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A CONSTRAINED PSEUDO-NEWTON CONTROL STRATEGY FOR NONLINEAR SYSTEMS

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Abstract

The duality of control theory and the theory on the solution of operator equations has been exploited to develop a nonlinear control law. The control law is developed by analogy to Newton's method and several related properties are shown. To handle the state and control variable constraints, the method is generalized to include a Successive Quadratic Programming (SQP) algorithm. Stability properties for this algorithm have been studied. It was found that with large enough but finite sampling time, systems that are open-loop asymptotically stable in the large can approach the setpoint monotonically. Simulation results of two example problems demonstrate the effectiveness of the proposed strategy.

1. Introduction

While linear control theory has been used in virtually all process control applications, the nonlinear character of many chemical processes is becoming increasingly appreciated. However, until recently, nonlinear process control has seen relatively few results. In fact, most of the current work relating to nonlinear processes can be divided into two categories: transformation methods and optimization methods. The former approach stems from transforming process models with inherent nonlinearities to those with linear structures. An empirical approach to these transformations was described by Georgakis (1986) using extensive variables. Later, using the global transformation results of Hunt, Su and Meyer (1983), Hoo and Kantor (1986) described a number of applications for which at least partial transformations can be made to reduce the nonlinear problem to linear form. A very readable account of this approach to nonlinear control, as well as its limitations, is given by Kantor (1987).

Optimization methods by themselves are not new. Here the on-line solution of state-space variational problems is considered using state-of-the-art nonlinear programming algorithms. This approach has been advanced by Sargent and coworkers and has seen considerable recent development due to simultaneous approaches to solving the differential equation model together with the optimization problem (see Biegler, 1984; Renfro et al., 1986; Eaton and Rawlings.1988). Unlike linear approaches and transformation approaches, optimization based approaches currently lack the familiar theoretical background of stability analysis and robustness. Nevertheless, this approach is very general and has yielded excellent performance.

Given the limited class of nonlinear models for which these transformations can be applied as well as the theoretical limitations of the optimization-based approaches, we consider instead an approach for nonlinear process control based on operator theory. Here the classes of nonlinear models that can be considered is unlimited and not restricted by size or other properties such as involutivity (see Kantor, 1987). Moreover, the duality between control theory and the theory on the solution of operator equations

has been observed previously by a number of researchers (e.g. Astrom and Wittenmark, 1984) and has been used to establish strong quantitative results. For instance, Kalman and Bertram (1960) were first to use contraction principle arguments to study the stability of autonomous discrete systems. Zames(1966) used the same principles to derive the so-called small gain, circle and conicity stability conditions for continuous input-output systems. Later, the singular value decomposition method, originally introduced in the study of the sensitivity of inverting linear operators, was employed by Doyle and Stein (1981), and Lehtomaki (1981) to establish a theory on the robustness of linear feedback structures.

However, practically all these results are confined to analysis issues, such as stability and robustness. The implications for synthesis and design still require further study.

To address these issues, Economou et al (1986) formulate the controller design problem as an operator equation. Both analysis and synthesis aspects are thus benefitted from a relatively well developed theory on the solution of operator equations. Here, controllers induced by Newton's method are also introduced and the corresponding stability characteristics are studied. This paper extends these concepts by taking advantage of the local and global convergence characteristics of strategies based on Newton's method. For example, it is well known that Newton's method has a fast rate of convergence in some neighborhood of the solution. Perhaps less well known is a global convergence property of Newton's method which can be guaranteed by choosing a suitable stepsize along the Newton search direction. Similar arguments can thus be made regarding stability (i.e. convergence) properties of nonlinear control algorithms.

In addition to developing these operator properties we also exploit the extension of this nonlinear algorithm to constrained, nonlinear process models. Except with optimization based approaches, control and state constraints are frequently overlooked in the development of control laws, especially where nonlinear systems are involved. In fact, the only widely used control algorithm which successfully handles general process constraints is Dynamic Matrix Control (DMC) (Cutler and Ramaker, 1979, Prett and Gillette, 1979) and its second generation development, Quadratic Dynamic Matrix Control (QDMC) (Garcia and Morshedi, 1984). Both are model- predictive controllers, which deal only with linear descriptions of the process (and its constraints) and solve linearly constrained optimization problems on-line to produce new controller actions. In this paper we also an approach similar to optimization based approaches, and apply constraints directly to the nonlinear system.

To develop the topics introduced above, we first introduce some basic notation and concepts in the next section. In section 3, we derive the pseudo-Newton control law following the derivation of the well-known Newton method. Stability theorems for open-loop systems in state space are introduced in section 4 together with stability of closed-loop systems under an assumption of a perfect process model. Furthermore, the relationship between sampling time and closed-loop stability is analyzed to establish global convergence criteria for the pseudo-Newton control law. Finally, we extend the pseudo-Newton control law to its optimization-based analog by considering a fully converged model based controller. Here the stability analysis developed above can also be extended for this case.

Another objective of this paper is to present a straightforward nonlinear, constrained controller design procedure. Thus, the nonlinear control law is transformed to an optimization problem in section 5, and is solved using a variant of the Successive Quadratic Programming (SQP) algorithm. The advantage of this approach is that it can directly deal with general nonlinear models with process constraints. To demonstrate the effectiveness of the strategy, two example problems are analyzed and simulated in section 6. These results clearly illustrate the potential of the proposed algorithm. The concluding section summarizes the major points in the paper and suggests future work.

2. Preliminaries

In order to deal with nonlinear systems in a general manner, it is necessary to establish a mathematical framework to describe some basic concepts. The autonomous lumped parameter Multiple-Input-Multiple-Output (MIMO) nonlinear system studied in this paper can be modeled by a set of ordinary differential equations (ODEs). The vector form is as follows:

$$\S = M\text{«}(0) \qquad (2.1)$$

where x € R^n is the state of the system, and for every t € (0, «), u(t) € R^m is the input, with the corresponding output map (y *e* R^p) and p ≤ m:

$$y = g(x) \qquad (22)$$

In this study, several assumptions are made: 1) The solutions of the system (2.1) exist and are unique. 2) The model is perfect. 3) The system inputs are piecewise constant functions. These assumptions are made to make the controller design problem finite dimensional. In the discrete-time formulation the s^m sampling interval extends from f to t^{8*1}. T « t^{8*1} - t^8 is the constant sampling time; x^8 is the state at t^8; u^8 is the system input held constant over (t^8, t^{8*1}).

In the discrete setting of the study, xO^{1*1}; ^.x^8^^{8}) is the solution of equation (2.1) at time t^{8*1} for u(t)« u^8 (t^8 < t < t^{8*1}) and initial condition x(t^s; t^s,x^8,u^8)« x^8; x^8 will denote the state of the system at t - t^.i.e.x^{8*1}:

$$\chi^s \blacksquare x^{s+1} = \chi(t^s+T;\ t^s,x^s,u^s) \qquad (23)$$

Since (2.1) is autonomous, f does not depend on t explicitly, i.e., xtt^{8*} T; f.x.u) « $\chi(t^{s+1}+T;$ t^.x.u), time will be dropped from the parameter list and the following convention will be used:

$$X^s \;=\; x(T\!\!>\ **\!»'\!*) \;=\; X(\wedge\!+r;\ Pjfjf) \qquad (2.4)$$

The derivatives of x^s with respect to x^s and u^s will be defined as follows:

$$\Phi^s \;=\; \frac{\partial \chi^s}{\partial x^s} \qquad (2.5)$$

$$\Gamma^s \;=\; \frac{\partial \chi^s}{\ } \qquad (2.6)$$

$y*+^1«\ g(x^{s*1})$ is the system output at f^{+1} , The derivative of y^{5*1} with respect to x^{6*1} will be defined by:

$$C^s \quad m \quad \frac{zl}{\partial \chi^s} \;=\; \frac{Z2\pm l}{\partial \chi^s} \qquad (2.7)$$

The sensitivity matrices $4>^s$ and r^ are obtained from solution of two sets of linear time-variant ODE's(Economou, 1985):

$$\frac{\partial \Phi(t)}{\partial t} \;=\; \frac{\partial f(\zeta,\ \xi)}{\partial \zeta}\Big|_{\zeta=\chi(t;x^s,u^s),\ \xi=u^s}\ \Phi(t) \qquad (2.8a)$$

initial condition: $\quad *(fO \;=\; /$ (2.86)

$$\Phi^s \;=\; \Phi(t^s+T) \qquad (2.8c)$$

$$\frac{anO}{a<} \;=\; \frac{3AC\ \ 0}{ac}\Big|_{\zeta=\chi(t;x^s,u^s),\ \xi=u^s}\ r(0 \;+\; \frac{2f\wedge\ \xi)}{\partial \zeta}\Big|_{\zeta=\chi(t;x^s,u^s),\ \xi=u^s} \qquad (2\text{•}9a)$$

fm'ttd/ *condition:* $\quad T(f) \;=\; 0$ **(2.96)**

$$\mathbf{P} \;« \;\mathbf{r(r»+7)} \qquad (2.9c)$$

For a linear system:

$$\frac{\wedge}{dt} \;=\; Ax+Bu \qquad \mathbf{(2.10a)}$$

$$\mathbf{y} \;« \;Cx \qquad \mathbf{(2.106)}$$

where A ,B ,C are constant matrices. 4> and r can be evaluated explicitly:

$$\Phi = exp(AT) \tag{2.11}$$

$$\Gamma = [exp(AT)-I]A^{-1}B \tag{2.12}$$

If the system (2.10a) is open loop stable i.e. all eigenvalues of the matrix A are in the open left half plane, then <t>-»0, $T->-A^{-x}B$ *for* $T \rightarrow \infty$.

3. Pseudo-Newton Control Law

Before deriving the pseudo-Newton control law we briefly introduce a few related concepts.

System Operator Under the existence and uniqueness assumptions, systems governed by (2.1) generate a well defined operator N, which maps states x^s at the beginning of a sampling interval t* and inputs u^s constant over that sampling interval, to states x^{s*1}« xO"»^{xS}.u^s) and outputs y^{8*1}« $g(x^{s+1})$ at t^{8*1}.

$$(x,\ u) \in R^n x R^m \xrightarrow{N} (x,\ y) \in R^n x R^m \tag{3.1}$$

Control Objective In this study, the basic control objective, which is used to formulate the control problem as an operator equation problem, is to drive the system output y^{s+1} to a desired level y*,i.e., y^{*+1} - $y^{\#}$. In general we may require y^{s+k} = y\ where k is a fixed number of forward steps.

Control Operator Equations The operator equation corresponding to this control objective is:

$$[0\ IJNQc.\ ii) * JVjC*.\ ii) = I \tag{3.2}$$

where I_m is the m x m identity matrix.

Control law computations to achieve the objective can be based on iterative algorithms for the solution of (3.2). Potential gains of this approach stem from a well developed theory on algorithms for the solution of operator equations, especially in the case of control theory for nonlinear systems.

In order to simplify the notation of a pseudo-Newton control law, we assume dim(u) - dim (y) * m i.e. system (2.1) is square. Let us postulate that the control objective is met at time s+2 i.e.

$$y^{s+2} - I = g(x^{s+2}) - y^* = g(\chi(x^{s+1}, u^{s+1})) - I - 0. \tag{3.3}$$

and expand this expression in a Taylor series around the state x « x^s, u • u^6:

$$0 = g(\chi(T;x^{s+1},u^{s+1})) - y^* = g(\chi(T;x^s,u^s)) - y^*$$
$$+\frac{\partial g(\zeta)}{\partial \zeta} \qquad \partial \chi(T;x^s,u^s)$$
$$\frac{Jg(O}{\wedge 5^{\zeta}}|_{\zeta=\chi(T;x^s,u^s)} \frac{\partial \chi(T;x^s,u^s)}{\partial u^s}(u^{s+1}-u^s)$$
$$+O[\|(x^{s+1}-x^s,\ u^{s+1}-u^s)^T\|^2] \qquad (3.4)$$

In the context of the Newton methods, in order to compute u^{s+1} that solves (3.4) to first order, the higher order terms in (3.4) are set to zero. Furthermore, x^{s+1} is substituted from (2.3) and the notation of section 2 is introduced, yielding:

$$0 = y^{s+1}-y^*+C^s\Phi^s(\chi^s-x^s)+(C^s\Gamma^s)(u^{s+1}-u^s) \qquad (3.5)$$

Solving (3.5) for u^{s+1}, the following control law is obtained:

$$u^{s+1} = u^s+(C^s\Gamma^s)^{-1}[(C^s\Phi^s(x^s-\chi^s)+(y^{\#}-y^{s+}\ )] \qquad (3.6)$$

The algorithm is called a pseudo-Newton control law, because it is similar to a Newton method. However, it is not identical to Newton's method; with Newton's method #*[1] is specified by the solution of an equation of the form (3.1), while in the above derivation it was set equal to xCHx[8]^[8]) to conform with the evolution of the system states. The stability of this control law will be discussed in the next section.

