12-2012

Gray-Space Spectrum Sharing with Cellular Systems and Radars, and Policy Implications

Rathapon Saruthirathanaworakun
Carnegie Mellon University, rsaruthi@andrew.cmu.edu

Follow this and additional works at: http://repository.cmu.edu/dissertations
Part of the Engineering Commons, and the Public Policy Commons

Recommended Citation

This Dissertation is brought to you for free and open access by the Theses and Dissertations at Research Showcase @ CMU. It has been accepted for inclusion in Dissertations by an authorized administrator of Research Showcase @ CMU. For more information, please contact research-showcase@andrew.cmu.edu.
Gray-Space Spectrum Sharing with Cellular Systems and Radars, and Policy Implications

Submitted in partial fulfilment of the requirements for
the degree of
Doctor of Philosophy
In
Engineering and Public Policy

Rathapon Saruthirathanaworakun

B.Eng., Electrical Engineering, Chulalongkorn University
M.Eng., Electrical Engineering, Chulalongkorn University

Carnegie Mellon University
Pittsburgh, PA

December, 2012
Gray-Space Spectrum Sharing with Cellular Systems and Radars, and Policy Implications

Rathapon Saruthirathanaworakun

Co-Supervisors: Doctor Jon M. Peha
Doctor Luis M. Correia

Thesis approved in public session to obtain the PhD degree in Engineering and Public Policy

Jury Final Classification:

Jury
Chair: Doctor Jon M. Peha

Members of the Committee:
Doctor Luis M. Correia
Doctor Marvin Sirbu
Doctor António A. Moreira
Doctor Taieb Znati

2012
To my family, teachers, friends, and colleagues:

Thanks for shaping me into who I am today.
This page is intentionally left blank.
Acknowledgements

I would like to acknowledge the Fundação para a Ciência e a Tecnologia, i.e., Portuguese Foundation for Science and Technology, for the financial support provided through the Carnegie Mellon - Portugal Program under Grant SFRH/ BD/33594/2008.

While working toward my Ph.D., I have received the greatest help, dedication, patience, and guidance from my two advisors: Professor Jon M. Peha of Carnegie Mellon University (CMU), and Professor Luis M. Correia of Instituto Superior Técnico (IST). I would also like to thank for suggestion and feedback from my Ph.D. committee members: Professor Marvin Sirbu of CMU, Professor António A. Moreira of IST, and Professor Taieb Znati of the University of Pittsburgh.

I would like to express my gratitude for the help and support that Ana Mateus, Victoria Finney, Barbara Bugosh, Patti Steranchak, Adam Loucks, and the rest of EPP and CMU-Portugal administrative staff have given me.

Without great friends and colleagues, being thousands of kilometres away from my home country would be much more difficult. My deepest appreciation goes to Engineer and Public Policy (EPP) students, both in Pittsburgh and Lisbon, members and staffs of the Group for Research on Wireless (GROW) at IST, and other friends for their invaluable inspiration and friendship. To mention some: Aweewan Mangmeechai, Anny Huang, Ivonne Peña, Caitlin Forschner, Alexandre M. Mateus, Qiwei Han, Rodrigo Belo, Moinul Zaber, Patrick Agyapong, Miguel Godinho de Matos, Ryan Turner, Mohammed Alotaibi, João Madeira and his family, Ema Catarré, Vera de Almeida, Mónica A. Branco, Carla Oliveira, Daniel Sebastião, Lucio S. Ferreira, Sina Khatibi, Michal Mackowiak, Nitin Shukla, Darya Snihirova, Andrea Mazzolari, Tiago Sá Reis, Feng Zhou, Liu Tomin, Pham Tien Cuong, and Wang Lei.

I would also like to thank Pralom Boonrussamee, Kitsanaphong Supsenee, Paisarn Sonthikorn, Chatklaw Jareanpon, Nutnath Jiraramanytwesin, Sirima Mayusiri, Natnaree Kanjanawattana, Jayakorn Vongkulbhisal and the rest of Thai communities in Pittsburgh, Lisbon, and Oporto for making the US and Portugal be like my second home. I am also grateful to Rachada Danponchareon, Suparat Chuechte, Wantane Viriyasitavat, and Pra-ornsiri Boonsue for their kind support and encouragement.

Last but not least, I would like to thank my parents Jaral and Pornthip Sarutiratanaworakun, my brother Rattapong, my sister Ritarot, and the rest of my family members for their never-ending care, support, and love.
This page is intentionally left blank.
Abstract

This dissertation considers gray-space primary-secondary spectrum sharing, in which secondary devices are allowed to transmit when primary transmissions are strong enough that additional interference would be tolerable, rather than when the primary transmissions are weak or absent so the spectrum is considered unused, as occurs in white-space sharing. To avoid causing harmful interference (i.e., interference causing disruptions in services), transmit power of a secondary device is dynamically adjusted. Various novel sharing mechanisms are proposed for two different types of primary system: cellular systems, and rotating radars. Both cases when primary and secondary systems cooperate (cooperative sharing), and when they do not (coexistent sharing) are considered. Based on analyses and extensive Monte Carlo simulations, this dissertation shows that useful secondary transmissions are possible even when the shared spectrum is considered 100% utilized by the primary system under conventional approaches to spectrum management. For example, in spectrum sharing with cellular systems, even when the primary system is 100% utilized, a modest extent of transmissions of around 0.01–0.03 bps/Hz is achievable for secondary transmitter and receiver that are 400 m apart. In spectrum sharing with radars, even in the scenario where radars are the most densely packed, a secondary transmitter can get almost 1.2 bps/Hz on average, when 5% of the transmitters are competing for the shared spectrum. This dissertation also shows the potential of sharing models in which a secondary system has information about a primary system, but does not cooperate in real time; such arrangements are not typically considered today. For sharing with radars, the case in which an OFDMA-based cellular system operates as the secondary spectrum-user in non-contiguous cells is considered, as might occur with a broadband hotspot service, or a cellular system using shared spectrum to supplement its dedicated spectrum. It is found that even with fluctuations and interruptions in secondary transmissions while radars rotate, the shared spectrum could be used efficiently for applications that generate much of the traffic on mobile Internet, including non-interactive video on demand, peer-to-peer file sharing, large file transfers, and web browsing, but not for applications such as real-time transfers of small files, and VoIP. For sharing with cellular systems, the efficiency of cooperative and coexistent sharing is compared based on performance of the secondary system measured as achievable transmissions, and performance of the cellular system measured as power consumption of a mobile device, which may be increased to compensate for additional interference from secondary transmissions. When both achievable secondary transmissions and primary power consumption are of concern, coexistent sharing is found to be as effective as cooperative sharing.

Keywords

Primary-secondary spectrum sharing; Coexistent; Cooperative; Gray-space; Cellular; Radars
This page is intentionally left blank.
# Table of Contents

Acknowledgements ........................................................................................................... v  
Abstract .............................................................................................................................. vii  
Table of Contents .............................................................................................................. ix  
List of Figures .................................................................................................................... xiii  
List of Tables ..................................................................................................................... xvi  
List of Acronyms ............................................................................................................. xvii  
List of Symbols ................................................................................................................. xviii  

1. Introduction .................................................................................................................. 1  
   1.1 Spectrum Scarcity and Alleviation ........................................................................... 2  
   1.2 Motivations and Research Overview ...................................................................... 3  
      1.2.1 Spectrum Sharing with a Cellular System ....................................................... 4  
      1.2.2 Spectrum Sharing with Radars ....................................................................... 5  
   1.3 Contributions and Research Methodology ............................................................. 6  
   1.4 Dissertation Organization ..................................................................................... 6  

2. Sharing with a Cellular ............................................................................................... 7  
   2.1 Link Loss Overview ............................................................................................... 8  
   2.2 Related Work on Spectrum Sharing with a Cellular System ................................. 9  
   2.3 Related Work on Spectrum Sharing with Rotating Radars .................................. 10  

3. Sharing with a Cellular ............................................................................................... 11  
   3.1 Overview ............................................................................................................... 12  
   3.2 Research Questions ............................................................................................... 13  
   3.3 Sharing Scenarios ................................................................................................. 14  
   3.4 Basic Approach to Control Transmit Power of a Secondary System to Prevent Harmful Interference .............................................................................................................. 15  
   3.5 Cooperative Sharing............................................................................................... 17
3.6 Coexistent Sharing ................................................................. 18
3.6.1 Estimation of Primary Downstream Transmit Power \( (PB) \) \( \) ........................................ 18
3.6.2 Estimation of Total Tolerable Interference Level \( (TTIL, \text{ i.e., } IT) \) .................. 19
3.7 Numerical Results ................................................................. 22
3.7.1 Assumptions for Numerical Results ........................................ 22
3.7.2 Extent of Secondary Transmissions and Performance of STIL Estimation ...... 24
3.7.3 Effect of the Spectrum Sharing on Transmit Power of a Primary User ........ 26
3.7.4 Comparison of Performance of Cooperative Sharing vs. Performance of Coexistent Sharing ..................................................... 27
3.7.5 Sensitivity Analyses .............................................................. 29
3.7.5.1 Variation of Intercell Interference at a Base Station ....................... 30
3.7.5.2 Allocated Spectrum ......................................................... 30
3.7.5.3 Channel Bandwidth .......................................................... 31
3.7.5.4 Cell Radius ................................................................. 31
3.8 Conclusions ............................................................................. 33
4. Sharing between a Single Cell and a Single Rotating Radar ...... 35
4.1 Overview and Research Questions ............................................. 36
4.2 Sharing Scenarios .................................................................... 37
4.3 Maximum Allowable Transmit Power of a Secondary System and Achievable SINR ......................................................................... 39
4.4 Performance Measurement ...................................................... 41
4.5 Numerical Results ................................................................... 43
4.5.1 Assumptions for Numerical Results ........................................ 44
4.5.2 Overall Performance of Secondary Transmissions ....................... 45
4.5.3 Performance of Non-Interactive Video Streaming ....................... 48
4.5.4 Performance of Voice over IP ................................................. 49
4.5.5 Performance of File upload- and Downloading Applications .......... 49
4.5.5.1 Fluctuations in Secondary Perceived Data Rate ....................... 50
4.5.5.2 Implications of the Fluctuations for Various Applications .......... 51
4.5.6 Sensitivity Analysis .............................................................. 52
4.5.6.1 Sensitivity of Average Extent of Secondary Transmissions ........ 52
4.5.6.2 Sensitivity of Fluctuations in Secondary Perceived Data Rate ........ 55
4.6 Conclusions ............................................................................. 56
5. Sharing between Multiple Cells and a Single Rotating Radar ..... 61
5.1 Overview and Research Questions ............................................. 62
5.2 Sharing Scenarios ............................................................... 63
5.3 Controlling Transmit Power of a Secondary System to Prevent Harmful Interference ............................................................ 64
  5.3.1 Basic Approach ............................................................... 64
  5.3.2 Regional Resource Allocation ........................................ 64
  5.3.3 Local Power Control ......................................................... 66
5.4 Numerical Results ............................................................. 67
  5.4.1 Assumptions for Numerical Results .................................. 67
  5.4.2 Extent of Secondary Transmissions ................................... 69
  5.4.3 Fluctuations in Perceived Data Rate and Implications on Performance Experienced by Various Applications .................................. 71
5.5 Conclusions ..................................................................... 72
6. Sharing between Multiple Cells and Multiple Rotating Radars .... 75
  6.1 Overview and Research Questions ....................................... 76
  6.2 Sharing Scenarios ............................................................... 77
  6.3 Controlling Transmit Power of a Secondary System to Prevent Harmful Interference ............................................................ 78
  6.4 Performance Measurement ................................................ 79
  6.5 Numerical Results ............................................................... 81
    6.5.1 Assumptions for Numerical Results .................................. 81
    6.5.2 Extent of Secondary Transmissions ................................... 84
    6.5.3 Fluctuations in Perceived Data Rate and Performance of the Sharing as Perceived by Various Applications .................................. 87
    6.5.4 Sensitivity Analyses ........................................................ 88
      6.5.4.1 Sensitivity of Average Extent of Secondary Transmissions .................................. 89
      6.5.4.2 Sensitivity of Fluctuations in Secondary Perceived Data Rate ........................... 94
  6.6 Conclusions ..................................................................... 98
7. Conclusions, Policy Implications, and Future Work ............ 101
  7.1 Contributions ................................................................ 102
  7.2 Conclusions and Policy Implications ................................... 102
  7.3 Policy Issues Associated with Gray-Space Sharing .............. 105
  7.4 Future Work .................................................................... 107
Appendix A. Equation Derivations ........................................... 111
Appendix B. Data Rate Derived from 3GPP Data Regression ....... 115
Appendix C. Figures Omitted from Detailed Discussions in the
Dissertation ................................................................. 119
Appendix D. Equation Derivations................................. 133
References......................................................................... 137
List of Figures

Figure 3.1 Mean STIL, with 95% confidence interval, vs. Number of active (primary) mobile terminals per channel .........................................................24
Figure 3.2 Mean Secondary Tolerable Interference Level (STIL) vs. Primary utilization .................................25
Figure 3.3 Mean secondary transmission rate vs. Primary utilization, when the secondary device cause 100% STIL ........................................................................26
Figure 3.4 Mean transmit power of a mobile terminal vs. Primary utilization, when the secondary device cause 100% STIL ........................................................................26
Figure 3.5 Trade-off between Extent of secondary transmissions vs. Mean transmit power of a mobile terminal, secondary transmissions are limited using the %STIL approach ................................................................................28
Figure 3.6 Mean derivative of secondary transmission rate with respect to mean transmit power of a mobile terminal vs. %STIL, the results are from coexistent spectrum sharing ................................................................................29
Figure 3.7 Mean secondary transmission rate vs. Primary utilization, when primary utilization in all cells are the same vs. when primary utilization in all neighboring cells = 100% ................................................................................30
Figure 3.8 Mean secondary transmission rate vs. Amount of spectrum allocated to the primary system, when the secondary device cause 100% STIL, and the primary utilization is at 10% ................................................................................31
Figure 3.9 Mean secondary transmission rate vs. Primary channel bandwidth, when the secondary device cause 100% STIL, and the primary utilization is at 10% .................................................................................32
Figure 3.10 Mean secondary transmission rate vs. Cell radius, when the secondary device cause 100% STIL, and the primary utilization is at 10% ................................................................................32
Figure 4.1 Example of radiation pattern of a radar antenna, the antenna is theoretically uniformly distributed aperture type, adapted from Chapter 7 of [Skol01] ................................................................................36
Figure 4.2 Sharing Scenario: One cell of a secondary system is sharing spectrum with one radar ..................................................................................................................................................38
Figure 4.3 Extent of secondary transmissions vs. Distance between a base station and a radar ..................................................................................................................................................47
Figure 4.4 Possibility that a VoIP latency is less an acceptable Level vs. Distance between a base station and a radar ......................................................................................................................................49
Figure 4.5 Perceived downstream data rate vs. Size of a file being transferred .................................................................................................................................................................50
Figure 4.6 First percentile, and mean perceived downstream data rate vs. distance between a base station and a radar ......................................................................................................................................52
Figure 4.7 Sensitivity of extent of transmissions on cell radius .................................................................................53
Figure 4.8 Sensitivity of average data rate on radar transmit power .................................................................................................................................................................54
Figure 4.9 Sensitivity of average data rate on radar tolerable interference level .................................................................................................................................................................55
Figure 4.10 Sensitivity of perceived secondary data rate on secondary cell radius ........................................................................................................................................................................57
Figure 4.11 Sensitivity of perceived secondary data rate in the upstream ........................................................................................................................................................................58
Figure 5.1 Scenario used to obtain numerical results, when multiple cells share spectrum with one radar ..................................................................................................................................................68
Figure 5.2 Mean downstream data rate per active cell with 95% confidence interval vs. Distance between a base station and the radar ......................................................................................................................................70
Figure 5.3 Mean downstream data rate per active cell with 95% confidence interval vs.
Distance between a base station and the radar, for different system rate limits............70
Figure 5.4 The first percentile of perceived data rate vs. Average perceived data rate, the
user is at the cell edge closest to the radar.........................................................71
Figure 6.1 Layout of a given radar in relative to its six adjacent neighbours, and area that
secondary cells can be deployed on an infinite plane...........................................81
Figure 6.2 Extent of secondary transmissions with 95% confidence interval vs. Distance
between a base station and its nearest radar.........................................................85
Figure 6.3 Extent of secondary transmissions with 95% confidence interval vs. Percentage
of active cells ...........................................................................................................86
Figure 6.4 The first percentile perceived data rate with 95% confidence interval vs. Mean
perceived data rate ..................................................................................................87
Figure 6.5 Sensitivity of extent of transmissions on distance between adjacent radars
\((dRd)\), 95% confidence interval is within \(\pm 1\%\) of the results shown .......................90
Figure 6.6 Sensitivity of extent of transmissions on cell radius of the secondary system,
95% confidence interval is within \(\pm 1\%\) of the results shown ..................................91
Figure 6.7 Sensitivity of extent of transmissions on distance between a radar and its
nearest cell \((dmin)\), 95% confidence interval is within \(\pm 1\%\) of the results shown ......92
Figure 6.8 Sensitivity of extent of transmissions on radar maximum tolerable Interference
to Noise Ratio (INR), 95% confidence interval is within \(\pm 1\%\) of the results shown ....93
Figure 6.9 Sensitivity of fluctuations in downstream perceived data rate on cell radius of
the secondary system, 95% confidence interval is within \(\pm 9\%\) of the results shown............................95
Figure 6.10 Sensitivity of fluctuations in downstream perceived data rate on radar rotating
period, 95% confidence interval is within \(\pm 10\%\) of the results shown .........................95
Figure 6.11 Sensitivity of fluctuations in downstream perceived data rate on radar transmit
power, 95% confidence interval is within \(\pm 11\%\) of the results shown .......................96
Figure 6.12 Sensitivity of fluctuations in downstream perceived data rate................................96

Figure B.1 Data rate vs. SINR from the regressions, and from the approximation that data
rate is a fraction of Shannon’s limit, assuming that bandwidth = 3 MHz ..................117
Figure C.1 Comparison of the four approaches used to limit transmission of a secondary
system, discussed in Section 3.7.4 Comparison of Performance of Cooperative
Sharing vs. Performance of Coexistent Sharing..........................................................120
Figure C.2 Mean derivative of secondary transmission rate with respect to mean transmit
power of a mobile terminal vs. %STIL; the results are from cooperative sharing, omitted from Section 3.7.4 Comparison of Performance of Cooperative Sharing vs. Performance of Coexistent Sharing .................................................................121
Figure C.3 Perceived downstream data rate vs. Size of a file being transferred, omitted from Section 4.5.5.1 Fluctuations in Secondary Perceived Data Rate........................122
Figure C.4 Sensitivity of average data rate on radar transmit power, omitted from Section
4.5.6.1 Sensitivity of Average Extent of Secondary Transmissions ..............................122
Figure C.5 Sensitivity of average data rate on radar tolerable interference level, omitted from Section 4.5.6.1 Sensitivity of Average Extent of Secondary Transmissions......123
Figure C.6 Sensitivity of perceived secondary data rate in the downstream, omitted from Section 4.5.6.2 Sensitivity of Fluctuations in Secondary Perceived Data Rate........123
Figure C.7 Percentage of time that a base station can transmit vs. Radar maximum
tolerable Interference to Noise Ratio (INR), 95% confidence interval is within
\(\pm 1\%\) of the results shown, omitted from Section 6.5.4.1 Sensitivity of Average Extent of Secondary Transmissions .................................................................125
Figure C.8 Sensitivity of extent of transmissions on radar transmit power, 95% confidence
interval is within \(\pm 1\%\) of the results shown, omitted from Section 6.5.4.1
Sensitivity of Average Extent of Secondary Transmissions ........................................125
Figure C.9  Effect of the six system parameters on fluctuations in perceived downstream data rate, when 20% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.9(a) in which a BS is at 50% of distance between two radars from its nearest radar, in the other figures, a BS is 140 km away from its nearest radar .................................................................126

Figure C.10  Effect of the six system parameters on fluctuations in perceived downstream data rate, when 4% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.10(a) in which a BS is at 36% of distance between two radars from its nearest radar, and Fig. C.10(c) in which a BS is at 120 km from its nearest radar, in the other figures, a BS is 100 km away from its nearest radar .........................................................................................129
List of Tables

Table 3.1 Parameters for Base-Case Numerical Results for Spectrum Sharing with Cellular Systems ...............................................................23
Table 4.1 Parameters for Base-Case Numerical Results for Spectrum Sharing with Radars (Unless stated otherwise.) .................................................45
Table 4.2 Ranges of Parameters Considered for Sensitivity Analysis for Sharing with Radars ........................................................................47
Table 5.1 Additional Parameters for Numerical Results When Multiple Cells Share Spectrum with One Radar, Extended from Table 4.1 (Unless stated otherwise.) ....68
Table 6.1 Additional Parameters for Base-Case Numerical Results When Multiple Cells Share Spectrum with Multiple Radars, Extended from Table 4.1 (Unless stated otherwise.) .......................................................................83
Table 6.2 Ranges of Additional Parameters Considered for Sensitivity Analysis When Multiple Cells Share Spectrum with Multiple Radars, Extended from Table 4.2 ........83
List of Acronyms

ATC  Air Traffic Control
BS   Base Station
CLT  Central Limit Theorem
DFS  Dynamic Frequency Selection
DS-CDMA  Direct-Sequence Code Division Multiple Access
EPA  Extended Pedestrian A
FAA  Federal Aviation Administration
FCC  Federal Communications Commission
FDD  Frequency Division Duplex
LoS  Line of Sight
LTE  Long Term Evolution
MIMO Multiple input Multiple Output
MT   Mobile Terminal
NTIA  National Telecommunications and Information Administration
PC   Personal Computer
QAM  Quadrature Amplitude Modulation
QPSK Quadrature Phase Shift Keying
Ofcom Office of Communications
OFDMA Orthogonal Frequency Division Multiple Access
P2P  Peer to Peer File Sharing
QoS  Quality of Service
SINR  Signal to Interference plus Noise Ratio
STIL  Secondary Tolerable Interference Level
TDD  Time Division Duplex
TTL  Total Tolerable Interference Level
UB   Upper Bound
VoIP  Voice over IP
## List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Probability that a cell will be active</td>
</tr>
<tr>
<td>$d_{min}$</td>
<td>Minimum distance between a radar and its nearest base station</td>
</tr>
<tr>
<td>$d_{Rd}$</td>
<td>Distance between two radars</td>
</tr>
<tr>
<td>$d_{Rd-BS}$</td>
<td>Distance between a radar and a base station</td>
</tr>
<tr>
<td>$d_{Rd-BS,i}$</td>
<td>Distance between a radar and base station $i$</td>
</tr>
<tr>
<td>$d_{Rd-MT}$</td>
<td>Distance between a radar and a mobile terminal</td>
</tr>
<tr>
<td>$d_{Rd-RX}$</td>
<td>Distance between a radar and a secondary receiver</td>
</tr>
<tr>
<td>$d_{Rd-S}$</td>
<td>Distance between a radar and a secondary device</td>
</tr>
<tr>
<td>$d_{Rd-TX}$</td>
<td>Distance between a radar and a secondary transmitter</td>
</tr>
<tr>
<td>$d_{TX-RX}$</td>
<td>Distance between a transmitter and its receiver</td>
</tr>
<tr>
<td>$f$</td>
<td>An average of the ratio between noise plus interference power density at a given mobile terminal, and link loss between the base station and the mobile terminal</td>
</tr>
<tr>
<td>$f_L$</td>
<td>A low estimate of the parameter $f$</td>
</tr>
<tr>
<td>$f_o$</td>
<td>Objective function</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Noise plus interference density at mobile terminal $i$</td>
</tr>
<tr>
<td>$n_0$</td>
<td>Background noise power spectral density</td>
</tr>
<tr>
<td>$n_S$</td>
<td>Background noise power spectral density of a secondary receiver</td>
</tr>
<tr>
<td>$p_{intrp}$</td>
<td>Probability of interruptions during video streaming</td>
</tr>
<tr>
<td>$p_{b_s \geq b_{req}}$</td>
<td>Fraction of time that a secondary data rate is higher than a required value</td>
</tr>
<tr>
<td>$E_b / I_0$</td>
<td>Bit energy to interference plus noise ratio</td>
</tr>
<tr>
<td>$G_r$</td>
<td>Antenna gain of a receiver</td>
</tr>
<tr>
<td>$G_{Rd,j}$</td>
<td>Antenna gain of radar $j$</td>
</tr>
<tr>
<td>$G_t$</td>
<td>Antenna gain of a transmitter</td>
</tr>
<tr>
<td>$I_i$</td>
<td>Interference caused to a given radar by cell $i$</td>
</tr>
<tr>
<td>$I_i^*$</td>
<td>Interference level that base station $i$ is allowed to caused</td>
</tr>
<tr>
<td>$I_{intc}$</td>
<td>Power spectral density of intercell interference</td>
</tr>
<tr>
<td>$I_{limit,i}$</td>
<td>Limitation on interference level cell $i$ is allowed to caused to a given radar</td>
</tr>
<tr>
<td>$I_{pw_limit,i}$</td>
<td>Interference caused to a given radar by base station $i$ when the base station transmits with its equipment power limit</td>
</tr>
<tr>
<td>$I_{rate_limit,i}$</td>
<td>Interference caused to a given radar by base station $i$ when the base station transmits with power that on average yields the maximum allowed data rate</td>
</tr>
<tr>
<td>$I_S$</td>
<td>Secondary tolerable interference level (STIL)</td>
</tr>
</tbody>
</table>
\( I_{\text{tol}} \) Tolerable interference level of a radar
\( I_T \) Total tolerable interference level of a given base station

\( \bar{I}_T \) Conservative estimate of total tolerable interference level of a given base station
\( \bar{I}_{T,1} \) Conservative estimate of total tolerable interference level of a given base station in Case 1: number of users in a cells is less than a threshold
\( \bar{I}_{T,2} \) Conservative estimate of total tolerable interference level of a given base station in Case 2: number of users in a cells is larger than a threshold

\( K \) Number of active mobile terminals in a cell
\( K_T \) A threshold on number of users \( K \)
\( K_T^* \) Optimal threshold on number of users

\( L_{BS} \) Link loss between a base station and a secondary device
\( L_i \) Link loss between a given base station and sensor \( i \)
\( L_1 \) Link loss
\( L_{1,Bd-S} \) Link loss between a radar and a secondary device
\( L_{1,S} \) Link loss between a secondary transmitter-receiver pair
\( L_{\min} \) Smallest link loss
\( L_{\min,LR} \) Lowest possible value of the smallest link loss \( L_{\min} \)
\( L_{M,i} \) Link loss between a given base station and its mobile terminal \( i \)
\( L_{Rd,i-MT} \) Link loss between a radar \( j \) and a mobile terminal
\( L_{Rd-BS,i} \) Link loss between a given radar and base station \( i \)
\( L_{Rd-MT,i} \) Link loss between a given radar and a mobile terminal in cell \( i \)
\( L_{S,i}(d_5) \) Link loss between base station \( i \) and its mobile terminal \( d_5 \) away
\( L_{T,j} \) Link loss threshold of a link between a secondary device and radar \( j \)
\( \bar{L}_{i,j} \) Expected link loss between base station \( i \) and radar \( j \)
\( \bar{L}_{1,Bd-S} \) Expected link loss between a radar and a secondary device

\( M_s \) Margin on transmit power of a secondary device
\( N_a \) Number of active cells sharing spectrum with a given radar
\( N_m \) Number of measurements
\( N_{Rd,i} \) Number of radars nearby base station \( i \)
\( N_{sr} \) Number of sensors
\( N_{st} \) Total number of starting time \( t_{st} \) considered
\( P_B \) Transmit power of a base station
\( P_{BS} \) Allowable transmit power of a base station
\( P_{BS,e,max,i} \) Equipment transmit power limit of base station \( i \)
\( P_{BS,i} \) Allowable transmit power of base station \( i \)
\( P_i \) Transmit power from a given base station to its mobile terminal \( i \)
\( P_{M,e,max} \) Equipment transmit power limit of a mobile terminal
Received power at a receiver
Power of transmissions from a given base station received at a secondary device
Power of radar pulses received at a secondary receiver
Transmit power of radar $j$
Maximum power that a secondary device is allowed to transmit
Equipment transmit power limit of a secondary device
Transmit power of a transmitter
Cell radius
Maximum allowed data rate
Data rate achieved by active cell $i$
Required data rate
Data rate per resource block of an LTE system
Secondary data rate
Perceived secondary data rate
Required data rate for video streaming
Required data rate for VoIP
Average secondary data rate
File transfer time
Rotating period of a radar
Amount of data in the buffer, of a streaming, at time $t$ given starting time $t_{st}$
Amount of data needed for initial buffering in video streaming
Size of a file
Bandwidth of secondary transmissions

Fraction of Shannon's limit
Mean
A dummy binary variable
Acceptable $E_b/I_0$ of a primary system in cooperative sharing
Acceptable $E_b/I_0$ of a primary system in coexistent sharing
SINR of base station $i$'s transmissions
Maximum correlation coefficient
SINR of secondary transmissions
Standard deviation of a random variable representing shadowing effect
Variance
Time period $i$-th that a secondary data rate is higher than a required value
Interrupted time experienced by a VoIP user
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>Azimuthal direction of a radar's main beam</td>
</tr>
<tr>
<td>$\bar{\phi}$</td>
<td>Vector of direction of radars' main beams</td>
</tr>
<tr>
<td>$\phi_j$</td>
<td>Direction of radar $j$'s main beam</td>
</tr>
<tr>
<td>$\Delta L_p$</td>
<td>Variation in path loss due to factors such as fading and shadowing</td>
</tr>
</tbody>
</table>
This page is intentionally left blank.
Chapter 1

Introduction
1.1 Spectrum Scarcity and Alleviation

Concern over spectrum scarcity is increasing. This is due, especially, to the expected large amount of spectrum needed for cellular and mobile broadband [FCC10]. Making more spectrum available for these cellular systems would make it possible to provide these services at lower cost. Alleviating this spectrum scarcity has become priority for some nations. For example, the US government has pledged to make 500 MHz of spectrum newly available for wireless broadband by year 2020 [FCC10].

