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Labour Supply and Taxation: A New Look

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Labour Supply and Taxation: A New Look

Guy Laroque

Sciences Po, University College London and Institute for Fiscal Studies

July 2015

R. Blundell, A. Bozio and G. Laroque, 'Extensive and Intensive Margins of Labour Supply: Work and Working Hours in the US, the UK and France', Fiscal Studies, March 2013

G. Laroque and S. Osotimehin, 'Fluctuations in hours of work and employment across age and gender', IFS Working Paper W15/03

Figure 1: Mean annual hours per individual aged 16-74
Plan of the presentation

1. Description
   1.1 International comparisons of hours worked
   1.2 Behaviour during recessions: the USA

2. Women labour supply
   2.1 The two offer model
   2.2 Identification: preference parameters and offer distribution
   2.3 Application to UK women
Figure 2: Employment rate (per population)
Figure 3: Mean annual hours per worker
Figure 4: Employment rate by age (male 1977)
Figure 5: Employment rate by age (male 2007)
Figure 7: Employment rate by age (female 1977)
Figure 8: Employment rate by age (female 2007)
Figure 9: Employment rate by age (female 2012)
Figure 10: Mean annual hours by age (female 1977)
Figure 11: Mean annual hours by age (female 2012)
Plan

Description
International comparisons of hours worked
Behaviour during recessions: the USA

Women labour supply
The two offer model
Identification: preference parameters and offer distribution
Application to UK women
Demographics and the business cycle

It is difficult to get monthly data on a long enough period to analyse fluctuations at business cycle frequencies.

This can only be done for the US, with the Current Population Survey.
Figure 12: Mean annual hours: men

The vertical lines mark recession dates, defined as peaks and troughs of aggregate hours worked per person.
Figure 13: Mean annual hours: women
Figure 14: Contribution of the demographic groups to the recessions
Plan

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Our model

We develop a two-offer model in which each individual is assumed to face two independent hours offers - the one at which they are observed to work, if they are working positive hours, and the one they turned down.

The latter may be identical to the former, in which case the individual would be completely constrained, with no hour choice to make.

The option of not working is always available.

As the number of offers increases the specification approaches that of the standard labour supply model at which observed choices coincide with the fully optimal choice over all hours options.
An intertemporal model with nonlinear budget constraints and fixed costs

Consumer program at date $t$

$$E_t \int_t^T u_t(c_{\tau}, h_{\tau})d\tau$$

subject to an intertemporal budget constraint

$$\int_t^T \exp[-r(\tau - t)]\{c_{\tau} - R(w_{\tau}, h_{\tau}) + b_{\tau}1_{h_{\tau} > 0}\}d\tau \leq S_t.$$ 

Given the Lagrange multiplier $\lambda_t$, current consumption and hours of work maximize

$$u_t(c, h) + \lambda_t [R(w_t, h) - c].$$
Intensive and extensive labor supply

Let \((c^e, h^e)\), the optimal choice of the working household, \(c^o\) the consumption of the household with the worker out of the labour market.

The decision is not to participate on the labour market whenever

\[ u_t(c^e, h^e) - \lambda_t[c^e - R(w_t, h^e) + b_t] < u_t(c^o, 0) - \lambda_t[c^o - R(w_t, 0)]. \]
Our specification choice

We specialize the model to

\[ u_t(c, h) = \frac{c^{1-\gamma}}{1-\gamma} + \frac{(L-h)^{1-\phi}}{1-\phi} a_t = \frac{c^{1-\gamma}}{1-\gamma} + a_t v(h), \]

with \( \gamma \) and \( \phi \) non-negative, \( L \) physiological bound on weekly hours (say 100)

\[ \ln(a_t) = Z_t^a \beta^a + \sigma^a \varepsilon^a, \]
\[ b = Z^b \beta^b + \sigma^b \varepsilon^b. \]
Necessary conditions for optimality, unrestricted choice

\[ \lambda = c^{-\gamma}, \]

The optimal hours \( h^e \) when working maximize

\[ av(h) + c^{-\gamma} R(w, h), \quad (1) \]

and the household chooses to stay out of the labour market whenever

\[ av(h^e) + c^{-\gamma} [R(w, h^e) - b] < av(0) + c^{-\gamma} R(w, 0). \]
Figure 15: A standard example of budget constraint
Manipulating the revealed preference inequalities

