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# Selling to Strategic Customers: Opaque Selling Strategies

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# Selling to Strategic Customers: Opaque Selling Strategies

Kinshuk Jerath, Serguei Netessine, and Senthil K. Veeraraghavan

**Abstract** Over the past few years, firms in the travel and entertainment industries have begun using novel sales strategies for revenue management. In this chapter, we study a selling strategy called opaque selling, in which firms guarantee one of several fully specified products, but hide the identity of the product that the consumer will actually obtain until after the purchase is completed. Several firms such as Hotwire, Priceline and Mystery Flights engage in opaque selling of travel products. The academic literature in this area is recent and evolving. We first survey the nascent literature on opaque selling strategies. After presenting the current state of theory and practice, we analyze in-depth a model of competing firms selling horizontally differentiated products through an opaque channel. Consumers strategically time their purchases by developing rational expectations about future availability in the opaque market, keeping in mind that demand is uncertain and product supply could be limited. This model helps illustrate the conditions under which opaque selling can increase firm profits. We conclude the chapter by discussing ongoing research and charting out future research directions.

## 1 Introduction

The emergence of electronic commerce has had a massive impact on the travel industry. In just over a decade after online ticket sales were introduced in the travel industry, more than 50% of airline tickets are sold online (SITA 2007), and this proportion is increasing at a fast rate – from 2001 to 2003, online leisure travel

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bookings in the United States more than tripled (Tedeschi 2005). The major players in the online travel market are Expedia, Travelocity, Orbitz, Hotwire and Priceline. The first three companies sell regular or *transparent* tickets: consumers see a posted price against each ticket that is available and make their purchase decisions. In addition, technological advancements brought about by electronic commerce have also enabled firms to employ other creative selling strategies. For instance, Hotwire and Priceline offer opaque tickets, whereby consumers are not given the full details of the ticket (e.g., specific airline, time of departure, number of stops) until after they have purchased it, but they do often pay a much lower price because of this uncertainty. The consumers are guaranteed that the ticket they receive will meet certain conditions (e.g., the range of departure and arrival times might be guaranteed), but they can receive one of several tickets from a host of carriers that meet these conditions. A particular consumer might prefer one of the tickets over the other, and there is a possibility that she will receive this preferred ticket. However, she also runs the risk of receiving a ticket she does not prefer.

There are several examples of firms engaging in opaque selling. For instance, several airlines (e.g., Delta, Northwest Airlines, United Airlines, US Airways, etc.) supply tickets to an intermediary Hotwire which sells them as opaque tickets at discounted prices. A potential customer at Hotwire keys in the details of the route she wants to fly and the time frame. In response, Hotwire provides the option to purchase an opaque ticket at a discounted price in which it does not reveal the name of the airline and the exact itinerary<sup>1</sup> along with several options for transparent tickets. The consumer then makes her purchase or no-purchase decision. Likewise, Priceline sells opaque tickets but with one major difference – it asks the user to bid the price that she wants to pay, which is known as Name-Your-Own-Price strategy. Besides airline tickets, Hotwire and Priceline also use opaque selling for hotel rooms and rental cars, in both cases again partnering with major companies in these businesses. Beyond Hotwire and Priceline, firms such as Norwegian Cruise Lines sell opaque tickets for cruises, whereby a customer pays a discounted price which guarantees a minimum class of cabin and is promised an upgrade (if available), but the details of the upgrade or the exact location of the cabin are revealed later based on availability. Mystery Flights, a firm in Australia, sells opaque tickets in which the starting and ending times of the itinerary are specified, but the destination is not revealed and it can be one of several pre-specified destinations. In this case, the opacity is with respect to a different aspect of the ticket attribute as compared to Hotwire, i.e., the destination is opaque, rather than the flight time.

An essential feature of opaque selling is that it requires at least two differentiated products to credibly hide the identity of the final product the consumer will receive. A monopolist selling several differentiated products can therefore choose to sell opaque products (e.g., a day ticket or a night ticket on the same route) but competing firms can also sell opaque products through intermediaries. Under the latter arrangement, an intermediary, such as Hotwire, is authorized to sell an opaque ticket with the final service delivered by one of the participating firms, such as US

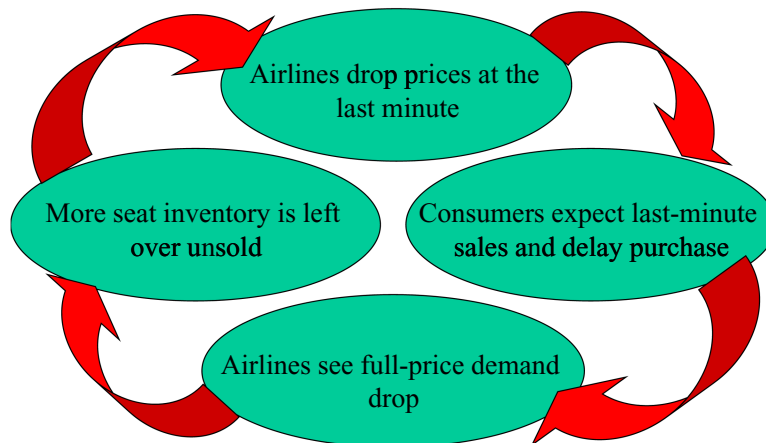
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<sup>1</sup> It is typically possible to determine the name of the airline from the exact itinerary and vice versa, so that opaque sellers hide both of these details.

Airways or United Airlines. A salient feature of opaque products in the travel industry is that they are only available late in the selling horizon. In other words, for a particular route, typically only transparent tickets (with full product information) are available several weeks before the date of the flight, but a few days before the flight one can observe opaque sales as well. This observation indicates that the opaque selling strategy is often used as a mechanism to sell capacity that could not be sold at higher prices, which is consistent with opinions of several industry experts we communicated with. Thus, the opaque selling strategy is an important tool for clearing unsold inventory of seats/rooms.

The practice of selling opaque products has generated a lot of debate in the industry. Travel companies are always on the lookout for innovative revenue management strategies and companies practicing opaque selling strategies argue that they augment revenues because they “enables airlines to generate incremental revenue by selling distressed inventory cheaply without disrupting existing distribution channels or retail pricing structures” (Smith et al. 2007). However, other experts argue that selling cheaper opaque tickets amounts to introducing another channel that competes with the full-price channel, which is harmful for the industry since it “starts a cycle of price degradation that will eventually lead to ... destroying the airlines” (Sviokla 2004). The argument for price degradation is captured in Figure 1.

To explain the picture, traditionally the revenue management literature as well as many real-life revenue optimization engines make simplistic assumptions regarding customer arrival patterns. Namely, customers are assumed to be passive to the different pricing strategies used by firms and their propensity to buy tickets is traditionally described by the exogenously specified stochastic arrival process. However, there is growing evidence that customers are strategic and, realizing that prices can decrease over time if there is unsold capacity late in the selling horizon, they learn



**Fig. 1** Last minute price discounting hurts airline revenues as more leisure customers continue to anticipate last minute sales.

to wait for these low prices. If a lot of customers wait in this manner, there will be excess unsold capacity close to flight departure and this can become a self-fulfilling cycle: more and more capacity will remain unsold at the full price, causing more discounts, which causes more customers to wait for discounts etc. This is the “cycle of price degradation” referred to in the picture above, and an argument can be made that opaque selling is one way of giving such last minute discounts. However, the argument can also be made that opaque selling strategies help break the cycle of degradation because the consumer has to anticipate which company will be the ultimate service provider. Thus, opaqueness introduces the additional level of price discrimination and makes last-minute discounts harder to exploit by strategic consumers.

The academic study of this novel selling strategy is nascent but growing. This stream of literature lies at the intersection of the study of revenue management strategies and the study of strategic consumer behavior. Currently most papers appear in the marketing domain and all of these papers are very recent. Formal research is needed to understand the impact of opaque selling strategies on strategic consumer behavior, as well as to compare effectiveness of opaque selling relative to other selling strategies. In this chapter, we survey the papers in the academic literature that study this phenomenon and we attempt to answer the question: Under what conditions, and why, is opaque selling attractive to firms? Broadly speaking, at least three different explanations emerge: (1) A monopolist using opaque selling can weakly improve profits by using opaque selling as a price discrimination strategy, (2) Under competition, when opaque selling is introduced simultaneously with transparent selling, it is profitable only if there is a large-enough class of brand-loyal consumers for each airline, and (3) Under competition, even if the assumption of brand-loyalty is not relied upon, opaque sales can still be profitable when introduced late in the selling horizon as a mechanism to sell off distressed inventory, and this happens without disrupting sales in the regular transparent channels. Overall, our chapter suggests that there are reasons to believe in viability of opaque selling strategies but it is also evident that this literature is just beginning to emerge and much more research is needed in this area. We discuss directions of potential future avenues of research in this chapter.

The rest of the chapter proceeds as follows. In Section 2, we place the literature on opaque selling within the related literature in Economics, Marketing and Operations Management. The focus of this section is on discussing five recent papers on opaque selling (Jiang 2007, Fay and Xie 2008, Fay 2008, Shapiro and Shi 2008, and Jerath et al. 2008). We then provide extensive coverage of the model in Jerath et al. (2008) with deterministic (Section 3) and stochastic (Section 4) demand. With the help of this model, we uncover the mechanism behind the opaque selling strategy and show how the profitability of opaque sales varies with demand uncertainty and customer valuation for the product. In Section 5, we summarize our conclusions from the current literature and provide directions for future work.

## 2 Literature Review

In this section, we survey the recent literature on opaque selling strategies as well as related work in other areas. We focus on papers that model opaque products as sold by Hotwire, so that a price for the opaque product is posted and consumers decide whether to make the purchase or not, rather than as sold by Priceline, which asks consumers to bid the price they want to pay and their bids can be accepted or rejected.<sup>2</sup> This helps us to narrow our focus down to five papers that we will discuss: Jiang (2007), Fay and Xie (2008), Fay (2008), Shapiro and Shi (2008) and Jerath et al. (2008). But first we place the literature on opaque selling strategies within the larger literature in Economics, Marketing and Operations Management and only then review the papers above in greater detail.

The study of opaque selling strategies is closely related to the literature on price discrimination (e.g., Narasimhan 1984) and market self-segmentation (e.g., Moorthy 1984). In these settings, firms offer a menu of products and customers self-select into classes based on their product preference. Since opaque sales also have a temporal aspect to them, as in the model in Jerath et al. (2008), they are related to the literature on inter-temporal pricing. The seminal work on inter-temporal sales is Coase (1972) which demonstrates that, given a durable product with an infinite number of selling opportunities over time, a monopolist will eventually decrease a product's price to its marginal cost because consumers will anticipate this decrease and will wait for discounts (the famous Coase conjecture). Numerous papers that followed laid out conditions in which the Coase conjecture may not hold (Stokey 1979, Besanko and Winston 1990 and DeGraba 1995). In particular, DeGraba (1995) showed that, under uncertain demand and capacity constraints, there is a threat of unavailability in the future and all consumers will not wait so higher prices can be charged. Note that both uncertain demand and limited short-term capacity are features of the travel industry.

The strategy of selling products both directly and through an opaque channel is related to the "damaged goods" literature (Deneckere and McAfee 1996) in which a high-quality product is sold with different options by disabling (or "damaging") some of its features. This is similar to versioning (Varian 2000) in which the same product is sold in different versions. While these are related to opaque selling, the main difference is that here the consumer knows that she is obtaining a lower-quality product and she knows exactly what is wrong with the product. On the other hand, with opaque selling the consumer can obtain her preferred product with a positive probability. In other words, opaque selling introduces "buyer uncertainty" in terms of product assignment. In that vein, opaque selling is related to the strategy of advance selling (Xie and Shugan 2001) which utilizes a different kind of buyer uncertainty by selling to consumers before they learn their valuations. Finally, opaque selling is related to the literature on revenue management (Talluri and van Ryzin

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<sup>2</sup> There is a rich literature studying the Name-Your-Own-Price selling format e.g. Terwiesch et al. (2005). The reader is referred to this paper for references.

2004) and it is also related to the strategic consumer behavior literature in operations management strategies which is described elsewhere in this book.

We now proceed to review the five papers on opaque selling mentioned earlier. All of these papers use economic modeling to study opaque sales. We first classify them according to their modeling framework. The main dimension of differentiation is monopoly vs. competition models: while a majority of opaque sales currently happens under competition between service providers (e.g., Hotwire sells air tickets from competing airlines), there are also cases like Norwegian Cruise Lines such that a single firm sells several of its own products as opaque. Papers by Jiang (2007) and Fay and Xie (2008) model a monopolist selling opaque products while Fay (2008), Shapiro and Shi (2008) and Jerath et al. (2008) model competing firms selling opaque products through an intermediary. The second dimension is demand uncertainty. In practice, demand for travel services is highly uncertain and, as a result, supply may not always match the demand. The uncertainty in demand and capacity constraints are reflected in models of Fay and Xie (2008) and Jerath et al. (2008). The third dimension is whether the model is dynamic (multi-period) or static (single-period). Only work of Jerath et al. (2008) incorporates dynamic considerations: the two competing firms first sell transparent tickets at full prices and then they may sell leftover capacity through the opaque intermediary. This modeling approach is meant to reflect an often-observed practice of selling opaque tickets only close to the date of travel service occurrence. Finally, in cases when there is an intermediary selling opaque tickets (i.e., when firms compete), the intermediary can be strategic (i.e., it makes pricing decisions) or passive. Works of Fay (2008) and Jerath et al. (2008) model strategic intermediaries. The classification in the previous discussion is summarized in Table 1. We now proceed to analyze opaque literature by discussing each paper in details.

In the monopoly setting, Jiang (2007) uses a single-period model with horizontally differentiated products and deterministic demand. The motivation is that the firm sells a morning flight (M) and an afternoon or night flight (N) on the same route. Even though the customer buying the opaque ticket knows the firm that is selling him the ticket, he does not know the departure time. The customers are uniformly distributed along the Hotelling line with each flight (M and N) located at the end points of the line. The monopolist can sell both transparent and opaque tickets. The main assumption made in the paper is that, although the flight information is not revealed for the opaque tickets, consumers expect an equal probability of ob-

**Table 1** Taxonomy of papers on opaque selling.

|                        | Competition | Demand<br>Uncertainty | Single<br>Period | Strategic<br>Intermediary |
|------------------------|-------------|-----------------------|------------------|---------------------------|
| Jiang (2007)           | No          | No                    | Yes              | –                         |
| Fay and Xie (2008)     | No          | Yes                   | Yes              | –                         |
| Shapiro and Shi (2008) | Yes         | No                    | Yes              | No                        |
| Fay (2008)             | Yes         | No                    | Yes              | Yes                       |
| Jerath et al. (2008)   | Yes         | Yes                   | No               | Yes                       |

taining M or N ticket independent of the actual allocation made by the firm. In other words, equal availability is assumed (i.e., there is equal number morning and afternoon flights sold in the opaque market). This is a common assumption in this stream of literature that is largely driven by the absence of capacity constraints and the absence of demand uncertainty which, if present, could lead to different proportions of M and N tickets sold as opaque.

Jiang shows that, since the seller imposes consumption uncertainty on the buyers, the buyers trade off consumption values for price savings (i.e., opaque tickets are priced lower, which is consistent with business reality). Due to buyer heterogeneity, some buyers believe that the difference between two product groups (opaque and transparent) is significant while others do not. Opaque selling can therefore help the monopolist increase profits by discriminating among these groups. Jiang (2007) also conducts welfare analysis of the effect of selling opaque tickets in the market and shows that, when buyer heterogeneity is high enough, selling opaque products can improve social welfare. Both the firm and customers can benefit from the dual-market strategy such that the firm sells both opaque and transparent tickets, so Pareto improvements are achieved. On the other hand, when very few customers in the market differentiate strongly between opaque tickets and transparent tickets, the firm might choose to sell only transparent tickets and serve only the high-value customers in the market.

Fay and Xie (2008) refer to opaque selling as “probabilistic selling.” They begin by considering a monopolist offering two products with consumers distributed on a Hotelling line as in Jiang (2007). The products have identical production costs. The seller considers two selling strategies: *traditional* selling (TS) and *probabilistic* selling (PS). Under traditional selling each good is offered at certain price. Under probabilistic selling, the seller offers one *probabilistic* good, which has a chance to be one of the two traditional goods. PS is essentially the same strategy as opaque selling, since the customer does not know the actual product until it is purchased. The traditional products are located at each end of the Hotelling line of unit length with the utilities normalized to one and all consumers have the same travel cost  $t$ . A customer located at  $x$  receives utility  $1 - xt$  from buying product 1, and he receives utility  $1 - (1 - x)t$  from buying product 2. Each customer needs only one good, and chooses the good that maximizes his or her expected surplus based on the prices of those goods. Thus, each customer can buy either of the traditional goods, or the probabilistic good, or not buy at all. An important feature of their model is that the consumers are rational and forward looking (see Muth 1961). In other words, consumers form expectations about each product’s allocation to the opaque channel by the monopolist, and these expectations are consistent in equilibrium.

