correlation $\rho_{\chi\xi}$.

When the equilibrium factor is constant the two factor model (1)-(3) reduces to the one factor model of Jaillet et al. [30] (see Schwartz and Smith [44, p. 894]), in which the short term factor $p_t \equiv \chi_t$ mean reverts to the constant risk adjusted equilibrium level ξ^*, rather than zero, and the deseasonalized spot price at time t is $\exp(\chi_t)$ (seasonality is modeled as in the two factor model). The risk neutral dynamics of this short term factor are

$$d\chi_t = \kappa(\xi^* - \chi_t)dt + \sigma dz^*, \quad (4)$$

where κ and σ are the speed of mean reversion and the volatility of this factor, and dz^* is an increment to a standard Brownian motion.

We account for regasification sales during time period i by multiplying the released quantity, net of regasification fuel losses (explained in detail below), by the natural gas price prevailing at time t_i. We also assume that the quantity sold does not affect the market price of natural gas and that the price state vector and the shipping processes evolve independently. These two assumptions are realistic for modeling an LNG system whose regasification terminal is located in the southern part of the U.S., e.g., Louisiana, where the natural gas spot market is fairly liquid.

There are operating costs and fuel requirements associated with the physical flows along the chain (Flower [24]). We denote by $\underline{h} \in \Re_+$ the per unit and time period physical inventory holding cost charged against the inventory x_i available at time t_i at the downstream terminal. We let ϕ be the fuel needed to regasify one unit of LNG, that is, the LNG to natural gas yield is $1 - \phi$. We denote by c the cost of unloading (handling) one unit of LNG at the downstream terminal.

Remark 1 (Units of measurement)**.** LNG capacity is typically measured in MTPA, cargos in cubic meters (CM), and LNG downstream storage space and regasification capacity in BCF and BCF/day. For simplicity, we assume that all the physical quantities are expressed as functions of million British thermal units (MMBTU), since the natural gas price in the U.S. is expressed in U.S. dollars per million British thermal units ($/MMBTU). (See Table 4 in Online Appendix A.)

MDP. We now formulate an optimization model to control the inventory level at the downstream facility. We assume that the stochastic evolution of the shipping process is uncorrelated with the evolution of the price of the market portfolio. Thus, the statistical and risk neutral dynamics of the shipping process can be taken to be identical. This allows us to formulate our model using risk neutral valuation (see, e.g., Smith [46]).

The state of the system at time t_i is $(x_i, z_i, \underline{s}_i, p_i)$. We denote a control policy by π and let Π be the set of feasible policies. We define the reward obtained in time period i as $\underline{r}_i(x_i, \underline{u}_i(x_i, z_i), \underline{q}_i, p_t) := g_i(p_i)(1-\phi)\underline{q}_i - \underline{h}x_i - c\underline{u}_i(x_i, z_i)$. We denote by $W_{I+1}(x_{I+1}, z_{I+1}, \underline{s}_{I+1}, p_{I+1})$ the salvage value of the LNG that is onboard the ships or is available at the regasification facility at time t_{I+1}. The objective is to solve the following optimization model:

$$\max_{\pi\in\Pi} \mathbb{E}\left[\sum_{i\in\mathcal{I}} \underline{\delta}^{i-1}\underline{r}_i(\tilde{x}_i^\pi, \underline{u}_i(\tilde{x}_i^\pi, \tilde{z}_i^\pi), \underline{\tilde{q}}_i^\pi, \tilde{p}_i) + \underline{\delta}^I W_{I+1}(\tilde{x}_{I+1}^\pi, \tilde{z}_{I+1}^\pi, \underline{\tilde{s}}_{I+1}^\pi, \tilde{p}_{I+1}) \mid x_1, z_1, \underline{s}_1, p_1\right], \tag{5}$$

where $\underline{\delta}$ denotes the one period risk free discount factor and $\mathbb{E}[\cdot|x_1, z_1, \underline{s}_1, p_1]$ denotes expectation with respect to the joint risk neutral probability distribution of the relevant random variables induced by feasible policy π, a dependence indicated by superscripting π, conditional on $(x_1, z_1, \underline{s}_1, p_1)$.

Denoting by $W_i(x_i, z_i, \underline{s}_i, p_i)$ the optimal value function in state $(x_i, z_i, \underline{s}_i, p_i)$ at time t_i, model (5) can be reformulated recursively as follows:

$$\begin{aligned} W_i(x_i, z_i, \underline{s}_i, p_i) \quad = \quad & \max_{\underline{q}_i\in\underline{\mathcal{Q}}(x_i,z_i)} \underline{r}_i(x_i, \underline{u}_i(x_i, z_i), \underline{q}_i, p_i) \\ & \qquad + \underline{\delta}\mathbb{E}\left[W_{i+1}(x_i + \underline{u}_i(x_i, z_i) - \underline{q}_i, \tilde{z}_{i+1}, \underline{\tilde{s}}_{i+1}, \tilde{p}_{i+1}) \mid x_i, z_i, \underline{s}_i, p_i\right], \\ & \forall i \in \mathcal{I}, x_i \in \mathcal{X}, z_i \in \mathcal{Z}, \underline{s}_i \in \underline{\mathcal{S}}_i, p_i \in \Re^P. \end{aligned} \tag{6}$$

The value of the regasification terminal at time t_1 is $W_1(x_1, z_1, \underline{s}_1, p_1)$; the value of its storage component at this time is this value minus the regasification terminal value under the policy that regasifies as much as possible in every time period. However, the curse of dimensionality makes model (5) computationally intractable, and thus approximations are needed for valuation purposes.

4 Strategic Valuation Model

In this section we describe our approximate model for the strategic valuation of a downstream LNG storage and regasification terminal. Given our objective of strategic valuation, instead of attempting to approximately solve the operational model (6), we formulate a simplified model: Specifically, we simplify (1) the coupling between the shipping process and the management of the inventory at the terminal, brought about by the LNG unloading step, and (2) the evolution of the

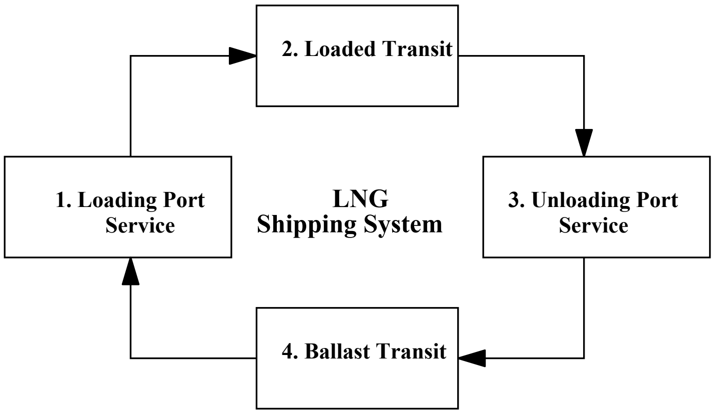

Figure 1: Representation of an LNG shipping system.

price state vector. Although simpler than model (6), the resulting model remains difficult to solve, but it paves the way for developing the computationally efficient heuristic that we discuss in §5.

Time Aggregation and Shipping and Terminal Operations. The valuation of the terminal involves time horizons exceeding 10 years. Thus, we increase the time scale of the operational model by aggregating the time periods in set $\mathcal{I}$ into J longer time periods in set $\mathcal{J} := \{1, \ldots, J\}$; the start of time period j is time t_j and $t_{J+1} := T$. All the aggregate time periods have the same length, which is application dependent; we take it to be one month in §7.

This aggregate time scale no longer allows us to track the detailed evolution of the interaction between LNG shipping, storage, and regasification; we are only able to track this interaction at the level of the number of ships that request to unload their cargos during an aggregate time period. That is, we treat all cargoes unloaded in this period identically; they all can be regasified and sold during the period at the prevailing price at the start of the period. Thus, the quantity of interest to us now is the distribution of the number of cargos or, equivalently, the amount of LNG unloaded during an aggregate time period. We denote by $\tilde{u}_j$, with realization u_j, the random amount of LNG unloaded during aggregate time period j and assume that it can take values in set $\mathcal{U}_j(x_j) := \{0, C, \ldots, N_j^U(x_j)C\}$, where $N_j^U(x_j)$ is the *maximum* number of ships that can unload their cargos during aggregate time period j when the inventory level at time t_j is x_j. Given this model, we now need to specify $N_j^U(x_j)$ and the probability distribution of $\tilde{u}_j$.