For the linear system (2.10), the control law of (3.6) becomes:

$$u^{s+1} = u^s+[C(exp(AT)-I)A^{-1}B]^{-1}$$
$$\bullet[Cexp(AT)(x^s-\chi^s)+y^*-y^{s+1}] \qquad (3.7)$$

The control law (3.7) is an output deadbeat controller that drives the system output to y* within one sampling interval. The proof can be found in the Appendix A. Its properties are well studied (Kuo,1980; Franklin and Powell, 1981).

Moreover, by including a relaxation factor X in the control law, equations (3.6) and (3.7) become:

$$u^{s+1} = u^s+\lambda(C^s\Gamma^s)^{-1}[(C^s\Phi^s(x^s-\chi^s)+(y^*-y^{s+1})] \quad (3.8)$$

$$u^{s+1} = u^s+\lambda[C(exp(AT)-I)A^{-1}B]^{-1} \bullet[Cexp(AT)(x^s-\chi^s)+y^*-y^{s+1}] \quad (3.9)$$

The relaxation factor λ not only changes the shape and speed of the system response, but also expands the region of stability of the closed-loop system, as we will discuss in the next section.

Control Algorithm Summary

0. Set s = 0; initialize model state x^s, e.g. based on process measurements; implement u^s
1. Compute χ^s from the model (2.1) and compute Φ^s,Γ^s, C^s from (2.7) to (2.9).
2. Measure y^{s+1} and set $\lambda = 1$.
3. Set $u^{s+1} = u^s+\lambda(C^s\Gamma^s)^{-1}[C^s\Phi^s(x^s-\chi^s)+(y^*-y^{s+1})]$

   a)Evaluate $y^{s+2} = g(\chi(x^{s+1}, u^{s+1}))$

   b)*If* $\|y^{s+2}-y^*\| - \|y^{s+1}-y^*\| \leq -\varepsilon\lambda$ (Armijo Inequality, see LI and Biegler (1988))

   for some small $\varepsilon > 0$, go to step 4

   Else set $\lambda = \gamma\lambda$, where $\gamma \in (0,1)$ and is found by quadratic interpolation

   go to step 3.

4) s = s+1, go to step 1.

For the algorithm to be successful, a nonzero λ must be found that satisfies the inequality in step 3.b. In section 4 below, we will show that for sufficient large but finite sampling times that this relation will always be satisfied and the algorithm converges to its setpoint. The extension of the pseudo-Newton control law to handle the process constraints will be discussed in section 5.

Computational Method for Sensitivity Functions

The pseudo-Newton control law (3.6) is based on the first order sensitivity functions Φ^s and Γ^s. If the dimensions of state and control variables are n and m, respectively, we need to integrate a system of n(1+n+m) ODEs. As the dimensions of the problem increase, the computational task becomes more difficult and time consuming. However, the sensitivity equations (2.8) and (2.9) have the special feature that they are linear, despite the nonlinearity of the state equations (2.1). Caracotsios and Stewart (1985) exploited this feature and developed an efficient algorithm to calculate the sensitivity functions. Here state

equations (2.1) are integrated first to obtain the state value at the end of the time step. Then a local interpolant of x[t} is used to represent the value of x(t) and equations (2.8) and (2.9) become two sets of time-variant linear ODE's which are independent of the state equation (2.1). Since the state equations are accurately calculated for a given time step, the sensitivities can be calculated simply by solving equation (2.8) and (2.9) for that step by using the same matrix factorizations that were calculated when solving the state equations. Thus, instead of integrating (2.8) and (2.9) along with the state equation, sensitivity functions are obtained at each step by solving linear equations after the state variables are determined.

4. Stability of Pseudo-Newton Control Law

This section is divided in three subsections. The first subsection provides a framework for the stability analysis. Next stability for the model reference case is analyzed, which is defined by assuming a perfect model. The effect on stability of sampling time T and a relaxation factor \ is then analyzed in subsection 4.3. Except for subsection 4.3 the proofs of the theorems and corollaries can be found in Economou (1986) and are omitted here.

4.1 A **Framework for Analysis**

For the stability analysis, we assume a fixed input u_f to the system (2.1) and reduce our attention to the system

$$\frac{dx}{dt} = f(x(t),u_f) \qquad (4.1a)$$

$$y = g(x) \qquad (4.1ft)$$

First we introduce a set of definitions which puts the stability discussion into a quantitative format.

Definition 4.1: Equilibrium State

A state vector x^ is an *equilibrium state* of the open loop system of (4.1) if it has the property:

$$f(x_{eq}^*,u_f) = 0 \qquad (42a)$$

Def. 4.1 and Eq. (4.1) imply that

$$\frac{\mathfrak{L}}{at} = 0, \quad for \quad x(0) = x_{eq}^* \qquad (42b)$$

Using the state evolution (2.4), an additional property of x^ can be written:

$$x_{eq} = \chi(T;\ x_{eq}, u_f) \qquad (4.2c)$$

Definition 4.2: Region of Attraction

A ball U(x°,r) is called a *region of attraction* for the equilibrium point x^ of the discrete system generated by (2.1) and u « u_{fl} if every trajectory starting at any initial state within U(x°,r) eventually converges to x^.

Definition 4.3: Asymptotically Stable in the Large

We first introduce a *nominal solution* $x_o(t)$ which satisfies system (4.1):

$$\frac{dx}{dt} = J\{x_o(t), M_f\} \qquad (4.3)$$

The nominal solution $x_o(t)$ of system (4.1) is *asymptotically stable in the large* if the following conditions are satisfied:

(a)For any \Q and any *z* > 0 there exists a 8(6,^) > 0 such that *Mt^XQ^W* £ 6 implies ||[x(/}-x_o(/)|| < e for all t £ \Q. *(Stable in the sense ofLyapunov)*

(b) For any *x{*\^ and any \Q

$$\text{IWfl-*6(0ll} \to 0 \qquad as\ t \to - \qquad (4.4)$$

Here IWI denotes any norm for the vector x.

Now let us introduce the stability criterion of an open-loop system in **a** region of attraction.

Theorem 4.1 Consider the discrete open loop system generated by (2.1) with u « u_f

$$x>+^1 = xiT^Mf) \ . \qquad (AS)$$

and a state x°. If

$$|w*u/)i| * II \frac{\partial n}{\partial x} < e < i \quad (4.6)$$

$$\forall x \in U(x^{\circ}S)$$

where

$$r \leq r^{\circ} \cdot \frac{\|x(T;x^{0},u_{p})-x^{0}\|}{(1-e)}$$

then the system has a unique asymptotically stable equilibrium state x_{eq} in $U(x^{\circ},r)$. Furthermore, $U(x^{o},r^{\circ})$ is a region of attraction for x_{eq}.

Corollary 4.1 A discrete linear system is stable if and only if

$$p(<D) = p\{exp(AT)) < 1 \quad (4.7)$$

where p(4>) denotes spectral radius of <t>.

Corollary 4.1 merely states that a discrete linear open loop system is stable if and only if the eigenvalues of the transition matrix * are inside the unit circle.

4.2 Model Reference Stability

In this subsection, a perfect model is assumed. The stability criterion of a closed-loop system with the control law of the following form :

$$u^{s+1} = IlCxVo\dot{y}\ ) \quad (4.8)$$

is established, where yj is any external input (usually a setpoint) and n is some finite dimensional operator from R^{n+m} to R^{m}.

In order to use control law of the form (4.8) and to apply the pseudo-Newton law derived in section 2, the state vector can be predicted at every sampling instant. This is because the perfect model of the system is available and, if the state of the system was known at some sampling instant (e.g. t « 0), the state vector could be inferred at any subsequent time by simulating the model of the system:

(4.9)

$$*_0 - XQ \qquad at \ t \ « \ 0$$

where z(t) is the *model* state. Under assumptions of the perfect model and available initial condition, the *model* state vector z(t) equals the process *state* vector x(t).

Consider the disaete closed loop system consisting of the open loop system (2.3) and feedback control law (4.8). Augmenting (2.3) by (4.8) generates an open loop system for the augmented state vector $(x^*,u^*)^T$. The stability of the closed loop system is equivalent to the stability of the augmented open loop system and is characterized by the following theorem.

Theorem 4.2 Consider the disaete dosed loop system generated by augmenting a sampled system of form (2.3) with feedback control law of the form (4.8) as well as a state $(x°, u°)^T$ of the resulting system. If

$$\left\| \begin{matrix} \frac{\partial \chi(T;x,\mu)}{\partial x} & \frac{\partial \chi(T;x,\mu)}{\partial u} \\ \frac{\partial \Pi(x,\mu)}{\partial x} & \frac{\partial \Pi(x,\mu)}{\partial u} \end{matrix} \right\| \leq 6 < 1 \qquad (4.10)$$

where

$$V \ (x,\mu) \ \in \ U((x^0,\mu^0),r)$$

$$r \geq r^0 = \frac{\|\chi(T;x,\mu)-x^0, \ \Pi(x^0,\mu^0)-u^0\|}{(1-\theta)}$$

the closed loop system has a unique asymptotically stable equilibrium state (x_{eq},u_{eq}) in U((x°,u°),r). Furthermore U((x°,u°VfJ^j6 a region for attraction tor (x^.u^J.

Corollary 4.2 A linear system with a linear control law

$$u^{n+1} = \Psi x^n + \Omega u^n + Py^* \qquad (4.11)$$

$$\bullet P \ 6 \ FT'^{\zeta R}, \quad Q \ € \ R\text{-x } m, \quad P \ e \ JT^{xn}$$

is stable if and only if

$$\rho\left(C^{r}\right) < 1 \qquad (4.12)$$

Corollary *42* implies that the dosed loop system will be stable, if and only if the feedback control law (4.8) places the closed loop eigenvalues of the discrete system inside the unit circle.

For the linear pseudo-Newton control law (3.9)

$$\Psi = -\lambda(CT)^{-1}C\Phi^{2} \qquad (4.13.a)$$

$$Q \ll (1\text{-}xy\text{-}Vcn^{\wedge}cor \qquad (4.13.b)$$

4.3 The Relationship Between Sampling Time and Closed-Loop Stability

For control of discrete time systems, the choice of sampling time T (over which the control variable is held constant) strongly influences the quality of control. As T increases, one effectively detunes the controller while for small T the computational burden of applying the control algorithm may be prohibitive. In the analysis presented next, we demonstrate an additional concern relating to the choice of T. Here we show that for some linear and nonlinear systems, the closed loop systems with the pseudo-Newton control are guaranteed to be stable for T above a finite critical sampling time. Theorem 4.3 shows the stability criterion for linear systems; theorem 4.4 shows the stability criterion for nonlinear systems.

Linear Systems

For open-loop stable linear systems, we can show that a closed-loop system with the pseudo-Newton controller may be unstable because of sampling times that are too short. Only when the sampling time is greater than a critical sampling time T* Is the stability of the closed-loop system guaranteed.

Introducing the relaxation factor X, on the other hand, has a similar effect on stability as increasing the sampling time. However, we first prove that adjusting the relaxation factor X alone cannot stabilize some of the closed-loop systems. Then we show with examples that adjustment of X will reduce the spectral radius and the critical sampling time for linear systems.

Theorem 4.3

For an open-loop stable linear system (2.10) there exists a finite critical sampling time T\ When the sampling time T is greater than the critical sampling time T\ the closed-loop system is stable with the pseudo-Newton control law (3.9). As limiting cases, the following results can be obtained:

(a) For X *m* 1, 2m eigenvalues approach zero; and n-m eigenvalues approach the system zeroes.
(b) For X -> 0, n eigenvalues approach the system poles; and m eigenvalues approach unity.
(c) For 0 < X £ 1, *as* *T* -> «>, n eigenvalues approach zero; and m eigenvalues approach (1-λ).

The proof of theorem 4.3 is in Appendix B.

Theorem 4.3 shows that increasing sampling time T alone can achieve closed-loop stability for linear systems. On the other hand, adjusting the relaxation factor *X* alone cannot guarantee stability. When *X* approaches zero, m eigenvalues approach one but they might do so from the outside of the unit disk. Therefore, stability of the closed-loop system is uncertain. (When *X* equals zero, the system becomes open loop.) When X equals one, n-m eigenvalues approach the system zeroes. Thus the closed-loop system becomes unstable if the zeroes of the discrete system are outside the unit circle. This can be the case even if all the zeroes of the continuous system are in the left half plane, which is demonstrated in the following examples.

I)An Inverse Response System

Consider the following example problem with the transfer function:

$$G(s) = \frac{y(s)}{u(s)} = \frac{(-3s+1)}{(s+1)(5s+1)} \qquad (4.14)$$

The spectral radius of the closed-loop system is a function of the sampling time. Figure 1 shows the spectral radius versus sampling time with the control law (3.9). Setting the relaxation factor X to unity, we see from the figure that the critical sampling time T* is 5.5 i.e., for a sampling time less than 5.5, the closed-loop system is unstable.

When varying the relaxation factor X from unity to a very small number, the critical sampling time T* is monotonically decreased. Figure 2 shows the critical sampling time T* versus relaxation factor X for the system (4.14). It shows that even when X approaches zero, the critical sampling time remains nonzero, i.e., too small a sampling time will make the closed-loop system unstable. We also note that the smallest X plotted in Figure 2 is 0.001, for which the corresponding critical sampling time is 3.3.