Fay and Xie (2008) find that opaque selling strictly improves the monopolist’s profit if production costs are sufficiently low. However, the advantage from opaque selling depends strongly on the magnitude of travel costs. When the travel costs are very small, the products are essentially substitutes and the equilibrium prices are very similar. Therefore, in this case probabilistic selling does not help improve profits since the opportunity to price discriminate is very limited. On the other hand, when there is significant differentiation between two products on the Hotelling line,



(i.e., when  $t$  is high) no customer wants to risk buying the probabilistic good. Thus, probabilistic or opaque selling does not add much profit when there is high differentiation between the two traditional goods. In summary, Fay and Xie (2008) show that profit advantage from opaque selling is highest when the horizontal differentiation of the products is at the intermediate level. Interestingly, Fay and Xie (2008) show that advantages of opaque selling do not depend so much on standard assumptions behind the classical Hotelling model. For instance, the demand distribution need not be uniform, and the preferences of the consumer population as a whole need not be symmetric. For example, when market demand for one product is more than for the other, the results continue to hold. They also extend the model to a special case of information uncertainty when the firm does not know which product has more demand than the other and all results still continue to hold. Finally, the authors confirm their results by considering a Salop circle (i.e., all customers are distributed along the circumference of a unit radius) while the seller offers  $N$  goods located equidistantly along the circumference. They show that offering probabilistic goods can reduce the seller's information disadvantage and lessen the negative effect of demand uncertainty on profit by significantly reducing the problem of mismatch between capacity and demand.

Shapiro and Shi (2008) model a circle-shaped city (Salop's circle) with  $N$  firms located equidistantly (similar to the aforementioned extension considered in Fay and Xie 2008). The market size is fixed (i.e., the demand is deterministic) so the firms cannot attract more customers by lowering prices. Furthermore, all the firms are endowed with ample capacity so that each firm has enough capacity to supply the entire market. The customers are typified by two parameters: their location and their travel (transportation) cost. The location of the customer is specified by the standard Hotelling model. The travel cost is a binary variable. There are some customers of the high type (with high travel cost, e.g., business travelers), and the rest have the low type (with low travel cost, e.g., leisure travelers). The number of customers in each class can be unequal.

The competing firms can sell through their own channels or through an opaque intermediary. The intermediary is passive, i.e., if a firm decides to participate in the opaque channel, it dictates the opaque price to the intermediary. The intermediary posts prices from the different firms but hides the identity of the firm. In their model with the opaque intermediary, the customers can make reservations either through the direct channels, or they could use the opaque travel agency. In the former case, customers can choose a specific hotel, and, other things being equal, they would like to stay at the hotel that is closest to their preferred location. When customers make their reservations in the opaque market, they do not know the hotel's location and they simply prefer the hotel with the lowest price. The authors focus on the equilibria in which all  $N$  firms participate in the opaque market. Clearly, as  $N$  becomes larger, there is more uncertainty with respect to the ultimate product that the customer receives in the opaque channel. In theory, there could be several opaque and transparent prices but the authors restrict their attention to the symmetric equilibria in which prices, both transparent and opaque, are equal across all firms. Furthermore, the authors assume that the probability of receiving a product from any one

firm is the same and equal to  $1/N$  ( $N$  is assumed to be an even number). These assumptions significantly reduce the complexity of the analysis since a customer's location becomes immaterial if he makes a reservation with the opaque intermediary.

There are many possible candidates for the equilibria and some further restrictions are needed to analyze the problem. The authors assume that it is never the case that all customers buy only opaque products and the authors also do not consider the case in which no customer buys opaque products. Thereafter, the authors show an interesting result regarding the effect of the number of competitors on the opaque selling. (To our knowledge, this is the only known result regarding dependency of opaque selling on the number of competing firms). In particular, they show that, when  $N \geq 4$ , the high-type customers strictly prefer to use the non-opaque product. Thus, if too many firms sell in the opaque market, the high-type customers are too uncertain about the good they receive through the opaque channel, and hence they choose to buy in the regular transparent market instead. Thus, the rest of analysis is restricted to equilibria in which the high type customers only buy in the regular (transparent) market.

Subject to the aforementioned conditions, Shapiro and Shi (2008) focus on two possible equilibrium types. In the *Full Separation* equilibrium all the high-type customers use the transparent channel and all the low-type customers use the opaque channel because intense competition for opaque sales drives prices down. In the *Partial Separation* equilibrium some low-type customers might use the transparent channel to buy tickets. There exists some minimal distance  $S$  to the nearest product such that all the low-type customers buy from the firm directly, and the rest of the customers buy in the opaque channel. Shapiro and Shi (2008) conclude that, although the opaque feature virtually erases product differentiation and intensifies competition, service providers can differentiate between those customers who are sensitive to service characteristics and those who are not. As a result, competition intensifies for low-type customers, and it reduces for high-type customers. Reduced competition for more valuable customers enables providers to commit to a higher price for this lucrative segment which leads to higher profits overall.

Fay (2008) constructs a model of an opaque selling in which channel considerations are investigated in richer details by considering a wide variety of contracts between service providers and an opaque intermediary, including simultaneous contract offers by the participants, sequential offers by the intermediary, sequential offers by the firms etc. This is the only paper we are aware of that investigates in detail the contracting decisions made by the product sellers and the opaque intermediary. There are three firms in the model – two symmetric competing firms at the two ends of a Hotelling line of unit length, and an intermediary that sells opaque tickets. The consumers in the market are divided into three classes: those loyal to one firm, those loyal to the other firm, and those who are willing to buy from both firms. The latter consumers are distributed uniformly on the Hotelling line. The first two consumer classes are equal in size and may be thought of as being collocated with the firm, and/or having infinite travel costs. Essentially, they buy from their preferred firm or they do not buy at all. Such customers are called *brand-loyals* and form fraction  $\rho$  of

the population. The rest of the customers (i.e., fraction  $(1 - \rho)$ ) are called *searchers*. The total demand in the market is assumed to be fixed and deterministic.

Initially, both firms allocate some tickets to sell through the opaque intermediary in return for a lumpsum payment. Then, they set prices for their transparent tickets and the intermediary observes these prices and sets its own price in the opaque channel. From the intermediary's perspective, the products from both firms are perfect substitutes. As a result, equilibrium wholesale prices are equal and they cannot exceed marginal costs or else the intermediary would only buy from one firm. Effectively, the intermediary sets its own profit margin through its pricing ability in the opaque market. The *searchers* observe the prices charged by the firms and by the opaque intermediary and make their purchase decisions. An important assumption in the model is that, although the consumers cannot observe the number of tickets initially allocated by each firm to the opaque channel, when purchasing an opaque ticket they expect to obtain it from either firm with equal probability. This assumption, again, is due to absence of the capacity constraints as well as absence of demand uncertainty. Fay (2008) finds that a monopolist can improve profits by introducing an opaque good at a small discount and by raising the prices of the traditional goods. However, if there is competition between the selling firms then the dynamics are different. In case when there is little brand loyalty, an opaque product intensifies price competition and thus reduces industry profits. On the other hand, if there is a significant amount of brand-loyalty, an opaque good curtails price competition and thus increases industry profits. As a result, Fay finds that, with sufficient brand loyalty, opaque sales help reduce price rivalry in the market and increase industry profits.

Jerath et al. (2008) analyze opaque selling in a two-period model with demand uncertainty. They assume that two symmetric firms are located at the ends of a Hotelling line and offer horizontally differentiated products. Firms have limited capacities. An intermediary offers an opaque product. The market demand can be high with a certain probability and low with the remaining probability. The consumers are distributed uniformly over the Hotelling line. If demand turns out to be high, the firms run out of capacity, while if demand turns out to be low, the firms are left with excess capacity. In the first period, the distribution of demand is known to every player but the realization is not known. Both firms only sell transparent tickets in this period, and declare to sell opaque tickets through the intermediary in the second period if any tickets are left over. Consumers make their purchase or no-purchase decisions keeping in mind two factors: (1) they might be able to obtain cheap opaque tickets in the second period, and (2) if demand turns out to be high and enough others purchase transparent tickets in the first period, a consumer buying in the second period might not be able to obtain the ticket. Keeping in mind these factors, consumers develop rational expectations about future product availability on which they base their decisions, and these expectations are consistent in equilibrium.

Note that the model of demand uncertainty which is a feature of travel industries is a significant differentiator in the model of Jerath et al. (2008) compared to the models in Shapiro and Shi (2008) and Fay (2008). Furthermore, tickets are not as-

sumed to be allocated to the opaque channel a priori, but only if they are left over late in the selling horizon. Of course, the firms can also sell transparent tickets in the second period through their own channels. Jerath et al. (2008) consider this strategy as well, and compare the profits from both strategies to determine which strategy is more profitable under different conditions of demand uncertainty and customer valuations. They find that opaque selling does not distort sales in the regular channels but helps increase profits by inducing consumers who would otherwise not purchase at all to purchase in the second period. This happens because, by creating uncertainty regarding which product a consumer will obtain in the opaque channel, the ex-ante utility of purchasing the product is higher than with the transparent sales. Hence, the opaque channel acts as a distress-selling mechanism without disrupting sales in the regular channels. In comparison with the last-minute transparent sales strategy, the authors find that opaque selling is a more profitable strategy when the probability of high demand is significant, the customer valuation for the product is low and/or customers have a high fit/travel cost. In the next section, we delve deeper into the phenomenon of opaque selling by developing a simple economic model based on Jerath et al. (2008).

### 3 Firm's Selling Strategies Under Deterministic Demand

In this section, we explore the strategies of the firms when demand is deterministic. The firms can sell through their own channels and they have the option of offering different prices in each period of sale. The firms can also choose to sell opaque products in the second period, after sales in the first period has ended. We consider two possible scenarios for each strategy: low-demand scenario ( $J < K$ ), and high-demand scenario ( $J > K$ ). The deterministic-demand model helps us gain insights into the players' decisions when demand is lower/higher than capacity and serves as a logical building block for the more complex model with demand uncertainty (Section 4).

#### 3.1 Selling Through Firms' Direct Channels

The demand is deterministic and equals  $J$ . The firms and all consumers know  $J$ . Assume that firm  $i$  (where  $i \in \{A, B\}$ ) charges  $p_i^1$  in the first period and  $p_i^2$  in the second period. Each consumer buys a product, if available, from the firm that provides him with the highest net utility (conditional on it being positive), either in the first period or the second period. In this case, we find that each firm charges the same price in the two periods.<sup>3</sup> This is formalized in the following Lemma.

<sup>3</sup> Prices would not be identical across periods if consumers discounted their second period utility. However, the *discount-adjusted* prices would be identical across periods. Introducing discounting makes the analysis more tedious, while all the insights continue to hold.

**Lemma 1.** *When customers are rational, the equilibrium prices are such that  $p_i^1 = p_i^2 = p_i, i \in \{A, B\}$ .*

Intuitively, if the firms were to try and charge a higher price in the first period and a lower price in the second period, the consumers, being strategic and having full information about demand, would wait to buy products until the prices were lowered. (In case of the uncertain demand, we will see that this result changes.) Employing this result ( $p_A^1 = p_A^2 = p_A$  and  $p_B^1 = p_B^2 = p_B$ ), we now analyze the cases of low and high demand.

### 3.1.1 Low Demand ( $J < K$ )

Firms  $A$  and  $B$  set revenue maximizing prices  $p_A$  and  $p_B$  and accrue profits  $\pi_A = p_A x_A J$  and  $\pi_B = p_B (1 - x_B) J$ , where  $x_A$  and  $x_B$  represent the locations of the farthest consumers who bought products from firms  $A$  and  $B$ , respectively, on the Hotelling line. The solution to the game is formalized in Proposition 1.

**Proposition 1.** *When demand is deterministic, there is ample capacity ( $J < K$ ) and firms sell only through their own channels, the optimal prices, market coverage and profits in the equilibrium are as follow:*

| $\frac{V}{t}$                      | Prices<br>$p_A, p_B$ | Market Coverage<br>$x_A, (x_B = 1 - x_A)$ | Profits<br>$\pi_A, \pi_B$                  |
|------------------------------------|----------------------|---|--|
| $\frac{1}{2} \leq \frac{V}{t} < 1$ | $\frac{V}{2}$        | $\frac{V}{2t}$                            | $\frac{V^2}{4t} J$                         |
| $1 \leq \frac{V}{t} < \frac{3}{2}$ | $V - \frac{t}{2}$    | $\frac{1}{2}$                             | $\left(V - \frac{t}{2}\right) \frac{J}{2}$ |
| $\frac{V}{t} \geq \frac{3}{2}$     | $t$                  | $\frac{1}{2}$                             | $\frac{t}{2} J$                            |

When  $1/2 \leq V/t < 1$ , each firm finds it optimal to cover less than  $1/2$  of the market (Hotelling line) and there are some leftover products. When  $1 \leq V/t < 3/2$ , each firm finds it optimal to cover exactly  $1/2$  of the market at the price  $V - t/2$ . In both of these cases, the firms act as local monopolies. In the third case, when  $V/t > 3/2$ , the competitive equilibrium emerges. Hence, as  $V/t$  increases, the prices, market coverage and firm revenues increase up to the point where the market becomes competitive.

### 3.1.2 High Demand ( $J > K$ )

Since demand is larger than capacity available in this scenario, full market coverage cannot occur. To maximize revenues, each firm will then cover the  $K/2$  consumers located closest to it. The location of the farthest consumer covered by firm  $A$  (when valuation is high enough) is, therefore,  $x_A = K/(2J) < 1/2$ . (Similarly,  $x_B = 1 - K/(2J) > 1/2$ .) The following proposition lays out the solution to the game.

**Proposition 2.** *When demand is deterministic, capacity is a constraint ( $J > K$ ), and firms sell only through their own channels, the optimal prices, market coverage and profits in the equilibrium are as follow:*

| $\frac{V}{t}$                                | Prices<br>$p_A, p_B$ | Market Coverage<br>$x_A, 1 - x_B$ | Profits<br>$\pi_A = \pi_B$                  |
|--|----------------------|-----------------------------------|---|
| $\frac{1}{2} \leq \frac{V}{t} < \frac{K}{J}$ | $\frac{V}{2}$        | $\frac{V}{2t}$                    | $\frac{V^2}{4t}J$                           |
| $\frac{K}{J} \leq \frac{V}{t}$               | $V - \frac{K}{2J}t$  | $\frac{K}{2J}$                    | $\left(V - \frac{K}{2J}t\right)\frac{K}{2}$ |

When  $1/2 \leq V/t < K/J$ , the optimal price charged by the firm is such that not all  $K/2$  closest consumers have positive valuation to buy from the firm, and there are some leftover products. When  $V/t \geq K/J$ , each firm has more than  $K/2$  consumers who are willing to buy the products. Hence, the firm sells all its inventory at a high price  $(V - Kt/(2J))$ , and the farthest consumer who buys a product is located at  $K/(2J)$ .

### 3.2 Opaque Selling

As we described in the introduction, firms often sell products/services through opaque intermediaries (such as hotwire.com and priceline.com in the travel industry). Further, opaque products typically go on sale only very close to the terminal time, i.e., after consumers have bought in the transparent channel but the firms still have some inventory of products left over. In the model described, after sales have been resolved in the transparent channel, the firms can accomplish opaque selling through an intermediary  $I$  that obtains products from the firms and sells them to maximize its own profits. For every product that the intermediary sells at price  $p_I$ , the airline receives  $\delta p_I$ .

Before purchasing an opaque product, a consumer does not know which firm will eventually provide it. However, every consumer develops expectations about the probability of obtaining the product from firm  $A$  or  $B$ . Since the opaque sales are based on remaining capacity, the expectations are regarding the left over capacity after transparent sales have concluded. Therefore, the probabilities of obtaining products from each firm develop *endogenously* in the game according to the following sequence of events.

1. Firms  $A$  and  $B$  set prices  $p_A$  and  $p_B$  in the direct-to-consumers channel and declare that they might sell through an opaque channel later in the selling horizon (e.g., hotwire.com lists all airlines that sell products through its web site). Firms will engage in opaque selling only if there are products that are left unsold through their own direct channels.
2. Given prices  $p_A$  and  $p_B$  and his expectations about future availability from both firms, every consumer makes a purchase decision in the transparent channel.

3. After the transparent channel sales are over, the leftover products are made available to the opaque intermediary  $I$  by both firms. The opaque intermediary sets a price  $p_I$  for the opaque product. Consumers who did not buy in the transparent channel now make their buying decisions in the opaque channel. A consumer may not obtain an opaque product if the number of leftover products is less than the number of consumers who are willing to buy at price  $p_I$ . We denote the probability that the consumer can obtain an opaque product by  $\beta$  so that each consumer desiring a product is equally likely to obtain it. Consumers considering the opaque channel form expectations about the probabilities that the product they will obtain will be from firm  $A$  (denoted by  $\gamma_A^e$ ) or firm  $B$  (denoted by  $\gamma_B^e$ ). Hence, any consumer who is considering buying an opaque product has an ex-ante expected utility given by

$$\beta (V - p_I - \gamma_A^e t x - \gamma_B^e t (1 - x)).$$

Based on the price  $p_I$ , the probabilities  $\gamma_A^e$  and  $\gamma_B^e$  and position  $x$  on the line, each consumer decides whether to purchase a product from the opaque channel or not.