We do this by extending the CQN model of the LNG shipping system proposed by Koenigsberg and Lam [33] and illustrated in Figure 1. Different from the shipping system described in §3, this system combines into a single unloading activity the activities of entering/exiting the downstream port and unloading (we discuss this CQN in detail in §5). We define $N_j^U(x_j) := \lfloor (X + Q - x_j)/C \rfloor$, where Q denotes the regasification capacity available during an aggregate time period. Consistent with §3, this definition implicitly assumes any operational policy seeks to maximize the utilization of the available regasification capacity at the downstream facility. The probability distribution of $\tilde{u}_j$ is determined by the transition laws of the CQN with the restriction that no more than $N_j^U(x_j)$ ships are allowed to transition from the unloading to the ballast transit stage during aggregate time period j. If more than $N_j^U(x_j)$ ships request to unload during this time period, the excess ships are blocked at the downstream facility at least until time t_{j+1}. This contrasts with the CQN of Koenigsberg and Lam [33] that does not include regasification capacity or the maximum inventory limit at this facility; that is, our CQN models both *congestion* at the loading and unloading stages, arising from uncertainty in the CQN as in Koenigsberg and Lam [33], and *blocking* at the unloading stage, arising from constraints on the regasification capacity and the maximum inventory space.

The vector s_j denotes the state of the shipping system at time t_j, which comprises the number of ships in the various stages, including those blocked; s_j is necessary to obtain the probability distribution of $\tilde{u}_j$. The set of these possible shipping states is $\mathcal{S}_j$.

We denote by $q_j \in \mathcal{Q}(x_j, u_j)$ the amount of LNG that can be feasibly regasified and sold during aggregate time period $j \in \mathcal{J}$ given $x_j \in \mathcal{X}$ and $u_j \in \mathcal{U}_j(x_j)$; here $\mathcal{Q}(x_j, u_j)$ constrains q_j as follows: given t_j, x_j, u_j, and q_j, the inventory level at time t_{j+1} is $x_{j+1} = x_j + u_j - q_j$, so that the restrictions $x_{j+1} \in \mathcal{X}$ and $q_j \in [0, Q]$ imply that $\mathcal{Q}(x_j, u_j) = [\max\{0, x_j + u_j - X\}, \min\{Q, x_j + u_j\}]$.

Discrete Time and Space Price State Process. The domain of the price state vector in model (6) is unbounded. From now on, we approximate the risk neutral evolution of the vector p_t using a discrete time and space stochastic process that at each time t_j, with $j \in \mathcal{J} \cup \{J+1\}$, can take values in the finite set $\mathcal{P}_j \subset \Re^P$; we discuss the generation of this stochastic process in §5.

MDP. Our strategic valuation model is an MDP with stage set $\mathcal{J} \cup \{J+1\}$. Its state in stage j is (x_j, s_j, p_j). We let $h \in \Re_+$ denote the per unit physical inventory holding cost during an aggregate time period and δ the risk free discount factor for this time period. The reward obtained during aggregate time period j is defined as $r_j(x_j, u_j, q_j, p_j) := g_j(p_j)(1-\phi)q_j - hx_j - cu_j$. We denote by $V_j(x_j, s_j, p_j)$ the optimal value function of our strategic valuation model in stage j and state (x_j, s_j, p_j). This function is defined as follows:

$$V_j(x_j, s_j, p_j) = \mathbb{E}\left[v_j(x_j, s_j, \tilde{u}_j, p_j) \mid x_j, s_j\right], \ \forall j \in \mathcal{J}, x_j \in \mathcal{X}, s_j \in \mathcal{S}_j, p_j \in \mathcal{P}_j, \tag{7}$$

$$v_j(x_j, s_j, u_j, p_j) := \max_{q_j \in \mathcal{Q}(x_j, u_j)} r_j(x_j, u_j, q_j, p_j) + \delta \mathbb{E}[V_{j+1}(x_j + u_j - q_j, \tilde{s}_{j+1}, \tilde{p}_{j+1}) \mid x_j, s_j, u_j, p_j], \tag{8}$$

with boundary conditions $V_{J+1}(x_{J+1}, s_{J+1}, p_{J+1}) := [g_{J+1}(p_{J+1})(1-\phi) - h]x_{J+1}$, for all $x_{J+1} \in \mathcal{X}$,

$s_{J+1} \in \mathcal{S}_{J+1}$, $p_{J+1} \in \mathcal{P}_{J+1}$; that is, at the final time, t_{J+1}, any remaining inventory is released and sold (for simplicity, we account for the holding cost using the coefficient h). During each aggregate time period $j \in \mathcal{J}$, an amount $u_j \in \mathcal{U}_j(x_j)$ of LNG is unloaded at the downstream terminal at cost cu_j, and becomes available for regasification and sale during this time period; holding cost hx_j is charged against the initial inventory x_j; an optimal amount of LNG from the total available inventory $x_j + u_j$ is regasified, and a fraction $1 - \phi$ of this amount is sold into the wholesale natural gas spot market at the prevailing price $g_j(p_j)$. A stochastic transition to the next aggregate period accounts for the uncertainty in the natural gas spot price and the shipping process, taking into account the inventory dynamics (as in §3, the stochastic evolution of the shipping process is independent of that of the price state vector and does not require any risk adjustment).

Comparing models (6) and (7)-(8) reveals two notable effects of our modeling simplification. (1) The quantity z_i is a state variable in the operational model but not in the strategic model, as at the aggregate time scale the precise status of the inventory onboard an unloading ship is insignificant. (2) The strategic model has factored expectations, that is, the maximization on the right hand side of (8) is taken over $\mathcal{Q}(x_j, u_j)$, assuming the quantity to be unloaded during aggregate time period j is known; this is supported by the fact that in actual operations one can fairly reliably schedule ship unloading during an aggregate time period, e.g., one month. This assumption is required to remove the dependence of the set of feasible regasification decisions from the evolution of the shipping system *within* an aggregate time period, that is, to make our time aggregation function.

Storage Valuation. We denote by $V_1^G(x_1, s_1, p_1)$ the value in the initial stage and state of a greedy inventory release policy that in each stage regasifies and sells as much as possible of each incoming cargo upon receipt. We define the value of storage in the initial stage and state as

$$S_1(x_1, s_1, p_1) := V_1(x_1, s_1, p_1) - V_1^G(x_1, s_1, p_1). \tag{9}$$

5 Heuristic Strategic Valuation Model

In this section we describe our heuristic strategic valuation model.

Heuristic Model Definition. Fundamentally, what continues to make model (7)-(8) computationally difficult is the curse of dimensionality brought about by the coupling between the shipping/unloading process and the management of the inventory at the terminal, which may result in ship blocking. Our heuristic model deals with the coupling of these activities in two steps. First, it decouples them by assuming away the possibility of ship blocking, which eliminates the dependence between the unloading and inventory processes. This makes computing a heuristic inventory release policy tractable. Second, it reinstates this coupling by including the possibility of ship blocking at an aggregate time scale when computing the value of storage when using this policy. Our heuristic model performs these steps by integrating (1) the shipping model, (2) the price evolution model, (3) the inventory release model, and (4) the storage valuation model.

Shipping Model. The shipping model derives a stochastic description of the number of LNG cargos unloaded during an aggregate time period. This model decouples the natural gas liquefaction and LNG shipping activities from the management of downstream LNG storage. In addition, we simplify our problem by modeling the unloading process as a sequence of independent and identically distributed random variables, based only on the overall shipping network configuration, not on the positions of individual ships. Each such random variable, denoted by $\tilde{u}$, represents the amount of LNG unloaded at the regasification facility by the ships in the network during an aggregate time period. The distribution of $\tilde{u}$, the set of nonzero probability realizations $\mathcal{U}$ and the probability $\Pr\{\tilde{u} = u\}$ for each $u \in \mathcal{U}$, is required in the inventory release model.

We determine this distribution in two steps by extending the original CQN of Koenigsberg and Lam [33]. This extension is a useful and flexible abstraction of LNG shipping: In its basic form with exponentially distributed processing times, this model provides a conservative estimate of the throughput of an LNG system (ignoring ship blocking); in a more advanced form with the shipping times modeled as multistage Coxian distributions, which can approximate the distribution of service times arbitrarily closely, our model computes a less conservative estimate of this throughput. Our shipping model can also accommodate other approaches to the modeling of the throughput of an LNG system. For example, in §7 we discuss results obtained with an essentially deterministic shipping model that, ignoring congestion, provides an optimistic estimate of throughput.

As an extension to Koenigsberg and Lam [33], suppose that the loading and unloading blocks in Figure 1 are first come first served (FCFS) exponential queues, and the transit blocks are ample server (AS) exponential or multistage Coxian queues; the latter case allows more flexibility in modeling variability than the exponential distribution (see Osogami and Harchol-Balter [37] for a discussion of Coxian modeling; Koenigsberg and Lam [33] only use exponential queues). With this representation, the shipping system is a particular CQN called a BCMP network having a closed form, product form stationary distribution, as defined and proved by Baskett et al. [5]. We denote by $\mathcal{N}$ the set of all possible states of the system, that is, the set of vectors that represent the number of ships in each stage. The steady state probability that the random variable state of the system, $\tilde{n}$, is equal to $n \in \mathcal{N}$ is $\gamma(n) := \Pr\{\tilde{n} = n\}$, and the CQN throughput can be computed by standard methods (see Baskett et al. [5]).