II) A High Order System

$$G \gg - \wedge j \qquad (4.15)$$

Now let us find the critical sampling time T* when the pseudo-Newton control law (3.9) is used. Setting the relaxation factor *X* to unity, the spectral radius of the dosed-loop system versus sampling time can be calculated. Figure 3 shows these results. We find that the critical sampling time is 1.84.

When decreasing the relaxation factor from one to a small number, the critical sampling time T* varies. Figure 4 shows the critical sampling time versus the relaxation factor X. From the plot we can find that when *X* approaches zero, the critical sampling time also approaches zero. It is worth noting that the smallest *X* plotted in Figure 4 is 10^{-8}, for which the corresponding critical sampling time is 0.017.

Astrom et al. (1984) studied the relationship between the zeroes of a continuous time system and its sampled version. They found that discrete systems with a zero order hold may have unstable zeroes as the sampling time is decreased, even though all the zeroes of the continuous system may be stable. The main results of their studies are limit theorems, which give the zero locations for small and large sampling times. It was shown that *all continuous time systems with pole excess larger than 2 will always give sampling systems with unstable zeroes provided that the sampling time is sufficiently small.* They found that the sampled version of system (4.15) has a zero outside the unit disc if $0 \leq T < 1.84$, which agrees with our result.

Nonlinear Systems

For nonlinear systems we show that there is a finite critical sampling time that lead to a descent direction for $II/+^{1}-/!!$ and thus we can guarantee stability with the pseudo-Newton control law of (3.8).

Unlike linear systems the sensitivities O^{s}, r* of nonlinear systems are dependent on the state x^{s} and input u^{s}. Varying sampling time alone cannot guarantee the global asymptotic stability of the closed-loop system. On the other hand, the nonlinear system can be locally approximated by a linearized system. Knowledge of sampling time affecting the stability of a linear closed-loop system can help us to understand the relationship between the sampling time and closed-loop stability fbr a nonlinear system. For a spedal dass of nonlinear systems, namely those *asymptotically stable in the large,* the system outputs are guaranteed to approach the setpoint monotonically when the sampling time is large enough.

Theorem 4.4

If an open-loop nonlinear system is *asymptotically stable in the large,* then the closed-loop system converges monotonically for $II/*^{1} - /\wedge 5$, where $8 > 0$, with a sampling time $T > Y\backslash$ where $T^{\#}$ is some finite critical sampling time, i.e:

$$\text{II}/^{+2} - y\text{\\} - \text{ll}/^{+1} - \text{y } \dot{\text{II}} * -\dot{y}^{\delta\lambda} \qquad (4.16)$$

A proof of this theorem is presented in Appendix C.

Corollary 4.3 If the assumptions of Theorem 4.4 hold, and S is such that the system is within the attraction region $U((x^\circ,u^\circ),r)$ for (x_{eq},u_{eq}) (i.e. equation 4.16 is satisfied by the closed-loop system), and the setpoint is the unique asymptotically stable equilibrium $(x_{eq}u_{eq})$ in $U(\{x^\circ_t u^\circ)_t ff$ i.e., y* « $g(x_{eq})$, then the system is asymptotically stable in the large.

Proof: Theorem 4.3 shows that linear systems can be stabilized by choosing T sufficiently large. Thus, also for nonlinear systems, for T sufficiently large there is a region of attraction which includes x^, u_{eq}. If 5 is chosen as indicated in Figure 5, the asymptotic stability in the large is ensured.

Theorem 4.4 guarantees that this convergence is monotonic up to $\|y(t)-\dot{y}\| \leq 5$. Figure 5 shows the relationship between 5 and r in a two dimensional state space.

Corollary 4.3 combines a number of properties. First, once the system is within the region of attraction and the setpoint is the unique equilibrium point in this region, the system output asymptotically approaches the setpoint. Moreover, this will occur because a large enough sampling time has been chosen and a descent direction is guaranteed for the line search.

For the case where $\|y-\dot{y}\| \leq 8$ we see theorem 4.4 applies and the setpoint deviation is reduced monotonically. Within the region of attraction $||y-/|| \leq$ r,*we know by theorem *42* that the pseudo-Newton control is stable for X * 1 and therefore converges to the setpoint.

The deadbeat structure of the pseudo-Newton control law for linear systems and the stability properties of this controller for nonlinear systems lead us finally to consider nonlinear algorithms that apply optimization approaches. The single step unconstrained analog to these is a model based controller where the setpoint is satisfied at the end of each timestep. If this can indeed be achieved for a specific problem (e.g. sampling times are large enough so that time delays, inverse and high order responses lead to implicit functions for the controller) then one could construct such an algorithm simply by applying step three of our algorithm iteratively to convergence. The above theoretical development can easily be extended to the case of this iterated control law by considering the following properties.

First, the monotonic convergence property of Theorem 4.4 is trivially satisfied because the setpoint is achieved at each sampling time. However, this, by itself does not imply asymptotic stability. For this we invoke Theorem 4.2 to state and prove the following theorem.

Theorem 4.5

If an open-loop system is asymptotically stable in the large, then there exists a finite critical sampling time T* such that for all sampling times larger than T* the closed loop system is asymptotically stable with the iterated control law.

The proof for this theorem can be found in Appendix D.

With the presentation of the stability properties for linear and nonlinear systems, the pseudo-Newton control law will be extended in the next section to deal with input and state variable constraints.

5. A Nonlinear Strategy for Handling Process Constraints

To extend the pseudo-Newton control law, optimization algorithms can be applied, which deal with linear and nonlinear constraints. Thus a possible approach to handle process constraints is to transform the control problem to an optimization problem. To motivate this approach, the control objective desaibed in section 2 can be written as follows:

$$\underset{\Delta u}{Min} \quad \| y(T;\ x^{*+}V^{+1}) - y^{\bullet} \| \tag{5.1}$$

If the first order approximation based on x^s and u^s is used to represent $y(T;x^{s+1}u^{s+1})$, y becomes:

$$y(T;\ x^{s+1},u^{s+1}) = y^{s+2} = y^{s+1}+C^{s}\Phi^{s}(\chi(T;x^{s},u^{s})-x^{s})+(C^{s}T^{s})(u^{s+1}-u^{s}) \tag{52}$$

Substituting (5.2) into (5.1) and rearranging yields the objective function:

$$\underset{\Delta u^{s}}{Min} \quad C^{T}Au? + \tfrac{1}{2}hitoffHto^{*} \tag{5.3}$$

$$An^{*} = u^{s+1} - u^{s}$$

where

$$C^{T} \ll -[C'^{\wedge}x^{*} - x^{*})+(\ 7^{\bullet} - y^{*1})]\ ^{r}(CT0 \tag{5.4}$$

$$H = (crrficr^{*}) \tag{5.5}$$

The above Quadratic Programming (QP) problem without any constraints is identical to the pseudo-Newton control law. The proof is given by Li and Biegler (1988). Note that the Hessian of the objective function is positive semi-definite, which means that a global minimum value can be found. Moreover, within a QP formulation, nonsquare systems can also be controlled by this algorithm, i.e. dim(y) does not need to be equal to dim(u).

With quadratic programming and the quadratic form of the objective function, linear equality and inequality constraints of control variables can be handled easily. In this case problem (5.3) becomes:

$$\begin{aligned} &\underset{\Delta u^s}{Min} && C^T \Delta u^s + \frac{1}{2}(\Delta u^s)^T H \Delta u^s \\ &SJ && cf \lesssim j4(w^*+Azi) \lesssim a^u \end{aligned} \qquad (5.6)$$

where A e k x m is a constant matrix multiplying the control variables; V defines the number of constraints . When a| equals af (subscript i indicates that it is I^{th} row in a^1 and a^u), it becomes an equality constraint. In practice, we need to deal not only with constraints on the control variables, but also with constraints on the state variables. The latter are important to get a desired product or to avoid failure of the production process. Since simple bounds on state variables are the most common state constraints in chemical processes, our objective is to handle the following problem:

$$\begin{aligned} &\underset{*f}{Min} && C^T A\ll^* + \frac{1}{2}(\Delta u^s)^T H \Delta u^s \\ &s.t && cf \ S \ i4(\ll^J + Aw^y) \lesssim a^u \\ & && JC^7 \ 5 \ x(t) \ S \ *\ll \\ & && t^{s+1} \le t \le t^{s+2} \end{aligned} \qquad (5.7)$$

In order to avoid the computational difficulties associated with these state constraints, which have infinite dimension (the function x(t) from t^{8*1} to t^{s*2} is continuous), a new state variable x_{n+1} is defined to convert the path constraints into a terminal constraint (Sargent and Sullivan,1977). The state equation for x_{n+1} becomes:

$$\frac{dx_{n+1}(t)}{dt} = \sum_{i=1}^{n} [min(0,\ x_i(t) - x_i^L)]^2 + \sum_{i=1}^{n} [min(0,\ x_i^U - x_i(t))]^2 \qquad <5.8>$$

$$x_{n+1}(t^{s+1}; t^{s+1}, x^{s+1}, u^{s+1}) = x_{n+1}^{s+1} = 0$$

We then impose the constraint $x_{n+1}^{s+2} = 0$ and substitute this constraint for the path constraints of the state variables in (5.7). Here the nomenclature defined in section 2 is used. Now x_{n+1}^{s+2} is an implicit function of u and there is no explicit method to handle this nonlinear function. Thus an iteration algorithm is developed. We expand $x_{n+1,\ j}^{s+2}$ about u_{j-1}^{s+1} by using Taylor series and truncate the expansion after the second term. The letter j is used as a subscript to indicate the j-th (QP) iteration to solve (5.7).

$$x_{n+1,\ j}(T;\chi^{s+1},u_j^{s+1}) = x_{n+1,\ j-1}(T;\chi^{s+1},u_{j-1}^{s+1}) + \frac{\partial x_{n+1}}{\partial u}\Big|_{u=u_{j-1}^{s+1}}(u_j^{s+1}-u_{j-1}^{s+1}) + O(u_j^{s+1}-u_{j-1}^{s+1})^2 \quad (5.9a)$$

We define

$$K_{j-1}^{s+1} \equiv \frac{\partial x_{n+1}}{\partial u}\Big|_{u=u_{j-1}^{s+1}} \quad (5.9b)$$

$$\Delta u_j^s = u_j^{s+1} - u^s \quad (5.9c)$$

$$du_j^{s+1} = u_j^{s+1} - u_{j-1}^{s+1} \quad (5.9d)$$

The calculation of K_{j-1}^{s+1} can found in Li and Biegler (1988). Thus, the constraint $x_{n+1,\ j}^{s+2} = 0$ is linearized by the following equation:

$$x_{n+1,\ j-1}(T;\ \chi^{s+1},u_{j-1}^{s+1}) + K_{j-1}^{s+1}du_j^{s+1} = 0 \quad (5.10)$$

The QP problem at the j^{th} iteration is therefore:

$$\begin{aligned} \underset{\Delta u}{Min} \quad & C^T\Delta u_j^s + \frac{1}{2}(\Delta u_j^s)^T H \Delta u_j^s \\ s.t \quad & a^l \le A(u_{j-1}^s + \Delta u_j^s) \le a^u \\ & x_{n+1,\ j-1}(T;\ \chi^{s+1},u_{j-1}^{s+1}) + K_{j-1}^{s+1}du_j^{s+1} = 0. \end{aligned} \quad (5.11)$$

Finally we consider the formulation for two types of constraints. With **hard constraints** no dynamic violations of the bounds are allowed at any time. For **soft constraints** violations of bounds are tolerated for satisfaction of other criteria. In problem (5.11), for instance, the state constraint is treated as a hard constraint even though the feasibility of the QP is unknown. Although the feasibility of state constraints can be analyzed theoretically, the system states can easily be pushed out of the feasible region by disturbances in the real process. Therefore, we add a slack variable δ in constraint (5.10) to relax the hard constraint to a soft constraint. This is a common relaxation technique that is similar to the treatment in Dynamic Matrix Control (Morshedi et al., 1985). Palacios-Gomez et al. (1982) have also proposed

appropriate weighting factors for these slack variables in order to enforce feasibility on soft constraints. Upon adding a slack variable the constraint (5.10) then becomes:

$$x_{n+1,\ j-1}(T;\ \chi^{s+1}, u_{j-1}^{s+1}) + K_{j-1}^{s+1} du_j \ * \ 6 \qquad (5.12a)$$

$$8 \ * \ 0 \qquad (5.12*)$$

In a large scale system we may deal with hundreds or even thousands of state constraints. If we lump all state constraints into (5.8), the algorithm may be very difficult to converge. In this case the individual state constraints can be defined separately. This approach has another advantage; the importance of different state constraints can be distinguished by assigning different weighting factors for the corresponding slack variables in the objective function. The t^h state constraint is therefore:

$$\frac{dx_{n+l}(t)}{dt} = [min(0,\ x_l(t)-x_l^l)]^2 + [min(0,\ x_l^u-x_l(t))]^2 \qquad (5.13)$$

$$x_{n+l}^{s+1} = 0 \qquad / - 1A \ .... \ m_1$$

The dimension of state constraints is m_1 which is less than or equal to the state dimension n. A first order approximation of the new variable x_{n+}, is used as in (5.9) and a slack variable 5, is added in the inequality to relax the constraint. The *P** state constraint of (5.13) in the j* iteration becomes:

$$x_{n+l,\ j-1}(T;\ \chi^{s+1}, u_{j-1}^{s+1}) + K_{l,\ j-1}^{s+1}\ du_j^{s+1} \ \leq \ \delta_l \qquad (5.14)$$

where

$$K_{l,}^{s+}$$

The calculation form can be found in Li and Biegler (1988). The vector form of these state constraints is

$$N_j(T;\ \chi^{s+1}, u_{j-1}^{s+1}) \ \leq \ \delta_{M_1} \qquad (5.16)$$

where

$$N_{j-1}(T;\ \chi^{s+1}\mu_{j-1}^{s+1}) = \begin{bmatrix} x_{n+1,\ j-1}(T;\chi^{s+1}\mu_{j-1}^{s+1}) \\ x_{n+2,\ j-1}(T;\chi^{s+1}\mu_{j-1}^{s+1}) \\ \cdots \\ \cdots \\ x_{n+m_1,\ j-1}(T;\ \chi^{s+1}\mu_{j-1}^{s+1}) \end{bmatrix} + \begin{bmatrix} K_{1,\ j-1}^{s+1} \\ K_{2,\ j-1}^{s+1} \\ \cdots \\ \cdots \\ K_{m_1,\ j-1}^{s+1} \end{bmatrix} du_j^{s+1} \quad (5.17)$$

$$\delta_{M_1} = [\ \delta_1,\ \delta_2,\ \ldots,\ \delta_{m_1}\ ]^T \quad (5.18)$$

