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**Multiperiod LP Models for Simultaneous Production Planning
and Scheduling in Multiproduct Batch Plants**

by

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**MULTIPERIOD LP MODELS FOR
SIMULTANEOUS
PRODUCTION PLANNING AND SCHEDULING
IN MULTIPRODUCT BATCH PLANTS**

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Abstract

Production planning and scheduling are intimately linked activities. Since the production goals set at the planning level must account for the ability to implement them at the scheduling level, ideally both activities should be analyzed simultaneously. However, this in general is a very difficult task given the large combinatorial nature of just the scheduling problem in itself. In this work, based on a previously developed LP flowshop scheduling model by Birewar and Grossmann, (1989b) that can effectively aggregate the number of batches belonging to each product, a multiperiod LP model is proposed for the simultaneous production planning and scheduling of multiproduct batch plants that may consist of one or several non-identical parallel lines. Inventory costs, sequence dependent cleanup times and costs, and penalties for production shortfalls are readily accounted for in this model. The actual schedule to achieve the production goals predicted by the planning problem is derived by applying a graph enumeration method to the results from the simultaneous planning and scheduling model or by any other scheduling method. Several examples are presented to illustrate the proposed method.

Introduction

Production planning and scheduling in multiproduct batch plants are closely related activities. The major objective in production planning is to determine production goals over a specified time horizon given forecasts for prices and product demands, and considerations of equipment availability and inventories. Thus, planning is basically a *macro* level problem that is concerned with the allocation of production capacity and time, product inventories, as well as labour and energy resources so as to meet market demands by maximizing the total profit over an extended period of time into the future.

Scheduling on the other hand is the *micro* level problem that is embedded in the production planning problem. Scheduling involves deciding upon the sequence in which various products should be processed in each equipment so as to meet the production goals that are set by the planning problem. A major objective here is to efficiently utilize the available equipment among the multiple products to be manufactured to an extent necessary to satisfy the production goals.

Thus, it's clear that decisions made at the production planning level have a great impact at the scheduling level, while the scheduling in itself determines the feasibility of carrying out the production plans. Thus, ideally both activities should be analyzed and optimized simultaneously. However, this is in general a very difficult task given that, even optimizing the scheduling problem in isolation for fixed production demands, is a nontrivial problem.

Most optimization problems for scheduling have been shown to be NP-complete (Garey et al, 1976). This is for instance the case in *Flowshop scheduling problem* where each product follows the same sequence through a set of processing stages. Despite the apparent simplicity of flowshop scheduling, this problem can become computationally expensive to optimize for various scheduling policies like UIS [Unlimited Intermediate Storage], FIS [Fixed Intermediate Storage], NIS [No Intermediate Storage] and ZW [Zero Wait] is quite difficult to solve. UIS flowshop scheduling has been shown to be NP-complete for three or more stages (Garey et al.

1976). The ZW policy is NP complete for makespan minimization if number of stages is more than two (Graham et al, 1979). Suhami (1980) has shown that the NIS problem is NP-complete if the number of stages is greater than two. This implies that the computational effort required to solve these problems to optimality can be quite high given the potential exponential increase in computational time with problem size. Moreover, the considerations of set-up costs and precedence constraints will often complicate the solution procedures for the scheduling. Thus, the production planning problem in conjunction with the rigorous solution of scheduling problem poses a formidable challenge. Therefore, several approximate methods have been proposed for integrating the planning and the scheduling problem (e.g. see Reklaitis, 1982, Mauderli, 1979).

Since the simultaneous treatment of production planning and scheduling poses great computational difficulties, the main approach that has been used is a hierarchical decomposition scheme where the overall planning problem is decomposed into two levels (Bitran and Hax,1977). At the upper level the planning problem is represented with a multiperiod LP planning model that sets production goals to maximize profit. At the lower level the scheduling problem is reduced to a sequencing subproblem that must meet the goals set by the planning problem. The multiperiod LP model typically involves a simplistic representation of the scheduling problem where the actual sequencing of products is not accounted for (e.g. through single product campaigns). The integration of the two levels is then carried out in a heuristic fashion (e.g. see Jain et al, 1978), and is commonly implemented through a rolling schedule strategy (Hax, 1978; and Baker, 1977). Studies by these authors indicate that solution with hierarchical decomposition schemes generally lie within 10% of the optimal solution obtained from the single level planning problem. Thus, it is clear that there is an incentive to develop methods and approaches that can more effectively integrate planning and scheduling. As will be shown in this paper this goal can be accomplished with a new representation for scheduling in multiproduct batch plants.

In this paper new multiperiod LP formulations will be presented for the simultaneous planning and scheduling of multiproduct batch plants that comprise

one or several parallel lines (facilities). Each line consists of several processing stages with one unit per stage. A key feature of the proposed models is that the sequencing considerations for scheduling can be accounted for at the planning level with very little error. It will be shown that this can be accomplished using the scheduling models for Unlimited Intermediate Storage and Zero Wait policy, proposed by Birewar and Grossmann(1989b). Furthermore, by taking advantage of the structure of the scheduling model and a novel graph representation developed recently by Birewar and Grossmann (1989b), detailed production schedules can easily be derived from the solutions of the multiperiod LP model.

Problem Statement

The problem addressed in this paper can be stated as follows:

A multiproduct batch plant for producing N_p products is given. There are L manufacturing lines (or facilities) each being capable of producing a subset of N_p products. Each line consists of M stages each having one processing unit. The products use all the processing stages in the same order [*Le. multtproduct orjlowshop plant*). A finite long range horizon is also given which is subdivided into T time-periods. The end of each time-period corresponds to due dates on which product demands are specified. Fixed processing times, sequence dependent cleanup times and costs and precedence constraints (in form of precedences that are essential or on other hand forbidden, etc) if necessary, are assumed to be given. The goal is then to determine a plan for the production level in each time-period, as well as the detailed schedules over the entire horizon that maximizes profit after accounting for inventory costs and penalties for production shortfall.

It will be assumed that product demand is flexible in the sense that it is given by a range of values having a hard upper bound and a soft lower bound. The lower bounds depend on the internal consumption requirements and the orders booked by the sales department. Production shortfalls with respect to the lower bounds are treated through penalties that for instance reflect relative loss of consumer

satisfaction. Hard upper bounds correspond to the maximum *projected* demands for each corresponding time period. As for the scheduling, single product campaigns as well as two extreme cases for mixed product campaigns: i.e. Zero Wait (ZW) and Unlimited Intermediate Storage (UIS) policies will be considered.

Outline of The Approach

It is clear from the introduction section that in order to obtain an optimal solution for the integrated production planning and scheduling problem, a scheduling model that can be easily solved and incorporated into the planning problem is a necessity.

Production planning involves long range horizons typically ranging from the order of several months to one year. Time-periods into which this long range horizon is divided vary typically from the order of a few days to few weeks. In such a long range horizon, obviously the number of batches (N) is high, and hence rigorous scheduling methods are extremely expensive to apply. Furthermore, there is the additional complication that the total number of batches (N) is an unknown that is to be determined in the planning problem.

It should be noted, however, that there are two features in the scheduling problem that can be exploited. The first is the fact that although the total number of batches, N can be quite high, they can be aggregated into sets of N_p (Birewar and Grossmann, 1989a) products which are typically much smaller in number, and more importantly, are known a priori. Once these batches are grouped according to their product-identity, they can be aggregated in a way where variables dealing with each batch of each product or their combinations are not required. Instead only variables associated with various pairs of products are required. The second feature that can be exploited is that for relatively long time intervals the total cycle time yields a very good approximation to the makespan.

Consider first the simple case of single product campaigns (SPC) with ZW policy. Let n^i be the number of batches of product i to be processed on production facility I in

time period t t_{ij} the processing time of product i in stage J and production line I and H_t the length of the corresponding time period. Then the number of batches n^{\wedge} can be constrained by the total cycle time in the linear inequality (Sparrow et al, 1975):

$$\sum_{i \in I_t} n^{\wedge} T_{Lu} \leq H_t \quad t=1 \dots T, \quad l=1 \dots L \quad (1)$$

where $T_{Lu} = \sum_{i \in I_t} t_{ij}$, $\{ t_{ij} \}$ corresponds to the limiting cycle time of product t on parallel production line I and I_t is the set of products i that can be produced in line L . The inequality neglects the clean-up times and changeovers between successive campaigns and the head and tail of the makespan. However, the error in this approximation is small if the number of batches is relatively large.

While single product campaigns are easy to implement, they are often not as efficient as mixed product campaigns where batches of dissimilar products are sequenced for production. Although here the scheduling problem is more difficult, linear constraints can be developed as has been shown by Birewar and Grossmann (1989a).

Firstly, for the case of the ZW policy where one batch at a given stage must be immediately transferred to the next stage upon completion, idle times will arise in the schedule. Therefore, it is first necessary to determine the slack times SL^{\wedge} for product i followed by product k in stage j of production facility I from the data on processing times and clean-up times (see Birewar and Grossmann, 1989a). By then defining the variables $NPRS^{\wedge}_{ikl}$ to represent the number of batches of i followed by it in the production facility I in time period t the number of batches n^{\wedge} will be constrained by the interval length H_t as follows:

$$n^{\wedge} + \sum_{i \in I_t} \sum_{k \in I_t} NPRS^{\wedge}_{ikl} SL_{ikl} \leq H_t \quad j=1 \dots M, \quad t=1 \dots T, \quad l=1 \dots L \quad (2)$$

where again the left hand side corresponds to the total cycle time.