We propose the rolling forward method to compute the distribution of $\tilde{u}$. This method uses the stationary distribution $\gamma(\cdot)$ as a starting point. It transitions the CQN forward through time from its stationary distribution, tracking the distribution of unloaded amounts over an aggregate time period: we uniformize (see, e.g., Asmussen [3]), condition on the number of "events" that occur over this time period, and calculate the distribution of the number of ships unloaded given the number of total events conditioning on the initial state of the shipping system, as specified by $\gamma(\cdot)$. Hence, the distribution of random variable $\tilde{u}$ is a one dimensional table.

We compute this distribution analytically, that is, we do not use Monte Carlo simulation. Specifically, denote by $\tilde{a}$ the random number of events that occur during an aggregate time period, a Poisson random variable with appropriate mean, and by $\tilde{\eta}$ the random number of unloaded ships during this time period. Let A be such that $\Pr\{\tilde{a} > A\} \leqslant \epsilon$ for arbitrarily small $\epsilon \in \Re_+$. Denote by $\Pr\{\eta|a, n\}$ the probability that η ships are unloaded during an aggregate time period given that the system is initially in state n and a total of a events occur. We have developed analytical expressions for $\Pr\{\eta|a, n\}$ through a forward recursion in η, but, since they are somewhat lengthy, in the interest of space we do not present them here. We compute the distribution of $\tilde{\eta}$, and hence that of $\tilde{u} \equiv \tilde{\eta}C$, as $\Pr\{\tilde{\eta} = \eta\} = [\sum_{n\in\mathcal{N}} \gamma(n) \sum_{a=0}^{A} \Pr\{\tilde{a} = a\} \Pr\{\eta|a, n\}]/(1-\epsilon)$.

Price Evolution Model. The price evolution model generates a lattice representation of the stochastic evolution of the price state vector during the given time horizon. We use the approach described by Tseng and Lin [50, §3] to build two trinomial lattices for the two factor model (1)-(3) and one trinomial lattice for the one factor model (4), with a time step equal to the length of an aggregate time period. We also use multiplicative adjustment factors to calibrate these lattices to the natural gas forward curve observed at the beginning of the time horizon. These calibrated lattices are the stochastic process $\{p_j, j \in \mathcal{J} \cup \{J+1\}\}$ used in our inventory release model.

Inventory Release Model. The inventory release model incorporates the output of the shipping and price evolution models to determine an inventory release policy that can be used to make LNG regasification and natural gas sale decisions. This is a simplified version of model (7)-(8) whose state in each stage $j \in \mathcal{J} \cup \{J+1\}$ is the pair (x_j, p_j), for all $x_j \in \mathcal{X}$ and $p_j \in \mathcal{P}_j$; that is, the state of the shipping system is no longer part of the state as the aggregate shipping/unloading process is approximated through the distribution of $\tilde{u}$.

We make the assumption that in each aggregate time period it is possible to regasify all of the LNG unloaded from ships arriving in this time period, that is, $Q \geqslant U := \max\{u : u \in \mathcal{U}\}$. (We could also penalize "blocked" LNG if $Q \geqslant U$ were to be unreasonable.) This assumption ensures that ships cannot be blocked at the downstream facility and makes the set $\mathcal{Q}(x_j, u)$, defined in §4,

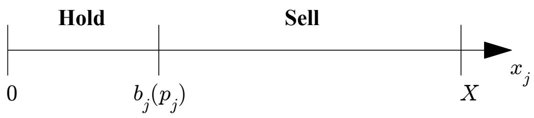

Figure 2: The structure of the optimal inventory release policy in stage j for a given value of the price state vector p_j.

nonempty for all $j \in \mathcal{J}$, $x_j \in \mathcal{X}$, and $u \in \mathcal{U}$.

We denote by $V_j^H(x_j, p_j)$ the value function of the inventory release model in stage j and state (x_j, p_j). This function is defined as

$$V_j^H(x_j, p_j) = \mathbb{E}\left[v_j^H(x_j, p_j, \tilde{u})\right], \ \forall j \in \mathcal{J}, x_j \in \mathcal{X}, p_j \in \mathcal{P}_j, \tag{10}$$

$$v_j^H(x_j, p_j, u) := \max_{q_j \in \mathcal{Q}(x_j, u)} r_j(x_j, u, q_j, p_j) + \delta \mathbb{E}\left[V_{j+1}^H(x_j + u - q_j, \tilde{p}_{j+1}) \mid p_j\right], \tag{11}$$

with boundary conditions $V_{J+1}^H(x_{J+1}, p_{J+1}) := [g_{J+1}(p_{J+1})(1-\phi) - h]x_{J+1}$, for all $x_{J+1} \in \mathcal{X}$ and $p_{J+1} \in \mathcal{P}_{J+1}$. This formulation is interpreted in a manner analogous to that of model (7)-(8).

Structural Analysis. We now study the structure of the optimal policy of model (10)-(11). This analysis greatly facilitates the computation of this policy and its efficient use in the storage valuation model, which makes our integrated model practical. Proposition 1 characterizes the optimal value function of model (10)-(11). (Online Appendix B includes the proofs for this section.)

Proposition 1 (Optimal value function)**.** *In every stage $j \in \mathcal{J} \cup \{J+1\}$, the function $V_j^H(x_j, p_j)$ is concave in $x_j \in \mathcal{X}$ for each given $p_j \in \mathcal{P}_j$.*

Turning to the optimal sale action, we define the quantity $q_j^*(x_j, p_j, u)$ as the largest element that optimizes the right hand side of (11). Any feasible sale cannot be smaller than $\max\{0, x_j + u - X\}$, because in aggregate time period j one must execute the forced sale $q_j^F(x_j, u) := \max\{0, x_j + u - X\}$ to avoid a tank overflow due to incoming cargos. We call the difference between the feasible sale $q_j(x_j, p_j, u)$ and the forced sale $q_j^F(x_j, u)$ the optional sale, and denote it by $q_j^O(x_j, p_j, u) := q_j(x_j, p_j, u) - q_j^F(x_j, u)$; that is, $q_j(x_j, p_j, u) \equiv q_j^F(x_j, u) + q_j^O(x_j, p_j, u)$.

Proposition 2 (Optimal policy structure)**.** *In every stage $j \in \mathcal{J}$, given $p_j \in \mathcal{P}_j$, define $b_j(p_j) \in \mathcal{X}$ as the smallest element of* $\arg\max_{x_{j+1} \in \mathcal{X}} \delta \mathbb{E}[V_{j+1}^H(x_{j+1}, \tilde{p}_{j+1}) \mid p_j] - g_j(p_j)(1-\phi)x_{j+1}$*. In stage $j \in \mathcal{J}$, it is optimal to try to sell down to $b_j(p_j)$, that is, $q_j^*(x_j, p_j, u) = q_j^F(x_j, u) + q_j^{O*}(x_j, p_j, u)$, with $q_j^{O*}(x_j, p_j, u) := \min\{\max\{x_j + u - q_j^F(x_j, u) - b_j(p_j), 0\}, Q - q_j^F(x_j, u)\}$.*

The quantity $b_j(p_j)$ can be interpreted as a basestock target for optimal optional sales; it is a target because it is constrained by the limited regasification capacity Q. Given price state vector

p_j, $b_j(p_j)$ partitions the feasible inventory set into two regions, one in which it is optimal to hold inventory and one in which it is optimal to sell down to the basestock target, as illustrated in Figure 2. We point out that when it is optimal to sell, it can be optimal to stop selling rather than draining the terminal as much as possible; formally $b_j(p_j) \in (0, X)$ is possible, as discussed in §7.

Under the assumptions of Proposition 3, which is related to Propositions 2 and 3 of Secomandi [45], computing an optimal inventory release policy can be done efficiently.

Proposition 3 (Optimal policy computation)**.** *Suppose that X, Q, and each $u \in \mathcal{U}$ are integer multiples of some maximal $L \in \Re_+$. Then, for every $j \in \mathcal{J} \cup \{J+1\}$, $V_j^H(x_j, p_j)$ is piecewise linear and continuous in $x_j \in \mathcal{X}$ for each given $p_j \in \mathcal{P}_j$, it changes slope at values that are integer multiples of L, and $b_j(p_j)$ is an integer multiple of L ($b_{J+1}(p_{J+1}) := 0$).*

The practical implication of this result is that in each stage one needs to compute the optimal value function only for a finite number of inventory levels, namely 0, L, $2L$, ..., X, and the search for an optimal basestock target can be limited to one of these values.