Let \( h^e \) be preferred to any other work time \( h \). Then for \( h < h^e \)

\[
c^\gamma a \leq \left\{ \frac{R(w, h) - R(w, h^e)}{v(h^e) - v(h)} \right\},
\]

while for \( h > h^e \)

\[
c^\gamma a \geq \left\{ \frac{R(w, h) - R(w, h^e)}{v(h^e) - v(h)} \right\}.
\]

\[
\min_{h \leq h^e} \frac{R(w, h) - R(w, h^e)}{v(h^e) - v(h)} \geq \max \left[ 0, \max_{h \geq h^e} \frac{R(w, h) - R(w, h^e)}{v(h^e) - v(h)} \right].
\]
How to deal with rejections of unrestricted model

Two inequalities: one is non parametric

$$\min_{h<h^e} \frac{R(w, h) - R(w, h^e)}{v(h^e) - v(h)} \geq 0.$$  

the other only depends on $\phi$

$$\min_{h<h^e} \frac{R(w, h) - R(w, h^e)}{v(h^e) - v(h)} \geq \max_{h>h^e} \frac{R(w, h) - R(w, h^e)}{v(h^e) - v(h)}$$

Traditional road : measurement error.

Here restricted set of offers: two independent offers.
Figure 16: Suboptimal hours, independently of the utility parameters
Figure 17: Suboptimal hours for some utility parameters
Plan

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    International comparisons of hours worked
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The restricted choice model

It is convenient to simplify the setup to a static, discrete choice case.

\[ U(h, Z, \beta, \varepsilon) \]

Here \( h \) belongs to a discrete set \( \mathcal{H} \) made of \( I \) elements \( \{h_1, \ldots, h_I\} \).

Given a subset of possible choices \( H \) in \( \mathcal{H} \), for each \( \beta \) and \( Z \), any distribution of \( \varepsilon \) yields a probability distribution on \( H \): \( p_i(H, Z, \beta) \) is the probability of choosing \( h_i \) in \( H \).

The standard choice model has \( H \) equal to \( \mathcal{H} \).
Identification of the structural parameters

We assume that given $U$, the observation of the family of probabilities $p_i(H, Z, \beta)$ identifies the parameter $\beta$, when $Z$ varies in the population, and the union of the family of (non singleton) choice sets $H$ for which the probabilities are observed covers the whole of $\mathcal{H}$. 
The two offer model

There is a distribution of offers, the probability of being offered $h_i$ being equal to $g_i$, $g_i > 0$, $\sum_{i=1}^{I} g_i = 1$.

Individuals draw independently two offers from $g$ and choose the one that yields the highest utility.

Distribution of observed choices $\ell_{2i}(Z, \beta)$

$$\ell_{2i}(Z, \beta) = g_i^2 + 2g_i \sum_{j \neq i} g_j p_i(\{i, j\}, Z, \beta),$$  \hspace{1cm} (2)
There are \( I \) equations, of which only \( I - 1 \) are independent: the sum of all the equations is identically equal to 1 (on the right hand side, this follows from the observation that

\[ p_i(\{i,j\}, Z, \beta) + p_j(\{i,j\}, Z, \beta) = 1 \text{ for all } i, j. \]

There are potentially \( I(I - 1)/2 + I - 1 \) unknowns on the right hand side: the ’structural’ choice probabilities \( p \) and the distribution of offers \( g \).
Recovering choices, knowing the distribution of offers

We specialize choices to the random utility model: utility of getting \( i \) is \( a_i - \varepsilon_i \). The joint distribution of the continuous variables \( \varepsilon_i \) is known to the econometrician.

\[
p_i(\{i, j\}) = F_{ij}(a_i - a_j).
\]

The aim is to recover the \( a_i \)'s, \( i = 1, \ldots, I - 1 \), the last alternative \( a_I \) being normalized to zero.
Lemma

Let $\ell$ and $g$ be two probability vectors in the simplex of $\mathcal{R}^l$, whose components are all positive. There exists at most a unique vector $a_i$ with $a_l = 0$ that satisfies the system of equations

$$
\ell_i = g_i^2 + 2g_i \sum_{j \neq i} g_j F_{ij} (a_i - a_j) \text{ for } i = 1, \ldots, l.
$$

(3)
Recovering the distribution of offers, knowing the choice probabilities

Given the probabilities $p_{ij}$ of choosing $i$ when both $i$ and $j$ are available for all $i$ different from $j$, we study whether the choices $\ell_i$ of agents getting two independent offers are constrained by the model, and whether the observation of $\ell$ allows to recover the probability of offers $g$.