As every product is manufactured r^{\wedge} times, it will appear exactly n^{\wedge} times in the first place and $r^{\wedge} n^{\wedge}$ times in the second place in the pairs (Uk) of products that are

manufactured during the production sequence. Therefore, the following two assignment constraints apply :

$$\sum_{k \in I_t} NPRS_{ikl} = n_{iil} \quad i \in I_t, \quad t=1...T, \quad l=1...L \quad (3)$$

$$\sum_{i \in I_t} NPRS_{ikl} = n_{kil} \quad k \in I_t, \quad t=1...T, \quad l=1...L \quad (4)$$

For the case of UIS policy there is no interaction among the stages, and hence the slacks or the forced idle times SL_{ikjl} are zero for the case of zero clean-up times. Thus the constraints in (2) reduce to,

$$\sum_{i \in I_t} n_{iil} t_{ijt} \leq H_t \quad j=1...M, \quad t=1...T, \quad l=1...L \quad (5)$$

where the left hand side corresponds to the total cycle time for the schedule. Note that the horizon constraint (5) for the UIS policy does not contain the term containing $NPRS_{iktl}$. That means that the cycle time requirement of any sequence of batches with UIS policy and zero clean-up times is equivalent. For the case of non-zero clean-up times, the total cycle time for UIS can be represented by constraints (2)-(4) by setting the slacks equal to the clean-up times.

The linear constraints in (2)-(5) introduce a very small error in the schedule. Firstly because they neglect heads and tails of the schedule; secondly because the assignment constraints (3) and (4) may allow subcycles (Birewar and Grossmann, 1989b).

It will be shown in the next section how the LP scheduling model for ZW policy given by (2)-(4), and for the UIS policy given by (2)-(4) or (3)-(5) can be used to build a simultaneous LP production planning and scheduling model. Since the available horizon is divided into various time periods depending on the due dates of various orders, the integrated planning and scheduling is a multi-period LP model.

Planning Model

The proposed multiperiod LP model for simultaneous production planning and scheduling over a set of T time periods each of fixed length H , $t=1\dots T$, is as follows.

The volumes, V_j^t , of equipment in all the stages $j=1\dots M$ and production lines (or facilities) $l=1\dots L$ constrains the batches of various products. In other words, the size of the batches, B_{ij} , in the time period t in production facility I times the appropriate size factor, S_{ij} , cannot exceed the corresponding equipment sizes:

$$B_{ij} S_{ij} \leq V_j^t \quad i \in I, \quad j=1\dots M, \quad t=1\dots T, \quad l=1\dots L \quad (6)$$

Also, the amount of product produced in each time period t in production facility I Q_{it} , is equal to the corresponding batch size multiplied by the number of batches, n^t :

$$B_{ij} n_{ij}^t = Q_{it} \quad i \in I, \quad j=1\dots M, \quad t=1\dots T, \quad l=1\dots L \quad (7)$$

Then from equations (6) and (7), it follows that capacity constraints can be expressed as:

$$Q_{it} S_{ij} \leq V_j^t n_{ij}^t \quad i \in I, \quad j=1\dots M, \quad t=1\dots T, \quad l=1\dots L \quad (8)$$

where the total production, Q_{it} , over all the lines is given by,

$$\sum_{i \in L_t} Q_{it} = Q_{it}^T \quad i=1\dots N_p, \quad t=1\dots T \quad (9)$$

where L_j is the set of lines I that are capable of producing product L . The inventory, IF_W at the end of each of the time interval will be equal to the inventory, IB_{it} , at the beginning of the time period t plus the quantity produced during that interval:

$$IF_W = IB_{it} + Q_{it} \quad i=1\dots N_p, \quad t=1\dots T \quad (10)$$

The inventory at the beginning of each interval will be the amount that is left from the previous interval after subtracting the total sales, QS_{it} , at the end of that interval:

$$IB_{it} = IB_{i,t-1} - QS_{it} \quad i=1..J^V, t=2..T \quad (8)$$

$$IB_{it} \geq 0 \quad i=1..J^V, t=1..T \quad (12)$$

Note that in the above IB_{iT} corresponds to the inventory left after satisfying the orders in the final time interval T.

The total sales, QS_{it} of product i in each of the time periods t are bounded by a hard upper bound and a soft lower bound. The upper bound is specified by the maximum projected market demand, QS_{it}^U .

$$QS_{it} \leq QS_{it}^U \quad i=1..J^V, t=1..T \quad (13)$$

The lower bound, QS_{it}^L of product i in interval t depends on the commitments made by the sales department and the internal product requirement for downstream processing. If these orders of product i , QS_{it}^L are not satisfied in an interval t then the shortfall, SF_{it} of product i in interval t is given by:

$$SF_{it} = QS_{it}^L - QS_{it} \quad i=1..N_p, t=1..T \quad (14)$$

$$SF_{it} \geq 0 \quad i=1..N_p, t=1..T \quad (15)$$

Failure to fulfil commitments can be quantified by using appropriate penalty functions. The penalty PN_{it} incurred for not meeting the commitments for product i in interval t can be expressed as a linear function in terms of the shortfalls SF_{it} . For example, it can be just some cost, λ_{it} , times the shortfall,

$$PN_{it} = \lambda_{it} \times SF_{it} \quad i=1..N_p, t=1..T \quad (16)$$

Such simple expression may be inadequate to represent the loss of consumer satisfaction and other penalties involved in not meeting the lower bounds on production. Piecewise linear functions can be used in place of (16) if necessary, as discussed in Appendix I.

Thus, it is assumed that given are the data on profit margins for each product i in time interval t , P_{it} , the orders booked, Q_{it} , the upper bounds on projected market demands for various products, Q_{it}^u , the cost of inventory, y_{it} , processing times, t_{ij} , size factors, S_{ij} ; the total available times in each of the intervals, H^l and information about lines that can produce various products, L_j . From equations (8) - (16) the model for simultaneous production planning and scheduling that maximizes profit (income minus profit minus inventory costs) is then given by the multiperiod LP problem:

$$\max \sum_{i=1}^N \sum_{t=1}^T Q_{it} P_{it} - PN_{it} - y_{it} \left(\frac{IB_{it} + IF_{it}}{2} \right) H_t$$

s.t.

$$Q_{itl} S_{ijt} \leq V_{jt} n_{itl} \quad i \in I_p, \quad j=1..M, \quad t=1..T, \quad l=1..L$$

$$\sum_{i=1}^N Q_{itl} = Q_{it} \quad i=1..N_p, \quad t=1..T$$

$$IF_{it} = IB_{it} + Q_{it}^u \quad i=1..N_p, \quad t=1..T$$

$$IB_{it+1} = IF_{it} - Q_{it} \quad i=1..N_p, \quad t=1..T$$

$$Q_{it} \leq Q_{it}^u \quad i=1..N_p, \quad t=1..T$$

$$SF_{it} \geq Q_{it} - Q_{it}^u \quad i=1..N_p, \quad t=1..T$$

$$SF_{it} \times a_{it} = PN_{it} \quad i=1..N_p, \quad t=1..T$$

$$gdn_{it} \leq 0 \quad i=1..T, \quad i \in L_i$$

$$Q_{it}, Q_{it}^u, Q_{it}^l, IF_{it}, IB_{it}, SF_{it}, n_{itl} \geq 0.$$

[LP]

where g^{ln} are the horizon constraints that depend on the type of scheduling policy to be implemented in the manufacturing facility. This gives rise to the following UP models :

a) Model LP1, with horizon constraint (1) for scheduling with single product campaigns (SPC) and Zero Wait policy.

b) Model LP2, with horizon and scheduling constraints in (2) - (4) for sequencing schedules with ZW policy.

c) Model LP3, with scheduling horizon constraints In (2)-(4) or (3)-(5) for sequencing schedules with UIS policy.

It should be noted though that the horizon constraints in LP1, LP2 and LP3 ensure that the "total cycle time" and not the "makespan" of the given production requirement is contained within the available horizon time. This however, should not pose difficulties since the makespan is underestimated by the total cycle time by a very small margin for relatively large number of batches.

Also, the number of predicted batches n^* , will in general not take integer values. To make them integer, the number of batches can simply be rounded down to the nearest integer. This serves another purpose, too. By rounding *down* the number of batches, the time requirement is reduced which counter-balances the error stemming from using cycle time as an approximation to the makespan. Also, as the lower bounds on the production are soft, rounding down the number of batches, i.e. manufacturing slightly less than stated by the initial LPs, does not make the LP problem infeasible. It will simply produce a decrease in the objective function which will be small if the number of batches is relatively large (see example). Hence, since the number of batches involved will often be relatively large, the above rounding scheme should produce solutions that are very near to the global optimum. Note that since the non-integer solution yields an upper bound to the profit, one can easily compute the maximum deviation of the rounded solution with respect to the upper bound on global optimum.

Solution Strategy

The proposed strategy for simultaneous planning and scheduling involves first solving the linear programming models LP1, LP2 and LP3 depending upon the scheduling policy that is going to be adopted during actual production. Data that are required are the size-factors, processing times for each product in each of the stages as well as amount of orders booked for each period and forecasts for amounts that can be sold in open market. Also needed is the data on penalty for shortfalls, sequence dependent clean-up times/costs, inventory costs and the profit margin for each of the products. These multiperiod LP models will give rise to problems of moderate to large size, which can be solved efficiently with standard LP codes (e.g. ZOOM, MPSX).