Storage Valuation Model. The storage valuation model determines the following lower bound estimate on the value of storage $S_1(x_1, s_1, p_1)$, defined by (9):

$$\hat{S}_1^B(x_1, s_1, p_1) := \hat{V}_1^B(x_1, s_1, p_1) - \hat{V}_1^G(x_1, s_1, p_1). \tag{12}$$

Here, $\hat{V}_1^B(x_1, s_1, p_1)$ is the estimate of the value function in the initial stage and state of the optimal basestock target policy of model (10)-(11) when used as a *heuristic* policy for model (7)-(8); that is, when ship blocking at the downstream facility can occur in the manner modeled in §4. This estimate is obtained by applying this policy within Monte Carlo simulations of the price state vector process, from the lattice representation of its evolution, and the shipping process with ship blocking. The quantity $\hat{V}_1^G(x_1, s_1, p_1)$ is the estimated value in the initial stage and state of the greedy policy, discussed at the end of §4, obtained in conjunction with $\hat{V}_1^B(x_1, s_1, p_1)$ by using common random numbers and multiplicative factors computed to adjust the sampled price state vectors to be consistent with the observed forward curve (these adjustment factors are not the same ones used in the calibration of the lattices).

The storage valuation model can be used to also estimate the values of the real option to store due to price seasonality and volatility. The former is the value of this real option with deterministic natural gas price evolution equal to the forward curve at time 0, which exhibits significant seasonality, as illustrated in §7. We estimate this value of storage in a manner analogous to the estimation of the value of storage, except that we compute an optimal inventory release policy using deterministic price dynamics. The latter is the difference between the value of storage and the value of storage due to price seasonality, and we estimate it accordingly.

6 Upper Bound

In this section we discuss a model that estimates an upper bound on the value of storage to complement our lower bound of §5. This model is an imperfect information dual upper bound model, in the sense of Brown et al. [11]. It uses the same periodic review setting as the strategic model (7)-(8) and employs Monte Carlo simulation to generate a set of sequences of unloaded cargos of the form $\{u_j, j \in \mathcal{J}\}$ by sequentially sampling from the stochastic shipping process used by the shipping model described in §5; that is, ignoring ship blocking. This model then computes an *optimized* inventory release policy for each of these sample sequences by using a variant of model (10)-(11) that assumes perfect foreknowledge of the ship arrival process, as described below.

A tank overflow at the regasification terminal would occur in stage j with inventory level x_j when $x_j + u_j - Q > X$. To prevent this, one would have to avoid unloading an amount $\max\{0, x_j + u_j - Q - X\}$ of LNG. To simplify the computation of our upper bound, we preprocess the sampled sequences of unloaded LNG by subtracting from each relevant u_j the quantity $\max\{0, u_j - Q\}$, which is the amount of LNG that would be blocked in stage j if the terminal were full. Then, the upper bound model values this amount of "blocked" LNG at $\max_{j' \in \{j, j+1, \ldots, J+1\}} \delta^{j'-j}(1-\phi)\mathbb{E}[g_{j'}(\tilde{p}_{j'}) \mid p_j]$: this LNG is allowed to be "virtually stored" for free and sold at the best possible price in the future (the cost of receiving u_j units of LNG is charged in stage j). Assuming perfect knowledge of all future shipments and truncating allows us to replace the expectation with respect to $\tilde{u}$ in (10) with a degenerate expectation with respect to the deterministic quantity $u_j - \max\{0, u_j - Q\}$. We average the value functions in stage 1 and state (x_1, s_1, p_1) over the generated sample paths to obtain the estimate $\hat{V}_1^{UB}(x_1, s_1, p_1)$ of an upper bound on $V_1(x_1, s_1, p_1)$. Finally, we estimate an upper bound on the value of storage $S_1(x_1, s_1, p_1)$, defined by (9), as follows:

$$\hat{S}_1^{UB}(x_1, s_1, p_1) := \hat{V}_1^{UB}(x_1, s_1, p_1) - V_1^G(x_1, s_1, p_1). \tag{13}$$

As estimating the upper bound does not require simulating the evolution of the price state vector, we do not estimate $V_1^G(x_1, s_1, p_1)$ in (13). Instead we take advantage of the linearity in price of the revenue of the greedy policy in every stage, and the property that the time 0 futures prices for the relevant stages (maturities), which are available at time 0, are the risk neutral expected spot prices in these stages given the information available at time 0. Hence, we use these futures prices directly to compute the value of the greedy policy in the initial stage and state.

7 Quantification of the Value of the Real Option to Store

In this section we report the results of a numerical study of the real option value of downstream LNG storage. After introducing the setting of the study and the estimation of the parameters of the natural gas price models used, we discuss our valuation results and related managerial insights.

Operational Parameters and Operating Costs. Table 1 summarizes the operational parameters and operating costs of the liquefaction, shipping, and regasification stages used in our numerical experiments. We consider a cargo size of $145,000$CM, a common size in the LNG industry (Flower [24]). We set the distance between the liquefaction and regasification facilities equal to 7,000 nautical miles (NM), which is roughly equal to the distance between Egypt, an LNG exporting country, and Lake Charles, Louisiana, which hosts an LNG terminal operated by Trunkline LNG (http://infopost.panhandleenergy.com/InfoPost/jsp/frameSet.jsp?pipe=tlng). We assume that the speed of each ship is 19 knots, a realistic value (Flower [24, p. 100], Cho et al. [15]), which makes a one way trip approximately 15 days long. The mean service times at the liquefaction and regasification facilities are one day each, which are representative of typical operations (EIA [19]). The unloading (handling) charge and the regasification fuel loss are the "Currently Effective Rates" of Trunkline LNG for firm terminal service as of 5/29/2009 for the unloading charge and 6/26/2006 for the regasification fuel loss (the 5/29/2009 rate sheet does not display a numerical figure for this quantity, so we employ the analogous figure from the 6/26/2006 rate sheet).

According to EIA [19], two cargos is the industry rule of thumb for the size of the receiving tanks. Since the Lake Charles terminal storage space amounts to roughly three cargos and some of the newly developed terminals in the U.S. have even larger sizes, we also consider larger values for this parameter. Moreover, for completeness we include one cargo storage size in our analysis. The send out capacity is 2BCF/day, which is consistent with the 2.1BCF/day peak capacity of the Lake Charles terminal (the capacity of the Sabine Pass, Texas, terminal, operated by Cheniere Energy, is 2.6BCF/day; Cheniere Energy has also proposed two other terminals with capacity equal to 2.6BCF/day and 3.3BCF/day, respectively). Apparently, Trunkline LNG and the other companies that manage the active regasification terminals in the U.S. do not charge a holding cost, so we set this to zero in our experiments.

We consider fleet sizes ranging from 1 to 10 ships in unitary increments. Their throughput levels vary from 0.7536MTPA to 7.5362MTPA, assuming that the ships are operated 365 days per year with deterministic processing (loading, unloading, and transit) times; with exponentially distributed processing times, ignoring ship blocking, these throughput figures are no more than

Table 1: Operational parameters and costs.

Liquefaction Average Loading Time			
1 Day			
Shipping			
Average One Way Transit Time	Distance	Speed	Ship Size
15 Days	7,000NM	19 Knots	145,000CM
Regasification			
Average Unloading Time	Capacity	Fuel Loss	Unloading Cost
1 Day	2BCF/Day	1.69%	\$0.0017/MMBTU

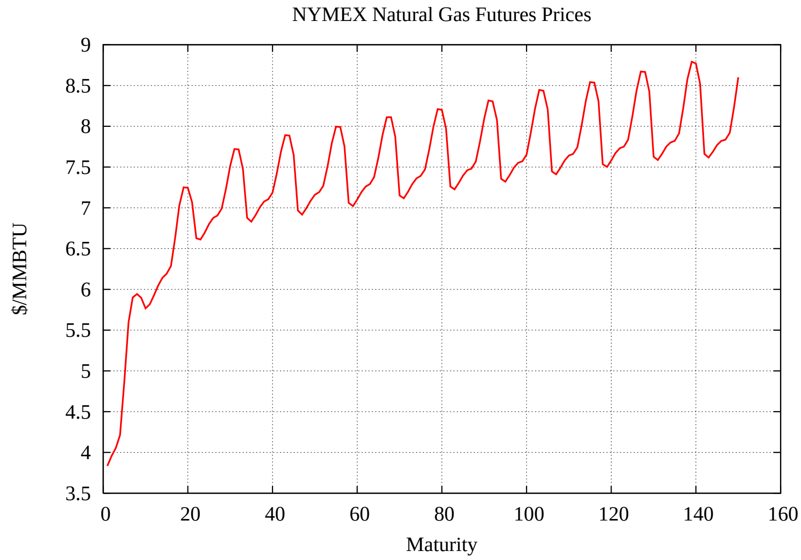

Figure 3: NYMEX natural gas futures prices traded on 5/29/2009.

2.22% lower, indicating a very low level of congestion in the systems that we analyze.

Estimation of the Parameters of the Natural Gas Price Models. We need to estimate the following parameters pertaining to the natural gas price models (1)-(3) and (4): κ_χ, λ_χ, σ_χ, $\mu_\xi^* := \mu_\xi - \lambda_\xi$, σ_ξ, $\rho_{\chi\xi}$, χ_0, and ξ_0, where the latter two are initial values for the two factors, and $f_1, \ldots, f_{12}$. We assume that regasified LNG is sold into the Louisiana wholesale natural gas spot market at the Henry Hub price. Thus, we use NYMEX data for estimation purposes and employ a dataset that includes natural gas futures prices and prices of call and put options on natural gas futures from 5/29/2009. We now describe our estimation approach.

