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by

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MULTIPERIOD INVESTMENT MODEL
FOR PROCESSING NETWORKS
WITH DEDICATED AND FLEXIBLE PLANTS

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ABSTRACT

A processing network is modelled as a combination of dedicated and flexible production facilities. The former produce a set of products in fixed proportions at all times, while the latter can accommodate different products at different times. Both continuous and batch operations may be involved. For such a processing network, a multiperiod MILP investment decision model is presented. The model considers the choice of technology, size of capacity additions, and allocation of resources over time in order to maximize the net present value of the project over a long range horizon. The application of the model is illustrated with a small example.

Introduction

A processing network is a combination of dedicated and flexible production facilities which are interconnected in an arbitrary manner. *Dedicated* production facilities produce a set of products in fixed proportions at all times and are usually used for the manufacturing of high volume chemicals. *Flexible* production facilities can manufacture different products at different times and are frequently required for the manufacturing of low volume chemicals. Both dedicated and flexible facilities can operate either *continuously* or in *batch* mode.

Today most of the industrial production facilities involve continuous dedicated units; the Kellogg ammonia synthesis process and the electrolytic production of caustic soda are just two examples. Paper mills which operate continuously and which can produce several paper types of different weight or color are examples of flexible continuous plants. Another example of flexible continuous plants are refineries which can accommodate different types of crude oils and in which the relative percentage of products changes depending on the operating conditions. Batch units have been traditionally used for the production of polymers and Pharmaceuticals. Most of these batch processes are flexible since in the same unit the production can be changed to accommodate different products which need to be produced in small amounts. Dedicated batch process can be found in the food industries where for example the production of beers, wines and liquors requires a dedicated fermenter. Detailed descriptions of the above and other processing networks can be found in standard reference books like Austin (1984).

The choice among competing dedicated and flexible technologies and the sizing of each type of facility to be used is an important concern at the level of planning the capacity expansion policy of a processing network. This paper addresses the following long range planning problem for processing networks. It is assumed that a network of processes and chemicals is given. This network includes an existing system as well as potential new processes and chemicals. The processes can be either dedicated or flexible manufacturing systems and they can operate either continuously or in a batch mode. Also given are forecasts for prices and demands of chemicals, as well as investment and operating costs over a finite number of time periods within a long range horizon. The problem then consists of determining the following items which will maximize the net present value over the given time horizon: (a) capacity expansion and shut-down policy for existing processes; (b) selection of new processes and their capacity expansion policy; (c) production profiles; (d) sales and purchases of chemicals at each time period.

As stated, the above is a multi-product, multi-facility, dynamic, location-allocation problem. Several planning models in the literature of capacity expansion of chemical processes address one or another aspect of this problem. A comprehensive survey of these models can be found in Sahinidis *et al.* (1989). These authors also presented a multiperiod MILP formulation for the long range planning problem of a chemical complex consisting of continuous dedicated processes. In this paper, it is shown that their model can be extended in order to account for production facilities which are flexible manufacturing systems operating in a continuous or in a batch mode. The suggested model provides a unified representation for the different types of process.

Problem Statement

A network consisting of a set of NP chemical processes which can be interconnected in a finite number of ways is assumed to be given. These processes may be dedicated or flexible and they may operate in a continuous or in a batch mode. The network also involves a set of NC chemicals which include raw materials, intermediates and products. Purchases and sales of chemicals are possible from NM different markets. For any process type, it will be assumed that the material balances can be expressed linearly in terms of the production amount of a main product. For continuous processes which are dedicated to the production of a set of products in fixed proportions, it will be assumed that the capacity of the plant is defined in terms of the production rate of the main product. For flexible production systems which can continuously produce a variety of products, it will be assumed that the capacity of the plant is defined in terms of the production rate of each of the main products which characterize the alternative production schemes. Each multiproduct batch process will be modelled as an independent unit without any considerations regarding inventory between stages. In this way the plant capacity will be defined by the size of the unit.

The objective function to be maximized is the net present value of the project over a long range horizon consisting of a finite number of NT time periods during which prices and demands of chemicals, and investment and operating costs of the processes can vary. The operating cost of a plant will be assumed to be proportional to the manufactured amount of its main product. As for the investment costs of the processes and their expansions, it will be considered that they can be expressed linearly in terms of the capacities with a fixed charge cost to account for the economies of scale.

Multiperiod MILP Model for Long Range Planning of Processing Networks

The processing network is modelled on a directed bipartite graph which involves two types of nodes: one for the processes and another for the chemicals. Let $\mathbb{B}$ and $\mathbb{C}$ denote the batch and continuous process nodes, respectively. The nodes of the network will be interconnected by a finite number of streams which represent the different alternatives which are possible for the processing and the purchases and sales from different markets.