Since the underlying model is a continuous LP problem, the solution to the overall planning problem will not necessarily result in integer number of batches. However, since the numbers will be typically large, a simple rounding scheme stated in the previous section can be applied. After the number of batches on all the production lines and time periods are rounded *down* to the nearest integer numbers, the respective LP models (LP1, LP2 or LP3) are resolved to calculate new values of the various variables involved. The solution will then determine the amounts of each product to be produced in each interval on each production line (or in each facility), amounts sold at the end of each time-period, the extent to which each order should be satisfied, inventory level to be maintained in each interval, number of batches and their batch sizes in each time interval, which products should be produced in what line and most importantly in what sequence.

After these LP models are solved, the actual detailed schedule that satisfies the production goals set by the model in the available production time, needs to be derived. Since the scheduling was explicitly included in the planning model with very small error, this task is guaranteed to have a feasible or very near feasible solution for the actual schedule on each production line in each of the time periods. For the case of Single Product Campaigns, any schedule containing campaigns of individual

products will represent a feasible or very near feasible solution. For the Mixed Product Campaigns with ZW or UIS scheduling policies, the detailed schedule is derived using the graph enumeration method developed recently by Birewar and Grossmann (1989b). This method is based on the graph representation of the schedule (Birewar and Grossmann, 1989b) specified by the values of variables, $NPRS_{ika}$ in the UP solutions. As the makespan was approximated by the total cycle time, the resultant schedule will be cycle rather than a sequence. This cycle can be arbitrarily broken at any point giving rise to a detailed sequence. The sequence thus generated will be very near to the globally optimal makespan solution.

In case it is desired that the schedules for each production line in each time period have a globally optimal makespan, the MILP models developed by Birewar and Grossmann (1989b) can be used. Appendix II presents a brief summary of the MILP models developed recently by above authors, that are particularly suitable for cases when the number of batches consist of relatively few products as would be the case in this planning problem.

Thus, the complete solution to the production planning and scheduling problem can be determined. The optimality of the suggested approach is subject only to the very small errors that are introduced by approximating the total makespan in the scheduling problem.

Remarks

Due to its ability to account for the effects of scheduling, the proposed LP production planning models are capable of giving better (more profitable) and more realistic production plans than other simplified multiperiod LP models. Their flexible and general nature allow various modifications or extensions that may result in a wide variety of applications.

The model can be modified to differentiate between the orders booked by different customers. This is possible by defining new production variables, QI_q for customer q and assigning them different weights or penalties for not satisfying orders according to the priority of the customer. Thus, potentially it can be used to derive schedules and plans to satisfy "important orders".

The other important modification can be to allow for backlogging. Presently the models LP1-LP3 assume that if a certain order is not satisfied in a given time interval, then some penalty is accrued. But sometimes this penalty can be reduced or eliminated, by producing that order in the next interval: in other words, by backlogging. This can be modelled as follows. Add the shortfall, SF_a in an interval t to the corresponding lower bound, QS_{t+1} , of the next interval $t+1$, with higher penalty. In this way it is necessary to first satisfy the backlog from previous interval, before satisfying the orders of current interval. The degree of desirability to achieve fulfilling of orders is controlled by the specifying proper values of penalties and functional forms for the penalty functions.

Since the models LP1-LP3 involve the solution of LP problems which are inexpensive, the proposed production planning and scheduling model can be used within a "rolling schedule strategy". After every shift or day, the production requirements, raw material availability, orders booked, etc can be updated and the LP model can be resolved to give new optimal schedule and production plans. Due to the computational ease with which the model can be solved, it is possible to use it to dynamically adjust the production plan and schedules as new orders are placed or

factors like equipment availability undergo unforeseen changes. In this way it is possible to almost continuously operate the plant at near optimal conditions.

Finally, another important use of this model can be as a tool to help in the decision making of the marketing department. As the above LP models accurately represent the actual capability of the manufacturing facilities, they can be used to evaluate the impact of accepting some new order that comes in, on the production plan, detailed schedule and the overall profitability of the plant. This can be done as follows :

1. Define the new time intervals based on time lapsed since last time the planning and scheduling procedure was carried out.
2. Add the new order, which has not been accepted yet, to the lower bound, Q_{fo} , with the data on the corresponding profit margin.
3. Define the current value of IB_U (the current inventories of various products), and horizon time H_t to reflect the time that is left for production.
4. Increase the penalty to a very large numbers for satisfying the orders that have been already comitted to.
5. Resolve the LP model for simultaneous production planning and scheduling to evaluate the ability to manufacture that extra production in the given time.

Examples for Planning and Scheduling

Example 1 :

This planning problem consists of manufacturing five different products A, B, C, D and E that are to be manufactured using three separate production lines each consisting of four manufacturing stages. Line 1 can process products A, B and C. Lines 2 and 3 are capable of processing products B,C, D and C, D, E, respectively (Figure 1). Data on the volumes of various processing stages in different processing lines, processing times and size factors are given in Tables 1(a) to 1(c), respectively. Zero clean-up times were considered here. The planning horizon spans one whole year and there are due dates for the orders of each product at the end of Spring, Summer, Fall and Winter seasons. The commitments made, i.e. the orders booked a priori for each product as well as the quantities for internal consumption are listed in Table 2(a). Table 2(b) lists the maximum projected demand in the open market for these products during the four seasons. Table 3(a) shows the profit per unit sales of products A-E during various seasons. The shortfall or the inability to meet the orders in Table 2(a) are penalized according to the proportionality constant (\$ per unit shortfall) shown in Table 3(b).

This problem was solved with formulation LP1 (Single Product Campaign), LP2 (ZW-MPC) and LP3 (UIS-MPC) using ZOOM (Marsten, 1986) through the modelling system GAMS (Brooke et al, 1988). LP1 required 137 variables and 153 constraints, LP2, 245 and 261, LP3, 245 and 261. The computer times were 19.56, 66.66 and 51.24 sec. of CPU time respectively on SUN 3/60. Solutions to LP1 through LP3 contained non-integer values for the variable, n_{itl} for the number of batches of products. These were rounded down to the nearest integer numbers and the linear programs LP1-LP3 were resolved for fixed number of batches n_{itl} . This time the LPs required 9.48, 25.76 and 18.52 seconds of CPU time, respectively. Table 4 shows the difference in the objective functions of the original LP model and the rounded solutions. The difference arising from the rounding procedure (1% or less) was small because the number of batches ranged between 6 to 135 for each time period.

As seen in Table 5, the optimal plan with UIS results in the highest profit, \$ 1,022,769.7. The plan with SPCs shows a profit of \$ 718,600.6 due to the poor utilization of available facilities among the three policies discussed. MPCs with ZW policy shows a profit of \$ 881,349.8, an increase of 22.6% compared to the SPCs. MPCs with UIS policy had an increase of 42.33% compared to the SPCs. Figure 2 shows how the MPCs with ZW and UIS policies consistently result in higher profit than SPCs with ZW policies in each of the quarter. These higher profits stem from the fact that better utilization of equipment with MPCs results in some idle time after satisfying the booked orders. This idle time is used to manufacture products to be sold in the open market. As seen in Figure 3, MPCs with UIS policy are able to satisfy all the orders that were booked; MPCs with ZW policy can satisfy 90% of the orders; in contrast SPCs with ZW could satisfy only 70% of the orders. In other words, planning with a better anticipation of scheduling can result in increased ability to satisfy consumer demands.

The actual schedules for each interval and each production line were then obtained. Here the globally optimal solutions for the scheduling were obtained. MILP1-MILP3, described in Appendix n were solved respectively to obtain globally optimal solutions for scheduling of SPCs with Zero Wait policy, MPCs with ZW and UIS policies, respectively. For each of the scheduling policies an MILP was solved for each production line in each time period. The solution of twelve MILPs (four time periods x three production lines) required a total of 23.06 seconds of CPU time when solved using ZOOM (Marsten, 1986) through the modelling system GAMS (Brooke et al, 1989) on a SUN 3/60. Schedule for each of the production lines in Spring is shown using the graph representation (Birewar and Grossmann, 1989) in Figures 4(a)-4(c).

Solutions for the twelve MILP2s for MPCs with ZW policies required a total of 17.44 seconds of CPU time. The schedule for the Spring time interval is shown in Figures 5(a)-5(c).

Solutions for the twelve MILP3s for MPCs with UIS policies required a total of 24.76 seconds of CPU time. These schedules are again shown using the graph

representation in Figures 6(a)-6(c).

The makespans of the required production with various scheduling policies for each production line in Spring time period as well as other relevant results regarding scheduling subproblems are listed in Table 6. It can be seen that all the makespans (various lines in each of the time period) are very close in value to the available horizon time of 1500 hrs. However, as noted earlier, some require slightly more time than 1500 hrs. This is because the cycle time is used as an approximation to the makespan. It should be noted, that the extra time requirement is very small (maximum 1.1 %), and hence has very little effect on the feasibility of the proposed problem.

Example 2 :

This example is much larger in size than the previous one and successfully demonstrates the capabilities of the planning and scheduling model presented earlier in the paper. The planning problem here consisted of manufacturing 10 different products (A - J). The manufacturing facility consisted of five non-identical parallel production lines, each consisting of 4 production stages. Figure 7 shows which of the products can be manufactured by each of the production lines. Processing times for various products in various stages ranged from 1 to 15 hrs. The data on size factors, volumes of stages in various production lines, commitments made, projected upper bound on market demands for various products in various time intervals, profit margins penalties on shortfalls for various products in various time intervals, inventory holding costs as well as the total time available on each production line in each time period can be found in Birewar (1989c). The total available time (6000 hrs) was divided into 12 equal time intervals as due dates were specified for each of the products at the end of each month. The problems of planning and scheduling were then solved using the proposed multiperiod LP models.