Consider model (1)-(3). Denote by $F(t,t')$ the time t price of a futures contract for delivery at time $t' \geqslant t$. Under this model, $\ln F(t,t')$ can be expressed as

$$\ln F(t,t') \quad = \quad \ln f_{m(t')} + e^{-\kappa_\chi(t'-t)}\chi_t + \xi_t + \mu_\xi^*(t'-t) - [1 - e^{-\kappa_\chi(t'-t)}]\frac{\lambda_\chi}{\kappa_\chi} + \frac{\breve{\sigma}^2(t,t')}{2}, \quad (14)$$

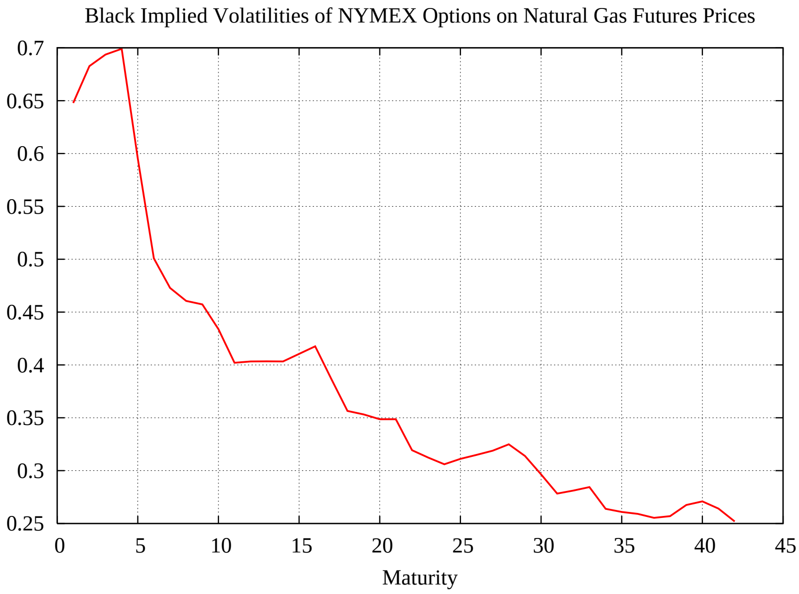

Figure 4: Black implied volatilities of NYMEX options on natural gas futures prices traded on 5/29/2009.

$$\check{\sigma}^2(t,t') \;:=\; [1-e^{-\kappa_\chi(t'-t)}]\frac{\sigma_\chi^2}{2\kappa_\chi} + \sigma_\xi^2(t'-t) + 2[1-e^{-2\kappa_\chi(t'-t)}]\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa_\chi}. \tag{15}$$

Closed form expressions for the time 0 prices of European call and put options on futures price $F(t,t')$ under model (1)-(3) depend on (15). Given the time 0 market price of a traded (call or put) European option on a futures and the futures price, one can use the well known Black [9] formula for the option price to compute, by means of standard techniques (Eydeland and Wolyniec [23, pp. 147-150]), the so called implied volatility parameter $\hat{\sigma}_B$, where subscript B stands for Black. We numerically compute a Black volatility for each option in our dataset. Under model (1)-(3) the price of a European call/put option on a futures price would match the market price of the traded option if $\hat{\sigma}_B^2 = \check{\sigma}^2(t,t')/t$.

Figures 3 and 4 illustrate the futures prices and Black implied volatilities in our dataset. The marked seasonality in futures prices is notable. The overall decline of the volatilities with increasing futures price maturity is typical; it is known as the Samuelson [42] effect. Following Clewlow and Strickland [16, p. 160], we estimate the price model parameters by minimizing the sum of the squared percent deviations of observed futures prices and implied volatilities subject to appropriate nonnegativity constraints and the constraint $\sum_{m=1}^{12} \ln f_m = 0$ (this is a normalization condition that uses a yearly cycle with constant seasonality factors within each month as in Jaillet et al. [30]). Table 2 displays the relevant estimates. The root mean squared errors (RMSEs) between the observed and estimated futures prices and implied volatilities are 0.0584 and 0.0280, respectively.

Table 2: Estimates of the parameters of the natural gas price models using NYMEX data from 5/29/2009.

Two factor model		One factor model	
Initial short term level (χ_0)	0.7417	Initial level (χ_0)	1.2721
Speed of mean reversion (κ_χ)	1.5245	Speed of mean reversion (κ)	1.0547
Short term volatility (σ_χ)	0.7388	Volatility (σ)	0.6696
Short term risk premium (λ_χ)	-2.1219	Risk adjusted long term level (ξ^*)	-2.0421
Initial long term level (ξ_0)	0.4724		
Risk adjusted long term drift (μ_ξ^*)	0.0038		
Long term volatility (σ_ξ)	0.1300		
Correlation ($\rho_{\chi\xi}$)	-0.0886		
January factor (f_1)	1.0826	January factor (f_1)	1.0809
February factor (f_2)	1.0773	February factor (f_2)	1.0761
March factor (f_3)	1.0441	March factor (f_3)	1.0433
April factor (f_4)	0.9537	April factor (f_4)	0.9532
May factor (f_5)	0.9467	May factor (f_5)	0.9464
June factor (f_6)	0.9547	June factor (f_6)	0.9547
July factor (f_7)	0.9693	July factor (f_7)	0.9674
August factor (f_8)	0.9717	August factor (f_8)	0.9711
September factor (f_9)	0.9690	September factor (f_9)	0.9697
October factor (f_{10})	0.9750	October factor (f_{10})	0.9767
November factor (f_{11})	1.0133	November factor (f_{11})	1.0156
December factor (f_{12})	1.0565	December factor (f_{12})	1.0590

We estimate the relevant parameters of model (4) in a manner analogous to the estimation of those of model (1)-(3). Table 2 reports also these estimates. The RMSEs of the futures prices and implied volatilities for this estimated model are 0.2266 and 0.0332, respectively. Thus, using two factors, instead of one, yields a more accurate fit of the data.

Valuation Results. We employ a valuation period of twelve years divided into monthly time periods. Although a time horizon of twenty years would be more in line with industry practices regarding the valuation of LNG projects (Flower [24]), our choice stems on the fact that the NYMEX natural gas forward curve spans approximately twelve years with monthly maturities. However, if one were willing to extend the forward curve beyond this time horizon our model could be applied to this longer time horizon at the expense of additional run time. Thus, we set $J := 143$. We let the time 0 price of natural gas be equal to the Henry Hub spot price traded on 5/29/2009, which is \$3.92/MMBTU. We use an annual risk free rate equal to 0.47%, the one year U.S. treasury rate on 5/29/2009. We build the lattices described in §5 using the parameter estimates shown in Table 2 and the futures prices displayed in Figure 3. At time t_1 no inventory is available in storage and all the ships are in the ballast stage; we set x_1 and s_1 accordingly.

The software implementation of our model features inventory and action sets expressed in num-

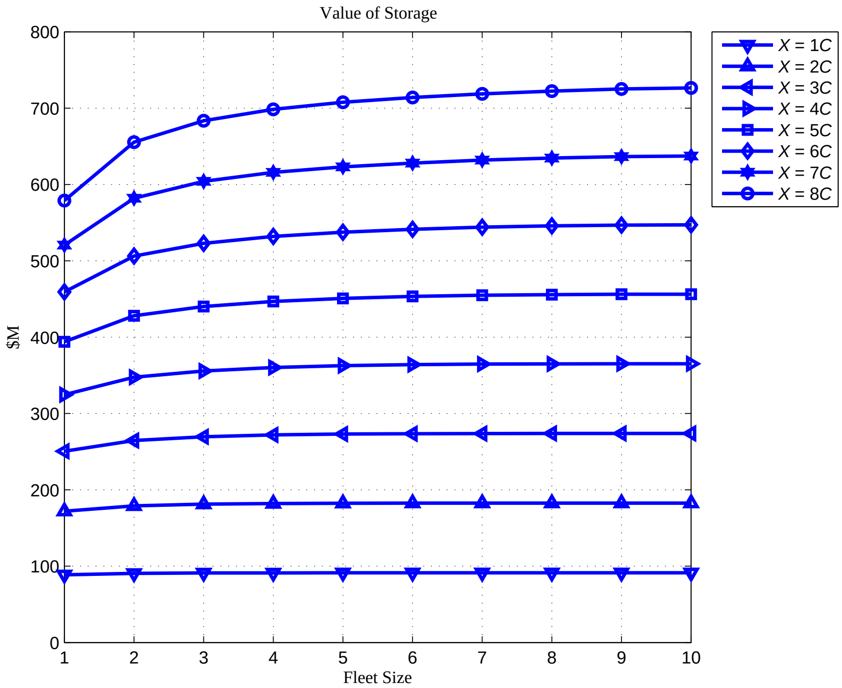

Figure 5: The estimated value of the option to store, $\hat{S}_1^B(x_1, s_1, p_1)$, using the two factor price model and exponentially distributed processing times ($M).

ber of cargos. This is justified by Proposition 3, whose assumptions are satisfied by the values of the relevant parameters used in this section. The Cpu times reported below pertain to computations performed on a 64 bits `Monarch Empro 4-Way Tower Server` with four `AMD Opteron 852 2.6GHz` processors, each with eight `DDR-400` SDRAM of 2 `GB` and running `Linux Fedora 11` (all the reported results were obtained using only one processor). Our model was coded in C++ and compiled using the compiler `g++ version 4.3.0 20080428 (Red Hat 4.3.0-8)`.