In the formulation of the problem, the variable Q_{it} represents the total capacity of the plant of process i which is available in period t, $t = 1,\mathrm{NT}$. The parameter Q_{i0} represents the existing capacity of a process at time $t = 0$. QE_{it} represents the capacity expansion of the plant of process i which is installed in period t. If y_{it} are the 0-1 binary variables which indicate the occurrence of the expansions for each process i at the beginning of each time period t, the constraints which apply are:

$$\left.\begin{array}{l} y_{it}\, QE^L_{it} \le QE_{it} \le QE^U_{it}\, y_{it} \\ y_{it} = 0,1 \end{array}\right\} \qquad i = 1, NP \qquad t = 1, NT \qquad (1)$$

$$Q_{it} = Q_{i,t-1} + QE_{it} \qquad i = 1, NP \qquad t = 1, NT \qquad (2)$$

In equation (1), QE^L_{it} and QE^U_{it} are lower and upper bounds for the capacity expansions. A zero-value of the binary variables y_{it} forces the capacity expansion at period t to zero, *i.e.* $QE_{it} = 0$. If the binary variable is equal to one, the capacity expansion is performed. Equation (2) simply defines the total capacity Q_{it} which is available at each time period t.

The raw materials, intermediates, and products will be represented by NC nodes of chemicals where purchases and sales are considered in one of several markets, $l = 1, NM$. If the variables for purchases and sales are represented, respectively, by P_{jlt} and S_{jlt}, they must satisfy the inequalities:

$$\left.\begin{array}{l} a^L_{jlt} \le P_{jlt} \le a^U_{jlt} \\ d^L_{jlt} \le S_{jlt} \le d^U_{jlt} \end{array}\right\} \qquad j = 1, NC \qquad l = 1, NM \qquad t = 1, NT \qquad (3)$$

where a^L_{jlt}, a^U_{jlt} are lower and upper bounds on the availabilities of the raw materials, and d^L_{jlt}, d^U_{jlt} are lower and upper bounds on the demands of the finished products.

Defining $I(j)$ as the index set of plants which consume chemical j, and $O(j)$ as the index set of plants which produce chemical j, the mass balances on the chemicals' nodes will be given by:

$$\sum_{l=1}^{NM} P_{jlt} + \sum_{i \in O(j)} W_{ijt} = \sum_{l=1}^{NM} S_{jlt} + \sum_{i \in I(j)} W_{ijt} \quad j = 1,NC \quad t = 1,NT \quad (4)$$

where the amounts of chemical *j* being consumed or produced in plant *i* during period *t* are represented by the variables W_{ijt}. According to this equation the total amount of a chemical's purchases from the various markets plus the amounts produced within the network must be equal to the sum of sales and the total consumption within the network.

In order to model the mass balances and capacity requirements for each process we consider two cases:

<u>Case I:</u> Continuous processes ($i \in C$).

Since the case of a dedicated continuous plant is a special case of a flexible continuous production facility, we will consider flexible processes first. Each flexible process *i* can operate at a number of alternative production schemes each of which is characterized by a main product For simplicity we assume that different production schemes are characterized by different main products. It will also be assumed that the production rate (r_{ijt}) of the main product *j* of each such scheme is proportional to the capacity of the plant:

$$r_{ijt} = \rho_{ij} Q_{it} \qquad i \in C \qquad j \in M_i \qquad t = 1,NT \quad (5)$$

where M_i denotes the index set of main products of flexible process *i* for all the alternative production schemes of this process. The dimensionless positive constants ρ_{ij} represent the relative production rates for each product *j* while the variables Q_{it} (units kg/hr) represent a multiplying capacity factor which uniformly increases or decreases the production rates. As an example consider a plant *i* with relative production rates of p_z*^ = 1 and P/g-^ ^ f°r chemicals A and B. Then if Q_{it} = 2 kg/hr, it implies that during period t plant / has production rates of r_{At} = 2 and r_{Bt} =2.4 kg/hr for products A and B, respectively.

By then defining for each process i the variable T_{ijt} as the time which during period t is allocated to the production scheme characterized by the main product j the amounts of main products produced by this process are given by:

$$W_{ijt} = (\rho_{ij} \; Q_{it}) \; T_{ijt} \qquad i \in C \qquad j \in M_i \qquad t = 1, NT \qquad (6)$$

Since the total allocation of production times cannot exceed the maximum available time of the flexible process:

$$\sum_{j \in M_i} T_{ijt} \leq H_{it} \qquad i \in C \qquad t = 1, NT \qquad (7)$$

where H_{it} is the time for which plant / is available for operation during period t.

Given the amounts of the main products ($j \in M_i$ for each process i), the amounts of all other products are given by the linear relations:

$$W_{ijt} = \sum_{j' \in M_i} \mu_{ijj'} \; W_{ij't} \qquad i \in C \qquad j \in L_i \qquad t = 1, NT \qquad (8)$$

where L_i is the index set of products which are inputs or outputs of process / and $\mu_{ijj'}$ are positive constants characteristic of each process.

Although equation (6) is nonlinear, the nonlinearities can be avoided by introducing variables θ_{ijt} which are defined as follows:

$$\theta_{ijt} = Q_{it}\, T_{ijt} \qquad i \in C \qquad j \in M_i \qquad t = 1, NT \qquad (9)$$

Then equations (6) and (7), respectively, become:

$$W_{ijt} = \rho_{ij}\; \theta_{ijt} \qquad i \in C \qquad j \in M_i \qquad t = 1, NT \qquad (10)$$

and

$$\sum_{j \in M_i} \theta_{ijt} \le Q_{it}\, H_{it} \qquad i \in C \qquad t = 1, NT \qquad (11)$$

both of which are linear in terms of the new variables θ_{ijt}.