In the conventional approach to spectrum management, i.e., spectrum licensing, different wireless systems are given exclusive access to blocks of spectrum as defined by their frequency range and geographic area such that the systems will not cause harmful interference to each other [FCC02]; operators of these wireless systems are called license holders. Harmful interference is defined as interference that causes significant service disruption [Marg03]. Spectrum licensing is an effective way to prevent mutual harmful interference among multiple wireless systems; however, it leads to inefficient use of spectrum and unnecessary spectrum scarcity. Many measurements, e.g., [FCC02][NAF03][SSC04], have found that even in city areas where extensive use of spectrum is expected, spectrum will have to sit idle when and where license holders are not active.

There are various ways that have been used to alleviate this spectrum scarcity:

- One way is to use new spectrum bands that have not been assigned to any wireless system, i.e., empty bands. However, currently, empty bands only exist at very high frequencies (e.g., spectrum bands in the range of 10 GHz and higher) that are not very useful for a number of technical reasons. For example, it is difficult and expensive to build hardware that operates at very high frequencies. Moreover, power of signals transmitted at very high frequency decreases quickly with distance along which the signals travel; this makes very high frequencies useful primarily for some wireless systems that do not require long transmission distance.

- Another way is to re-allocate spectrum bands used by some wireless systems to other wireless systems; i.e., spectrum re-allocation. In the US, examples are parts of the 700 MHz band that are re-allocated for cellular and public safety systems [FCC10], and various bands, used by federal agencies, that currently are under consideration to re-allocate for wireless broadband use [NTIA10]. However, spectrum re-allocation is a non-trivial, and time consuming process [FCC10]; those wireless systems of which spectrum bands are re-allocated from might still be in use, and will also need other bands to operate in.

- Alternatively, spectral efficiency can be improved by allowing multiple wireless systems to share the same band of spectrum in a manner that these systems will not cause harmful interference to each other, i.e., spectrum sharing.
1.2 Motivations and Research Overview

Among various forms of spectrum sharing, opportunistic primary-secondary spectrum sharing\textsuperscript{1} has the potential to substantially alleviate the growing problem of spectrum scarcity [Peha09]. In opportunistic primary-secondary spectrum sharing, a primary system will be protected from harmful interference, while one or more secondary systems are allowed to transmit in the same band when and only when these transmissions cause no harmful interference to the primary system.

Many of current opportunistic primary-secondary sharing models in both research and policy communities have focused on allowing a secondary device to transmit where and when the strength of primary transmissions is so weak that spectrum is considered “unused,” in the space or time domains, e.g., [Kahl11][Mora11][Yuce09][Tand09][Marq09][FCC08a]. This is the white-space sharing adopted in spectrum bands allocated for TV [FCC08a][Stev09]. The white-space sharing is also applied in Dynamic Frequency Selection (DFS) mechanism used by unlicensed devices operating in the same bands as radars, such as the 5 GHz band [FCC06]. The DFS mechanism will allow unlicensed devices to transmit where no radar transmissions are detected. Generally there is no limit to the number of unlicensed devices deployed in a designated band; however, there are limits on transmit power of those devices which will keep utilization low enough to limit mutual interference, and few restrictions [Peha09], such as the DFS mechanism applied in the 5 GHz band. White-space sharing is the easiest form of sharing, and may therefore lead to more near-term commercial successes. However, it is an increasingly common mistake to believe that this form of sharing is the most efficient in all bands that allow opportunistic sharing.

In contrast to the white-space sharing, this dissertation considers opportunistic sharing that will allow a secondary device to transmit at the same time as the primary system, and even when the device is geographically close to the primary system, i.e., gray-space sharing [Peha12]. With gray-space sharing, a secondary device is allowed to transmit as long as the signal strength of primary transmissions is strong enough that additional interference from secondary transmissions would be tolerable. Specifically, in the approaches considered in this dissertation, secondary devices will use power control to maximize transmit power to the extent possible without significant risk of causing harmful interference to the primary system. Power control has long been used to control devices within a single communication system, e.g., a cellular network [Chia08]. This dissertation uses power control for a different purpose: to allow different systems, under different administrative control, to share spectrum in an opportunistic primary-secondary arrangement.

This dissertation proposes and then evaluates various novel mechanisms to enable gray-space sharing with two different types of primary system:

\textsuperscript{1} This opportunistic sharing is different from the other type of primary-secondary sharing called underlay sharing [Peha09], in which a secondary device transmits at very low power that will never cause harmful interference to the primary system, e.g., when the secondary system uses ultra-wide-band technology.
o Cellular systems, see Section 1.2.1 for further discussions
o Radars, see Section 1.2.2 for further discussions.

As will be shown, the proposed sharing can even take spectrum that is considered 100% utilized by conventional spectrum management, and make it possible to use that spectrum for additional communications without any harmful interference to existing primary systems.

A secondary device can determine its allowable transmit power either by having a protocol for cooperation and explicit communications with the primary system, i.e. cooperative sharing, or by having no explicit communications and hence no need for protocol for cooperation, i.e. coexistent sharing. This dissertation considers both cooperative and coexistent approaches.

Generally, the research questions asked throughout the dissertation will be on:

o Performance and viability of a secondary system sharing spectrum with the primary systems considered, as might be determined from Quality of Service (QoS), and/or the extent of transmissions achievable
o Performance of the primary systems operating in shared spectrum, as might be measured as incurred risk of harmful interference and/or power consumption of a primary user.

Much of the recent research on primary-secondary sharing has assumed that secondary devices are unlicensed. However, other policies on secondary devices are also applicable to this dissertation [Peha09], including the following:

o Secondary License: A regulator gives exclusive rights to an entity to operate as a secondary system.

o Secondary Market: A primary license-holder authorizes an entity to operate as a secondary system, perhaps in return for payment. This is already allowed in the US, see footnote 237 of [FCC04].

In order to quantify the overall extent of secondary transmissions achievable, this dissertation does not consider how spectrum will be allocated among multiple secondary devices. Although the sharing concepts can be extended to the unlicensed-secondary policy by limiting secondary device density or through cooperation among secondary devices, the sharing concepts considered are more directly relevant to the other two policies.

1.2.1 Spectrum Sharing with a Cellular System

Using power control, a secondary device will be allowed to transmit at the same time as a cellular system, and even when the device is geographically close to the cellular system, as long as cellular transmissions are strong enough to tolerate interference from secondary transmissions. A similar approach could be adopted with other kinds of primary systems in which SINR (Signal to Interference plus Noise Ratio) at a primary receiver is larger than necessary in some but not all of the time.
To evaluate performance of the sharing model in general, a secondary device is assumed to be a generic fixed, or portable device.

Generally, spectrum utilization of a cellular system is efficient, so it is probably not the easiest case for gray-space sharing. Nevertheless, this dissertation will show that sharing is possible even in the peak hour when cellular utilization may reach 100%. Moreover, cellular utilization is typically near its peak for a few hours per day, and far more secondary activity is possible the rest of the time. In addition, communications systems using cellular technology for public safety have been deployed in some countries, and considered in other countries. Some systems are voice-only, and others allow data as well, e.g. [Tatt01][Peha07][FCC08b]. Such public safety communications systems have sufficient capacity for large emergency, but are lightly used most of the time; these may provide opportunities for the sharing.

1.2.2 Spectrum Sharing with Radars

Sharing spectrum with radar is an important research topic to alleviate spectrum scarcity because radar operates in a large amount of spectrum. For example, in the US over 1.7 GHz of spectrum from 225 MHz to 3.7 GHz “involves radar and/or radionavigation infrastructure,” [NTIA09], and around 1.1 GHz of this 1.7 GHz is used by fixed land-based radars in non-military applications [NTIA00]. Moreover, in the US, the National Telecommunications and Information Administration (NTIA) is now considering opening up spectrum in bands used by radars [NTIA10]. As a sign of the importance and urgency of this new research area, NTIA held a conference in 2011 dedicated to spectrum sharing with radars [NTIA11].

This dissertation evaluates the performance of the gray-space sharing concept with a radar for which the main beam rotates, i.e. a rotating radar. Examples of these radars include Air Traffic Control (ATC) radars operating in the [2.7, 2.9] GHz band, weather radars operating in the [2.7, 3.0] GHz band, and other surveillance radar operating in the [0.42, 0.45] GHz and [2.7, 3.5] GHz bands [NTIA00]. With a rotating main beam, the radar antenna gain seen by a fixed secondary device varies over time. Hence, there will be periods of time when the link loss, which includes antenna gains and path loss, between the device and the radar is high enough so that the device can transmit successfully, without causing harmful interference to the radar.

Meanwhile, Internet traffic in cellular systems is increasing rapidly [FCC10], and access to more spectrum could substantially reduce the cost of such services. This dissertation considers scenarios in which a secondary system provides point-to-multipoint transmissions in multiple cells, but those cells do not blanket a region. These scenarios apply when a cellular system provides broadband hotspots, or only uses shared spectrum when a temporary surge of traffic in a given cell requires more capacity than what is available from the cellular system’s dedicated spectrum.
1.3 Contributions and Research Methodology

Regarding contributions, this dissertation,

- proposes novel mechanisms that enable gray-space sharing with cellular systems, and rotating radars
- quantifies potential of the gray-space spectrum sharing model that relies on power control to avoid causing harmful interference to a primary system
- considers the gray-space sharing for both cooperative and coexistent approaches
- investigates the sharing by considering both the performance as perceived by a secondary system, and performance as perceived by a primary system
- is the first to address how to make coexistent (rather than cooperative) gray-space sharing possible with cellular systems, and radars.

To investigate the proposed sharing models,

- Mechanisms for gray-space sharing are devised under various different sets of assumptions on the primary and secondary systems, and on sharing approaches
- Analysis and/or extensive Monte Carlo simulations are used to assess the viability and performance of the sharing mechanisms.

1.4 Dissertation Organization

Basic concepts used throughout the dissertation, and related works on opportunistic gray-space spectrum sharing with cellular systems and rotating radars are reviewed in Chapter 2.

In Chapter 3, a model for gray-space sharing with a cellular system is proposed. The viability and performance of the sharing are investigated.

In Chapter 4, a model for sharing spectrum with rotating radars is proposed. The overall extent of achievable secondary transmissions, and performance of various applications in spectrum shared with radars are investigated. The scenario considered is simplified to when there is one cell of the secondary system sharing spectrum with one radar.

The simplified scenario in Chapter 4 is extended to when multiple cells are sharing spectrum with a single rotating radar, and with multiple rotating radars in Chapters 5 and 6, respectively.

Conclusions, policy implications and policy issues associated with gray-space sharing, and future work are discussed in Chapter 7.
Chapter 2

Basic Concepts and Literature Review
Throughout this dissertation, link loss will be used in calculating interference between primary and secondary systems; a brief overview of link loss is given in Section 2.1. Related works on opportunistic gray-space spectrum sharing with cellular systems, and rotating radars are reviewed in Sections 2.2, and 2.3, respectively.

### 2.1 Link Loss Overview

Link loss determines how much signal transmitted from a transmitter will be received at the corresponding receiver. Link loss (in dB) between a transmitter-receiver pair $L_{[dB]}$ is defined as a difference between transmit power $P_{t[dBm]}$ from a transmitter, and received power $P_{r[dBm]}$ at a receiver:

$$L_{[dB]} = P_{t[dBm]} - P_{r[dBm]}.$$  \hspace{1cm} (2.1)

Equivalently, $L_{[dB]}$ can also be written as

$$L_{[dB]} = G_{t[dBi]} + G_{r[dBi]} + \left[ \bar{L}_{p[dBi]}(d_{t-r}) + \Delta_{\ell_p[dBi]} \right],$$  \hspace{1cm} (2.2)

where,

- $G_{t[dBi]}$ is antenna gain, in dBi, of the transmitter
- $G_{r[dBi]}$ is antenna gain, in dBi, of the receiver
- $\bar{L}_{p[dBi]}(d_{t-r})$ is an expected path loss between the transmitter and the receiver which are $d_{t-r}$ apart
- $\Delta_{\ell_p[dBi]}$ is a zero-mean random variable (r.v.) that characterizes fluctuations in wireless channel between the transmitter and the receiver such as fading and shadowing.

Unless stated otherwise, the term “link loss” that will be referred to throughout the dissertation is the one defined by (2.1) and (2.2), but in absolute units, not in [dB].
2.2 Related Work on Spectrum Sharing with a Cellular System

The observation that gray-space primary-secondary sharing with a cellular system is possible is not new. Panichpapiboon and Peha [Pani03][Peha04], and subsequently other researchers [Lee09][Bakr08][Ghav08], have considered such models under different assumptions on policy, and technology used by cellular and secondary systems. The extent of secondary transmissions has been investigated in [Pani03][Peha04][Lee09]. Some researchers, [Pani03][Peha04], found that a significant amount of secondary transmissions is possible even when a cellular system is 100% utilized, others did not [Lee09].

The sharing model considered in this dissertation and each of the previous work use fundamentally different technologies and technical assumptions to enable the sharing. Geolocation is used to make the sharing possible in [Pani03][Peha04]. In [Lee09], a secondary device is allowed to transmit in unused time slots of the primary system; hence, the sharing approach is only applicable to a primary system using TDMA technology. Sharing based on directionality exploited through multi antenna beam forming capacity of a secondary device is considered in [Bakr08]. In [Ghav08], the authors considered only the scenario in which primary and secondary base stations are always collocated. The sharing model considered in [Ghav08] could be useful for capacity sharing between two categories of users within the same system and therefore under the same administrative control, but is problematic when the primary and secondary systems are independent, because collocation would usually not be possible.

All of the previous work, [Pani03][Peha04][Lee09][Bakr08][Ghav08], only considered the cooperative sharing. Thus, this dissertation is the first that proposes mechanisms allowing a secondary device to coexistently share spectrum (without direct communications) with a cellular system, and the first that compares coexistent and cooperative sharing by taking into account both the secondary transmissions achievable and the effect of resulting interference, from secondary transmissions, to the primary system.

There have also been other works that considered sharing spectrum with a cellular system, e.g. where the secondary devices operate in the same frequency band as a cellular system but are far enough away from the cellular coverage area that harmful interference can be avoided [Lars08], or where harmful interference is allowed such that the resulting interference perceived at the cellular system is below a pre-defined interference threshold, e.g., [Khos10][Huan09][Atta08]. Note that for the sharing approach that is based on a pre-defined threshold, at the time when SINR (Signal to Interference plus Noise Ratio) of primary transmissions is low, secondary transmissions could cause harmful interference to the primary system. Harmful interference is therefore avoided by making the cellular system more tolerant to interference, e.g., by decreasing cellular capacity. There may be situations in which slightly decreasing cellular capacity to allow secondary transmissions is useful. However, it is not consistent with the requirement of the sharing considered, in this dissertation, that performance of the primary system be unaffected by the presence of secondary devices.
2.3 Related Work on Spectrum Sharing with Rotating Radars

The existing models and current proposals for opportunistic spectrum sharing with radars are usually based on the white-space approach, e.g., the DFS mechanism used by unlicensed devices in the 5 GHz band. A similar sharing idea was investigated in the 2.8 GHz band used by ATC radars [Wang08][Rahm11]. In other bands such as the 3.5 GHz band, NTIA has proposed allowing non-radar systems to operate except in exclusion zones that can only be used by radars [NTIA10]. This exclusion zone is conservatively calculated such that the possibility of harmful interference between radars and secondary devices would be very low.

Only a few previous works have address gray-space spectrum sharing that will allow a secondary device to transmit even when transmissions from a rotating radar are detected. Marcus qualitatively discussed possibility of the sharing idea in [Marc09]. This dissertation is the first to quantitatively evaluate the extent of transmissions achievable from allowing a secondary device to transmit close to a radar, but with transmissions that are dynamically adjusted according to varying link loss between the device and the radar as the radar rotates. Because transmissions of a secondary device are controlled based on radar rotation, there could be fluctuations and interruptions in achievable secondary transmissions. This dissertation is also the first to study the impact of these interruptions and fluctuations on the performance of the sharing that a secondary user will perceive. It was also found that access opportunity for an unlicensed device, operating in the same band as a radar, could be increased using information on varying link loss between a rotating radar and the device [Terc11].
Chapter 3

Sharing with a Cellular System
3.1 Overview

This chapter investigates the feasibility of the gray-space sharing concept in a spectrum band that is heavily and efficiently used by a primary system. The scenarios, where a secondary transmitter sharing spectrum with the upstream\(^2\) of the primary system which is a cellular voice communications system, are considered.

As discussed in Section 1.2.1, a secondary device will be allowed to transmit at the same time as a cellular system, and even when the device is geographically close to the cellular system, as long as cellular transmissions are strong enough to tolerate interference from secondary transmissions.

The overall extent of secondary transmissions, achievable from both cooperative and coexistent sharing are quantified and compared; one pair of secondary transmitter-receiver sharing spectrum with a cellular system is considered; cooperative and coexistent sharing is described in Section 1.2.

To enable coexistent sharing, this chapter presents novel mechanisms through which secondary devices can determine how much interference they can generate by querying a sensor network rather than the primary system. With this novel approach, the sensor network estimates primary upstream communications from monitored primary downstream communications, and thus how much interference secondary devices can generate without causing harmful interference at a given primary base station.

This sensor network can be used to monitor and identify opportunity for spectrum sharing in multiple bands, and hence could be a cost-effective approach to coexistent sharing. A system of relatively low-cost sensors can simultaneously monitor many bands used by different types of primary systems, and direct any secondary device wishing to transmit to the most appropriate band. None of this complexity must be implemented in the secondary devices. Exchanging information among sensors, to facilitate spectrum sharing, is still a challenging research question which is beyond the scope of this dissertation. This sensor network could be deployed by various interested parties. For example, the sensor network could be deployed by a third party that manages how spectrum is utilized, i.e., a band manager, or that simply provides information on the availability of different spectrum bands, like the white space database operators emerging in the TV band in the US [FCC10b]. This third party might charge users or manufacturers of secondary devices for the service.

It is also assumed that the sensor network has a priori access to information about the primary system, as described further in Section 3.3. This happens easily with the secondary market policy, as operators of primary systems have strong incentive to share information. In the other policies, the regulator might mandate information sharing [Peha09]. (See Section 1.2 for details on relevant policy on a secondary system.) As this chapter shows, in the case where the primary is a cellular system, making technical information about the primary system available to designers of a secondary system is a useful and often overlooked way to promote efficient sharing.

\(^2\) The upstream is direction of transmissions from a cellular mobile terminal to its base station.
Research questions asked in investigating the gray-space sharing with a cellular system are discussed in Section 3.2. The sharing scenario is described in Section 3.3. Basic approach used to control transmission of a secondary transmitter to prevent a cellular system from harmful interference is explained in Section 3.4. Sections 3.5 and 3.6 give details on a secondary system can cooperatively and coexistently share spectrum with a cellular system, respectively. Section 3.7 shows and discusses numerical results. Conclusions are drawn in Section 3.8.

3.2 Research Questions

In investigating gray-space sharing in which a cellular system is a primary system, the following research questions are asked:

1) With the gray-space sharing concept, what is the extent of overall secondary transmissions achievable when spectrum is shared with a primary system such as a cellular system, which already utilizes spectrum efficiently?

2) How much is the impact of the spectrum sharing on the primary system?

3) How different is performance of cooperative sharing from that of coexistent sharing concerning both the secondary transmissions and the impact of resulting interference on the primary?

With explicit communications with a primary system, cooperative sharing might achieve higher secondary transmissions than coexistent sharing. However, cooperative sharing would also cause higher interference to the primary system.

Moreover, sharing with cooperation also has other disadvantages [Peha09]:

- Cooperation requires a protocol supported by both primary and secondary systems. This can be problematic, especially for legacy equipment not designed to interact with secondary systems.
- Cooperation is generally built on trust, so it may be inappropriate in cases where the primary users lack incentive to encourage sharing.
- Cooperative sharing can be vulnerable to the hidden terminal problem.
- One must consider the transaction costs and overhead associated with cooperation.

By quantifying extent of transmissions achievable and impact of the sharing on the primary system for both cooperative and coexistent sharing, whether the benefits of the cooperative sharing exceed the costs could be determined.
3.3 Sharing Scenarios

Spectrum sharing in a frequency band used for upstream transmissions of a Frequency Division Duplex (FDD) DS-CDMA\textsuperscript{3} (Direct-Sequence Code Division Multiple Access) cellular system is considered. Although interference from secondary devices does not cause harmful interference, it does increase interference at a primary Base Station (BS), causing Mobile Terminals (MT’s) to increase their transmit power to compensate for the higher interference.

The proposed sharing mechanisms basically assume the following:

1) The primary system utilizes power control to maintain $\frac{E_b}{I_0}$ (bit energy to interference plus noise ratio) in both up- and downstreams.

2) For coexistent sharing, the sensor network has following information about the primary system; one can easily find items a) to e) from the specifications of the primary system equipment:
   a) Required $\frac{E_b}{I_0}$ at a BS, and a MT
   b) Maximum transmit power of a MT
   c) Processing gain
   d) Bandwidth
   e) Background noise power spectral density
   f) Upper bound on intercell interference at a BS; this might be fixed, or varying with time of day
   g) Lowest possible value of link loss between a MT and its BS, i.e., link loss at cell edge
   h) Low estimate of parameter $f$, defined as the average (over all active MT’s) of the ratio between noise and interference density ($n_i$) at a given MT, and the link loss between the BS and this MT (i.e., $L_{BS,i}$). The primary system can provide this low estimate from measurement data or in some scenarios from analysis; further details are presented in Section 3.6.2.

3) For coexistent sharing, the sensor network can differentiate signals from different BS’s. The sensor network can achieve this if the primary system shares information about the short pseudo-noise sequence that a BS uses as its identity [Will00], or if the sensor network employs directional antennas.

4) For coexistent sharing, it is possible to determine link loss factor from a secondary device to a BS when one knows that from the BS to the device. This is generally true when the difference

\textsuperscript{3} For OFDMA-based primary systems, in cooperative sharing the primary systems need to inform the secondary systems which sub-carriers to use, in addition to the maximum allowable transmit power. A different approach would be needed for coexistent sharing. This approach for coexistent sharing, and the resulting extent of secondary communications are open research questions.
between upstream and downstream frequencies is small [Pera01].

In addition to the four assumptions above which are basic to the sharing approach, to quantitatively assess the sharing concept, the sharing scenario is simplified by having these additional specific assumptions:

1) There are only voice communications in the primary system\(^4\), as might happen in various communication systems used for public safety as discussed in Section 1.2.1.
2) Power control in the primary system is perfect.
3) The possibility of simultaneous secondary transmissions causing significant interference to a given BS is negligible. This might hold because secondary devices transmit infrequently—e.g., when secondary systems interconnect meter readers or alarm systems— or because there is coordination among secondary devices. Secondary transmissions quantified in this chapter paper are the upper bound on the extent of transmissions achievable when there is no coordination among secondary devices.
4) The number of sensors in a sensor network is sufficient to average out fading and shadowing that affect the sensor network’s measurement of primary downstream transmissions. As a sensor considered is a low-cost device, deploying multiple of these sensors does not seem to be a problem.
5) Locations of active MT’s in each cell are sufficiently independent to apply the Central Limit Theorem (CLT). CLT will be applied to the distribution of \( f \) parameter, as discussed further in Section 3.6.2.
6) A sensor network knows mean, variance, and maximum correlation coefficient of \( \frac{r_{ij}}{i_{M,j}} \). The primary system can provide the sensor network these three statistics from measurement data, or in some cases from analysis.

3.4 Basic Approach to Control Transmit Power of a Secondary System to Prevent Harmful Interference

In a primary cell, if a secondary device wants to transmit, the device has to determine the maximum transmit power at which it is allowed to transmit \( P_{3,\text{max}} \). The device can trivially calculate

\(^4\) The same general concepts may apply to a system carrying data as well as voice, since any CDMA system operating at less than full capacity can tolerate interference. However, data brings additional complexities. First, instantaneous utilization fluctuates more in a data system, so utilization must be averaged over some period, which should be carefully chosen. Second, correlation between up- and downstream transmission rates in a data system is less than 1. Thus, estimations of upstream rates made by observing the downstream will have greater uncertainty. The implications of these complexities are open research questions.
From

- Link loss between itself and the BS ($L_{BS}$)
- Maximum interference level from its transmissions that the BS can tolerate, called **Secondary Tolerable Interference Level** (STIL, i.e. $I_S$)

using

$$P_{S,max} = \frac{I_S}{L_{BS}}.$$  \hspace{1cm} (3.1)

$I_S$ is defined as the total interference a given BS can tolerate from all secondary devices. When there are multiple distinct autonomous secondary systems, some coordination among secondary systems would be needed. However, as it is intended to quantify the overall extent of secondary transmissions, such coordination is outside the scope of this dissertation. As assumed in Section 3.3, one secondary transmitter-receiver pair sharing spectrum with the cellular system is considered. When a secondary device is near a boundary of multiple primary cells, this secondary device's transmissions cannot cause $I_S$ to be exceeded in any of those cells.

For cooperative sharing, a secondary device learns $L_{BS}$ and $I_S$ from communications with the BS; see Section 3.5.

For coexistent sharing, a secondary device determines how strong it can transmit by asking a sensor network; the sensor network consists of fixed sensors deployed in the area where spectrum sharing is taking place. The sensor network monitors downstream transmissions from surrounding BS's. The sensor network tells the secondary device its estimate of the instantaneous $I_S$, and the instantaneous transmit power from the BS (i.e., $P_B$). This allows the secondary device to determine $L_{BS}$ using the definition of link loss given by (2.1):

$$L_{BS} = \frac{P_{r,B-S}}{P_B},$$ \hspace{1cm} (3.2)

where $P_{r,B-S}$ is power of the BS transmissions received at the secondary device; see basic assumption 4. Details on determining $I_S$ from $P_B$ are in Section 3.6.

Note that fading would make $L_{BS}$ fluctuate over time; hence $P_{r,B-S}$ would also fluctuate even when $P_B$ is constant. To ensure that the interference from secondary transmissions do not exceed $I_S$, in both cooperative and coexistent sharing a secondary device needs to either adjust its $P_{S,max}$ quickly enough to keep up with changes in $P_{r,B-S}$ due to fading, or put a margin on its transmit power to deal with the fluctuating $L_{BS}$. Using a margin to deal with fading is simple, but would result in lower extent of secondary transmissions. In contrast, adjusting transmissions of a secondary device to keep up with changes in channel due to fading adds more complexity to the sharing, but would achieve higher
transmissions; potential of a sharing model in which a secondary device adjusts its transmissions according to fading has been identified in [Ghas07]. \(I_s\) and \(P_B\) also change over time, and each secondary device must keep up with these changes as well. This is achieved in cooperative sharing through updates from the primary BS, and in coexistent sharing through updates from the sensor network. How often \(P_{S,max}\), \(I_s\) and \(P_B\) should be updated, and how much the fade margin should be used are beyond the scope of this dissertation.

### 3.5 Cooperative Sharing

As described in Section 3.4, the key to allowing secondary and primary devices to transmit simultaneously in the same spectrum band is the calculation of STIL (i.e., \(I_s\)). \(I_s\) of a given BS is derived from background noise power spectral density (\(n_0\)), intercell interference power spectral density, i.e., \(I_{intc}\), and Total Tolerable Interference Level (TTIL, i.e., \(I_T\)). \(I_T\) (in [W/Hz]) is defined as the maximum total interference that the BS can tolerate in addition to interference from active MT’s in its cell, i.e., intracell interference. Hence,

\[
I_s = I_T - [n_0 + I_{intc}].
\] (3.3)

\(I_s\) (in [W/Hz]) is determined such that a secondary device can avoid causing harmful interference to any MT in the cell considered. The upstream signal most vulnerable to harmful interference comes from the MT with the smallest link loss (\(L_{min}\)) of all \(K\) active MT’s in the cell. With the sharing approach considered, the MT with \(L_{min}\) may need to increase transmit power up to its equipment limit of \(P_{M,e,max}\). Due to power control in the cellular system, at the BS, \(\frac{P_B}{I_0}\) of transmissions from the MT with \(L_{min}\) will remain at the acceptable level \(\rho_{a,1}\), adapted from [Rapp02]:

\[
\rho_{a,1} = \frac{P_{M,e,\text{max}} \times L_{min} \times G}{W \times I_T + [K-1] \times P_{M,e,\text{max}} \times L_{min}},
\] (3.4)

where,

- \(G\) is processing gain of the (primary) cellular system
- \(W\) is bandwidth of the cellular system.