We first discuss the valuation results obtained using the two factor price model and a shipping model with exponentially distributed processing times (Table 5 in Online Appendix C reports the probability mass functions of the number of unloaded cargos per aggregate time period computed by the rolling forward method of §5 for different fleet sizes; computing this table takes less than 1 Cpu second). We compare these with results obtained with an essentially deterministic shipping system when discussing the shipping model effect, and with those obtained with the one factor model in our discussion of the price model effect.

The value of storage. Figure 5 displays the estimated value of the option to store for different fleet sizes and levels of storage space. The system configurations that we consider satisfy our assumption made in §5 that $\Pr\{\tilde{u} > U\} = 0$ holds. The estimates of the value of storage, obtained using 500,000 price and unloaded cargo sample paths, vary from $89M (1 ship and 1 cargo of

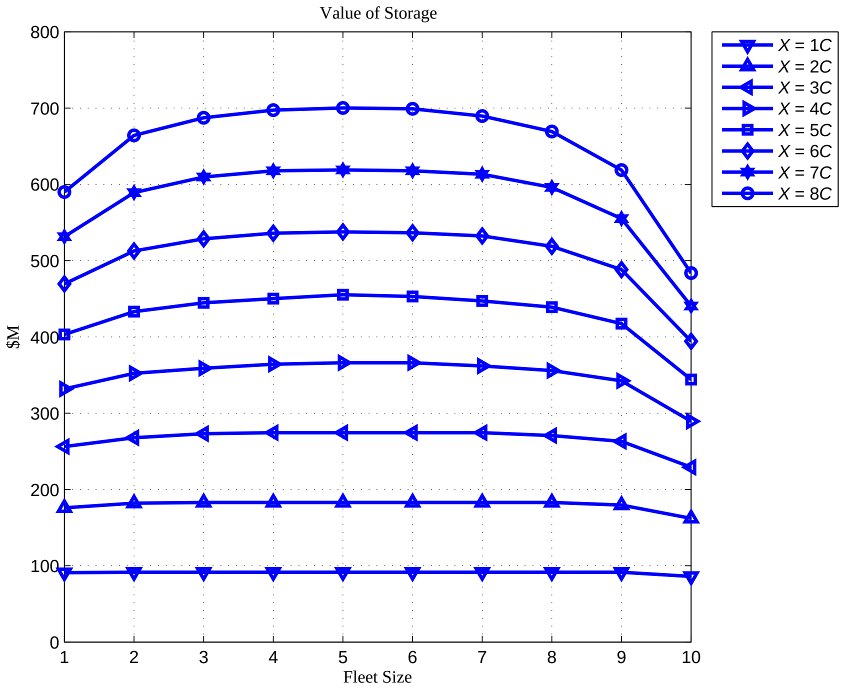

Figure 6: The estimated value of the option to store, $\hat{S}_1^B(x_1, s_1, p_1)$, using the two factor price model and the essentially deterministic shipping system when $Q = 1$BCF/day ($M).

storage space) to $726M (10 ships and 8 cargos of storage space), and their relative standard errors are 0.06% (we refer to a standard error expressed as a fraction of the estimate it pertains to as a relative standard error). The Cpu times required to compute these values vary from approximately 5 to 53 minutes; these times also include the estimation of the value of storage due to seasonality, discussed below, using 500,000 samples.

With the two factor price model, we use 1,000 unloaded cargo sample paths to estimate the upper bound, which takes between 61 and 144 minutes of Cpu time depending on the instance. We reject the hypothesis that $S_1^{UB}(x_1, s_1, p_1)$ is equal to $S_1^B(x_1, s_1, p_1)$ in favor of the alternative hypothesis that $S_1^{UB}(x_1, s_1, , p_1)$ is greater than $S_1^B(x_1, s_1, p_1)$ at the 5% significance level in 50 out of 80 combinations of fleet size and storage space. Moreover, the estimates of the value of storage are never lower than 99.11% of their upper bound estimates; on average this figure is 99.73%. These results suggest that our estimates of the value of storage are of very high quality, and reflect the very low ship blocking probability and low dependence between unloaded amounts in successive aggregate time periods in the systems that we consider.

Throughput and space effects. Figure 5 illustrates that the value of storage increases in the available space. This is intuitive, as more available space allows more effective exploitation of high natural gas prices. In addition, as one would expect, the marginal benefit of additional storage is

Table 3: Percent improvements of the basestock target policy relative to the myopic policy, using the two factor price model and exponentially distributed processing times.

	Storage Size (# of Cargos)							
# of Ships	1	2	3	4	5	6	7	8
1	0.36	2.27	5.16	8.60	12.42	16.60	21.14	26.05
2	0.03	0.26	0.95	2.00	3.31	4.83	6.51	8.29
3	0.01	0.04	0.21	0.60	1.18	1.89	2.73	3.67
4	0.00	0.00	0.05	0.18	0.45	0.84	1.30	1.84
5	0.00	0.00	0.01	0.05	0.17	0.38	0.66	0.99
6	0.00	0.00	0.00	0.01	0.06	0.16	0.33	0.56
7	0.00	0.00	0.00	0.00	0.02	0.06	0.16	0.31
8	0.00	0.00	0.00	0.00	0.01	0.02	0.06	0.15
9	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.06
10	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03

decreasing, as each additional unit of space is used less often. Although not immediately evident from Figure 5, it is more interesting that the value of storage can be nonmonotonic in the number of ships, or, equivalently, throughput: with 1 cargo and 2 cargos of storage space the value of storage decreases from \$91.3052M to \$91.3044M and from \$182.607M to \$182.601M when the number of ships increases from 9 to 10. These drops in storage value are minimal, but Figure 6 more forcefully illustrates this possibility (we obtain the values displayed in this figure by using the essentially deterministic shipping model described below when discussing the shipping model effect). This suggests that, due to regasification capacity constraints, after a critical level of throughput the value of storage *decreases*. Intuitively, as throughput grows large, forced sales approach the system capacity. As optional sales shrink, so does the ability to store, and consequently the value of storage (see Online Appendix C for a more detailed justification of this statement). The fundamental insight here is that discretionary regasification capacity must be available for an LNG terminal to have storage value. Otherwise, as throughput approaches the regasification capacity, an LNG terminal becomes a delivery mechanism with little or no storage value. This suggests that the discussion in Holcomb [29] (see §1) is pertinent to situations that satisfy this condition.

The storage values due to price seasonality and volatility. There is almost an equal split between the storage values due to price seasonality and volatility, with the former varying between 46% and 51% of the total option value (the relative standard error of the value due to seasonality is no more than 0.0186%). Considering the pronounced seasonality displayed by the natural gas forward curve (see Figure 3), it is remarkable that more value is not attributable to seasonality, or, put another way, that there is such significant value in adapting the inventory release policy to price volatility.

Forward looking optimization effect. As mentioned in §5, an optimal basestock target policy can be nontrivial, that is, the basestock targets may take values different from 0 and X. This is a consequence of the forward looking optimization nature of this policy. To assess the relevance of this feature of this policy, we compare the percent improvements of this policy relative to a myopic policy that in every stage sells as much as possible if the difference between the discounted price of the futures with maturity in the next stage and the spot price is positive, and holds as much inventory as possible otherwise (except for the forced sales); this is a basestock target policy whose targets are set naïvely either equal to 0 or X in every stage. Computing the myopic policy does not require solving an MDP. Table 3 reports these figures using the two factor price model and the exponential shipping system. The basestock target policy outperforms the myopic policy by no more than 26.05%, 8.29%, 3.67%, 1.84%, and 0.99% with fleets growing from 1 to 5 ships, respectively, and by less than 1% with 6 or more ships. For each fleet size, these improvements increase (weakly) in the available storage space. These results suggest that the simple myopic policy seems adequate for storage valuation when the throughput is sufficiently high relative to the storage space. Otherwise, there appears to be significant value from optimizing the inventory release policy by taking into account the entire future consequences of a current action.

Shipping model effect. As pointed out by Koenigsberg and Lam [33], exponential processing times, with a coefficient of variation equal to 1, are unrealistic, especially for shipping times whose averages are of the order of two or more weeks. Since our CQN model is a BCMP network, we could reduce the variability of the transit times by increasing the number of Erlang stages in the two shipping blocks (exponential times correspond to the case of a single Erlang stage). For simplicity, we consider the extreme case in which all the processing times are deterministic and equal to their means. In this case the throughput of the LNG chain, expressed in number of cargos per day, is equal to the fleet size divided by the sum of the average processing times.