For the case of only one production scheme (*i.e.* for a dedicated continuous process i), we have $\rho_{ij} = 1$ and the above equations (8), (10) and (11) can be simplified to give:

$$W_{m_i t} \le Q_{it}\, H_{it} \qquad i \in C \qquad j \in M_i \qquad t = 1, NT \qquad (12)$$

$$W_{ijt} = \mu_{ijm_i}\; W_{im_i t} \qquad i \in C \qquad j \in L_i \qquad t = 1, NT \qquad (13)$$

Here, the capacity Q_{it} of plant i is defined in terms of the production rate of its main product m_i whose production therefore cannot exceed the installed capacity (constraint (12)).

Case II: Batch processes ($i \in B$).

For batch processes each unit will be considered as a separate process with no considerations for inventory. Furthermore, no distinction will be made between dedicated and flexible units since their modelling is identical. Given the capacity Q_{it} (eg. size in liters) of the unit, an upper bound for the number of batches (N_{ijt}) which can be processed for product j in plant i during time period t is given by $(W_{ijt})(\sigma_{ij}) / (Q_{it})$ where the size factors σ_{ij} (eg. in liters/kg) define different batch sizes for different products in the same size of equipment. By letting %a denote the batch time for the production of product,/ in process í, the constraint that the total available time in unit *i* cannot be exceeded during period *t* is expressed as follows:

$$\sum_{j \in M_i} N_{ijt}\, \tau_{ij} \le H_{it} \qquad i \in B \qquad t = 1, NT \qquad (14)$$

which by substituting (W^Xc^y) / (Q/$_r$) for N$_z$y$_r$ gives:

$$\sum_{j \in M_i} W_{ijt}\, \sigma_{ij}\, \tau_{ij} \le Qit^{u}it \qquad "?B \qquad t = 1, NT \qquad (15)$$

Here we can relax the integrality requirements on N^ since for the planning model x$_z$y « W_{lt} and therefore the errors introduced by this simplification are negligible.

The production amounts of the secondary products in the batch plants are calculated similarly to the continuous case:

$$^{w}(/r= \sum_{j' \in M_i} Mijf\hat{}ift \qquad ie\% \qquad jeLi \qquad r=1, NT \qquad (16)$$

Note that by letting p^y = 1 / (a$_z$y x$_z$y) for all the batch processes and defining the variables 9y$_r$ = W$_£$y$_r$/p$_z$y as in (10), constraint (15) reduces to constraint (11) which was developed for continuous processes. In this way, the same constraint representation can be used for both continuous and batch processes provided the production rate coefficients p$_z$y and the corresponding capacities Q^ are defined accordingly.

Using the above defined variables, the net present value of the project is given by:

$$NPV = - \sum_{i=1}^{NP} \sum_{r=1}^{NT} (\alpha_{it} QE_{it} + \beta_{it} y_{it}) - \sum_{i=1}^{NP} \sum_{j \in M[} \sum_{t=1}^{NT} \delta_{ijt} W_{ijt}$$
$$+ \sum_{y=1}^{NC} \sum_{/= i}^{NM} \sum_{/= i}^{NT} (\gamma_{jlt} S_{jlt} - \Gamma_{jlt} P_{jlt}) \qquad (17)$$

where the parameters a;, and $(3_{ZT}$ represent, respectively, the variable and fixed terms for the investment cost, Syf is the unit operating cost, and yn_{jlt} and Yn_{jlt} are the prices of sales and purchases of the chemical *j* in market /. All these parameters are discounted at the specified interest rate and include the effect of taxes in the net present value.

Additional constraints which can be considered include:

a) Limit on the number of expansions of some processes:

$$\sum_{/= 1}^{NT} y/r \leq NEXP(i) \qquad i \in I' \subset \{1,2, \dots NP\} \qquad (18)$$

b) Limit on the capital available for investment during some time periods:

$$\sum_{/=1}^{NP} (\overline{an}\ QE_{U} + \overline{P\&}\ Yu) \leq CI(0 \qquad t \in V \subset \{1,2,\dots NT\} \qquad (19)$$

where $\overline{a//}$, $\overline{P/r}$ are non-discounted cost coefficients corresponding to period r.

Finally, by using the unified representation of equations (10) and (11) for the dedicated continuous processes and for the batch processes, the complete multiperiod MILP model for the long range planning problem of processing networks is as follows:

Model LREl

$$NPV = -\sum_{i=1}^{NP}\sum_{t=1}^{NT}(\alpha_{it}QE_{it} + p_{it}y_{it}) - \sum_{j=1}^{NP}\sum_{j \in A_i}\sum_{t=1}^{NT}\delta_{ijt}W_{ijt} + \sum_{j=1}^{NC}\sum_{l=1}^{NM}\sum_{t=1}^{NT}(y_{jlt}S_{jlt} - r_{jlt}P_{jlt}) \qquad (17)$$

S.t.