Thus, from (3.4) even the most vulnerable MT is safe with
A BS knows all information needed to calculate $I_T$ and $I_S$ shown in (3.3) and (3.5), e.g., $n_0$, $I_{intc}$, $K$, $L_{min}$ etc. Thus, the BS can tell a secondary device how much $I_S$ the BS can tolerate.

### 3.6 Coexistent Sharing

With coexistent sharing, secondary devices must still know $I_S$. Without communications with the primary system, the secondary system cannot know the smallest link loss $L_{min}$ nor the number of active primary users $K$, both of which are needed to determine $I_T$ using (3.5), and then $I_S$ using (3.3).

In coexistent sharing, the sensor network gives the secondary system a conservative estimate of $I_S$ calculated by using $n_0$ and an Upper Bound (UB) on $I_{intc}$; $n_0$ and an upper bound on $I_{intc}$ can be provided to the sensor network by the primary system, see basic assumptions in Section 3.3. What is left for getting the conservative estimate of STIL, is to estimate TTIL. The sensor network conservatively estimates TTIL from,

- Downstream transmit power of a BS ($P_B$) which is also determined by the sensor network. $P_B$ is the power of signals sent to all $K$ active MT’s in the cell; hence, the sensor network can infer information about $K$ from $P_B$.
- Some technical information about the primary system, see basic assumptions in Section 3.3.

How the sensor network estimates $P_B$, and TTIL are explained in Sections 3.6.1 and 3.6.2, respectively.

### 3.6.1 Estimation of Primary Downstream Transmit Power ($P_B$)

From the power level of primary downstream transmissions received at a sensor, any sensor $i$ can trivially determine current downstream power $P_B$ using a link loss from the BS to itself ($L_i$). With $P_B$ changing over time, determining $L_i$ from measurements at a single sensor is impossible, but can be achieved from the measurements of multiple sensors at multiple synchronized times. If there are $N_{sr}$ sensors and each has measurements at $N_m$ synchronized times, in total there are $N_{sr} \times N_m$ measurements, and $N_{sr} + N_m$ unknowns. This can be solved as a standard least-square problem.

Because sensors and the BS are fixed, the sensor network can estimate $L_i$ before spectrum sharing starts. This $L_i$ estimation does not need to be rapid. The effect of fading and shadowing on the sensor network’s measurement of primary transmissions will be averaged out when there are
sufficient sensor readings; see additional assumption 4 in Section 3.3.

3.6.2 Estimation of Total Tolerable Interference Level (TTIL, i.e., \( I_T \))

As with cooperative sharing, \( I_T \) is calculated using (3.5). However, the number of active MT’s in a cell \( (K) \) is obtained not through cooperation but as a function of \( P_B \), noise and interference power spectral density from other cells that each MT \( i \) perceives \( (n_i) \), and link loss between the BS and the MT \( (L_{M,i}) \). In general, \( K, L_{M,i}, \) and \( n_i \) determine how strong \( P_B \) is.

Downstream power \( P_B \) is the summation of transmit powers to each MT \( i \) (i.e. \( P_i \)) over all \( K \) active MT’s. Due to power control, \( \frac{\hat{P}_B}{l_0} \) at each MT equals the required level \( \rho_{a,2} \). Thus, \( \frac{\hat{P}_B}{l_0} \) of MT \( i \) with path loss factor \( L_{M,i} \), and noise plus interference (power spectral density) \( n_i \) would be at the required level

\[
\rho_{a,2} = \frac{P_i \times L_{M,i} \times G}{W \times n_i + [P_B - P_i] \times L_{M,i}}.
\] (3.6)

By taking summation of \( P_i \) over all \( K \) users, and defining \( f \) as the average of \( \frac{n_i}{L_{M,i}} \) across all \( K \) users:

\[
f = \frac{\sum_{i=1}^{K} \left( \frac{n_i}{L_{M,i}} \right)}{K},
\] (3.7)

a relationship between \( K \) and \( P_B \) is obtained from (3.6). With the \( K \)-and-\( P_B \) relationship and (3.5), the equation relating \( I_T \) with \( P_B \) together with parameter \( f \) can be derived; \( f \) accounts for uncertainty in relating \( K \) with \( P_B \). The derivation of (3.8) is given in Appendix A.

\[
I_T = \frac{P_{M,x_{max}} \times L_{min}}{W} \times \left[ \frac{G}{\rho_{a,1}} - \frac{G + \rho_{a,2}}{\rho_{a,2}} \frac{1}{1 + W \times \frac{\hat{P}_B}{l_0}} - 1 \right].
\] (3.8)

Note from (3.8) that \( I_T \) is an increasing function of \( L_{min} \). Thus, a conservative estimate of \( I_T \) can be obtained by using the lowest possible value of \( L_{min} \), i.e. \( L_{min,1B} \), as would be appropriate if one MT was at the edge of the cell.

The remaining challenge is to estimate \( f \). From (3.8), \( I_T \) is an increasing function of \( f \). So, to obtain a conservative estimate of TTIL, a low estimate of \( f \), i.e., \( f_i \) is needed. \( f \) is the average of \( K \)
random variables (r.v.’s) $\frac{n_j}{t_{m,j}}$. Although noise may be correlated from one MT’s location to another, the magnitude of this r.v. is far more a function of path loss factors from the BS to MT locations, which are likely to be independent across MT’s. Thus, in cases where $K$ is large, i.e.,

**Case 2:** $K \geq$ a positive real threshold ($K_T$), the Central Limit Theorem (CLT) is applied to find a conservative but fairly tight $f_L$.

On the other hand, in cases where $K$ is so small that the CLT bound would be too low to be useful, i.e.,

**Case 1:** $K < K_T$, an even more conservative assumption on $K$ can be made for the estimated TTIL.

Since this is when more sharing is possible anyway, the more conservative assumption on $K$ is less problematic.

As the sensor network can determine the primary downstream power $P_B$, but does not know $K$, the estimated TTIL from both cases will be calculated, and then the smaller value of the two is selected as the conservative estimate of TTIL, i.e., $\hat{I}_T$. $\hat{I}_T$ is a function of $K_T$, and for a given $P_B$, $K_T$ is chosen to maximize $\hat{I}_T$ over all.

From (3.5), $\hat{I}_T$ is a decreasing function of $K$. In Case 1, the actual TTIL is guaranteed to be larger than the estimated TTIL (i.e. $\hat{I}_{T,1}$) for all $K$ if $\hat{I}_{T,1}$ is calculated with $K = K_T$. Thus, from (3.5) and $L_{min,LB}$,

$$\hat{I}_{T,1} = \frac{P_{M,b,\text{max}} \cdot t_{m,\text{min},LB}}{W} \left[ \frac{\alpha}{\rho_{a,1}} - \frac{G}{[K_T - 1]} \right]. \quad (3.9)$$

In Case 2, $\hat{I}_T$ as shown in (3.8) increases with $f$. From (3.8), $L_{min,LB}$, and $f_L$, the estimated TTIL in this case (i.e. $\hat{I}_{T,2}$) would be

$$\hat{I}_{T,2} = \frac{P_{M,b,\text{max}} \cdot t_{m,\text{min},LB}}{W} \left[ \frac{\alpha}{\rho_{a,1}} - \frac{G + \rho_{a,2}}{\rho_{a,2}} \left( \frac{1}{1 + \frac{f_L}{P_{B,0}}} \right) - 1 \right]. \quad (3.10)$$
\( f_L \) will be found by applying CLT to the distribution of \( f \).

From standard normal distribution, define \( \beta \) as a positive real number that makes \( \text{Prob}\left\{ \frac{f-E(f)}{\sqrt{\text{Var}(f)}} < -\beta \right\} \) negligible, where \( E(f) \) and \( \text{Var}(f) \) are mean and variance of \( f \), respectively. \( f \) corresponding to \( \beta \), i.e. \( f_{\text{min}} \), that makes \( \text{Prob}(f < f_{\text{min}}) \) negligible is then

\[
f_{\text{min}} = E(f) - \beta \sqrt{\text{Var}(f)}.\tag{3.11}\]

Recall that \( f \) is a sample mean of \( \frac{n_i}{l_{M,i}} \) from \( K \) active MT's. With \( E\left\{ \frac{n_i}{l_{M,i}} \right\} = \mu \), then \( E(f) = \mu \). With \( \text{Var}\left\{ \frac{n_i}{l_{M,i}} \right\} = \sigma^2 \), and \( \rho_{ij} \) is the correlation coefficient of \( \frac{n_i}{l_{M,i}} \) between any two MT's, \( \text{Var}(f) \) is a function of \( \sigma^2, K \), and \( \rho_{ij} \). As \( K \) grows large, \( f \) will get closer to \( E(f) \) with smaller \( \text{Var}(f) \). Hence in Case 2, a non-negative low estimate of \( f_{\text{min}} \), i.e., \( f_L \), is obtained using

1. \( K \) = the threshold \( K_T \)
2. \( \max_{\forall i \neq j} \rho_{ij} \), i.e., \( \rho_{\text{max}} \).

From Section 3.3, \( \mu \), \( \sigma^2 \), and \( \rho_{\text{max}} \) are known to the secondary system. Using the resulting \( f_L \) and (3.10), the estimated TTIL for Case 2 (\( \hat{T}_{T,2} \)) as a function of \( K_T \) is:

\[
\hat{T}_{T,2} = \frac{P_{M,e,\text{max}}l_{\text{min,LB}}}{W} \times \left[ \frac{G + \rho_{a,2}}{\rho_{a,1}} - \max\left\{ \frac{1}{1 + W \times \frac{\rho_{a,2}}{\rho_{a,1}}}, 0 \right\} - \frac{1}{\rho_{a,2}} \right].\tag{3.12}\]

Note that for a given primary downstream power \( P_B \), \( \hat{T}_T \) is the minimum of the two estimated TTIL shown in (3.9) and (3.12), one of which decreases with \( K_T \) while the other increases with \( K_T \). Hence, \( \hat{T}_T \) is a function of \( K_T \), and there is an optimum non-negative threshold \( K_T^* \) that results in the maximum possible \( \hat{T}_T \). \( K_T^* \) is the point where \( \hat{T}_{T,1} = \hat{T}_{T,2} \). This is a quadratic equation with the following solution:

\[
K_T^* = \frac{-\left(1 - \rho_{\text{max}}\right)^2 C^2 + 2AD \pm \sqrt{\left(1 - \rho_{\text{max}}\right)^2 C^2 + 2AD} \left[1 + 4A^2 \left(\rho_{\text{max}} R^2 C^2 - B^2\right)\right]}{2 \left(\rho_{\text{max}} R^2 C^2 - B^2\right)},\tag{3.13}
\]
where,

- \( A = \frac{g \cdot P_{a,2}}{\rho_{a,2}} \)
- \( B = \frac{W}{P_B} \)
- \( C = \beta \sigma \)
- \( D = 1 + \mu B \).

The derivation of (3.13) is given in Appendix A.

### 3.7 Numerical Results

Assumptions used to obtain numerical results are summarized in Section 3.7.1. Section 3.7.2 evaluates and compares the extent to which secondary communications are possible. The impact of secondary transmissions on the primary system’s transmit power is quantified in Section 3.7.3. Section 3.7.4 studies tradeoffs between performance of the secondary system vs. performance of the primary system. The performance of the secondary system is measured as Secondary Tolerable Interference Level (STIL) and secondary data rate; that of the primary system is measured as transmit power of a primary Mobile Terminal (MT). Section 3.7.5 investigates the effect on the amount of secondary transmissions of various parameters, including intercell interference at the BS, amount of spectrum allocated to the primary system, channel bandwidth of the primary system, and a primary cell radius.

### 3.7.1 Assumptions for Numerical Results

Results are from computer simulations using the following assumptions; unless stated otherwise, the 95% confidence interval is within ±5% of the presented numbers:

1) The primary cell layout is a highway model. All cells have the same radius, and are deployed along a straight line; e.g., a highway. This assumption would make intercell interference within the primary system lower than the level occurs in a typical cell layout; sensitivity analysis of the results on the intercell interference is investigated in Section 3.7.5.1.

2) MT’s are randomly placed with uniform distribution along the highway.

3) Secondary devices are located on the highway.

4) Secondary devices operate in all neighboring cells to the cell of interest, using the coexistent sharing. This assumption allows a fair comparison between the efficiency of cooperative sharing and that of coexistent sharing. However, this assumption would make secondary communications from cooperative sharing higher than what it would be if spectrum sharing in neighboring cells is based on cooperation.

5) The primary system uses load balancing mechanism, allowing number of active MT’s in a
channel to differ from those in other channels by at most one.

6) 100% primary utilization, is when the primary system in a cell is at busy hour with 2% call blocking probability, based on the Erlang-B formula, see Appendix A of [Rapp02].

7) Unless stated otherwise, the secondary system transmits as much as possible, i.e., 100% STIL.

8) Without loss of generality, the extent of secondary transmissions together with the impact of the sharing on the primary system is investigated in one cell of the primary system.

9) The free space path loss model with path loss coefficient greater than 2 is adopted for path loss represented as \( L_p \) in (2.2), see Chapter 3 of [Rapp02] for more details on the path loss model.

10) Data rates are calculated at the Shannon’s limit, assuming that the effect of interference from the primary system on a secondary receiver has the same impact as that of white noise.

11) Parameters used to obtain base-case results are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Cell radius [km] (^5)</td>
<td>8</td>
</tr>
<tr>
<td>2) Distance between a secondary transmitter and a given BS [% of cell radius]</td>
<td>85</td>
</tr>
<tr>
<td>3) Distance between a secondary transmitter and its receiver [m]</td>
<td>400</td>
</tr>
<tr>
<td>4) Center frequency [MHz] (^6)</td>
<td>880</td>
</tr>
<tr>
<td>5) Antenna gain [dBi]</td>
<td>18</td>
</tr>
<tr>
<td>- at the base station (^7)</td>
<td>0</td>
</tr>
<tr>
<td>- at a mobile terminal</td>
<td>0</td>
</tr>
<tr>
<td>- at a secondary device</td>
<td>0</td>
</tr>
<tr>
<td>6) Effective gain-and-loss in a link between a BS and a MT [dB] (^8)</td>
<td>3.44</td>
</tr>
<tr>
<td>7) Channel bandwidth of the primary system [MHz]</td>
<td>5</td>
</tr>
<tr>
<td>8) Spectrum allocated to the primary system [MHz]</td>
<td>10</td>
</tr>
<tr>
<td>9) Capacity per channel of the primary system with a 5-MHz channel [number of active mobile terminals] (^9)</td>
<td>131</td>
</tr>
<tr>
<td>10) Path loss coefficient</td>
<td>3</td>
</tr>
<tr>
<td>11) Probability of harmful interference to the primary system [%]</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>12) Processing gain of the primary system with a 5-MHz channel (^10)</td>
<td>512</td>
</tr>
<tr>
<td>13) Equipment power limit of a mobile terminal ( P_{M.e, max} ) [dBm]</td>
<td>24</td>
</tr>
<tr>
<td>14) Required ( E_b/I_0 ) at the base station ( \rho_{a,1} ), and a mobile terminal ( \rho_{a,2} ) [dB]</td>
<td>4.5</td>
</tr>
<tr>
<td>15) Background noise power spectral density [dBm/Hz]</td>
<td>-174</td>
</tr>
</tbody>
</table>

\(^5\) From data in Table 3 of [Newm06], it is estimated that a typical cell radius ranges from 1 to 15 km.

\(^6\) http://wireless.fcc.gov/services/index.htm?job=service_bandplan&id=cellular

\(^7\) http://www.globalsources.com/manufacturers/Base-Station-Antenna.html

\(^8\) This is due to antenna, connector, and miscellaneous losses. The number is adjusted from Table 3 of [Newm06], using receiver sensitivity of a base station = -121 dBm, see p. 192 of [Kari02].

\(^9\) The number is calculated using the ratio between intercell interference and intracell interference at a
3.7.2 Extent of Secondary Transmissions and Performance of STIL Estimation

Fig. 3.1 shows the performance of the STIL estimation, described in Section 3.6, by comparing mean STIL achieved from coexistent sharing with that from cooperative sharing which is the actual STIL that the BS can tolerate. As expected, the actual STIL decreases as the spectrum is more utilized by the primary system (i.e., when there are more active primary users in the cell $K$), and the estimated STIL is always lower than the actual one. The estimated STIL is almost at the same level across the range of number of users $K$ shown in the figure due to the conservative estimation used. Fig. 3.1 shows that this STIL estimate is conservative when there is only one or two active MT’s in the channel; the difference between the actual and the estimated STIL when there is only one MT can exceed eight orders of magnitude. However, this difference is less than an order of magnitude when the number of active MT exceeds three.

![Figure 3.1 Mean STIL, with 95% confidence interval, vs. Number of active (primary) mobile terminals per channel](image)

In practice, $K$ fluctuates as calls begin and end. It is assumed that $K$ is modeled with Erlang-B distribution; see assumptions for numerical results in Section 3.7.1. Fig. 3.2 shows mean STIL as a function of primary utilization. As would be expected (from Fig. 3.1), the difference between the STIL

---

base station = 0.2. From p. 131 of [Bell99], when a cell is surrounded by six adjacent cells, a typical value of the interference ratio for omni-directional antenna is 0.6. Because in a highway model, for each cell there are only two adjacent cells, the interference ratio of 0.2 is used.

Parameters 12) to 15) are from [AbuR07].
estimated with coexistent sharing and the actual value from cooperative sharing is larger with low utilization, but the difference is far less pronounced than that in Fig. 3.1. With coexistence, mean STIL is significant until primary utilization approaches 100%.

![Graph showing Mean Secondary Tolerable Interference Level (STIL) vs. Primary utilization](image)

**Figure 3.2** Mean Secondary Tolerable Interference Level (STIL) vs. Primary utilization

Another meaningful way to assess the two sharing approaches is by quantifying the achievable data rate of secondary devices. Using base-case distances from the secondary transmitter to the BS, and to the secondary receiver, Fig. 3.3 shows mean secondary data rate as a function of primary utilization. From the figure, there is little difference between cooperative and coexistent sharing in achievable data rates. Secondary data rate with coexistence is only around 0.1 bps/Hz less than that with cooperation for the entire range of primary utilization considered.

When primary utilization is low, as would often be the case outside the busy hour, secondary data rates can be high, e.g., around 4 bps/Hz at 10% utilization, and around 0.5 bps/Hz at 50% utilization. Even when the primary is 100% utilized, the secondary data rate is still modest; 0.01 bps/Hz and 0.03 bps/Hz for sharing with coexistence and with cooperation, respectively. Qualitatively, this finding is consistent with the results in [Pani03][Peha04][Lee09], in which the authors found that significant secondary transmissions are possible under likely conditions. (Specifically, the authors of [Pani03][Peha04] considered sharing in the downstream of a cellular system, and found that the sharing was possible even when the cellular system was at 100% utilization. However, in this chapter and [Lee09], the extent of secondary transmissions at 100% primary utilization was modest, but would be significant at lower primary utilization.) As the previous works on gray-space sharing with a cellular system, as discussed in Section 2.2, made fundamentally different assumptions to enable the sharing, fair quantitative performance comparisons will be challenging, if not impossible, and beyond the scope of this dissertation.
3.7.3 Effect of the Spectrum Sharing on Transmit Power of a Primary User

As described in Section 3.3, to compensate for higher interference from spectrum sharing, a MT must increase transmit power. This increase can be significant, as shown in Fig. 3.4 which shows the mean power at which a MT transmits with and without spectrum sharing. The figure is obtained assuming that a secondary device would transmit at the maximum level allowed, i.e., 100% STIL.

![Image of Figure 3.3](image3.3.png)

**Figure 3.3** Mean secondary transmission rate vs. Primary utilization, when the secondary device cause 100% STIL

![Image of Figure 3.4](image3.4.png)

**Figure 3.4** Mean transmit power of a mobile terminal vs. Primary utilization, when the secondary device cause 100% STIL
Without spectrum sharing, mean transmit power increases with primary utilization. With sharing, the opposite is true. This occurs because sharing forces the furthest MT in the cell to transmit at maximum power, and when utilization is low, the furthest MT is more likely to be close to the BS. Thus, with sharing the difference in mean MT transmit power is smaller at higher utilization. With cooperation, at 10% utilization mean MT transmit power could be as high as eight times of that with no sharing, versus around two times when utilization is 100%. With coexistence, this effect is much smaller. Indeed, at 100% utilization, coexistent sharing has little impact on primary devices.

### 3.7.4 Comparison of Performance of Cooperative Sharing vs. Performance of Coexistent Sharing

As shown in Sections 3.7.2 and 3.7.3, secondary devices are often able to transmit at significant power levels without causing harmful interference, but this increases the mean transmit power of cellular MT’s. Practically, the secondary system does not need to generate interference as high as 100% of STIL as assumed in Section 3.7.3. This section shows the effect of limiting the interference generated by the secondary system to less than STIL.

Four simple approaches are considered to limit interference from the secondary system by allowing the secondary system to generate interference:

1) At STIL if and only if primary downstream power ($P_b$) is less than a threshold
2) At STIL if the STIL is less than a threshold, and at the threshold otherwise
3) At a fixed percentage of STIL (i.e., %STIL approach)
4) At STIL if and only if $P_b$ is larger than a threshold.

The third approach is adopted, and denoted as %STIL approach, because at any level of primary utilization, it gives the highest secondary data rate with low variance. (These numerical results are omitted but are available in Appendix C.)

For the %STIL approach, increasing the percentage of STIL increases secondary transmit power and data rate, and also primary transmit power. This tradeoff is demonstrated in Fig. 3.5 obtained by varying this fixed percentage. Fig. 3.5(a) shows the secondary data rates, and 3.5(b) shows mean secondary transmit power, as a function of mean transmit power of a (primary) MT.

With both forms of spectrum sharing, Fig. 3.5(a) shows that allowing secondary devices to transmit at modest power levels can at first yield significant data rates, with relatively little impact to the primary. As the secondary transmit power increases further, the secondary system gains less in improved data rate per unit increase of mean transmit power of a MT; this shows a diminishing return. Thus, if reducing the transmit power of a MT is important, there are good reasons to design secondary devices that transmit at levels below the maximum allowable power.

Once designers decide to reduce the secondary transmit power to benefit the primary system, there is virtually no difference between coexistent and cooperative sharing. Fig. 3.5(b) shows that for
a given mean transmit power of a primary MT, cooperation can enable only slightly greater transmit power for secondary devices. As shown in Fig. 3.5(a), this yields no discernible difference between coexistent and cooperative sharing in achievable data rate of the secondary system.

![Graph](image)

(a) Mean Secondary Transmission Rate

![Graph](image)

(b) Mean Secondary Transmit Power

**Figure 3.5** Trade-off between Extent of secondary transmissions vs. Mean transmit power of a mobile terminal, secondary transmissions are limited using the %STIL approach

Cooperation does allow secondary devices to transmit at a greater power. However, due to the diminishing return, the resulting gain in secondary data rate is relatively small, while the impact on the primary system is relatively large. For example, at 10% primary utilization, the maximum secondary
data rate achieved by coexistent spectrum sharing is around 3.8 bps/Hz while the maximum data rate achieved by cooperative sharing is only around 8% more. However, this requires a MT to increase its mean transmit power by more than 50%.

Fig. 3.6 shows the mean derivative of secondary transmission rate with respect to mean transmit power of a MT versus a fixed percentage of STIL (i.e. %STIL) used to limit secondary transmit power in the %STIL approach. The curves shown are from coexistent sharing. Results from cooperative sharing show the same trend; thus are omitted for brevity, but are available in Appendix C. The derivative can be seen as the marginal benefit of spectrum sharing.

![Graph showing the mean derivative of secondary transmission rate with respect to mean transmit power of a mobile terminal vs. %STIL, the results are from coexistent spectrum sharing](image)

Figure 3.6 Mean derivative of secondary transmission rate with respect to mean transmit power of a mobile terminal vs. %STIL, the results are from coexistent spectrum sharing

Fig. 3.6 shows that the marginal benefit of sharing is greater when primary utilization is lower; this is another reason why spectrum sharing may be more attractive in off-peak hours. Also, due to the diminishing returns, the marginal benefit curves in Fig. 3.6 decrease as transmit power of secondary devices increases. This effect is much stronger when primary utilization is low than when the utilization is high. For example, when primary utilization is 20%, marginal benefit drops by two orders of magnitude as secondary transmit power increases from 0 to its maximum allowable level; in contrast, it drops by less than one order of magnitude at 100% primary utilization.

3.7.5 Sensitivity Analyses

This section investigates the effect of four important parameters on the extent of secondary transmissions achievable; the parameters are

- Intercell interference at a given BS
- Amount of spectrum allocated to the primary system
- Channel bandwidth of the primary system
- Cell radius of the primary system.

### 3.7.5.1 Variation of Intercell Interference at a Base Station

This section shows how primary utilization in neighboring cells, constituting intercell interference, would affect secondary transmissions. Low-average secondary data rates at different primary utilizations in the cell are obtained by assuming that utilizations in all neighboring cells are at 100%. Then, the low-average data rates are compared to the data rates obtained when assuming that primary utilization in all cells are the same; see Fig. 3.7. The figure shows that intercell interference has little impact on the secondary data rates. Thus, the results presented in this chapter are not highly dependent on assumptions about intercell interference at the BS.

![Figure 3.7](image)

**Figure 3.7** Mean secondary transmission rate vs. Primary utilization, when primary utilization in all cells are the same vs. when primary utilization in all neighboring cells = 100%

### 3.7.5.2 Allocated Spectrum

Fig. 3.8 shows secondary data rates as a function of the amount of spectrum allocated to the primary system. The figure is obtained by assuming the secondary is transmitting as much as possible (i.e., at 100% STIL), and 10% primary utilization. With more spectrum, the primary system can support more traffic, and thus less secondary communications are expected. Fig. 3.8 shows secondary data rate decreasing only slightly with the amount of spectrum allocated. Although not shown here, results are similar at greater utilizations. Thus, base-case results are relevant with larger spectrum allocations.
3.7.5.3 Channel Bandwidth

Fig. 3.9 shows secondary data rates as a function of bandwidth of the primary’s channel, when the total amount of spectrum allocated to the primary system is held constant. The figure is obtained by assuming that the secondary is transmitting at 100% STIL, and the primary is at 10% utilization. Even though the number of active primary Mobile Terminals (MT’s) per MHz remains constant regardless of the size of primary channel bandwidth, Fig. 3.9 shows that achievable secondary data rate decreases with increasing channel bandwidth. Similar results are found at greater utilizations. The reason for this counterintuitive trend is that, even though the mean number of MT’s per MHz is independent of channel bandwidth, the variance of this number is greater when channels are small. Greater variance means that there are more times when channel utilization is well below average, and secondaries can take advantage of these times to increase their data rates.

3.7.5.4 Cell Radius

For a capacity-limited cell, changing cell radius while keeping primary traffic constant would not change the number of active MT in the cell. Hence, changing cell radius has no impact on STIL. However, decreasing cell radius reduces distance between a MT and a secondary receiver, thereby increasing interference from the MT’s to secondary receivers. Hence, the achievable secondary data rate would decrease with decreasing cell radius, see Fig. 3.10. The figure shows secondary data rates at different cell radii assuming the secondary transmitting at 100% STIL, and 10% primary utilization. From the figure, the secondary data rate drops from around 6 bps/Hz to only 0.5 bps/Hz.
when cell radius decreases from 15 km to 1 km. 

**Figure 3.9** Mean secondary transmission rate vs. Primary channel bandwidth, when the secondary device cause 100% STIL, and the primary utilization is at 10%

![Mean Secondary Transmission Rate vs. Primary Channel Bandwidth](image)

**Figure 3.10** Mean secondary transmission rate vs. Cell radius, when the secondary device cause 100% STIL, and the primary utilization is at 10%

![Mean Secondary Transmission Rate vs. Cell Radius](image)

However, if the mean number of simultaneous secondary transmissions per cell is constant, decreasing cell radius would increase the number of secondary devices per area faster than it would decrease secondary data rates. For example, decreasing cell radius from 15 km to 1 km would decrease transmission rate per secondary device by a factor of 12, but would increase the number of devices per square km by a factor of 225. Hence, overall, the secondary system might get higher total
communications with decreasing cell radius.

3.8 Conclusions

This chapter studies gray-space (primary-secondary) spectrum sharing when the primary system is a cellular system. Sharing is allowed if primary communications can withstand additional interference, rather than if there is no primary communications. Both when the sharing could happen with communications with the primary system, i.e., cooperative sharing, and without the communications, i.e., coexistent sharing, are considered.

With cooperation, it is relatively easy to determine exactly how much interference secondary devices can generate without causing harmful interference to the primary system, whereas for coexistent spectrum sharing, novel mechanisms for that purpose has been devised. In coexistent sharing, secondary devices query a sensor network which observes primary downstream communications to estimate how much additional interference the primary system can tolerate. Numerical results show that these estimates are conservative when there is only one or two primary devices transmitting, but are reasonably accurate with more primary devices.