In computing the value of storage in this case, we modify our deterministic assumption slightly to satisfy the conditions of Proposition 3 in §5: we use a two point unloading random variable that takes values equal to the floor and the ceiling of the throughput with probabilities determined to make the mean of this random variable equal to this throughput. We refer to this case as the essentially deterministic system. For consistency, we continue to use 500,000 samples to estimate the value of storage and that due to price seasonality; this takes between 1.4 and 1.7 Cpu minutes.

The estimate of the value of storage obtained under the exponential assumption is at least 98% of the value of storage computed in the essentially deterministic case (the relative standard errors of the value of storage in the latter case are roughly 0.06%); on average this figure is 99.37%. That the value of storage is lower in the former case is intuitive, since an essentially deterministic shipping system allows for easier planning of released inventory. That this value is not dramatically lower stems from the low ship blocking probabilities and congestion in the exponential shipping system. Similar to the exponential case, the estimated values due to price seasonality approximately range from 47% to 51% of the option value; their relative standard errors are no more than 0.0031%. Thus, a simple version of the shipping model may be adequate to value storage for the range of parameters that we consider.

Price model effect. We reexamine all the previous comparisons, but now for the one factor price model. In this case the estimates of the value of storage are between 83% and 84% of those obtained using the two factor price model, both with the exponential and essentially deterministic shipping systems (the relative standard errors with 500,000 samples are 0.05% for both systems). Hence, the value of storage due to price seasonality, which is the same with the one and two factor models, is relatively more important with a single factor, varying between 55% and 62% of the value of storage (the relative standard errors of the values due to price seasonality are no more than 0.0189% in the exponential case and 0.0031% in the essentially deterministic case).

Estimating the value of storage is less computationally demanding with a one factor model, and so is estimating our upper bound. Thus, we also estimate the upper bound using 500,000 samples of unloaded cargo sequences, rather than using 1,000 samples as in the two factor model case. With the exponential shipping system it takes from about 2 to 38 Cpu minutes to run our valuation model and from about 7 to 28 Cpu minutes to compute the upper bound; in the essentially deterministic shipping system it takes about 26 Cpu seconds to run our valuation model. For the exponential shipping system, we reject the hypothesis that $S_1^{UB}(x_1, s_1, p_1)$ is equal to $S_1^{B}(x_1, s_1, p_1)$ in favor of the alternative hypothesis that $S_1^{UB}(x_1, s_1, , p_1)$ is greater than $S_1^{B}(x_1, s_1, p_1)$ at the 5% level of significance in 53 out of 80 cases; the estimated value of storage is at least 99% of its upper bound and on average it is 99.71% of this value. The value of storage so computed is at least 97.84% of that obtained with the essentially deterministic system and on average it is 99.33% of this value.

These results suggest that a quicker but appreciably lower estimate of the value of storage can be obtained by using a one factor model, in which case our integrated model continues to deliver very high quality storage valuations relative to our upper bound estimate.

8 Conclusions

Motivated by current developments in the LNG industry, we develop a real option model for the strategic valuation of downstream LNG storage. Unique to our model is the integration of models of natural gas liquefaction and LNG shipping, natural gas price evolution, and LNG inventory regasification and sale into the wholesale spot market. This provides a heuristic strategic valuation of the real option to store LNG at a regasification terminal. We apply our model to real and estimated data and find these valuations to be highly accurate. We also investigate how these values depend on the level of stochastic variability in the shipping model, the type of natural gas price model used, the LNG throughput, and the type of inventory control policy employed.

Our model has the potential to be applied in practice as it is both computationally manageable and highly accurate. It could be used by LNG players to assess the value of leasing contracts on regasification facilities, or as an economic valuation model by different parties involved in the development of LNG projects (see the discussion in Flower [24, p. 120]). Moreover, while our focus has been on LNG, our model and analysis have potential applicability in other commodity industries that exhibit uncertainty in the commodity production or shipping processes, for example those characterized by random yield and/or spot price fluctuations.

For further research, one could assess the dependence of our valuation results on the type of multifactor model used to represent the evolution of the price of natural gas. In particular, it would be interesting to study the case when this evolution is captured using an equilibrium model, such as that of Routledge et al. [41], or reduced form models with more than two factors, such as those presented by Schwartz [43], Casassus and Collin-Dufresne [13], and Lai et al. [34].

Another additional research area is the development of approximate dynamic programming algorithms (Bertsekas [7, Chapter 6], Adelman [2], Powell [39]) for solving the operational model discussed in §3, perhaps extended to include the status of the liquefaction facility and the inventory available at this location. This would allow one to obtain a policy for tactical and operational control. The work of Besbes and Savin [8] is pertinent here.

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Online Appendix

A Units of Measurement and Conversion Factors

Table 4 reports relevant units of measurements and conversion factors.

B Proofs for §5

Proof of Proposition 1 (Optimal value function). By induction. The claimed property clearly holds in stage $J+1$. Make the induction hypothesis that this property holds also in stages $j+1,\ldots,J$. Consider stage j. The function $V^H_{j+1}(x_{j+1}, p_{j+1})$ is concave in $x_{j+1} \in \mathcal{X}$ for each given $p_{j+1} \in \mathcal{P}_{j+1}$. This implies that $\mathbb{E}[V^H_{j+1}(x_{j+1}, \tilde{p}_{j+1}) \mid p_j]$ is concave in $x_{j+1} \in \mathcal{X}$ for each given $p_j \in \mathcal{P}_j$. Fix $p_j \in \mathcal{P}_j$, $u \in \mathcal{U}$, x^1_j, $x^2_j \in \mathcal{X}$ with $x^1_j \neq x^2_j$, and q^1_j, $q^2_j \in \mathcal{Q}(x_j, u)$ with $q^1_j \neq q^2_j$. Define $x^1_{j+1} := x^1_j + u - q^1_j$, $x^2_{j+1} := x^2_j + u - q^2_j$, and $x^\theta_{j+1} := \theta x^1_{j+1} + (1-\theta)x^2_{j+1}$ for some $\theta \in [0,1]$. Since x^1_{j+1}, $x^2_{j+1} \in \mathcal{X}$, the convexity of $\mathcal{X}$ implies that $x^\theta_{j+1} \in \mathcal{X}$. The concavity of $\mathbb{E}[V^H_{j+1}(x_{j+1}, \tilde{p}_{j+1}) \mid p_j]$ in x_{j+1} for given p_j and the linearity of expectation imply that

$$\mathbb{E}[V^H_{j+1}(x^\theta_{j+1}, \tilde{p}_{j+1}) \mid p_j] \geqslant \theta\mathbb{E}[V^H_{j+1}(x^1_{j+1}, \tilde{p}_{j+1}) \mid p_j] + (1-\theta)\mathbb{E}[V^H_{j+1}(x^2_{j+1}, \tilde{p}_{j+1}) \mid p_j].$$

Thus, the definitions of x^1_{j+1} and x^2_{j+1} imply that $\mathbb{E}[V^H_{j+1}(x_j + u - q_j, \tilde{p}_{j+1}) \mid p_j]$ is jointly concave in x_j and q_j for given p_j and u, and so is $g_j(p_j)(1-\phi)q_j - hx_j - cu + \delta\mathbb{E}[V^H_{j+1}(x_j + u - q_j, \tilde{p}_{j+1}) \mid p_j]$. This property, the convexity of set $\mathcal{A}(u) := \{(x,q) : \ x \in \mathcal{X}, \ q \in \mathcal{Q}(x,u)\}$, and Proposition B-4 in Heyman and Sobel [53, p. 525] imply that $v^H_j(x_j, p_j, u)$ is concave in x_j for given p_j and u. The concavity of $V^H_j(x_j, p_j)$ in x_j for given p_j follows since $V^H_j(x_j, p_j) = \mathbb{E}[v^H_j(x_j, p_j, \tilde{u})]$, and the property holds in all stages by the principle of mathematical induction. □

Proof of Proposition 2 (Optimal policy structure). Consider arbitrary stage $j \in \mathcal{J}$. Since finding an optimal action is equivalent to finding an optimal optional sale after having performed

Table 4: Units of measurement and conversion factors.