$$y_{it}QE_{it}^{L} \le QE_{it} \le QE_{it}^{U}y_{it} \qquad i=1,NP \quad t=1,NT \qquad (1)$$

$$Q_{it} = Q_{i,t-1} + QE_{it} \qquad i=1,NP \quad t=1,NT \qquad (2)$$

$$\sum_{l=1}^{NM}P_{jlt} + \sum_{i \in O(j)} W_{ijt} = \sum_{l=1}^{NM}S_{jlt} + \sum_{i \in I(j)} W_{ijt} \qquad j=1,NC \quad t=1,NT \qquad (4)$$

$$W_{ijt} = \rho_{ij}\,\theta_{ijt} \qquad i=1,NP \quad j \in M_i \quad t=1,NT \qquad (10)$$

$$\sum_{j \in M_i}\theta_{ijt} * Q_{it} \le H_i \qquad i=1,NP \quad t=1,NT \qquad (20)$$

$$W_{ijt} = \sum_{j' \in M_i}\mu_{ijj'}\,W_{ij't} \qquad i=1,NP \quad j \in L_i \quad t=1,NT \qquad (21)$$

$$\sum_{t=1}^{NT} y_{it} \le NEXP(i) \qquad i \in IC \subseteq \{1,2,\ldots NP\} \qquad (18)$$

$$\sum_{i=1}^{NP} (\alpha_{it}\, QE_{it} + \beta_{it}\, y_{it}) \le CI(t) \qquad t \in T' \subseteq \{1,2, \ldots NT\} \qquad (19)$$

$$\left.\begin{array}{l} a^L_{jlt} \le P_{jlt} \le a^U_{jlt} \\ d^L_{jlt} \le S_{jlt} \le d^U_{jlt} \end{array}\right\} \qquad j = 1, NC \quad l = 1, NM \quad t = 1, NT \qquad (3)$$

$$y_{it} = 0 \text{ or } 1 \qquad i = 1, NP \quad t = 1, NT \qquad (22)$$

$$QE_{it},\ W_{ijt},\ P_{jlt},\ S_{jlt},\ \theta_{ijt} \ge 0 \qquad (23)$$

where the parameter ρ_{ij} is defined as follows:

(a) $\rho_{ij} = 1$ for a continuous dedicated plant i,

(b) ρ_{ij} is a specified constant for a continuous flexible plant i,

(c) $\rho_{ij} = 1 / (\sigma_{ij}\, \tau_{ij})$ for a batch process i (dedicated or flexible).

The production times allocated to different production schemes on a flexible unit can be determined after the solution of this model from (9): $T_{ijt} = \theta_{ijt} / Q_{it}$.

Discussion of the Model

The following comments can be made about model (LRP) regarding its nature and methods for its solution.

1. The above model is a multiperiod mixed integer linear programming model.

2. For this type of multiperiod MILP problems, Sahinidis *et al.* (1989) have compared the performance of several computational strategies including branch and bound, strong cutting planes followed by branch and bound, Benders decomposition and strong cutting planes

followed by Benders decomposition. Among these, a combination **of** strong cutting planes and **branch and bound seems to be the most successful for large problems.**

3. **The development of nonstandard formulations for multiperiod MELP** models for planning **and scheduling has proved to be quite successful in some applications (see** Sahinidis and **Grossmann, 1989, 1990), Following those approaches, we can disaggregate the capacity expansion variables of our model by introducing new variables** $<P[_{tx}$ **to denote capacity expansion of plant** i **made in period** t **in order to serve production requirements up to period T (T> t). In this case, we also need to introduce the following additional constraints:**

$$QE/_r \geq cp/,_T \qquad / = 1.NP \quad t = 1,NT \quad \% > t \qquad (24)$$

$$9/rr \ \wedge \ ^c in \ \ Yit \qquad i = 1.NP \quad t = 1.NT \quad r > t \qquad (25)$$

and substitute (20) by the linear constraint:

$$\sum_{\tau=1}^{t} <Pm \geq \frac{1}{HT_U} \ X_{j \in M_i} \ ^Q ijt \sim 0/0 \qquad i = 1.NP \quad t = 1,NT \qquad (26)$$

Although this alternative model requires more constraints and variables and the ***a*** *priori* **calculation of the coefficients** $C/_{rr}$ **in (25), it is expected that it will exhibit a tighter** linear **programming relaxation than the** more **standard** formulation presented above and therefore have **lower computational requirements** for its solution.

4. **In** the **development** of the constraints describing the material balances for flexible production facilities considerations of transition times and costs, and inventory holding costs were omitted. Although these can constitute an important factor for these processes, their

inclusion would introduce many nonlinearities into the model (see for example Sahinidis and Grossmann, 1990).

Example

In order to illustrate the application of the multiperiod MILP model, consider the processing network shown in Fig. 1. Products C and D are to be produced by process 2 or 3, and 3 or 4, respectively. Processes 2 and 4 are dedicated respectively to the production of products C and D only. On the contrary, the continuous process 3 has the flexibility of producing either product C or product D and to switch the production between these two products. The feedstock B to processes 2, 3 and 4 is either bought or manufactured in process 1. There are no processes initially installed and no limits on investment were specified. Three time periods were considered each of two years length.

The data for this example are shown in Tables 1 to 8. The objective function coefficients have already been discounted at a specified interest rate and include the effect of taxes in the net present value. Two different scenarios for the demands of products C and D were considered as shown in Table 8. The market demands for products C and D increase with time in scenario 1. In scenario 2, the demand for C decreases while the demand for D increases with time. In each case, the MILP model involved 12 binary variables, 100 continuous variables and 97 constraints. Each problem was solved in 1.2 CPU seconds on an IBM-3083 using MPSX. The results for Scenario 1 are presented in Tables 9 and 10 while those for Scenario 2 are presented in Tables 11 and 12.