From the numerical results, even in highly utilized spectrum like a cellular band, a significant amount of secondary communications is possible. Secondary data rates are modest when the primary system is 100% utilized: around 0.01–0.03 bps/Hz, for a 400-meter link. However, real primary systems would be near 100% utilization for a few hours per day at most. The secondary data rates increase drastically with decreasing primary utilization. For example, the data rate increases to 0.5 bps/Hz, and 4.0 bps/Hz when primary utilization is at 50%, and 10%, respectively.

Although cooperation requires standardization of sharing protocols, greater trust, and modification to legacy systems, some argue for sharing based on cooperation because it yields better performance. If only the amount of communications achievable is of concern, cooperation always yields more than coexistence. However, increasing secondary communications also causes primary mobile terminals to transmit at greater powers, especially when primary utilization is low. If the transmit power of mobile terminals is also an important design objective, there is little benefit to cooperation because marginal improvements in secondary communications diminish as secondary transmit power increases. The decision of whether to choose cooperation or coexistence should therefore be based on non-performance factors which are beyond the scope of this dissertation.

Note that even without direct communications with primary systems, the surprisingly good performance of coexistent sharing (compared to cooperative sharing) comes in part from the assumption that secondary devices have a priori access to basic information about the technology used by primary systems in the band. Rather than focusing entirely on secondary devices that know only what they can sense, policymakers and license-holders should consider making some information of this type available to help secondary systems more effectively avoid interfering with primary
systems.

These results appear to be relatively insensitive to some potentially important factors, including intercell interference, and the amount of spectrum allocated to the primary system. Sharing with primary system utilizing smaller channel bandwidth would achieve higher secondary data rate. Decreasing cell radius while keeping the mean number of secondary devices per cell constant will decrease the data rates per secondary device. However, it will increase the number of secondary devices per area even more, so this is presumably not problematic.

More generally, these results demonstrate the potential value of gray-space spectrum sharing in which secondary devices transmit when received signal from the primary system is strong, rather than when it is weak. This approach is worth considering, regardless of whether the primary system happens to be a cellular system. In addition, this study shows the potential value of sharing models in which the secondary system has information about the primary system, but does not cooperate with the primary system in real time. Such arrangements are not typically considered today.
Chapter 4

Sharing between a Single Cell and a Single Rotating Radar
4.1 Overview and Research Questions

The viability and performance of spectrum sharing with rotating radar are investigated. Radar is another promising candidate for a primary system due to large amount of spectrum it is operating in. The secondary system considered provides point-to-multipoint transmissions in multiple cells, but those cells do not blanket a region. This might happen when a cellular system only uses shared spectrum when a temporary surge of traffic in a given cell requires more capacity than the one available from its dedicated spectrum, or when an Internet service provider in hotspots.

Existing model and current proposals to sharing spectrum with radars usually allow secondary transmissions in a region where spectrum is detected as unused [FCC06][Wang08][NTIA10][Rahm11]. In contrast, with gray-space sharing, a secondary device is allowed to transmit near a radar, but only when and with a transmit power that will not cause harmful interference. The maximum transmit power of a secondary device changes over time based on the varying link loss between the device and the radar. So, secondary transmissions are possible, even though with fluctuations and interruptions while the radar rotates. Fig. 4.1 shows an example of the gain of radar antenna as a function of direction of the radar’s main beam relative to location of a given secondary device; azimuthal angle $\varphi$ of $0^\circ$ is when the main beam is pointing at the secondary device. The sharing can be either cooperative (with direct communications with the primary system) or coexistent (without the communications), see Section 1.2, and both are considered.

Figure 4.1 Example of radiation pattern of a radar antenna, the antenna is theoretically uniformly distributed aperture type, adapted from Chapter 7 of [Skol01].

$^{11}$ Fig. 4.1 is obtained assuming that the antenna’s 3-dB elevation and azimuthal beamwidths are 4.7°, and 1.4°, respectively [ITUR03]; the main beam gain is 33.5 dBi [ITUR03], and the front-to-back ratio is 38 dB [Skol01].
The investigation of the gray-space sharing with rotating radars is based on two research questions:

1) How much is the extent of secondary transmissions achievable in sharing spectrum with rotating radars?
2) Could a secondary system provide services- e.g., voice and/or data transmissions- using only the spectrum shared with radars?

Overall extent of secondary transmissions is quantified, and the effect of interruptions and fluctuations on secondary transmissions is characterized, for both up- and downstreams. Moreover, performance of the sharing is analyzed for six applications from the four classes of services [3GPP10],

1) Voice-over-IP (VoIP): belongs to conversational class of service
2) Non-interactive video on demand: belongs to streaming class of service
3) Peer-to-Peer file sharing (P2P): belongs to background class of service
4) Automatic meter reading (e.g., electricity meter reading): belongs to background class of service
5) File transfer: belongs to interactive class of service, with traffic in both up- and downstreams
6) Web browsing: belongs to interactive class of service, with traffic mostly in the downstream.

The sharing scenario is described in Section 4.2. The power level that a secondary device will be allowed to transmit and the resulting SINR (Signal to Interference plus Noise Ratio) are derived in Section 4.3. Parameters used to evaluate performance of the sharing, numerical results, and conclusions are discussed in Sections 4.4, 4.5, and 4.6, respectively.

### 4.2 Sharing Scenarios

To quantify the overall extent of secondary transmissions, a canonical case is considered in which there is one radar, one secondary Base Station (BS), and one secondary Mobile Terminal (MT), as shown in Fig. 4.2. The BS is \(d_{Rd-BS}\) away from the radar, and the MT is \(d_{BS-MT}\) from the BS.

The radar uses the same antenna for transmission and reception. The radar’s main beam rotates, hence, the radar’s radiation pattern seen by a secondary device (i.e., a MT or a BS), depends on the angle between its main beam and the device. The rotation can be either mechanical as when a radar antenna rotates, or electronic as when phased array is used. The radar transmits a series of pulses with constant power, and detects echoes from its surroundings [Skol01].

A secondary device can take advantage of the radar’s changing antenna gain, as long as it has some information on current state of the main beam rotation. It is assumed that
o A secondary device knows either the instantaneous antenna gain of the radar together with the expected value of path loss between itself and the radar, or the summation of the two.

o The device can remain synchronized with the main beam rotation.

Figure 4.2 Sharing Scenario: One cell of a secondary system is sharing spectrum with one radar

The instantaneous path loss may differ from the expected value due to fading and other factors, such as error in determining the expected path loss. The secondary device can determine this loss in a variety of ways, depending on the sharing approach and type of radar. For example, for a typical ATC radar which rotates horizontally with a constant speed, in a sharing scheme based only on coexistence, the device may combine any a priori knowledge of the radar with observations over numerous rotations to determine the periodic pattern of the expected link loss, which includes antenna gains of the secondary device and the radar, and the expected path loss. Alternatively, some observations can be replaced by cooperative sharing, which allows the radar to explicitly inform secondary device about its rotation patterns. The situation is more complicated with a weather radar, which scans in both horizontal and vertical planes at a speed that may change when a storm is approaching [ITU-R03], or a tracking radar, which may suddenly change the direction of its main beam in response to a target’s movements. Cooperative schemes are more useful in such cases. With or without cooperation, the secondary device should stop transmitting, and resynchronize if the monitored rotation pattern appears to change unexpectedly. The possibility of synchronization errors is beyond the scope of this dissertation.

A radar insidiously misdetects targets if the interference level is high enough to disrupt its echo receptions [Bedf07]. To protect radars from harmful interference, a maximum in-band Interference-to-Noise Ratio (INR) is defined (e.g., by [ITU-R03]) below which the radar’s detection performance is largely unaffected, and INR must be kept below this limit with sufficiently high probability. Many radar systems are used in applications that affect people’s lives, so preventing harmful interference in these cases is especially important. For example, recently in the UK there has been discussion on the risk of inadvertent interference to aviation radars operating in the 2.7 GHz band from LTE operating in 2.6 GHz [Ofco09].
It is assumed that the secondary system uses Orthogonal Frequency Division Multiple Access (OFDMA), which is used in wireless communication systems such as LTE. However, the proposed sharing model could be applicable to other secondary systems as well. The sharing model assumes that:

1) Some information of the radar, such as pulse power, rotating period, and tolerable interference level, is known to the secondary system. These parameters rarely change over time.

2) The secondary system:
   a) uses Time Division Duplex (TDD)
   b) will use as much available bandwidth as possible
   c) can always transmit signaling traffic without causing harmful interference to a radar. This could easily happen, e.g., if signaling is transmitted in a frequency band different from the one shared with the radar.

There are additional assumptions on the applications considered for the secondary system:

1) Signaling traffic is negligible. Typically, the amount of this traffic is small compared to that of the application content.
2) Time used to transfer data within networks (i.e., network delay) is negligible compared to the time used to transfer the data between MT and BS.
3) MT’s using video on demand have enough memory for buffering, and a constant streaming rate is used.

### 4.3 Maximum Allowable Transmit Power of a Secondary System and Achievable SINR

A secondary device determines its maximum allowable transmit power, $P_{S,max}$, from the radar’s tolerable level $I_{tol}$ (which is known to the device, assumption 1 of the sharing model in Section 4.2), and the expected link loss between itself and the radar, which varies depending on azimutal angle between the radar’s main beam and the device, $\phi$. When one secondary transmitter is using as much bandwidth as possible (assumption 2 of the sharing model), from $I_{tol}$, the device at distance $d_{Rd-S}$ from a radar can determine $P_{S,max}$, based on the definition of link loss given by (2.1), as

$$P_{S,max}(\phi, d_{Rd-S}) = \min \left\{ \frac{I_{tol}}{S_{L} I_{tol} - S_{Rd-S}(\phi, d_{Rd-S})}, P_{S,max} \right\},$$

(4.1)
where,

- $P_{s_e,\text{max}}$ is transmit power limit of the secondary transmitter
- $L_{i,\text{Rd-S}}(\varphi, d_{\text{Rd-S}})$ is the link loss, between the radar and the transmitter, that the transmitter expects
- Margin $M$ provides interference protection for any difference between the instantaneous path loss and the one expected by the secondary transmitter, including differences from signal fading and from any measurement error during the start-up phase.

As discussed in Section 4.2, there are various ways that a secondary transmitter can determine $\bar{L}_{i,\text{Rd-S}}$, e.g., when coexistent sharing is employed, $\bar{L}_{i,\text{Rd-S}}$ could be estimated during a start-up phase by averaging over repeated samples of instantaneous link loss between the radar and the transmitter. $L_{i,\text{Rd-S}}$ accounts for the radar’s antenna gain, antenna gain of the secondary transmitter, and path loss between the radar and the transmitter. When radar pulse power $P_{\text{Rd}}$ and rotating period are known to the secondary system (see sharing model assumption 1 in Section 4.2), using the definition of link loss defined by (2.1), the secondary transmitter determines $L_{i,\text{Rd-S}}$ from

$$L_{i,\text{Rd-S}}(\varphi, d_{\text{Rd-S}}) = \frac{P_{\text{Rd-S}}(\varphi, d_{\text{Rd-S}})}{P_{\text{Rd}}}, \quad (4.2)$$

where $P_{\text{Rd-S}}(\varphi, d_{\text{Rd-S}})$ is the instantaneous power of radar pulses received at the secondary transmitter. With this assumption, coexistent and cooperative sharing - in which a radar tells $P_{\text{Rd}}$ to a secondary device - achieve the same $L_{i,\text{Rd-S}}$, and hence the same $P_{s,\text{max}}$.

When the difference from the expected path loss is high, to keep the probability of harmful interference at a particular small level, a higher margin will be needed; this higher margin will result in more conservative extent of secondary transmissions. Even with an adequate margin, gray-space sharing inherently introduces some risks that are not presented with white-space sharing, e.g., the power control mechanisms in a secondary system might be hacked and made to interfere with the primary system, or some system bug might cause a secondary transmitter to inaccurately calculate $P_{s,\text{max}}$. To mitigate the effects of interference in such cases, a new approach to regulation and governance is required, which is beyond the scope of this dissertation, but discussed in [Peha11][Peha12].

From (4.1), it is possible to calculate SINR, $\rho_s$, of the secondary device’s transmissions, which will in turn determine achievable data rates. This calculation is based on the assumptions that 1) the time between radar pulses during which the secondary will not experience interference from the radar, which is typically less than 1 ms [ITUR03], can be neglected, 2) power spectral density of radar is constant over the shared spectrum band, 3) a secondary device transmits with equal power spectral density across the band, and 4) adjacent channel interference is negligible, for both adjacent channel
interference into and out of the band. Assumptions 1) and 2) make the data rates that can be achieved by secondary systems lower than would be possible with more realistic assumptions. Other ways to allocate power spectral density of a secondary device over the shared spectrum might also be possible, but those are not considered here. If adjacent channel interference were significant, this would decrease achievable secondary transmissions. When a secondary transmitter and receiver are $d_{rd-TX}$ and $d_{rd-RX}$ away from the radar, and the secondary transmitter and receiver are $d_{TX-RX}$ apart, $\rho_S$ is

\[
\rho_S(\phi, d_{rd-TX}, d_{rd-RX}, d_{TX-RX}) = \frac{L_{LS}(d_{TX-RX}) \times P_{s}/P_{max}(\theta_{rd-TX})}{n_s W_S + L_{LS}(\phi, d_{rd-RX}) \times P_{rd}},
\]

(4.3)

where,

- $L_{LS}(d_{TX-RX})$ is link loss between the secondary transmitter-receiver pair
- $n_s$ is background noise power spectral density at the secondary receiver
- $W_S$ is bandwidth of secondary transmissions.

### 4.4 Performance Measurement

The parameters used to evaluate the performance of spectrum sharing with radar are:

- Secondary data rate, $R_{b,S}$, calculated from $\rho_S$, which is shown in (4.3)
- Fraction of time that a secondary device can achieve a required data rate $R_{b,req}$
- Statistics and distribution of time that $R_{b,S} < R_{b,req}$, i.e., interrupted time.

From the SINR in (4.3), the resulting $R_{b,S}(\rho_S)$ is calculated using a set of equations, obtained from regression analyses on 3GPP data [Jaci09]; see Appendix B for details on the equations. The data are obtained assuming 1) an urban or suburban environments with fairly small cell and low delay spread (i.e., Extended Pedestrian A channel model); 2) a very low speed user (i.e., 5-Hz Doppler frequency); and 3) Multiple Input Multiple Output in both up- and downstreams. The resulting $R_{b,S}(\rho_S)$ will be the maximum rate obtained among QPSK, 16QAM, and 64QAM modulation schemes.

From (4.3), $R_{b,S}$ varies periodically, and is a function of $\phi$, $d_{rd-TX}$, $d_{rd-RX}$, and $d_{TX-RX}$. For a given $R_{b,req}$, the interrupted time together with its statistics, and the fraction of time that $R_{b,S} \geq R_{b,req}$, $p_{R_{b,S} > R_{b,req}}$, in a rotating cycle can be obtained.
where,

- $\tau_{R,b,S\geq R_{b,req}}$ is the $i$-th period, in a rotating cycle, that $R_{b,S} \geq R_{b,req}$
- $T_{rd}$ is the rotating period of the radar.

Different service classes of applications require different quality measurements [3GPP10]. For the considered applications, discussed in Section 4.1, different parameters are used to evaluate performance: (As described in Section 4.2, network delay is assumed to be negligible.)

- VoIP requires a symmetric constant data rate $R_{b,VoIP}$ in both up- and downstreams, and an interrupted time $\tau_{VoIP}$ lower than an acceptable level.
- Video on demand initially requires an average downstream rate over a rotating cycle $\overline{R_{b,S}}$ larger than the constant streaming rate $R_{b,Video}$; as a buffer is used to maintain streaming continuity, the probability that the buffer will be empty is also evaluated.
- P2P, file transfer, web browsing, and meter reading, require no minimum data rate. For P2P and file transfers, achievable data rate is an appropriate measure. For an interactive application like web browsing, web page downloading time is a more appropriate measure.

For VoIP, samples of $\tau_{VoIP}$ across a rotating cycle, and its distribution, were obtained from instantaneous secondary data rate and $R_{b,VoIP}$. As VoIP is bi-directional, instantaneous secondary data rate for VoIP is the minimum of the up- and downstream values.

For video on demand, given the amount of data needed for initial buffering, $V_{bf,\text{init}}$, the amount of data buffered at time $t$ after streaming starts (or resumes) $V_{bf}(t, t_{st})$ is

$$V_{bf}(t, t_{st}) = V_{bf,\text{init}} + \int_{t_{st}}^{t} R_{b,S,\delta}(t') \, dt' - R_{b,Video},$$  

(4.5)

where,

- $R_{b,S,\delta}(t')$ is the instantaneous downstream secondary data rate
- $t_{st}$ is the streaming starting time
- $R_{b,Video}$ is a required constant data rate for streaming.

From (4.5), for a given start time $t_{st,i}$, one can determine if and when the buffer would be empty. Based on multiple values of $t_{st,i}$ across a rotating cycle, the probability that the buffer will be empty,
i.e., probability of interruptions, $p_{intrp}$, during streaming can be obtained from,

$$p_{intrp} = \frac{\sum_{i=1}^{N_{st}} v_i}{N_{st}},$$

(4.6)

where,

- $v_i$ is a binary variable that equals 1 if $V_{bf} = 0$, and 0 otherwise
- $N_{st}$ is the total number of $t_{st,i}$ considered.

Note that video on demand operates when $R_{b,s} > R_{b,video}$, hence, if $V_{bf}(t, t_{srt,i}) \neq 0$ in a rotating cycle, it will never go to zero during streaming.

For P2P, file transfer, web browsing, and meter reading, a wide range of file sizes is transferred. Because secondary transmissions can be interrupted, files of different sizes experience different perceived data rates $R_{b,s,p}(t_{st})$, which depend on when a file is transferred $t_{st}$.

$$R_{b,s,p}(t_{st}) = \frac{v_f}{T_{f}(t_{st})},$$

(4.7)

where,

- $v_f$ is size of a file
- $T_{f}(t_{st})$ is the time that a secondary device uses to transfer the file, i.e., file transfer time.

Regarding the transfer of each file, it is assumed that 1) each file has to be transmitted completely before another file can be transmitted, and 2) only transmission delay is considered; time period for which each file waits in a queue before it is transferred (i.e., queuing delay) is not considered. Distribution of $R_{b,s,p}$ can be obtained by choosing different $t_{st}$'s.

### 4.5 Numerical Results

Parameters used to obtain numerical results are summarized in Section 4.5.1. The performance of secondary transmissions averaged across a cell is evaluated in Section 4.5.2; this average performance is measured in terms of both mean data rate and percentage of time that a secondary device can transmit. Performance for video streaming and VoIP are evaluated in Sections 4.5.3 and 4.5.4, respectively. The characteristics and performance of secondary transmissions for file upload- and downloading services are shown in Section 4.5.5. Sensitivity of these results on important system parameters including (secondary) cell radius, radar transmit power, radar tolerable
interference level, and radar rotating period is investigated in Section 4.5.6.

4.5.1 Assumptions for Numerical Results

Numerical results are obtained using the following assumptions:

1) Antenna of the radar is uniformly-distributed aperture type [Skol01]. Directivity of this theoretically derived antenna pattern is usually lower than antennas used by radars, and hence would result in more conservative extent of secondary transmissions achievable.

2) The secondary system’s duplex technique is symmetric TDD.

3) The ITU-R P.1546 path loss model [ITU09] is adopted between the radar and the secondary system. The path loss model is valid in the frequency ranging from 300 MHz to 3 GHz, and distances from 1 up to 1,000 km. Conservatively, flat terrain is assumed; even though the assumption would result in longer distances that the radar and the secondary system can affect each other, it will increase interference between the radar and the secondary system, and reduce the extent of secondary transmissions.

4) For the path loss between a BS and MT, the COST 231 Walfisch-Ikegami model [Kurn99] is adopted.

5) Rayleigh fading [Cave02] is considered in a link between the radar and a MT; as the difference between the height of radar and that of a MT is typically large, Line of Sight (LoS) between radar and a MT rarely exists.

6) For a link between the radar and a BS, Ricean fading [Cave02] with two different K factors is assumed for a link within LoS distance from the radar, and for a link beyond the LoS distance.

7) Fading considered in 5) and 6) has equal impact on all sub-carriers of the secondary system. In practice, at a given time some sub-carriers might experience high fading while the others experience low fading; one might take advantage of this to achieve higher secondary data rate than what is quantified in this dissertation.

8) The difference between instantaneous link loss, and the value expected by a secondary transmitter \( L_{i,Rd-s} \) in (4.1), that is due to measurement error, is negligible compared to the difference due to fading.

9) Unless stated otherwise, the values of parameters used to obtain base-case numerical results are summarized in Table 4.1.

10) The ranges of parameters considered for sensitivity analysis are summarized in Table 4.2.

To evaluate a typical extent of secondary transmissions achievable in a cell, data rate and percentage of time that a secondary device can transmit, averaged across all directions of the radar’s main beam and across a cell, need to be considered. Hence, the case when the location of the secondary user is uniformly located across the cell is considered. In contrast, to evaluate the performance of sharing for a given application, the location of a secondary user needs to be specified, and the worst case is considered when the user is fixed at the cell edge in the worst direction, which is
toward the radar.

Table 4.1 Parameters for Base-Case Numerical Results for Spectrum Sharing with Radars (Unless stated otherwise.)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Radars [ITUR09]:</strong></td>
<td></td>
</tr>
<tr>
<td>Operating Frequency [GHz]¹²</td>
<td>2.8</td>
</tr>
<tr>
<td>Bandwidth [MHz]</td>
<td>3.0</td>
</tr>
<tr>
<td><strong>Antenna Characteristics</strong></td>
<td></td>
</tr>
<tr>
<td>- Elevation 3-dB Beamwidth [degree]</td>
<td>4.7</td>
</tr>
<tr>
<td>- Azimuthal 3-dB Beamwidth [degree]</td>
<td>1.4</td>
</tr>
<tr>
<td>- Main Beam Gain [dBi]</td>
<td>33.5</td>
</tr>
<tr>
<td>- Front to Back Ratio [dB], [Skol01]</td>
<td>38</td>
</tr>
<tr>
<td>- Height [m]</td>
<td>30</td>
</tr>
<tr>
<td>Rotating Period $T_{rd}$ [s]</td>
<td>4.7</td>
</tr>
<tr>
<td>Transmit Power $P_{rd}$ [MW]</td>
<td>0.45</td>
</tr>
<tr>
<td>Interference to Noise Ratio (INR) [dB]</td>
<td>-10</td>
</tr>
<tr>
<td>Background Noise [dBm]</td>
<td>-106</td>
</tr>
<tr>
<td><strong>Cellular System [Holm09]:</strong></td>
<td></td>
</tr>
<tr>
<td>Antenna Gain of a MT, Omni-Directional [dBi]</td>
<td>0</td>
</tr>
<tr>
<td>Antenna Gain of a BS, Sectorized [dBi]</td>
<td>18</td>
</tr>
<tr>
<td>Equipment Power Limit of a MT [dBm]</td>
<td>23</td>
</tr>
<tr>
<td>Equipment Power Limit of a BS [dBm]</td>
<td>46</td>
</tr>
<tr>
<td>Cell Radius $R$ [m]</td>
<td>800</td>
</tr>
<tr>
<td>Background Noise Power Spectral Density $n_s$ [dBm/Hz]</td>
<td>-174</td>
</tr>
<tr>
<td>Noise Figure at a Receiver [dB]</td>
<td>5</td>
</tr>
</tbody>
</table>

4.5.2 Overall Performance of Secondary Transmissions

Fig. 4.3 shows the mean secondary data rate over a rotating cycle, and the fraction of time that a secondary device can transmit, as a function of the distance between the BS and the radar. The data rate and the fraction of transmission time are averaged across the cell.

---

¹² ATC radars operate in this band.

¹³ The antenna is up-tilted to reduce ground-reflected signals, and hence its gain in the horizontal direction is 5 dB lower than the main beam gain [Hink76].
Table 4.1 (Contd.) Parameters for Base-Case Numerical Results for Spectrum Sharing with Radars
(Unless stated otherwise.)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>COST 231 Walfisch-Ikegami Model(^{14}) [Kurn99]:</td>
<td></td>
</tr>
<tr>
<td>Building Height [m](^{15})</td>
<td>15</td>
</tr>
<tr>
<td>MT Antenna Height [m]</td>
<td>1.7</td>
</tr>
<tr>
<td>BS Antenna Height [m](^{16})</td>
<td>30</td>
</tr>
<tr>
<td>Other Parameters:</td>
<td></td>
</tr>
<tr>
<td>Ricean K Factor [dB]</td>
<td></td>
</tr>
<tr>
<td>- Within LoS from a Radar(^{17})</td>
<td>10</td>
</tr>
<tr>
<td>- Beyond LoS from a Radar (i.e., Rayleigh Fading)</td>
<td>0</td>
</tr>
<tr>
<td>Margin (M_2) in (4.1) [dB]</td>
<td></td>
</tr>
<tr>
<td>- For Ricean K Factor = 10 dB</td>
<td>5</td>
</tr>
<tr>
<td>- For Ricean K Factor = 0 dB</td>
<td>8.4</td>
</tr>
<tr>
<td>Probability of Harmful Interference [%](^{18})</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Applications Considered for the Secondary System:</td>
<td>15</td>
</tr>
<tr>
<td>Required Data Rate for VoIP (R_{b,VoIP}) [kbps], [3GPP10]</td>
<td></td>
</tr>
<tr>
<td>Video on Demand [Netf12][Zamb09]</td>
<td></td>
</tr>
<tr>
<td>- Required Streaming Rate [Mbps]</td>
<td>1.6</td>
</tr>
<tr>
<td>- Content is initially buffered enough to play for [s]</td>
<td>2</td>
</tr>
</tbody>
</table>

Fig. 4.3 shows that spectrum shared with a radar can support high average data rates, even when a secondary device is close to the radar. Consider the conventional non-opportunistic approach. The distance between the BS and the radar that allows the secondary device to transmit all the time at the data rate achieved in dedicated spectrum, i.e., system rate limit, is around 400 km; the required separation is around 100 km for downstream, and 400 km for upstream. Alternatively, if only

\(^{14}\) Other required parameters including width of street, building separation, and orientation between the street and the wave are 17.5 m, 35 m and 90°, respectively, as suggested in [Kurn99].

\(^{15}\) This number represents a building with small to medium height.

\(^{16}\) With this height of BS antenna, the cell radius can be up to 1.5 km [Hom09]. In the US, a nationwide mean height of a commercial cellular tower is around 60 m (the tower portfolio is from a major US tower company, American Towers, see Table 1 of [Hall10]). Hence, 30 m represents a reasonable compromise between an on-tower antenna and a rooftop one.

\(^{17}\) With the considered heights of radar and BS, the LoS distance is around 20.8 km, see Chapter 2 of [Pars00].

\(^{18}\) The value selected is less than radar’s required misdetection probability which is around 1% [Pint11].
the radar is to be protected [Wang08], with a 8.4 dB fade margin, the minimum separation is 286 km. In contrast, by taking advantage of main beam rotation, significant downstream transmissions are possible at a fraction of this 400 km distance. At 50 km from the radar, which is just 12.5% of 400 km, in the downstream the BS can transmit almost all the time with an average data rate near the system limit of 10.8 Mbps. In the upstream, at 19% of 400 km, a secondary device can transmit 90% of the time, with an average rate around 62.5% of the 8.0 Mbps rate limit.

Table 4.2 Ranges of Parameters Considered for Sensitivity Analysis for Sharing with Radars

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell Radius $R$ [km], [Holm09]</td>
<td>0.2</td>
</tr>
<tr>
<td>Radar Transmit Power $P_{\text{Rd}}$ [MW], [ITUR03]</td>
<td>0.025</td>
</tr>
<tr>
<td>Radar Interference to Noise Ratio (INR) [dB], adapted from [Bedf07]</td>
<td>-13</td>
</tr>
<tr>
<td>Radar Rotating Period $T_{\text{Rd}}$ [s], [ITUR03]</td>
<td>4</td>
</tr>
</tbody>
</table>

![Figure 4.3](image.png)

(a) Mean Data Rate, with 95% Confidence Interval

Figure 4.3 Extent of secondary transmissions vs. Distance between a base station and a radar
(b) Percentage of Time that a Secondary Device can Transmit, with 95% Confidence Interval

**Figure 4.3 (Contd.)** Extent of secondary transmissions vs. Distance between a base station and a radar

Fig. 4.3 also shows that the secondary system uses spectrum more efficiently in the downstream than in the upstream, in terms of data rate per MHz of spectrum, and fraction of time that the system can transmit. If the goal is to maximize spectral efficiency, spectrum sharing with radar might be more suitable for applications that have more traffic in the downstream, which is a typical characteristic of many current applications.

### 4.5.3 Performance of Non-Interactive Video Streaming

One important measure of performance for video on demand is average downstream data rate. The previous section showed that even at small distance from the radar, the achievable average downstream rate is very close to the rate one might get in dedicated spectrum.