The multiperiod model LP1 for SPCs with ZW policy consisted of 781 variables and 901 constraints. The solution required 293.8 seconds of computing time when

solved using ZOOM through the modelling system GAMS on SUN 3/60. Resolving the LP1 after rounding down the number of batches to the nearest integer numbers required 193.3 seconds of CPU time. The total profit was \$ 2,192,345.2 in the total available time of one year (6000 hrs).

The problem of production planning and scheduling for MPCs with ZW policy, the multiperiod model LP2 consisted of 1345 variables and 1441 constraints. The optimal solution was obtained on a SUN 3/60 using ZOOM in 817.14 seconds of CPU time. Rounding down the number of batches and resolving the multiperiod LP2 required a CPU time of 477.26 sec. The total profit for MPCs with zero wait policy was higher by 7.14% at \$2,348,848.45.

For MPCs with UIS policy the problem was modelled using LP3. It required 781 variables and 1081 constraints. The solution using ZOOM required a CPU time of 338 seconds. LP3 after rounding down the number of batches required the CPU time of 220.74 seconds. The profit this time increased by 18.11% to \$2,589,342.1 per year.

Thus, the profits made by mixed product campaign scheduling with both ZW and UIS policies were consistently higher in all the 12 time periods (Figure 8). Out of possible 120 orders, the number of shortfalls for MPCs with ZW and UIS policies were 14 and 9 respectively, significantly less than the 27 shortfalls for the SPCs (Figure 9). The results for Example 2 are summarized in Table 7. Also Table 8 shows the difference in the objective functions between the non-integer and rounded solutions. It can be seen that these are very small.

Detailed schedules for each of the 5 parallel production lines in each of the 12 time-periods can be derived first by representing the schedule specified by optimal values of $NPRS^i$ by the graph representation and then by expanding it in the form of a detailed cycle using the graph enumeration scheme. For instance, the Zero Wait (MPC) schedule for one of the parallel production lines in first time period is shown in form of a compact graph representation in Figure 10. Note that for this schedule the optimal cycle time is 489 hrs. and that the upper and lower limits on the optimal makespan are 497 hrs and 493 hrs respectively. The lower and upper bounds were

evaluated using the analytical expressions by Birewar and Grossmann (1989b). It is clear from this example that use of MILP2 to derive globally optimal makespan solution is hardly justifiable as the lower and the upper bounds are quite close and upper bound (497 hrs) is lower than the total available time (500 hrs) in that interval.

Conclusions

This paper has presented new multiperiod LP formulations for *simultaneous planning and scheduling* of multiproduct batch (flowshop) plants that consist of parallel production lines with one unit per stage. These models explicitly account for the effect of scheduling by Mixed Product Campaigns with Zero Wait and Unlimited Intermediate Storage policies as well as the Single Product Campaigns with ZW policy. The inventory costs, sequence dependent clean-up times and costs as well as set-up times and costs are readily accounted for. Penalty is incurred for not satisfying the orders booked. The penalty function can be made to increase as the percentage shortfall increases. The proposed models presented here can also handle batch plants with more than one dissimilar parallel production lines. The UP models also decide which orders are to be satisfied and to what degree in case there is a competition for resources between two different orders. Also the amount of inventory to be carried over from one interval to the next is accounted for.

The LP models set the production goals for each time period and each production facility for various products. Also a feasible schedule to achieve these goals is derived simultaneously for Mixed Product Campaigns with ZW as well as UIS policy. For the case of SPCs, changeovers from one product to the another can be neglected for large number of batches with relatively smaller number of products, as the changeovers will occur only when switching from one campaign to the another. Thus, as the scheduling and planning activities are analyzed simultaneously with approximations that introduce very little error, solutions that are very near to the global optimum are achieved. As seen from examples presented in this paper, significant increase in profits can be achieved due to better utilization of equipment in case of MPCs compared to the SPCs. Scheduling at the planning stage allows better utilization of the existing capabilities of manufacturing often resulting in the increased ability to satisfy orders booked as shown by the examples presented. Thus explicit accounting for scheduling at the production planning level results in plans that are more reliable and represent more accurately the actual capabilities of the existing

production facilities.

Acknowledgments

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APPENDIX I: A More General Penalty Function for Production Shortfalls

The penalty function described by constraints in (16) may not be able to adequately represent loss of consumer satisfaction due to the shortfalls in orders booked. For example, it is more realistic to assume that the degree of consumer dissatisfaction will increase as the percentage shortfall increase (Figure 1(a)). In other words the constant of proportionality would rise as the percentage shortfall increases. For example, if the shortfall, SF_{af} , for product f in interval t is less than a/t , then the penalty is proportional to the shortfall and the constant of proportionality or the penalty constant is Q/t . For shortfall between a/t and of/t the penalty constant is given by Q/t . For shortfall a greater than af/t the penalty constant is given by nf/t .

The total penalty for each product i in interval t is defined by following three groups of constraints,

$$PN_U \geq SF_U \times n^1_u \quad \wedge L..JV,, \quad i=i...r \quad (17)$$

$$PN_U \geq cl \times nl + (SF_U - c/t) \times nl \quad \hat{1}=1..^ \quad M..T \quad (18)$$

$$PN_U \geq \sigma \times n/t + \sigma^2 \times \Omega^2_u + (SF_i - \sigma^2_u) \times \Omega^3_u \quad i=1...N_p, \quad t=1...T \quad (19)$$

provided

$$Cl/t \leq Of/t < Ql \quad i=1...N_F, \quad isl...r \quad (20)$$

Similarly any such group of linear constraints can be used to replace the penalty constraints in models LP1-LP3.

Appendix II : Exact Scheduling Method for Minimizing Makespan

The MILP model by Birewar and Grossmann(1989b) can be used to find a sequence with minimum makespan for each time period on each production line. Alternatively, an approximate makespan minimization scheme consisting of solving an LP and evaluation of some analytical bounds proposed in the same paper (Birewar and Grossmann, 1989), may be used. The MILP1 for rigorous minimization of makespan for ZW policy is stated below for a given production line at time period t (Refer to the original paper by Birewar and Grossmann for detailed explanation) :

$$\begin{aligned}
 & \min MS_{it} \\
 & \text{s. t.} \\
 & \sum_{k \in I_t} NPRS_{ikl} = n_{it} \quad i \in I_t, \quad t=1\dots T, \quad l=1\dots L \\
 & \sum_{i \in I_t} NPRS_{ik} = n_{kt} \quad k \in I_t, \quad t=1\dots T, \quad l=1\dots L \\
 & \sum_{i \in I_t} \sum_{k \in I_t} Y_{ikl} = 1 \quad t=1\dots T, \quad l=1\dots L \\
 & Y_{ikl} \leq NPRS_{ikl} \quad i, k=1\dots N_p, \quad t=1\dots T, \quad l=1\dots L \\
 & MS_{it} \geq \sum_{i \in I_t} n_i t_{ij} + \sum_{i \in I_t} \sum_{k \in I_t} NPRS_{ik} SL_{ikj} + \\
 & \quad \sum_{i \in I_t} \sum_{k \in I_t} \left(\left(\sum_{j < i} t_{ij} + \sum_{j > i} t_{ij} \right) - SL_{ikj} \right) YP_{ik} \quad j=1\dots M, \quad t=1\dots T, \quad l=1\dots L \\
 & NPRS_{ik} \geq 0, \quad Y_{ik} = 0,1, \quad i, k = 1\dots N_p, \quad MS \geq 0.
 \end{aligned}
 \tag{MILP1}$$

Solution to this MILP1 determines the values for the variable $NPRS_{ikl}$ that will determine the optimal cycle of batches. The product-pair (i,k) that will break the cycle to form an optimal sequence, will be denoted by the non-zero binary variable Y_{ik} . The detailed sequence then can be derived using the graph enumeration method by

Birewar and Grossmann, (1989b).

For the single product campaigns with zero wait policy, it is important to note that each product will be produced in one campaign. Thus, there will be exactly $n_m - 1$ pairs consisting of product i followed by another batch the same product i . The last batch of each of the product will be followed by the first batch of the product belonging to next campaign. Thus following constraint must be satisfied :

$$NPRS_{iUl} = n_{ul} - 1 \quad i=1..N_p, \quad l=1..T, \quad l \in L_i \quad (17)$$

Adding (17) to MILP1 gives MILP2 for rigorous minimization of makespan of single product campaigns with zero wait policy.

For MPCs with UIS policy, it has been proved that all the schedules have the same cycle time (Birewar and Grossmann, 1989). Also, as stated in the section on outline of approach, by fixing the idle times to 0, the zero wait horizon constraints reduce to the constraints for UIS policy; i.e. following conditions need to be satisfied for the UIS policy :

$$SL_{ikjt} = 0 \quad Lk=1..N_{py}, \quad y=1..J, \quad t=1..T, \quad l \in L_4. \quad (18)$$

Adding (18) to MILP1 gives MILP3 for rigorous minimization of makespan of mixed product campaigns with UIS policy.

