Single and double peripheral production of sigmas in proton-proton collisions

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I. INTRODUCTION

The Regge formalism is successful for treating high energy elastic scattering and diffractive hadronic processes, and all known hadrons seem to lie on Regge trajectories, however, it has long been known that the treatment of these high energy processes is not consistent with the Regge meson trajectories [1]. Pomeron exchange dominates these high energy processes, which means in terms of Quantum Chromodynamics (QCD) that the Pomeron is not related to mesonlike \( \bar{q}q \) states, but is gluonic in nature. There has been a great deal of work in recent years in studying the gluonic structure of the Pomeron. QCD perturbation theory has been used to derive the hard or BFKL Pomeron (see Ref. [2] used to treat high momentum processes. The soft Pomeron, which we call the Pomeron, which can account universally for elastic and peripheral process, must be treated by nonperturbative QCD (see Ref. [3] for a review of nonperturbative QCD treatments, including the use of QCD instantons [4]).

Peripheral processes, which correspond to the production of low-momentum particles from Regge trajectories, at high energies are given by the emission of the peripheral particles from the Pomeron, often referred to as Double Pomeron Exchange. Since the Pomeron is gluonic in nature, peripheral production at high energy is given by the coupling of the peripheral particles to the gluonic field.

In the present work we study single and double-sigma peripheral production in high energy proton proton (pp) collisions, using an external field method to derive these processes. It is based on the model [5,6] that there exists a light scalar glueball strongly coupled to the I = 0 two-pion system, which we call the glueball/sigma model, and our recent work [7] that this system might lie on the daughter trajectory of the Pomeron. The sigma/glueball model is proposed based on three observations: (i) at low energies the scalar-isoscalar \( \pi - \pi \) system is observed in \( \pi - \pi \) scattering to be a Breit-Wigner resonance [8], which we call the sigma; (ii) the two-sigma channel is large in scalar glueball decay [9]; and (iii) in QCD sum rule calculations we find [5] a light scalar glueball far below the coupled scalar glueball-meson systems, which we find correspond to the \( f_0(1370) \) and \( f_0(1500) \). Our proposed glueball/sigma resonance is a coupled-channel glueball-2\( \pi \) system with a mass and width both about 400 MeV. With this picture it was predicted [6] that there will be found a large branching ratio for the decay of the \( F_{11}(1440) \) baryon resonance to a sigma and a nucleon.

Three quantities are necessary for the calculation of the cross sections: (i) the Pomeron-Nucleon vertex, (ii) the Pomeron propagator, and (iii) the sigma-Pomeron vertex. In our earlier work [7] we showed that the coupling of the Pomeron to the nucleon can be predicted using the glueball/sigma model with no free parameters, and that this coupling agrees within expected errors with a phenomenological Pomeron exchange model [10] that is consistent with many high energy experiments. We use this phenomenological model of the Pomeron-Nucleon vertex. The sigma-gluonic coupling, obtained from the glueball/sigma picture of Ref. [6] gives the sigma-Pomeron vertex. Since in the present work we give our results as the ratio cross sections of the peripheral production processes to the elastic scattering with similar kinematics, many details of the Pomeron formalism are not important. We also note that recent work suggests[11] that the \( \phi/(2230) \), if it turns out to be a tensor glueball, might lie on the Pomeron itself, while in our formalism the tensor glueball lies on the trajectory of the light scalar glueball [7], and should be found at about 2.8 GeV.

In Sec. II we briefly review the Pomeron formalism for elastic pp scattering. In Secs. III and IV we derive the cross section for single and double-sigma peripheral production, respectively. In Sec. V we give our conclusions that there are large branching ratios that can be tested in experiment.

II. ELASTIC PP SCATTERING VIA POMERON EXCHANGE

In this section we briefly review the formalism for elastic pp scattering at high energy, which is known to be dominated by Pomeron exchange. The two ingredients of the quite successful phenomenological model of Pomeron ex-
change for high energy elastic nucleon-nucleon scattering [10] are the Pomeron-Nucleon vertex, $V_{\mu N}^{P-N}$,
\begin{equation}
V_{\mu N}^{P-N} = \beta \gamma_{\mu} F_1(t),
\end{equation}
and the Pomeron propagator, $D^p$. The constant $\beta$ is known to be approximately $6$ GeV$^2$, but will not be needed in the present work. As depicted in Fig. 1(a), the form of the elastic proton-proton scattering amplitude with Pomeron exchange is given by
\begin{equation}
A^{PP} = \tilde{V}(t) D^p(t, s) V(t),
\end{equation}
where $t = (p_1 - p_1')^2$, $s = (p_1 + p_2)^2$, and $V(t)$ is given by $\beta$ and $F_1(t)$ of Eq. (1). We write the Pomeron propagator $D^p$ using the notation of QCD sum rules that is used to study glueballs [5]
\begin{equation}
D^p(q, s) = \int d^4x e^{i(q - x)} < 0|T[[G(x)G(0)][G(0)G(0)]]|0>,
\end{equation}
where $[G(x)G(x)]$ is a symbolic form for the current of the Pomeron.

