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Preliminary ideas in 2D skeletonization

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Abstract

This report describes some preliminary ideas in the automatic generation of the Medial Axis Transform (MAT) from discretized objects. The methods presented arc similar in spirit to thinning algorithms in two and three dimensions described by various authors over the past twenty years. However, it appears that most of the past work was motivated by domains other than design, thus allowing it to remain at the level of a discrete representation. The goal of this woric is to generate continuous skeletons of a truly lower dimension, which can then serve as approximations to the MAT.

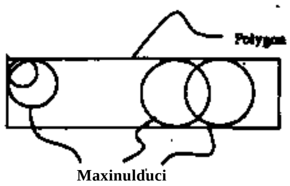


Figure 1: Definition of 2D MAT

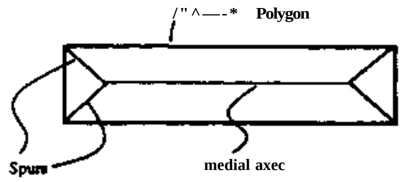


Figure 2: Example of MAT for a simple polygon

1 Introduction

For the purposes of modeling, an object will be regarded as a compact[1] in space. A compact can be represented as the union of some finite number of open sets (by definition), or by representing its boundary. Most geometric modelers conform to this. However, there is an equivalent way of representing compacts in space, called the Medial Axis Transform (MAT), defined first in [Blum]. By equivalent we mean that *any* compact in space can be represented using the MAT.

For a two-dimensional (2D) object, the medial axes are the locus of the centers of maximal discs which can be fit into the object. Figure 1 shows how to determine medial axes by the fitting of maximal discs into a polygon. A radius function is defined at each point on the medial axes, giving the minimal distance from each point on the medial axes to the original object boundary. Figure 2 shows a polygon and its medial axes.

The MAT definition is similarly extended to three-dimensional objects. The MAT then consists of a set of medial surfaces (instead of axes) and a radius function. The medial surfaces are identified from the boundary of the solid using a set 6f maximal spheres (instead of discs). Figure 3 shows a rectangular plate, and it's MAT, which consists of four triangles, eight trapezia, and a rectangle.

The MAT of an object facilitates analyses in several domains. In particular, such a representation is very useful in automating manufacturability analysis for net-shape manufacturing of thin-walled parts, as in injection molding and die-casting [Hall]. Briefly, many numerical analysis packages take advantage of the plate-like nature of such parts, and simplify the analysis by assuming laminar flow. With this assumption, it is now possible to approximate the pan by a collection of so-called 2^D patches, with associated thickness information (see figure 4). It is possible to use portions ot the MAT as the elements of such a 2|D representation. The MAT can also play a very important role in extracting shape features useful in a knowledge-based approach to moldability analysis (see [Hall], [Corney].)

Despite this, it is neither convenient nor easy to use the MAT as the primary representation of

[1]A compact is a standard concept in topology. Refer to a text such as [Noll].

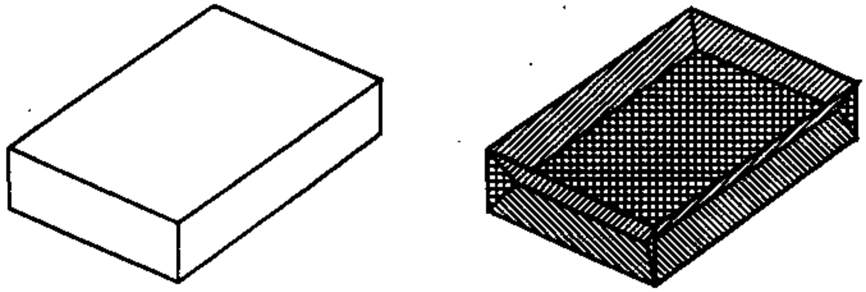


Figure 3: A rectangular plate and the 3D MAT

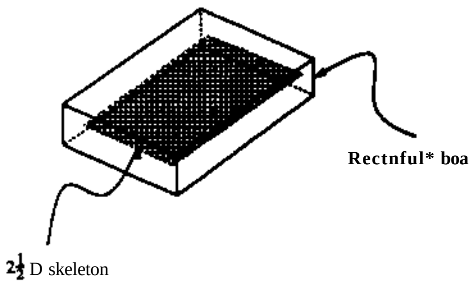


Figure 4: A rectangular plate and it's 2.5D skeleton.