The solutions to MILP1-MILP3 or the solutions of LP1-LP3 combined with analytical bounds (Birewar and Grossmann, 1989b), yield the solutions for the optimal or near optimal scheduling, respectively in each of the time interval on each production line. The information gotten from these solutions though, does not give rise to the actual, exact sequence to be followed. These solutions represent a family of solutions, all of which will have equivalent performance (same makespan). The detailed sequence then can be generated from the information obtained above by using the graph representation of the solution and the graph enumeration algorithm developed by Birewar and Grossmann (1989b).

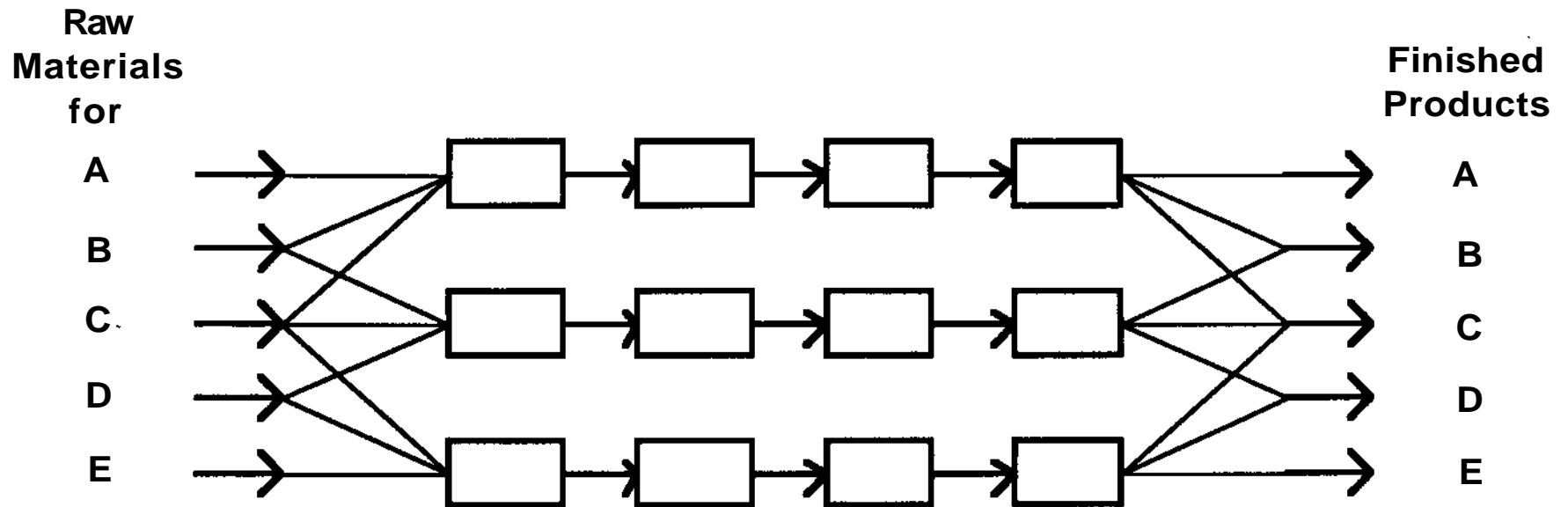
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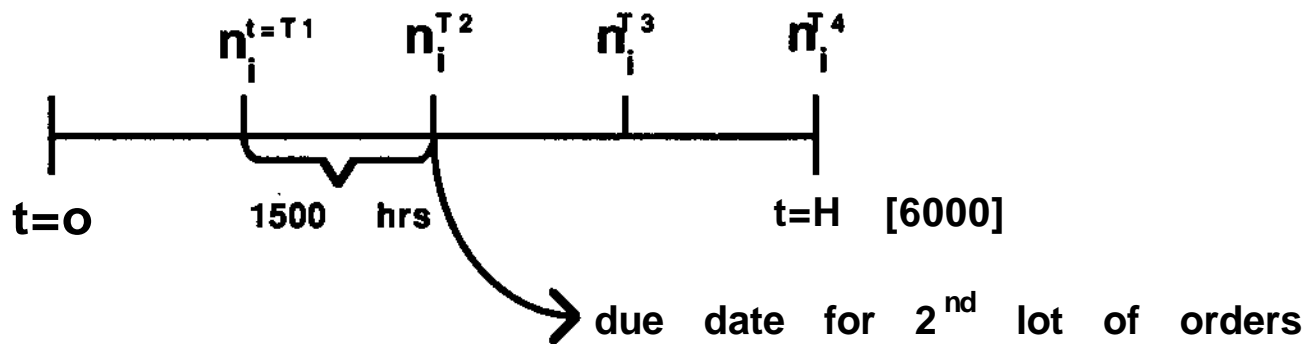
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Production Line 1.



(a)



(b)

Figure 1 : (a) Batch Processing Plant for Example 1.
 (b) Time Horizon and Intervals.

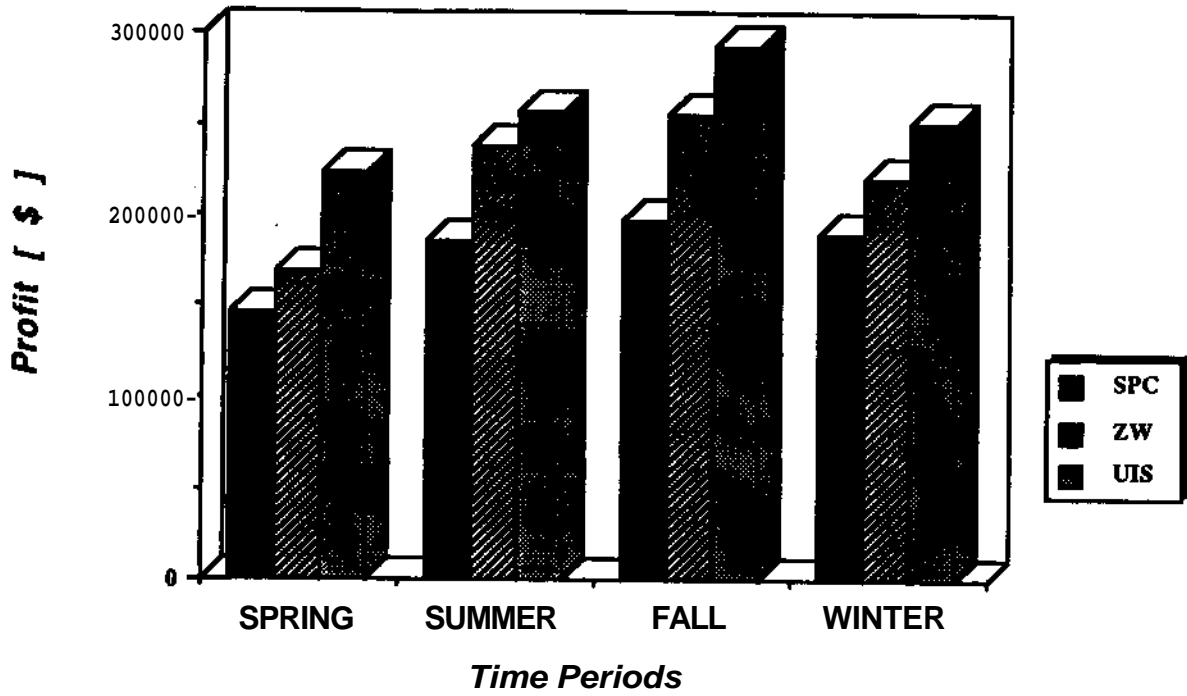


Figure 2: Profit With Various Scheduling Policies (Example 1)