Let us now assume the soft constraints for control variables are

$$d \diamond A_s(u^s+\Delta u^s) \leq a'' \quad (5.19)$$

where A, € R^m2 and m_2 is the number of soft constraints. Since these constraints can be violated to satisfy the hard constraints, the slack variables $Su_{...2}$ need to be added in the inequality (5.19) to increase the feasible region. On the other hand, including the summation of all elements of 5_M in the objective function, SM - [^ $_1$ $5u_{...2}$]. the algorithm can determine the control to satisfy soft constraints whenever it is feasible. Then the soft constraints of (5.19) become:

$$a^l-\delta_{M_2} \leq A_s(u^s+\Delta u^s) \leq a''+6_{w\,2} \quad (5.20)$$

Since the importance of each constraint may not be the same, weighting factors w_s can be included to reflect the differences, with the value of the weighting factor w_4 tuned on-line. Including all process constraints, the quadratic programming problem at j* iteration then becomes:

$$\begin{aligned} \underset{\Delta t}{Min} \quad & C^T\Delta u_j^s+\frac{1}{2}(\Delta u_j^s)^T H\Delta u_j^s+\sum_{i=1}^{m} w_i\delta_i \\ SJ \quad & a^l \leq A(u_{j-1}^s+\Delta u_{j-1}^s) \leq a^u \qquad \{521) \\ & N(T;\ \chi^{s+1}\mu_{j-1}^{s+1}) \leq \delta_{M_1} \\ & a^l-\delta_{M_2} \leq A_s(u_{j-1}^s+\Delta u_{j-1}^s) \leq a^u+\delta_{M_2} \\ & \delta_M \geq 0. \\ & t^{s+1} \leq t \leq t^{s+2} \end{aligned}$$

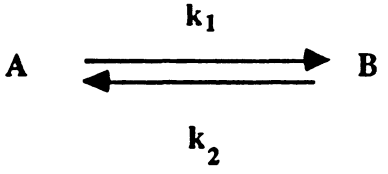
where $m = m_1 + m_2$ is the dimension of all of the soft constraints. The detailed description of this control algorithm can be found in Li and Biegler, (1988).

6. Examples

Two example problems are simulated to demonstrate the effectiveness of the strategy proposed in this work. The first example problem was modified from Economou and Morari (1986), which describes a first order reversible reaction in a well stirred tank reactor. Here their original problem was extended to a three dimensional system with constraints. The second example problem was adapted from the paper of Orava and Niemi (1974), which models a pH control process in two tanks connected in series. The control objective of both problems is to operate the processes as closely as possible to the set point. Both example problems are highly nonlinear with process constraints. The nonlinear controller with constraints proposed in this work controls both systems very well, as seen by the results of our simulations.

Example 1: A Stirred Tank Reactor

The first order reversible exothermic reaction



is carried out in an ideal stirred tank shown in Figure 6.

We assume that the combined concentration of A and B is constant,.i.e., $C = C_A + C_B = 1.0$ and the tank is well stirred with liquid outlet determined by the liquid height in the tank. The nonlinear ODE's are derived from differential mass and energy balances.

$$\frac{dx_1}{dt} = f_1(x; u) = -0.16x_1x_3^{-1.0}u_1 + K_1(1.0 - x_1) - K_2x_1$$

$$\Delta = f_2(x, u) = 0.16u_2x_3^{-1.0} - 0.16x_2x_3^{-1.0}u_1 + 5.0\times[K_1(1.0 - x_1) - K_2x_1]$$

$$\frac{dx_3}{dt} = f_3(x; u) = 0.16u_1 - 0.4x_3^{5} \qquad (6.1)$$

$$K_1 = 5.0\times10^{3}\exp\left(-\frac{5.0\times10^{3}}{x_2}\right)$$

$$K_2 = 1.0\times10^{6}\exp\left(-\frac{7.5\times10^{3}}{x_2}\right)$$

$y_x = x_1$, $y_1 = 0.508$

$y_2 = x_3$ $f_2 = 0.160$

As analyzed in Economou et al. (1986), this problem has a sign change in the steady state gain and therefore can cause difficulties for linear control algorithms. In fact, in their analysis, Economou et al. (1986) showed that linear Internal Model Control (IMC) (Garcia and Morari, 1982) becomes unstable for certain initial conditions. For this reason Economou et al. proposed a nonlinear IMC control law similar to the one developed here.

In the model, x_1, x_2, and x_3 denote the concentration of reactant, temperature, and liquid height in the tank, respectively. u_1 and u_2 denote flow rate and temperature in the inlet stream, respectively. The control objective is to operate the reactor output as closely as possible to the setpoint subject to the process constraints. The initial condition is set to $x_{10} = 0.41$, $x_{20} = 503.0$, $x_{30} = 0.20$, $u_{10} = 1.12$, $u_{20} = 504.0$. At this initial condition, the *linear* Internal Model Control] (IMC) linearized around the setpoint, with a constant flowrate, was shown by Economou et al to become unstable. Using the algorithm developed in section 5 (see also Li and Biegler, 1988), we simulated this example problem with and without control variable and state variable constraints for comparison. Figures 7 and 8 show system outputs y_1 and y_2 versus time, respectively. Figures 9 and 10 show control variables u_1 and u_2 versus time, respectively. Two curves were plotted in each figure. Curve 1 has a constraint on x_1; Curve 2 has no state variable constraints. The parameters for these constraints are listed in Table 1.

From Figure 7, Curve 1 with the constraint on x_1 performs better than one without the state constraint in terms of having small system errors in the initial stage of the system response. The price to be paid for having better control on x_1 is that one has to tolerate sluggish response on x_3. In other words, one can manipulate the state variable constraints to change the response speed of the individual state variable for a MIMO system. As seen, in problem (5.21), the hard constraints on the control variables are always satisfied during the iterations due to the structure of the problem. This is also supported by the simulation results plotted in Figures 9 and 10.

The constrained controller moves for this example problem usually require 3 to 4 QP iterations per time step except at the first sampling interval, where more iterations are required to find a solution for (5.21). To simulate and control 10 sampling times (sampling time T * 1.0 min.) in this example required about 2.0 CPU minutes on a MicroVax II. Of course problems without state variable constraints require much less computation since only one QP solution is required for a time step. Note also that convergence to the setpoint requires at most only three to four sampling times, which makes its computational effort quite comparable to optimization based approaches.

Example 2: A pH Control Process

Consider an assembly of a stirred tank reactor, an intermediate tank, and a pH measuring chamber which is illustrated in Figure 11. The control objective is to operate the reactor at a desired pH value. The pH value is only measured in the measuring chamber. Mixing in the tanks and in the chamber is perfect so that the fluid phase is homogeneous. If the equilibrium of H* and OH' is assumed to hold at any time, and a variable C is used to denote the difference of concentration I-T and OH', we have:

$$C = //+ - Otr = //+ - \frac{K_a}{} \qquad (tla)$$

where $K_a = 10^{-14}$ is the equilibrium constant. By solving the quadratic equation (6.2.a) for H+ and taking the positive root, we have:

$$//\bullet = 0.5[C+(C^2+4.0\text{x}10^{-14})^{0.5}] \qquad (62b)$$

Using material balances and the assumptions made by Orava and Niemi (1974), the process can be modelled as a set of ODEs:

$$\frac{dx_1}{dt} = ffa\ u) = -5.0\text{x}10^{-6}+0.\backslash u-(.05+u)x_1$$

$$\wedge \ll * /_2 0\text{r},\ u) \ll (.05+ii)\text{x}(x_r\wedge) \qquad (63)$$

$$\wedge \ll \ll /_3(\text{r},\ u) \ll 100\text{c}\wedge)$$

$$y = -\log_{10}\{.5[x_3+(4+4.0\times10^{-1}] \} \quad / \ll 7.0$$

Here x_v x_2, x_3 are values of C in the reactor, intermediate tank, and measuring chamber,

respectively, u is the acid flow rate Q_1 (pH « 1). The inlet flow Q_o is constant with pH value of 10. The control objective is to operate at a desired pH value in the reactor. The difficulty is that the pH value can only be measured in the chamber (system output), and therefore lags behind the pH value in the reactor. The initial conditions are x_{10} « x^ - x ^ « 10^{-4}, i.e., the pH value in the process is equal to the pH value of the inlet flow; the initial inlet flow u_0 is set to zero. Table 2 lists numerical values of all parameters for this system.

Using the pseudo-Newton control law developed in section 3. the example problem was simulated without process constraints. Since we assume the model is perfect, the process measurements are equal to predicted system outputs.

From Figures 12 and 13 we can see that the controller is able to achieve offset free output even though it has some oscillations. Here, it is worth noting that if the sampling time is too small, the system outputs can have severe oscillations. This may be caused by the lag between the system output (pH value in the measuring chamber) and the state variable (pH value) in the reactor. Here the controller overcorrects the offset outputs, which causes the oscillations. Basically, the pseudo-Newton control law cannot handle the oscillations in the system outputs because the control objective is to force the system output to meet the setpoint at each sampling time and not between the time interval. The acid flow rate, control variable u, versus time is shown in Figure 14. Note that from Figure 13 the output exactly equals the setpoint and thus performs the same way as if the output were converged to the setpoint at the end of each time step. Here we see that a single step optimization- based approach would not yield any different results.

When the system outputs oscillate with a frequency faster than our sampling frequency, the controller may not eliminate these oscillations, which then deteriorate the control performance. Thus it would be desirable to formulate the control problem so as to allow an additional *integral control term* which would help eliminate these oscillations. This can be done by modifying the control objective when the constrained pseudo-Newton type controller is used:.

$$\begin{aligned} &Min \ \| \ y\text{»}^2 \ - \ / \ \text{II}^2 \ + \ \text{I} \qquad \text{IwCO} \ - \ / \ \text{II}^2 \ dt \\ &St. \quad \frac{\text{£}}{dt} = Mtirtt)) \qquad (6.4) \\ &\qquad yit) = g(x(t)) \\ &\qquad \text{x}^{**1} \bullet known \end{aligned}$$

Here we augment the system outputs to include p_t (pj £ *p)* new variables where

$$\frac{dz_i}{dt} = w_i(y_i(t) - y^*)^2 \qquad (6.5)$$

$$z_i(t^0) = 0$$

$$i = 1, 2 \ldots p_1$$

which are those system outputs for which integral control action is desired. Zj (i « 1, 2,... p_t) are those newly defined variables. Comparing Eq. (6.5) with the objective function, one can intuitively argue that y_{jf} a tuning parameter, is proportional to the inverse of sampling time. By increasing value of Vj. the controller may reduce the degree of oscillation for corresponding system outputs by decreasing the average of the norm of the difference between system outputs and the setpoint. However, this may slow down the system response and cause the system outputs to have a larger offset at the end of a given time interval. At the present time, there is no analytical method to choose the optimal value of y_{if} and therefore, the value of % has to be tuned on-line.

The system output $\bar{y}$ of dimension $p + p_1$ is

$$y = [y. \; zf \qquad (6.6a)$$

$$f = |y| \; if \qquad (6.66)$$

and

$$i = 0 \qquad (6.6c)$$

Thus the objective function is also modified to accommodate the new system outputs, i.e.

$$\underset{\Delta u}{Min} \; \| y+^l - y^* \|^2 \qquad (6.7)$$

Then the algorithm proposed in previous sections can be employed to handle this problem.