T	Metric Tons (LNG)
MTPA	Million Metric Tons per Annum
CM	Cubic Meters (LNG)
NM	Nautical Miles
MMBTU	Million British Thermal Units
BCF	Billion Cubic Feet
1T = 51.98237MMBTU	
1Knot = 1NM per Hour	
1CM = 23.6863MMBTU	
1BCF = 1,100,000MMBTU	

the forced sale, we denote by $y_j(x_j, u)$ the post forced sale inventory level given x_j and u:

$$y_j(x_j, u) := \begin{cases} X & x_j + u \in (X, X+U], \\ x_j + u & x_j + u \in [0, X]. \end{cases}$$

This inventory level can only take values in set $\mathcal{X}$. The costs hx_j and cu_j do not affect the choice of an optimal optional sale in stage j, and we can restrict our attention for this purpose to state $(y_j, p_j) \in \mathcal{X} \times \mathcal{P}_j$. To find an optimal optional sale in this state, we first consider the relaxed problem of finding an optimal optional sale by ignoring the capacity restriction. Thus, the feasibility set is simply equal to $[0, y_j]$ for each $y_j \in \mathcal{X}$, and the problem to be solved is $\max_{q_j^O \in [0,y_j]} \nu_j(y_j, q_j^O, p_j)$, where $\nu_j(y_j, q_j^O, p_j) := g_j(p_j)(1-\phi)q_j^O + \delta\mathbb{E}[V_{j+1}^H(y_j - q_j^O, \tilde{p}_{j+1}) \mid p_j]$.

Define $\check{q}_j^O(y_j, p_j)$ as the largest quantity in set $\arg\max_{q_j^O \in [0,y_j]} \nu_j(y_j, q_j^O, p_j)$. Following Porteus [54, p. 67], by letting $x_{j+1} = y_j - q_j^O$, it clearly holds that

$$\begin{aligned} \max_{q_j^O \in [0,y_j]} \nu_j(y_j, q_j^O, p_j) &= \max_{x_{j+1} \in [0,y_j]} \nu_j(y_j, y_j - x_{j+1}, p_j) \\ &= g_j(p_j)(1-\phi)y_j \\ &\quad + \max_{x_{j+1} \in [0,y_j]} \left\{ \delta\mathbb{E}[V_{j+1}^H(x_{j+1}, \tilde{p}_{j+1}) \mid p_j] - g_j(p_j)(1-\phi)x_{j+1} \right\}. \end{aligned}$$

Notice that any element of $\arg\max_{x_{j+1} \in \mathcal{X}} \nu_j(y_j, y_j - x_{j+1}, p_j)$, and so its smallest one denoted by $\check{x}_{j+1}(p_j)$, does not depend on y_j. In particular, it holds that

$$\check{q}_j^O(y_j, p_j) = \begin{cases} 0 & y_j \in [0, \check{x}_{j+1}(p_j)], \\ y_j - \check{x}_{j+1}(p_j) & y_j \in (\check{x}_{j+1}(p_j), X]. \end{cases}$$

This implies that we can define $b_j(p_j) := \check{x}_{j+1}(p_j)$. Imposing the capacity constraint on optimal optional sale $\check{q}_j^O(y_j, p_j)$ yields the stated expression for $q_j^{O*}(x_j, p_j, u_j)$. $\square$

Proof of Proposition 3 (Optimal policy computation). By induction. The claimed properties clearly hold in stage $J+1$. Make the induction hypothesis that they hold also in stages $j+1, \ldots, J$. Consider stage j. Fix $p_j \in \mathcal{P}_j$ and $u \in \mathcal{U}$. Recall that set $\mathcal{P}_{j+1}$ is finite. This and the induction hypothesis imply that, given p_j, $\delta\mathbb{E}[V_{j+1}^H(x_{j+1}, \tilde{p}_{j+1}) \mid p_j]$ is piecewise linear and continuous in $x_{j+1} \in \mathcal{X}$ and changes slope in x_{j+1} only at integer multiples of L. It is easy to show that, given p_j and u, $g_j(p_j)(1-\phi)q_j - hX - cu + \delta\mathbb{E}[V_{j+1}^H(X + u - q_j, \tilde{p}_{j+1}) \mid p_j]$ changes slope in q_j only at integer multiples of L, which implies that $b_j(p_j)$ is an integer multiple of L. It is now shown that the function $V_j^H(x_j, p_j)$ is piecewise linear and continuous in $x_j \in \mathcal{X}$ and changes slope in x_j only at integer multiples of L. By Proposition 2, the following cases need to be considered: (H) $x_j + u \in [0, b_j(p_j))$ and (S) $x_j + u \in [b_j(p_j), X+U]$. Case (H): $v_j^H(x_j, p_j, u) = -cu - hx_j + \delta\mathbb{E}[V_{j+1}^H(x_j + u, \tilde{p}_{j+1}) \mid p_j]$. Case (S): if $b_j(p_j)$ can be reached from

Table 5: Probability mass functions of the number of unloaded cargos in one aggregate time period (1 month) computed by the rolling forward method for the shipping model with exponential processing times.

Unloaded	# of Ships in the Fleet									
Cargos	1	2	3	4	5	6	7	8	9	10
0	0.2809	0.0791	0.0223	0.0063	0.0018	0.0005	0.0002	0.0000	0.0000	0.0000
1	0.5216	0.2935	0.1241	0.0468	0.0166	0.0057	0.0019	0.0006	0.0003	0.0001
2	0.1776	0.3726	0.2728	0.1462	0.0672	0.0282	0.0111	0.0042	0.0016	0.0006
3	0.0189	0.1956	0.3037	0.2514	0.1565	0.0829	0.0395	0.0176	0.0074	0.0030
4	0.0010	0.0512	0.1877	0.2627	0.2332	0.1609	0.0947	0.0501	0.0245	0.0113
5	0.0000	0.0074	0.0699	0.1762	0.2346	0.2181	0.1621	0.1037	0.0595	0.0316
6	0.0000	0.0006	0.0166	0.0792	0.1649	0.2136	0.2054	0.1618	0.1105	0.0680
7	0.0000	0.0000	0.0026	0.0248	0.0834	0.1548	0.1971	0.1946	0.1604	0.1158
8	0.0000	0.0000	0.0003	0.0055	0.0310	0.0846	0.1458	0.1836	0.1852	0.1586
9	0.0000	0.0000	0.0000	0.0008	0.0087	0.0355	0.0842	0.1376	0.1722	0.1769
10	0.0000	0.0000	0.0000	0.0001	0.0018	0.0116	0.0385	0.0828	0.1302	0.1624
11	0.0000	0.0000	0.0000	0.0000	0.0003	0.0030	0.0141	0.0403	0.0807	0.1234
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0005	0.0041	0.0160	0.0413	0.0782
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0010	0.0052	0.0175	0.0415
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0014	0.0062	0.0186
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0004	0.0019	0.0070
16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0005	0.0023
17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0006
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

$x_j + u$ then $v_j^H(x_j, p_j, u) = g_j(p_j)(1-\phi)[x_j + u - b_j(p)] - cu - hx_j + \delta\mathbb{E}[V_{j+1}^H(b_j(p_j), \tilde{p}_{j+1}) \mid p_j]$, otherwise $v_j^H(x_j, p_j, u) = g_j(p_j)(1-\phi)Q - cu - hx_j + \delta\mathbb{E}[V_{j+1}^H(x_j + u - Q, \tilde{p}_{j+1}) \mid p_j]$. It is an easy task to verify that, for given p_j and u, $v_j^H(x_j, p_j, u)$ is piecewise linear and continuous in x_j and changes slope in x_j only at integer multiples of L. Hence, $V_j^H(x_j, p_j)$ satisfies this property because it is a convex combination of a finite number of functions that also satisfy this property. By the principle of mathematical induction, the claimed properties hold in every stage. □

C Supporting Material for §7

Table 5. Table 5 displays the probability mass functions of the number of unloaded cargos per aggregate time period (1 month) computed by the rolling forward method when the loading, unloading, and shipping times are exponentially distributed according to the parameters displayed in Table 1.

Explanation of the Decreasing Value of Storage for High Throughput. Consider a degenerate unloading random variable equal to u. Momentarily, impose the constraint that the inventory level at time $t_{J+1} \equiv T$ be zero, i.e., $x_{J+1} = 0$. Suppose that $u = Q$, so that the LNG rate into the terminal is equal to the maximum rate out of it. In this case, the value of storage must be zero because no amount of LNG can be stored in any aggregate time period. Thus, when u is sufficiently high, as it approaches Q from below, the value of storage decreases to zero.

Now, remove the constraint $x_{J+1} = 0$, so that the only conditions imposed on x_{J+1} are $0 \leqslant x_{J+1} \leqslant X$. Finally, make the realistic assumption that $QJ \geqslant X$, that is, a full terminal at time $t_1 \equiv 0$ can be emptied by time T. If $u = Q$, any amount of LNG not released in some aggregate time period $j \in \mathcal{J}$ must be stored until time T, and the maximum amount of stored LNG during the entire planning horizon is $\min\{uJ, X\} = \min\{QJ, X\} = X$. Thus, the value of storage for any level of throughput that allows one to store at least an amount X of LNG during the entire planning horizon (that is, for any $u \leqslant Q$ such that $uJ \geqslant X$) must be at least the one obtainable when $u = Q$. In other words, $u = Q$ is the level of throughput that minimizes the value of storage among all those that satisfy $uJ \geqslant X$. Therefore, as $u \geqslant X/J$ approaches Q, obviously from below, the value of storage decreases in a neighborhood of Q.

When the unloading random variable is not degenerate, explaining the decreasing value of storage after some level of throughput is more involved, but the main intuition provided here remains relevant.

References

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