4. The opaque intermediary keeps a fraction  $1 - \delta$  of the revenues from the opaque channel. The remaining fraction  $\delta$  is distributed between firms  $A$  and  $B$  in proportion to the products sold for each firm, i.e., firm  $A$  obtains a fraction  $\delta \gamma_A$  and firm  $B$  obtains a fraction  $\delta \gamma_B$  of the total opaque channel revenues.<sup>4</sup>

We now discuss how consumers purchasing in the opaque channel form their expectations. Let  $x_A^e$  and  $x_B^e$  denote the points on the Hotelling line such that every consumer believes that, in the transparent channel, the consumers in the interval  $[0, x_A^e]$  bought products from  $A$  and the consumers in the interval  $[x_B^e, 1]$  bought products from  $B$ . Thus, every consumer believes that the number of products left over for firm  $A$  to sell in the opaque channel is  $l_A^e = \max\{K/2 - x_A^e J, 0\}$  and the number of products left over for firm  $B$  is  $l_B^e = \max\{K/2 - (1 - x_B^e)J, 0\}$ . In line with these expectations, consumers perceive that, if they buy in the opaque channel, they will obtain a product from firm  $A$  with probability  $\gamma_A^e = l_A^e / (l_A^e + l_B^e)$  and from firm  $B$  with probability  $\gamma_B^e = 1 - \gamma_A^e = l_B^e / (l_A^e + l_B^e)$ . Consequently, for the consumer at  $x_A$  who is indifferent between buying from firm  $A$  and buying in the opaque channel, the following condition holds:

$$V - p_A - t x_A = \beta (V - p_I - \gamma_A^e t x_A - \gamma_B^e t (1 - x_A)).$$

Note that  $p_I^e$  is a function of  $\gamma_A^e$  and  $\gamma_B^e$ , and the consumers rational beliefs are imposed on availabilities. To solve for the rational expectations equilibria under high and low demand, we first characterize the equilibrium beliefs of the consumers by the following lemma. Recall that, in equilibrium, the profit maximizing prices set by the firms are expected rationally by consumers. Further, in equilibrium, expectations of all consumers regarding the number of consumers that buy in the first (and

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<sup>4</sup> This revenue sharing contract with opaque intermediaries is consistent with observations and industry practice (see Phillips 2005).

second) period, are consistent, i.e., it should match the actual number of consumers buying in both periods.

**Lemma 2.** *When the capacities of the firms are equal, the equilibrium expectations of the fraction of opaque products from each firm are  $\gamma_A = \gamma_A^e = 1/2 = \gamma_B = \gamma_B^e = 1/2$ .*

Lemma 2 is a significant result. It shows that, if the firms have equal capacities, then it is rational for consumers to expect that, in the opaque channel, half of the products come from one firm and the other half from the other. Furthermore, the lemma also specifies that any other expectations about product availability are either irrational or inconsistent or both. Conditional on the event that a consumer has received an opaque product, we allow for asymmetric consumer expectations about its source, but they are not sustained in equilibrium when the firms are identical. Suppose that the customers have asymmetric expectations about the product availability. For such asymmetric availability to be an equilibrium, (the realization of) second period left-over inventory from both the firms must be unequal. This in turn implies that one of the firms had poorer market coverage in the first period. Therefore, the prices were asymmetric in the first period. However, in such a case, the firm charging the higher price would unilaterally deviate to a lower price to increase its coverage in the market. The equilibrium occurs at symmetric prices.

This result implies that, in the equilibrium, at price  $p_I$  the expected utility of each consumer from buying in the opaque channel is  $V - p_I - t/2$ . Without loss of generality, we focus on  $\delta = 1$ ; any  $\delta \in [0, 1]$  yields same insights.

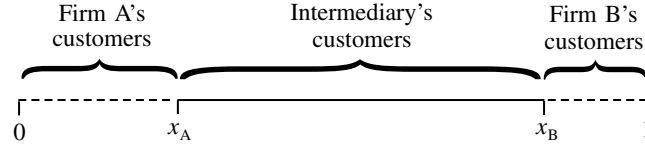
### 3.2.1 Low Demand ( $J < K$ )

Suppose that the prices  $p_A$  and  $p_B$  in the transparent channels are such that consumers located in the interval  $[0, x_A]$  buy from firm  $A$  and consumers in  $[x_B, 1]$  buy from firm  $B$ . After the firms have sold through their direct channels, the intermediary has access to consumers in the range  $[x_A, x_B]$  as shown in Figure 2. Note that the opaque market will exist only if there are some products left over after sales in the transparent channel have concluded. Since demand is low, there will be enough units in the opaque channel to cover the remaining market so that each consumer in the opaque channel will definitely obtain the product (i.e.,  $\beta = 1$ ). If the intermediary charges a price  $p_I$ , the total revenue from the opaque channel is  $\pi_I = p_I(x_B - x_A)J$ , and each firm obtains a part of it.

Since the firms now have the opaque channel to “clear up” the remaining market in the second period, they can raise prices in the first period (sell to fewer consumers at higher prices) which can lead to higher profits.

Note, however, that consumers are strategic. They recognize that the firms could rely upon the opaque channel and increase prices in the transparent channels. Further, consumers know that they can prevent the firms from implementing the opaque channel if they delay their purchases (in the extreme, delay purchases until right before the selling horizon ends). Effectively, through this strategic behavior, the con-





**Fig. 2** Consumers to the left of  $x_A$  buy from firm A and consumers to the right of  $x_B$  buy from firm B.  $x_A$  denotes the location of the consumer indifferent to buying from firm A or from the opaque channel.  $x_B$  denotes the location of the consumer indifferent to buying from firm B or the opaque channel. Consumers between  $x_A$  and  $x_B$  are the target consumers for the intermediary.

sumers will make the firms charge a price no higher than the price in the direct channel equilibrium (Section 3.1.1). In this equilibrium, however, the firms can still use the opaque channel when  $1/2 \leq V/t < 1$  the market is not covered with transparent prices. In the equilibrium, the firms will charge a price  $V - t/2$  in the opaque channel to all remaining consumers, who will all buy products. The following proposition characterizes the equilibrium in this case.

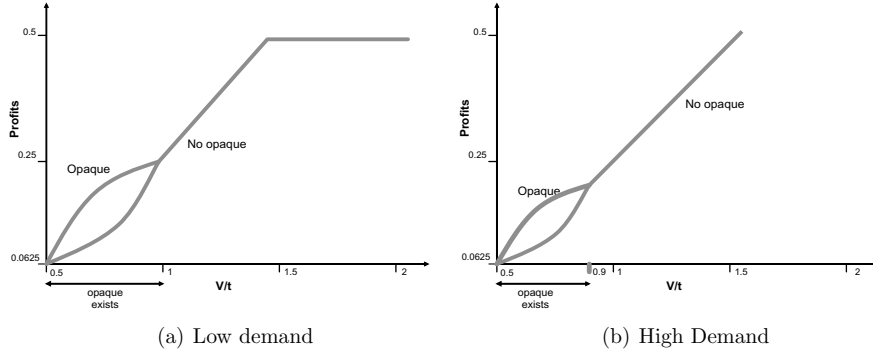
**Proposition 3.** *When demand is deterministic, there is ample capacity ( $J < K$ ), and firms can utilize the opaque channel, the equilibrium prices charged in the first period by the firms, the price charged in the opaque channel by the intermediary, and the opaque market coverage in the equilibrium are as follow:*

| $\frac{V}{t}$                      | First-period prices<br>$p_A = p_B$ | Opaque prices<br>$pt$ | Opaque coverage<br>$x_B - x_A$ |
|------------------------------------|------------------------------------|-----------------------|--------------------------------|
| $\frac{1}{2} \leq \frac{V}{t} < 1$ | $\frac{V}{2}$                      | $V - \frac{t}{2}$     | $1 - \frac{V}{t}$              |
| $1 \leq \frac{V}{t} < \frac{3}{2}$ | $V - \frac{t}{2}$                  | —                     | —                              |
| $\frac{3}{2} \leq \frac{V}{t}$     | $t$                                | —                     | —                              |

Proposition 3 shows that (compared to Proposition 1) using the opaque channel increases the total market coverage (and profits) given the same valuation  $V$  and strength of brand preferences  $t$  when the ratio  $V/t$  is small ( $1/2 \leq V/t < 1$ ). For higher  $V/t$ , the firms cover the full market through the transparent channel and there is no leftover capacity for the opaque channel.

### 3.2.2 High Demand ( $J > K$ )

When demand is deterministic and higher than available capacity, some consumers do not obtain products. The right-most consumer that firm A can cover through its own channel is located at  $K/(2J) < 1/2$ . Similarly, the left-most consumer that firm B can cover is located at  $1 - K/(2J) > 1/2$ . Further, the insight that the firms will not be able to leverage the opaque channel to increase prices in the first period holds in the high-demand case also. Hence, as before, the equilibrium in this case will



**Fig. 3** Figure (a) shows the equilibrium profits of one firm with and without opaque channels when demand is deterministic and lower than capacity. In this figure  $\delta = 1, t = 1, J = 1, K > J$ . Figure (b) shows the equilibrium profits of a firm with and without opaque channels when demand is deterministic and higher than capacity. In this figure  $\delta = 1, t = 1, J = 1, K = 0.9$ .

be similar to that in Section 3.1.2, except that when the firms do not cover the full market in the first period they resort to opaque sales in the second period to clear up remaining inventory.

**Proposition 4.** *When demand is deterministic, capacity is a constraint ( $J > K$ ) and firms can utilize the opaque channel, the equilibrium prices charged in the first period by the firms, the price charged in the opaque channel by the intermediary, and the opaque market coverage in the equilibrium are as follow:*

| $\frac{V}{t}$                                   | First-period prices<br>$p_A = p_B$ | Opaque prices<br>$p_I$ | Opaque coverage<br>$x_B - x_A$ |
|---|------------------------------------|------------------------|--------------------------------|
| $\frac{1}{2} \leq \frac{V}{t} \leq \frac{K}{J}$ | $\frac{V}{2}$                      | $V - \frac{t}{2}$      | $K - J \frac{V}{t}$            |
| $\frac{K}{J} \leq \frac{V}{t}$                  | $V - \frac{t}{2}$                  | —                      | —                              |

### 3.3 Comparison of Strategies Under Deterministic Demand

We now compare the profits that the firms make with and without the opaque channel. Note that in both high- and low-demand scenarios, the opaque channel acts as a “clean up” mechanism to dispose of unsold products, without disturbing the pattern of sales in the transparent channels. Hence, if the opaque channel exists (when the market is not fully covered by the transparent channels), it will strictly improve firm profits (as in Fay 2008). We demonstrate this observation in Figures 3a and 3b.

In the opaque channel, the ex-ante expected utility from buying a product is zero for all consumers. To see this, consider a consumer who is located at  $x$ . His net expected utility from buying in the opaque channel is

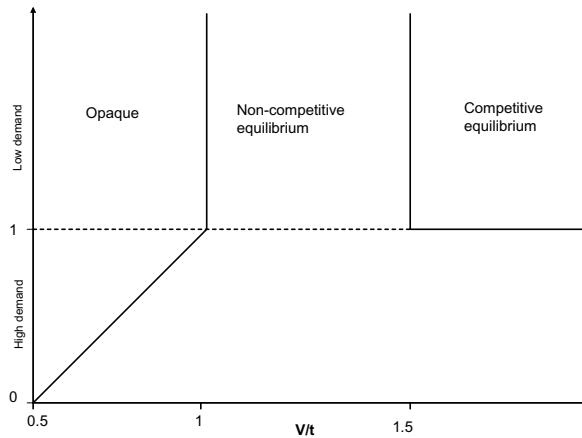
$$V - p_I - \gamma_A^e t x - \gamma_B^e t(1 - x)$$

which is zero in equilibrium (in equilibrium,  $\gamma_A^e = \gamma_B^e = 1/2$  and  $p_I = V - t/2$ ). Therefore, all consumers who have not yet purchased a product and find one available do purchase it. In other words, by hiding the identity of the product, the opaque channel helps sell products at lower prices to the consumers who are not willing to buy directly from the firms because direct prices are too high.

Ex post, however, under the assumption that products in the opaque channel are allocated randomly, half the consumers obtain positive valuations from the products they bought (since they obtain a product from the firm they prefer more), and the other half obtain negative valuations (since they obtain a product from the firm they prefer less). This is consistent with the practical observation that although consumers pay lower prices when they buy opaque products, sometimes they experience dissatisfaction because the product does not meet their preferences.

Figure 4 depicts the optimal strategies for the firms given different values of consumer valuations (the ratio  $V/t$ ) and inventory availability relative to demand (the ratio  $K/J$ ). As we discussed earlier, as  $V/t$  increases, the market becomes more competitive because either the consumers' valuations ( $V$ ) for flying are high, or the strength of brand preference in the market ( $t$ ) is low, or both.

Under both high and low demand, firms sell products through the opaque channel only if  $V/t$  is small enough because in this case the firms do not cover the full market in the transparent channels and use opacity as a mechanism to dispose of unsold products. As the ratio  $V/t$  increases above a threshold, the firms have the option of using an opaque channel, but price in the transparent channels to cover the market anyway, and do not need to resort to selling cheaper opaque products. Figure 4 also shows that if demand is high, opaque sales will be seen less frequently (for a smaller range of  $V/t$ ), than if demand is low. This is consistent with the notion



**Fig. 4** Strategy space for different valuations and capacity/demand scenarios.

that the opaque channel is used to dispose of distressed inventory (Harrison 2006). Finally, as we argued in Sections 3.2.1 and 3.2.2, when demand is deterministic, strategic consumer behavior prevents the firms from leveraging the opaque channel to increase prices in the transparent channel by adopting a strategy of waiting. This is consistent with the Coase conjecture.

#### 4 Modeling Uncertain Demand: The Effect of Uncertainty on Opaque Selling Strategies

Uncertainty in demand volume is a pervasive feature in the travel industry. Firms usually can estimate the demand distribution for a given airline route or hotel using historical records but the precision of such estimates is quite limited (see Talluri and van Ryzin 2004). As the departure date approaches, the firms can improve the forecast and therefore project with a higher degree of confidence whether the demand for the route is higher or lower than the available capacity. Building on the analysis in previous sections, this section extends our model to incorporate demand uncertainty.

Due to the presence of demand uncertainty, consumers cannot always credibly adopt a strategy of waiting in the early stages of the game because market demand could be high and tickets could be unavailable later. However, a consumer can form rational expectations about future availability and buy early if the expected utility from doing so is higher than the expected utility from waiting. These dynamics capture the practical consideration that not all consumers wait for last-minute discounts and allow us to derive several insights beyond the model with deterministic demand. As mentioned in the introductory sections, the possibility of capacity shortage is one of the counter arguments to the Coase conjecture.

The specifications of the model remain the same, except that the level of demand is now variable. We assume that, with probability  $\alpha$  the total number of consumers in the market is  $H(> K)$  and, with probability  $1 - \alpha$  the total number of consumers in the market is  $L(< K)$ . As before, each firm has capacity  $K/2$ . The parameters  $\alpha, L, H$  and  $K$  are common knowledge. The selling horizon is divided into two periods. In the first period, the firms and the consumers know the distribution of demand, but do not know the state of nature (whether demand is  $H$  or  $L$ ). At the end of the first period, but before the second period begins, the realization of demand is observed by the firms and the consumers. This assumption is clearly a simplification of reality. In practice, some residual uncertainty in demand would remain. There are also several ways to extend this assumption. We discuss some future research prospects in our concluding section.

We assume that, in any selling period, if the number of consumers who are willing to buy a seat is higher than the capacity available, tickets are allocated randomly to the consumers. In other words, if a certain number of consumers desire tickets at the announced price but the number of tickets available is lower than the number of tickets demanded (which can be the case if demand is high), it is possible that

consumers with a lower expected (but positive) surplus obtain tickets at the expense of consumers with a higher expected surplus. In the following sections, we analyze the two strategies of selling through the firms' direct channels ("Last-Minute Sales" Strategy, or LMSS) and opaque selling ("Opaque Sales" Strategy, or OpSS).

We consider the following two selling strategies of the firms:

1. "Last-minute sales" Strategy (LMSS): In the first period, firms sell tickets at prices  $p_A^1$  and  $p_B^1$ . In the second period, they sell the leftover tickets at the "last minute" prices based on the demand realization. If the demand is high, they sell the unsold seats at prices  $p_A^{2H}$  and  $p_B^{2H}$  and, at prices  $p_A^{2L}$  and  $p_B^{2L}$  if the demand is low.
2. "Opaque sales" Strategy (OpSS): In the first period, firms sell tickets using their own channels at prices  $p_A$  and  $p_B$ . In the second period, the firms provide access to the unsold tickets to an intermediary,  $I$ , who sells opaque tickets at price  $p_I^H$  if the demand is high, and at price  $p_I^L$  if the demand is low.

Under each strategy, consumers might postpone their purchase based on prices in the first period and their expectations for availability and prices in the second period, which in turn are influenced by the fraction of consumers who postpone the purchase. In equilibrium, the fraction of consumers who postpone should be consistent with the belief that each consumer has about the fraction of consumers who have postponed purchasing their ticket. We ensure this by solving for the rational expectations equilibrium.

In the first period, the firms and the consumers know the distribution of demand, but do not know the state of nature (whether demand is  $H$  or  $L$ ). At the beginning of the first period, the firms announce their first period prices and their second period selling strategies.<sup>5</sup> The consumers strategically decide whether to buy in the first period itself, or postpone their purchase to the later period. For consumers who postpone the purchase, there is a possibility that they may not be able to obtain tickets in the second period if the demand turns out to be high, or if the firms charge a very high price. At the end of the first period, but before the second period starts, the realization of demand is observed. During the second period, the demand realization is known to the firms and consumers. Depending on their strategies, the leftover tickets are sold by the firms through their own channels at "last minute" prices (which can be high or low) or through an opaque intermediary. The consumers obtain tickets at the prices offered only if available.

#### ***4.1 Selling Through Firms' Direct Channels***

The following is the order of events in the game when firms adopt a LMSS.

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<sup>5</sup> Both firms announce the same strategy. This is imperative if they want to sell through an opaque intermediary.