We now reconsider Example 2 and modify the system output to include the integral control term discussed above:

$$\S \; « \; V(y - y)^2 \qquad (6.8a)$$

$$I?] = [y. \; zf \qquad (6.86)$$

After including the new defined variable z in our system output, the process is then controlled by the Newton-type control algorithm with the same initial conditions and sampling time as above. Figures 15 and 16 show pH values in the reactor and in the measuring chamber versus time, respectively. Figure 17 shows acid flow rate, u (control variable), versus time. There are three curves in every figure because three different values of parameter y are used to adjust the integral control action. The values of y are chosen to be 0.1, 0.0125, and 0.008 , corresponding to Curve 1, Curve 2, Curve 3.

From these figures we observe that when the value of y increases, the integral control action also increases in terms of smoothing the system response. When y equals 0.1, the system response, Curve 1, has no oscillations. However, the rise time increases with increasing the value of y. At present, we have no analytical method to choose the value of y to satisfy the desired control criterion, but choosing a value of y proportional to the inverse of the sampling time gives an intuitive guideline.

To simulate and control 10 sampling times (sampling time T « 40.0 sec.) in this example problem required about one CPU minute on a MicroVax II.

7. Conclusions

A nonlinear, constrained controller design procedure for stationary lumped-parameter MIMO nonlinear systems is presented and analyzed. The derivation of this pseudo-Newton cotrol law closely parallels that of the well-known Newton method for solving nonlinear equations. Moreover, global and local convergence properties of Newton's method have similar analogies to the stability properties of the control law. Here, the stability criteria for both state-feedback and model reference controllers are established for general nonlinear systems. These properties can be sharpened when only linear cases are considered. In addition, the relationship among the sampling time, relaxation factor, and system closed-loop stability has been shown. In particular, we note that for nonlinear systems that are *asymptotically stable in the large,* the closed-loop system with the pseudo-Newton control law converges monotonically to the setpoint for sufficiently large, but finite, sampling times. Furthermore, while the appropriate selection of the relaxation factor alone will not guarantee stability for a given sampling time, it does allow smaller sampling times to be used for stable performance.

The same results also apply to closed-loop linear systems with the pseudo-Newton control law. Again, it was shown for sampling times larger than some critical sampling time, the closed-loop system is stable. However, in contrast to nonlinear systems, this critical sampling time can be calculated a priori by evaluating the spectral radius of the closed-loop system.

The pseudo-Newton control law also extends naturally to deal with state and control variable constraints. This extension is formulated as an optimization problem which can be solved efficiently by tailoring the Successive Quadratic Programming (SQP) algorithm. These formulations can also be generalized to include both hard and soft constraints on the process. As demonstrated in the previous

section, the method is very efficient as it requires little iteration of QP solutions for each time step. Moreover, simulation results for two nonlinear control problems show the effectiveness of this algorithm.

Finally, two issues that have not been considered in much depth in this paper, and remain for future work, are the algorithm's behavior in the presence of model mismatch and the extension of the pseudo-Newton method to a multi-step algorithm. To handle the first issue, an efficient parameter estimation scheme can be applied to reduce the model mismatch. Note that the algorithm can still be applied here as long as a descent direction is obtained for the pseudo-Newton controller. Consequently, the analysis for Theorem 4.4 can be extended to include the presence of model mismatch and to include the application of parameter estimation to reduce its effect. To deal with the second issue, the pseudo-Newton structure can be extended to a multistep predictive algorithm that follows a moving horizon. This can be viewed as a nonlinear analog to several well-known linear control schemes such as Dynamic Matrix Control (DMC). This extension would be especially useful for nonlinear processes with time delay. Development of this multistep approach will be described in a forthcoming paper.

APPENDIX A

We now prove that when a pseudo-Newton law (3.7) is employed to control the linear system (2.10), the controller is deadbeat, i.e. the system output is offset free at all sampling instants after the first.

The state of system (2.10) at $t = t^{s+1}$ can integrated analytically, yielding:

$$x^{s+1} = e^{AT}x^s + (e^{AT} - I)A^{-1}Bu^s \qquad (i.a)$$

$$y^{s+1} = Cx^{s+1} \qquad (i.b)$$

Substituting (2.11) and (2.12) into (i.a), we obtain:

$$x^{s+1} = \Phi x^s + \Gamma u^s \qquad (ii.a)$$

or

$$x^{s+1} - x^s = (\Phi - I)x^s + \Gamma u^s \qquad (ii.b)$$

Similarly, we have:

$$\begin{aligned} x^{s+2} &= \Phi x^{s+1} + \Gamma u^{s+1} \\ &= \Phi(\Phi x^s + \Gamma u^s) + \Gamma u^s \\ &= \Phi^2 x^s + \Phi\Gamma u^s + \Gamma u^{s+1} \end{aligned} \qquad (iii.a)$$

$$\begin{aligned} y^{s+2} &= Cx^{s+2} \\ &= C\Phi^2 x^s + C\Phi\Gamma u^s + C\Gamma u^{s+1} \end{aligned} \qquad (iii.b)$$

The pseudo-Newton control law (3.7) can be written as follows:

$$\begin{aligned} u^{s+1} &= u^s + (C\Gamma)^{-1}[C\Phi(x^s - x^{s+1}) + (y^* - y^{s+1})] \\ &= u^s + (C\Gamma)^{-1}[(-C\Phi^2 + C\Phi)x^s - C\Phi\Gamma u^s + (y^* - C\Phi x^s - C\Gamma u^s)] \\ &= u^s + (C\Gamma)^{-1}(-C\Phi^2 x^s - C\Phi\Gamma u^s + y^* - C\Gamma u^s) \\ &= -(C\Gamma)^{-1}(C\Phi^2 x^s + C\Phi\Gamma u^s - y^*) \end{aligned} \qquad (iv)$$

Substituting the control law (iv) into Eq.(iii.b), the system output y^{s*2} becomes:

$$y^{*+2}\ m\ c\lhd\rhd V + corv - crtccn'kc\hat{}V + corv - /)]$$

$$= /\qquad \text{(v)}$$

Thus, we prove that the controller is deadbeat.

APPENDIX B

Proof of Theorem 4.3

Referring to corollary 4 2 , the stability condition of linear closed-loop systems with the pseudo-Newton control law (3.9) is determined by the eigenvalues of the following matrix:

$$\begin{bmatrix} \Phi & \mathbf{r} \\ \Psi & a \end{bmatrix} \qquad (0$$

where

$$* = exp(AT) \qquad * \; e \; R^{HXH} \qquad (\ddot{\cdot}.a)$$

$$\mathbf{r} = [exp(ATH]A\sim^{J}B \qquad T \; € \; R^{nxm} \qquad (ii.b)$$

$$\Psi = -\lambda(C\Gamma)^{-1}C\Phi^{2} \qquad V \; 6 \; R^{m\times n} \qquad (ii.c)$$

$$Q = (1-\lambda)I-\lambda(C\Gamma)^{-1}C\Phi\Gamma \qquad a \; e \; R^{m\times m} \qquad (ii.d)$$

When sampling time T is small, we have

$$O - (1+87V \qquad (iii.a)$$

$$\mathbf{r} - \delta TA^{-1}B \qquad (iii.b)$$

$$\Psi = -\lambda(1+\delta T)^{2}(\delta TCA^{-1}B)^{-1}C \qquad (iii.c)$$

$$\mathbf{O} - (I\text{-}\backslash)I\text{-}\lambda(1+\delta T)(\delta TCA^{-1}B)^{-1}CA^{-1}B \qquad QUJ)$$

Obviously, the matrix (i) has nontrivial eigenvalues, m(T).

(a) $\lambda = 1$

In order to analyze the distribution of eigenvalues for matrix (i) under this condition, the following similarity transformation is employed, which leaves the eigenvalues unchanged:

$$\begin{bmatrix} I & 0 \\ (C\Gamma)^{-1}C\Phi & I \end{bmatrix} \begin{bmatrix} \Phi & \Gamma \\ \Psi & \Omega \end{bmatrix} \begin{bmatrix} I & 0 \\ -(C\Gamma)^{-1}C\Phi & I \end{bmatrix} \qquad (iv)$$

$$= \begin{bmatrix} \Phi-\Gamma(C\Gamma)^{-1}C\Phi & \Gamma \\ 0 & 0 \end{bmatrix} \qquad (v)$$

It is obvious that m eigenvalues of matrix (v) are zero. Using Schur's formula (Kailath 1980), the characteristic equation of matrix Φ-$\Gamma(C\Gamma)^{-1}C\Phi$ can be written:

$$det\ [\mu I-\Phi+\Gamma(C\Gamma)^{-1}C\Phi)]$$

$$= det \begin{bmatrix} (\mu I-\Phi) & \Gamma \\ -C\Phi & C\Gamma \end{bmatrix} \frac{1}{det(C\Gamma)}$$

$$= det \begin{bmatrix} I & 0 \\ C & -\mu I \end{bmatrix} \begin{bmatrix} (\mu I-\Phi) & \Gamma \\ C & 0 \end{bmatrix} \frac{1}{det(C\Gamma)} = 0 \qquad (vi)$$

Note that the determinant of the second matrix in the bracket defines the zeros of the system {C, Φ, Γ}. It follows from (vi) that m eigenvalues are zero. Thus, when $\lambda = 1$, 2m eigenvalues go to zero; and n-m eigenvalues go to the system zeroes.

(b) $X \to 0$

When *X* approaches zero, matrix (i) becomes:

$$\begin{bmatrix} \Phi & \Gamma \\ 0 & I \end{bmatrix} \qquad \text{(vü)}$$

m eigenvalues approach one; and n eigenvalues approach the system poles.

(c) $0 < X \leq 1$

Now let us show that when sampling T approaches infinity, n eigenvalues of matrix (i) approach zero and m eigenvalues approach (1-X).

$$\lim_{T\to\infty} \Phi = 0 \qquad \textit{(VUIM)}$$

$$\lim_{T\to\infty} \Gamma = -A^{-1}B \qquad \textit{(yüi.b)}$$

$$\lim_{T\to\infty} (CT)^{-1} * -(CA^{-1}B)^{-1} \qquad \textit{(yiiix)}$$

$$\lim_{T\to\infty} \Psi = \lim_{T\to\infty} -\lambda(CT)^{-1}\Phi^2 = 0 \qquad \textit{(viii.d)}$$

$$\lim_{T\to\infty} a = \lim_{T\to\infty} (1-\lambda)I - \lambda(CT)^{-1}C\Phi\Gamma = (1-\lambda)I \qquad \textit{(viiix)}$$

When T approaches infinity, **the** characteristic equation for matrix (i) becomes:

$$\lim_{T\to\infty} \det \begin{bmatrix} \Phi & \Gamma \\ \Psi & n \end{bmatrix} - \mu I = \det \begin{bmatrix} 0 & -A^{-1}B \\ 0 & d\text{-}xy \end{bmatrix} - \mu I = 0 \qquad \textit{(ix)}$$

Thus, n eigenvalues approach zero; and m eigenvalues approach (1-X). Since all elements in *, r, y, and *a* are continuous functions of sampling time T, the spectral radius is also a continuous function of T. Therefore, there exists *f* < -, sothatp(T) < l forr > *f.*

APPENDIX C

The notation defined in section 2 is used here. We make following assumptions:

(1) The system (2.1) is *asymptotically stable in the large* for all feasible control variables u e R^m

(2) All gradients with respect to u* and x^8 are bounded and continuous.

Proof of Theorem 4.4

Let us prove first that all gradients with respect to x^s approach zero when the sampling time approaches infinity. Let us introduce a new variable F+^{2} « xfx^.u^8), i.e., the system state at t = *f+^{2}* if the input is held constant at u • u^s over the (s+1)th sampling interval. Expand 3c^{J+2}($x^{S+1,1}$,u^s) in its Taylor series around (x^s,u^s):

$$\bar{x}^{s+2}(x^{s+1},u^s) = x^{s+1}+\Phi^s(x^{s+1}-x^s)+\frac{1}{2!}(x^{s+1}-x^s)^T\Phi^s_{xx}(x^{s+1}-x^s)+ \quad \text{(i.a)}$$

where

$$\frac{\partial\Phi^s}{\partial x^s} = \frac{\partial^2\chi(T;x^s,u^s)}{\partial(x^s)^2} \quad \text{(i.b)}$$

Using assumption (1), we have

$$\lim_{T\to\infty} \bar{x}^{s+2}(x^{s+1},u^s) = x^{s+1}(x^s,u^s) \quad \text{(ii)}$$

Eq.(ii) also implies that when the sampling time T approaches infinity, every term except the first in Eq.(i.a) goes to zero. Therefore, we can write:

$$\lim \quad (*\underline{Q}'' = 0 \qquad f \bullet 1, 2, \ldots \quad \text{(iii.a)}$$

where

$$(\Phi^s_x)^i = \frac{\partial^i\chi(T;x^s,u^s)}{\partial(x^s)^i} \quad \text{(iii.b)}$$

The pseudo-Newton control law (3.8) can be rewritten as follows:

$$\Delta u^s = u^{\wedge'}\text{-}w^* - MPrThCVOf\text{-}xr^{\wedge}y\text{-}y^{\dagger r}*] \quad \text{(iv)}$$