a design. Firstly, any form-function relationships designers might use to create the design would be in terms of the spatial properties of the point-set to be represented, and, in general, it is not easy to visualize this from the MAT. Secondly, in manufacturing processes where the surface of the design must be created by shaping or removing material, a representation which directly yields this information must surely be preferred.

Since the MAT is potentially of use in manufacturability analysis, and yet is not necessarily a good choice for the primary representation of a design, we feel that an abstraction process which can *derive* the MAT of an object from a conventional representation would greatly enhance the power of any design-support system. Unfortunately, this has proved to be very difficult. Many people have investigated the problem, and although success has been reported for two-dimensional objects (see [Gursoy],) the three-dimensional case has proved much harder. Dutta and Hoffman have reported some preliminary work in this area (see [Dutta], [Hoffman]), but it is not clear whether their approach, while sophisticated, is powerful enough to yield a closed form solution to the problem.

We expect, however, that the MAT of a designed object will be used mainly in analyses of the sort mentioned above. We feel that accuracy is not crucial for such purposes (see [Yu]). It is our goal to develop a simple, robust technique of generating *approximate* MATs of solids. We expect to be able to show that the process is convergent, although only to a subset of the exact MAT.

2 Digital Thinning

The process of obtaining skeletal objects presented here is related to an approach called digit.il thinning. The process begins with a digital, or discrete representation of an object. The modeling

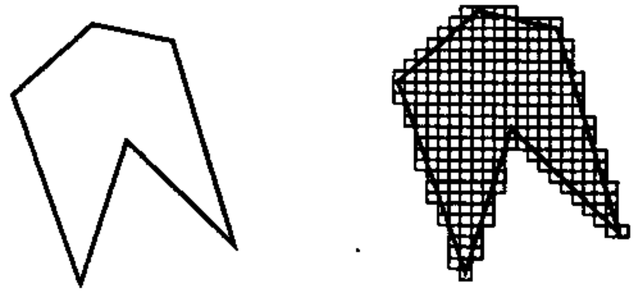


Figure 5: (a) A polygon, and (b) The discrete representation obtained by "filling-in" squares which are paritally or wholly in the polygon

(a) (b) (c)

Figure 6: The 2D thinning process, (a) The discrete object, (b) First set of boundary pixels (shown dotted) and (c) The "skeletal" pixels shown shaded (the object is shown for reference)

space is Z\ where *n* is the dimension of the space. Thus, the elements of the representation are points on a lattice which spans space. The object, then, is a subset of Z\ The objective of thinning is to reduce the number of points in the object, while preserving its topology and shape. Topology here is understood to mean the number (and location) of connected components and cavities, as also the number of handles or tunnels for solids. Shape is understood to be preserved by preserving topology, and by ensuring that end-points of arcs and'edges of plate-like components of the skeleton (3D) are not deleted. The literature in digital thinning is vasr, for a good overview of the field, the reader is referred to [Kong 89].

Figure 5 illustrates one way of obtaining a discrete representation from a continuous one. Figure 6(b) shows the so-called simple points (see [Morgenthalef]) dotted. Iteratively, such simple points are identified and removed. Depending on the specific choices made in defining the thinning process, a possible outcome could be the object shown in figure 6(c).

Thus, the objective of the thinning process has traditionally been set at reducing the discrete object to a set of points (pixels/voxels) which have thickness as close to unity as possible. An additional consideration has been the preservation of components of the skeleton, so that they do not get shortened.