1. In the first period, firm  $A$  prices its tickets at  $p_A^1$  and firm  $B$  prices its tickets at  $p_B^1$  and both firms declare that there might be last-minute sales.
2. All consumers form expectations about the number of consumers purchasing in the first period (and therefore the corresponding future prices and availability) and strategically make or postpone their purchase.
3. At the end of period 1 and before period 2 begins, demand uncertainty is fully resolved. The level of demand is determined as  $H$  or  $L$ , and is observed by both the firms and the consumers.
4. The firms then set their prices (e.g., firm  $A$  sets price  $p_A^{2L}$  if demand is low and  $p_A^{2H}$  if demand is high, and similarly for firm  $B$ ).
5. The consumers who postponed their purchase in the first period decide to purchase or not in the second period at the announced prices.

The rational expectations equilibrium solution for the above game is provided in the following proposition.

**Proposition 5.** *When the firms sell tickets through their own channels, the following equilibrium always exists: In the first period both firms set prices  $p_A^1 = p_B^1 = V - Kt/(2H)$  and cover  $x_A = 1 - x_B = K/(2H)$  of the market. There are no tickets sold at the last minute if demand is high. Consumers located between  $x_A = K/(2H)$  and  $x_B = 1 - K/(2H)$  buy only in the second period if demand is low. The second-period market is characterized by the following prices and coverage:*

| $\frac{V}{t}$                                     | Second-period prices when demand is low ( $p_A^{2L} = p_B^{2L}$ ) | Second-period coverage when demand is low               |
|---|---|---|
| $\frac{1}{2} \leq \frac{V}{t} < 1 - \frac{K}{2H}$ | $\frac{1}{2} \left( V - \frac{K}{2H}t \right)$                    | $\frac{1}{2} \left( \frac{V}{t} - \frac{K}{2H} \right)$ |
| $1 - \frac{K}{2H} \leq \frac{V}{t} < \frac{3}{2}$ | $V - \frac{t}{2}$   | $\frac{1}{2} \left( 1 - \frac{K}{H} \right)$            |
| $\frac{V}{t} \geq \frac{3}{2}$                    | $t$   | $\frac{1}{2} \left( 1 - \frac{K}{H} \right)$            |

In the equilibrium, all consumers who attempt to buy a ticket in the first period obtain a ticket, but pay the high price  $V - Kt/(2H)$ . If demand is high, firm  $A$  sells to  $K/(2H) \cdot H (= K/2)$  consumers in the first period and thus exhausts its capacity so there are no tickets sold in the second period through last-minute sales. If demand is low, firm  $A$  sells to  $K/(2H) \cdot L (< K/2)$  in the first period and will have some seats left over. (The situation is symmetric for firm  $B$ .) Moreover, there are more of these leftover seats than the number of unserved consumers in the market in the second period. Therefore, the consumers who waited for the “last-minute” tickets obtain them at lower prices only if demand is lower than capacity.

To summarize, in the first period all consumers with “high brand preference” (locate in the interval  $[0, K/(2H)]$ ) buy at a high price from firm  $A$ . If there are any leftover tickets, the consumers with “low brand preference” (located in the interval  $[K/(2H), 1/2]$ ) buy from firm  $A$  during the last-minute sales at lower prices. If there are no leftover tickets, there are no sales in the second period. In effect, the firms are separating out consumers who are ready to pay a higher price under the treat

of stockout and making most of their profits from the high prices charged to the high-preference consumers in the first period.

## 4.2 Opaque Selling

The following is the order of events in the game when the firms adopt an opaque sales strategy.

1. In the first period, firm  $A$  prices its tickets at  $p_A^1$  and firm  $B$  prices its tickets at  $p_B^1$  and both firms declare intention of sales through an opaque channel.
2. Consumers develop expectations about availability in the second period and the firm they will probably obtain a ticket from in the opaque channel, and strategically purchase or postpone purchasing a ticket.
3. At the end of period 1 and before period 2 begins, demand uncertainty is resolved, the level of demand is determined as  $H$  or  $L$  and is observed by the firm and the consumers.
4. The leftover seats, if any, are made available to the opaque intermediary  $I$ , who then sets a price  $p_I^H$  if the demand realization is  $H$  or a price  $p_I^L$  if the demand realization is  $L$ .
5. Consumers who have not purchased in the transparent channel now make their buying decision in the opaque channel.
6. For every ticket sold, the opaque intermediary keeps a fraction  $1 - \delta$  of the revenue accrued from the opaque channel. The intermediary commits to a credible opaque strategy and sells tickets from both firms at price  $p_I$  with equal preference. It distributes the remaining fraction  $\delta$  to firm  $A$  or  $B$  whose ticket it sold.

We now discuss how consumers purchasing in the opaque channel form their expectations about the probabilities of ticket availability. The consumers do not know which firm will ultimately provide the service, but they form expectations about the locations of the right-most and left-most consumers on the Hotelling line that buy tickets from  $A$  and  $B$ , respectively, in the transparent channel.

1. If the level of demand is low, then leftover seats for firm  $A$  must be  $l_A^{L,e} = \max\{K/2 - x_A^e L, 0\}$  and leftover seats for firm  $B$  must be  $l_B^{L,e} = \max\{K/2 - (1 - x_B^e)L, 0\}$ . In line with these expectations, consumers perceive that if they buy in the opaque channel then they will obtain a ticket from  $A$  with probability  $\gamma_A^{L,e} = l_A^{L,e} / (l_A^{L,e} + l_B^{L,e})$  and from  $B$  with probability  $\gamma_B^{L,e} = l_B^{L,e} / (l_A^{L,e} + l_B^{L,e})$ .
2. If the level of demand is high, then the expected leftover for firm  $A$  must be  $l_A^{H,e} = \max\{K/2 - x_A^e H, 0\}$  and the expected leftover for firm  $B$  must be  $l_B^{H,e} = \max\{K/2 - (1 - x_B^e)H, 0\}$ . In line with these expectations,  $\gamma_A^{H,e} = l_A^{H,e} / (l_A^{H,e} + l_B^{H,e})$  and from  $B$  with probability  $\gamma_B^{H,e} = l_B^{H,e} / (l_A^{H,e} + l_B^{H,e})$ .

Based on the price  $p_I$ , the expectation probabilities  $\gamma_A^{L,e}$  and  $\gamma_B^{L,e}$  and his position  $x$  on the line, every consumer decides whether to purchase a ticket or not. In equilibrium, for all consumers the outcomes  $\gamma_A$  and  $\gamma_B$  must be consistent with their beliefs

$\gamma_A^{i,e}$  and  $\gamma_B^{i,e}$ . The equilibrium prices in the rational expectations equilibrium of the above game are provided in Proposition 6. (To keep results simple, we present the case with  $\delta = 1$ . The analysis for any  $\delta \in [0, 1]$  yields similar insights.)

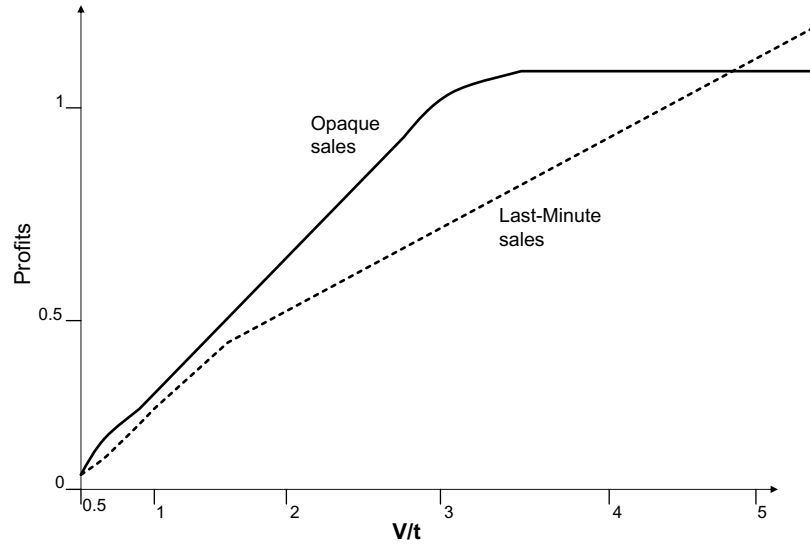
**Proposition 6.** *When the firms sell tickets through the opaque intermediary, the following equilibrium always exists:*

| $\frac{V}{t}$   | First-period prices<br>$p_A = p_B$                                   | Opaque prices<br>$p_I^L, p_I^H$    |
|---|--|------------------------------------|
| $\frac{1}{2} \leq \frac{V}{t} \leq \frac{K}{H}$   | $\frac{V}{2}$  | $V - \frac{t}{2}, V - \frac{t}{2}$ |
| $\frac{K}{H} < \frac{V}{t} \leq \frac{K}{H} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}$  | $V - \frac{K}{2H}t$  | $V - \frac{t}{2}, -$               |
| $\frac{K}{H} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L} < \frac{V}{t} < 1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}$   | $\frac{V}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}t$   | $V - \frac{t}{2}, -$               |
| $1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L} \leq \frac{V}{t} < \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}$ | $V - \frac{t}{2}$  | $-, -$                             |
| $\frac{V}{t} \geq \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}$   | $\left(1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}\right)t$ | $-, -$                             |

Under deterministic demand we saw that the opaque channel was primarily a clearance mechanism when the entire market could not be covered by the firms using transparent prices. However, in the deterministic demand case, the consumers know the state of demand and adopt a strategy of waiting if the firms charge high prices. In contrast, when demand is uncertain, the consumers do not know the state of demand in the first period and therefore may not wait because of the imminent possibility of the firms stocking out if demand is high. In other words, if a consumer has positive utility in the first period at the price offered by a firm, then he will purchase the ticket, inferring that he might not obtain it at all if the demand turns out to be high. This consideration allows the firms to charge higher prices in the first period. Consequently, if demand is low, only a few tickets will be sold in the first period. However, in this eventuality, the firms can use the opaque channel to “clean up” the leftover seats if any. Selling to a smaller population at higher prices in the first period helps the firms to increase the expected profit across two periods.

The above argument naturally leads to the interesting insight that, as the probability of high demand increases, the firm will rely more and more on the opaque channel. The reason is that, if there is a greater chance that demand is high, the “competition for tickets” among consumers in the first period will be higher, which means that the firms will be able to raise the first-period prices. If demand turns out to be high, the firms will exhaust their capacities. On the other hand, even if demand turns out to be low, there will still be some consumers left in the market because of high first-period prices. Consequently, there will be some leftover tickets, and the firms will sell them through the opaque channel.



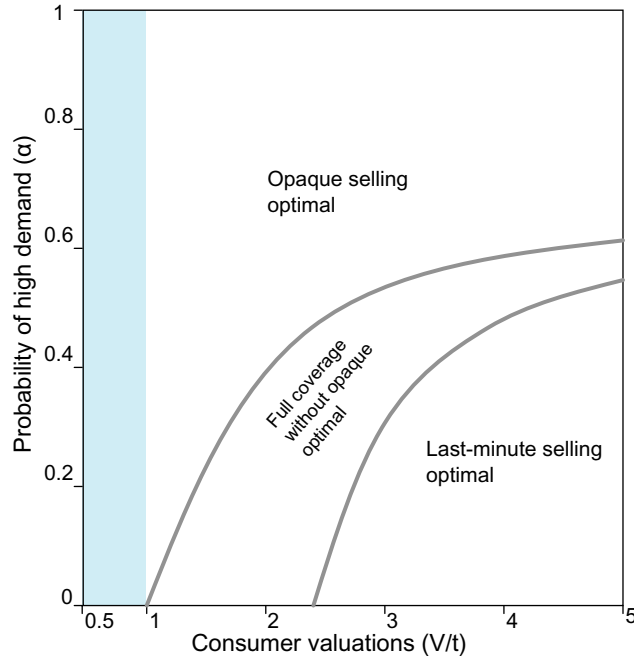


**Fig. 5** Profits accrued by a firm ( $\pi_A, \pi_B$ ) under uncertain demand, when the firm employs the last-minute sales strategy and the opaque sales strategy. For the figure, we use  $\alpha = \frac{1}{2}$ ,  $K = 1$ ,  $L = \frac{1}{2}$ ,  $H = \frac{3}{2}$ ,  $t = 1$ ,  $\delta = 1$ .

### 4.3 A Comparison Of Two Selling Strategies

We saw in the previous two sections that both LMSS and OpSS can increase firms' profits. In this section, we seek to answer the question: When should firms employ LMSS versus OpSS? For expositional simplicity, we provide a graphical illustration of the profits of the firms for these two strategies for a representative set of parameter values ( $\alpha = 1/2, K = 1, L = 1/2, H = 3/2, t = 1$ ) in Figure 5. If  $V$  is low, the profits from OpSS are higher than the profits from LMSS. However, as  $V$  increases, the profits from OpSS flatten out, while the profits from LMSS keep increasing. Above a certain threshold for  $V$ , LMSS profits become higher than OpSS profits.

To see why the above result holds, note that under LMSS the bulk of a firm's profits comes from tickets sold in the first period to the consumers that are closer to the firm on the Hotelling line. If the valuation for flying in the market is high (i.e.,  $V$  is high), this price ( $V - tK/(2H)$ ) is high. However, if the valuation for flying is low, the first-period prices are very low, the second-period prices are even lower, and hence profits from LMSS are low. In OpSS, on the other hand, the first-period prices are higher than in LMSS for low  $V$  because each firm is choosing to cover only a small portion of the market in the transparent channel and the rest using the opaque channel. Moreover, note that the second-period prices in the opaque channel (if opaque sales are present) are higher than the second-period prices for LMSS because the firms collude via the intermediary to sustain these higher second-period prices.



**Fig. 6** Strategies the firm should adopt for different consumer valuations ( $\frac{V}{t}$ ) and various probabilities of high demand ( $\alpha$ ). In the intermediate region, the firm is able to cover the entire market without actually selling opaque tickets, even though it declares opaque sales as its strategy. For the figure, we use  $K = 1$ ,  $L = \frac{1}{2}$ ,  $H = \frac{3}{2}$ ,  $t = 1$ ,  $\delta = 1$ .

As  $V$  increases, the revenue from LMSS increases faster, because the firms are able to separate out the consumers with a high preference for a particular firm and charge these consumers higher prices even if demand is low. In OpSS, on the other hand, prices are such that the firms cover a large portion of the market at lower prices if demand is low. In fact, if  $V$  is high enough, the firms are in a competitive equilibrium under OpSS when demand is low, so that prices are very low. (In Figure 5, this is the region where the OpSS profits level off.) Hence, when  $V$  is high, LMSS yields higher profits because it allows the firms to “milk” the high-preference consumers in the first period, even if it has to charge lower prices in the second period when demand turns out to be low.

We now investigate the effect of increasing probability of high demand realization. As we discussed earlier for OpSS, as the probability of high demand realization increases, consumers are under a higher threat of stockout in the first period. Thus, many more consumers prefer to buy in the first period and therefore the firms increase prices. In other words, not only is there a higher chance that demand is high, the prices are high also. If demand turns out to be low, the first-period sales suffer, but the leftover capacity is cleared through the opaque channel. Over the two periods, expected profits increase. In LMSS, however, the firms charge a first-period

price  $V - Kt/(2H)$  irrespective of the probability of high demand. Further, consumers with low firm preferences buy only if demand is low, which now happens with lower probability. Hence, even though expected profits increase (because there is a higher chance of high demand) the increase is slower than in OpSS. Figure 6 summarizes the comparison between the opaque strategy and the last-minute direct sales strategy for various probabilities of high demand ( $\alpha \in [0, 1]$  on the y-axis) and consumer valuations ( $V$  on the x-axis). The shaded area denotes the region where the opaque selling market exists for deterministic low demand (i.e., when  $V/t \leq 1$ ).

#### 4.4 Concluding Discussion

When product/service demand is uncertain and available capacity cannot be changed easily in the short term, companies often end up with one of the two extremes – a shortfall of capacity due to high demand, or leftover unused (and expensive) capacity due to low demand. To deal with the mismatch between demand and supply, firms have implemented a variety of strategies, and two of the most prominent strategies are direct last-minute sales at reduced prices and sales through an opaque intermediary. However, consumers are becoming more and more strategic – they have learned to anticipate this last-minute distress selling and might decide to postpone their purchase in expectation of future lower prices. The risk the consumers face while making this decision is of not being able to obtain a product if demand turns out to be high.

Several papers have tried to model this strategic interaction between competing firms and consumers to understand different selling mechanisms. The key question that we posed to address is: when should firms offer last-minute sales through an opaque intermediary? In this chapter we presented one model that helps answer this question and which compares opaque selling with direct-to-consumer selling. The answer that we find depends on at least three factors: (1) the valuations that consumers have for the service, (2) the strength of brand preference that consumers have for competing firms (alternatively, the extent of service differentiation between competing firms), and (3) the probability that demand in the market exceeds capacity. If consumer valuation for product is high and/or the strength of brand preference of the consumers in the market is low, firms prefer direct last-minute sales over opaque sales. Furthermore, as the probability of high demand increases, firms start to prefer opaque sales over direct last-minute sales. At the extreme, if market demand is deterministic, direct last-minute sales are never offered while opaque sales can be offered if consumer valuations for travel are very low. These findings immediately translate into empirically testable hypotheses.