Scenario 1

The analysis of the results indicates that the dedicated processes 2 and 4should be preferred to the flexible process 3. This happens because as seen in Tables 1, 2 and 4 the flexible process 3 is more expensive to build and operate than each of the dedicated processes 2 and 4. In particular, the optimal solution with a net present value of $15,404.6 x 10^ involves installing processes 1, 2 and 4 in time period 1 (see Fig. 2). Also, as seen in Table 9, all processes operate below maximum capacity in periods 1 and 2 and are fully utilized only during the last time period 3. This illustrates the effect of economies of scale on the optimal solution: the cost of maintaining an idle (partially used) process is outbalanced by the savings of a large installation (the more capacity that is purchased, the less the price per unit of capacity). Finally, it can be seen in Table 10 that chemical B is not only purchased from the external market but also produced within the complex.

Scenario 2

The results for scenario 2 indicate that the flexible process 3 should be preferred to the dedicated processes 2 and 4, despite of the fact that the flexible process 3 is more expensive to build and operate than either of the dedicated processes. In particular, the optimal solution with a net present value of $8,784.3 x 10^ involves installing processes 1 and 3 in time period 1 (see Fig. 3). This happens because, as seen in Table 8, the market demand for product C decreases with time. As a result of this, if product C were to be produced in a significant amount, a dedicated process for the production of this product would have to operate well below full capacity during a significant portion of the planning horizon. On the other hand, the flexible process can easily accommodate the dynamic requirements of the market especially since while the market demand for product C decreases with time, the demand for product D increases during the same time. In this way, the flexible process will be used to mostly accommodate the demand of product C during the first time period and that of product D during the final time period. In particular, the following percentage of the capacity will be allocated for product C during the three time periods: 68%, 37% and 5% , while-the corresponding figures for product D are 10%, 43% and 95%. As in scenario 1, here too, part of the capacity remains unused during the first two time periods.

Conclusions

This paper has presented an extension of the multiperiod MILP planning model of Sahinidis *et al.* (1989) for dedicated continuous processes. It has been shown that the case of processing networks with flexible processes, either continuous or batch, can be incorporated into this model under a unified mathematical representation. In order to illustrate the potential of the model, two different scenarios for product demands were considered for a small example problem involving a processing network with flexible and dedicated processes.

Nomenclature

Indices:

i	**process 0 = 1, NP);**
j	chemical (7=1,NC);
I	market (/=1,NM);
$^m i$	main product of process /;
t, x	**time period *it,* x = 1, NT).**

Sets:

B	set of batch processes;
C	set of continuous processes;
Ki)	the index set of processes which consume chemical *j;*
$^L i$	the index set of products which are inputs or outputs of process /;
Mi	the index set of main products of flexible process / for all the alternative production schemes;
O(j)	the index set of processes which produce chemical *j.*

Parameters:

a/r	variable term of investment cost;
P;r	fixed term for the investment cost;
$\bar{a}$/r» $\bar{P}$/7	non-discounted investment cost coefficients;
γ_{jlt}	prices of sales of the chemical; in market / during time period r,
Γ_{jlt}	prices of purchases of the chemical; in market / during time period *t*;
δ_{ijt}	unit operating cost for process *i* [$ / unit of production amount of product;];
$\mu_{ijj'}$	material balance coefficients characteristic of each process /, main product;'

and product/',

p^y dimensionless production rate coefficients characteristic of each process / and main product *j;* they are equal to one for continuous dedicated processes, they are given constants for continuous flexible processes, while for batch processes they are defined as $p_z y$ = 1 / ($a_z y$ x^);

$a_z y$ size factors for batch process *i* and product/;

if; batch production time for process *i* and main product;;

dfjl$_t$, a& lower and upper bounds for availability of raw materials;

tfl't»^jlt l ower anc* upper bounds for demand of finished products;

C_z-$_{rr}$ upper bounds for disaggregated capacity expansions, defined in (25);

CI(r) the capital investment limitation corresponding to period r,

Hj-$_r$ the time for which plant *i* is available for operation during period r;

NC number of chemicals in the network;

NEXP(z) the maximum allowable number of expansions for process /;

NM number of markets;

NP number of processes in the network;

NT number of time periods considered;

Q_{i0} existing capacity of process *i* at time r = 0;

QE_{it}^L lower bounds for the capacity expansions;

QE_{it}^U upper bounds for the capacity expansions.

Variables:

θ_{ijt} production amounts defined in (9);

$\varphi_{it\tau}$ capacity expansion of plant *i* made in period r in order to serve production requirements up to period r (T ≥ r);

N_{ijt} number of batches of product; in plant / during time period r;

NPV net present value;

P_{jt} amount of product j purchased from market l at the beginning of period t;

Q_{it} total capacity of the plant of process i which is available in period t;

QE_{it} capacity expansion of the plant of process i which is installed in period t;

r_{ijt} the production rate of the main product j in process i during period t;

S_{jlt} amount of product j sold to market l at the beginning of period t;

T_{ijt} time which during period t is allocated in process i to the production scheme characterized by the main product j;

W_{ijt} amount of flow of product j to/from process i during time period t;

y_{it} decision variable which is 1 whenever there is an expansion for process i at the beginning of time period t, and 0 otherwise.