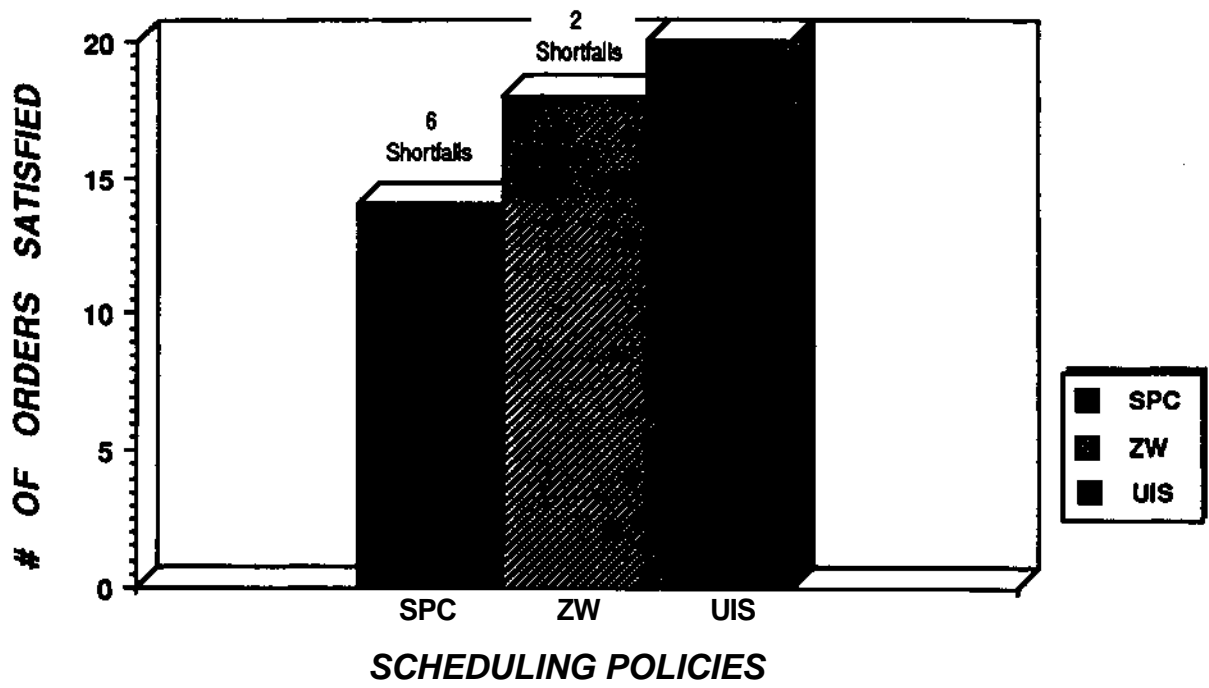
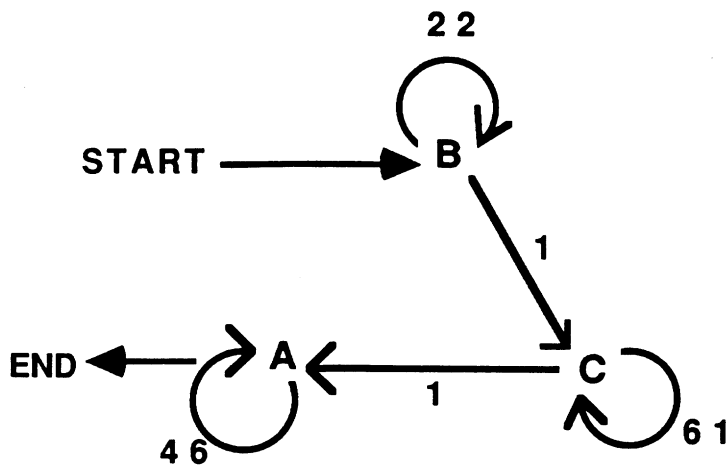
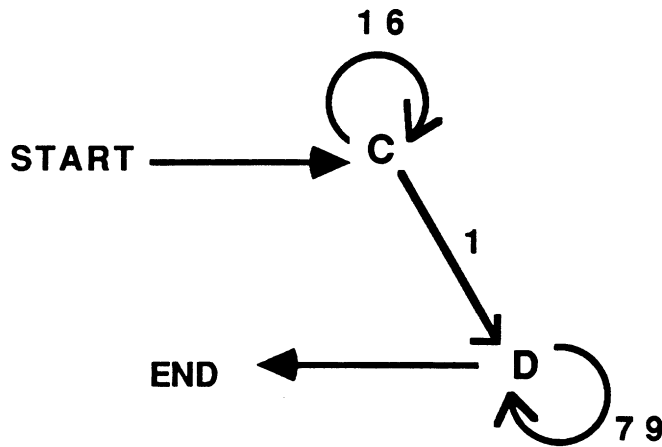


Figure 3 : Shortfalls for Various Scheduling Policies (Example 1)



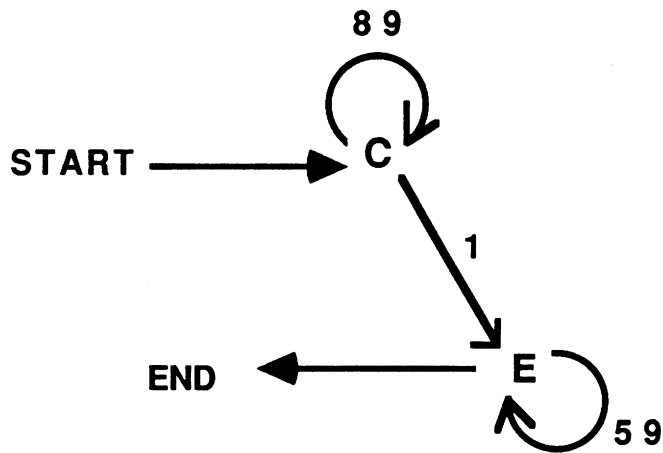
Makespan = 1498 hrs

(a)



Makespan = 1492 hrs

(b)



Makespan = 1516 hrs

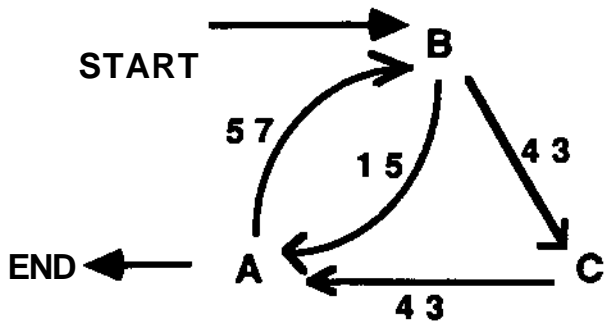
(c)

Figure 4 : Single Product Campaigns for Spring season

(a) Production Line 1

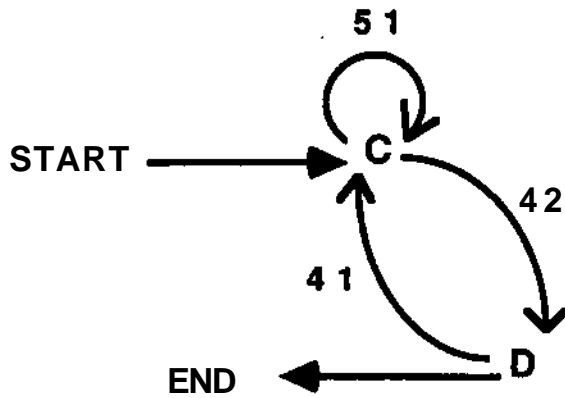
(b) Production Line 2

(c) Production Line 3



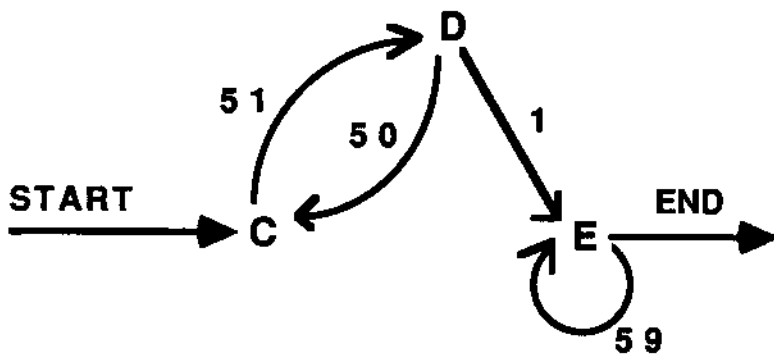
Makespan = 1488 hrs

(a)



Makespan = 1509 hrs

(b)



Makespan = 1503 hrs

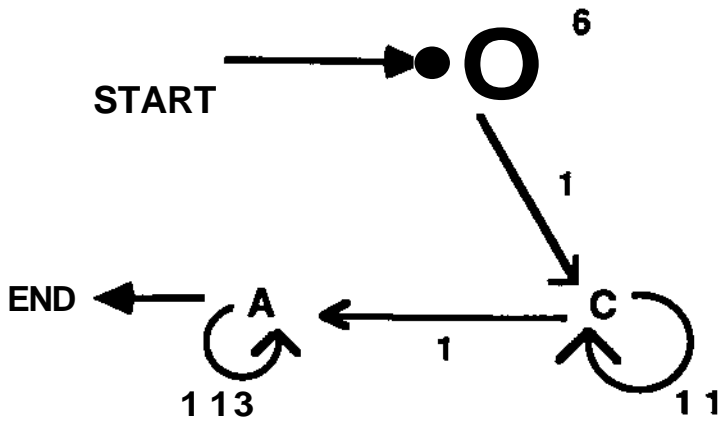
(c)

Figure 5 : MPCs with ZW policy for Spring season

(a) Production Line 1

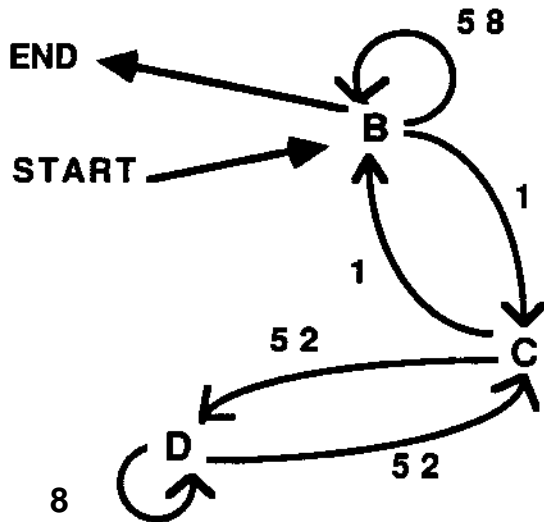
(b) Production Line 2

(c) Production Line 3



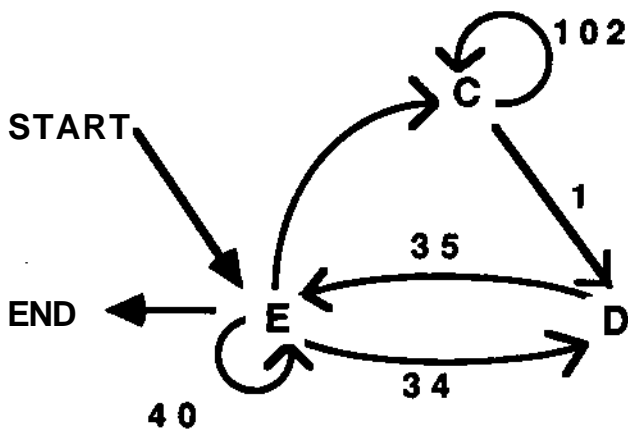
Makespan = 1508 hrs

(a)