Expanding $y^{6*2\wedge*1}$, u^{*+1}) in a Taylor series around (x»,u») gives:

$$y^{s+2} = y^{s+1} + C^s\Phi^s(x^{s+1}-x^s) + C^s\Gamma^s(u^{s+1}-u^s) \qquad (v.a)$$
$$+ (x^{s+1}-x^s)^T W_{xu}(u^{s+1}-u^s) + \frac{1}{2}(x^{s+1}-x^s)^T W_{xx}(x^{s+1}-x^s)$$
$$+ \frac{1}{2}(u^{s+1}-u^s)^T W_{uu}(u^{s+1}-u^s) + O(\|(x^{s+1}-x^s),\ (u^{s+1}-u^s)\|^3)$$

where

$$W_{xu} = (\Phi^s)^T C_x^s \Gamma^s + C^s \Phi_u^s \qquad (v.b)$$
$$W_{xx} = (\Phi^s)^T C_x^s \Phi^s + C^s \Phi_x^s \qquad (vx)$$
$$W_{uu} = C^s \Gamma_u^s \qquad (v.d)$$

$$\Phi_u^s = \frac{\partial \Phi^s}{\partial u^s} = \frac{\partial^2 \chi(t;x^s,u^s)}{\partial u^s \partial x^s} \qquad (v.e)$$
$$\Gamma_u^s = \frac{\partial \Gamma^s}{\partial u^s} = \frac{\partial^2 \chi(t;x^s,u^s)}{\partial (u^s)^2} \qquad (v.f)$$
$$C_x^s \triangleq \frac{\partial^2 y^{s+1}}{\partial (x^s)^2} \qquad (v.g)$$

Subtract y* from both sides, apply the pseudo-Newton control law (iv), and take norms of both sides to get:

$$\|y^{s+2}-y^*\| \le (1-\lambda)\|y^{s+1}-y^*\| + (1-\lambda)\|C^s\Phi^s(x^{s+1}-x^s)\| \qquad (vi)$$
$$+ \|x^{rtl}-x^f\|\ \|i\wedge'-if\|\ WJH + \|x^{s+1}-x^s\|^2\|W_{xx}\|$$
$$+ \frac{1}{2}\|u^{s+1}-u^s\|^2\|W_{uu}\| + O(\|(x^{s+1}-x^s),\ (u^{s+1}-u^s)\|^3)$$

We now have from above:

$$\lim_{T\to\infty} \Phi^s \to 0 \qquad (vii.a)$$

$$\lim_{T\to\infty} \Phi^s_x \to 0 \qquad (vii.b)$$

$$\lim_{T\to\infty} \Phi^s_u \to 0 \qquad (vii.c)$$

Therefore

$$\lim_{T\to\infty} W_{xu} \to 0 \qquad (vii.d)$$

$$\lim_{T\to\infty} W_{xx} \to 0, \quad \textit{and} \qquad (vii.e)$$

$$\|u^{s+1}-u^s\|^2 = \lambda^2\|(C^sT^s)^{-1}\|^2\|C^s\Phi^s(x^s-x^{s+1})+y^*-y^{s+1}\|^2 \qquad (vii.f)$$

For high order terms $O(\|(x^{s+1}-x^s),\ (u^{s+1}-u^s)\|^3)$, all gradients with respect to x^s go to zero when sampling time T approaches infinity. All gradients with respect to u^s are bounded and the terms involving u are multiplied by a power of the relaxation factor λ. Using eq. (vii), eq.(vi) can be written as follows :

$$\|y^{s+2}-y^*\| - \|y^{s+1}-y^*\| \le -\lambda\|y^{s+1}-y^*\| + (1-\lambda)O(\Phi^s) + O(\lambda\Phi^s) + O(\Phi^s)^2 + O(\Phi^s_x) + O(\lambda^2) + O((\Phi^s)^3,\ \lambda^3) \qquad (viii)$$

We now choose a finite T so that Φ^s,Φ^s_x and λ are small enough to satisfiy:

$$(1-\lambda)O(\Phi^s) + O(\lambda\Phi^s) + O(\Phi^s)^2 + O(\Phi^s_x) + O(\lambda^2) + O((\Phi^s)^3,\ \lambda^3) \le \frac{\delta\lambda}{2} \le \frac{\lambda}{2}\|y^{s+1}-y^*\| \qquad (ix)$$

Then we have

$$\|y^{s+2}-y^*\| - \|y^{s+1}-y^*\| \le -\frac{\lambda}{2}\|y^{s+1}-y^*\| \le -\frac{\delta\lambda}{2} \qquad (x)$$

which proves the theorem.

APPENDIX D

Proof of Theorem 4.5

The assumptions of *the asymptotically stable In the large* system **and** continuous and bounded gradients in Appendix C still hold here.

Referring to Theorem *42,* the discrete closed-loop system has a unique stable equilibrium state $(x_{eq}, u$«q>^{inu}«*°' $u^0)'^r>^{Hmethe}$ Wowing condition is satisfied:

$$\left\| \begin{matrix} \frac{\partial \chi(T;x,\mu)}{\partial x^*} & \frac{\partial \chi(T;x,\mu)}{du} \\ \frac{\partial \Pi(x,\mu)}{\text{at}} & \frac{dUjxju)}{du} \end{matrix} \right\| \leq \theta < 1 \qquad (4.10)$$

where

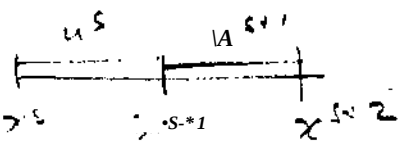
$$\forall\ (x,\mu) \in U((x^0,\mu^0),r)$$

$$r \geq r^0 = \frac{\|\chi(T;x,\mu_j)-x^0,\ \Pi(x^0,\mu^0)-u^0\|}{(1-\theta)}$$

If we assume that the system output reaches the setpoint at the end of the first sampling time by using the iterative control law, then the following relation

$$u^{s+1} = \Pi^s(x^s,\mu^{s+1},y^*|g(x^{s+2})=y^*) \qquad (i)$$

holds. At the optimal condition, we also have:



$$\frac{\partial g(x^{s+2})}{\partial x^{s}} + \frac{\partial g(x^{s+2})}{\partial u^{s+1}} \frac{\partial u^{s+1}}{\partial x^{s}} = 0 \qquad (ii.a)$$

which is equivalent to the following equation:

$$C^{s+1}\Phi^{s+1}\Phi^{s} + C^{s+1}\Gamma^{s+1} \frac{1}{\partial x^{s}} = 0 \qquad (ii.b)$$

where

$$\Phi^{s+1} = \frac{\partial \chi^{s+1}}{\partial x^{s+1}}$$

$$\Gamma^{s+1} = \frac{\partial \chi^{s+1}}{\partial u^{s+1}}$$

$$C^{s+1} = \frac{\partial y^{s+2}}{\partial \chi^{s+1}}$$

Then the derivatives of n* with respect to x* and u* become:

$$\frac{\partial u^{s+1}}{\partial x^{s}} = \frac{\partial \Pi^{s}}{\partial x^{s}} = -(C^{s+1}\Gamma^{s+1})^{-1}C^{s+1}\Phi^{s+1}\Phi^{s} \qquad (iii.a)$$

$$\frac{\partial}{\partial u^{s}} = 0 \qquad (iii.b)$$

The contraction mapping matrix is:

$$\begin{bmatrix} \Phi^{s} & \Gamma^{s} \\ -(C^{s+1}\Gamma^{s+1})^{-1}C^{s+1}\Phi^{s+1}\Phi^{s} & 0 \end{bmatrix} \qquad (iv)$$

As T approaches infinity, the eigenvalues go to zero, which proves Theorem 4.5.

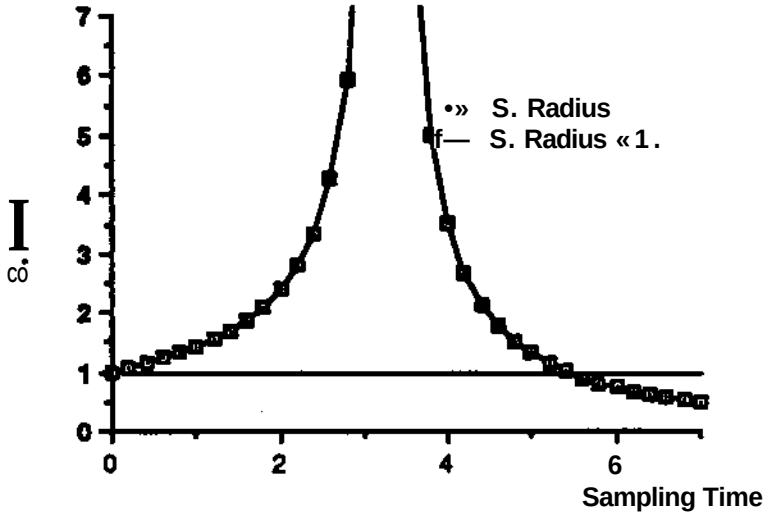
	x_1^l	x_1^u	x_2^l	x_2^u	u_2^l	u_2^u
Curve 1	0.49	0.51	300.0	550.0	300.0	550.0
Curve 2	no	no	no	no	300.0	550.0

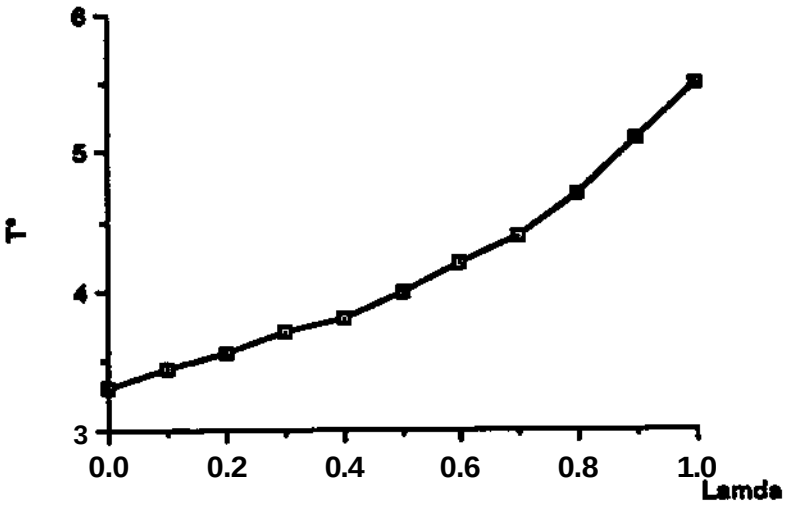
$$x_1^l \le x_1 \le x_1^u$$
$$x_2^l \le x_2 \le x_2^u$$
$$u_2^l \le u_2 \le u_2^u$$

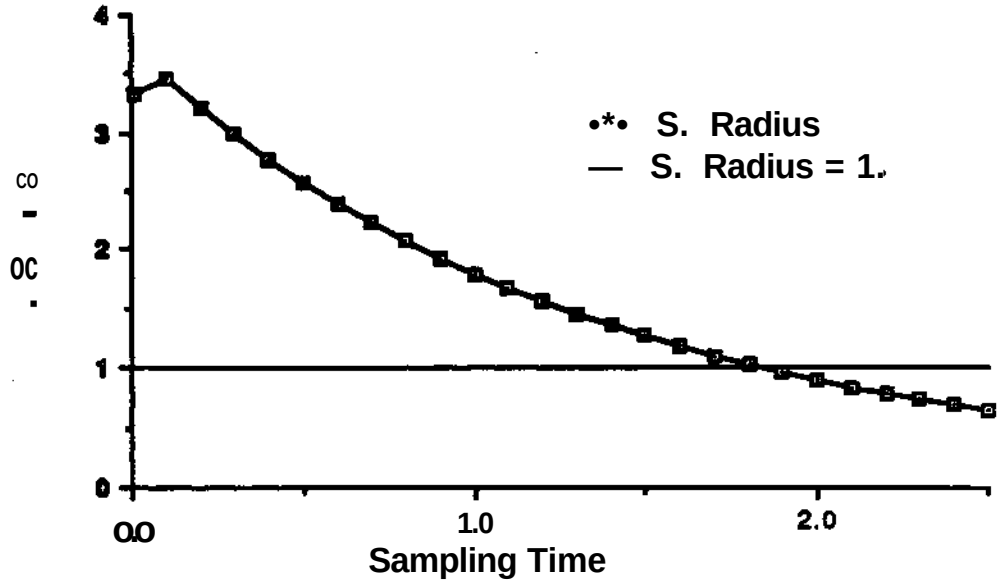
Table 1: Process constraints for example 1