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list of Tables:

Table 1. Variable investment coefficients (a^ [=] 10^2 \$/(ton yr^{1})).

Process	Time Period 1	2	3
1	1.58	1.36	1.28
2	4.40	3.12	2.52
3	4.64	3.24	2.56
4	2.64	2.24	1.56

Table 2. Fixed investment coefficients ($\hat{p}$ [=] 10^6 \$).

Process	Time Period 1	2	3
1	112	95	85
2	102	82	73
3	114	97	89
4	128	115	100

Table 3. **Prices of raw materials and products (lO^ $/ton).**

Chemical		Period		
		1	2	3
Raw materials				
	A	7.32	5.24	4
	B	13.52	11.52	9.6
Products				
	C	45	40	36
	D	58	50	47

Table 4. **Operating expenses coefficients (5;/ $_t$ [=] 10^ $/ton).**
"ATI*

Process	Time Period		
	1	2	3
1	0.6	0.5	0.4
2	0.8	0.7	0.6
3-scheme 1	0.9	0.8	0.7
3-scheme 2	0.8	0.7	0.6
4	0.7	0.6	0.5

Table 5. Mass balance coefficients (μ_{ijm_i}).

Process	Chemical			
	A	B	C	D
1	1.11	1^*		
2		1.05	1^*	
3-scheme 1		1.05	1^*	
3-scheme 2		1.05		1^*
4		1.05		1^*

*: Denotes main product

Table 6. Production rate coefficients, process availability per period, capacity expansion bounds.

$\rho_{3C} = 1$; $\rho_{3D} = 1.1$;

$H_{it} = 2$ yr ;

$QE^L_{it} = 0$; $QE^U_{it} = 200$ kton/yr

Table 7. **Upper** bounds for raw material availabilities (kton/yr).

	Chemical	Period		
		1	2	3
Availability				
	A	30	40	45
	B	100	125	150

Table 8. Upper bounds for product demands (kton/yr).

	Chemical	Period		
		1	2	3
Scenario 1				
	C	65	75	90
	D	85	95	100
Scenario 2				
	C	65	35	5
	D	10	45	100

Table 9. Selected Processes and Production Profiles (kton/yr) for Scenario 1.

Process		Period		
		1	2	3
1	Capacity	20.3	20.3	20.3
	Production	13.5	18	20.3
2	Capacity	40.7	40.7	40.7
	Production	18	29.2	40.7
3	Capacity	0	0	0
	Production	0	0	0
4	Capacity	50	50	50
	Production	42.5	47.5	50

Table 10. Purchases and Sales (kton/yr) for Scenario 1.

	Chemical	Period		
		1	2	3
Purchases				
	A	15	20	22.5
	B	50	62.5	75
Sales				
	C	18	29.2	40.8
	D	42.5	47.5	50

Table 11. Selected Processes and Production Profiles (kton/yr) for Scenario 2.

Process		Period 1	2	3
1	Capacity	20.3	20.3	20.3
	Production	13.5	18	20.3
2	Capacity	0	0	0
	Production	0	0	0
3	Capacity in terms of C:	48	48	48
	Production of C:	32.5	17.5	2.5
	Capacity in terms of D:	52.8	52.8	52.8
	Production of D:	5	22.5	50
4	Capacity	0	0	0
	Production	0	0	0

Table 12. Purchases and Sales (kton/yr) for Scenario 2.

	Chemical	Period 1	2	3
Purchases				
	A	30	40	45
	B	51.7	48	69.7
Sales				
	C	32.5	17.5	2.5
	D	5	22.5	50