Another performance measurement is the probability that streaming will be interrupted. It is found that this probability is sufficiently low, and thus unlikely to be a problem. Even when the average downstream rate $\bar{R}_{b,S}$ is only 4% higher than the streaming rate (which is assumed to be 1.6 Mbps), and the BS is just 9.6 km from the radar. With the initial buffering of 2 s the possibility of interruption is less than 0.001. The hypothesis testing on this probability used 10,000 samples; the resulting p-value is 0.0008. The result is obtained when the user is at the cell edge closest to the radar. From the assumption on the initial buffering, the amount of content the application needs to initially buffer for 1.6 Mbps streaming rate is 400 kB. With only 3 MHz of spectrum, the transfer time for the initial-buffer content is at most 3 s even at only 9.6 km from the radar.

Due to the high achievable average downstream data rate, and the small chance of being interrupted, even with only a few seconds of initial buffering, and when the secondary is very close to
the radar, video on demand is a very promising application for spectrum that is shared with radar.

4.5.4 Performance of Voice over IP

One important requirement for VoIP is a symmetric data rate in both up- and downstreams. Hence, the asymmetric data rate, as shown in Fig. 4.3(a), will limit performance of VoIP. Moreover, VoIP performance depends on instantaneous data rate, and the application cannot tolerate long interruptions. In shared spectrum, interruptions sometimes cause the instantaneous secondary data rate to be much lower than the average value, and could be a problem for VoIP.

With a required instantaneous rate of 15 kbps, Fig. 4.4 shows the probability that the resulting interrupted time (i.e., latency) $\tau_{\text{VolP}}$ would be less than an acceptable level $\tau_{\text{VolP,a}}$, as a function of distance of secondary transmissions from a radar; $\tau_{\text{VolP,a}}$ is taken as 80 and 150 ms [3GPP10], and the results are obtained when the user is at the cell edge. Fig. 4.4 shows that $\tau_{\text{VolP,a}}$ is always satisfied beyond 70 km from the radar. Note from Fig. 4.3(a) that, at this distance, the average upstream rate is around 5 Mbps; however, the application can only obtain at least 15 kbps instantaneous data rate with the acceptable interrupted time. Compared to VoIP operating in dedicated spectrum, VoIP is relatively inefficient in spectrum shared with radar, and hence is not attractive for such sharing.

![Figure 4.4 Possibility that a VoIP latency is less an acceptable Level vs. Distance between a base station and a radar](image)

4.5.5 Performance of File upload- and Downloading Applications

As occurs with VoIP, the fluctuations in data rate can also affect performance of file up- and downloading services, such as file transfers, Peer-to-Peer file sharing (P2P), meter reading, and web
browsing. Files with different sizes would experience different ranges of perceived data rate, defined in Section 4.4. The fluctuation and its implications on applications in this service class are investigated, in Sections 4.5.5.1 and 4.5.5.2, respectively.

4.5.5.1 Fluctuations in Secondary Perceived Data Rate

Fig. 4.5 shows maximum, mean and minimum perceived downstream data rate of a user at the cell edge as a function of size of a file being transferred. The results from the upstream show a similar trend, and thus are omitted but can be found in Appendix C.

Fig. 4.5 shows that a small file would experience a wider range of perceived data rate than a large one, because a small file is more likely to be transferred in less than a rotating period. Due to sporadic interruptions in transmissions, file transfer time, and hence, perceived data rate, would highly depend on when the transfer starts. In contrast, the fluctuations are unnoticeable and appear the same as if this were dedicated spectrum when the file size is above a certain threshold that makes file transfer time much longer than the rotating period; this threshold is much smaller for secondary devices that are closer to the radar. From Fig. 4.5, at 4 km from the radar, the fluctuation in perceived data rate tends to be insignificant for any file larger than 100 kB, while at 10 km, the fluctuation starts to be insignificant for a file larger than 1 MB.

Thus, fluctuations in perceived data rate depend on file size, and distance between a secondary device and the radar. As long as radar transmissions still affect secondary transmissions, these fluctuations will be most noticeable when a secondary device far from the radar transfers a small file. The fluctuations could be a problem for some applications, but not all.

![Figure 4.5 Perceived downstream data rate vs. Size of a file being transferred](image-url)
4.5.5.2 Implications of the Fluctuations for Various Applications

Start with P2P and transfers of a large file (e.g., a song which is typically much larger than 1 MB), for which the perceived data rate is the typical measure of performance. P2P is often used for transfers of large files, and transfers that do not have strict delay requirements. Because file size is large, interruptions in secondary transmissions would have little impact on perceived secondary data rate, as shown in Fig. 4.6. Moreover, Fig. 4.3(a) shows that a secondary user could achieve a high average data rate even close to the radar. Hence, P2P and transfers of large files could also be promising for spectrum sharing, although it might not be quite as promising as video on demand, of which traffic is mostly in the downstream.

Guaranteeing high data rates for small files is more challenging. Fig. 4.6 shows the 1st-percentile and mean perceived downstream rate, of a user at the cell edge, as a function of distance of a BS from the radar. Fig. 4.6 shows that transferring files with different sizes would experience approximately the same mean perceived rate at any distance from the radar. However, for small files (i.e., smaller than 1 MB), the 1st-percentile perceived rate can be much lower than the mean, hence, for transfers of small files, this could be a problem, if users would not tolerate fluctuations in perceived data rate. Hence, such applications would not be suitable for sharing spectrum with radar. Upstream results are similar, and thus are omitted.

There are also applications for which file transfer time, rather than perceived data rate, is the important performance measure. Although transferring small files, the applications would still work well in spectrum shared with radar. One example is web browsing, since users probably expect a web page to be retrieved just as quickly, even if it contains many more bytes. Time for downloading a web page is suggested to be less than 2 to 4 s, with a preferred target of 0.5 s [3GPP10].

Fig. 4.6 shows that just beyond 10% of the 286 km distance at which secondary transmissions will not affect the radar, a user downloading a 1 MB web page would experience the 1st-percentile perceived rate of around 8 Mbps. The 90th-percentile webpage size in 2010 was 660 kB [Rama11], hence, even with 3 MHz of spectrum, for 99% of the time, most of transfers would experience file transfer time less than 1 s. For the web pages larger than 1 MB, the file transfer time would be larger than 1 s; however, this transfer time will be fairly close to the one when transmissions occur in dedicated spectrum. For example, with the 10.8 Mbps downstream rate limit, the transfer time for a web page larger than 1 MB would be larger than 740 ms. Thus, quality of service is good for web browsing even close to the radar.

There are also applications that transfer small files, but can tolerate interruptions during transmissions, e.g., automatic meter reading. Generally, the system consists of many meters installed at users’ premises; each of these devices intermittently transfers a small amount of information, in the order of tens of kB, to an aggregation point. Because significant delays are tolerable, an average amount of data that can be transferred in a given period is the important performance measure. As Fig. 4.3(a) shows that the secondary system can achieve high average data rate, spectrum shared with radar would also work well with this application.
Figure 4.6 First percentile, and mean perceived downstream data rate vs. distance between a base station and a radar

4.5.6 Sensitivity Analysis

In this section, the sensitivity of the average extent of secondary transmissions, and fluctuations in perceived secondary data rate are investigated on four important parameters including,

- Secondary cell radius $R$
- Radar transmit power $P_{rad}$
- Radar tolerable interference level represented as maximum INR
- Radar rotating period $T_{rd}$.

4.5.6.1 Sensitivity of Average Extent of Secondary Transmissions

Unlike the other parameters considered, changing radar rotating period $T_{rd}$ will not change the amount of data a secondary device can transfer in a given period of time, and thus, it will have no impact on the average extent of secondary transmissions. Hence, those graphical results will be omitted. Similarly to the previous sections, the average extent of secondary transmission is calculated across a cell.

Fig. 4.7 shows the average secondary data rate, and the percentage of time that a secondary device can transmit for both up- and downstreams, as a function of the cell radius $R$. Fig. 4.7 shows that, as expected, the average data rate and percentage of time that a secondary device can transmit at a given distance from the radar decreases with increasing cell radius. It is still the case that extensive communications are possible for a cell that is relatively close to the radar, but for larger...
cells, the cell must be somewhat farther from the radar.

Fig. 4.7(a) shows that in the upstream, at 70 km from the radar, the average data rate of a cell with 0.8 km radius is around 60% of the system rate limit, as defined in Section 4.5.2; 70 km is around 24% of the 286 km distance. For a larger cell, with 1.5 km radius, the same level of upstream data rate can be achieved at around 100 km from the radar, i.e., 35% of 286 km. Moreover, in the downstream, at only 14% of the 286 km distance, a cell with 1.5 km radius can achieve almost 100% of the system rate limit. Hence, the secondary system can still achieve high data rates close to the radar, even with a fairly large cell.

(a) Average Secondary Data Rate

(b) Percentage of Time that a Secondary Device can Transmit

Figure 4.7 Sensitivity of extent of transmissions on cell radius
Fig. 4.8 shows the average data rate as a function of radar transmit power $P_{Ru}$. Changes in radar transmit power have a similar effect on both data rate and the percentage of time that a secondary device can transmit, similar to what was observed with changes in cell radius in Fig. 4.7. Hence, results showing the percentage of time that a secondary device can transmit are omitted for brevity, but can be found in Appendix C.

Fig. 4.8 shows that the data rate decreases with increasing transmit power of the radar. However, high average data rates close to a radar can still be attained, even when the radar transmit power is high, if the cell is a bit farther from the radar. For example, in the upstream when the radar transmit power increases from 0.5 to 1.4 MW, the distance from a cell to the radar needs to be increased from around 70 to 90 km, so that the achievable data rate is still around 60% of the system rate limit; 90 km is still only 31% of the 286 km. In the downstream, the increase in radar transmit power only slightly decreases the average data rate, hence, the secondary system can achieve high data rates at fairly short distance to a radar transmitting with high power.

![Figure 4.8 Sensitivity of average data rate on radar transmit power](image)

Fig. 4.9 shows the average data rate as a function of the radar’s tolerable INR. The results for percentage of time that a secondary device can transmit shows the same trend as its average data rate counterpart, and hence is omitted, but can be found in Appendix C. As expected, the data rate slightly decreases with a more stringent requirement on radar tolerable interference level, i.e., smaller tolerable INR. For example, in the downstream at 20 km from the radar, reducing INR from -10 to -13 dB would decrease the average data rate from 93% to 88% of the rate limit. Note from Fig. 4.9 that no impact of INR on the average upstream data rate is observed because a secondary device can transmit with power higher than its equipment limit without causing harmful interference to a radar. Hence, the device will still transmit at its equipment power limit even when the radar tolerable interference level changes; this is more likely to happen when there are only a few MTs in a cell, as
assumed in this scenario. However, when there are many users, it is expected that reducing radar’s INR would decrease the data rate in both up- and downstreams.

Similarly to Fig. 4.7 and Fig. 4.8, the secondary system can still achieve high data rates under more stringent INRs when a cell is a bit farther from the radar, e.g., by moving a cell from 20 to only 30 km from the radar, the downstream data rate, for INR = -13 dB, is 96% of the rate limit. Thus, the secondary system can also achieve high data rates close to the radar even under radar’s stringent tolerable interference requirements.

**Figure 4.9** Sensitivity of average data rate on radar tolerable interference level

In addition, for a typical range of cell radius, transmit power and tolerable interference level of the radar, comparing Fig. 4.7, Fig. 4.8 and Fig. 4.9 shows that cell radius would relatively have higher impact on the extent of secondary transmissions compared to the other parameters. For example, at 20 km from the radar, in the downstream, the average data rate increases from 70% to 99% of the rate limit, when cell radius decreases from 1.5 to 0.2 km. At the same distance from the radar, the average downstream data rate increases from 88% to 96% of the rate limit, when INR increases from -13 to -7 dB, while the average downstream data rate slightly increases from 93% to 94% of the rate limit, when the radar transmit power decreases from 1.4 to 0.025 MW. Hence, at least for an ATC radar, if one adjusts design parameters within a range that might be considered reasonable, the extent of secondary transmissions achievable seems to be affected more by the radius of the cell rather than by the transmit power and tolerable interference of the radar that the secondary system will share spectrum with.

### 4.5.6.2 Sensitivity of Fluctuations in Secondary Perceived Data Rate

Fig. 4.10 shows fluctuations in perceived data rate as a function of file size: in the upstream at 70 km from the radar, and in the downstream at 20 km. The fluctuations in perceived data rate are
evaluated for a user at the edge of a cell in the direction toward the radar. It is seen, from Section 4.5.5, that when radar transmissions still affect transmissions of the secondary system, the farther away from the radar, the higher fluctuations in perceived data a secondary user would experience. At 70 km from the radar for upstream and at 20 km for downstream, secondary transmissions are still highly affected by the radar; Fig. 4.3 shows that the increasing rate, i.e., slope, of the extent of secondary transmissions with distance from the cell to the radar starts to decrease after around 70 and 20 km for up- and downstreams, respectively.

Fig. 4.10(a) shows that, as expected, a user in a smaller cell would experience higher fluctuations in perceived data rate than that in a larger cell. However, the increase in fluctuations in perceived data rate with decreasing cell radius will not be a problem, as a user uploading large files (i.e., larger than 1 MB) would still experience insignificant fluctuations even when the cell radius is as small as 200 m. For the downstream, Fig. 4.10(b) shows that at only 20 km from the radar, a user downloading large files would also experience insignificant fluctuations in perceived data rate even when the cell radius is 200 m. Hence, a user transferring a large file would still experience insignificant fluctuations in perceived data rate even in a small cell.

The effect of the other parameters, including transmit power, INR, and rotating period, on the fluctuations in downstream perceived data rate are similar to the effect of cell radius as shown in Fig. 4.10(b). Hence, the results on fluctuations of perceived data rate in the downstream are omitted, but can be found in Appendix C. For the upstream, Fig. 4.11 shows the perceived data rate as a function of file size when (a) the radar transmit powers are 0.025, 0.45, and 1.4 MW, (b) INR is -13, -10, and -7 dB, and (c) the rotating periods are 4, 4.7, and 6 s; the results are from a cell 70 km away from the radar. Fig. 4.11 shows that changes in these three parameters have only marginal impact on the fluctuations in data rate perceived when files larger than 1 MB are uploaded. Hence, when one adjusts these parameters within this reasonable range, a user transferring a large file would still experience insignificant fluctuations in perceived data rate. Note from Fig. 4.11(b) that, as it is expected from the sensitivity analysis on average extent of secondary transmissions, when a small number of users are assumed in a cell, changing radar INR would have no impact on the fluctuations in upstream perceived data rate.

4.6 Conclusions

This chapter studies gray-space primary-secondary spectrum sharing between one rotating radar and one cell of a secondary system. The secondary system provides communications in non-contiguous cells around the radar, as might be appropriate if the secondary system provides broadband hotspots, or is a cellular system utilizing the shared spectrum when a traffic surge temporarily exceeds what can be supported in dedicated spectrum. A secondary device is allowed to transmit as long as the resulting interference does not exceed the tolerable level of the radar. The model will be extended to multiple cells and radars in later chapters.
Unlike existing models of sharing with radar, the proposed model allows secondary devices to adjust to variations in radar antenna gain as the radar’s main beam rotates, thereby making extensive secondary transmissions possible, although with some interruptions and fluctuations. Thus, sharing spectrum with rotating radar is a promising option to alleviate spectrum scarcity. Additional technical and governance mechanisms are needed, e.g. to address interference from malfunctioning devices [Peha11][Peha12].
It is also found that the secondary system will utilize spectrum more efficiently in the downstream than in the upstream, where efficiency may be measured in data rate per MHz of spectrum as compared to what can be achieved in dedicated spectrum, and fraction of time that a secondary device can transmit. Hence, spectrum sharing with radar would be more appropriate for applications that require more capacity in the downstream, which is a typical characteristic of many applications. However, if needed, the upstream rate could be increased by dedicating more spectrum to the upstream.

**Figure 4.11** Sensitivity of perceived secondary data rate in the upstream
Moreover, with the dynamic sharing model considered, the secondary system can achieve extensive transmissions in large areas that otherwise would be unavailable with current approaches to sharing with radar which is more static. For example, with the base-case assumptions, a secondary device that does not adjust its transmit power as the radar’s main beam rotates must be at least 286 km from the radar to prevent harmful interference. With the sharing model, at 27% of the distance at which secondary transmissions will not affect a radar, the secondary system can transmit all the time in the downstream with an average data rate almost equal to that achieved in dedicated spectrum, and roughly 90% of the time in the upstream with an average rate equal to 63% of the dedicated-spectrum data rate. It is also found that the secondary system can still achieve extensive transmissions even when key system parameters, including cell radius, radar transmit power, radar tolerable interference level and radar rotating period, change from the base-case assumptions.

Although average data rate is roughly the same for all file sizes, because of interruptions as the radar’s main beam rotates, perceived data rate fluctuates for smaller files while appearing fairly constant for larger files. The magnitude of this fluctuation is also more noticeable when a secondary device far from the radar transmits small files. The fluctuations in perceived data rate make sharing spectrum with radar attractive for applications that can tolerate interruptions in transmissions, such as video on demand, peer-to-peer file sharing, and automatic meter reading, or applications that transfer large enough files so the fluctuations are not noticeable, such as song transfers. Moreover, even with changes in cell radius, radar transmit power, radar tolerable interference level, and radar rotating period, spectrum shared with radar is still attractive to these applications.

Especially for video on demand, because currently the application is the fastest rising traffic class in the Internet [Cisc11], it appears to be a very promising application that can share spectrum
with radar. It is also found that this form of sharing works well with an application such as web browsing for which file transfer time rather than perceived data rate is the most appropriate performance measure. In contrast, spectrum shared with radar would be unattractive for interactive exchanges of small pieces of data, e.g., packets or files, of which instantaneous data rate matters, such as VoIP and small file transfers.
Chapter 5

Sharing between Multiple Cells and a Single Rotating Radar
5.1 Overview and Research Questions

This chapter extends the sharing model considered in Chapter 4, when one cell (of a secondary system) is sharing spectrum with one rotating radar, to when multiple cells are sharing spectrum with one rotating radar.

The case where, at any given time, there may be multiple cells close enough to a radar for harmful interference to be a concern, but these active cells do not blanket the region is considered. This can occur if a cellular system only uses shared spectrum when a temporary surge of traffic in a given cell requires more capacity than the one available from its dedicated spectrum. The scenario is also applicable to an Internet service provider in hotspots.

Only downstream cellular transmissions are considered, upstream being left for future work. Although upstream traffic can also be supported, an Internet service provider may choose to use this shared spectrum only for downstream. This is in part because downstream traffic greatly exceeds upstream one in Internet access, and because in shared spectrum, downstream was found, in Chapter 4, to be more spectrally efficient.

When multiple cells share spectrum with radar, two complementary mechanisms for controlling secondary transmissions are proposed. The first one allocates the shared spectrum resource to each cell using regional information obtained across all active cells. These allocations change relatively slowly, because they do not depend on current radar directions. Using the allocated resource, the other mechanism locally adjusts the maximum transmit power of a Base Station (BS) as the radar rotates to avoid harmful interference. Similarly to Chapter 4, the sharing approach can be either cooperative (through explicit communications with the radar) or coexistent (through monitoring, but without explicit communications with the radar).

The same research questions as in Chapter 4 are asked when the scenario is extended from when one cell of a secondary system shares spectrum with one radar to when multiple cells shares spectrum with the radar:

1) How much is the extent of secondary transmissions in sharing spectrum with rotating radars?
2) Could a secondary system provide services- e.g., voice and/or data transmissions- using only the spectrum shared with radars?

The resulting extent of secondary transmissions per active cell is quantified, and the impact of interruptions in secondary transmissions on the performance of various applications is investigated. It is shown that similar to what has been observed in Chapter 4, even with the interruptions, shared spectrum works well for the applications that generate the majority of mobile Internet traffic, including video streaming, web browsing, and peer-to-peer file sharing, but not so well for some other applications.
The sharing scenario and the two complementary mechanisms are explained in detail in Sections 5.2 and 5.3, respectively. Numerical results and conclusions are discussed in Sections 5.4 and 5.5, respectively.

5.2 Sharing Scenarios

The sharing model from Chapter 4 is extended to when multiple cells of an OFDMA cellular system are sharing spectrum with a radar. Secondary transmissions occur simultaneously in some, but not all, cells around the radar.

Characteristics of radar considered are the same as those studied in Chapters 4, see Section 4.2. (A radar uses the same antenna for transmission and reception. The radar transmits a series of pulses with constant power, and detects echoes of the pulses from its surroundings.)

The sharing model assumes,

1) Some technical information of a radar—such as tolerable interference level, pulse power, and rotating period—is known to the secondary system

2) The cellular system:
   a) will use as much available bandwidth as possible
   b) can always transmit signaling traffic without harming a radar, which could easily happen, for example, if signaling is transmitted in a frequency band different from the one shared with radars.

This chapter considers a specific case defined by the following additional assumptions:

1) Inter-cell interference among cells is negligible. As secondary transmissions occur in some, but not all, cells, it is unlikely that all neighboring cells will interfere with each other as it would occur in a typical cellular system. Moreover, interference among neighboring cells can also be reduced further by some mechanisms, such as those used in LTE (Long Term Evolution) to mitigate inter-cell interference.

2) To quantify overall transmissions achieved per active cell, all users are collocated in each cell.

3) When mean data rate is considered, the achievable secondary data rate is estimated as a fraction of Shannon’s limit, where this fraction was selected to roughly approximate what can be observed in an OFDMA-based system, such as LTE.
5.3 Controlling Transmit Power of a Secondary System to Prevent Harmful Interference

5.3.1 Basic Approach

When multiple cells (of a secondary system) have active downstream channels in the same band as the radar, the transmit power of each BS needs to be controlled, such that the total interference is not harmful. A BS can determine the maximum allowable transmit power using two complementary mechanisms: regional resource allocation and local power control.

The regional resource allocation mechanism allocates a portion of the shared spectrum resource to each cell, possibly using information from across the region, such as the link loss between each active BS and the radar. In particular, this mechanism specifies an Upper Bound (UB) on how much interference each BS can ever cause to the radar, such that there is little risk that cumulative interference to the radar will be harmful. These allocations among cells are relatively static, i.e., they do not change as the radar rotates, but change only when an active BS becomes inactive, or vice versa.

The local power control mechanism dynamically adjusts maximum allowable transmit power of a BS, based on the direction of the radar's main beam, to keep interference below the specified UB's obtained from the first mechanism. Only local information is used, so these adjustments can be made quickly, and without coordination among cells.

5.3.2 Regional Resource Allocation

The regional resource allocation will set interference UB’s so as to maximize the mean data rate per active cell, with a constraint to protect the radar from harmful interference. The effect of imposing a constraint on the maximum data rate that a cell is allowed is also considered. This constraint prevents some cells from gaining too much capacity at the expense of others. Other methods of enhancing fairness among cells are possible, but are not considered here.

The following optimization problem can be formed to allocate the interference power each BS $i$ can cause, $I_i$, so that the total interference from active BS’s is less than the radar tolerable level $I_{tot}$. The objective function $f_o$ considered is the total mean data rate achievable by all active cells; in the perspective of a system designer, regardless of types of applications offered by the secondary system, the designer might want the system to achieve as much transmissions as possible in the shared spectrum.
\[
\begin{align*}
\text{maximize: } & f_0 = \sum_{i=1}^{N_a} E_{\text{cell}} \left\{ E_{\phi} \left( R_{b,i}(\phi) \right) \right\} \\
\text{s.t. } & \sum_{i=1}^{N_a} l_i \leq I_{\text{tol}}, \\
& l_i \leq I_{\text{limit},i},
\end{align*}
\]

(5.1)

where,

- \( I_{\text{limit},i} \) is limitation on interference level BS \( i \) is allowed to cause to the radar
- \( N_a \) is the number of active BS’s sharing spectrum with the radar
- \( R_{b,i}(\phi) \) is cell \( i \)’s data rate, which is calculated from the achievable SINR
- \( \phi \) is the angle between the radar’s main beam and the BS
- \( E_{\text{cell}} \left\{ E_{\phi} \left( R_{b,i}(\phi) \right) \right\} \) is the mean data rate achievable by BS \( i \), the expectation is calculated across the cell area and \( \phi \).

In addition to the constraint used to protect a radar from harmful interference, i.e., the first constraint of (5.1), \( l_i \) is limited by: i) \( l_{\text{ts},i} \): the maximum interference a BS causes (to the radar) when transmitting at the maximum power \( P_{\text{max},i} \) that the BS equipment can achieve; ii) \( l_{\text{rate},i} \): the interference a BS causes when transmitting at SINR’s that, on average, yield the maximum allowed data rate \( R_{b,\text{limit}} \). This data rate limit is used to improve fairness among cells.

Similarly to Chapter 4, the SINR of secondary transmissions is calculated by assuming that 1) time between radar pulses is negligible, so the radar transmits continuously, 2) power spectral density of radar is constant over the shared spectrum band, 3) a secondary device transmits with equal power spectral density across the band, and 4) adjacent channel interference into and out of that band is negligible. Assumptions 1) and 2) reduce SINR, and thus achievable data rates. Other ways to allocate power spectral density of a secondary device over the shared spectrum might be possible, but are not considered here. If adjacent channel interference were significant, this would decrease achievable secondary transmissions. With the assumption that:

- in each cell, users will transmit as much as possible
- the secondary data rate is approximated as a fraction \( \gamma \) of Shannon’s limit
- inter-cell interference among active cells is negligible,

the mean data rate \( E_{\text{cell}} \left\{ E_{\phi} \left( R_{b,i}(\phi) \right) \right\} \) achieved by BS \( i \), shown in (5.1), can be written as

\[
E_{\text{cell}} \left\{ E_{\phi} \left( R_{b,i}(\phi) \right) \right\} = \gamma W_3 E_{\text{cell}} \left\{ E_{\phi} \left( \log_2 \left( 1 + \frac{L_{\phi}(d_2)\times l_i}{nS+W+L_{\phi}(d_2)\times l_{\phi}(d_2)} \right) \right) \right\},
\]

(5.2)
where,
- $W_s$ is the bandwidth of secondary transmissions
- $L_{o,i}(d_s)$ is link loss between BS $i$ and its Mobile Terminals (MT's) $d_s$ away from the BS
- $I_i$ is the power interference level that the BS causes on the radar
- $n_s$ is background noise power spectral density at MT's
- $P_{rd}$ is the radar transmit power
- $L_{rd-BS,i}(\varphi)$ and $L_{rd-MT,i}(\varphi)$ are instantaneous link loss between the radar and the BS, and between the radar and MT's, respectively. These link losses account for the radar’s antenna gain $G_{rd}(\varphi)$, antenna gain of a secondary device (i.e., BS or MT), and path loss between the radar and the device, which is a function of distance, but for brevity, this dependence is omitted in (5.2).

As discussed in Chapter 4, when radar pulse power $P_{rd}$ and rotating period are known, the secondary system can determine $L_{rd-BS,i}(\varphi)$ and $L_{rd-MT,i}(\varphi)$ using (4.2); with this assumption, coexistent and cooperative sharing, in which the radar informs $P_{rd}$ to the secondary system, achieves the same data rate.

An algorithm to allocate $I_i$ is developed by solving the optimization problem in (5.1)-(5.2); the algorithm maximizes any objective function $f_o$ for which $\frac{\partial}{\partial I_i} f_o > 0$, and $\frac{\partial^2}{\partial I_i^2} f_o < 0$. (See Appendix D for details on $\frac{\partial}{\partial I_i} f_o$ and $\frac{\partial^2}{\partial I_i^2} f_o$ of the objective function considered.)

### Algorithm Proposed to Allocate $I_i$

1: Turn all BS transmitters off

2: Increase interference allocated to the BS(s) that have the greatest transmission efficiency, until the allocated interference cannot increase further without exceeding the constraints in (5.1). Transmission efficiency is defined as the increased data rate per unit increase of interference to the radar, i.e., $\frac{\partial}{\partial I_i} f_o$

3: Repeat step 2 with the other BS’s, until it is impossible to increase allocated interference of a BS without violating any of the constraints

### 5.3.3 Local Power Control

The local power control at each BS $i$ calculates the maximum allowable transmit power using the allocated interference $I_i$ obtained from the regional resource allocation, and the link loss between the radar and BS $i$. This maximum allowable transmit power $P_{BS,i}$ is a function of the distance between the radar and the BS, i.e., $d_{rd-BS,i}$ and $\varphi$. 
\[ P_{BS,i}(\varphi, d_{Rd-BS,i}) = \frac{1}{M_i \bar{l}_{Rd-BS,i}(\varphi, d_{Rd-BS,i})} \]  

(5.3)

where

- \( \bar{l}_{Rd-BS,i} \) is mean link loss between the radar and the BS, which is a function of \( \varphi \), and \( d_{Rd-BS,i} \)
- \( M_i \geq 1 \) is a system margin used to deal with fluctuations in link loss between the radar and the BS

Similarly to Chapter 4, using (4.2) the BS can determine the instantaneous link loss between itself and the radar, and hence \( \bar{l}_{Rd-BS,i} \). System designers can determine \( M_i \) from the distribution of total interference from secondary transmissions, such that the risk of harmful interference to the radar is negligible.