The dynamics underlying the functioning of opaque selling strategy are intriguing. In general, each firm prices in the earlier periods so that only consumers with high preference for the firm buy the product. Thus, each firm derives the bulk of its profits primarily by charging high prices to these consumers, while second-period prices are very low (however, these cheap products are available only if demand

turns out to be lower than capacity). Quite differently, in the opaque selling strategy, if the consumer valuations are very low, the firms set first-period prices to extract maximum profits from consumers and then clear any remaining products through the opaque channel. When valuations are high, the firms price in the first period to ensure that the number of consumers who want to buy products exceeds supply, introducing clamor for the limited number of products, and leveraging the risk of product shortage to charge higher first-period prices. To summarize, the direct last-minute sales strategy can be construed as extracting profits from high-preference consumers, while the opaque sales strategy can be thought of as creating a frenzy for products to raise prices. Clearly, opaque selling is far from simple “inventory clearance mechanism” – such strategies are indeed responses by the firms to consumers making strategic purchasing decisions.

## 5 Other Related Considerations and Future Research

Opaque sales and last-minute sales are encountered in a variety of practical situations, many of which are not fully reflected in the stylized model described above. Below we outline some of the interesting modeling considerations that are quite promising to be considered by future research in this area

**Different selling mechanisms:** In our model we assume that the opaque intermediary operates using a posted price regime. This assumption quite accurately reflects the way *hotwire.com* conducts business but it is not reflective of NYOP price regime of *priceline.com*. The reason different opaque intermediaries utilize different pricing strategies is probably to avoid direct competition with each other. Nevertheless, we expect that NYOP selling has a potential to further increase attractiveness of opaque selling because it allows for finer price-discrimination of consumers than the last-minute selling does.

**Heterogeneous values for the core product:** In the described model, consumers were homogeneous in their preference for the core product, i.e., value  $V$  does not vary by consumer. In practice, some companies (e.g., airlines) derive significant profits by discriminating between “business” and “leisure” travelers who typically have drastically different preferences for travel, time preferences, attitudes towards risk of not getting a ticket or all of them together. This is the subject of voluminous revenue management literature in operations, see Talluri and van Ryzin (2004), which often models consumer preferences as evolving over time. We ignore such considerations since consumers with high utility for product consumption are likely to purchase the product at a full price, and would not participate in either opaque or last-minute sales channels. Thus, our model focuses exclusively on price-conscious consumers with relatively low value for the product itself. It is, however, straightforward to incorporate into our model consumers that differ in their core value for the product. For example, we could introduce a second Hotelling line with a much higher core value  $V$  representing consumers with high valuation for the product. Since these consumers have high

willingness to pay, the firms will allocate capacity to satisfy these consumers first, and then sell to consumers with lower  $V$ . Essentially, demand from high-valuation consumers can be subtracted from firms' capacities and the remaining problem is solved as described above with insights unchanged. An even more realistic model would incorporate a continuous distribution of values  $V$ . In this case our results above indicate that higher core values  $V$  make last-minute sales preferable over opaque sales. Therefore, depending on the distribution of values  $V$ , we expect that opaque selling will be preferred when there are more consumers with low valuations and direct last-minute selling will be preferred when there are more consumers with high valuations. In either case, our insights will remain qualitatively unchanged.

**Concentrated versus monopolistic markets:** In the above model there is competition in the transparent market but the opaque intermediary is a monopoly. In practice this may or may not be true. For example, for several years priceline.com enjoyed near monopoly in selling opaque products although recently it has experienced competition from other opaque intermediaries such as hotwire.com. It is possible to have situations in which both transparent and opaque markets are either monopolistic or competitive. For example, Norwegian cruise lines offer both specific staterooms on their ships as well as opaque staterooms which guarantee certain minimal amenities but not specific location on the ship. To analyze the impact of market competitiveness on our findings, we considered a situation in which both transparent products A and B are managed by the same firm which maximizes the total profit. We find that the monopoly firm is able to derive higher profit from the last-minute direct sales arrangement due to its ability to charge higher prices. Thus, without competition, the last-minute direct sales strategy becomes preferred over opaque selling for a larger range of problem parameters.

**Multiple hidden product attributes:** In the opaque literature the products are characterized by a single attribute. In practice, however, products may differ in multiple dimensions. Hotel rooms purchased on hotwire.com differ in size, location and amenities. Airline tickets differ in the number of stops, departure times and trip lengths. All these different attributes can be hidden from or revealed to consumers in the opaque selling channel. Some opaque intermediaries allow consumers to select the level of opacity: e.g., priceline.com lets its consumers specify whether a "red eye" flight is acceptable and also allows to set the upper bound on the number of stops. The issue of selecting the optimal level of opacity and the right attributes to hide provides potential for future research but is outside the scope of this study.

**Vertical product differentiation:** We believe adding multiple dimensions of opacity might shift consumer preferences towards direct last-minute sales channel. In the literature we have surveyed, the consumers are certain that the firms are selling products of identical valuations in both the channels. However, if there is additional uncertainty about exact features of the product purchased from the opaque channel, then the consumers will be more likely to purchase directly from the firms.

**Queueing for Semi-Opaque Products:** In practice we often encounter examples of semi-opaque products. For example, theaters in New York Broadway area sell the left-over tickets on the day of the show through a service run by an intermediary called TKTS. The decision-making process of theater customers is somewhat different from the model considered in this chapter. Local customers who are keen on seeing a particular performance will buy directly from the theaters. However, leisure customers, say tourists visiting New York for a weekend, might consider buying at TKTS, because they are fine with watching one of several shows. This is not to say that those customers have no preferences between the shows, but as long as they can purchase one of the shows they would like to see, the customers receive a positive value. TKTS sells mainly unsold (and some rationed tickets) on the last day of the show. The customers have to queue up, based on their ex-ante expectations of getting a ticket for one of their preferred shows. The queues are generally quite long, and therefore represent a significant waiting cost for an unclear final outcome. However, at the end of the queue, the customers get to *choose* the show they would like to see, as long as tickets for the show are available. In this sense, the products sold by TKTS are only semi-opaque. Clearly, there are some dynamic issues involved here and the customer may not always get the show he lined up for. In fact, sometimes none of the shows a customer stood in line for might be available. In such cases, she might have to just quit at (or close to) the head of the line.

**Dynamic pricing decisions:** In all the models that we surveyed, the pricing decisions made by the firms are simplified. There are only two selling opportunities: one “regular” and one “sales”. In practice, for example, airlines offer many fares, and prices tend to increase until the very last moment when last-minute sales are announced. Incorporating such considerations into modern decision support systems remains a challenge and an exciting avenue for dynamic pricing research.

**Empirical work:** Finally, we note the existence of rich opportunities for empirical modeling in the airline revenue management literature. For example, *easy* (see Easy.com 2008) brand owned by Stelios Haji-Ioannou, operates on the idea of increasing prices as the selling horizon matures. This pricing strategy is beneficial in two ways: first, it discourages customers from waiting for last-minute deals, and second, it encourages early purchases thus providing the airline with advanced demand information so that it can target its price increases accordingly. Although numerous studies have modeled airline revenue management decisions, there have been very few attempts to verify these findings empirically. See Koenigsberg et al. (2008) for an exception which analyzes EasyJet pricing strategy. They find that the last minute sales are likely to be offered when the capacity levels are intermediate (i.e., not too high or low relative to demand), and when there are many flight segments. Our perception is that empirical studies in the revenue management area tend to be limited by data availability. Although airlines share lots of data with regulatory authorities (Federal Aviation Administration and the Department of Transportation), these data are not precise enough to distill specific pricing strategies employed by an airline. See also work by Cho et al. (2008).

## 6 Appendix A: Deterministic Demand

### 6.1 Proof of Lemma 1

*Proof.* We begin the proof by observing that the two periods are identical in the information all the players (firms and consumers) have and there is no stochastic component in demand or utilities. Further, there is no discounting.

Consider the case of firm  $A$ . First, consider the case when demand is low ( $J < K$ ). Suppose, in equilibrium, the consumer located at  $x_A$  is indifferent between buying from firm  $A$  in the first period at price  $p_A^1$  and in the second period at price  $p_A^2$ . For this consumer, the following indifference condition holds when he is making his purchase or postpone decision in the first period ( $p_A^{2,e}$  is his first-period expectation of the second-period price):

$$V - p_A^1 - tx_A = V - p_A^{2,e} - tx_A$$

which implies that  $p_A^1 = p_A^{2,e}$  in equilibrium, regardless of the location of the indifferent customer. Further, in any rational expectations equilibrium, the expectations will be correct, i.e.,  $p_A^{2,e} = p_A^2$ . Therefore, firm  $A$  will offer the same price ( $p_A^1 = p_A^2 = p_A$ ) in both periods.

When demand is high ( $J > K$ ), there are two kinds of consumers based on their locations – those who will obtain firm  $A$ 's product (located in the region  $[0, K/(2J)]$ ), and those who will not obtain firm  $A$ 's product (located to the right of  $K/(2J)$ ). When demand is high, firm  $A$  maximizes revenues by selling to the consumers located to the left of  $K/(2J)$ . The second kind of consumers therefore would be unable to buy products in the high-demand scenario. For the first set of consumers, the indifference condition above again holds and we have  $p_A^1 = p_A^2$ . Further, firm  $A$  sets these prices so that the consumer at  $K/(2J)$  is indifferent between purchasing and not purchasing a ticket, i.e.,  $V - p_A^1 - tK/(2J) = V - p_A^2 - tK/(2J) = 0$ , which yields  $p_A^1 = p_A^2 = V - K/(2J)$ .

Similar arguments apply for prices offered by firm  $B$ .

### 6.2 Proof of Proposition 1

*Proof.* We prove the proposition for low demand. Note that the total capacity of the two firms ( $K$ ) is more than the total demand ( $J$ ). Let  $V/t \geq 1/2$  as described in the paper.

First, consider the case in which the firms are acting as local monopolies. We consider the decision of firm  $A$  in detail, and the analysis will be identical for firm  $B$ . If firm  $A$  chooses the price  $p_A$ , the right-most consumer to buy from the firm will be at  $x_A$  such that  $V - tx_A - p_A = 0$ , i.e., the utility of the consumer at  $x_A$  is zero. The price charged by the firm to all consumers will then be  $p_A = V - tx_A$ , and the

demand will be  $x_A J$ . Thus, the profit for the firm will be  $\pi_A = p_A x_A J = (V - t x_A) x_A J$ . This profit is maximized at  $x_A = V / (2t)$ , and the maximized profit is given by  $\pi_A = J V^2 / (4t)$ . However, to ensure that the firms are local monopolies, we need to ensure that at the optimum  $x_A < 1/2$ , which yields  $V/t < 1$ .

When  $V/t \geq 1$ , the above equilibrium does not hold, since the firms are not local monopolies (the optimal coverage for each firm will be  $> 1/2$ ). We propose that for  $1 \leq V/t < 3/2$  both firms charge prices  $p_A = p_B = V - t/2$  in equilibrium, cover half the market and make profits  $\pi_A = \pi_B = (V - t/2) J / 2$ . We now show that this is the unique equilibrium. Suppose firm  $A$  raises its price and charges  $p_A^+ = V - t/2 + \varepsilon t$  where  $\varepsilon > 0$ , while firm  $B$  still charges  $p_B = V - t/2$ . Then, firm  $A$  covers  $x_A = 1/2 - \varepsilon$  and makes a profit  $(1/2 - \varepsilon)(V - t/2 + \varepsilon t) J$ . However, under the condition  $V/t \geq 1$ , this profit is lower than the equilibrium profit, so that the firm does not have an incentive to raise its price above the equilibrium price. Now, consider the case in which the firm lowers its price and charges  $p_A^- = V - t/2 - \varepsilon t$ . The point  $\tilde{x}$  at which the indifferent consumer is located is then found by solving the condition  $V - p_A^- - t\tilde{x} = V - p_B - t(1 - \tilde{x})$ , which yields  $\tilde{x} = (1 + \varepsilon)/2$ , and the profit for firm  $A$  is given by  $\frac{1}{2}(1 + \varepsilon)(V - t/2 - \varepsilon t) J$ . However, under the condition  $V/t < 3/2$ , this profit is always lower than the equilibrium profit, so that the firm does not have an incentive to lower its price below the equilibrium price. Hence, the equilibrium proposed above is indeed an equilibrium for the range  $1 \leq V/t < 3/2$ .

Now consider the case in which the two firms are in direct competition. Firm  $A$  charges a price  $p_A$  and firm  $B$  charges a price  $p_B$ . Assume that the indifferent consumer is located at  $\tilde{x}$ . Since this consumer is indifferent to buying from  $A$  or  $B$ , the following condition holds for him:  $V - p_A - t\tilde{x} = V - p_B - t(1 - \tilde{x})$ , which gives  $\tilde{x} = 1/2 + (p_B - p_A) / (2t)$ . The profits for firms  $A$  and  $B$  are given, respectively, by  $\pi_A = p_A \tilde{x} J$  and  $\pi_B = p_B (1 - \tilde{x}) J$ . Maximizing the profits jointly, we obtain  $p_A = p_B = t$ ,  $\tilde{x} = 1/2$  and  $\pi_A = \pi_B = J t / 2$ . Under our assumption that the outside utility of a consumer is zero, we need to ensure that  $V - p_A - t\tilde{x} = V - p_B - t(1 - \tilde{x}) \geq 0$ , which gives the condition  $V/t \geq 3/2$ .

This specifies the equilibrium for all values of  $V/t \geq 1/2$  and completes the proof.

### 6.3 Proof of Proposition 2

*Proof.* Note that  $V/t \geq 1/2$ . In this proposition, we analyze the high demand case. The total capacity of the two firms ( $K$ ) is less than the total demand ( $J$ ) and firms will act as local monopolies. Again, we consider firm  $A$  and the analysis is identical for firm  $B$ . If firm  $A$  chooses the price  $p_A$ , the right-most consumer to buy from the firm will be at  $x_A$  such that  $V - t x_A - p_A = 0$ . The price charged by the firm to all consumers will then be  $p_A = V - t x_A$ , and the demand will be  $x_A J$ .

Thus, the profit for the firm will be  $\pi_A = p_A x_A J = (V - t x_A) x_A J$ . This profit is maximized at  $x_A = V / (2t)$ , and the maximized profit is given by  $\pi_A = J V^2 / (4t)$ .



However, to ensure that the firms do not stock out, we need to ensure that at the optimum  $x_A \leq K/(2J)$ , which gives  $V/t \leq K/J$ .

For  $V/t > K/J$  each firm will charge the price  $p_A = p_B = V - tK/(2J)$ , cover  $x_A = 1 - x_B = K/(2J)$  and make profits  $\pi_A = \pi_B = (V - Kt/(2J))(K/2)$ . Note that the firm cannot lower its price below this level, since it does not have the capacity to serve the expanded market. It can be easily shown, using an  $\varepsilon$ -deviation argument as in the proof of proposition 1, that the firm does not have an incentive to lower its price below this level. This specifies equilibria for all values of  $V/t \geq 1/2$  and completes the proof.

## 6.4 Proof of Lemma 2

*Proof.* We first prove that, when demand is deterministic, the rational expectations equilibrium does not exist for  $\gamma_A^e \in [0, 1] \setminus \{\frac{1}{2}\}$ , and only  $\gamma_A^e = \gamma_B^e = 1/2$  are supported in equilibrium. We first consider the deterministic low demand case, and then the high demand case. In both cases, we establish the rational expectations equilibrium by first analyzing the second period, and then the first period. In all cases,  $V/t \geq 1/2$  as before.

**Low demand:** We consider the case in which the firms have ample capacity, i.e.,  $J < K$ . Let us consider the *second* period. Without loss of generality, let  $x_A, x_B$  be the location of the consumers closest to the firm who did not buy in the first period. Hence interval  $[x_A, x_B]$  denotes the location of all the consumers remaining in the second period. Consider any consumer located at  $x \in [x_A, x_B]$ .  $x_A$  and  $x_B$  are the left-most and right-most points on the line available to the intermediary to sell opaque products.

Since we are in the second period, demand realization has occurred, and opaque seller has announced price  $p_I$ . The consumer has expectations over availability. Upon buying the opaque product at price  $p_I$ , the surplus a consumer located at  $x$  expects to attain is

$$V - p_I - \gamma_A^e tx - \gamma_B^e t(1 - x)$$

which, using  $\gamma_B^e = 1 - \gamma_A^e$ , can be written as

$$V - p_I - t(1 - \gamma_A^e) + (1 - 2\gamma_A^e)tx.$$

$0 < \gamma_A^e \leq 1/2$ : It suffices to consider  $0 < \gamma_A^e \leq 1/2$  because, if  $\gamma_A^e = 0$ , the market is not opaque since the consumers believe that all the products in the opaque channel are coming from firm  $B$ . The analysis for  $1/2 \leq \gamma_A^e < 1$  is identical to the analysis for  $0 < \gamma_B^e \leq 1/2$  (which is the same as the analysis below by symmetry). For a given  $p_I$ , the surplus for a consumer purchasing in the opaque market is increasing in his location  $x$ , as long as  $\gamma_A^e < 1/2$ . In other words, the minimum surplus is obtained by the consumer located at  $x_A$ . We consider two cases, namely,

when the intermediary wants to cover the full market from  $x_A$  to  $x_B$  and when the intermediary considers covering this interval partially.