Makespan = 1509 hrs

(b)



Makespan = 1507 hrs

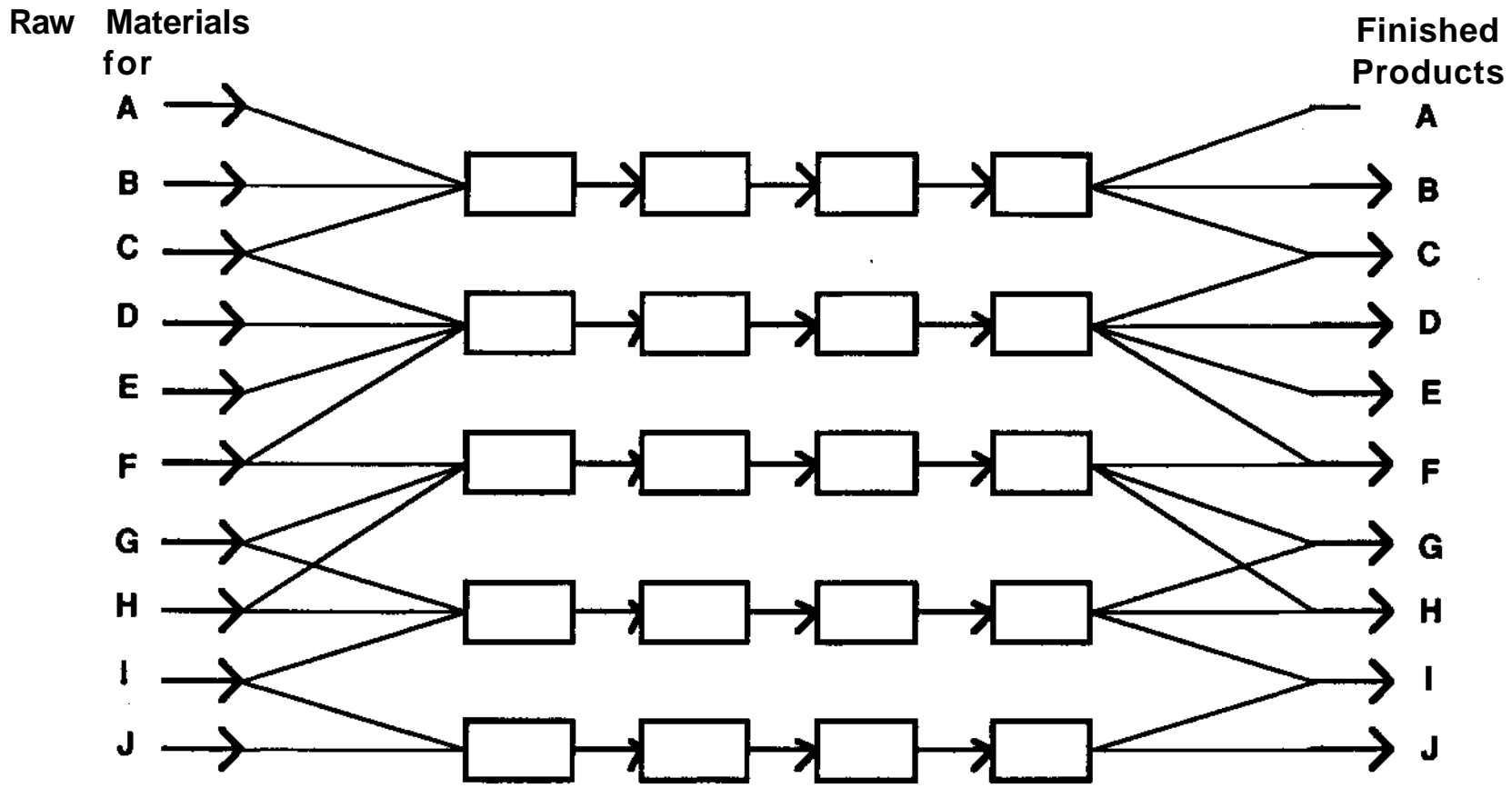
(c)

Figure 6 : MPCs with UIS policy for Spring season

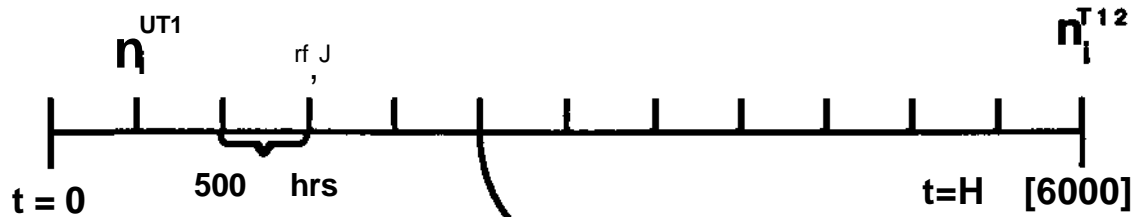
(a) Production Line 1

(b) Production Line 2

(c) Production Line 3



(a)



(b)

due date for 5th lot of orders

Figure 7 : (a) Batch Processing Plant for Example 2.
 (b) Time Horizon and Intervals.

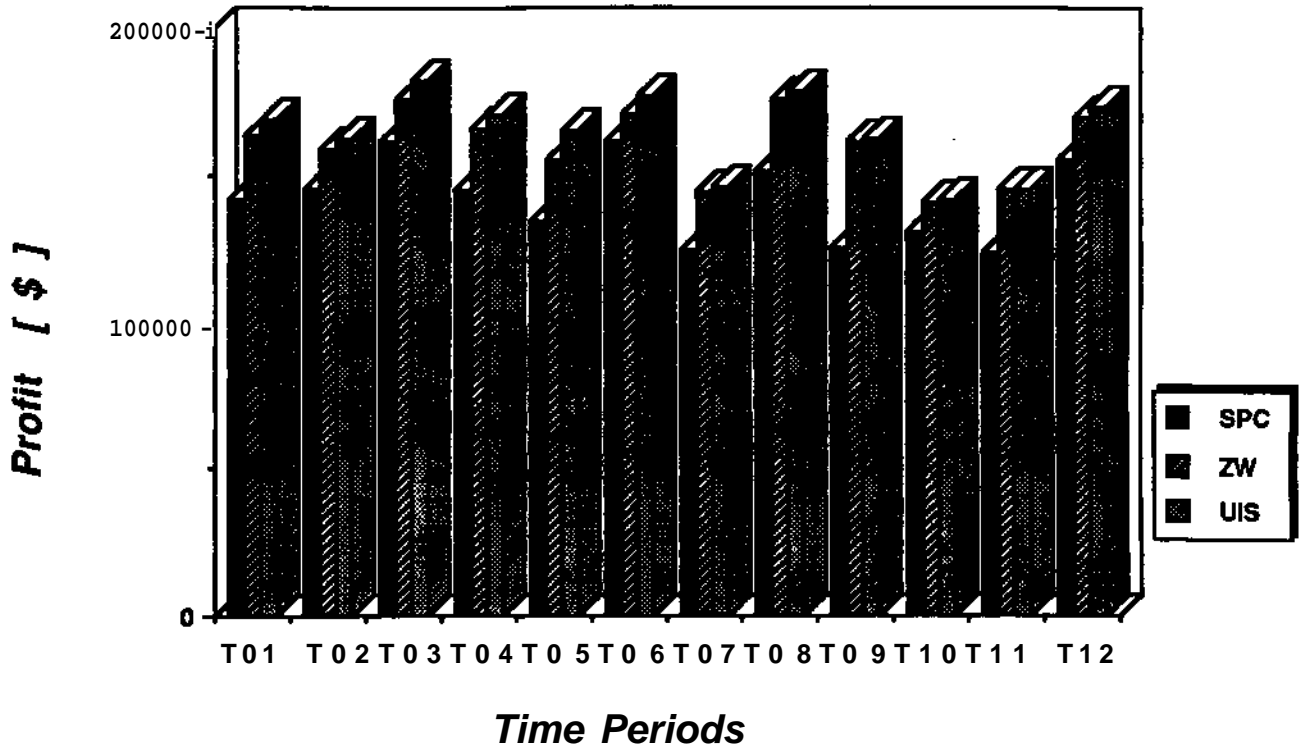


Figure 8 : Profits With Various Scheduling Policies (Example 2)