$$\ell_i = g_i^2 + 2g_i \sum_{j \neq i} g_j p_{ij}$$

(4)

where for all couples $(ij)$, $i \neq j$,

$$p_{ij} + p_{ji} = 1.$$  

(5)
The distribution of offers

Lemma

Given the choice probabilities $p_{ij}$, $p_{ij} \geq 0$ satisfying (5), for any observed probability $\ell_i$ in the simplex of $\mathcal{R}^I$, there exists a unique offer probability $g_i$ in the simplex of $\mathcal{R}^I$ which satisfies (4).
Assume that each $a_i$ for $i = 1, \ldots, I-1$ is a smooth function of a finite parameter vector $\gamma$ and, in a similar fashion, the offer probability $g_i$ is a smooth function of finite parameter vector $\beta$, where
\[
\dim[\gamma : \beta] \leq I - 1.
\]
The likelihood has the form
\[
Q_i = -\ell_i + g_i(\beta)^2 + 2g_i(\beta) \sum_{j \neq i} g_j(\beta) F_{ij}(a_i(\gamma) - a_j(\gamma)) \text{ for } i = 1, \ldots, I-1.
\]
A sufficient condition for local identification

For identification we require full column rank of the matrix

$$\Pi = \left[ \frac{\partial Q}{\partial a}, \frac{\partial Q}{\partial g} \right] \cdot \left[ \frac{\partial Q}{\partial \gamma'}, \frac{\partial g}{\partial \beta} \right]. \quad (7)$$

where the matrix of derivatives relating to the $Q_i$ has elements of the form

$$\frac{\partial Q_i}{\partial a_i} = 2g_i \sum_{j \neq i} g_j f(a_i - a_j)$$

$$\frac{\partial Q_i}{\partial a_j} = -2g_i g_j f(a_i - a_j)$$

$$\frac{\partial Q_i}{\partial g_i} = 2g_i + 2 \sum_{j \neq i} g_j F(a_i - a_j)$$

$$\frac{\partial Q_i}{\partial g_j} = 2g_i F(a_i - a_j)$$
Plan

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Application


<table>
<thead>
<tr>
<th>Education level</th>
<th>Percent</th>
<th>Number child</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low educated</td>
<td>56.4</td>
<td>1</td>
<td>38.6</td>
</tr>
<tr>
<td>Middle Educated</td>
<td>25.8</td>
<td>2</td>
<td>41.5</td>
</tr>
<tr>
<td>Higher Education</td>
<td>17.8</td>
<td>3+</td>
<td>19.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spouses status</th>
<th>Percent</th>
<th></th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Woman inwork</td>
<td>62.6</td>
<td>London</td>
<td>9.7</td>
</tr>
<tr>
<td>Spouse inwork</td>
<td>56.4</td>
<td>Cohabitant</td>
<td>76.9</td>
</tr>
<tr>
<td>Both inwork</td>
<td>41.2</td>
<td>Youngest child 0-4</td>
<td>29.3</td>
</tr>
<tr>
<td>Both out of work</td>
<td>22.1</td>
<td>Youngest child 5-10</td>
<td>30.3</td>
</tr>
</tbody>
</table>
Model specification recalled

\[ u_t(c, h) = \frac{c^{1-\gamma}}{1-\gamma} + \frac{(L-h)^{1-\phi}}{1-\phi} a_t = \frac{c^{1-\gamma}}{1-\gamma} + a_t v(h), \]

with \( \gamma \) and \( \phi \) non negative, \( L \) physiological bound on weekly hours (say 100)

\[ \ln(a_t) = Z_t^a \beta^a + \sigma^a \epsilon^a, \]

\[ b = Z^b \beta^b + \sigma^b \epsilon^b. \]
Estimation results

Table 1: Labour supply parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Two offer model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>12.4 (1.7)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1 (0.3)</td>
</tr>
<tr>
<td>$a$: constant</td>
<td>51.0 (7.9)</td>
</tr>
<tr>
<td>$a$: cohab</td>
<td>0.7 (0.2)</td>
</tr>
<tr>
<td>$a$: youngest 0-4</td>
<td>2.7 (0.3)</td>
</tr>
<tr>
<td>$a$: youngest 5-10</td>
<td>1.2 (0.2)</td>
</tr>
<tr>
<td>$b$: constant</td>
<td>19.4 (3.0)</td>
</tr>
<tr>
<td>$b$: cohab</td>
<td>-12.4 (2.8)</td>
</tr>
<tr>
<td>$b$: number of kids</td>
<td>12.7 (1.2)</td>
</tr>
<tr>
<td>Likelihood</td>
<td>-2.83</td>
</tr>
<tr>
<td>Number of observations</td>
<td>11481</td>
</tr>
</tbody>
</table>
Estimation results: continued