### 5.4 Numerical Results

The assumptions used to obtain numerical results are summarized in Section 5.5.1. The achievable mean data rate is evaluated in Section 5.5.2. Fluctuations in perceived data rate, and their implications on how various prevalent applications on the Internet- including video streaming, web browsing, file downloading, downstream Peer-to-Peer (P2P) file sharing, and Voice-over-IP (VoIP)- would work are investigated in Section 5.5.3.

#### 5.4.1 Assumptions for Numerical Results

Numerical results are obtained using Monte Carlo simulations. A plane covered with cellular cells is considered. The location of the radar is randomly selected across the plane based on a uniform distribution, and the radar has to be at least \( d_{\text{min}} \) away from a BS. All cells have the same probability of being active \( a \), and the cases where \( a \) is sufficiently low so that inter-cell interference is negligible are considered. Fig. 5.1 shows areas around a given radar where secondary cells can and cannot be deployed.

The following assumptions are also used:

1) The impact of cells further than 100 km from the radar is negligible. From Chapter 4, when there is one cell sharing spectrum with radar, achievable secondary transmissions in the downstream are very close to that achieved in dedicated spectrum at only around 50 km from the radar.

2) Similarly to Chapter 4, the ITU-R P.1546 path loss model is adopted for a link between the radar and the cellular system. Conservatively, flat terrain, which increases interference, and reduces the extent of secondary transmissions is assumed.

3) The COST 231 Walfisch-Ikegami model is adopted for path loss between a BS and its MT's.
4) Values of parameters characterizing radar, cellular system, and the COST 231 Walfisch-Ikegami model are the same as those summarized in Table 4.1. Additional parameters needed for this extended scenario are summarized in Table 5.1.

**Figure 5.1** Scenario used to obtain numerical results, when multiple cells share spectrum with one radar

**Table 5.1** Additional Parameters for Numerical Results When Multiple Cells Share Spectrum with One Radar, Extended from Table 4.1 (Unless stated otherwise.)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cellular System:</strong></td>
<td></td>
</tr>
<tr>
<td>Minimum Distance to a Radar (d_{\text{min}}) [km]</td>
<td>1</td>
</tr>
<tr>
<td><strong>Other Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Fraction of Shannon’s limit (\gamma) in (5.2)(^{19})</td>
<td>0.53</td>
</tr>
<tr>
<td>Margin (M_z) in (5.3)(^{20})</td>
<td>1</td>
</tr>
</tbody>
</table>

From \(P_{BS_i}\) in (5.3), the resulting SINR, and hence data rate that BS \(i\) achieves can be calculated. As observed in Chapter 4, due to interruptions and fluctuations in instantaneous data rate

\(^{19}\) This value results in minimum mean square error between the estimated data rate and the data rate obtained from 3GPP data regressions; see Fig. B.1 in Appendix B.

\(^{20}\) No fading is considered. Effect of variations in wireless channel including fading and shadowing will be included in Chapter 6.
of secondary transmissions as the radar rotates, data rate experienced by a user transferring files, i.e., perceived data rate $R_{p,s,p}$, as defined by (4.7), can be significantly different from average data rate. Performance of the sharing is measured as an achievable mean data rate per active cell, and fluctuations in perceived data rate that a secondary user will experience.

To obtain the mean data rate per active cell, a realistic scenario, wherein multiple cells are sharing spectrum with radar, is considered. The mean data rate per active cell can be quite different when resources are shared among multiple active cells from when there is only one active cell as considered in Chapter 4. It is also assumed that the location of the collocated (secondary) users is uniformly distributed across each active cell, as this assumption is appropriate when calculating expected data rate achievable across the cell. The mean data rate achieved across cell $i$ can then be obtained by substituting the allocated interference $l_i^*$ into (5.2).

In order to determine how quality of service will be perceived by a given user, the case when the collocated users are at a fixed location in a cell, and (similarly to Chapter 4) the resulting data rate is the maximum among those obtained from QPSK, 16QAM and 64QAM modulation schemes is considered. The relationship between data rate and SINR is obtained from the regressions on 3GPP data. The fluctuations in perceived data rate, experienced by users located at different distances from the radar, are quantified. Within a cell, the users are at the edge closest to the radar; this results in the worst-case data rate and fluctuations.

### 5.4.2 Extent of Secondary Transmissions

The mean data rate that a BS can achieve in the downstream, defined by (5.2), is analyzed in what follows. Fig. 5.2 shows the mean data rate per active cell, and the 95% confidence interval, from simulations, as a function of the distance between a BS and the radar. The results are from when fractions of cells active $a$ at a given time are 4%, 12%, and 20%.

Fig. 5.2 shows that, at around 20 km from the radar, a BS can achieve a mean data rate that approaches the system downstream rate limit of 10.8 Mbps achieved in dedicated spectrum, see Section 4.5.2. When no fading is considered and only one active cell is sharing spectrum with the radar, the BS must be 215 km away from the radar, to ensure that the BS will never cause harmful interference on it, even when in the radar's main beam [Saru12]. This distance is even greater when there are multiple active cells. Hence, with this opportunistic gray-space sharing, high mean data rates are possible even close to the radar, although with interruptions and fluctuations as the radar rotates.

As discussed in Section 5.3.2, the maximum data rate per active cell can be limited to enhance fairness in transmissions among the cells. Fig. 5.3 shows mean data per active cell together with the 95% confidence interval as a function of distance between a BS and the radar. The results are from three different data rate limits are imposed: 10.8, 8.1 and 5.4 Mbps/cell, i.e., 4, 3, 2 bps/Hz, respectively. When the rate limit decreases, a BS can transmit closer to the radar. Hence, fairness in transmissions among secondary cells can be improved by limiting the data rate at which each cell can
transmit. Moreover, there is a tradeoff: reducing the maximum data rate per cell allows cells in an even larger area to achieve high mean data rates. The overall transmissions achievable might be further improved if interference resource allocated to BS’s very close to the radar could be re-allocated to BS’s further away, when interference from the radar is so high that the BS’s very close to the radar can only achieve marginal data rate.

Figure 5.2 Mean downstream data rate per active cell with 95% confidence interval vs. Distance between a base station and the radar

Figure 5.3 Mean downstream data rate per active cell with 95% confidence interval vs. Distance between a base station and the radar, for different system rate limits
5.4.3 Fluctuations in Perceived Data Rate and Implications on Performance Experienced by Various Applications

As has been observed from Chapter 4, for some applications, a high mean data rate may not be sufficient to meet QoS requirements. This section investigates fluctuations in perceived data rate experienced by a given user, and whether these fluctuations will be problematic for various applications prominent on the Internet.

The perceived data rate is highly dependent on the size of the file being transferred. This is clear from Fig. 5.4, which shows the first percentile of perceived data rate as a function of the perceived data rate averaged across the radar rotating cycle, when files of different sizes, ranging from 1 kB to 10 MB, are transferred. When files larger than 1 MB are transferred, if the average data rate is good enough to meet an application’s QoS requirements, then, the fluctuations are unlikely to be a problem. Indeed, at 10.8 Mbps, there are no noticeable fluctuations. However, for files of just 1 kB, the perceived data rate is sometimes more than an order of magnitude less than the average one. Thus, for applications that transfer small files, and require reliably high data rates to meet QoS requirements, these fluctuations in perceived data rate will be a problem.

![Figure 5.4](image)

**Figure 5.4** The first percentile of perceived data rate vs. Average perceived data rate, the user is at the cell edge closet to the radar

The fluctuations in perceived data rate make the shared spectrum attractive for applications that transfer sufficiently large files so that the fluctuations are not noticeable, such as video downloads. Shared spectrum is also attractive for applications that can tolerate interruptions in transmissions, such as P2P.

It is found in Section 4.5.3 that when there is one cell, with only a few seconds of buffering, fluctuations in perceived data rate are not sufficient to cause disruption in video streaming. This is
also true when there are multiple cells, because although the presence of additional cells affects a cell’s interference allocation, and therefore mean transmission delay, other cells do not affect the power control mechanism that causes fluctuations in data rate. As found in Section 5.4.2, even with multiple cells, a BS can achieve high downstream rate close to the radar. Hence, shared spectrum is also attractive for video streaming.

Although it transfers some small files, web browsing would also work well in spectrum shared with a radar. As discussed in Section 4.5.5.2, for web browsing, file transfer time rather than perceived data rate is the important performance measure. Fig. 5.4 shows that when the average data rate is around 10 Mbps, a user downloading a 1 MB web page will experience the 1st-percentile perceived rate around 8 Mbps. Hence, even with 3 MHz of spectrum, most (i.e., 99%) of the time, more than 90% of transfers would experience file transfer time less than 1 s. As of Section 4.5.5.2, webpage downloading time is suggested to be less than 2 to 4 s, and the 90th-percentile webpage size in 2010 was 660 kB. For web pages larger than 1 MB, the file transfer time will not be very different from that in dedicated spectrum. Thus, in shared spectrum, QoS is still good for web browsing.

In contrast, when users do not tolerate fluctuations in perceived data rate, spectrum shared with a radar will be problematic for interactive exchanges of small pieces of data (e.g., packets or files), such as small file downloads. Moreover, spectrum shared with radar is expected to be unattractive for VoIP, as the spectrum will be inefficiently used even when only one cell is sharing spectrum with radar, see Section 4.5.4.

As a result, when multiple cells are sharing spectrum with a radar, the fluctuations will not be a problem for video streaming, large file download, web browsing, and downstream P2P, although they can be problematic for applications such as small file download, and VoIP. Hence, even with the fluctuations in data rate, the majority of traffic expected on the Internet will work well in shared spectrum even close to a radar.

### 5.5 Conclusions

This chapter studies opportunistic gray-space primary-secondary spectrum sharing between a rotating radar and a cellular system. The sharing scenario from Chapter 4 is extended to where at any given time, some, but not all, cells (of the secondary system) share spectrum with the radar. The sharing can be achieved if the secondary system either senses the primary system’s behavior or explicitly communicates with the primary system. The extent of secondary transmissions in the downstream is investigated. Sharing with multiple radars will be addressed in Chapter 6, and the secondary upstream transmissions will be investigated in future work.

Unlike existing models of sharing with radar, the proposed sharing model allows secondary devices to adjust to variations in radar antenna gain as the radar rotates. This makes extensive
secondary transmissions possible, even close to the radar, although with some interruptions and fluctuations occur when the radar rotates. For example, when 20% of the base stations are active, beyond only 20 km from the radar, they can achieve a mean data rate that approaches the rate obtained in dedicated spectrum. (The distance that active base station will never cause harmful interference to the radar, even in the radar’s main beam, is expected to be larger than 215 km.) Thus, sharing spectrum with a rotating radar is a promising option to alleviate spectrum scarcity.

It is found that fairness in transmissions among secondary cells can be improved by limiting the data rate at which each cell can transmit. Hence, reducing the maximum data rate per cell allows cells in an even larger area to achieve high mean data rates. Moreover, it is expected that the overall transmissions achievable could be further improved if interference resource allocated to BS’s very close to the radar is re-allocated to BS’s further away, when interference from the radar is so high that the BS’s very close to the radar can only achieve marginal data rate; the idea will be addressed in Chapter 6.

It is found that the perceived data rate is highly dependent on the size of the file being transferred. The fluctuations will not be a problem for video streaming, large file download, web browsing, and downstream Peer-to-Peer file sharing. However, the fluctuations can be problematic for some other applications, such as small file download and VoIP, that are sensitive to interruptions and fluctuations in data rate. Hence, even with the fluctuations in data rate, spectrum sharing close to a radar will work well for the majority of traffic expected on the Internet, including video streaming, web browsing, and (downstream) Peer-to-Peer file sharing.
This page is intentionally left blank.
Chapter 6

Sharing between Multiple Cells and Multiple Rotating Radars
6.1 Overview and Research Questions

This chapter extends the sharing model considered in Chapter 5, when multiple cells are sharing spectrum with one rotating radar, to when multiple cells are sharing spectrum with multiple rotating radars using the same frequency band.

Similarly to Chapter 5,

- Multiple cells of the secondary system do not blanket a region considered; as might occur when a cellular system provides broadband hotspots, or only uses shared spectrum when a temporary surge of traffic in a given cell requires more capacity than what is available from the cellular system’s dedicated spectrum.
- The spectrum sharing model can be applied to both cooperative and coexistent sharing.
- Only downstream secondary transmissions are considered, see detailed discussions in Section 5.1.
- Two-step mechanisms, i.e., 1) regional allocation and 2) local power control, are proposed to control transmissions of each cell so that the resulting cumulative interference will not be harmful to any radar.

The first mechanism allocates the shared spectrum resource to each cell using regional information obtained across all active cells. These allocations change relatively slowly, because they do not depend on current radars’ directions. Using these resource allocations, the second mechanism locally adjusts transmissions of secondary devices as the radars rotate to avoid harmful interference. There are various ways to implement these two-step mechanisms. For example, in Chapter 5 the regional allocation allocates interference level that each active cell can cause to a given radar. Then, based on the allocated interference level, the local power control adjusts transmit power of a Base Station (BS) to avoid harmful interference while radar rotates. Differently, the mechanisms proposed in this chapter will either allow a secondary device to transmit with its equipment power limit, or not transmit at all depending on link loss level between the device and its nearby radars. As discussed in Section 5.4.2, the overall extent of secondary transmissions could be improved if BS’s close to a radar stop transmitting whenever interference from the radar is too high that the BS’s can achieve only marginal transmissions. Doing this will allow BS’s further away from the radar to cause more interference, and hence, improve overall transmissions achievable.

The two research questions from Chapters 4 and 5 are asked in this extended scenario:

1) How much is the extent of secondary transmissions achievable in sharing spectrum with rotating radars?
2) Could a secondary system provide services- e.g., voice and/or data transmissions- using only the spectrum shared with radars?
The sharing scenarios, and the two complementary mechanisms are explained in detail in Sections 6.2 and 6.3, respectively. Parameters used to measure sharing performance are described in Section 6.4. Numerical results are presented in Section 6.5, and conclusions are drawn in Section 6.6.

6.2 Sharing Scenarios

The sharing model considered in Chapters 4 and 5, is extended to when multiple cells of an OFDMA cellular system share spectrum with multiple radars; these radars use the same frequency band. Secondary transmissions occur simultaneously in some but not all cells around the radars.

The same type of radar as in Chapters 4 and 5 is considered. (A radar uses the same antenna for transmission and reception. The radar transmits a series of pulses with constant power, and detects echoes of the pulses from its surroundings.) Rotation speed, and main beam direction of one radar need not be the same as those of other radars.

Similarly to Chapters 4 and 5, to protect radars from harmful interference, radars’ Interference to Noise ratio (INR) must be kept below an established value with sufficiently high probability. A secondary device can take advantage of the radar’s changing antenna gain, as long as it has some information on current state of the main beam rotation. It is assumed that

- A secondary device knows either the instantaneous antenna gain of a given radar together with the expected value of path loss between itself and the radar, or the summation of the two. The secondary device can determine this loss in a variety of ways, depending on the sharing approach and type of radar; see Section 4.2 for detailed discussions.
- The device can remain synchronized with the main beam rotation. The possibility of synchronization errors is beyond the scope of this dissertation.

Similarly to Chapter 5, the sharing model assumes,

1) Some technical information of a radar - such as tolerable interference level, pulse power, and rotating period- is known to the secondary system
2) The cellular system:
   a) will use as much available bandwidth as possible
   b) can always transmit signaling traffic without harming a radar, which could easily happen, if, for example, signaling is transmitted in a frequency band different from the one shared with radars.

Moreover, a specific case is considered, in which inter-cell interference among cells is negligible. (As secondary transmissions occur in some but not all cells, it is unlikely that all
neighboring cells will interfere each other as it would occur in a typical cellular system. In addition, interference among neighboring cells can also be reduced further by mechanisms such as those used in LTE to mitigate inter-cell interference.)

6.3 Controlling Transmit Power of a Secondary System to Prevent Harmful Interference

When multiple secondary cells have active downstream transmissions in the same band as radars, transmissions of each active BS’s need to be controlled, such that the total interference is not harmful. A BS can determine when it can transmit and at what power using two complementary mechanisms: regional resource allocation and local power control.

The regional resource allocation allocates a portion of the shared spectrum resource to each active cell, possibly using information from across the region, such as the probability that each BS will be active, and link loss between BS’s and radars. These allocations are relatively static, i.e., do not change as radars rotate. Each BS has a local power control that adjusts its maximum transmit power or simply stops transmissions temporarily based only on local information, so adjustments can be made quickly, and without coordination among cells. The mechanisms are also applicable to a more complicated scenario in which multiple radars use different frequency bands. If a secondary system has multiple bands that are shared with radars, and the locations of radar differ from band to band, then the two-step mechanisms described in this chapter would operate independently in each band. At any given instant, achievable throughput will vary considerably from band to band. It is possible to use OFDMA and some form of spectrum aggregation to achieve a total throughput that is greater than the sum of throughput achieved from each band. Moreover, aggregating across multiple bands where radars operate independently will also reduce the variance of perceived data rate experienced by a secondary user. Hence, data rate per MHz and the impact of data rate fluctuations on quality of service are both worse when sharing one spectrum band with radars as opposed to several spectrum bands. However, these potential benefits of spectrum aggregation are not investigated in this dissertation.

There are multiple ways to do this, but the proposed regional resource allocation mechanism specifies a threshold on link loss \((L_{ij})\) between each radar \(j\) and all BS’s within range of radar \(j\). The local power control mechanism allows a BS to transmit at the equipment limit power \(P_{BS,e,max}\) when the expected link loss (in absolute units, not in dB) is below this threshold, and no transmissions otherwise. While decisions depend on expected link loss, note that actual link loss can differ from expected link loss due to factors like fading. The threshold does not change as the radar rotates, but changes with a change in the fraction of active BS’s, or if there were a substantial shift in the geographic clustering of active cells.

BS \(i\) will transmit when its expected link loss to each of its nearby radars is below the
associated thresholds. At any current main beam direction of each of the radars nearby, represented here as vector $\bar{\phi}$, transmit power of the BS, $P_{BS,i}(\bar{\phi})$, is

$$P_{BS,i}(\bar{\phi}) = \begin{cases} P_{BS,e,\max} & \text{if } \forall j, L_{i,j}(\phi_j) < L_{T,j} \\ 0 & \text{otherwise} \end{cases} \tag{6.1}$$

where,

- $L_{i,j}(\phi_j)$ is the expected link loss between nearby radar $j$ and BS $i$
- $\phi_j$ is the main beam direction of radar $j$.

$L_{i,j}$ accounts for the radar’s antenna gain $G_{rad,j}(\phi_j)$, the BS antenna gain, and expected path loss between the radar and the BS.

The threshold is determined such that the risk that cumulative interference to any of the associated radars could be harmful is sufficiently low. Even when there is no change in the fraction of cells that are active, and therefore no change in the expected value of this cumulative interference, the instantaneous value of cumulative interference varies over time due to factors such as

- fading and shadowing
- which specific cells are active
- whether each of these active cells has enough data to send with maximum power.

The link loss threshold will exceed the expected value enough to protect the radar from these fluctuations. The threshold is derived from the distribution of this cumulative interference, which can be obtained analytically, empirically, or by simulation.

### 6.4 Performance Measurement

Performance in shared spectrum will be evaluated from the extent of transmissions achievable on average, and how interruptions and fluctuation in secondary transmissions affect quality of service of various applications as radars rotate. The extent of transmissions is measured as an achievable data rate, and fraction of time that a cell is allowed to transmit.

The data rate, achievable by an active cell $i$, is calculated as the mean data rate averaged across the cell area $E_{cell}(R_{b,i}(\bar{\phi}))$. The data rate $R_{b,i}(\bar{\phi})$ is a function of SINR $\rho_i(\bar{\phi})$, which depends on the current direction $\bar{\phi}$ of the $N_{rad,i}$ radars nearby. Given that inter-cell interference among active cells is negligible, $E_{cell}(R_{b,i}(\bar{\phi}))$ is calculated using these additional assumptions, which are also used in Chapter 5:

1) The SINR of secondary transmissions is calculated as if a) time between radar pulses is
negligible, so the radar transmits continuously, b) power spectral density of radar is constant over the shared spectrum band, c) a secondary device transmits with equal power spectral density across the band, and d) adjacent channel interference into and out of that band is negligible. Assumptions a) and b) result in lower data rates for secondary systems than what is expected for radars that transmit intermittently. Other ways to allocate power spectral density of a secondary device over the shared spectrum might be possible, but are not considered in this dissertation. If adjacent channel interference were significant, this would decrease achievable secondary transmissions.

2) The location of collocated users is uniformly distributed throughout each cell. As often occurs with LTE-like systems, each user gets an equal share of spectrum, but those who are closer to the BS achieve higher data rates.

3) The data rate is approximated as a fraction $\gamma$ of Shannon’s limit, where $\gamma$ is selected to roughly approximate what can be observed on an OFDMA-based system, such as LTE.

With the transmit power of BS $i$ (i.e., $P_{BS,i}$) as shown in (6.1),

$$E_{cell}(R_{b,i}(\varphi)) = \gamma W_S E_{cell}(\log_2(1 + \rho_i(\varphi))) \quad (6.2)$$

$$\rho_i(\varphi) = \frac{L_{S,i}(d_S) \times P_{BS,i}(\varphi)}{n_S \times W_S + \sum_{\mu=1}^{\mu_{MT}} \big[ L_{Rd,j-MT}(\varphi_j) \times P_{Rd,j} \big]} \quad (6.3)$$

where,

- $W_S$ is the bandwidth of secondary transmissions
- $L_{S,i}(d_S)$ is link loss between BS $i$ and its Mobile Terminals (MT’s) $d_S$ away
- $n_S$ is background noise power spectral density at the MT’s
- $P_{Rd,j}$ is transmit power of radar $j$
- $L_{Rd,j-MT}(\varphi_j)$ is link loss between the radar and the MT’s.

As has been discussed in Chapter 4, because secondary transmissions can be interrupted, a user, at a given location in a cell, will experience different perceived data rates $R_{b,s,p}$ when transferring files of different sizes. $R_{b,s,p}$ is defined in Section 4.4.

Similarly to Chapter 5, to determine how quality of service will be perceived by a user, the case when collocated users are at a fixed location in a cell is considered. The fluctuations in perceived data rate are quantified, when the users are at different distances from their nearest radar. Within a cell, the users are at the edge closest to the radar; this results in the worst-case data rate and fluctuations.
6.5 Numerical Results

The assumptions used to obtain numerical results are summarized in Section 6.5.1. The extent of secondary transmissions is evaluated in Section 6.5.2. Fluctuation in perceived rate and its implication on performance of prominent applications on the Internet are investigated in Section 6.5.3. Sensitivity of these results on important system parameters are investigated in Section 6.5.4.

6.5.1 Assumptions for Numerical Results

Monte Carlo simulations are used to determine the extent of cellular communications achievable. Radars are located on an infinite plane, under the constraint that no two radars can be less than $d_{rd}$ apart. Thus, each radar is precisely $d_{rd}$ away from its six adjacent radars. This same plane is blanketed by cellular cells of equal size, each of which is active with probability $a$, independent of which other cells are active. $a$ is sufficiently low that inter-cell interference is negligible. The only areas that are not covered by (cellular) cells are those in the immediate vicinity of radar, so no cell is deployed within $d_{min}$ of any radar. From all radars uniformly deployed on the infinite plane, Fig. 6.1 shows layout of a given radar in relative to its six neighboring radars; secondary cells can be deployed in the shaded area which is at least $d_{min}$ away from any radar.

![Figure 6.1 Layout of a given radar in relative to its six adjacent neighbours, and area that secondary cells can be deployed on an infinite plane](image)

The following assumptions are also adopted:

1) To be conservative, path loss from radar to cell is assumed to follow the ITU-R P.1546
model [ITUR09] in flat terrain, which will increase interference between radar and BS, thereby reducing the extent of transmissions achievable.

2) Because of the shorter distances, path loss from BS to MT follows the COST 231 Walfisch-Ikegami model [Kurn99].

3) A wireless channel is subject to shadowing and multipath fading as represented by $\Delta_{L_p}$ in (2.2). The shadowing effect is assumed to be log-normal distributed with unit mean and standard deviation $\sigma_{Sh}$, and the multipath fading is Ricean distributed with different K factors for a link within Line of Sight (LoS) from a radar, and a link beyond the LoS.

4) Similarly to Section 4.5.1, the fading considered has equal impact on all sub-carriers of the secondary system. The extent of secondary transmissions quantified in this dissertation might be lower than when different sub-carriers experience different levels of fading.

The link loss threshold of a given radar $j$ ($L_{R,j}$) is set such that even if every active cell transmits at the equipment power limit whenever its link losses to all radars are below the thresholds (see (6.1)), it is very unlikely that total interference to any radar will exceed the radar’s tolerable limit. The thresholds are determined as follows:

- For a given threshold, the joint distributions for the following parameters are determined via simulation
  - the fraction of time that an active cell $i$ is allowed to transmit
  - the amount of interference that cell $i$ would cause to each of the $N_{Rd,i}$ radars within its range if the BS transmits at its equipment power limit
  - the maximum data rate achievable by cell $i$.

- Cell $i$’s location is selected randomly on the infinite plane using a uniform distribution, except within radius $d_{min}$ from any radar where no BS is deployed.

- From this distribution of interference to radar derived through simulation, it is possible to determine the relationship between the probability that total interference will exceed the radar tolerable limit and the percentage of cells that can be active $a$ for a given threshold.

The relationship is determined under the simplifying assumptions that i) all cells are equally likely to be active, and ii) interference from different active cells is independent so the sum of these independent random variables can be approximated with a normal distribution.

To obtain base-case results, the values of parameters characterizing radar, cellular system, and the COST 231 Walfisch-Ikegami model are the same as those summarized in Table 4.1. Additional parameters needed for this extended scenario are summarized in Table 6.1.

The ranges of various parameters studied in sensitivity analysis are the same as those summarized in Table 4.2; these include cell radius, radar transmit power, radar tolerable interference level, and radar rotating period. The ranges of two additional parameters, which are distance between two radars and minimum distance between a radar and its nearest cell, are summarized in Table 6.2.
Table 6.1 Additional Parameters for Base-Case Numerical Results When Multiple Cells Share Spectrum with Multiple Radars, Extended from Table 4.1 (Unless stated otherwise.)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Radar (FAA07):</strong></td>
<td></td>
</tr>
<tr>
<td>Distance between Adjacent Radars $d_{rd}$ [km]</td>
<td>280</td>
</tr>
<tr>
<td><strong>Cellular System:</strong></td>
<td></td>
</tr>
<tr>
<td>Minimum Distance to a Radar $d_{min}$ [km]$^{21}$</td>
<td>5</td>
</tr>
<tr>
<td><strong>Other Parameters:</strong></td>
<td></td>
</tr>
<tr>
<td>Fraction of Shannon’s limit $\gamma$ in (6.2)$^{22}$</td>
<td>0.53</td>
</tr>
<tr>
<td>Shadowing Parameter $\sigma_{Sh}$ [dB]$^{23}$</td>
<td>7</td>
</tr>
<tr>
<td>Ricean K Factor [dB]</td>
<td></td>
</tr>
<tr>
<td>- Within LoS from a Radar$^{24}$</td>
<td>10</td>
</tr>
<tr>
<td>- Beyond LoS from a Radar (i.e., Rayleigh Fading)</td>
<td>0</td>
</tr>
<tr>
<td>Probability of Harmful Interference [%]$^{25}$</td>
<td>&lt;0.1</td>
</tr>
</tbody>
</table>

Table 6.2 Ranges of Additional Parameters Considered for Sensitivity Analysis When Multiple Cells Share Spectrum with Multiple Radars, Extended from Table 4.2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>Distance between Adjacent Radars $d_{rd}$ [km]</td>
<td>280</td>
</tr>
<tr>
<td>Minimum Distance to a Radar $d_{min}$ [km]</td>
<td>5</td>
</tr>
</tbody>
</table>

Note from Table 6.1 that the value of $d_{rd}$ chosen results in the worst case scenario in which radars are as densely packed as possible on the infinite plane; according to the US Federal Aviation Administration (FAA), distance between two radars using the same frequency should be at least

$^{21}$ It is found from Chapter 5 that secondary transmissions rarely happen in a very close vicinity to a given radar, e.g., 1 km away from the radar.

$^{22}$ As discussed in Chapter 5, this value results in minimum mean square error between the estimated data rate and the data rate obtained from 3GPP data regressions.

$^{23}$ Adapted from [Cave02].

$^{24}$ As discussed in Chapter 4, with the considered heights of radar and BS, the LoS distance is around 20.8 km.

$^{25}$ The value is the same as that used in Chapters 4.
280 km [FAA07]. Hence, in the base-case scenario, this dissertation considers sharing with a spectrum band that would be considered 100% utilized by radars, by the standards of conventional spectrum management. White-space sharing is not possible, but as will be shown, gray-space sharing can be extensive.

6.5.2 Extent of Secondary Transmissions

The extent of secondary transmissions is measured as the fraction of time that a BS can transmit, and mean data rate achievable by an active cell. The mean data rate per active cell is $E_{cell}(R_{k,i}(\bar{g}))$, shown in (6.2), averaged across all active cells, and all directions of all nearby radars’ main beams. Moreover, the impact of three parameters of the sharing model on the extent of secondary transmissions achievable, including distance between a BS and its nearest radar, percentage of cells that are active ($\alpha$), and acceptable risk of harmful interference to radars are investigated.