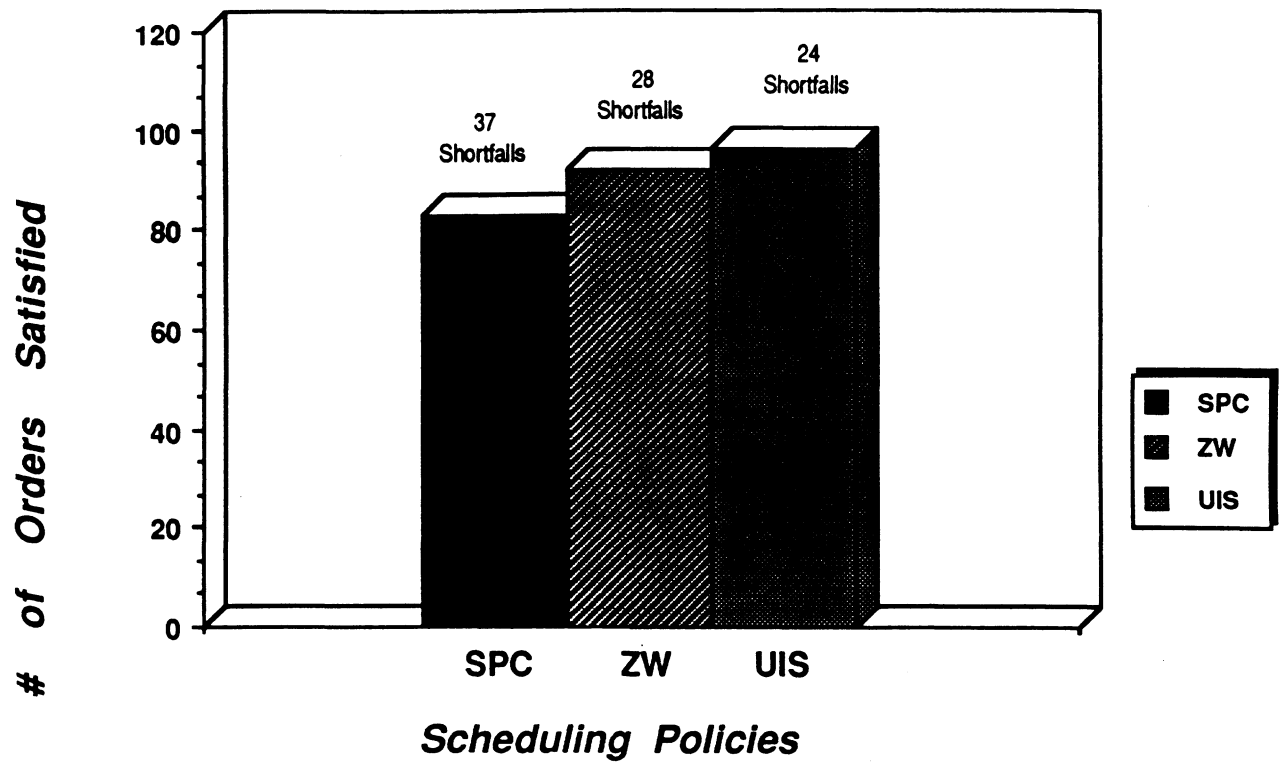
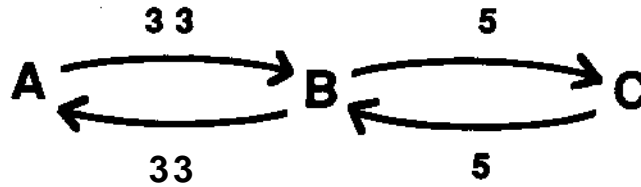


Figure 9 : Shortfall for Various Scheduling Policies (Example 2)



Cycle Time = 489 hrs

Lower Bound on Makespan = 493 hrs

Upper Bound on Makespan = 497 hrs

Figure 10 :MPCs with ZW policy for Spring season :
For Production Line 1

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-

Table 1 : Data for Example 1

(a) Total Horizon Time
(H_t hrs/Time Period)

| | Fall | Winter | Spring | Summer |
|-------|------|--------|--------|--------|
| H_t | 1500 | 1500 | 1500 | 1500 |

(b) Processing Times
($t_{i,j}$ hrs)

| $i \backslash j$ | STG 1 | STG 2 | STG 3 | STG 4 |
|------------------|-------|-------|-------|-------|
| A | 10 | 4 | 10 | 1 |
| B | 3 | 10 | 6 | 12 |
| C | 4 | 12 | 6 | 10 |
| D | 16 | 3 | 8 | 4 |
| E | 7 | 2 | 5 | 3 |

(c) Size Factors
(S_u liters/kg)

| $i \backslash j$ | STG 1 | STG 2 | STG 3 | STG 4 |
|------------------|-------|-------|-------|-------|
| A | 2 | 3 | 2 | 6 |
| B | 7 | 3 | 1 | 2 |
| C | 1 | 4 | 3 | 2 |
| D | 5 | 5 | 2 | 6 |
| E | 1 | 6 | 3 | 2 |

Table 2 : Data for Example 1 (contd)

(a) Orders Booked
 (QS_{it}^L kg)

| $i \backslash t$ | FALL | WINTER | SPRING | SUMMER |
|------------------|--------|--------|--------|--------|
| A | 1,500 | 5,500 | 5,000 | 3,500 |
| B | 2,000 | 7,000 | 5,000 | 4,500 |
| C | 15,000 | 16,500 | 19,000 | 14,500 |
| D | 6,500 | 5,500 | 7,500 | 8,000 |
| E | 4,000 | 4,500 | 5,500 | 7,500 |

(b) Maximum Projected Market Demand
 (QS_{it}^U kg)

| $i \backslash t$ | FALL | WINTER | SPRING | SUMMER |
|------------------|--------|--------|--------|--------|
| A | 6,500 | 6,500 | 7,000 | 5,500 |
| B | 7,000 | 9,000 | 7,500 | 8,500 |
| C | 17,000 | 19,000 | 22,000 | 16,000 |
| D | 7,500 | 6,000 | 8,500 | 8,500 |
| E | 5,000 | 5,000 | 6,500 | 9,000 |

Table 3 : Data for Example 1 (contd)

(a) Profit (P_{it} \$/kg)

| $i \backslash t$ | FALL | WINTER | SPRING | SUMMER |
|------------------|------|--------|--------|--------|
| A | 4.5 | 5.0 | 5.0 | 4.5 |
| B | 4.5 | 5.0 | 4.5 | 4.5 |
| C | 6.0 | 6.5 | 7.0 | 5.5 |
| D | 6.0 | 6.0 | 6.0 | 6.5 |
| E | 4.0 | 4.5 | 4.0 | 5.5 |

(b) Penalty (PN_{it} \$/kg-shortfall)

| $i \backslash t$ | FALL | WINTER | SPRING | SUMMER |
|------------------|------|--------|--------|--------|
| A | 2.25 | 2.5 | 2.5 | 2.25 |
| B | 2.25 | 2.5 | 2.25 | 2.25 |
| C | 3.0 | 3.25 | 3.5 | 2.75 |
| D | 3.0 | 3.0 | 3.0 | 3.25 |
| E | 2.0 | 2.25 | 2.0 | 2.75 |

Table 4 : Comparison of the LP solutions with integer and non-integer number of batches (Example 1)

| Scheduling Policies | Profit with Non-Integer Number of Batches [\$] | Profit with Integer Number of Batches [\$] |
|--|---|---|
| Single Product Campaigns Zero Wait Policy | 724,164.6 | 718,600.6 |
| Mixed Product Campaigns Zero Wait Policy | 890,245.0 | 881,349,8 |
| Mixed Product Campaigns Unlim. Int. Stor. | 1,028,097.7 | 1,022,887.1 |

Table 5 : Results for Example 1.

| Scheduling Policies | Total Profit [1000 \$/yr] | % Profit Increase Over SPCs | % Orders Satisfied | Number of Variables | Number of Equations | CPU Time* [sec] |
|--|---------------------------|-----------------------------|--------------------|---------------------|---------------------|-------------------|
| Single Product Campaigns Zero Wait Policy | 718.60 | --- | 70 | 137 | 153 | 52.1 |
| Mixed Product Campaigns Zero Wait Policy | 881.35 | 22.60 | 90 | 245 | 261 | 109.9 |
| Single Product Campaign Unlim. Int. Stor. | 1022.77 | 42.33 | 100 | 245 | 261 | 94.5 |

* For first solving the relaxed LP and then the LP with rounded number of batches followed by the 12 MILPs for finding globally optimal scheduling solutions for each production line in each time period on SUN 3/60 using ZOOM through modelling system GAMS.

Table 6 : Makespans (hrs) for Globally Optimal Schedules
in Spring Time Period

| Scheduling Policies | Line 1 | Line 2 | Line 3 |
|---|---------------|---------------|---------------|
| Single Product Campaigns Zero Wait Policy | 1498 | 1492 | 1516 |
| Mixed Product Campaigns Zero Wait Policy | 1488 | 1509 | 1503 |
| Single Product Campaigns Unlim. Int. Stor. | 1508 | 1509 | 1507 |

Table 7 : Results for Example 2.

| Scheduling Policies | Total Profit [1000 \$/yr] | % Profit Increase Over SPCs | % Orders Satisfied | Number of Variables | Number of Equations | CPU Time* [min] |
|--|----------------------------------|------------------------------------|---------------------------|----------------------------|----------------------------|--------------------------|
| Single Product Campaigns Zero Wait Policy | 1,700.7 | --- | 69.17 | 769 | 901 | 7.4 |
| Mixed Product Campaigns Zero Wait Policy | 1,926.3 | 13.26 | 76.67 | 1249 | 1417 | 18.5 |
| Single Product Campaign Unlim. Int. Stor. | 1,972.7 | 15.99 | 80.00 | 1249 | 1417 | 15.1 |

* For first solving the relaxed LP and then the problem with rounded number of batches followed on SUN 3/60 using ZOOM through the scheduling system GAMS.

Table 8 : Comparison of the LP solutions with integer and non-integer number of batches (Example 2)

| Scheduling Policies | Profit with Non-Integer Number of Batches [\$] | Profit with Integer Number of Batches [\$] |
|--|---|---|
| Single Product Campaigns Zero Wait Policy | 1,735,823.1 | 1,700,748.6 |
| Mixed Product Campaigns Zero Wait Policy | 1,953,021.0 | 1,926,293.9 |
| Mixed Product Campaigns Unlim. Int. Stor. | 1,983,314.5 | 1,972,711.4 |