From the design perspective, there is one fundamental shortcoming in such algorithms. These

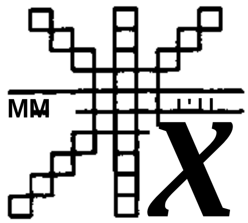


Figure 7: An example of a 2D thinned object which remains too dense to allow a continuous skeleton to be fit to it.

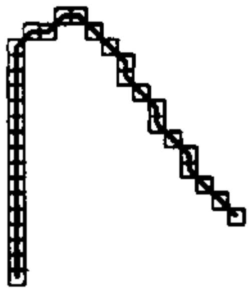


Figure 8: A continuous interpretation of the skeleton

algorithms consider the job to be complete when all points are either a single layer thick (so to speak), or no points can be deleted without changing the topological properties of the object. Note, however, that these are still points in a "discrete" space. If these points are regarded as being the centers of unit cells in 2D/3D (this is a common way of visualizing the object in /?*), there is no reduction in the dimension; and if they are regarded as points on a lattice, there is no continuous skeleton to be inferred in their distribution.

The problem of inferring a continuous line/surface on the borders of discrete objects has been addressed at some length in the literature (see [Kong 85] [Morgenthaler]). However, these papers describe methods for inferring continuous surfaces onto boundaries which are assumed to be 1-manifold and 2-manifold, for 2D and 3D models respectively. In general, a skeleton will not satisfy these assumptions. Furthermore, specific instances of objects can be found where no point can be removed while thinning (since that would change the topology of the object,) yet the points are so densely packed that no skeletal interpretation (in the continuous sense just mentioned) can be found. Figure 7 shows a 2D example.

The methods to be described here are intended to yield skeletons which are always of a lower dimensionality than the original model. Indeed, this shall be the criterion for terminating: while any of the original elements (pixels or voxels) remain, the process cannot be complete. Intuitively, in the 2D example shown in the figures so far, the process should yield a continuous skeleton as in figure 8, instead of that in figure 6(c).

3 Problem Statement

The goal of skeletonization (at least for use in engineering design/analysis) should be to produce an entity of lower dimensionality than the original model. The chief difficulty appears to be that in

Figure 9: The Delaunay diagram (shown solid) and its Voronoi dual (shown dotted)

Figure 10: A "diagonal" edge is introduced in the Voronoi diagram (shown dotted) to reflect 8-connectivity

order to maintain the topology of the object, it is necessary to keep certain points in the object (i.e., not delete them.) Sometimes (as in figure 7) this leads to incidental adjacencies, which obscure the nature of the skeleton. It then becomes necessary, in our view, to decouple the twin goals of thinning and topology preservation. If such a decoupling can be achieved, it might then be possible for thinning to proceed without getting deadlocked, and for the skeleton of the object to emerge in the data structure used for maintaining topology.

Since the desired skeleton is a continuous one, we propose to use a data structure representing a continuum to model the adjacencies between points. Thus, it is hoped, the skeleton will be found to reside in this data structure when the thinning process is complete.

4 Skeletonization in Two Dimensions

For the rest of this paper, our terminology will be consistent with that of [Kong 89]. To aid the development of the new ideas to be presented, we shall use terms from graph theory, specifically the notions *graph dual, Delaunay diagrams,* and *Voronoi diagrams.* Our ideas shall first be presented in two-dimensional space. Throughout this section, we assume the object to be 8-connected, and the complement to be 4-connected.

We shall consider objects to represent unit squares, located in R^n such that the center of the square is located at coordinates given by the integer tuple representing the point in Z^n. When thus interpreted, the diagram of the object is referred to as its *Delaunay*[1] diagram. The vertices, edges, and faces of this diagram are called Delaunay vertices, edges, and faces. The *Voronoi* diagram of the object is *defined* to contain those components of the *dual* graph which is *contained* in the Delaunay diagram. Figure 9 shows our Delaunay diagram for an object, and its Voronoi diagram.

Since the object is required to be 8-connected, we extend the usual definition of a Voronoi dual graph to include a diagonal edge to reflect the connectivity between two vertex-neighboring pixels when both their common edge-neighbors are absent. In a sense, the Delaunay vertex shared by the two points is regarded as an edge having infinitesimal length.