Table 2: Restricted choice and correlation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Two offer model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>15.2 (0.2)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>9.4 (0.3)</td>
</tr>
<tr>
<td>$m_2$</td>
<td>38.0 (0.2)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>4.4 (0.1)</td>
</tr>
<tr>
<td>$p_1$</td>
<td>0.7 (0.0)</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>60.0 (1.9)</td>
</tr>
<tr>
<td>$\rho(\varepsilon^a, \varepsilon^c)$</td>
<td>-0.3 (0.0)</td>
</tr>
<tr>
<td>$\rho(\varepsilon^a, \varepsilon^w)$</td>
<td>-0.7 (0.0)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>3.1 (0.4)</td>
</tr>
</tbody>
</table>
When is the restricted model useful?

<table>
<thead>
<tr>
<th>Observations...</th>
<th>...not rejecting the Unrestricted Model</th>
<th>...rejecting the Unrestricted Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion among ‘in work’ women</td>
<td>0.91</td>
<td>0.09</td>
</tr>
<tr>
<td>Age at end of studies</td>
<td>17.43</td>
<td>16.85</td>
</tr>
<tr>
<td>Age</td>
<td>37.95</td>
<td>37.17</td>
</tr>
<tr>
<td>Hourly wage</td>
<td>7.40</td>
<td>5.05</td>
</tr>
<tr>
<td>Marginal tax rate</td>
<td>2.21</td>
<td>0.61</td>
</tr>
<tr>
<td>Usual weekly hours</td>
<td>27.29</td>
<td>21.06</td>
</tr>
<tr>
<td>Log of consumption</td>
<td>5.52</td>
<td>5.07</td>
</tr>
<tr>
<td>Number of kids</td>
<td>1.74</td>
<td>1.86</td>
</tr>
<tr>
<td>A kid younger than 4</td>
<td>0.25</td>
<td>0.22</td>
</tr>
<tr>
<td>Youngest kid between 5 and 10</td>
<td>0.31</td>
<td>0.37</td>
</tr>
<tr>
<td>London</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>Cohabitant</td>
<td>0.86</td>
<td>0.46</td>
</tr>
<tr>
<td>Spouse inwork</td>
<td>0.70</td>
<td>0.21</td>
</tr>
<tr>
<td>Out of work income</td>
<td>349.04</td>
<td>201.45</td>
</tr>
<tr>
<td>In work income</td>
<td>503.94</td>
<td>245.96</td>
</tr>
</tbody>
</table>
Figure 18: Hours distributions and rejection of the unrestricted model

![Graph showing hours distributions and rejection of the unrestricted model]
Model fit: hours distribution

[Graph showing the actual hours distribution and the two-offer model.]

Actual Hours Distribution

Two-offer Model
### Table 3: ‘Long-run’ Elasticities

<table>
<thead>
<tr>
<th>Budget Constraint</th>
<th>Linear budget</th>
<th>Non-linear Δw = 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{\text{Int}} )</td>
<td>0.31</td>
<td>0.74</td>
</tr>
<tr>
<td>( \varepsilon_{\text{Int}} : p1% )</td>
<td>0.09</td>
<td>-1.67</td>
</tr>
<tr>
<td>( \varepsilon_{\text{Int}} : p5% )</td>
<td>0.11</td>
<td>0.00</td>
</tr>
<tr>
<td>( \varepsilon_{\text{Int}} : p10% )</td>
<td>0.12</td>
<td>0.00</td>
</tr>
<tr>
<td>( \varepsilon_{\text{Int}} : p50% )</td>
<td>0.22</td>
<td>0.36</td>
</tr>
<tr>
<td>( \varepsilon_{\text{Int}} : p75% )</td>
<td>0.37</td>
<td>0.54</td>
</tr>
<tr>
<td>( \varepsilon_{\text{Int}} : p90% )</td>
<td>0.59</td>
<td>0.83</td>
</tr>
<tr>
<td>( \varepsilon_{\text{Int}} : p95% )</td>
<td>0.93</td>
<td>1.25</td>
</tr>
<tr>
<td>( \varepsilon_{\text{Int}} : p99% )</td>
<td>1.53</td>
<td>5.71</td>
</tr>
</tbody>
</table>
Conclusion of the modeling exercise

Assess the robustness of the example: Monte-Carlo, introduction of measurement error, extend the analysis on a long period.

For the future:
many offers, link with 'consideration sets' or search and matching.