This formalism for elastic scattering will be extended to peripheral production using an effective field method in the following two sections.

III. SINGLE SIGMA PERIPHERAL PRODUCTION IN PROTON PROTON COLLISIONS

In this section we derive the cross section for peripheral sigma production using our glueball/sigma model. With a notation similar to Eq. (2), the amplitude for the peripheral production of $\sigma \rightarrow \pi\pi$ as shown in Fig. 1(b) is given by
\begin{equation}
A^{P\sigma\sigma} \approx \tilde{V}(t_1) \tilde{D}_\sigma^p(t_1, t_2, s) V(t_2),
\end{equation}
where $\tilde{D}_\sigma^p$ is the propagator of the exchanged Pomeron coupled to a propagating $\sigma$ (which decays to the $I = 0$ 2$\pi$ resonance), which is often called a double Pomeron, $t_1 = (p_1 - p_1')^2$ and $t_2 = (p_2 - p_2')^2$. Although $\tilde{D}_\sigma^p$ depends on the momentum transfers to the two interacting nucleons, $t_1$ and $t_2$, which are different from the momentum transfer, $t$, for elastic scattering with the same $s$, the sigma meson for peripheral production carries a very small momentum $p_\sigma$, so that $t_1 \approx t_2 \approx t$ unless $t_1, t_2 \approx M_\sigma^2$. As an example, for $p_\sigma = 0$ and $M_\sigma = 400$ MeV, in the center of mass system $t_1 = t_2 = 0.16$ GeV$^2$. For this reason we use the approximation that
\begin{equation}
\tilde{D}_\sigma^p(t_1, t_2, s) \approx \tilde{D}_\sigma^p(t, p_\sigma, s),
\end{equation}
with $t = (t_1 + t_2)/2$ and $p_\sigma$ is the momentum of the $\sigma$.

Even for modest energies and small momentum transfers, such as the 1985 CERN experiment looking for glueballs in pp collisions at $\sqrt{s} = 63$ GeV [12], one could use an average value, as in Eq. (5), with only small changes in our results. Note that our calculations are for 50 GeV protons and the sigma momentum is of the order of 0.3 GeV, and that in our work we use low-energy theorems, so that only very low-energy $\sigma$ production can be treated in our model. This also allows the use of the external field method, which is described below.

The validity of our model requires the coupling of the Pomeron to the proton to be consistent with the gluonic-quark coupling in our model. Writing the scalar glueball current as $J_{GB} = 3g \cdot G/(4\pi^2)$, a low-energy theorem [13] for the glueball-quark coupling gives
\begin{equation}
\int d^4y < T[q(0) \bar{q}(0) J_{GB}(y)] > \approx \frac{32}{9} < 0|\bar{q}(0)q(0)|0 >.
\end{equation}

Using this theorem it was shown [7] that the pomeron-nucleon coupling of the phenomenological Donnachie-Landshoff model [10] is approximately given by our gluonic/Pomeron picture. Our model is also consistent with the low-energy theorem [13]
\begin{equation}
i\int d^4x < 0|T[G \cdot G(x)G \cdot G(0)]|0 > \approx \frac{29\pi^3}{b} < 0|\alpha_s G \cdot G|0 >,
\end{equation}
with $b = 11 - 2N_f/3$.