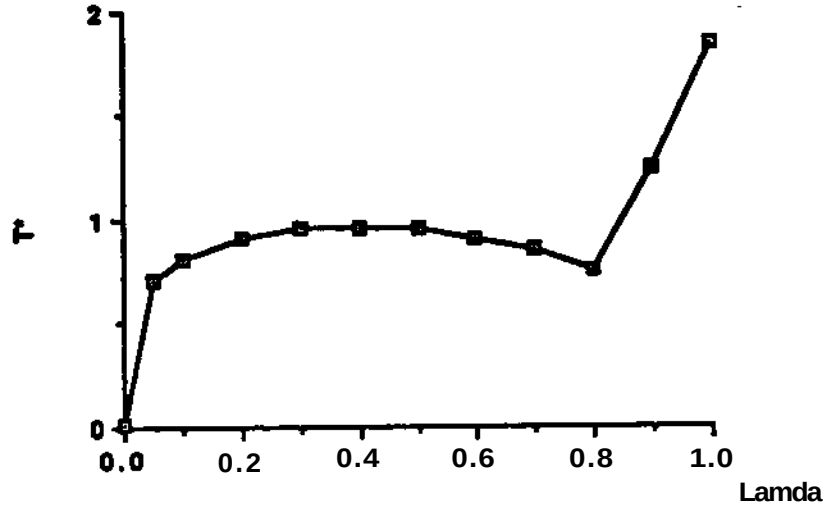
Q_0	inlet flow rate	S.01 /sec.
Qi	acid flow rate	control variable
QM	inlet flow rate for the measuring chamber	0.1 1/sec.
VR	volume of the reactor	100.01
VI	volume of the intermediate tank	100.01
VM	volume of the measuring chamber	0.01 1
c_0^+	H^+ concentration of the inlet flow in the reactor	10.0 mol /I
c_0	H + concentration of the acid flow in the reactor	10.0 mol /I

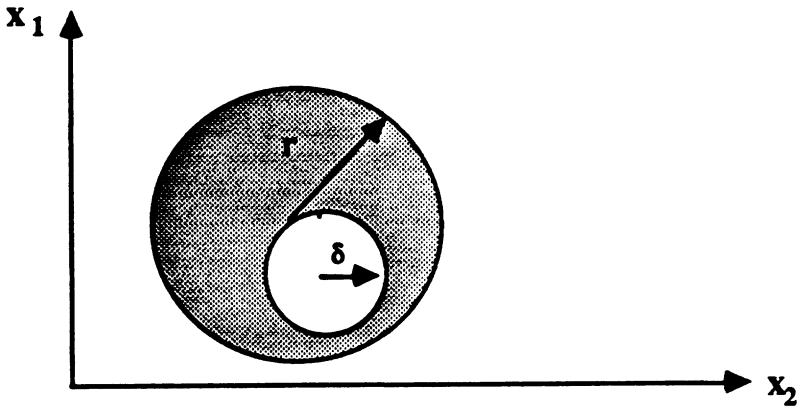
Table 2: Numerical values of all parameters for example 2

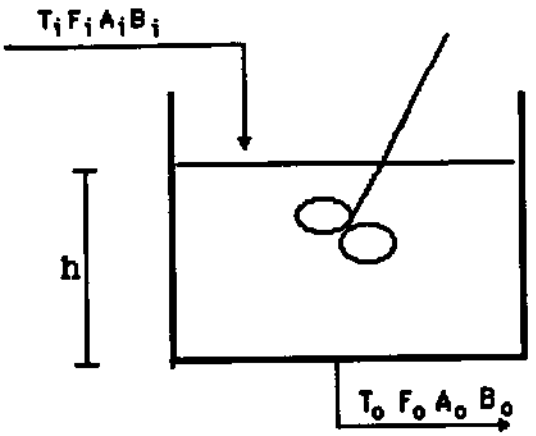

7
6
5
4
3
2
1
0
0
2
4
6
I
co
•» S. Radius
f— S. Radius «1.
Sampling Time

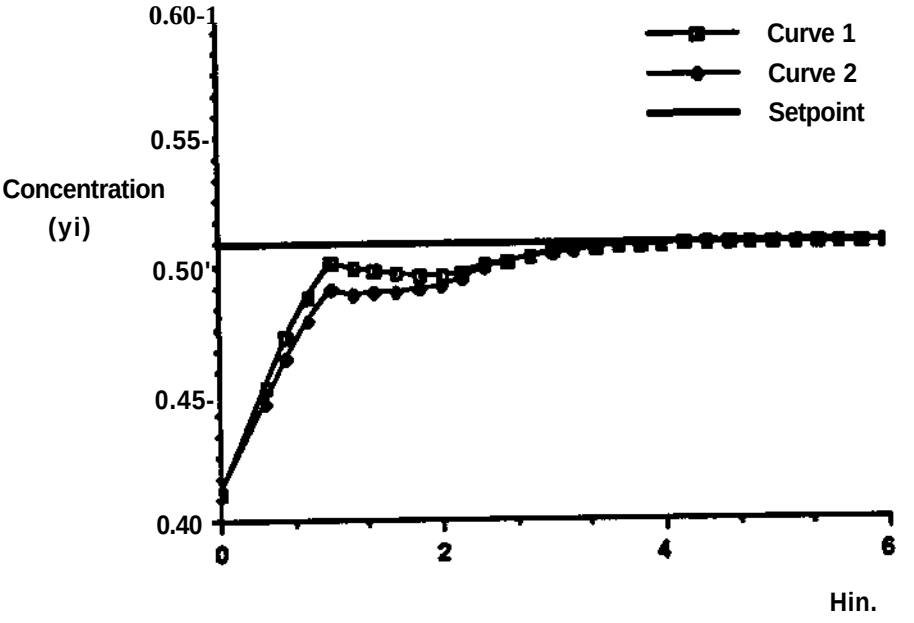

6
5
4
3
T*
0.0
0.2
0.4
0.6
0.8
1.0
Lamda

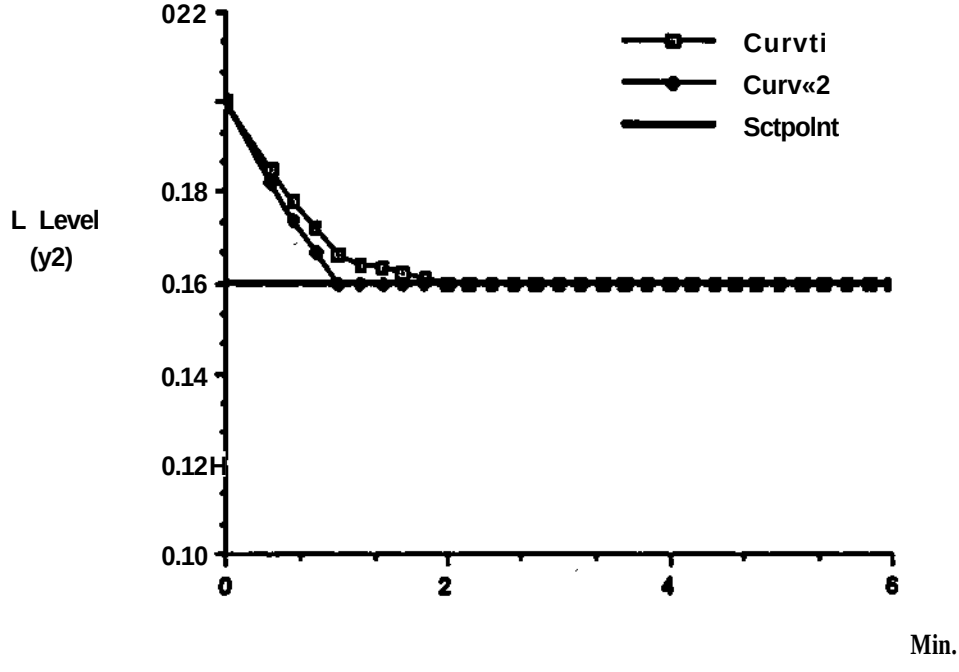

4
3
2
1
0
CO
OC
0.0
1.0
2.0
Sampling Time
•*• S. Radius
— S. Radius = 1.

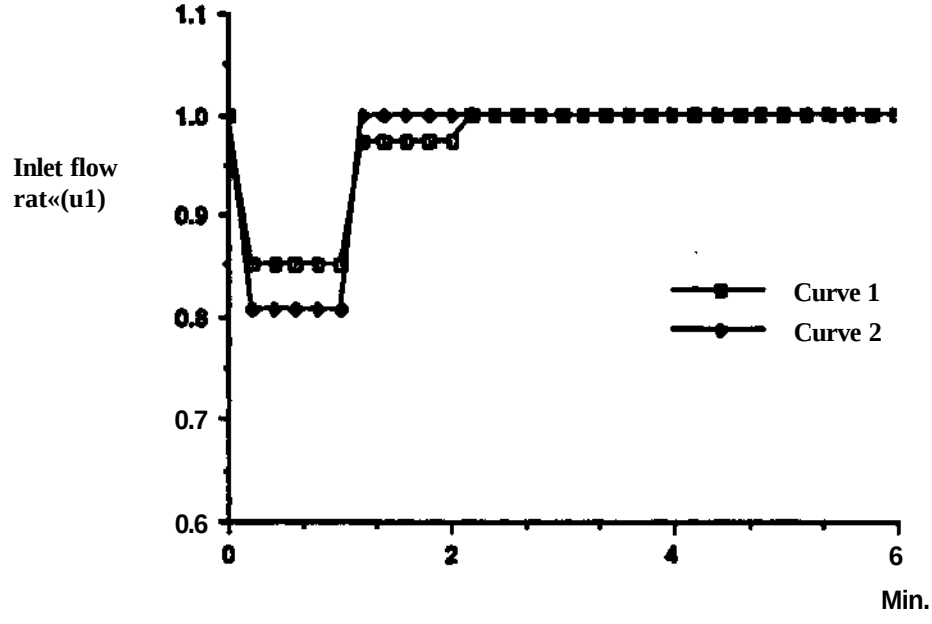

2
1
0
0.0
0.2
0.4
0.6
0.8
1.0
Lamda

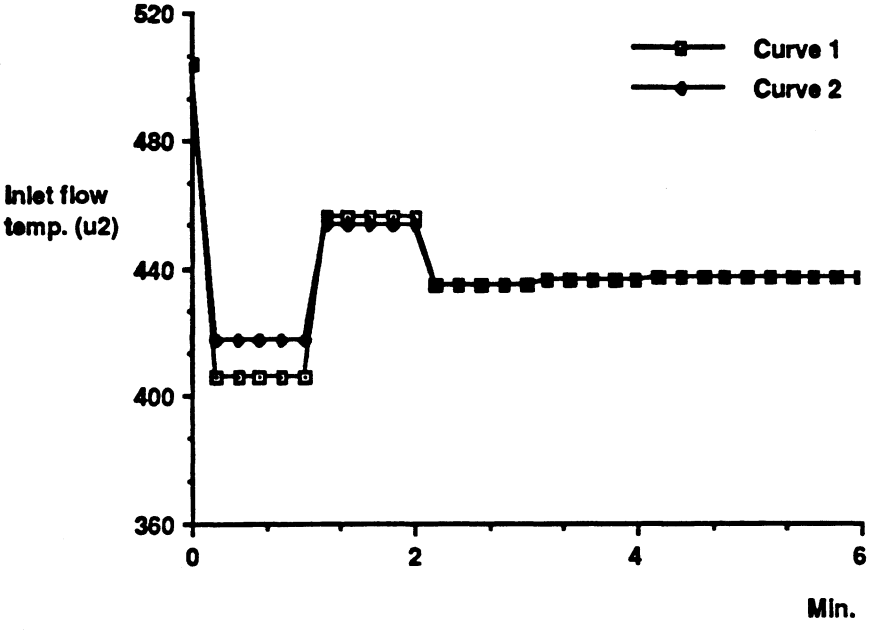

x1
r
δ
x2

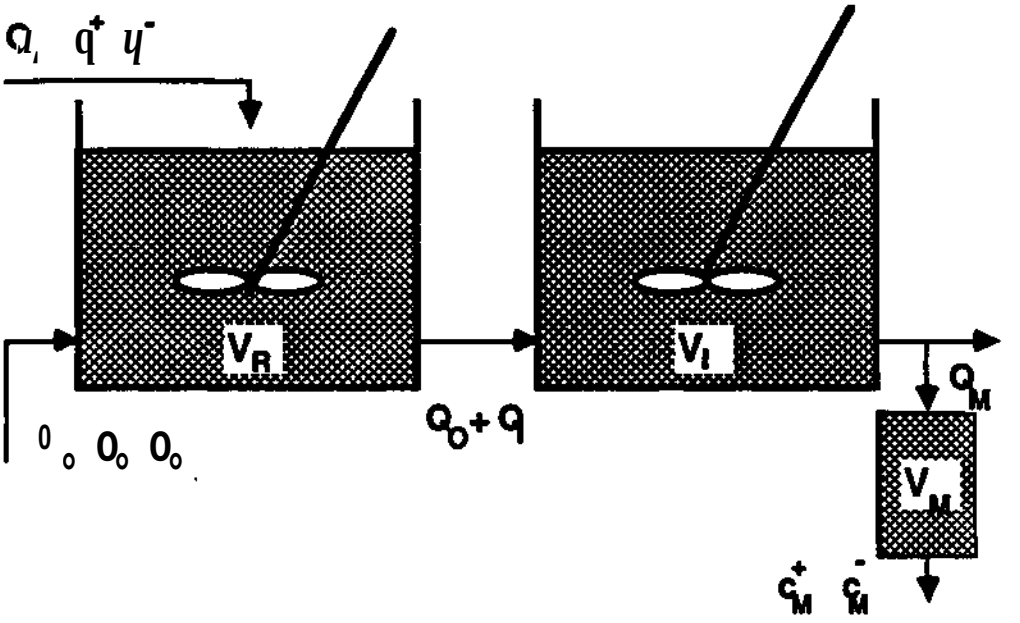

$T_i F_i A_i B_i$
h
$T_o F_o A_o B_o$

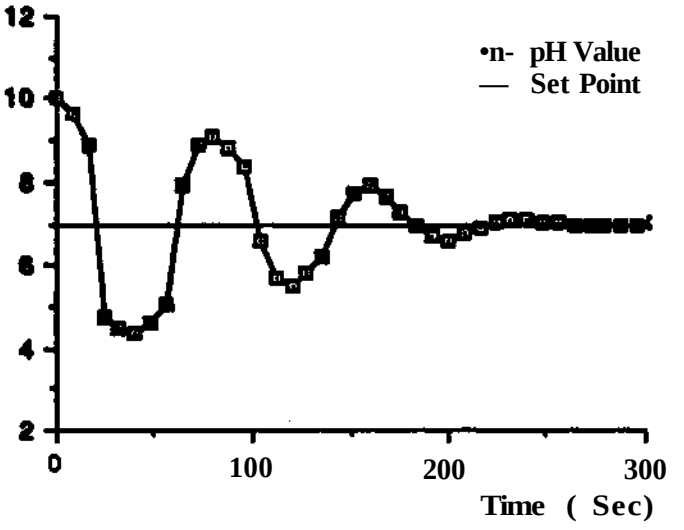

0.60-1
0.55-
0.50'
0.45-
0.40
Concentration
(yi)
0
2
4
6
Hin.
Curve 1
Curve 2
Setpoint