Fig. 6.2 shows (a) percentage of time that a BS is allowed to transmit, and (b) mean data rate per active cell as a function of distance between a BS and its nearest radar, when 4%, 12% or 20% of cells are active. The results are from the worst scenario in which radars are packed as closely as possible, so that white-space sharing with the radars is not possible. Moreover, every active BS always transmits with its maximum power whenever it is allowed. Hence, the interference those cells can cause to radars is at maximum, and the criteria used to protect radars from harmful interference has to be very stringent.

Fig. 6.2 shows that distance between a BS and a radar is one important factor that affects the extent of transmissions achievable. The transmissions are low when a BS is very close to a radar (e.g., at 10 km away), but quite high when the BS is tens of km away. Even in this extremely limited scenario, gray-space sharing makes secondary transmissions possible in an area where white-space sharing is not possible. Fig. 6.2 shows that at 100 km from the nearest radar, a BS can transmit 28% of the time when the percentages of cells active is 4%, and that the resulting mean data rate per active cell is 28% of that achievable in dedicated spectrum (i.e., system rate limit). With the assumptions used to obtain base-case numerical results, the system rate limit in the downstream is around 3.6 bps/Hz, see Section 4.5.2. Even with 20% of cells active, 0.54 bps/Hz data rate is high enough to support medium-quality video streaming in just 3 MHz of shared spectrum, with required streaming rate of 1.6 Mbps as assumed in Chapter 4. Interruptions during transmissions would not be a problem as long as 3 to 4 s of content can be buffered because typical rotating period of the radar is only around 4.7 s [ITU03]. 4 s of content with 1.6 Mbps streaming rate is less than 1 MB of buffering. Thus, the proposed sharing scheme would enable extensive communications for secondary systems that would otherwise have been impossible with white-space sharing.

Fig. 6.2 also shows that even though a cell cannot transmit all the time, the data rate that the cell can achieve when it is allowed to transmit is very close to what is achievable in dedicated spectrum. For example, at 12% of cells active, when a BS is allowed to transmit, it transmits at
around 3.5 bps/Hz, which is close to a data rate achievable in dedicated spectrum. This is true regardless of the percentage of active cells.

![Graph](image)

(a) Percentage of Time that a Base Station can Transmit

![Graph](image)

(b) Mean Downstream Data Rate per Active Cell

Figure 6.2 Extent of secondary transmissions with 95% confidence interval vs. Distance between a base station and its nearest radar

Fig. 6.3(a) shows mean data rate per active cell as a function of percentage of cells active when the risk of harmful interference to radar, if all cells transmit at equipment limit power at all times, is either 0.1% or 0.5%. As expected, Fig. 6.3(a) shows that increasing the percentage of cells active would reduce the achievable data rate per active cell. However, Fig. 6.3(b) shows that increasing the percentage of cells active increases total system throughput, as represented by the summation of mean data rate in every active cell divided by total area in which secondary cells can be deployed (i.e.,
Thus, even though the data rate per active cell goes down when many cells are active, the total benefit derived from the spectrum actually goes up with percentage of cells active. Hence, this form of sharing can be useful regardless of whether the number of cells active at any given time is large or small. Moreover, there is a tradeoff between the total system throughput and the throughput achievable by an active cell. System designers can choose to allow many cells to share the spectrum if they want to achieve high total system throughput, or to limit number of cells sharing the spectrum if they want to guarantee a certain level of transmissions achievable by each cell.

![Graph (a)](image-a.png)

**Figure 6.3** Extent of secondary transmissions with 95% confidence interval vs. Percentage of active cells

Finally, Fig. 6.3 shows that the level of acceptable risk of harmful interference to a radar has a
small impact on the extent that a BS can transmit. Hence, it is possible to provide high levels of protection to radars with little reduction in achievable secondary transmissions.

6.5.3 Fluctuations in Perceived Data Rate and Performance of the Sharing as Perceived by Various Applications

As has been discussed in Chapters 4 and 5, mean data rate is a good performance measure for some applications, such as video streaming, but not all. This section investigates fluctuations in perceived data rate experienced by a given user, and whether these fluctuations will be a problem for prominent Internet applications including file transfers, Peer-to-Peer file sharing (P2P), web browsing, and Voice-over-IP (VoIP) [Cisc11]. The perceived data rate is defined in Section 4.4.

Fig. 6.4 shows the 1st percentile perceived data rate as a function of mean perceived data rate that a user at the edge of the cell experiences. Results are shown for files of different size, ranging from 10 kB to 10 MB. Fig. 6.4 shows that transferring a small file is more susceptible to high fluctuations in perceived data rate. More than 99% of the time a user downloading large files (i.e., files larger than 1 MB) would perceive data rate which is close to the mean; however, a user downloading small files, e.g., 10 kB files, might perceive data rate that is an order of magnitude lower than the mean even when the mean is high. The fluctuations make spectrum sharing attractive for applications transferring files large enough so that the fluctuations are not noticeable, such as song and video downloads. The sharing is also attractive for applications that can tolerate interruptions, such as P2P. However, the fluctuations will be problematic for interactive exchanges of small pieces of data, each of which must be received within a small period. Meeting these requirements even when perceived data rate per MHz is small would require much more shared spectrum; this is spectrally inefficient.

Figure 6.4 The first percentile perceived data rate with 95% confidence interval vs. Mean perceived data rate
Although it involves transferring small files, web browsing can be supported with acceptable QoS at high spectral efficiency in the shared spectrum. File transfer time rather than perceived data rate is the important performance measure for web browsing. It has been suggested that the downloading time for a webpage should not exceed 4 s [3GPP10]. As shown by Fig. 6.4, this is achieved 99% of the time with as little as 3 MHz of shared spectrum for a webpage not larger than 1 MB. Note that 1 MB is large for a webpage, given that the 90th percentile webpage size in 2010 was 660 kB, see Section 4.5.5.2. As a user transferring large files will experience less fluctuations in data rate, a user downloading webpages larger than 1 MB would obviously experience longer downloading time; however, the webpage downloading time would be fairly close to that achievable in dedicated spectrum. Thus, in shared spectrum, required QoS can still be maintained for web browsing at high spectral efficiency.

In contrast, meeting QoS requirements of VoIP is only possible at very low spectral efficiency. Indeed, this is true even in the simpler case when a cell must only concern itself with one radar. From Section 4.5.4, consider the case of constant data rate VoIP that requires a latency less than 150 ms. It was found that the required latency could not be met unless the VoIP constant data rate is below 0.005 bps/Hz.

To summarize, when multiple cells are sharing spectrum with multiple radars, the fluctuations will not be a problem for video streaming, large file download, web browsing, and downstream P2P, although they can be problematic for applications such as small file download, and VoIP. Hence, even with the fluctuations in data rate, the majority of traffic expected on the Internet will work well in shared spectrum in an area where white-space sharing is not possible.

6.5.4 Sensitivity Analyses

This section investigates sensitivity of the base-case results on various important system parameters including

- Distance between adjacent radars \(d_{Rd}\)
- (Secondary) cell radius \(R\)
- Minimum distance between a radar and its nearest cell \(d_{\text{min}}\)
- Radar tolerable interference level represented as maximum tolerable Interference to Noise Ratio (INR)
- Radar transmit power \(P_{Rd}\)
- Radar rotating period \(T_{Rd}\).

The impacts of these parameters on the average extent of secondary transmissions, and fluctuations in perceived data rate are investigated in Sections 6.5.4.1, and 6.5.4.2, respectively.
6.5.4.1 Sensitivity of AverageExtent of Secondary Transmissions

As discussed in Section 4.5.6.1, unlike the other parameters considered, changing radar rotating period $T_{rd}$ will not change the amount of data a secondary device can transfer in a given period of time, and thus, it will have no impact on the average extent of secondary transmissions. Hence, those graphical results will be omitted. Similarly to the previous sections, the average extent of secondary transmission of an active cell is calculated across a cell and directions of radars’ main beams.

Fig. 6.5 shows (a) the percentage of time that a BS can transmit, and (b) the mean downstream data rate achievable, as a function of $d_{rd}$. The achievable transmissions increase quickly with increasing distance between adjacent radars. For example, Fig. 6.5(b) shows that increasing $d_{rd}$ by a factor of 2, from 280 to 560 km, increases secondary mean data rate per active cell by a factor of 2.2 when 4% of cells are active, and by a factor of 3.8 when 20% of cells are active. Note that the base-case results are obtained from the worst-case scenario, in which radars are 280 km apart and thus are the most densely packed as possible. In practice, radars (using the same frequency) would actually be further apart than in the theoretical worst-case scenario. Hence, in practice, significantly higher achievable transmissions than those achieved in the base-case scenario are expected.

Fig. 6.6 shows (a) the percentage of time that a BS can transmit, and (b) the mean downstream data rate achievable, as a function of cell radius of the secondary system. Increasing cell radius reduces the number of BS’s competing for the shared spectrum. This, combined with the assumption, that all these competing BS’s will always transmit at their equipment power limit whenever they are allowed to transmit, means that an increase in cell radius will increase the percentage of time that a BS can transmit, and the mean data rate per active cell. In contrast, mean data rate per active cell does not increase with cell size in dedicated spectrum. Even though mean data rate per active cell increases with radius, total throughput decreases because the number of cells per area decreases, and so does the number of active cells per area. This can be seen in Figure 6.6(b), which shows how cell radius affects the data rate per area averaged over all active cells, i.e., mean data rate per active area.

Cellular networks in the future are likely to have both large cells so mobile devices make fewer handoffs and to fill in holes in coverage, and small cells for greater capacity and spectral efficiency. Spectrum shared with radar can be used effectively for both large and small cells, but other sharing models also work for very small cells. Given that finding spectrum for large cells is likely to be more of a problem for operators, and that secondary transmissions increase with cell size, the operators may wish to use this shared spectrum for large cells. (Mean data rate per active cell and fraction of time when transmissions are possible increase with cell size, which yields fewer interruptions and better quality of service.)
Fig. 6.7 shows the effect of distance between a radar and its nearest cell \((d_{\text{min}})\) on the extent of secondary transmissions achievable. As \(d_{\text{min}}\) increases, the number of cells competing for the shared spectrum decreases. Hence, as expected, Fig. 6.7(a) and 6.7(b) show that the percentage of time that a BS can transmit, and the corresponding mean data rate achievable per active cell increase with larger \(d_{\text{min}}\). However, because the number of cells that compete for the shared spectrum decreases with increasing \(d_{\text{min}}\), system total throughput might also decrease; the system total throughput is measured as throughput averaged across the entire infinite plane considered, including both area that BS’s can, and cannot be deployed. Fig. 6.7(c) shows a tradeoff between the system...
total throughput and $d_{min}$. If $d_{min}$ is too large, the system total throughput will start to decrease with increasing $d_{min}$. Fig. 6.7(c) also shows that $d_{min}$ has only a little impact on the system total throughput. For example, increasing $d_{min}$ by a factor of 20, from 5 km to 100 km, will decrease the system total throughput only by a factor of 0.8 when 4% of cells are active, and by a factor of 0.9 when 20% of cells are active. Hence, increasing $d_{min}$ is, in essence, a shifting of capacity between cells close to radars and cells far from radars, without a huge impact on total transmissions that the secondary system can achieve. Nevertheless, a system designer might not want to choose too large $d_{min}$ because doing so will reduce area that spectrum sharing can occur, and decrease the overall transmissions achievable.

![Graph](image)

(a) Percentage of Time that a Base Station can Transmit

![Graph](image)

(b) Mean Downstream Data Rate per Active Area

*Figure 6.6* Sensitivity of extent of transmissions on cell radius of the secondary system, 95% confidence interval is within ±1% of the results shown
Figure 6.7 Sensitivity of extent of transmissions on distance between a radar and its nearest cell \((d_{min})\).

95% confidence interval is within ±1% of the results shown.

Fig. 6.8 shows the mean downstream data rate achievable per active cell as a function of radar’s maximum tolerable INR. Similarly to what has been observed in Fig. 6.5, the percentage of time that a BS can transmit shows the same trend as the mean achievable data rate per active cell, and hence is omitted for brevity but can be found in Appendix C. As expected, Fig. 6.8 shows that the extent of secondary transmissions decreases with more stringent protection of radars from harmful interference, i.e., smaller tolerable INR. The extent of transmissions achievable is quite sensitive to the value of INR used. For example, decreasing the INR by 3 dB, from -10 to -13 dB, would reduce
mean data rate per active cell by a factor of 1.6 at 4% cells active, and by a factor of 1.4 at 20% cells active. Hence, radar’s tolerable interference level is an important factor that determines the extent of transmissions achievable in the sharing.

Figure 6.7 (Contd.) Sensitivity of extent of transmissions on distance between a radar and its nearest cell ($d_{\text{min}}$), 95% confidence interval is within ±1% of the results shown

Figure 6.8 Sensitivity of extent of transmissions on radar maximum tolerable Interference to Noise Ratio (INR), 95% confidence interval is within ±1% of the results shown

Regarding the impact of radar transmit power on the extent of secondary transmissions, it is found that radar transmit power has insignificant impact on secondary transmissions. This could be because with the proposed sharing mechanisms, a BS will transmit when link losses between the BS
and its nearby radars are high enough that interference to and from any nearby radar is insignificant; hence, interference from the radars has insignificant impact on transmissions of the secondary system. The graphical results are omitted, but can be found in Appendix C.

6.5.4.2 Sensitivity of Fluctuations in Secondary Perceived Data Rate

This section investigates the impact of six parameters on the fluctuations in perceived downstream data rate: cell radius, radar rotating period, radar transmit power, distance between a radar and its nearest cell, radar maximum tolerable INR, and distance between adjacent radars. Each of Fig. 6.9 to Fig. 6.12 shows the 1st percentile, the 99th percentile, and the mean perceived data rates as a function of size of files being transferred. Unless stated otherwise, results are shown for a cell 140 km from the radar (where a cell is close enough to be strongly affected by radar, as previously shown in Fig. 6.2) and with 4% of cells active. The results from when 20% of cells are active, and at different distances from the radar show similar trends, and hence are omitted, but can be found in Appendix C.

Fig. 6.9 shows the impact of cell radius (of the secondary system) on the fluctuations of perceived data rate, when cell radii are 0.2 and 1.5 km. Regardless of cell size, perceived data rate can be vastly lower than its mean for small files, but not for larger files, e.g., files larger than 1 MB. Hence, as observed from the base-case results, the sharing mechanisms will work well for a large class of applications over a very large range of cell sizes. Fig. 6.9 also shows that a user in a small cell will experience more fluctuation in perceived data rate than a user in a larger cell, primarily because devices in small cells are more likely to see times when perceived data rate is far better than average. However, quality of service is more dependent on low perceived data rate, and the difference between the 1st percentile and mean data rates are similar regardless of radius. Thus, the conclusions about quality of service for different applications in the base case also apply for cells of different sizes.

Fig. 6.10 shows the impact of radar rotating period on the fluctuations of perceived data rate, when the rotating periods are 4 and 6 s. As discussed in Section 6.5.4.1, rotating period is irrelevant for applications for which only throughout matters. If fluctuations of data rate are of concern, Fig. 6.10 shows that shorter rotating periods will result in less fluctuation in perceive data rate. However, the fluctuations when rotating periods are 4 and 6 s are not very different. Hence, for the range of rotating period considered, rotating period has insignificant impact on the fluctuations in perceived data rate; the range of rotating period selected characterizes radars such as ATC radars, see Section 4.5.1 for more details.

Fig. 6.11 shows the lack of impact of radar transmit power on the fluctuations of perceived data rate, when the radar transmit powers are 0.025 and 1.4 MW. As previously discussed in Section 6.5.4.1, with the proposed sharing mechanisms, interference from the radars would have insignificant impact on secondary transmissions because a BS will transmit only when link losses between the BS and its nearby radars are high enough that interference to and from any nearby radar is insignificant. Hence, as expected, the curves with transmit power of 0.025 MW and the curves with transmit power
of 1.4 MW are so close that they are almost indistinguishable. Clearly, radar transmit power does not affect how data rate fluctuates.

Figure 6.9 Sensitivity of fluctuations in downstream perceived data rate on cell radius of the secondary system, 95% confidence interval is within ±9% of the results shown.

Figure 6.10 Sensitivity of fluctuations in downstream perceived data rate on radar rotating period, 95% confidence interval is within ±10% of the results shown.

Regarding the other three parameters, Fig. 6.12 shows the impact of the following parameters on fluctuations in perceived data rate:

(a) Distance between a radar and its nearest cell \(d_{\text{min}}\), when \(d_{\text{min}}\)’s are 5 and 100 km
(b) Radar’s maximum tolerable INR, when the INR’s are -13 and -7 dB
(c) Distance between adjacent radars \( (d_{\text{Rd}}) \), when \( d_{\text{Rd}} \)'s are 280 and 560 km, and a BS is at 50% of \( d_{\text{Rd}} \).

Similarly to what has been observed from the previous three parameters as shown by Fig. 6.9 to 6.10, \( d_{\text{min}} \), radar’s INR, and \( d_{\text{Rd}} \) have no significant impact on the fluctuations in data rate that a user transferring files of different sized will perceived. In Fig. 6.12(a), the curve when \( d_{\text{min}} \) is 5 km and the curve when \( d_{\text{min}} \) is 100 km are so close that they are almost indistinguishable.

Figure 6.11 Sensitivity of fluctuations in downstream perceived data rate on radar transmit power, 95% confidence interval is within ±11% of the results shown

Figure 6.12 Sensitivity of fluctuations in downstream perceived data rate
(b) when Radar Maximum Tolerable Interference to Noise Ratios (INR) are -13 and -7 dB, 95% Confidence Interval is Within ±8% of the Results Shown

(c) when Distances between Two Radars ($d_{rad}$) are 280 and 560 km, 95% Confidence Interval is Within ±11% of the Results Shown

Figure 6.12 (Contd.) Sensitivity of fluctuations in downstream perceived data rate

To summarize, within the ranges of the six parameters considered, a user transferring large files (i.e., file larger than 1 MB) will experience insignificant fluctuations in perceived data rate while the fluctuations experienced by a user transferring smaller files could be significant. Hence, similar to what have been observed from the base-case results, within these reasonable ranges of the six parameters, the fluctuations in perceived data rate will be a problem for interactive exchanges of small pieces of data, each of which must be received within a small period, but will not be a problem for applications transferring files large enough so that the fluctuations are not noticeable, such as song
and video downloads.

6.6 Conclusions

This chapter quantitatively demonstrates the potential of opportunistic gray-space sharing between multiple rotating radars and multiple broadband cells. Although the cells that are active at a given instant in time do not entirely blanket a region, this might be valuable for a cellular network that uses shared spectrum in a given cell only at those times when demand exceeds the capacity available in dedicated spectrum. This sharing scenario also is applicable when a secondary system provides a hotspot service.

In general, this chapter proposes that such sharing should be provided using a two-step mechanism; a regional resource allocation mechanism can use information gathered from many cells to provide every active cell with relatively static parameters that reflect the current level of activity across the region, while a local power control mechanism can make more dynamic decisions about when it is safe to transmit and at what power using these static parameters and local information, such as where the nearby radars are currently pointing their main beams. Thus, decisions about transmit power, which must be made quickly and often, are based entirely on local information regardless of what is happening in other cells. The particular regional resource allocation mechanism considered provides every active cell with a threshold for expected signal loss, and the local power control mechanism ceases transmission during those periods when current expected signal loss falls below that threshold.

Even in the worst possible deployment scenario— in which spectrum is 100% utilized as viewed by traditional spectrum management approaches, and radars are packed to the theoretical maximum; each radar is surrounded by six other radars, all separated by the minimum allowable distance— on average sufficiently high data rates can be achieved in these cells to be of significant benefit to a cellular network. For example, if instantaneous load exceeds what a cellular network can carry over its dedicated spectrum in 5% of its cells, that cellular network can get almost 1.2 bps/Hz on average from the shared spectrum. Even greater total throughput is possible when more cells are active, although the achievable data rate per active cell would be lower. Although dedicated spectrum can support data rates higher than 1.2 bps/Hz, this is impressive from spectrum that is already so heavily utilized. Moreover, it should be possible to support much greater transmission rates in practice, as it is nearly impossible to place every radar at the absolute minimum distance from six other radars, and achievable mean transmission rates increase rapidly with an increase in distance between radars.

These extensive transmissions can be achieved with an interference risk to radar that is likely to be well below the interference risk that these rotating radars already pose to each other, in part because cellular performance was found to be relatively insensitive to the level of risk to radars. Hence, it is possible to provide high levels of protection to radars with little reduction in achievable
secondary transmissions.

In contrast to dedicated spectrum, data rate in shared spectrum fluctuates. These fluctuations may be problematic for a few applications, most notably VoIP and urgent transfers of small files, but it is found that fluctuations and interruptions are not a problem for applications such as video streaming, web browsing, peer to peer file sharing, and large file transfers which collectively account for most wireless Internet traffic.

None of the conclusions above with respect to either mean data rate or perceived data rate in the face of fluctuations change significantly when varying important system parameters, including radar rotation period, radar transmit power, distance between adjacent radars, distance between radar and the closest cell, and cell size. The level of interference that radar can tolerate has greater impact on the performance that the cellular system can achieve, but achievable transmission rates are still high within the expected range.

Cellular systems can make good use of spectrum shared with radar for both large and small cells. Given that finding spectrum for large cells is likely to be more problematic for operators, and high mean data rates are possible for large cells in shared spectrum, operators may wish to use this shared spectrum for large cells.

The ability to achieve a significant data rate on average, and quality of service that meets the needs of most applications, over spectrum that otherwise is inaccessible, is encouraging. This may motivate discussion of spectrum reforms that would make this possible, as discussed in [Peha12].
Chapter 7

Conclusions, Policy Implications, and Future Work
Contributions of the dissertation are summarized in Section 7.1. Section 7.2 presents findings and policy implications found throughout the dissertation. Policy issues related to gray-space sharing are discussed in Section 7.3. Future work is discussed in Section 7.4.

7.1 Contributions

Regarding the contributions, this dissertation,

- proposes various novel mechanisms that enable gray-space sharing with cellular systems, and rotating radars
- quantifies and shows potential of the gray-space spectrum sharing model that uses power control to avoid causing harmful interference to a primary system
- is the first to address how to make coexistent, rather than cooperative, gray-space sharing possible with cellular systems, and radars
- demonstrates the potential of spectrum sharing models in which a secondary system has information about a primary system, but does not cooperate in real time
- is the first to compare coexistent and cooperative sharing by taking into account both the performance as perceived by a secondary system, and performance as perceived by a primary system, see Chapter 3
- is the first to quantitatively evaluate both the extent of transmissions achievable in spectrum shared with radars, and the impact of interruptions and fluctuations in secondary transmissions on the performance of various prominent applications on the Internet, see Chapters 4 to 6.

7.2 Conclusions and Policy Implications

This dissertation investigates the potential of gray-space primary-secondary spectrum sharing. In this form of sharing, a primary spectrum user will be protected from harmful interference, i.e., interference causing disruptions in services. A secondary spectrum user will be allowed to transmit as long as transmissions of the primary system are strong enough that additional interference from secondary transmissions would be tolerable, rather than only when the primary transmissions are weak or absent so the spectrum is considered unused, as occurs in white-space sharing. Specifically, in this dissertation, harmful interference to the primary system is avoided by dynamically adjusting transmit power of a secondary device.

Two different types of primary system are considered: cellular systems, and rotating radars, e.g., ATC radars and weather radars. Both cases when primary and secondary systems cooperate (cooperative sharing), and when they do not (coexistent sharing) are considered. In particular, to
enable coexistent sharing, this dissertation assumes that a secondary system has some technical information about a primary system.

Depending on types of the primary system, this dissertation considers spectrum sharing in different scenarios, and proposes various novel sharing mechanisms. Specifically,

- When the primary system is radar, spectrum sharing with OFDMA-based cellular system, operating as the secondary system in non-contiguous cells, is considered. This might be valuable for a cellular network that uses shared spectrum in a given cell only at those times when demand exceeds the capacity available in dedicated spectrum, and for a secondary system providing a hotspot service. This dissertation proposes that the sharing should be provided using a two-step mechanism:
  1. **Regional resource allocation mechanism** uses information gathered from many secondary transmitters to provide every transmitter with relatively static parameters that reflect the current level of activity of the secondary system across the region.
  2. **Local power control mechanism** makes more dynamic decisions about when it is safe to transmit and at what power using the static parameters and local information, such as current directions of the nearby radars’ main beams. Thus, decisions about transmit power, which must be made quickly, are based entirely on local information regardless of activity of other secondary transmitters.

- When the primary system is a cellular system, spectrum sharing in the uplink of a voice CDMA cellular system is investigated, assuming that a secondary device is a fixed or portable device. To enable coexistent spectrum sharing, the coexistent sharing model is proposed, in which secondary devices query a sensor network which observes primary downstream communications to estimate how much additional interference the primary system can tolerate in the upstream.

Using analyses and extensive Monte Carlo simulations, it is shown that useful secondary transmissions are possible, even when the shared spectrum is considered 100% utilized by the primary system under conventional approaches to spectrum management. For example, it is found that:

- In spectrum sharing with radars, even in the scenario in which radars are packed to the theoretical maximum density, if instantaneous load exceeds what a cellular network can carry over its dedicated spectrum in 5% of its cells, the secondary system can get almost 1.2 bps/Hz on average from the shared spectrum. Even greater total throughput is possible when more cells are active, although the achievable data rate per active cell would be lower. Moreover, significantly transmissions higher than those achieved from the worst-case scenario are expected in practice, as actually radars would rarely be as closely packed as possible.

- In spectrum sharing with cellular systems, when the primary system is 100% utilized, a modest extent of transmissions of around 0.01–0.03 bps/Hz is achievable for secondary transmitter and receiver that are 400 m apart. Moreover, the data rate increases to
0.5 bps/Hz, and 4.0 bps/Hz when primary utilization is at 50%, and 10%, respectively.

These extensive transmissions show potential of the proposed spectrum sharing mechanisms for both when the primary systems are cellular systems, and rotating radars.

More generally, the ability to achieve a significant data rate on average over spectrum that otherwise is inaccessible by the white-space sharing is encouraging. This also shows that the white-space sharing is not the only way to increase efficiency in spectrum use, and, thus, may motivate discussion of spectrum policy reforms that would make other kinds of sharing possible, including the gray-space sharing utilizing power control considered in this dissertation.

Moreover, this dissertation shows the potential of spectrum sharing models in which a secondary system has information about a primary system, but does not cooperate in real time; such arrangements are not typically considered today. Rather than focusing entirely on secondary devices that know only what they can sense, policymakers and license-holders should consider making some information of primary systems available to help secondary systems more effectively avoid interfering with the primary systems.

From spectrum sharing with radars, it is also found that the transmissions achievable are relatively insensitive to the level of risk of harmful interference to radars. Hence, it is possible to provide high levels of protection to radars with little reduction in achievable secondary transmissions.

Even though high transmissions can be achieved, secondary transmissions in the shared spectrum will fluctuate, and can be interrupted as radars rotate. It is found that these sporadic interruptions, and fluctuations may be problematic for some applications, most notably VoIP and urgent transfers of small files. However, these high levels of fluctuations and interruptions are tolerable for applications such as video streaming, web browsing, peer to peer file sharing, and large file transfers which collectively account for the majority of mobile Internet traffic. Hence, cellular operators may use the shared spectrum for the applications that are tolerable to the fluctuations and interruptions in transmissions, while use their own dedicated spectrum for the other applications that are sensitive to the fluctuations and interruptions.

None of the conclusions above on either mean data rate or perceived data rate in the face of fluctuations change significantly when varying important system parameters, including radar rotation period, radar transmit power, distance between adjacent radars, distance between radar and the closest cell, and cell size. The level of interference that radar can tolerate has greater impact on the extent of transmissions achievable. However, the achievable transmission rates are still high within the expected range of radars’ tolerable interference level; hence, this would not be a problem.

The extent of transmissions achievable and quality of services experienced by users of various applications show promising potential of spectrum shared with radars for alleviating the spectrum scarcity problem, especially when large demand of spectrum is expected for cellular and mobile broadband.
From spectrum sharing with cellular systems, the efficiency of cooperative and coexistent sharing is compared based on performance of the secondary system measured as achievable transmissions, and performance of the cellular system measured as power consumption of a mobile device, which may be increased to compensate for additional interference from secondary transmissions. If only the amount of transmissions achievable is of concern, sharing with cooperation is always more efficient than sharing with coexistence. However, if both achievable secondary transmissions and primary power consumption are of concern, coexistent sharing is found to be as efficient as cooperative sharing. Hence, the decision of whether to choose sharing with cooperation or coexistence might need to be based on non-performance factors.

These results (from sharing spectrum with cellular systems) appear to be relatively insensitive to some potentially important system parameters, including intercell interference, and the amount of spectrum allocated to the primary system. Moreover, sharing with the primary system using smaller channel bandwidth would achieve higher secondary data rate. Decreasing cell radius of the primary system while keeping the mean number of secondary devices per cell constant will decrease data rate achievable per secondary device. However, it will increase the number of secondary devices per area even more, so this is presumably not problematic.