Using the external field method with the sigma treated as an external field we write the Pomeron propagator coupled to a $\sigma$ as
\begin{equation}
\tilde{D}_\sigma^p(q, p_\sigma, s) = \int d^4x e^{i(q \cdot x)} < 0|T[[G(x)G(x)] \times [G(0)G(0)]]|0 >.
\end{equation}

Assuming factorization, we use $[[G(x)G(x)] \times [G(0)G(0)]]|_\sigma = [G(x)G(0)][G(x)G(0)]|_\sigma$, with
\begin{equation}
<T\tilde{G}(x)G(0)>\text{ an external field expression for the sigma coupled to the gluon, giving}
\end{equation}
\begin{equation}
\tilde{D}_\sigma^p(q, p_\sigma, s) = \int d^4k \tilde{D}_\sigma^p(k, s) g_\sigma F_\sigma(q - k, p_\sigma),
\end{equation}
with $F_\sigma(q - k, p_\sigma)$ the sigma-gluon vertex form factor and $g_\sigma$ the coupling constant of the sigma to the gluon.
value of $g_\sigma$ was extracted from experiment in Ref. [6] from the glueball/sigma model, with its latest version including instanton effects [14] in essential agreement with the earlier model. The method is to use the external field method for the sigma current coupled to the gluon propagator [6]:

$$<G_{\mu\nu}^{a}J_{\sigma}^{a}G_{\mu\nu}^{a}> = g_{\sigma} <G_{\mu\nu}^{a}G_{\mu\nu}^{a}>$$

Assuming that the sigma $\pi\pi$ resonance arises from light glueball pole, $g_\sigma$ is given by the width of the pole, or $g_\sigma \approx 400$ MeV. For low-momentum sigmas, which we are assuming, $F_\sigma(q-k) \approx \delta^4(q-k)$. Including propagation of the $\sigma$ to the $\pi\pi$ vertex we obtain for the Pomeron propagator with a produced $\sigma$ with momentum $p_\sigma$:

$$\hat{D}_\sigma(p_t, p_\sigma, s) \approx g_\sigma G_\sigma(p_\sigma)D^P(i, s).$$

In Eq. (10) $G_\sigma$ is the Breit-Wigner resonance propagator of the sigma. Introducing the appropriate phase space factors we find from Eqs. (2), (4), and (10) the relationship

$$\frac{d^2\sigma_{pp\sigma}}{dtdE_{\sigma}} \frac{d\sigma_{pp}}{dt} = \frac{1}{4\pi^2} \frac{g_\sigma^2 F_\sigma^2}{t^2} \frac{1}{F^2(i)} \mathcal{F}(t_1, t_2, i),$$

where $t_1 = t_2 + p_\sigma^2 - 2p_\sigma \cdot (p_2 - p_1)$, and we have included the fact that both gluons can radiate sigmas. The energy/momentum structure of Eq. (11) can be understood by considering the dispersion relation for $D^P$, and recognizing that in our approximations we assume that it is dominated by the pole and that at low momentum for the $\sigma$ the numerator of the pole is independent of energy. The function $F(i)$, which gives the $t$-dependence of the Pomeron-quark vertex, is taken from the phenomenological fits of Ref. [10]:

$$F(i) = \frac{4m_{\pi}^2 - 2.79t}{4m_{\pi}^2 - t} \frac{1}{(1 - t/7.1)^2}.$$

The results for the ratio of cross sections in the center of mass frame are shown in Fig. 2. As shown by comparing the dashed curve to the solid curve in Fig. 2, for values of the sigma momentum consistent with peripheral production, the effects of $t_1 \neq t_2$ are very small. For this reason we drop the factor $\mathcal{F}(t_1, t_2, i)$ in the rest of the calculations.

For comparison with experiment we rewrite our results in terms of the $\sigma$ rapidity distribution which is a function of the transverse momentum of the $\sigma$. With the standard definition the rapidity of the $\sigma$ and the $\sigma$ energy, they are given by

$$y = \tan^{-1}\left(\frac{p_{\sigma\perp}}{E_{\sigma}}\right),$$

$$E_{\sigma} = \sqrt{m_{\sigma}^2 + p_{\sigma\perp}^2} \cosh(y).$$

FIG. 2. Ratio of differential cross sections of $pp \rightarrow ppp\sigma$ process to elastic $pp \rightarrow pp$ process. Dashed line shows the ratio without the factor $\mathcal{F}$, Eq. (12).

where $p_{\sigma\perp} = \sqrt{p_x^2 + p_y^2}$ is the transverse momentum of the $\sigma$. Using the approximation that the $t$ variable is the same as for elastic p-p scattering for peripheral production, as explained above, we integrate over the $t$ variable to obtain from Eq. (11)

$$\frac{d\sigma_{pp\sigma}}{dy} = \sigma_{tot} \frac{g_\sigma^2}{4\pi^2} \sinh(y)$$