The proposed technique operates by taking the Voronoi dual of the Delaunay diagram representing the object. Faces of the Voronoi diagram are also unit squares, and can be regarded as a thinner version of the same object; it is then possible to take the Voronoi dual of this diagram.

[2]Graph theorists with a choleric disposition are advised to read further under sedation.

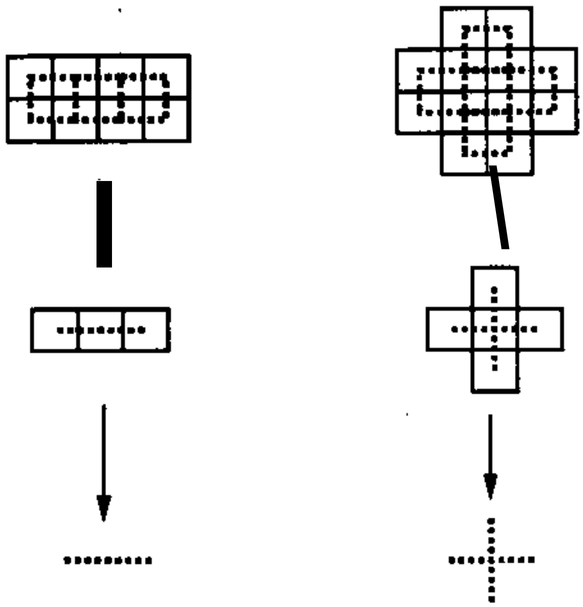


Figure 11: The thinning process for **two** objects

The procedure is thus repeated until no unit squares can be identified in the diagram. Figure 1 l(a) illustrates this process **for** two simple objects. The *skeleton* **of** the initial object must then include **the Voronoi edges and** vertices (found at some iteration in the above process.) This is illustrated in **figure** 1 **l(b) for the objects** shown in figure 11.

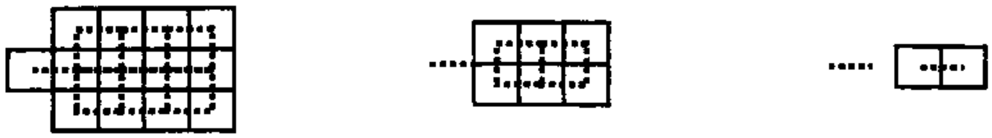
4.1 Connectivity Preservation

We believe that this technique can be shown to always yield skeletons which are thin in the sense **of a genuine reduction** in dimension. It is tempting, then, to define a thinning process purely on this basis. Unfortunately, however, the process does not preserve topology. This is illustrated in the thinning **of** the simple object of figure 12. Hence it is clear that additional edges must be **introduced to maintain the topology.**

Note, however, that those edges of **the** Voronoi dual which *do not* border a face contribute no **further to the process of** thinning. They are skeletal, and we propose to capture them in a separate **representation. Then, we** feel, additional edges can be introduced in this representation to maintain **connectivity. We next** propose a procedure for maintaining connectivity.

Consider the middle diagram of figure 12(a). This marks the stage after which the skeleton would get disconnected. Note that at this stage, the skeleton is connected to a Delaunay vertex. We identify two possible ways in which such a minimal connection can occur.between the skeleton **(as** identified till some stage in thinning) and the Delaunay diagram of the rest of the object. These two possibilities **are** shown in figure 13.

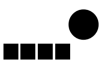
To handle the first situation, we propose to add an edge to the skeleton from the center of the pixel to the Delaunay vertex shared by the skeleton and the rest of the object. For the second case. we add an edge from the Delaunay vertex to the midpoint of the (as yet unformed) edge joining the dual vertices corresponding to the two pixels. Note that this edge is not yet formed, but *must*



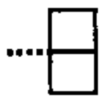
(a)

(b)

Figure 12: The process of generating "skeletal" edges causes the topology to be changed, (a) The thinning process, (b) The (disconnected) skeleton



(a)

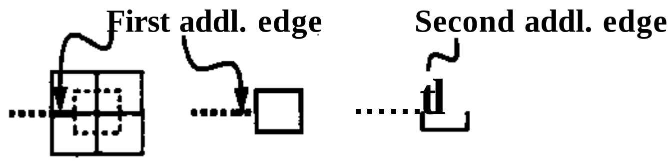


(b)

Figure 13: Minimal connection between the skeleton and the Delaunay diagram of the rest of the object.