The intermediary may not necessarily cover the full market  $[x_A, x_B]$  available to him. Suppose that the intermediary only aims to cover the market  $[x', x_B]$ , where  $x' > x_A$ . Note that the surplus of a consumer is increasing in his location  $x$ . Therefore the opaque intermediary will price such that  $p_I = V - (1 - \gamma_A^e)t + (1 - 2\gamma_A^e)t x'$ ,  $x_A < x' < x_B$ . Then the consumers in the interval  $[x_A, x']$  (which is defined to be null if  $x' < x_A$ ) do not buy because they have negative utility. The consumers in the interval  $[x', x_B]$  buy in the opaque channel because they have non-negative utility. The profit of the intermediary is then  $\pi_I = (V - (1 - \gamma_A^e)t + (1 - 2\gamma_A^e)t x')(x_B - x')J$ . To maximize this profit the intermediary sells to the market  $[x^{J*}, x_B]$  where

$$x^{J*} = \frac{(1 - \gamma_A^e)t - V + (1 - 2\gamma_A^e)t x_B}{2(1 - 2\gamma_A^e)t}.$$

This implies that  $p_I = \frac{1}{2}(V - (1 - \gamma_A^e)t + (1 - 2\gamma_A^e)t x_B)$ .

Now consider the analysis for firm  $A$  selling in the transparent channel. The person located at  $x_A$  has negative utility in the opaque channel. Thus, to this consumer, firm  $A$  selling in the transparent channel can charge  $p_A = V - x_A t$  and make him indifferent between buying and not buying. This gives firm  $A$  a profit of  $\pi_A = (V - t x_A)x_A J + \delta \gamma_A^e (V - (1 - \gamma_A^e)t + (1 - 2\gamma_A^e)t x_B)(x_B - x')$ .

Now consider firm  $B$ . The consumer at  $x_B$  has to be indifferent between purchasing in the first period and in the second period. The consumer at  $x_B$  solves  $V - p_B - t(1 - x_B) = V - p_I - \gamma_A^e t x_B - (1 - \gamma_A^e)t(1 - x_B)$ , which, using the value of  $p_I$  from above, gives  $p_B = \frac{1}{2}(V + t(-1 + x_B - \gamma_A^e + 2x_B \gamma_A^e))$ . The profit for firm  $B$  is  $\pi_B = p_B(1 - x_B)J + \delta(1 - \gamma_A^e)p_I(x_B - x')J$ . Maximizing  $\pi_A$  and  $\pi_B$  w.r.t.  $x_A$  and  $x_B$  simultaneously, we obtain

$$x_A = \frac{V}{2t} \quad \text{and} \quad x_B = \frac{V(1 + (-1 + \gamma_A^e)\delta) + t(-2 + \delta + (\gamma_A^e)^2\delta - \gamma_A^e(3 + 2\delta))}{t(-2 + \delta + 2(\gamma_A^e)^2\delta - \gamma_A^e(4 + 3\delta))}.$$

Using these values of  $x_A$  and  $x_B$ , we obtain

$$\gamma_A^{\text{realized}} = \frac{K/2 - x_A J}{K/2 - x_A J + K/2 - (1 - x_B)J}.$$

In the rational expectations equilibrium, the beliefs have to be consistent with the outcome. It must be that  $\gamma_A^{\text{realized}} = \gamma_A^e$ . Imposing this condition we solve for  $\gamma_A^{\text{realized}} = \gamma_A^e = \gamma_A^e(V, t, J, K, \delta)$ . The value of  $\gamma_A(V, t, J, K, \delta)$  is algebraically complicated and we do not present it here. However, we check that imposing the condition  $0 < \gamma_A^e < 1/2$  implies  $V/t < 1/2$ , which is a contradiction. (Recall that we require  $V/t \geq 1/2$  as a ‘‘sanity condition’’ to ensure that if the firms sell products for free, then everybody in the market will have positive evaluation to obtain the product from at least one of the firms. In other words the condition ensures some market coverage at zero prices.) Thus, when the intermediary sets

prices such that the intermediary's market coverage is partial, then the rational expectations equilibrium does not exist.

Let us now analyze the case when the intermediary prices to cover the entire market  $[x_A, x_B]$ . If the intermediary wants to cover the full opaque market, he will price so as to make the surplus of the consumer at  $x_A$  equal to zero, i.e.,  $p_I = V - t(1 - \gamma_A^e) + x_A(1 - 2\gamma_A^e)t$ .

Since the consumer  $x_A$  is indifferent between the opaque and first period market, firm  $A$  sets its price  $p_A$  by solving the following equation:

$$V - p_A - tx_A = V - p_I - \gamma_A^e tx_A - (1 - \gamma_A^e)t(1 - x_A).$$

To extract maximum revenues in the opaque market, the intermediary sets  $p_I$  such that the right hand side of the above equation is zero. Therefore the value that  $x_A$  receives is zero.

Therefore in the first period, if firm  $A$  covers the interval  $[0, x_A]$ , the price is  $p_A = V - tx_A$ . In the first period firm  $A$  then maximizes its profit  $\pi_A = (V - tx_A)x_A J + \delta \gamma_A^e p_I (x_B - x_A) J$ , where  $p_I$  is as above.

Firm  $B$  solves

$$V - p_B - t(1 - x_B) = V - p_I - \gamma_A^e tx_B - (1 - \gamma_A^e)t(1 - x_B).$$

Restricting to  $\gamma_A^e \in (0, 1/2]$  and using  $p_I$  above, we obtain  $p_B = V + t(-1 + x_A - 2x_A \gamma_A^e + 2x_B \gamma_A^e)$ . The profit for firm  $B$  is given by  $\pi_B = p_B(1 - x_B)J + \delta(1 - \gamma_A^e)p_I(x_B - x_A)J$ .

Maximizing  $\pi_A$  and  $\pi_B$  simultaneously for the firms w.r.t.  $x_A$  and  $x_B$  gives:

$$\begin{aligned} x_A &= [V(4 - (1 + 2\gamma_A^e)\delta) + (1 - 3\gamma_A^e + 2(\gamma_A^e)^2)\delta^2 \\ &\quad + t\delta(1 - \delta + 2(\gamma_A^e)^3\delta + 4\gamma_A^e(1 + \delta) - (\gamma_A^e)^2(8 + 5\delta))] \\ &\quad \cdot [t(8 + (1 + 4\gamma_A^e - 12(\gamma_A^e)^2)\delta + (1 - 2\gamma_A^e)^2(-1 + \gamma_A^e)\delta^2)]^{-1} \\ x_B &= [t(2 - 2\delta + 2(\gamma_A^e)^4\delta^2 - 5(\gamma_A^e)^3\delta(2 + \delta) + (\gamma_A^e)^2\delta(1 + 4\delta) \\ &\quad + \gamma_A^e(4 + 5\delta - \delta^2)) + V(3(-1 + \delta) + (\gamma_A^e)^2(4 - 3\delta)\delta \\ &\quad + 2(\gamma_A^e)^3\delta^2 + \gamma_A^e(2 - 6\delta + \delta^2))] \\ &\quad \cdot [t\gamma_A^e(8 + (1 + 4\gamma_A^e - 12(\gamma_A^e)^2)\delta + (1 - 2\gamma_A^e)^2(-1 + \gamma_A^e)\delta^2)]^{-1} \end{aligned}$$

Using the above values, we obtain

$$\gamma_A^{\text{realized}} = \frac{K/2 - x_A J}{(K/2 - x_A J) + (K/2 - (1 - x_B)J)}.$$

In the rational expectations equilibrium,  $\gamma_A^{\text{realized}} = \gamma_A^e$ . Upon solving this, we obtain  $\gamma_A^{\text{realized}} = \gamma_A^e = 1/2$  as the only real-valued solution. Hence, the equilibrium does not exist for  $0 \leq \gamma_A^e < 1/2$  when the intermediary wants to cover the full market between  $[x_A, x_B]$ . Only  $\gamma_A^e = \gamma_B^e = 1/2$  can be supported in the equilibrium.

$1/2 < \gamma_A^e \leq 1$ : When  $1/2 < \gamma_A^e \leq 1$ , for a given  $p_I$ , the surplus decreases with  $x$ . In other words, the minimum surplus is obtained by the consumer located at  $x_B$ . The analysis proceeds as above, except the subscripts  $A$  and  $B$  are suitably interchanged. Using identical arguments, we show that there is no equilibrium such that  $1/2 < \gamma_A^e \leq 1$ . Further, the analysis for  $\gamma_A^e > 1/2$  is the same as the analysis for  $\gamma_B^e < 1/2$ .

In summary, all consumers develop rational expectations  $\gamma_A^e = \gamma_B^e = 1/2$ , which are realized in equilibrium.

In other words, the rational expectations equilibrium does not exist for  $\gamma_A \in [0, 1] \setminus \{\frac{1}{2}\}$  and only  $\gamma_A^e = \gamma_B^e = 1/2$  are supported in the rational expectations equilibrium.

Since  $\gamma_A^e = \gamma_B^e = 1/2$  for every consumer in the market and the probability of getting an opaque product  $\beta = 1$  (since there is ample capacity), the ex-ante expected surplus for each consumer buying from the opaque channel is simply  $V - p_I - t/2$  and is independent of the location of the consumer. Therefore intermediary prices at  $p_I = V - t/2$  and attains the revenue  $\pi_I = (1 - \delta)(V - t/2)(x_B - x_A)J$  by selling to the entire remaining market. Note that the revenue-maximizing action in the opaque channel is independent of the fraction of revenues  $(1 - \delta)$  held by the intermediary.

**Limited capacity/High demand:** In the case in which the firms have limited capacity, i.e.,  $J > K$ , we need to impose the conditions  $x_A \leq K/(2J)$  and  $1 - x_B \leq K/(2J)$  while optimizing the profits for firms  $A$  and  $B$  respectively, which does not change the procedure of the preceding proof. We sketch the argument below. Let us consider the second period when opaque products are being offered. Wlog, let  $x_A, x_B$  be the locations of the consumers who were indifferent between purchasing and not purchasing from the firms  $A$  and  $B$  in the first period, respectively. Hence  $[x_A, x_B]$  denotes the interval of all the consumers remaining in the second period. Now, consider any consumer located at  $x \in [x_A, x_B]$ .

Since we are in the second period, demand realization has occurred, and opaque seller has announced price  $p_I$ . The consumer located at  $x$  has beliefs over availability. Upon buying the opaque product at price  $p_I$ , the surplus he expects to attain is

$$V - p_I - \gamma_A^e t x - \gamma_B^e t (1 - x)$$

which, using  $\gamma_B^e = 1 - \gamma_A^e$ , can be written as

$$V - p_I - t(1 - \gamma_A^e) + (1 - 2\gamma_A^e)t x.$$

If capacity is not binding, then the analysis is no different from the previous analysis of the low demand case. (i.e., in equilibrium, the expectations  $\gamma_A = \gamma_B = 1/2$ ).

Suppose capacity is limited in the second period. In other words, the residual capacity in the second period is less than the unfulfilled demand in the second period, i.e.  $(K - x_A J - (1 - x_B)J) < (x_B - x_A)J$ . Let us assume that the opaque intermediary covers interval  $[x', x'']$  where  $x' > x_A$  and  $x'' < x_B$ .

Consider  $0 < \gamma_A^e < 1/2$ .  $p_I$  is obtained by solving  $V - p_I - t(1 - \gamma_A^e) + (1 - 2\gamma_A^e) \cdot tx' = 0$ . Since the indifferent consumer is located at  $x_A < x'$ , we have that the net valuation of the consumer purchasing from firm A is zero. (If the consumer has positive utility, then a consumer to the right of  $x_A$  would also buy with positive utility.) The optimal revenue of firm A is achieved by maximizing  $\pi_A = (V - tx_A)x_A J + \delta \gamma_A^e p_I (x'' - x') J$  w.r.t.  $x_A$ . This implies  $x_A^* = \min\{K/(2H), V/(2t)\}$ .

Since capacity is binding, all the seats with the opaque intermediary are sold. Hence,  $x'' < x_B$  is determined by  $(K - x_A J - (1 - x_B)J) = (x'' - x')J$ . Then firm B maximizes  $\pi_B = (V - t(1 - x_B))(1 - x_B)J + \delta(1 - \gamma_A^e)p_I(x'' - x')J$  w.r.t.  $x_B$ . This implies  $x_B = 1 - \min\{K/(2H), V/(2t)\}$ . Hence  $x_A = 1 - x_B$ , and the rational expectations regarding the probabilities of availability of the left over products from each firm will be symmetric. Hence only  $\gamma_A = \gamma_B = 1/2$  is sustained in equilibrium in the high-demand environment.

### 6.5 Equilibrium Characterization For the Low-Demand Case (If Consumers Do Not Strategically Wait)

From proof of Lemma 2 in Appendix 6.4, we have  $\gamma_A^e = \gamma_A = 1/2$  and  $\beta = 1$  for all consumers in the market. The ex-ante expected surplus for each consumer buying from the opaque channel is simply  $V - p_I - t/2$  and is independent of the location of the consumer. Therefore, the intermediary prices at  $p_I = V - t/2$  and attains the revenue  $\pi_I = (V - t/2)(x_B - x_A)J$  by selling to the entire remaining market.

We now analyze the optimal choices of the firms in their own transparent channels before the opaque sales. The consumers located between  $x_A$  and  $x_B$  prefer to buy from the opaque channel. As before, for firm A the consumer located at  $x_A$  must be indifferent between buying from the firm now or in the opaque channel later. In a low-demand state, the leftover products are sufficient to cover all the remaining demand. Hence, we have:

$$V - p_A - tx_A = V - p_I - \gamma_A^e tx_A - \gamma_B^e t(1 - x_A).$$

Since  $\gamma_A^e = 1/2$  and  $p_I = V - t/2$ , the right-hand side of the equation above is zero. Hence, the price charged by firm A is  $p_A = V - tx_A$ . Firm A, which covers the market till  $x_A$  and charges price  $p_A = V - tx_A$ , makes a profit of

$$\pi_A = (V - tx_A)x_A J + \delta \gamma_A^e \left(V - \frac{t}{2}\right) (x_B - x_A) J.$$

The value of  $x_A$  that maximizes the profit for firm A is

$$x_A = \frac{V}{2t} - \frac{\delta}{4} \left(\frac{V}{t} - \frac{1}{2}\right),$$

which is decreasing in  $\delta$ . As the ability to earn more revenues from the opaque channel increases, the firm chooses to cover less through its own channels. This does not imply that the firm is generating smaller revenue through its own channel. The corresponding optimal price  $p_A = V - tx_A = V/2 + (\delta/4)(V/t - 1/2)t$  is increasing in  $\delta$ . Because of the presence of the opaque channel, firms sell fewer products in their own channels at *higher* prices. Proceeding with a similar analysis for firm  $B$ , we obtain

$$x_B = 1 - \left( \frac{V}{2t} - \frac{\delta}{4} \left( \frac{V}{t} - \frac{1}{2} \right) \right).$$

Thus, the coverage by the intermediary is

$$x_B - x_A = 1 - \frac{V}{t} + \frac{\delta}{2} \left( \frac{V}{t} - \frac{1}{2} \right).$$

For the above expression, we need to ensure that

$$x_A \leq \frac{1}{2} \Rightarrow \frac{V}{t} \leq \frac{4 - \delta}{2(2 - \delta)}.$$

For  $V/t \geq 3/2$ , the competitive equilibrium holds, with both firms covering exactly half the market at prices  $p_A = p_B = t$  and obtaining profits  $\pi_A = \pi_B = tJ/2$  while  $\pi_I = 0$ . For  $(4 - \delta)/[2(2 - \delta)] \leq V/t \leq 3/2$  we construct the non-competitive equilibrium as both firms charging  $p_A = p_B = (V/t - 1/2)t$ , covering exactly half the market, and therefore making profits  $\pi_A = \pi_B = (V/t - 1/2)(t/2)J$  with  $\pi_I = 0$  (see table below). Thus, for  $V/t \geq (4 - \delta)/[2(2 - \delta)]$  it turns out that nothing is allocated to the opaque channel in equilibrium.

|  | Profit from the first period   | Profit from opaque sales   |
|--|--|--|
| $\frac{1}{2} \leq \frac{V}{t} \leq \frac{4 - \delta}{2(2 - \delta)}$ | $\left( \frac{V}{2} + \frac{\delta}{4} \left( \frac{V}{t} - \frac{1}{2} \right) t \right) \cdot \left( \frac{V}{2t} - \frac{\delta}{4} \left( \frac{V}{t} - \frac{1}{2} \right) \right) J$ | $\delta \left( \frac{V}{t} - \frac{1}{2} \right) \left( 1 - \frac{V}{t} + \frac{\delta}{2} \left( \frac{V}{t} - \frac{1}{2} \right) \right) \frac{t}{2} J$ |
| $\frac{4 - \delta}{2(2 - \delta)} \leq \frac{V}{t} \leq \frac{3}{2}$ | $\left( \frac{V}{t} - \frac{1}{2} \right) \frac{t}{2} J$   | 0  |
| $\frac{3}{2} \leq \frac{V}{t}$                                       | $\frac{t}{2} J$  | 0  |

The analysis above shows that (compared to the table in Proposition 1), using the opaque channel increases the total market coverage (and profits) given the same valuation  $V$  and strength of brand preferences  $t$  when the ratio  $V/t$  is small. However, the firms utilize the opaque channel only when the consumers' willingness to pay is low. For a higher willingness to pay, i.e., when  $V/t \geq (4 - \delta)/[2(2 - \delta)]$ , the firms price in the transparent channel so as to cover the entire market and there are no sales through the opaque channel. Note that, in the above case, when  $V/t \geq (4 - \delta)/[2(2 - \delta)]$ , the profits are identical to what a firm makes when it does not use an opaque channel and  $V/t \geq 1$  (Proposition 1). Also, note that

$1 \leq (4 - \delta)/[2(2 - \delta)] \leq 3/2$  for all values of  $\delta \in [0, 1]$ , which means that using the opaque channel increases profits in the non-competitive regime. Therefore, firms charge a higher price in the first period, cover less through their own channels after the introduction of the opaque channel, but “clear up” the remaining market using the opaque channel (albeit at a lower price), which leads to higher overall profits.