Using the experimental fit to the total elastic p-p cross section at high energy [15], $\sigma_{tot} = 21.70 s^{0.008} + 56.08 s^{-0.4525}$ mb, which is similar to the fit obtained with the Pomeron/Reggeon model of Ref. [10], we obtain the results shown in Fig. 3. As seen in Fig. 3, in our model the

FIG. 3 (color online). Rapidity and transverse momentum distribution for sigma production.
cross section for peripheral production near \(y = 1.0\) and low-momentum transfer has a peak of about 4.0 mb. Note that the charged \(\pi^+\pi^-\) channel might not be as satisfactory because of the \(\rho\)-meson background from processes at the nucleon vertices. Even though \(\rho\)-mesons cannot be produced in Pomeron diffractive production, they can be produced by the scattered nucleons at a Pomeron-nucleon vertex, which makes the interpretation of experiment more difficult in some reactions.

Recently there have been extensive experiments by the WA102 collaboration in which centrally produced \(\pi^+\pi^-\) and \(\pi^0\pi^0\) systems have been analyzed [16]. Although the publications have not discussed the detection of what one now calls \(\sigma\), one can see structure in the low-energy \(\pi\pi\) systems that resemble the recent charm D+ decay experiments of the E791 collaboration [17] and the \(\tau^-\) decay of the CLEO collaboration [18], where the broad 400 MeV \(\sigma\) was seen. It is also interesting to note that a Regge pole analysis [19] is in agreement with the low-energy behavior of the CERN AFS experiment [12]. Althought the physics of that fit to the data is quite different from our glueball/sigma picture, the parameterization of the \(\pi\pi\) amplitude is similar to ours and our model should also be consistent with the data if the branching ratio of the low-energy \(\pi\pi\) channel were extracted. Also, very recently the sigma, with the mass and width parameters consistent with our glueball/sigma model has been observed [20] in the \(J/\Psi \rightarrow \omega \pi^+\pi^-\) decay.

IV. DOUBLE PERIPHERAL PRODUCTION OF SIGMARS IN PROTON PROTON COLLISIONS

In this section we study the high energy reaction \(pp \rightarrow pp\sigma\sigma\) with low-momentum peripheral production via Pomeron exchange. Using the same methods as in the previous section, the amplitude for this peripheral double-sigma production can be written as

\[
A^{p p \sigma \sigma}(t_1, t_2, s) = V(t_1)\bar{D}^P_{\sigma_1 \sigma_2}(t_1, t_2, s)V(t_2),
\]

where \(\bar{D}^P_{\sigma_1 \sigma_2}\) is the propagator of the exchanged Pomeron coupled to two sigmas, \(t_1 = (p_1 - p_1')^2, t_2 = (p_2 - p_2')^2\), and \(p_{\sigma_1}, p_{\sigma_2}\) are the momenta of the two sigmas. Note that our calculations are for 50 GeV protons and the sigma momenta are of the order of 0.3 GeV for sigma peripheral production, so that the momentum transfer at each nucleon vertex is approximately \(t\), the momentum transfer for elastic pp scattering. Taking advantage of the very low momentum of the sigmas, we use the effective field method to estimate the Pomeron propagator coupled to two sigmas.

Corresponding to Fig. 4(b), the Pomeron propagator with a sigma coupled to each gluonic leg is

\[
\bar{D}^P_{\sigma_1 \sigma_2}(\bar{\tau}, s) = g^2_{\sigma} G_{\sigma}(p_{\sigma_1}) G_{\sigma}(p_{\sigma_2}) D^P(\bar{\tau}, s),
\]

Using factorization and neglecting the form factor for the sigma-gluonic coupling as part of our low-momentum calculation, we find

\[
\bar{D}^P_{\sigma_1 \sigma_2}(\bar{\tau}, s) = g^2_{\sigma} G_{\sigma}(p_{\sigma_1}) G_{\sigma}(p_{\sigma_2}) D^P(\bar{\tau}, s),
\]

with \(\bar{\tau} = (t_1 + t_2)/2\) and \(p_{\sigma}\) is the momentum of a \(\sigma\), and the Pomeron propagator is evaluated at \(\bar{\tau} \approx t_1 \approx t_2\) (as we discuss below). \(g_{\sigma}\) is the \(\sigma\)-gluon coupling constant derived in Ref. [6] and \(G_{\sigma}\) is the Breit-Wigner resonance propagator of the sigma. Using the factorization approximation it is easy to see that the process with both sigmas coming from the same gluonic leg, pictured in diagram of Fig. 4(a) gives an equal contribution. There is also an equal contribution from the process with the two sigmas coupled to the other leg. This gives us for the amplitude of the double-sigma peripheral production

\[
A^{p p \sigma \sigma}(t_1, t_2, s) = 3g^2_{\sigma} G_{\sigma}(p_{\sigma_1}) G_{\sigma}(p_{\sigma_2}) A^{p p}(\bar{\tau}, s).
\]