(a) (b)

Figure 14: Additional edges (shown bold) are introduced to maintain connectivity of the skeleton



(a) (b) (c)

Figure 15: Third possible minimal contact case, and its solution

get formed in the next stage. Both these cases are shown in figure 14.

Case (b) in figure 14 introduces a third possible "minimal" contact between the skeleton and the rest of the object. If the (unformed) edge between the pixels of figure 13(b) in fact adjoins a Voronoi face after the next iteration, as shown in figure 15(a), then another type of minimal contact comes into play: that shown in figure 15(b). To handle this, we propose to add an edge between the midpoint of the edge in question and the Voronoi vertex which is the dual (at the *next* stage) of the Voronoi face of *this* stage. This is shown in figure 15(c).

As an example, figure 16 shows how the "deadlocked" object of figure 7 would get resolved by our technique.

4.2 Convergence to MAT

There is another, more subtle drawback to this new thinning technique. It is found that the skeleton it yields is not symmetric with respect to the boundaries of the object, and thus does not converge to the MAT. Figure 17 illustrates this. Note that the "legs" are at 45° to the vertical. In the MAT, this angle would be smaller. The problem is that the proposed thinning technique always generates skeletal edges aligned with one of the axes, or at 45* to both. However, the skeleton would nevertheless be contained in the original Delaunay diagram of the object. Depending on the

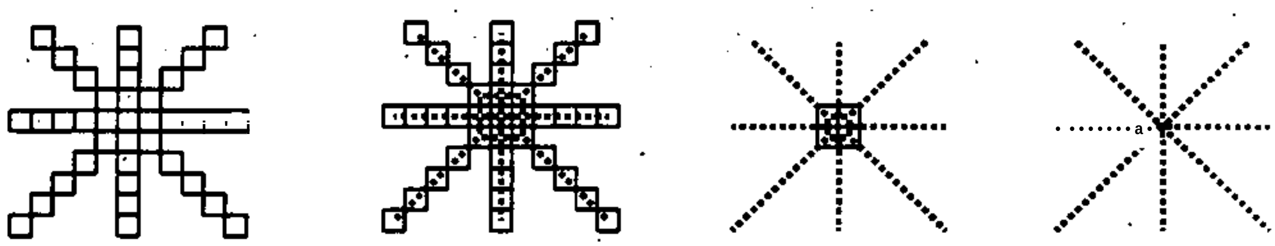


Figure 16: A digital asterisk and the skeletal representation.

application in which the skeleton is to be used, this non-convergence may not be a concern. If it *is* a concern, we propose to carry out thinning in two stages: in the first, thinning proceeds by the traditional techniques (see [Kong 89]). After this is done, the methods proposed here should be used to yield a continuous skeleton of lower dimension.

5 Future Work

The next goal for this work is to describe the proposed process formally, and attempt to prove the correctness of the ideas presented so far. We wish to show that the suggested procedure will always produce continuous skeletons of lower dimension, and that these skeletons are topologically equivalent to the object (i.e., they have the same number (and approximate location) of connected components and cavities as the original object.

Once this has been achieved, we shall attempt to extend our ideas to three dimensions. The portion of our technique concerned only with thinning the object extends immediately to three dimensions. Components of the skeleton contributed directly by the adjacency of *voxels* can be inferred by taking graph duals of Delaunay edges and (where no suitable edge is present), of Delaunay vertices. How to maintain connectivity, while also maintaining the number of cavities and handles (tunnels), is the focus of our current research. This shall be the subject of a forthcoming paper.