### 6.6 Proof of Proposition 3

*Proof.* When  $1/2 \leq V/t < 1$ , the firms find it optimal to charge a price  $V/2$  and cover  $V/(2t)$ , which is less than  $1/2$ . The remaining  $1 - V/t$  portion of the market (between  $V/(2t)$  and  $1 - V/(2t)$ ) is covered in the opaque channel by charging a price  $V - t/2$ . The proof is along the lines of the proof in Appendix 6.5. When  $V/t \geq 1$ , the full market is covered in the transparent channel in the first period itself, and there are no products left to be allocated to the opaque channel.

### 6.7 Equilibrium Characterization for the High-Demand Case (If Consumers Do Not Strategically Wait)

Consider a consumer at  $x \in [x_A, x_B]$  and note that  $K < J$ . The probability this consumer obtains a product is

$$\beta = \frac{K - x_A J - (1 - x_B) J}{(x_B - x_A) J}.$$

If this consumer obtains a product from the opaque seller at price  $p_I$ , the surplus he attains is

$$V - p_I - \gamma_A^e t x - \gamma_B^e t (1 - x).$$

Again, using Lemma 2,  $\gamma_A^e = 1/2$ . In this case, the expected surplus in the equilibrium is simply  $V - p_I - t/2$  and is independent of the location of the consumer. The opaque intermediary prices the products at  $p_I = V - t/2$  and the total revenue accrued in the channel is  $\pi_I = (V - t/2)(K/J - x_A - (1 - x_B))J$ . Note that the intermediary can sell to *any* consumer located between  $x_A$  and  $x_B$  even though he is unable to cover the full market between these two points due to constrained capacity.

Now consider firm  $A$  in the transparent channel. The consumer at  $x_A$  (the right-most consumer that buys from  $A$ ) solves

$$V - p_A - t x_A = \beta(V - p_I^e - \gamma_A^e t x_A - \gamma_B^e t (1 - x_A)).$$

Since  $\gamma_A^e = 1/2$  and  $p_I^e = p_I = V - t/2$ , the right-hand side of the equation above is zero.

The rest of the analysis proceeds exactly as above in the proof for Proposition 3, except that the firm stocks out if the optimal value of  $x_A$  is greater than  $K/(2J)$ . After imposing  $x_A \leq K/(2J)$  in the solution above due to capacity constraints, we obtain

$$\frac{V}{2t} - \frac{\delta}{4} \left( \frac{V}{t} - \frac{1}{2} \right) < \frac{K}{2J} \Rightarrow \frac{V}{t} \leq \frac{4K/J - \delta}{2(2 - \delta)}.$$

Upon imposing the condition  $V/t \geq 1/2$ , we obtain a lower bound on  $K/J$ , i.e.,  $K/J > 1/2$ . Thus, for

$$\frac{1}{2} \leq \frac{V}{t} \leq \frac{4K/J - \delta}{2(2 - \delta)}$$

(ensuring the firm does not stock out in the transparent channel) we obtain

$$\begin{aligned} \pi_A = \pi_B &= \left( \frac{V}{2} + \frac{\delta}{4} \left( \frac{V}{t} - \frac{1}{2} \right) t \right) \left( \frac{V}{2t} - \frac{\delta}{4} \left( \frac{V}{t} - \frac{1}{2} \right) \right) J \\ &\quad + \frac{\delta}{2} \left( \frac{V}{t} - \frac{1}{2} \right) \left( K - \left( \frac{V}{t} - \frac{\delta}{2} \left( \frac{V}{t} - \frac{1}{2} \right) \right) J \right) t, \\ \pi_I &= (1 - \delta) \left( \frac{V}{t} - \frac{1}{2} \right) \left( K - \left( \frac{V}{t} - \frac{\delta}{2} \left( \frac{V}{t} - \frac{1}{2} \right) \right) J \right) t. \end{aligned}$$

For the case in which  $V/t \geq (4K/J - \delta)/[2(2 - \delta)]$ , we construct the non-competitive equilibrium as follows: both firms charge a price  $p_A = p_B = (V/t - K/(2J))t$  and cover  $x_A = 1 - x_B = K/(2J)$ . The profits are given by

$$\pi_A = \pi_B = \left( \frac{V}{t} - \frac{K}{2J} \right) \frac{K}{2} t \quad \text{and} \quad \pi_I = 0.$$

When demand is higher than capacity available, the profits accrued by the firms from the sales through their own channels and opaque channels are as follow.

| $V/t$  | Profit from the first period   | Profit from opaque sales  |
|--|--|---|
| $\frac{1}{2} \leq \frac{4K/J - \delta}{2(2 - \delta)}$ | $\left( \frac{V}{2} + \frac{\delta}{4} \left( \frac{V}{t} - \frac{1}{2} \right) t \right) \cdot \left( \frac{V}{2t} - \frac{\delta}{4} \left( \frac{V}{t} - \frac{1}{2} \right) \right) J$ | $\delta t \left( \frac{V}{t} - \frac{1}{2} \right) \left( K - \left( \frac{V}{t} - \frac{\delta}{2} \left( \frac{V}{t} - \frac{1}{2} \right) \right) J \right)$ |
| $\frac{4K/J - \delta}{2(2 - \delta)} \leq \frac{V}{t}$ | $\left( \frac{V}{t} - \frac{K}{2J} \right) \frac{t}{2} J$  | 0   |

## 6.8 Proof of Proposition 4

*Proof.* When  $1/2 \leq V/t < K/J$ , the firms find it optimal to charge a price  $V/2$  and cover  $V/(2t)$ , which is less than  $1/2$ . The remaining  $K/2 - VJ/t$  portion of the market  $((V/(2t), K/(2J)]$  and  $(1 - K/(2J), 1 - V/(2t))$  is covered in the opaque



channel by charging a price  $V - t/2$ . The proof is along the lines of the proof in Appendix 6.7.

When  $V/t \geq K/J$ , the full market is covered in the transparent channel in the first period itself (i.e., the firms sell all  $K/2$  products), and there are no products left to be allocated to the opaque channel.

## 7 Appendix B: Uncertain Demand

### 7.1 Proof of Proposition 5

*Proof.* Consider any consumer at some position  $x \leq 1/2$  who has the following beliefs about the location of the indifferent consumers: The consumer  $x$  believes that the consumer indifferent between buying in the first period from firm  $A$  and buying in the second period from firm  $A$  is located at  $x_A^e$ . Further,  $x$  believes the consumer indifferent between buying and not buying from  $A$  in the second period is located at  $y_A^{H,e} \geq x_A^e$  when demand is high and at  $y_A^{L,e} \geq x_A^e$  when demand is low.

**Second Period:** In the second period, if demand is high, the number of products available is  $(K/2 - x_A^e H)^+$  and the number of products demanded is  $(x_A^e H - K/2)^+ + (y_A^{H,e} - x_A^e)H$ . Hence, the probability of obtaining a product is

$$\min \left\{ 1, \frac{\min \left\{ (K/2 - x_A^e H)^+, (x_A^e H - K/2)^+ + (y_A^{H,e} - x_A^e)H \right\}}{(x_A^e H - K/2)^+ + (y_A^{H,e} - x_A^e)H} \right\}.$$

If demand is low, the number of products available is  $(K/2 - x_A^e L)^+$  and the number of products demanded is  $(x_A^e L - K/2)^+ + (y_A^{L,e} - x_A^e)L$  so that the probability of obtaining a product is  $\min \left\{ 1, (K/2 - x_A^e L)^+ / ((x_A^e L - K/2)^+ + (y_A^{L,e} - x_A^e)L) \right\}$ .

The expected surplus for the consumer at  $x$  for the second period is, therefore,

$$\begin{aligned} &= \alpha \min \left\{ 1, \frac{\min \left\{ (\frac{K}{2} - x_A^e H)^+, (x_A^e H - \frac{K}{2})^+ + (y_A^{H,e} - x_A^e)H \right\}}{(x_A^e H - \frac{K}{2})^+ + (y_A^{H,e} - x_A^e)H} \right\} (V - xt - p_A^{2H}) \\ &\quad + (1 - \alpha) \min \left\{ 1, \frac{(\frac{K}{2} - x_A^e L)^+}{(x_A^e L - \frac{K}{2})^+ + (y_A^{L,e} - x_A^e)L} \right\} (V - xt - p_A^{2L}). \end{aligned}$$

**First Period:** In the first period, if demand is high, the probability that a consumer will obtain a product is  $(\min\{\frac{K}{2}, x_A^e H\} / (x_A^e H))$  and, if demand is low, the probability that this consumer will obtain a product is 1. Hence, the expected surplus for the consumer at  $x$  from buying in the first period is

$$[\alpha (\min\{K/2, x_A^e H\} / (x_A^e H)) + (1 - \alpha)] (V - xt - p_A^1).$$

Let  $x_A$  be the actual location of the indifferent consumer in the first period. Therefore, in the equilibrium, let  $x_A = x_A^e$  be the position of the indifferent consumer so we can write

$$\begin{aligned} & \left( \alpha \left( \frac{\min\{\frac{K}{2}, x_A^e H\}}{x_A^e H} \right) + (1 - \alpha) \right) (V - x_A^e t - p_A^1) \\ &= \alpha \min \left\{ 1, \frac{\min \left\{ \left( \frac{K}{2} - x_A^e H \right)^+, \left( x_A^e H - \frac{K}{2} \right)^+ + (y_A^{H,e} - x_A^e) H \right\}}{\left( x_A^e H - \frac{K}{2} \right)^+ + (y_A^{H,e} - x_A^e) H} \right\} \\ & \quad \cdot (V - x_A^e t - p_A^{2H}) \\ & \quad + (1 - \alpha) \min \left\{ 1, \frac{\left( \frac{K}{2} - x_A^e L \right)^+}{\left( x_A^e L - \frac{K}{2} \right)^+ + (y_A^{L,e} - x_A^e) L} \right\} (V - x_A^e t - p_A^{2L}). \quad (1) \end{aligned}$$

Note that, trivially, we can ensure  $y_A^{L,e} \geq x_A^e$  and  $y_A^{H,e} \geq x_A^e$ . The condition above looks quite imposing to solve, but we can simplify it considerably by dividing it into two cases: (1) when  $x_A^e \leq K/(2H)$  and (2) when  $x_A^e > K/(2H)$ .

Assuming  $x_A = x_A^e < K/(2H)$ , the above simplifies to

$$V - x_A t - p_A^1 = \alpha(V - x_A t - p_A^{2H}) + (1 - \alpha)(V - x_A t - p_A^{2L}).$$

Assuming  $x_A = x_A^e > K/(2H)$  (and  $y_A^{L,e} L < K/2$ , i.e., no stockout in the low-demand state), the above simplifies to

$$\left( \alpha \frac{K/2}{x_A H} + (1 - \alpha) \right) (V - x_A t - p_A^1) = (1 - \alpha)(V - x_A t - p_A^{2L}).$$

We analyze the above cases separately in the following subsections. We consider firm  $A$  but the analysis for firm  $B$  is identical.

- $x_A = x_A^e < K/(2H)$

Suppose all consumers correctly believe that  $x_A = x_A^e < K/(2H)$ . In the first period, denote the price charged by firm  $A$  by  $p_A^1$ . In the second period, the firms know the state of demand to be high or low. Denote the prices charged by firm  $A$  in high- and low-demand states by  $p_A^{2H}$  and  $p_A^{2L}$ , respectively. Let the indifferent consumer be located at  $x_A$ . For this consumer, in the equilibrium and when expectations are consistent, we have

$$\begin{aligned} V - p_A^1 - t x_A &= \alpha(V - p_A^{2H} - t x_A) + (1 - \alpha)(V - p_A^{2L} - t x_A) \\ \Rightarrow p_A^1 &= \alpha p_A^{2H} + (1 - \alpha) p_A^{2L}. \end{aligned}$$

Now consider a consumer at  $x_A + \delta$  where  $\delta > 0$  and such that  $x_A + \delta < K/(2H)$ . This consumer has the same belief  $x_A^e$ , which is consistent. Moreover, for this consumer the net utility from buying a product in the first period is  $U_1 = V - p_A^1 - t(x_A + \delta)$ , and the net expected utility from waiting to buy in the second

period is  $U_2 = \alpha(V - p_A^{2H} - t(x_A + \delta)) + (1 - \alpha)(V - p_A^{2L} - t(x_A + \delta))$ . Using the fact that  $p_A^1 = \alpha p_A^{2H} + (1 - \alpha)p_A^{2L}$ , these utilities are equal. Hence, the consumer at  $x_A + \delta$  is also indifferent between buying in the first period or waiting to buy in the second period. This argument can be extended to any consumer in the range  $[0, K/(2H))$ , which means that the belief  $x_A^e$  is incorrect in the equilibrium. Hence, a rational expectations equilibrium with  $x_A = x_A^e < K/(2H)$  does not exist.

- $x^A = x_A^e > K/(2H)$

The goal of this section is to show that this equilibrium does not exist for all values of the parameters  $V, t, K, L, H$  and  $\alpha$ .

Consider the second period. Suppose demand is high. Then the firm stocks out in the first period, because the number of products is less than the demand in the first period  $x^A H > HK/(2H) = K/2$ . Now, suppose demand is low. We consider firm  $A$  and limit ourselves to the case when, even in low demand, it is a local monopoly and covers the line till  $y_A^L \leq 1/2$ . The firm charges a price  $p_A^{2L} = V - y_A^L t$  where  $y_A^L \leq 1/2$  and sells  $(y_A^H - x_A)L$  products to make a profit of  $\pi_A^{2L} = (V - y_A^L t)(y_A^L - x_A)H$ , which is maximized at  $y_A^L = (V + x_A t)/(2t)$ , with  $p_A^{2L} = (V - x_A t)/2$  and  $\pi_A^{2L} = (V - x_A t)^2 L/(4t)$ .

Next, consider the equation for the indifferent consumer. Under the assumption  $x_A^e \geq K/(2H)$  and  $y_A^{L,e} L < K/2$ , i.e., no stock out in the low-demand state, the indifference condition is

$$p_A^1 = \frac{\alpha K/(x_A^e H) + (1 - \alpha)}{\alpha K/(x_A^e H) + 2(1 - \alpha)}(V - x_A t).$$

Writing the expression for the total expected profit of firm  $A$  as

$$\pi_A = p_A^1(\alpha K/2 + (1 - \alpha)x_A L) + (1 - \alpha)\pi_A^{2L}$$

and differentiating w.r.t. to  $x_A$ , we obtain

$$x_A = \frac{\alpha K(VL(1 - \alpha) - \alpha K t - x_A^e H(1 - \alpha)t}{L(1 - \alpha)t(2H(1 - \alpha)x_A^e + 3K\alpha)}.$$

In the rational expectations equilibrium, we have  $x_A^e = x_A$ , which yields

$$x_A = x_A^e = \frac{\sqrt{\alpha K(\alpha K(H^2 + 9L^2 - 2HL) + 8(V/t)HL^2(1 - \alpha) - \alpha K(H + 3L))}}{4HL(1 - \alpha)}.$$

Note that we need the following conditions to hold:  $V/t \geq 1/2$ ,  $x_A \geq K/(2H)$  and  $y_A^L \leq 1/2 \Rightarrow V/t \leq 1 - x_A$ . This equilibrium does not always exist. For instance, when  $L = 1/2$ ,  $K = 1$ ,  $H = 3/2$ ,  $\alpha = 1/2$ ,  $t = 1$  and  $V = 2/3$ , the equilibrium does not hold.

- $x^A = x_A^e = K/(2H)$

Suppose all consumers correctly hold the belief that  $x^A = x_A^e = K/(2H)$ . Consider the indifferent consumer at  $x_A = K/(2H)$ . In the first period, irrespective of demand being high or low, this consumer can obtain a product at price  $p_A^1$ . In

the second period, if demand is high, no products are being sold. If demand is low, the consumer can obtain a product at price  $p_A^{2L}$ . For this consumer, we can therefore write

$$\begin{aligned} V - tx_A - p_A^1 &= (1 - \alpha)(V - tx_A - p_A^{2L}) \\ p_A^1 &= \alpha(V - Kt/(2H)) + (1 - \alpha)p_A^{2L}. \end{aligned}$$

Next, consider a consumer to the left of this indifferent consumer, at  $x_A - \varepsilon$ ,  $\varepsilon > 0$ , who holds the belief  $x_A = x_A^e = K/(2H)$ . In the first period, this consumer can obtain a product irrespective of high or low demand at price  $p_A^1$ , which gives him net utility  $U_1 = V - t(x_A - \varepsilon) - p_A^1$ . If the consumer waits for the second period, he can obtain a product only in the case of low demand at price  $p_A^{2L}$ . His net expected utility from waiting is  $U_2 = (1 - \alpha)(V - t(x_A - \varepsilon) - p_A^{2L})$ . Then,  $U_1 - U_2 = \alpha\varepsilon t > 0$ , which means that this consumer prefers to buy in the first period rather than wait.