LIST OF FIGURES

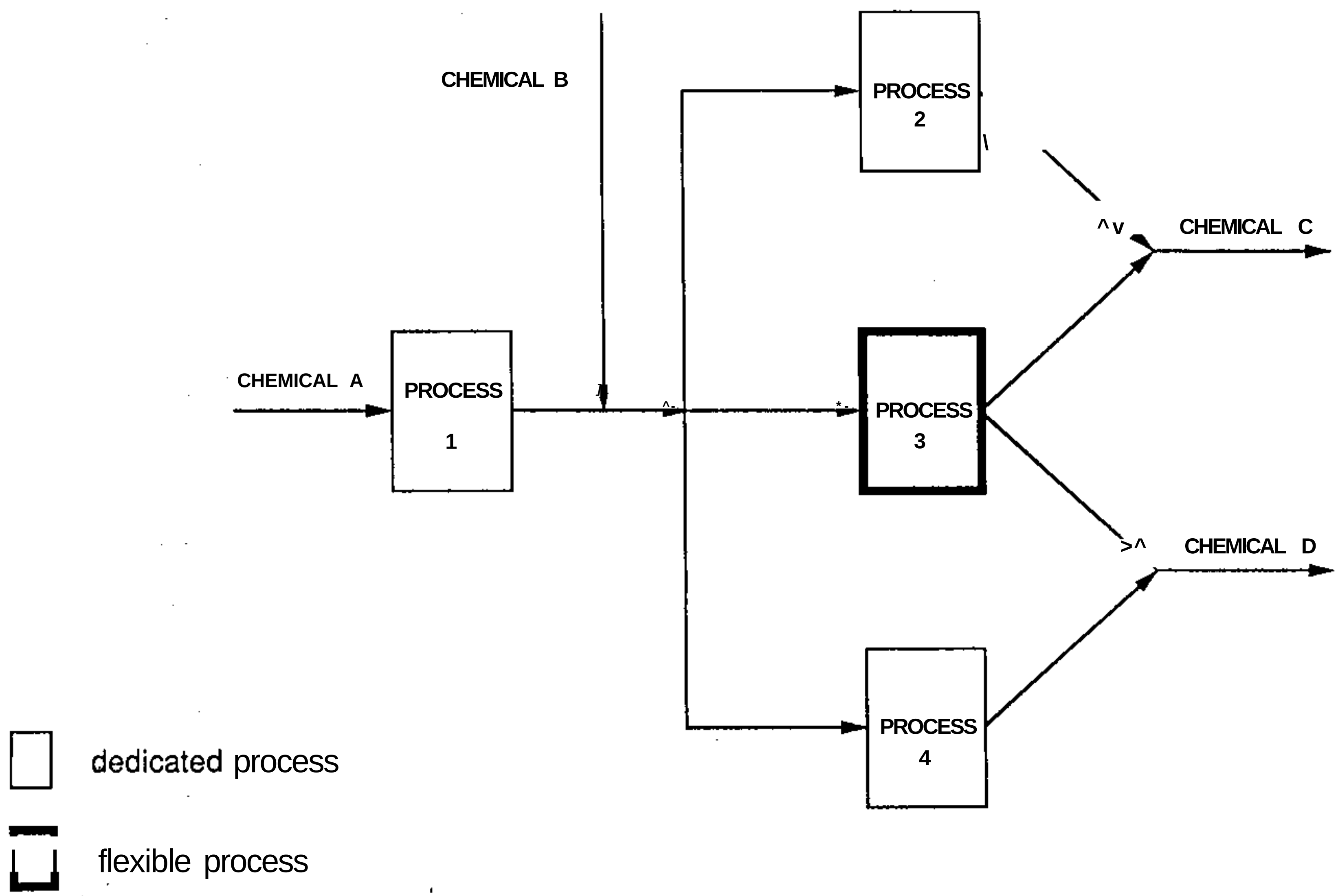

Fig. 1: Processing Network.