Introducing the appropriate phase space factors we find for the ratio of the double peripheral sigma production to the pp elastic scattering

\[
F = \frac{A^{p p \sigma \sigma}}{A^{p p}}
\]

![Fig. 4.](image_url)

(a) Peripheral production from each gluonic leg of pomeron; (b) Double peripheral production from one leg.

![Fig. 5.](image_url)

The Factor \(F\) of Eq. (11) for peripheral production.
FIG. 6 (color online). Ratio of differential cross sections of \( pp \to pp \sigma_1 \sigma_2 \) process to elastic \( pp \to pp \) process.

\[
\frac{d^2\sigma^{pp\sigma\sigma}}{dy_1dy_2} = \frac{9g_\sigma^4}{4\pi^2}\frac{p_{\sigma_1}}{p_{\sigma_1}^4 + M_{\sigma_1}^2 \Gamma_{\sigma_1}^2} \times \frac{p_{\sigma_2}}{p_{\sigma_2}^4 + M_{\sigma_2}^2 \Gamma_{\sigma_2}^2} \mathcal{F}(t_1,t_2,\bar{t}),
\]

with \( \mathcal{F}(t_1,t_2,\bar{t}) \) given in the previous section. For \( p_{\sigma} \) limited to 0.3 GeV to be consistent with peripheral production, the factor \( \mathcal{F} \) is close to unity, as shown in Fig. 5. As one can see the factor is close to unity, and we drop in the rest of this paper.

In Fig. 6 we show the ratio defined in Eq. (21) in terms of the transverse momenta of the two sigmas. For comparison with experiment we rewrite our results in terms of the rapidity distributions of the two \( \sigma \). Using the approximation that the \( t \) variable is the same as for elastic pp scattering for peripheral production, and integrating over the \( t \) variable one obtains from Eq. (21)

\[
\sigma_{pp}^{pp\sigma\sigma} = \frac{9g_\sigma^4}{(2\pi)^4} \sinh(y_1) \sinh(y_2) \times \sqrt{(m_\sigma^2 + p_{\sigma_1 \perp} E_{\sigma_1} - m_2^2)(E_{\sigma_1} - m_\sigma^2)^2 + m_\sigma^4 \Gamma_{\sigma}^2} \times \sqrt{(m_\sigma^2 + p_{\sigma_2 \perp} E_{\sigma_2} - m_2^2)(E_{\sigma_2} - m_\sigma^2)^2 + m_\sigma^4 \Gamma_{\sigma}^2},
\]

with the standard definition of rapidity, \( y = \tan^{-1}(p_{\sigma_1}/E_{\sigma}), E_{\sigma} = \sqrt{m_\sigma^2 + p_{\sigma_\perp}^2 \cosh(y)} \).

For our numerical plots shown in Figs. 7 and 8 we also use the experimental fit to the total elastic p-p cross section at high energy [15], \( \sigma_{pp}^{tot} = 21.70\pm0.0808 \pm 56.085\pm0.4525 \) mb.

In Fig. 7 The double rapidity distribution for the double peripheral sigma production is shown with the transverse momentum of each sigma approximately 0.3 GeV.

In Fig. 8 the differential cross section with respect to the first sigma’s rapidity is shown for important values of its transverse momentum, with the second sigma having transverse momentum approximately 0.3 GeV and rapidity 0.7.

The plot is obtained from Eq. (22) with \( y_1 \) fixed.

It is seen that there is a large branching ratio for the double peripheral production of the sigma \( \pi \pi \) resonance. The experimental observation of this process would be very valuable for improving our understanding for gluonic interactions and the nature of the pomeron.

V. CONCLUSIONS

We have derived the cross section for diffractive single and double-sigma production in high energy proton-proton
collisions. By mapping out the low-energy spectrum of the two $\pi$'s one will find the sigma resonance if this glueball/sigma model is correct. If so, the production of sigmas can be used as a signal for glueballs and hybrids as well as the Pomeron. There also would be important implications for sigma effects in charm quark systems, where gluonic effects are known to be important.

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