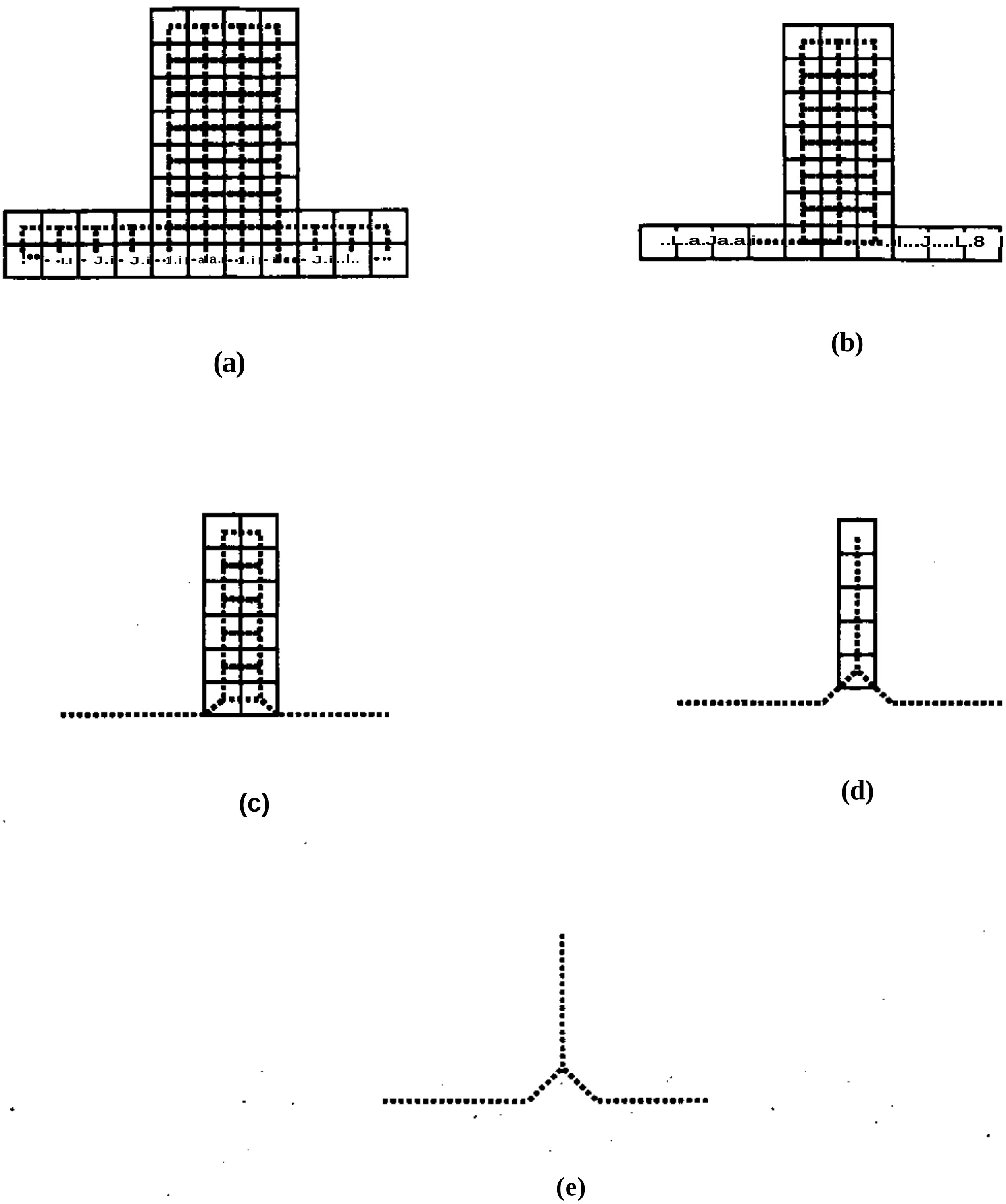


Figure 17: The edges meeting at the junction are at 45 deg, and thus not convergent to the MAT

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