7.3 Policy Issues Associated with Gray-Space Sharing

This dissertation has shown that gray-space spectrum sharing can potentially increase spectral efficiency by allowing a secondary device to transmit at the same time as primary devices, and even close to the primary devices, without causing harmful interference.

With gray-space sharing, this higher spectral efficiency is generally achieved by making the primary and secondary systems technically interdependent [Peha12], e.g., the sharing mechanisms proposed in Chapters 3 to 6 are highly dependent on technology used by the primary and secondary systems. Hence, existing spectrum sharing policies, that do not require much interaction between primary and secondary systems, would be not appropriate for this gray-space sharing.

Moreover, as discussed in Section 4.3, by allowing a secondary device to transmit in the same geographical area, and at the same time as primary devices, gray-space sharing creates higher risk of unintended harmful interference compared to the existing spectrum sharing models, such as the white-space sharing. (For example, there might be a bug that causes a secondary device to miscalculate its transmit power to avoid causing harmful interference to primary users.) Therefore, mechanisms that will promptly identify and stop secondary devices from causing this unintended harmful interference are also needed in gray-space sharing.

Hence, to enable these forms of sharing, some entities have to be responsible for [Peha12]:

- granting and withholding permission to deploy a secondary system in shared spectrum
- sharing of information on technical design and operation between primary and secondary
systems
- overseeing appropriate modification(s) on sharing mechanisms, when primary and/or secondary systems are changed in a way that could affect spectrum sharing
- approving testing procedures to ensure that sharing will not cause harmful interference
- solving harmful interference that might occur, by accepting complaints about harmful interference, identifying the source(s) of harmful interference, and promptly requiring secondary systems to be reconfigured or simply turned off to end harmful interference
- encouraging primary spectrum users to share spectrum
- encouraging confidence in secondary spectrum users that their secondary rights will be sufficiently stable over time to warrant long-term investment.

Existing spectrum sharing policies, such as secondary license, secondary market, and unlicensed secondary devices discussed in Section 1.2, might be used to facilitate this gray-space sharing in a few specific cases. Currently, a form of gray-space sharing might be legal in some countries. For example, in the US, gray-space sharing would be legal under the secondary market rulings; however, the secondary market is allowed only in some but not all spectrum bands [FCC04]. More generally, some modifications on the existing spectrum sharing policies, or new policies might be needed depending on how fast technology used in primary systems changes, and number of primary and secondary spectrum users participates in the sharing [Peha12].

In cases where technology used by primary systems is relatively static- i.e., does not change over a short period of time as occurs in broadcasting- there are scenarios in which a regulator may be able to make spectrum available for secondary use, and hence the secondary license policy is applicable. In these cases, a regulator might enforce sharing of technical and operational information between the primary and secondary systems.

In other cases where technology used by a primary system is dynamic, and a single entity is licensed to be the primary spectrum user over a large area, compared to area where secondary users are operating, the existing secondary market policy (e.g., [FCC04]) could be modified to facilitate the sharing. Unlike a regulator, a license-holder (which is the primary spectrum user) has detailed information on its current technology and system upgrade plans, which it can share with secondary users. The license-holder also has the ability to detect harmful interference much faster than the regulator, and the ability and motivation to promptly stop secondary transmissions if harmful interference occurs. However, in gray-space sharing, different systems operate in the same block of spectrum, and in the same geographic area; this does not fit the existing secondary spectrum market policy wherein blocks of spectrum with clear frequency and geographic boundaries are exchanged for extended periods. Moreover, existing secondary market policy requires secondary spectrum users to operate in compliance with any restriction on the license-holder. This could be a problem in cases where the primary license has restrictions on technology used and transmitter location; the secondary market policy in its current form would not allow the license-holder to make gray-space sharing arrangements.
In some cases, the combination of the existing spectrum sharing policies might be appropriate for facilitating gray-space sharing. For example, when multiple primary systems share their spectrums with a single secondary system, one possible approach is to grant exclusive secondary rights to the secondary spectrum user, and allow that user to bargain for gray-space sharing with each primary system on a bilateral basis. The scenarios could be applied to when a single cellular operator provides service throughout a region, and want to share spectrum with multiple radar systems, with primary rights, used by different government agencies. Moreover, recently, the US Federal Communication Commission (FCC) has proposed spectrum sharing in the 3.5 GHz band. In the FCC’s proposal [FCC12], government users, as primary users, will share spectrum with secondary spectrum users providing service in small cells. Similar to the scenarios considered in this dissertation for spectrum sharing with radars, those small cells would typically not blanket a region. In the sharing, two groups of secondary spectrum users share spectrum with the primary users. The first group is licensed secondary users, and the other group is unlicensed secondary users who would be required to accept interference from the primary users and the licensed secondary users.

For more complicated cases in which multiple primary systems, under different administrative control and primary technology is not static, share spectrum with multiple secondary systems, new policies are needed. For such cases, regulators might not competent enough to manage the complexity of preventing interference among multiple diverse and changing systems, which are close enough together to potentially cause interference. Hence, a new governing body should be established to serve as a band manager for gray-space sharing. The band manager must have the ability, authority, and responsibility for introducing the sharing while protecting primary users as discussed earlier in this section. This includes determining where sharing is possible, ensuring that technical information is shared when necessary, managing technology changes over time, addressing any harmful interference problem that might occur, and encouraging potential primary and secondary spectrum users to participate in the sharing.

These policy issues related to gray-space sharing are discussed in more details in [Peha12].

7.4 Future Work

This dissertation has investigates potential of the gray-space sharing, based on various specific scenarios, and research questions. Additional studies under different scenarios and research questions are still open for future work. Some of the future work is discussed below.

Generally, this dissertation considers one technique to enable gray-space spectrum sharing, by employing power control to avoid causing harmful interference to a primary system. Depending on various factors- such as types of the primary and secondary systems, and whether the sharing is based on cooperation or coexistence- other techniques might also be possible for gray-space sharing, for example, by using information about relative locations of a primary and secondary devices, i.e.,
geolocation information, or by using beam-forming technology which would allow a secondary device to transmit in directions that will not cause harmful interference, etc. Performance of gray-space sharing using different techniques, and comparisons of the performance based on corresponding advantages and disadvantages of each sharing techniques would give regulators insight and useful information on potential of the sharing, and how gray-space sharing should be allowed.

For spectrum sharing with radars, except in Chapter 4 in which the simplified scenario when one cell shares spectrum with one radar is considered, this dissertation only investigates transmissions achievable in the downstream of the secondary system. This is, in part, because it is found in Chapter 4 that the shared spectrum is used more efficiently for the downstream than for the upstream transmissions of the secondary system. However, to compare efficiency of the transmissions achievable between the up- and downstream in more realistic scenario, the upstream transmissions of the secondary system, when there are multiple radars, need to be investigated.

As discussed in Sections 4.3, 5.3 and 6.4, performance of the proposed sharing mechanisms are evaluated assuming that adjacent channel interference is negligible, for both adjacent channel interference into and out of the band shared by radar and a secondary system. Significance of this adjacent channel interference, and its impact on performance of the secondary system in the shared spectrum need to be evaluated.

The numerical results in this dissertation are obtained assuming characteristics of one type of rotating radars, i.e., ATC radars. The potential of spectrum sharing with other types of rotating radars could also be considered, such as weather radars for which rotation pattern might change when storm approaches, and radars using phased array, etc. Using phased array, a radar might be more tolerant to interference from secondary transmissions as it can re-scan area that it might misdetect a target more quickly, compared to a radar with mechanical rotating antenna. However, direction of the radar’s main beam will not be deterministic, and hence, cooperative sharing might be more appropriate. This study would help indentify types of rotating radars that have high potential to share spectrum with.

Moreover, this dissertation considers the sharing in specific scenarios in which multiple cells of a secondary system share spectrum with multiple radars using the same frequency, and the percentage of cells active are low enough that intercell interference among the secondary cells can be negligible. These specific scenarios could be extended to other scenarios, which are applicable to some other real situations, such as when

- percentage of cells active is high enough that the analyses need to consider intercell interference among the secondary cells
- multiple radars use different frequencies; as discussed in Section 6.3, higher total system throughput, and smaller variations in data rate perceived by a secondary user could be achieved by using OFDMA and some form of spectrum aggregation. The benefit of using spectrum aggregation with this more complicated scenario should be investigated and quantified.
For spectrum sharing with cellular systems, this dissertation considers the scenarios in which a secondary system shares spectrum with the upstream of a CDMA voice communications system. The sharing models could be extended to when the primary system uses other prevailing technologies such as OFDMA, and/or carries both voice and data. The sharing model could also be extended to when the secondary system shares spectrum with the downstream of the primary system. Sharing in the downstream of a cellular system will add more complexity into analyses because in each cell secondary transmissions will interfere with mobile terminals which usually spread throughout the cell.

In Section 3.4, this dissertation has discussed various methods that might be used to deal with fading in wireless channel between the primary and the secondary systems. Further investigations on advantages, and disadvantages of these methods are needed.

Moreover, even though this dissertation quantifies and demonstrates the benefit of implementing a sensor network to enable coexistent sharing with a cellular system, the cost of implementing the sensor network, such as amount of communications needed to facilitate the sharing has not been quantified yet. The communications includes exchanging of information among the sensors, and exchanging of information between the sensor network and the secondary system. Quantifying this amount of communications could be useful for a more concrete feasibility analysis of implementing the sensor network for spectrum sharing.
This page is intentionally left blank.
Appendix A

Equation Derivations
This Appendix collects derivations of various equations that are omitted in the main body of the dissertation.

**Derivation of (3.8)**

From (3.6), power of signals transmitted from a given Base Station (BS) to its $i$-th mobile terminal $P_i$ is

$$P_i = \frac{\rho_{a,2}}{G+\rho_{a,2}} \left[ W \times \frac{n_i}{l_{M,i}} + P_B \right]. \quad \text{(A.1)}$$

By taking summation of $P_i$ over all $K$ users,

$$\sum_{i=1}^{K} P_i = \frac{\rho_{a,2}}{G+\rho_{a,2}} \sum_{i=1}^{K} \left[ W \times \frac{n_i}{l_{M,i}} + P_B \right]$$

$$= \frac{\rho_{a,2}}{G+\rho_{a,2}} \left[ W \times \sum_{i=1}^{K} \frac{n_i}{l_{M,i}} + K \times P_B \right]. \quad \text{(A.2)}$$

As $\sum_{i=1}^{K} P_i = P_B$, and by defining $f$ as the average of $\frac{n_i}{l_{M,i}}$ across all $K$ users: $f \triangleq \frac{\sum_{i=1}^{K} \frac{n_i}{l_{M,i}}}{K}$, from (A.2) can be re-written as,

$$P_B = \frac{\rho_{a,2}}{G+\rho_{a,2}} \left[ W \times K \times f + K \times P_B \right]. \quad \text{(A.3)}$$

Hence,

$$K = \frac{G+\rho_{a,2}}{\rho_{a,2}} \frac{P_B}{P_B+W\times f}. \quad \text{(A.4)}$$

(3.8) is then obtained by substituting $K$ as of (A.4) into (3.5). #
Derivation of (3.13)

As discussed in Section 3.6.2, the optimal threshold on number of user $K_T^*$ is obtained when the estimate of TTIL (Total Tolerable Interference Level) of a given BS from Case 1 equals to that from Case 2, i.e., $\hat{I}_{T,1} = \hat{I}_{T,2}$. From $\hat{I}_{T,1}$ and $\hat{I}_{T,2}$ as shown by (3.9) and (3.12), respectively

\[
\frac{P_{M,e,\text{max} \times L_{\text{min},\text{LB}}}}{W} \times \left( \frac{G}{P_{a,1}} - \left[ K_T - 1 \right] \right) = \frac{P_{M,e,\text{max} \times L_{\text{min},\text{LB}}}}{W} \times \left( \frac{G + \rho_{a,2}}{\rho_{a,2}} \right) \left[ \frac{1}{1 + \frac{\rho_{\text{a,2}}}{\rho_{\text{a,1}}}} \frac{1}{\mu \sqrt{\frac{1}{K_T} + \left( \frac{1 + (K_T - 1)\rho_{\text{max},a}}{\rho_{\text{a,1}}} \right)}} \right] - 1
\]

(A.5)

From (A.5),

\[
K_T = \frac{G + \rho_{a,2}}{\rho_{a,2}} \frac{1}{1 + \frac{\rho_{a,2}}{\rho_{a,1}} \frac{1}{\mu \left( \frac{1}{K_T} + \frac{1 + (K_T - 1)\rho_{\text{max},a}}{\rho_{\text{a,1}}} \right)}}
\]

(A.6)

Define

- $A = \frac{G + \rho_{a,2}}{\rho_{a,2}}$
- $B = \frac{W}{\rho_{a,1}}$
- $C = \beta \sigma$
- $D = 1 + \mu B$
- $x = K_T$

(A.6) can be re-written as

\[
x = \frac{A}{1 + B \left[ \mu - C \right] \sqrt{\frac{1 + (x - 1)\rho_{\text{max}}}{x}}}
\]

\[\left[1 + \mu B\right] x - BCx \sqrt{\frac{1 + (x - 1)\rho_{\text{max}}}{x}} = A\]

\[BC \sqrt{x + x(x - 1)\rho_{\text{max}}} = Dx - A\]

\[BC \left[ x + x(x - 1)\rho_{\text{max}} \right] = Dx^2 - 2ADx + A^2\]

\[\left[ B^2 \rho_{\text{max}} - D^2 \right] x^2 + \left[ (1 - \rho_{\text{max}})B^2 C^2 + 2AD \right] x - A^2 = 0. \]
(A.7) has $x = K_T$ as a variable, and is in a quadratic form of which a solution is given by (3.13). ##
Appendix B

Data Rate Derived from 3GPP Data Regression
This Appendix gives details on a set of equations mapping from SINR of a cellular user to the resulting data rate, which is used in Chapters 4 and 5. The equations are from regression on data from 3GPP [Jaci09], which are obtained by assuming

1) An urban or suburban environments with fairly small cell and low delay spread (i.e., Extended Pedestrian A (EPA) channel model
2) A very low speed user, i.e., 5-Hz Doppler frequency
3) Multiple Input Multiple Output (MIMO) 2 × 2

The relationships between SINR $\rho_S$ and the resulting data rate per resource block (in bps) $R_{b,\text{BB}}$ of an LTE system for three different modulations, i.e. QPSK, 16QAM, and 64QAM are

For QPSK with a coding rate of 1/3:

$$R_{b,\text{BB},\text{QPSK}}[\text{bps}] = \begin{cases} 
(0.019\rho_S^3 - 0.1455\rho_S^2 + 0.3516\rho_S) \times 10^4, & -2 < \rho_S < 2 \\
(0.0063\rho_S + 9.6009) \times 10^4, & 2 < \rho_S < 4 
\end{cases} \quad (B.1)$$

For 16QAM with a coding rate of 1/2:

$$R_{b,\text{BB},\text{16QAM}}[\text{bps}] = \begin{cases} 
(-0.000945\rho_S^4 + 0.0103\rho_S^3 - 0.0141\rho_S^2 + 0.1696\rho_S + 1.0083) \times 10^5, & -2 < \rho_S < 6 \\
(0.0048\rho_S^3 - 0.1503\rho_S^2 + 1.5644\rho_S - 2.4858) \times 10^5, & 6 < \rho_S < 12 \\
293820, & \rho_S > 12 
\end{cases} \quad (B.2)$$

For 64QAM with a coding rate of 3/4:

$$R_{b,\text{BB},\text{64QAM}}[\text{bps}] = \begin{cases} 
(-0.1292\rho_S^3 + 1.3299\rho_S^2 - 0.4279\rho_S + 0.3036) \times 10^4, & 0 < \rho_S < 6 \\
(-0.1018\rho_S^2 + 2.92\rho_S + 3.8494) \times 10^4, & 6 < \rho_S < 10 \\
(0.0585\rho_S^3 - 1.0032\rho_S^2 + 6.4581) \times 10^5, & 10 < \rho_S < 16 \\
(0.4354\rho_S^2 - 1.6099) \times 10^5, & 16 < \rho_S < 18 \\
(-0.0241\rho_S^3 + 1.0214\rho_S^2 - 4.33555) \times 10^5, & 18 < \rho_S < 22 \\
647085, & \rho_S > 22 
\end{cases} \quad (B.3)$$

Fig. B.1 shows data rate as a function of corresponding SINR curves assuming transmission bandwidth is 3 MHz. Fig. B.1 compares the curve obtained from (B.1)-(B.3), with the other curve
which is from the approximation that data rate is a fraction $\gamma$ of Shannon’s limit; $\gamma$ that results in minimum mean squared error between the curve from regressions and the approximation is around 0.53.

![Graph](image)

**Figure B.1** Data rate vs. SINR from the regressions, and from the approximation that data rate is a fraction of Shannon’s limit, assuming that bandwidth = 3 MHz
This page is intentionally left blank.
Appendix C

Figures Omitted from Detailed Discussions in the Dissertation
This Appendix collects figures that are omitted from discussions throughout the dissertation. The original section of a figure is given at the end of each figure caption.

Figures Omitted from Chapter 3:

Fig. C.1 compares performance of the four simple approaches, used in Section 3.7.4, to limit interference from a secondary system; the figure shows results at 10% and 60% primary utilization for both cooperative and coexistent sharing. These approaches will allow the secondary system to generate interference:

1) At (Secondary Tolerable Interference Level) STIL if and only if primary downstream power ($P_B$) is less than a threshold, i.e., P-UL approach
2) At STIL if the STIL is less than a threshold, and at the threshold otherwise, i.e., STIL-UL approach
3) At a fixed percentage of STIL, i.e., %STIL approach
4) At STIL if and only if $P_B$ is larger than a threshold, i.e., P-LL approach

![Figure C.1](HW(20000S), BW/SP=5/10MHz,R/r=11km/400m,Coop)

(a) From Cooperative Sharing

**Figure C.1** Comparison of the four approaches used to limit transmission of a secondary system, discussed in Section 3.7.4 Comparison of Performance of Cooperative Sharing vs. Performance of Coexistent Sharing
From Coexistent Sharing

**Figure C.1 (Contd.)** Comparison of the four approaches used to limit transmission of a secondary system, discussed in Section 3.7.4 Comparison of Performance of Cooperative Sharing vs. Performance of Coexistent Sharing

**Figure C.2** Mean derivative of secondary transmission rate with respect to mean transmit power of a mobile terminal vs. %STIL; the results are from cooperative sharing, omitted from Section 3.7.4 Comparison of Performance of Cooperative Sharing vs. Performance of Coexistent Sharing
Figures Omitted from Chapter 4:

**Figure C.3** Perceived downstream data rate vs. Size of a file being transferred, omitted from Section 4.5.5.1 Fluctuations in Secondary Perceived Data Rate

**Figure C.4** Sensitivity of average data rate on radar transmit power, omitted from Section 4.5.6.1 Sensitivity of Average Extent of Secondary Transmissions
Figure C.5 Sensitivity of average data rate on radar tolerable interference level, omitted from Section 4.5.6.1 Sensitivity of Average Extent of Secondary Transmissions

Figure C.6 Sensitivity of perceived secondary data rate in the downstream, omitted from Section 4.5.6.2 Sensitivity of Fluctuations in Secondary Perceived Data Rate
Figure C.6 (Contd.) Sensitivity of perceived secondary data rate in the downstream, omitted from Section 4.5.6.2 Sensitivity of Fluctuations in Secondary Perceived Data Rate
Figures Omitted from Chapter 6:

**Figure C.7** Percentage of time that a base station can transmit vs. Radar maximum tolerable Interference to Noise Ratio (INR), 95% confidence interval is within ±1% of the results shown, omitted from Section 6.5.4.1 Sensitivity of Average Extent of Secondary Transmissions

**Figure C.8** Sensitivity of extent of transmissions on radar transmit power, 95% confidence interval is within ±1% of the results shown, omitted from Section 6.5.4.1 Sensitivity of Average Extent of Secondary Transmissions
(b) Mean Downstream Data Rate per Active Cell

Figure C.8 (Contd.) Sensitivity of extent of transmissions on radar transmit power, 95% confidence interval is within ±1% of the results shown, omitted from Section 6.5.4.1 Sensitivity of Average Extent of Secondary Transmissions

Figure C.9 Effect of the six system parameters on fluctuations in perceived downstream data rate, when 20% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.9(a) in which a BS is at 50% of distance between two radars from its nearest radar, in the other figures, a BS is 140 km away from its nearest radar
(b) Cell Radius, Confidence Interval is within ±11% of the results shown

(c) Distance between a Radar and its Nearest Cell ($d_{min}$), Confidence Interval is within ±14% of the results shown

**Figure C.9 (Contd.)** Effect of the six system parameters on fluctuations in perceived downstream data rate, when 20% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.9(a) in which a BS is at 50% of distance between two radars from its nearest radar, in the other figures, a BS is 140 km away from its nearest radar
(d) Radar Maximum Tolerable interference to Noise Ratio (INR). Confidence Interval is within ±14% of the results shown

(e) Radar Rotating Period, Confidence Interval is within ±9% of the results shown

**Figure C.9 (Contd.)** Effect of the six system parameters on fluctuations in perceived downstream data rate, when 20% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.9(a) in which a BS is at 50% of distance between two radars from its nearest radar, in the other figures, a BS is 140 km away from its nearest radar.
(f) Radar Transmit Power, Confidence Interval is within ±8% of the results shown

**Figure C.9 (Contd.)** Effect of the six system parameters on fluctuations in perceived downstream data rate, when 20% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.9(a) in which a BS is at 50% of distance between two radars from its nearest radar, in the other figures, a BS is 140 km away from its nearest radar

(a) Distance between Adjacent Radars \(d_{rad}\), Confidence Interval is within ±4% of the results shown

**Figure C.10** Effect of the six system parameters on fluctuations in perceived downstream data rate, when 4% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.10(a) in which a BS is at 36% of distance between two radars from its nearest radar, and Fig. C.10(c) in which a BS is at 120 km from its nearest radar, in the other figures, a BS is 100 km away from its nearest radar
(b) Cell Radius, Confidence Interval is within ±6% of the results shown

(c) Distance between a Radar and its Nearest Cell ($d_{min}$), Confidence Interval is within ±10% of the results shown

**Figure C.10 (Contd.)** Effect of the six system parameters on fluctuations in perceived downstream data rate, when 4% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.10(a) in which a BS is at 36% of distance between two radars from its nearest radar, and Fig. C.10(c) in which a BS is at 120 km from its nearest radar, in the other figures, a BS is 100 km away from its nearest radar.
(d) Radar Maximum Tolerable interference to Noise Ratio (INR), Confidence Interval is within ±7% of the results shown

(e) Radar Rotating Period, Confidence Interval is within ±3% of the results shown

Figure C.10 (Contd.) Effect of the six system parameters on fluctuations in perceived downstream data rate, when 4% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.10(a) in which a BS is at 36% of distance between two radars from its nearest radar, and Fig. C.10(c) in which a BS is at 120 km from its nearest radar, in the other figures, a BS is 100 km away from its nearest radar
(f) Radar Transmit Power, Confidence Interval is within ±4% of the results shown

Figure C.10 (Contd.) Effect of the six system parameters on fluctuations in perceived downstream data rate, when 4% of cells are active, omitted from Section 6.5.4.2 Sensitivity of Fluctuations in Perceived Data Rate; except for Fig. C.10(a) in which a BS is at 36% of distance between two radars from its nearest radar, and Fig. C.10(c) in which a BS is at 120 km from its nearest radar, in the other figures, a BS is 100 km away from its nearest radar.
Appendix D

Objective Function Considered in Chapter 5
This Appendix shows two characteristics of the objective function \( f_o \) considered in Chapter 5 which is the total mean data rate from all active cells \( N_a \): \( \frac{\partial}{\partial I_i} f_o > 0 \), and \( \frac{\partial^2}{\partial I_i^2} f_o < 0 \). These characteristics are necessary for the algorithm proposed to allocate interference level that Base Station (BS) \( i \) can cause to a given radar-as described in Section 5.3.1- to be applicable.

With the mean data rate shown by (5.2), the objective function of the optimization problem described by (5.1) can be re-written as

\[
f_o = \gamma W_s \sum_{i=1}^{N_a} E_{\text{cell}} \left\{ E_\phi \left\{ \log_2 \left( 1 + \frac{L_{S,i}(d_{S,i})}{n_s N_s + L_{Rd-MT,j}(\phi) \times P_{Rd}} \right) \right\} \right\},
\]

(D.1)

For brevity, the dependency of link loss on distance, i.e., \( L_{S,i}(d_S) \) in (D.1), will be omitted. From (D.1), \( \frac{\partial f_o}{\partial I_i} \) is

\[
\frac{\partial f_o}{\partial I_i} = \gamma \times W_s \times \left[ \frac{\partial}{\partial I_i} E_{\text{cell}} \left\{ E_\phi \left\{ \log_2 \left( 1 + \frac{L_{S,i}(d_S)}{n_s N_s + L_{Rd-MT,j}(\phi) \times P_{Rd}} \right) \right\} \right\} \right] + \sum_{\nu \neq i} \left[ \frac{\partial}{\partial I_i} E_{\text{cell}} \left\{ E_\phi \left\{ \log_2 \left( 1 + \frac{L_{S,i}(d_S)}{n_s N_s + L_{Rd-MT,j}(\phi) \times P_{Rd}} \right) \right\} \right\} \right].
\]

(D.2)

Because \( \sum_{\nu \neq i} \left[ \frac{\partial}{\partial I_i} E_{\text{cell}} \left\{ E_\phi \left\{ \log_2 \left( 1 + \frac{L_{S,i}(d_S)}{n_s N_s + L_{Rd-MT,j}(\phi) \times P_{Rd}} \right) \right\} \right\} \right] \) in (D.2) is always 0, (D.2) is reduced to

\[
\frac{\partial f_o}{\partial I_i} = \gamma \times W_s \frac{\partial}{\partial I_i} E_{\text{cell}} \left\{ E_\phi \left\{ \log_2 \left( 1 + \frac{L_{S,i}(d_S)}{n_s N_s + L_{Rd-MT,j}(\phi) \times P_{Rd}} \right) \right\} \right\}.
\]

(D.3)

As both \( E_{\text{cell}} \{ \} \) and \( E_\phi \{ \} \) are done independently of \( I_i \),

\[
\frac{\partial f_o}{\partial I_i} = \gamma \times W_s \times E_{\text{cell}} \left\{ \frac{\partial}{\partial I_i} \log_2 \left( 1 + \frac{L_{S,i}(d_S)}{n_s N_s + L_{Rd-MT,j}(\phi) \times P_{Rd}} \right) \right\}.
\]

(D.4)

\[
= \frac{\gamma \times W_s}{\ln 2} E_{\text{cell}} \left\{ \frac{L_{S,i}(d_S)}{L_{Rd-MT,j}(\phi) \times P_{Rd}} \left[ \frac{1}{[L_{S,i}(d_S)]^{1/2} + \sqrt{[L_{S,i}(d_S)]}} \right] \right\}.
\]

With the feasible region defined by the constraints in (5.1),
\[ \frac{L_{S,I}}{L_{Rd-BS},(\varphi)} \frac{1}{[n_S \times W_S + L_{Rd-MT},(\varphi) \times P_{Rd} + [L_{S,I} \times l_I / L_{Rd-BS},(\varphi)]]} \] in (D.4) is always larger than 0. Hence, \[ \frac{\partial}{\partial l_I} f_o > 0. \#

From (D.4),

\[
\frac{\partial^2}{\partial l_I^2} f_o = y x W_S \frac{E_\varphi}{m_2} E_{\text{cell}} \left\{ \frac{L_{S,I}}{L_{Rd-BS},(\varphi)} \left[ \frac{1}{[n_S \times W_S + L_{Rd-MT},(\varphi) \times P_{Rd} + [L_{S,I} \times l_I / L_{Rd-BS},(\varphi)]]} \right] \right\} 
\]

\[
= -y x W_S \frac{E_\varphi}{m_2} E_{\text{cell}} \left\{ \left[ \frac{L_{S,I}}{L_{Rd-BS},(\varphi)} \right]^2 \frac{1}{[n_S \times W_S + L_{Rd-MT},(\varphi) \times P_{Rd} + [L_{S,I} \times l_I / L_{Rd-BS},(\varphi)]]} \right\} \] (D.5)

As \[ \left[ \frac{L_{S,I}}{L_{Rd-BS},(\varphi)} \right]^2 \frac{1}{[n_S \times W_S + L_{Rd-MT},(\varphi) \times P_{Rd} + [L_{S,I} \times l_I / L_{Rd-BS},(\varphi)]]^2} \] in (D.5) is always larger than 0, \[ \frac{\partial^2}{\partial l_I^2} f_o < 0. \#

This page is intentionally left blank.
References


[FCC06] US FCC, In the matter of Revision of Parts 2 and 15 of the Commission’s Rules to Permit


[ITUR09] ITU, Recommendation ITU-R P.1546-4: Method for point-to-area predictions for terrestrials services in the frequency range 30 MHz to 3,000 MHz, 2009.