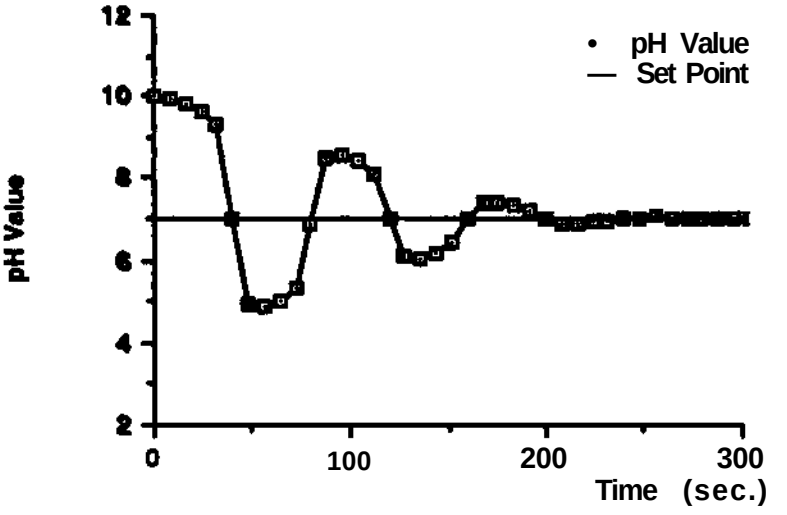

022
0.18
0.16
0.14
0.12H
0.10
L Level
(y2)
Curvti
Curv«2
Sctpolnt
0
2
4
6
Min.

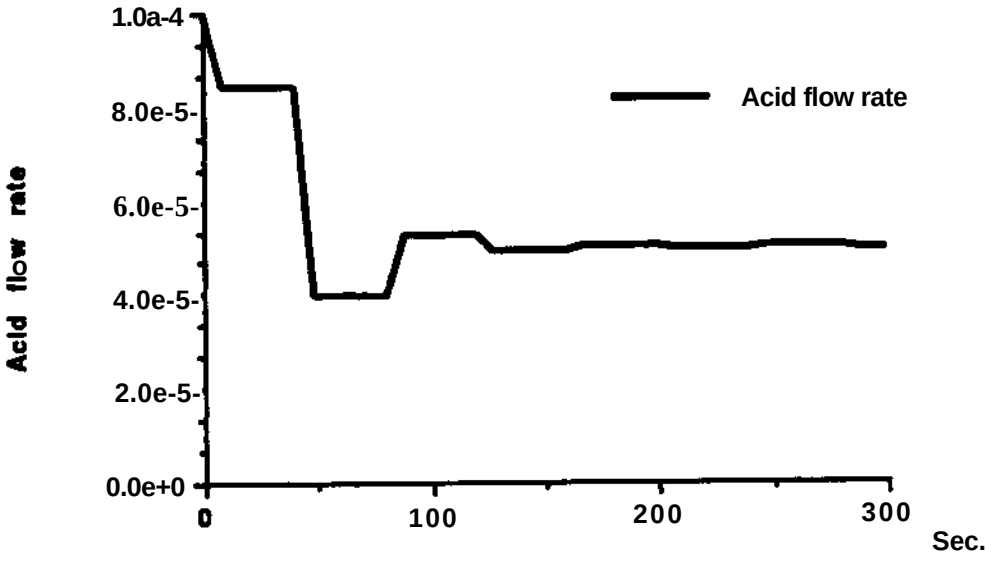

Inlet flow rat«(u1)
1.1
1.0
0.9
0.8
0.7
0.6
0
2
4
6
Min.
Curve 1
Curve 2

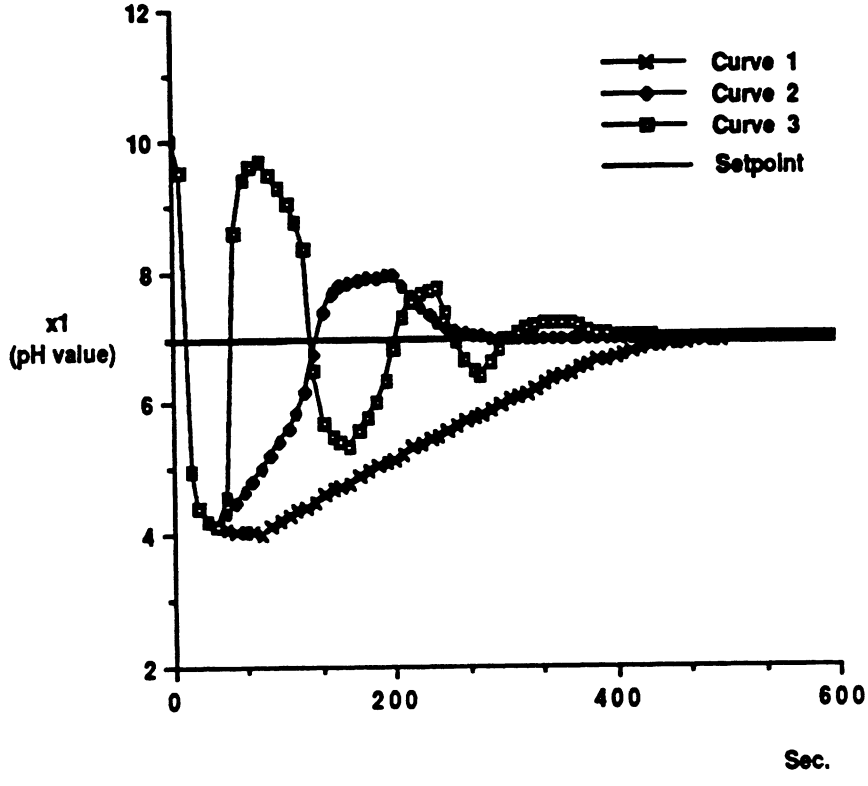

520
480
440
400
360
Inlet flow
temp. (u2)
0
2
4
6
Min.
Curve 1
Curve 2

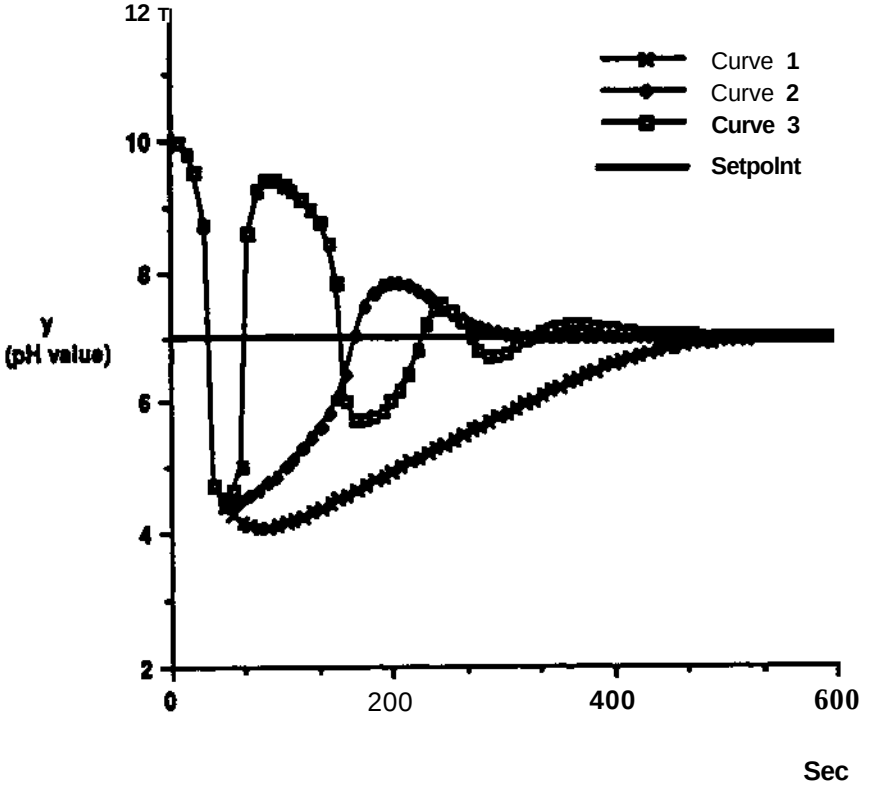

Q_i q^+ q^-
V_R
V_I
Q_M
V_M
Q_0+q
c_M^+ c_M^-

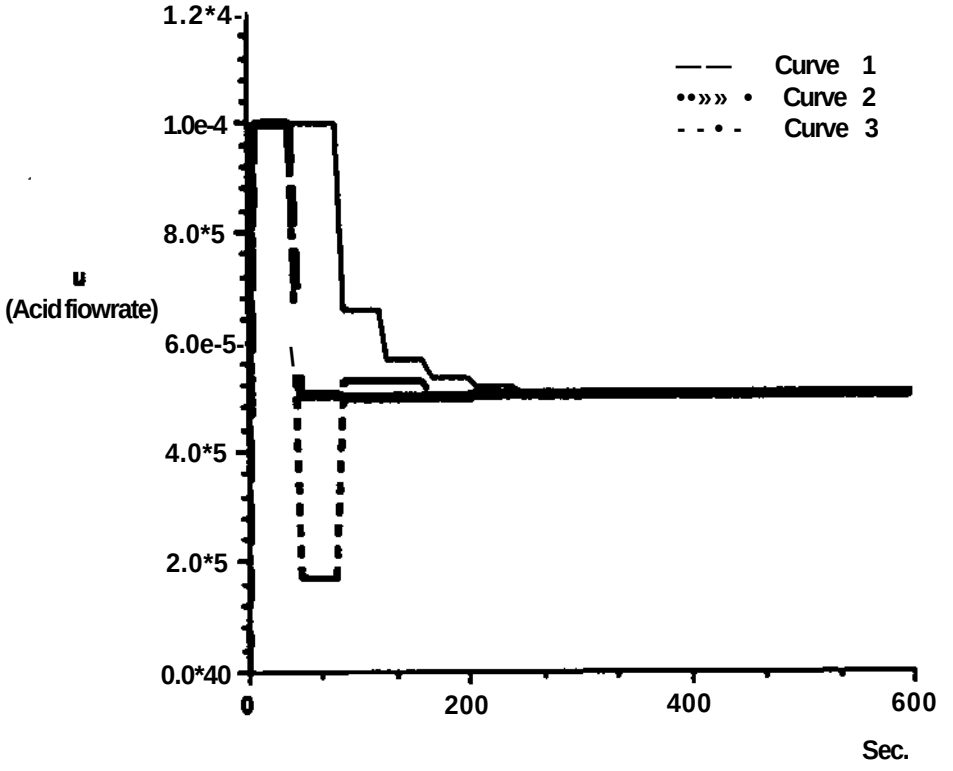

12
10
8
6
4
2
0
100
200
300
Time (Sec)
•n- pH Value
— Set Point


12
10
8
6
4
2
pH Value
0
100
200
300
Time (sec.)
• pH Value
— Set Point


1.0a-4
8.0e-5-
6.0e-5-
4.0e-5-
2.0e-5-
0.0e+0
Acid flow rate
0
100
200
300
Sec.
Acid flow rate


12
10
8
6
4
2
x1
(pH value)
0
200
400
600
Sec.
Curve 1
Curve 2
Curve 3
Setpoint


12
10
8
6
4
2
y
(pH value)
0
200
400
600
Sec
Curve 1
Curve 2
Curve 3
Setpoint


1.2*4-
1.0e-4
8.0*5
6.0e-5-
4.0*5
2.0*5
0.0*40
u
(Acid flowrate)
0
200
400
600
Sec.
—— Curve 1
••»» • Curve 2
- - • - Curve 3

References

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