Next, consider a consumer to the right of the indifferent consumer, at  $x_A + \varepsilon$ ,  $\varepsilon > 0$  who holds the belief  $x_A = x_A^e = K/(2H)$ . In the first period, this consumer can obtain a product if demand is high. (In the case of high demand,  $K/2$  products are being bought by  $K/(2H) \cdot H = K/2$  consumers and if this consumer wants to buy a product,  $K/2$  products will be bought by  $K/2 + \varepsilon'$  consumers, and he will obtain a product with probability  $\lim_{\varepsilon' \rightarrow 0} [(K/2)/(K/2 + \varepsilon')] = 1$ .) The consumer can also obtain a product in the low-demand state. In other words, he can obtain a product in the first period irrespective of high or low demand at price  $p_A^1$ , which gives him net utility  $U_1 = V - t(x_A + \varepsilon) - p_A^1$ . If he waits for the second period, he can obtain a product only in the case of low demand at price  $p_A^{2L}$ . His net expected utility from waiting is  $U_2 = (1 - \alpha)(V - t(x_A + \varepsilon) - p_A^{2L})$ . Then,  $U_1 - U_2 = -\alpha\varepsilon t < 0$ , which means that this consumer prefers to wait and buy in the second period.

Hence, a consumer to the left of the indifferent consumer prefers to buy in the first period, and a consumer to the right of the indifferent consumer prefers to wait for the second period, which is consistent with equilibrium beliefs. Hence, this equilibrium always exists. It now remains to characterize the equilibrium.

We first limit ourselves to the case in which, even in the low-demand state, each firm is a local monopoly. If demand is low, the firm charges a price  $p_A^{2L} = V - y_A^L t$  where  $y_A^L < 1/2$ , and sells  $(y_A^H - K/(2H))L$  products to make a profit of  $\pi_A^{2L} = (V - y_A^L t)(y_A^L - K/(2H))H$ . This profit is maximized at  $y_A^L = [V + Kt/(2H)]/(2t)$ , with  $p_A^{2L} = (V - Kt/(2H))/2$  and gives  $\pi_A^{2L} = [(V - Kt/(2H))^2/(4t)]L$ . Using  $p_A^1 = \alpha(V - Kt/(2H)t) + (1 - \alpha)p_A^{2L}$ , we obtain  $p_A^1 = ((1 + \alpha)/2)(V - Kt/(2H)t)$ . The firm's first-period profit is given by  $\pi_A^1 = p_A^1(\alpha H + (1 - \alpha)L)K/(2H)$ , and total profit is given by  $\pi_A = \pi_A^1 + \pi_A^{2L}$ . However, we need to impose  $y_A^{2L} \leq 1/2$ , which gives the restriction  $V/t < 1 - K/(2H)$ .

For  $1 - K/(2H) \leq V/t < 3/2$ ,  $y_A^{2L} = 1/2$ ,  $p_A^{2L} = V - t/2$ ,  $p_A^1 = \alpha(V - Kt/(2H)) + (1 - \alpha)(V - t/2)$ ,  $\pi_A^{2L} = (1 - \alpha)(V - t/2)(1 - K/H)L/2$  and  $\pi_A^1 = p_A^1(\alpha H + (1 - \alpha)L)K/(2H)$ . This is the non-competitive equilibrium with each firm covering exactly half the line.

For  $V/t \geq 3/2$ ,  $y_A^{2L} = 1/2$ ,  $p_A^{2L} = t$ ,  $p_A^1 = \alpha(V - Kt/(2H)) + (1 - \alpha)t$ ,  $\pi_A^{2L} = (1 - \alpha)t(1 - K/H)L/2$  and  $\pi_A^1 = p_A^1(\alpha H + (1 - \alpha)L)K/(2H)$ . This is the competitive equilibrium with each firm covering half the line. This completely characterizes the equilibrium for all values of  $V/t$ .

## 7.2 Proof of Proposition 6

*Proof.* We characterize the equilibrium for the case  $\delta = 1$ ; the intuition remains similar for all  $\delta \in [0, 1]$ , because changing  $\delta$  only changes the profits transferred from the opaque intermediary to the firms.

The analysis below is for  $\gamma_A^{H,e} = \gamma_A^{L,e} = 1/2$ , which are rational expectations in equilibrium. To see why this is the case, assume that  $\gamma^{i,e} = 1/2$ . Consider the case when demand is low and consumers purchase in the opaque channel. Suppose that the intermediary has the market  $(x_A, x_B)$  available to it (and enough capacity to fulfil this demand) and offers a price  $p_I^L$ . For any customer at  $x \in (x_A, x_B)$ , the ex ante surplus from purchasing an opaque ticket is  $V - p_I^L - \gamma_A^{L,e}tx - \gamma_B^{L,e}t(1 - x) = V - p_I^L - t/2$ . This is independent of position  $x$ , or, stated differently, all consumers have the same ex ante utility from purchasing an opaque ticket. The intermediary will then price at  $p_I^L = V - t/2$ , since at this price all consumers will purchase.

Now, consider the case when demand is high and consumers purchase in the opaque channel. Suppose that the intermediary has the market  $(x_A, x_B)$  available to it (but not enough capacity to meet all this demand, so that a consumer who wants to purchase an opaque ticket will only get it with probability  $\beta$ ) and offers a price  $p_I^H$ . For any customer at  $x \in (x_A, x_B)$ , the ex ante surplus from an opaque ticket is

$$\beta(V - p_I^L - \gamma_A^{L,e}tx - \gamma_B^{L,e}t(1 - x)) = \beta(V - p_I^L - t/2).$$

Once again, the intermediary will then price at  $p_I^H = V - t/2$ , since at this price all consumers will want to purchase (while only a fraction  $\beta$  of them will actually get tickets).

Now, using these expressions, we solve for firm prices  $p_A$  and  $p_B$  in the first period. Finally, to confirm that  $\gamma^{i,e} = 1/2$  are equilibrium expectations, in each of the cases below we will confirm that the realized probabilities of availability,  $\gamma$ , are all equal to  $1/2$ .

- $1/2 \leq V/t \leq K/H$

The firms will not stock out even when demand is high. As in the deterministic demand case, consumers will not let the firms leverage the opaque channel to increase first-period prices, so that

$$\begin{aligned}
\pi_A &= p_A x_A (\alpha H + (1 - \alpha)L) \\
&\quad + \delta \gamma_A \left( \alpha p_I^H \left( \frac{K}{2} - x_A H + \frac{K}{2} - (1 - x_B)H \right) + (1 - \alpha) p_I^L (x_B - x_A)L \right), \\
\pi_B &= p_B (1 - x_B) (\alpha H + (1 - \alpha)L) \\
&\quad + \delta \gamma_B \left( \alpha p_I^H \left( \frac{K}{2} - x_A H + \frac{K}{2} - (1 - x_B)H \right) + (1 - \alpha) p_I^L (x_B - x_A)L \right) \\
\pi_I &= (1 - \delta) \left( \alpha p_I^H \left( \frac{K}{2} - x_A H + \frac{K}{2} - (1 - x_B)H \right) + (1 - \alpha) p_I^L (x_B - x_A)L \right).
\end{aligned}$$

The profit  $\pi_i$  is shared by firms  $A$  and  $B$  in proportion to the products sold by each. The no-stockout situation implies  $x_A \leq K/(2H)$  and  $x_B \geq 1 - K/(2H)$ . Firm  $A$  sets  $p_A = V - x_A t$  and firm  $B$  sets  $p_B = V - (1 - x_B)t$ . Optimizing the above two expressions w.r.t.  $x_A$  and  $x_B$ , we obtain  $x_A = 1 - x_B = V/(2t)$  and  $p_A = p_B = V/2$ . After imposing  $x_A \leq K/(2H)$  we obtain  $V/t \leq K/H$ , which we have already assumed. Moreover,  $1/2 \leq V/t \Rightarrow 1/2 \leq K/H \Rightarrow K/H \geq 1/2$ , which is required, but is a mild assumption.<sup>6</sup>

Using these values of  $x_A$  and  $x_B$ , we obtain

$$\begin{aligned}
\gamma_A^{L, \text{realized}} &= \frac{K/2 - x_A L}{K/2 - x_A L + K/2 - (1 - x_B)L} = \frac{1}{2} \quad \text{and} \\
\gamma_A^{H, \text{realized}} &= \frac{K/2 - x_A H}{K/2 - x_A H + K/2 - (1 - x_B)H} = \frac{1}{2}.
\end{aligned}$$

This confirms that  $\gamma_i^{e} = 1/2$  are indeed equilibrium expectations.

If demand is high, the capacity sold in the opaque channel is the leftover from the transparent channel which is  $K - VH/(2t)$ , and if demand is low, the amount sold in the opaque channel is  $L - VL/(2t)$ . The prices in the opaque channel are  $p_I^H = p_I^L = V - t/2$  and the profits are  $\pi_I^H = (V/t - 1/2)(K - VH/(2t))t$  and  $\pi_I^L = (V/t - 1/2)(1 - V/(2t))Lt$ . In equilibrium, half of the above profits will be transferred to each firm. Hence, the profits for firms  $A$  and  $B$  are:

$$\begin{aligned}
\pi_A = \pi_B &= \frac{V^2}{4t} (\alpha H + (1 - \alpha)L) \\
&\quad + \frac{1}{2} \left( \frac{V}{t} - \frac{1}{2} \right) \left( \alpha \left( K - \frac{V}{2t} H \right) + (1 - \alpha) \left( 1 - \frac{V}{2t} \right) L \right) t.
\end{aligned}$$

In the cases that follow, we construct the equilibria and it can be shown using an  $\varepsilon$ -deviation argument that these are indeed equilibria. Further, we do not explicitly show that  $\gamma_i^{\text{realized}} = \gamma_i^e$  in all these cases; the reader can, however, check that this always holds.

<sup>6</sup> This condition is required because we assume that  $V/t > 1/2$  to ensure that if tickets are free, consumers located farthest from both firms (at  $1/2$ ) have positive utility for them.

- $\frac{K}{H} < \frac{V}{t} \leq \frac{K}{H} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}$

Here, we construct the equilibrium as follows: both firms charge  $p_A = p_B = V - Kt/(2H)$  and cover exactly  $x_A = 1 - x_B = K/(2H)$  in both high and low demand. In high demand there is no leftover for the opaque channel, while in low demand the total leftover is  $(x_B - x_A)L = (1 - K/H)L$ . In high demand the opaque channel profit is zero (since nothing is left over to be allocated to it). In low demand the opaque channel price is  $p_I^L = V - t/2$  and the profit is  $\pi_I = (V/t - 1/2)(1 - K/H)Lt$ . The profit for firms  $A$  and  $B$  is therefore

$$\begin{aligned} \pi_A = \pi_B &= \left(V - \frac{K}{2H}t\right) \left(\alpha \frac{K}{2} + (1-\alpha) \frac{K}{2H}L\right) \\ &\quad + \frac{1}{2}(1-\alpha) \left(\frac{V}{t} - \frac{1}{2}\right) \left(1 - \frac{K}{H}\right) Lt. \end{aligned}$$

- $\frac{K}{H} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L} < \frac{V}{t} < 1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}$

Here, we construct the equilibrium as follows: both firms charge

$$p_A = p_B = \frac{V}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}t$$

and cover exactly  $K/(2H)$  when demand is high and

$$\frac{V}{2t} - \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}$$

when demand is low. (For  $V/t < 1 + (\alpha/(1-\alpha))K/(2L)$  this is  $\leq 1/2$ .) In the high-demand state there is no leftover capacity for the opaque channel, while in the low-demand state the total uncovered market is

$$1 - \frac{V}{t} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}.$$

Thus, in the high-demand state the profit from opaque channel is zero. In the low-demand state the opaque channel price is  $p_I^L = V - t/2$  and the profit is

$$\pi_I = \left(\frac{V}{t} - \frac{1}{2}\right) \left(1 - \frac{V}{t} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}\right) Lt.$$

The profit for firms  $A$  and  $B$  is therefore

$$\begin{aligned} \pi_A = \pi_B &= \left(\frac{V}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}t\right) \left(\alpha \frac{K}{2} + (1-\alpha) \left(\frac{V}{2t} - \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}\right)\right) \\ &\quad + \frac{1}{2}(1-\alpha) \left(\frac{V}{t} - \frac{1}{2}\right) \left(1 - \frac{V}{t} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}\right) Lt. \end{aligned}$$

- $1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L} \leq \frac{V}{t} < \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}$

We construct the equilibrium as follows: both firms charge  $p_A = p_B = V - t/2$  and cover exactly  $K/(2H)$  in the high-demand state and  $1/2$  in the low-demand state. In both high- and low-demand cases, there is no leftover for the opaque channel and the opaque channel profit is zero. The profit for firms  $A$  and  $B$  is therefore

$$\pi_A = \pi_B = \left(\frac{V}{t} - \frac{1}{2}\right) \left(\alpha \frac{K}{2} + (1-\alpha) \frac{L}{2}\right) t.$$

- $\frac{V}{t} \geq \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}$

We begin by assuming that the firms cover  $K/(2H)$  each in the high-demand state but are in the “competitive equilibrium” in the low-demand state. Thus, for prices  $p_A$  and  $p_B$ ,  $x_A = 1/2 + (p_B - p_A)/(2t)$  in the low-demand state. In either demand state, nothing is left over for the opaque channel. Thus, the firms’ profits are

$$\begin{aligned} \pi_A &= p_A \left( \alpha \frac{K}{2} + (1-\alpha) \left( \frac{1}{2} + \frac{p_B - p_A}{2t} \right) \right) \quad \text{and} \\ \pi_B &= p_B \left( \alpha \frac{K}{2} + (1-\alpha) \left( \frac{1}{2} + \frac{p_A - p_B}{2t} \right) \right). \end{aligned}$$

Optimizing the above expressions simultaneously w.r.t.  $p_A$  and  $p_B$ , we obtain  $p_A = p_B = (1 + (\alpha/(1-\alpha))(K/L))t$ ,  $x_A = x_B = 1/2$ , and the optimal profits are

$$\pi_A = \pi_B = \left(1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}\right) \left(\alpha \frac{K}{2} + (1-\alpha) \frac{L}{2}\right) t.$$

For the equilibrium to exist, the consumer located at  $1/2$  should have a non-negative utility. Mathematically,

$$V - p_A - \frac{t}{2} \geq 0 \Rightarrow \frac{V}{t} \geq \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L},$$

which we have already assumed.

The above analysis characterizes the equilibria for the full range of  $V/t \geq 1/2$ .

## References

1. Besanko D, Winston WL (1990) Optimal price skimming by a monopolist facing rational consumers. *Management Science* 36(5):555–567
2. Cho M, Fan M, Zhu Y-P (2008) An empirical study of revenue management practices in the airline industry. University of Washington, working paper
3. Coase RH (1972) Durability and monopoly. *Journal of Law and Economics* 15(1):143–149
4. DeGraba P (1995) Buying frenzies and seller-induced excess demand. *Rand Journal of Economics* 26(2):331–342



5. Deneckere RJ, McAfee RP (1996) Damaged goods. *Journal of Economics & Management Strategy* 5(2):149–174
6. Easy.com (2008) Portal site for easyGroup companies
7. Fay S (2008) Selling an opaque product through an intermediary: a case of disguising one's product. *Journal of Retailing* 84(1):59–75
8. Fay S, Xie J (2008) Probabilistic goods: a creative way of selling products and services. *Marketing Science* 27(4):674–690
9. Harrison J (2006) Traveling 'blind'. *The Ottawa Citizen*, September 26
10. Jerath K, Netessine S, Veeraraghavan SK (2008) Revenue management with strategic customers: last-minute selling and opaque selling. Carnegie Mellon University, working paper
11. Jiang Y (2007) Price discrimination with opaque products. *Journal of Revenue and Pricing Management* 6(2):118–134
12. Koenigsberg O, Muller E, Vilcassim NJ (2008) EasyJet pricing strategy: Should low-fare airlines offer last-minute deals? *Quantitative Marketing and Economics* 6(3):279–297
13. Moorthy KS (1984) Market segmentation, self selection, and product-line design. *Marketing Science* 3(4):288–307
14. Muth JF (1961) Rational expectations and the theory of price movements. *Econometrica* 29(3):315–335
15. Narasimhan C (1984) A price discrimination theory of coupons. *Marketing Science* 3(2):128–147
16. Phillips RL (2005) Pricing and revenue optimization. Stanford Business Books, Stanford, CA
17. Shapiro D, Shi X (2008) Market segmentation: The role of opaque travel agencies. University of North Carolina, working paper
18. SITA (2007) Airline IT Trends Survey 2007  
[http://www.sita.aero/News\\_Centre/Industry\\_surveys\\_and\\_trends/default/default.htm](http://www.sita.aero/News_Centre/Industry_surveys_and_trends/default/default.htm)
19. Smith BC, Darrow R, Elieson J, Gunther D, Rao BV, Zouaoui F (2007) Travelocity becomes a travel retailer. *Interfaces* 27(1):68–81
20. Stokey N (1979) Intertemporal price discrimination. *Quarterly Journal of Economics* 93(3):355–371
21. Sviokla J (2004) Value poaching. *Conference Board Review*, June
22. Talluri K, van Ryzin G (2004) The theory and practice of revenue management. Kluwer Academic Press, Boston, MA
23. Tedeschi B (2005) Online travel industry tries to adjust to maturity. *New York Times*, May 30
24. Terwiesch C, Savin S, Hann I (2005) Online haggling at a name-your-own-price retailer: theory and application. *Management Science* 51(3):339–351
25. Varian H (2000) Versioning information goods. In: *Internet publishing and beyond: The economics of digital information and intellectual property*. MIT Press, Cambridge, MA. 190–202
26. Xie J, Shugan SM (2001) Electronic products, smart cards and online prepayments: When and how to advance sell. *Marketing Science* 20(3):219–243