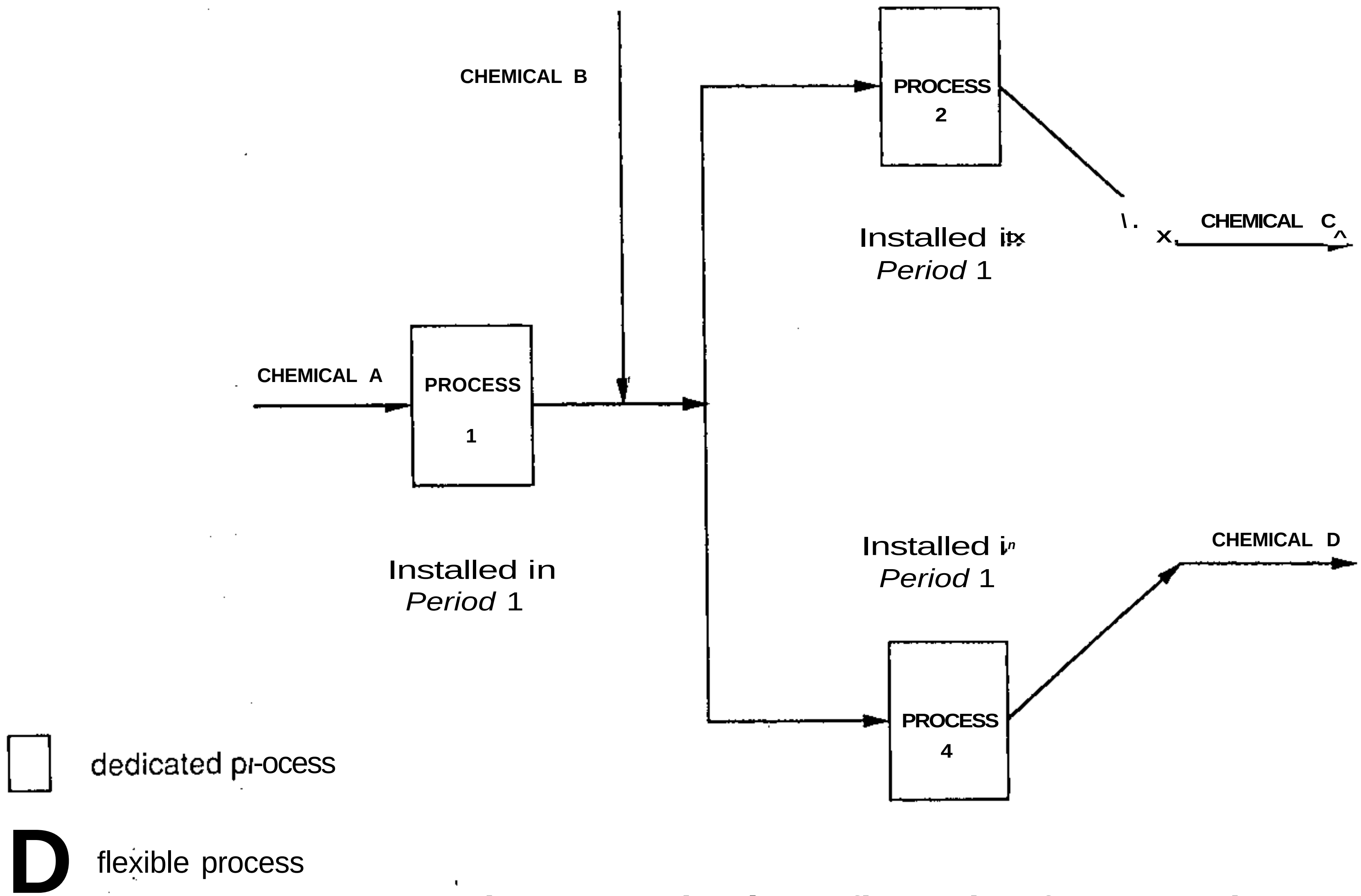

Fig. 2: Optimal Configuration for Scenario 1.

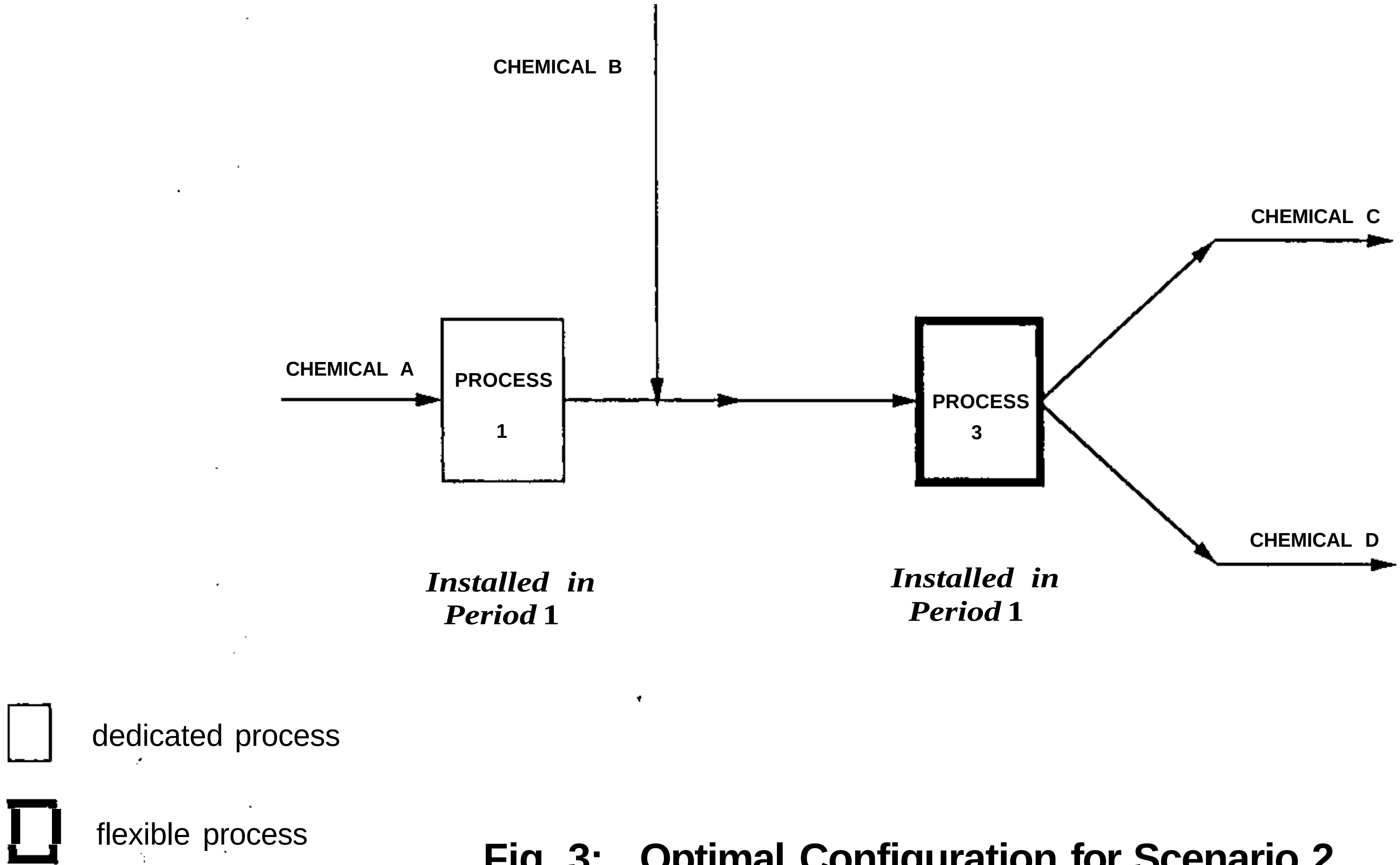

Fig. 3: **Optimal Configuration for Scenario 2.**